Magnetic field measurement system based on rotating PCB coils

Author: **Gianluca Nicosia**
Politecnico di Milano
Supervisor: **Joseph DiMarco**
Fermilab TD

September 24, 2014

POLITECNICO DI MILANO

Fermilab

Section 1

INTRODUCTION

AIM OF THE INTERNSHIP

Developing a magnetic field measurement system in LabVIEW and MATLAB implementing preexisting scripts and using it to analyze the performances of rotating PCB coils comparing them to more traditional machine-winded harmonic coils.

ROTATING COIL IN MAGNETIC FIELD

The system is based on Faraday's Law:

$$\mathcal{E} = -\frac{d\phi}{dt} = -\frac{d}{dt}\iint_A \mathbf{B}\cdot\mathbf{n}\mathrm{d}A = \tag{1}$$

$$\underbrace{-\iint_A \frac{d\mathbf{B}}{dt}\cdot\mathbf{n}\mathrm{d}A}_{\text{Time-varying field}} \qquad \underbrace{-\iint_{\partial A} \mathbf{v}\times\mathbf{B}\mathrm{d}l}_{\text{Displacement or deformation of the coil}} \tag{2}$$

If the geometry and the position of the coil are known, integrating the voltage, the flux is obtained.

$$\Phi - \Phi_0 = -\int\limits_0^t \mathcal{E}\mathrm{d}t \tag{3}$$

The field harmonics (multipoles) are derived using knowledge of the coil geometry.

HARMONIC DECOMPOSITION

Let's consider a region of space free of charges and current.

$$\nabla \cdot \mathbf{B} = 0 \tag{4}$$

$$\nabla \times \mathbf{B} = 0 \tag{5}$$

A magnetic field $\mathbf{B} = (B_x, B_y, B_z)$ with B_z constant and the other two components given by

$$B_y + iB_x = \overline{C_n}(x + iy)^{n-1} = \overline{C_n}z^{n-1} \quad \overline{C_n} \in \mathbb{C}, n \in \mathbb{N} \tag{6}$$

satisfies 4 and 5

HARMONIC DECOMPOSITION

A generic field is given by

$$B_y + iB_x = \sum_{n=1}^{\infty} C_n \left(\frac{z}{R_r}\right)^{n-1} \tag{7}$$

Harmonics can be easily measured starting form the flux

$$\Phi(\theta) = Re\left(\sum_{n=1} C_n K_n e^{in\theta}\right) \tag{8}$$

K_n is the winding sensitivity and is defined as:

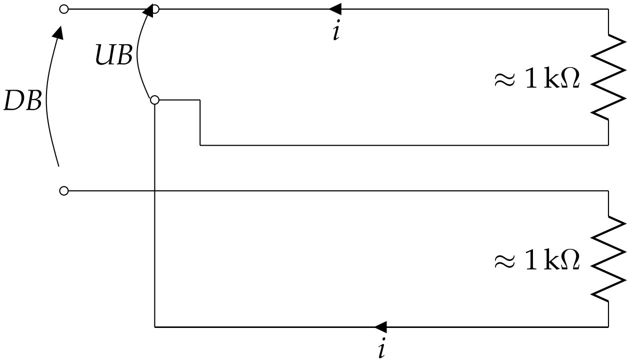
$$K_n = \sum_{j=1}^{N_{wires}} \frac{L_j R_r}{n} \left(\frac{x_j + iy_j}{R_r}\right)^n (-1)^j \tag{9}$$

Flux Fourier coefficients F_n

$$C_n = \frac{F_n}{K_n} \tag{10}$$

BUCKING

To accurately measure higher order harmonics it is necessary to connect the coils in such a fashion as to suppress the signal of the main field component. This will consequently suppress spurious harmonics due to coil vibrations. This technique is called *bucking*.



Section 2

SET-UP AND NOISE ANALYSIS

WORKING BENCH

MORGAN PROBE

DAQ (PXI-4462)

- Maximum sampling frequency: $204.8kHz$
- Differential inputs
- ADC resolution: $24bit$
- Input dynamic range set to $\pm 0.316\,\mathrm{V} \longrightarrow 30\,\mathrm{dB}$ gain
- Input resistance: $1\,\mathrm{M\Omega}$

$$LSB = \Delta = \frac{0.316\,\mathrm{V} \times 2}{2^{24}} \approx 37.67\,\mathrm{nV}$$

Quantization noise:

$$\sigma = \frac{\Delta}{\sqrt{12}} \approx 10.87\,\mathrm{nV}$$

Not infinite input resistance leads to signal loss of

$$PCB \approx 1 - \frac{1\,\mathrm{M\Omega}}{10\,\mathrm{k\Omega} + 1\,\mathrm{M\Omega}} \approx 1\%$$

$$Morgan \approx 1 - \frac{1\,\mathrm{M\Omega}}{10\,\Omega + 1\,\mathrm{M\Omega}} \approx \epsilon$$

DAQ NOISE

Channel	Mean [μV]			Standard deviation [μV]		
AI0	-10.24	-11.24	-12.04	0.39	0.44	0.46
AI1	5.42	5.35	5.34	0.35	0.36	0.36
AI2	0.57	0.87	0.53	0.39	0.38	0.44
AI3	4.63	4.64	4.64	7.18	7.12	7.08

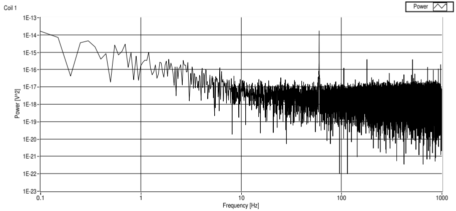
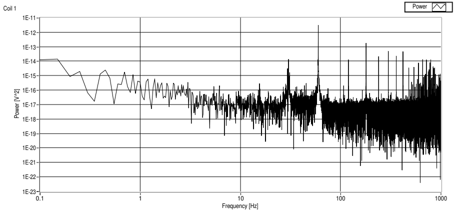


Figure 2 : AI0 Noise Spectrum

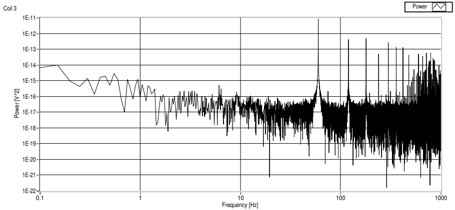
SENSORS

- **PCB probe** 5 signals: Unbucked (UB), Dipole Bucked (DB), Dipole Quadrupole Bucked (DQB), Dipole Quadrupole Sextupole Bucked (DQSB) and Unbucked Low Gain (UBL)
- **Morgan probe** 6 signals: Dipole (2P1), Quadrupole (4P1), Sextupole (6P1), Decapole (10P1) and Dodecapole (12P1) sensitive
- **Rotary encoder** 2 signals: index and encoder pulses

PCB PROBE PROPER NOISE

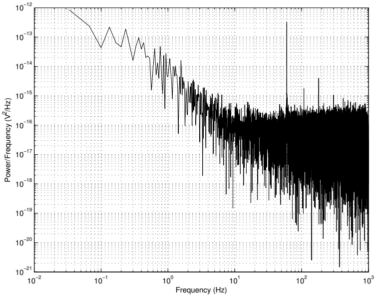


(a) UB coil

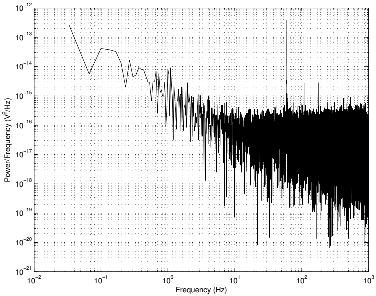


(b) DQB coil

MORGAN PROBE PROPER NOISE



(a) 2P1



(b) 12P1

PROBE NOISE COMPARISON

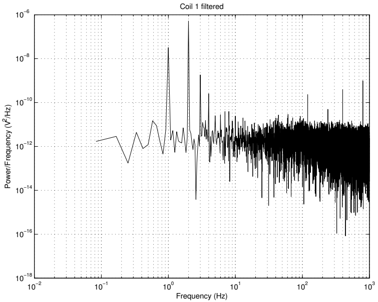
White noise level appears to be almost almost the same in both probes.

- **DAQ**: $\sqrt{S_f} \approx 1 \frac{nV}{\sqrt{Hz}}$
- **UB**: $\sqrt{S_f} = \sqrt{4kTR_{coil}} \approx \sqrt{4kT \times 1\,\mathrm{k}\Omega} \approx 4 \frac{nV}{\sqrt{Hz}}$ difficult to see on a log graph.
- **DQB**: $\sqrt{S_f} = \sqrt{4kTR_{coil}} \approx \sqrt{4kT \times 4.5\,\mathrm{k}\Omega} \approx 8.5 \frac{nV}{\sqrt{Hz}}$ slight increase visible
- **2P1** and **12p1**: resistance in the order of few Ω. Thermal noise negligible with respect to DAQ noise

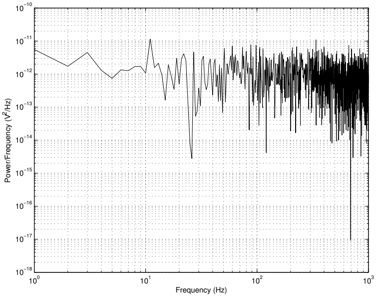
Conclusion: PCB coils are slightly noisier than Morgan coils.

STEPPER MOTOR

Probes are spun using a stepper motor. This kind of actuators are quite noisy.



(a) UB

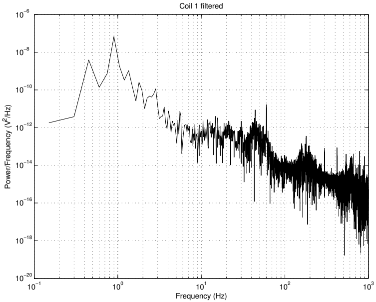


(b) 2P1

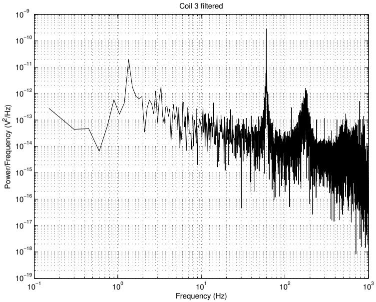
Noise raised from $\sqrt{S_f} \approx 1 \frac{nV}{\sqrt{Hz}}$, to $\sqrt{S_f} \approx 1 \frac{\mu V}{\sqrt{Hz}}$. No relation with the spinning frequency was found.

STEPPER MOTOR

Power spectra obtained spinning the probe manually confirm that the stepper motor is a dominant source of noise



(a) UB



(b) DQB

POWER SUPPLY

Magnets were powered using a Kepco BOP 36-12M DC bipolar power supply. Random fluctuations of the current generated by it can increase the uncertainty of the measures.

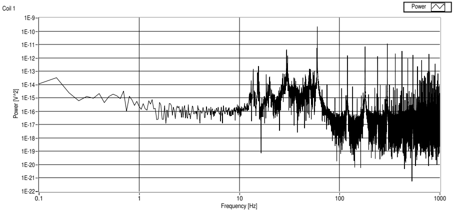


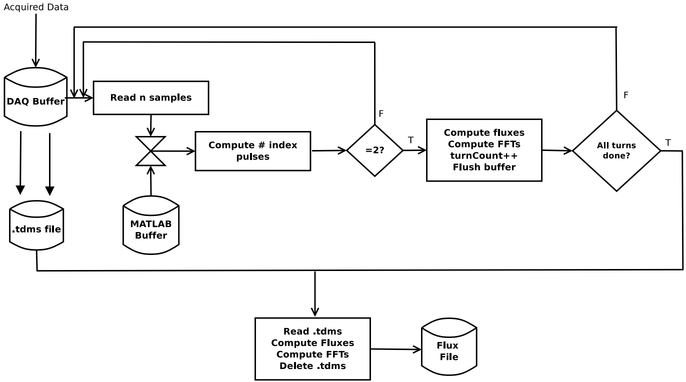
Figure 7 : UB coil. Power supply on

MAGNETS

Two magnets were employed to test the probes:

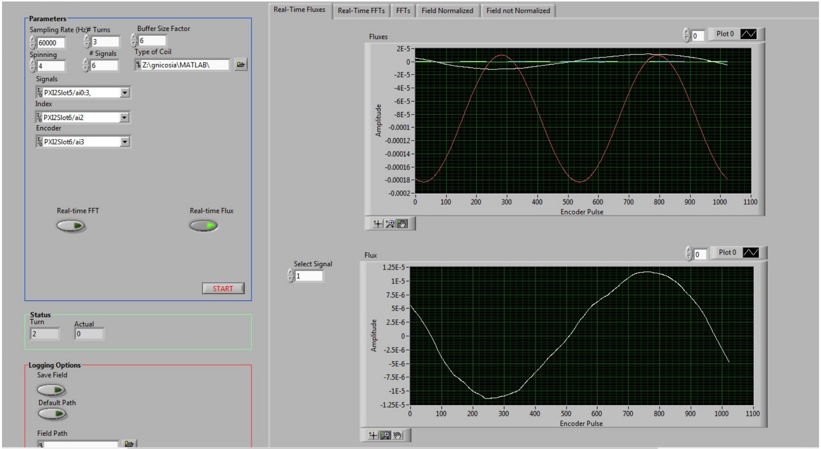
- **Dipole magnet**: 10 A → $C_1 \approx 71\,\mathrm{mT} \quad R_{ref} = 10\,\mathrm{mm}$
- **Quadrupole magnet**: 5 A → $C_2 \approx 2\,\mathrm{mT} \quad R_{ref} = 10\,\mathrm{mm}$

LABVIEW VI



Fluxes displayed after each turn. Harmonic analysis performed at the end of data acquisition.

LABVIEW VI



Parameters

Sampling Rate (Hz): 60000

Turns: 3

Buffer Size Factor: 6

Spinning: 4

Signals: 6

Type of Coil: Z:\gnicosia\MATLAB\

Signals: PXI2Slot5/ai0:3,

Index: PXI2Slot6/ai2

Encoder: PXI2Slot6/ai3

Status

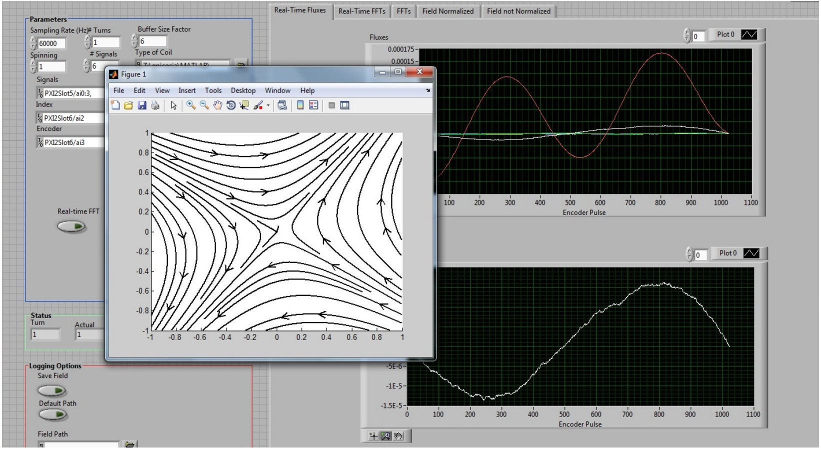
Turn: 2

Actual: 0

Logging Options

Select Signal: 1

LABVIEW VI



Parameters

Sampling Rate (Hz): 60000
Turns: 1
Buffer Size Factor: 6
Spinning: 1
Signals: 6
Signals: PXI2Slot5/ai0:3,
Index: PXI2Slot6/ai2
Encoder: PXI2Slot6/ai3

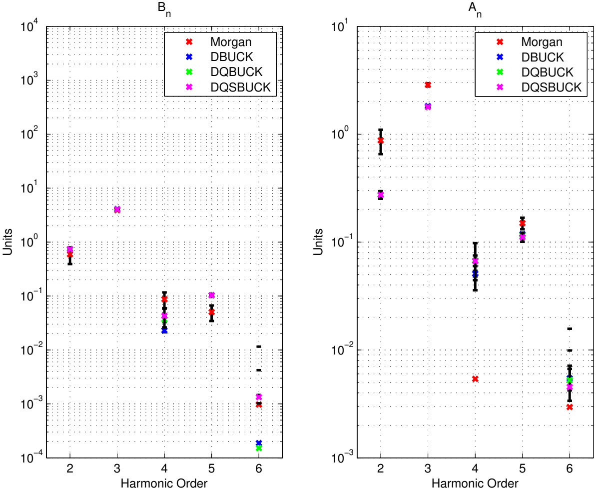
Status

Turn: 1
Actual: 1

Section 3

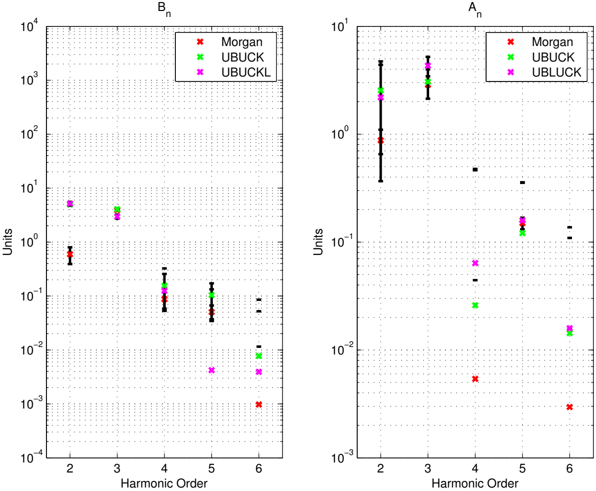
HARMONIC ANALYSIS

DIPOLE MAGNET: HARMONICS



Dipole harmonics comparison: normal component B_n and skew component A_n. Error as $\pm\sigma$

DIPOLE MAGNET: HARMONICS



Dipole harmonics comparison: normal component B_n and skew component A_n. Error as $\pm\sigma$

Dipole magnet: relative error

Relative error defined as

$$\epsilon = \frac{\sigma_{C_n}}{|C_n|}$$

Signal	f	2	3	4	5	6
Morgan	1 Hz	0.96	66.40×10^{-3}	2.1529	0.38	15.30
	2 Hz	0.28	26.44×10^{-3}	0.55	0.15	5.32
	4 Hz	0.11	11.71×10^{-3}	0.23	63.43×10^{-3}	0.57
UB	1 Hz	0.28	0.14	3.38	0.95	7.16
	2 Hz	0.39	0.18	2.94	1.52	8.01
	4 Hz	0.75	0.37	5.69	3.21	8.85
DB	1 Hz	0.12	8.1724×10^{-3}	0.21	45.17×10^{-3}	0.92
	2 Hz	35.04×10^{-3}	2.17×10^{-3}	62.37×10^{-3}	11.16×10^{-3}	0.27
	4 Hz	31.315×10^{-3}	1.83×10^{-3}	84.86×10^{-3}	16.15×10^{-3}	0.28
DQB	1 Hz	0.12	15.89×10^{-3}	0.21	62.40×10^{-3}	1.18
	2 Hz	35.04×10^{-3}	5.47×10^{-3}	0.11	21.441×10^{-3}	0.43
	4 Hz	31.315×10^{-3}	9.04×10^{-3}	0.21	50.70×10^{-3}	0.90
DQSB	1 Hz	0.12	15.89×10^{-3}	0.64	0.14	2.92
	2 Hz	35.04×10^{-3}	5.47×10^{-3}	0.44	77.05×10^{-3}	1.28
	4 Hz	31.315×10^{-3}	9.04×10^{-3}	0.92	0.19	3.26
UBL	1 Hz	0.31	0.15	3.35	1.05	5.21
	2 Hz	0.40	0.17	3.18	1.50	7.56
	4 Hz	0.75	0.36	6.27	3.15	8.73

DIPOLE MAGNET: ABSOLUTE ERROR

Standard deviation value σ_{C_n} in milliunits. Dipole magnet

Signal	f	2	3	4	5	6
Morgan	1 Hz	942.79	313.68	147.84	60.925	47.62
	2 Hz	302.1769	127.94	48.68	24.47	16.56
	4 Hz	118.72	56.38	20.29	10.01	5.80
UB	1 Hz	1639.4	722.62	343.60	169.56	97.81
	2 Hz	2192.6	941.77	461.47	242.49	130.27
	4 Hz	4347.6	1918.8	972.95	541.37	320.75
DB	1 Hz	91.46	36.64	12.31	7.06	4.32
	2 Hz	27.64	9.71	3.49	1.75	1.48
	4 Hz	24.50	8.20	4.72	2.57	1.52
DQB	1 Hz	91.46	70.71	17.06	9.62	5.36
	2 Hz	27.64	24.31	8.52	3.33	2.29
	4 Hz	24.50	40.10	16.49	7.97	4.77
DQSB	1 Hz	91.46	70.71	56.22	20.55	10.01
	2 Hz	27.64	24.31	35.22	11.70	6.05
	4 Hz	24.50	40.10	77.58	30.37	14.66
UBL	1 Hz	1840.0	775.48	354.72	165.92	90.27
	2 Hz	2234.9	922.66	446.30	237.70	123.91
	4 Hz	4312.0	1885.3	944.94	518.58	302.56

The PCB probe performs better than the Morgan one

TO DO

- Perform harmonic analysis on quadrupole magnet: a lot of problems arose when this analysis was performed.
- Repeat measures using a less noisy motor
- Understand the reason for differences in values of not allowed harmonics measured by the two probes
- Perform comparison using a preamplified PCB probe
- PCB probe behavior with ramping field