SUMMER SCHOOL FINAL REPORT

DESI EXPERIMENT

Barrel Design of Mayall Telesope

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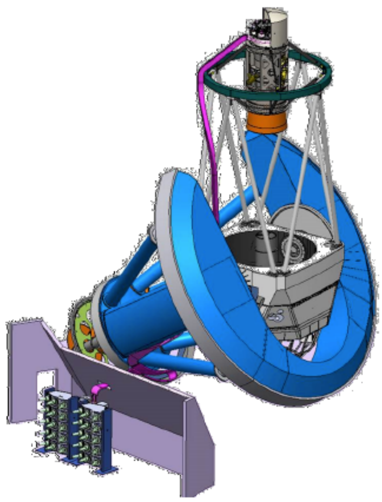
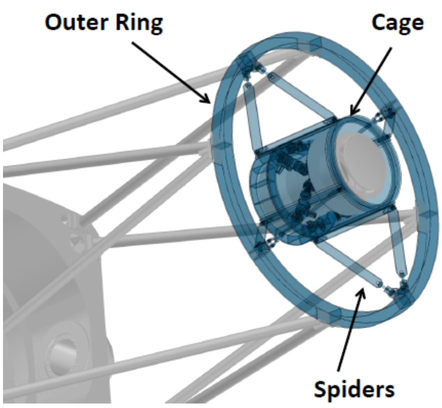
Contents

1 Introduction

DESI experiment will run in the 4 meter telescope in Kitt Peak known as the Mayall and Fermilab has been charged with designing and building the structure that will support the optics and the fiber positioners and with rebuilding the top end of the telescope.

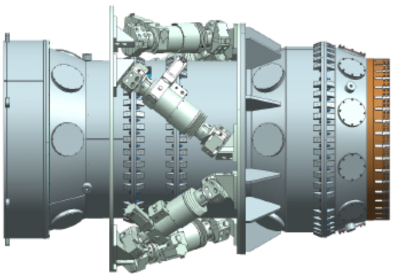
I have worked on different tasks which are all ralated to the design of the barrel of the Mayall Telesope.

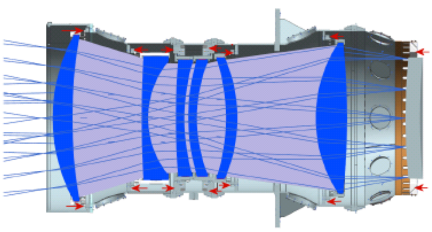




The main tasks are:

- Stiffness analysis of the gear systems for DESI application
- Cell design and FEM analysis
- Preliminary evaluation of stress due to Hertzian contact.





2 Stiffness analysis of the gear systems

The main goal of this work was to understand how to use the formula presented in *A common formula for the combined torsional mesh stiffness of spur gears.* In this paper the Authors, using a FEA model, have found a ralationship between the principal parameteres, which describe the gear, and the stiffness of the tooth, the body and the region of contact of the same gear. The paper's formulae are listed below:

$$K_{B,P} = c_B \cdot E \cdot w \cdot (ln(r_d - r_s))^{1.6} \cdot r_s^{1.6} \tag{1}$$

$$K_{T,P} = c_T \cdot E \cdot w \cdot m^2 \cdot z^2 \tag{2}$$

$$K_{C,P} = c_C \cdot E \cdot w \cdot m^{1.85} \cdot z^{2.2} \cdot \tau^{0.105} \tag{3}$$

where $C_B = 0.0009555$, $C_T = 0.000032$, $C_C = 0.000079365$, E is the young's modulus, w is the face width, r_s is the shaft radius, r_d is the dedendum radius, z is the number of theet and τ is the torque. The second equation is dimensionally correct, so it's possible to understand which units of measurement has to be used. The results, which are going to be presented, are obtained using millimiter for the lengths and MPa for the young's modulus. For example number one, presented in the paper, we obtained the following results:

$$K_B = 2.10 \cdot 10^3 Nm/rad \tag{4}$$

$$K_T = 3.83 \cdot 10^3 Nm/rad \tag{5}$$

$$K_B = 8.02 \cdot 10^3 Nm/rad \tag{6}$$

$$K_1 = 0.58 \cdot 10^3 Nm/rad \tag{7}$$

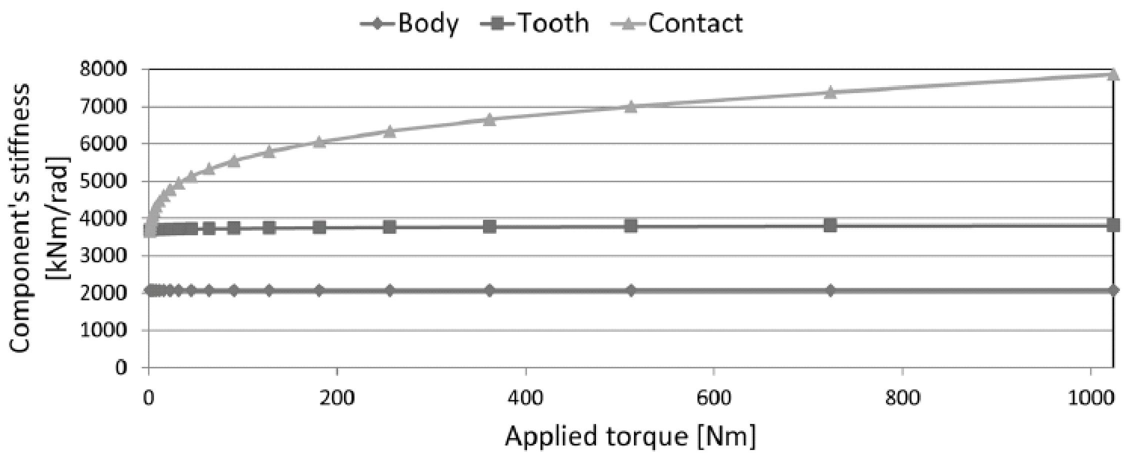


Figure 1: Example 1, Influence of the applied torque on body, teeth and contact stiffness

It's easy to see there is a difference of 10^3 between the results obtained and the results presented in the paper. Also for the second examples we find a difference of 10^3, in fact the value of the resulting stiffness is

$$K_2 = 0,152 \cdot 10^3 Nm/rad \tag{8}$$

For the examples 3 and 4, in which the gear ration it is different from 1, the results are

$$K_3 = 24.6 \cdot 10^3 Nm/rad \qquad e = +7,9\% \tag{9}$$

$$K_4 = 0.664 \cdot 10^3 Nm/rad \tag{10}$$

All the results are obtained with the formulae presented in the paper and using a spreadsheet.
Probably we need to use coefficient 1000 times greater than the values shown before. In order to be sure of this correction it's necessary to have a confirmation about the order of magnitude of the mesh stiffness . For this reason it will be presented the values of the mesh stiffness calculated using the procedure presented in the *DIN 3990 T1* mentioned as reference on the paper.

	A	B	C	D	E	F
1	Z1	13			Kb1	517,96
2	Z2	31			Kb2	2833,85
3	m (mm)	4				
4	W1 (mm)	16			Kt1	485,61
5	W2 (mm)	15			Kt2	3080,18
6	Rs1 (mm)	10				
7	Rs2 (mm)	20			Kc1	949,93
8	E(GPa)	210			Kc2	890,56
9					K1	198,31
10	Rd1	21			K2	555,43
11	Rd2	57			K	146,13
12	Cb	0,0009555				
13	Ct	0,000032				
14	Cc	0,000079				
15	T (Nm)	100				
16	Rp1	26				
17	Rp2	62				

Figure 2: Spreadsheet

2.1 DIN 3990 Procedure

In this standard the tooth stiffness is defined as the normal tooth load in the transverse section, which is necessary to deform by $1\mu m$ normal to the tooth involute in the transverse section one or more meshing error free tooth pairs with a $1mm$ facewidth.
In order to compute the value of the tooth stiffness (C'), we used the following formulae:

$$C' = C'_{th} \cdot C_M \cdot C_R \cdot C_B \qquad (N/mm \cdot \mu m) \tag{11}$$

where $C_M = 0.8$, $C_R = 1$ and $C_B = 1$

$$C'_{th} = (q')^{-1} \qquad (N/mm \cdot \mu m) \tag{12}$$

$$q' = C_1 + \frac{C_2}{z_1} + \frac{C_3}{z_2} \tag{13}$$

with $C_1 = 0.04723$, $C_2 = 0.15551$, $C_3 = 25791$

Once that it has been calculated the tooth stiffness, it's possible to compute the mesh stiffness C_γ using the formula:

$$C_\gamma = C' \cdot (0.75\varepsilon_\alpha + 0,25) \qquad (N/mm \cdot \mu m) \tag{14}$$

in which ε_α is the addendum overlap. The results which are going to be presented are computed for $\varepsilon_\alpha = 1.2$ and for $\varepsilon_\alpha = 1.9$.
In order to obtain the linear mesh stiffness K', it's necessary to moltiply C_γ by the length of the facewidth.

$$K' = C_\gamma \cdot w \qquad (N/mm) \tag{15}$$

At this point, in order to be able to compare the results obtained with this procedure with the results of the paper, we have to change the linear stiffness into a rotational stiffness (K). In the paper is stated that is possible to do this but it's not explained the procedure. Below it will be presented the procedure that it has been used for this purpose.
It will be called r_p the primitive radius of the gear, v the displacement of the tooth in the trasverse direction and θ is the rotation due to this displacement.

$$F_t = K'v \iff \tau = K' \cdot v \cdot r_p \tag{16}$$

Moreover $\theta \approx v/r_p$ and finally

$$\tau = K' \cdot v \cdot r_p^2 \iff K = K' \cdot r_p^2 \tag{17}$$

Using the same formulae we can easly compute the stiffness of the gear, then to calculate the total stiffness considering the two gears as spring in series.

$$K_{tot} = \frac{K_p \cdot K_g}{K_p + K_g} \tag{18}$$

In conclusion we can see that the order of magnitude of the results presented is consistent with the results presented in the paper. The uncertanties of this procudure are due to the translation from linear stiffness to rotational stiffness and in the choice of the parameters used in the *DIN 3990 Procedure*

Z1	56
Z2	266
m (mm)	3,6
W1 (mm)	25,4
W2 (mm)	25,4
Rs1 (mm)	13,5
Rs2 (mm)	434,7
E(GPa)	210

Rd1	96,3
Rd2	474,3
Cb	0,0009555
Ct	0,000032
Cc	0,000079
T (Nm)	100
Rp1	100,8
Rp2	478,8

Figure 3: Proposed gear system parameters for DESI application

Example	**1**	**2**	**3**	**4**
$K_{paper}(10^6$ Nm/rad)	0.652	0.159	24.0	0.103
$K_{\varepsilon-min}(10^6$ Nm/rad)	0.55 $e = -15\%$	0.12 $e = -22\%$	17 $e = -23\%$	0.12 $e = +18\%$
$K_{\varepsilon-max}(10^6$ Nm/rad)	0.80 $e = -15\%$	0.17 $e = +23\%$	25 $e = +14\%$	0.18 $e = +80\%$

2.2 ADC Trasmission System

In this section It will be computed the stiffness of the ADC Trasmission System using both the paper's formulae and the DIN 3990 Procedure.

2.2.1 Paper's Procedure

In order to use the formulae it's necessary to have the parameters listed in figure 3.1:

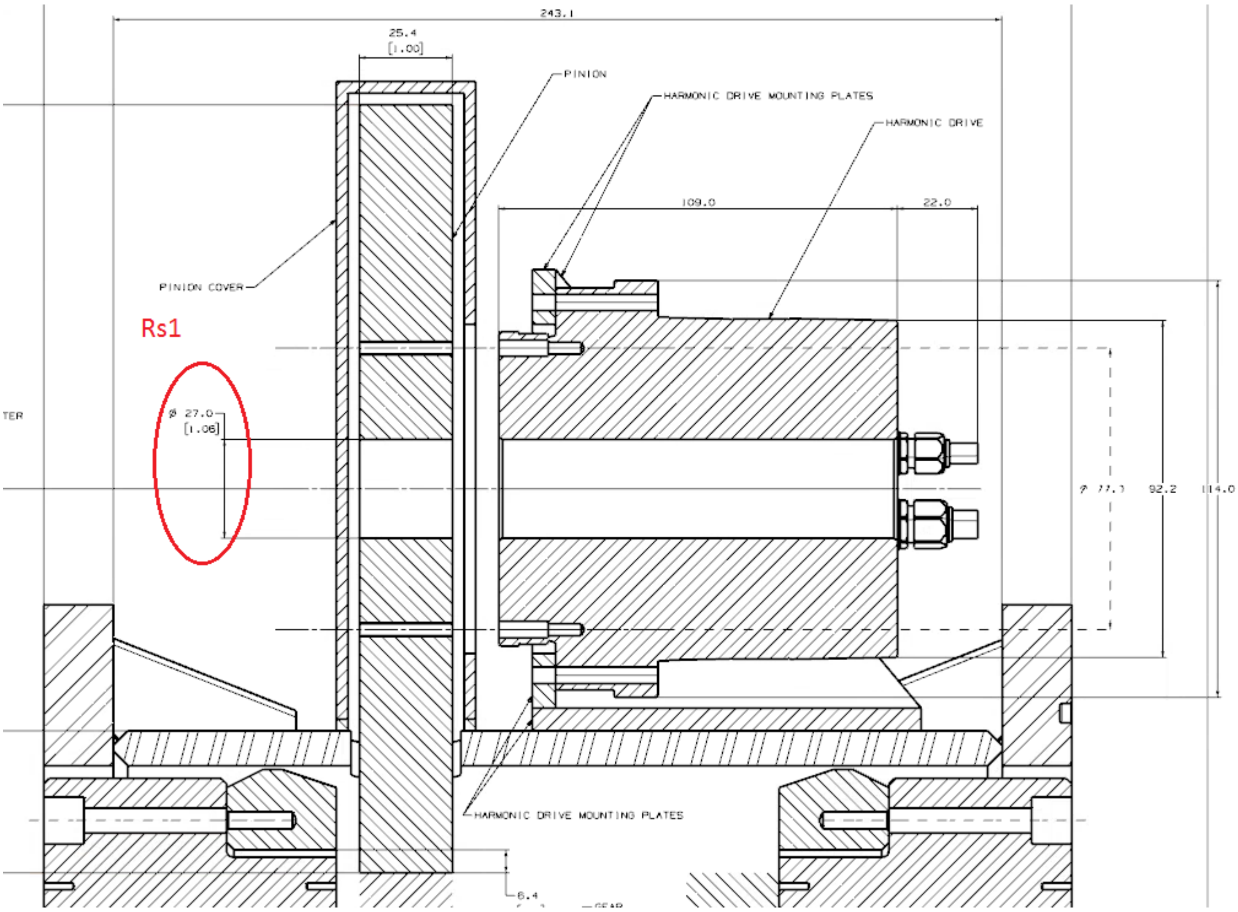


Figure 4: Pinion shaft radius of the DESI proposal design

The radius of the pinion shaft and the radius of the gear shaft aren't easly definable. In order to obtain a conservative value of the stiffness It has been chosen to adopt the minum values consistent with the system's drafting (figure 3.2-3.3). The value of the stiffness computed with this procedure, with the correction of is

$$K = 1,94 \cdot 10^6 Nm/rad \tag{19}$$

As we can easly see from the parameter, in this case the gear ratio is $u = 4.75$. This value is greater than 2.94 which is the maximum value of the gear ratio for wich the paper's formulae would give a reliable results.

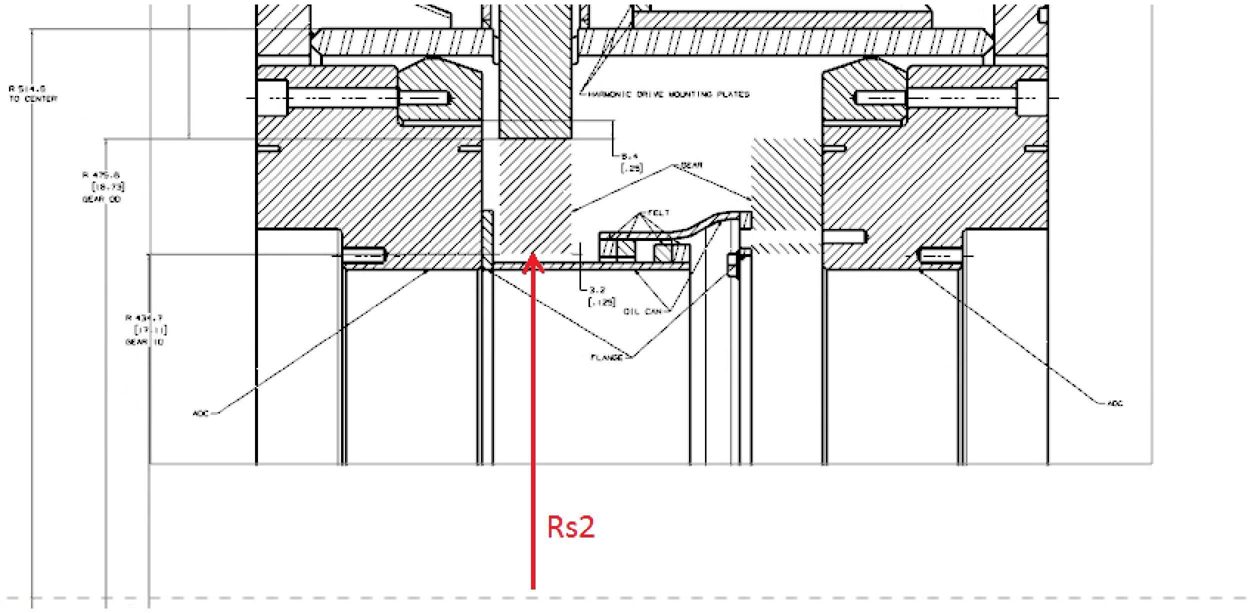


Figure 5: Gear shaft radius of the DESI proposal design

2.2.2 DIN 3990 Procedure

In this section we will calculated the stiffness of the system with the procedure explained before. The parameters are the same which are listed in figure 3.1. The result are compuetd for $\varepsilon_{\alpha} = 1.2$ in order to obtain a a conservative value of the stiffness.

$$K = 4,45 \cdot 10^{6} Nm/rad \tag{20}$$

The values of the stiffness calculated with these procedures have the same order of magnitude. The difference between these values could be due to the procedure to change the linear stiffness into a rotational stiffness, which it' is not presented in the paper, and to the choice of the parameters' values.

3 Cell design and FEM analysis

The goal of this task is to design a cell which will be used in the structural tests in which it will be evaluated the deformations of the barrel due to its weight. During this test, called Test Weight, the barrel will be loaded in two sections with a load of 500 Kg. The cell, which will be used in the tests has to have the same stiffness of the one that will be built on the telescope in order to get reliable results from the test. Moreover it has to be easier to be manufactured for a matter of costs. So the geometry it has been simplified as shown in the following figures:

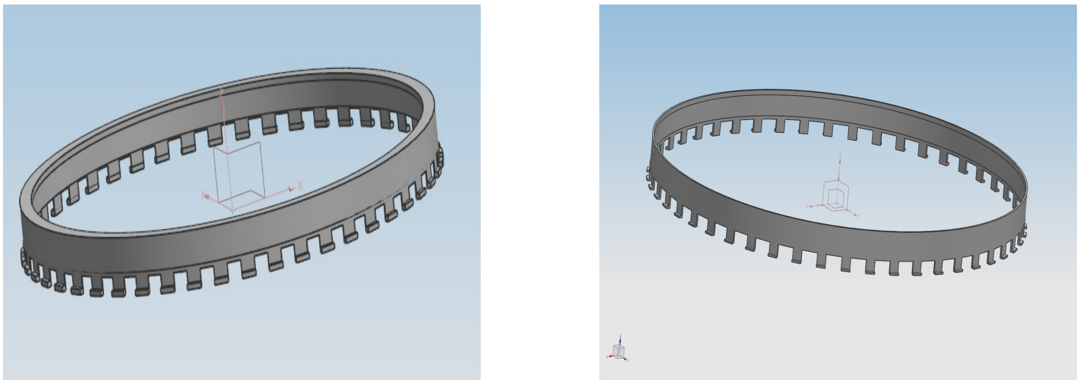


Figure 6: Original Cell and Cell for Test Weight

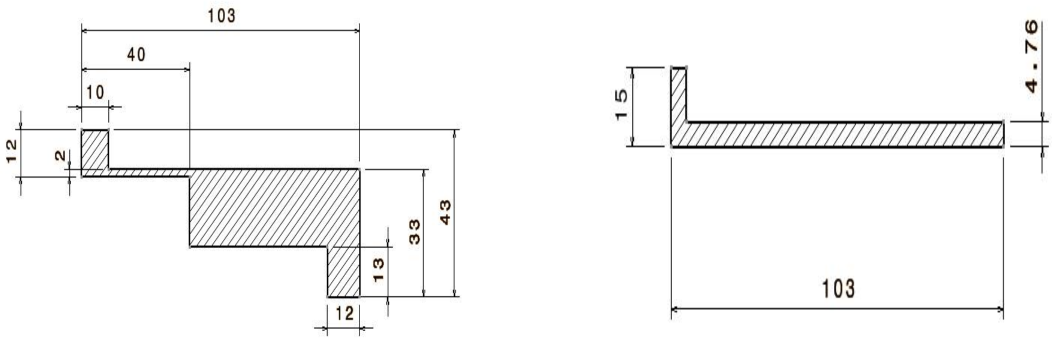


Figure 7: Geometry of the cells

3.1 First Model

In order to understand how the different geometry of the cells affect the deformation of the barrel it has been performed a FEM analysis.

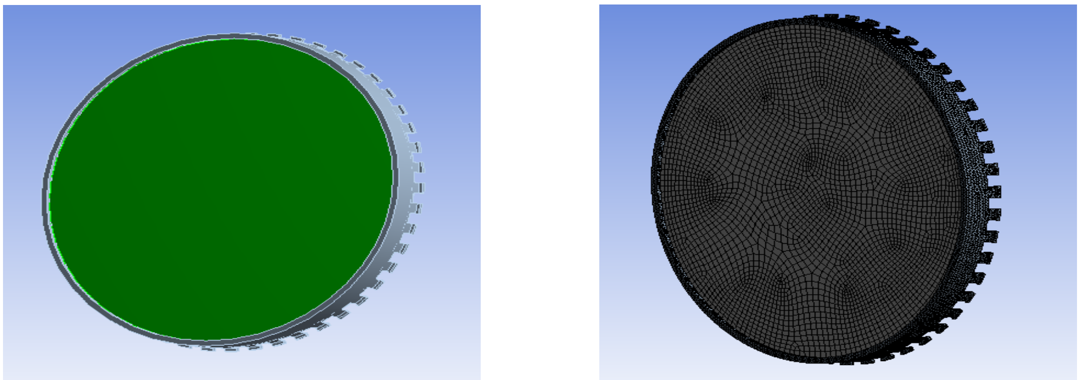


Figure 8: CAD models and Mesh for FEM Analysis

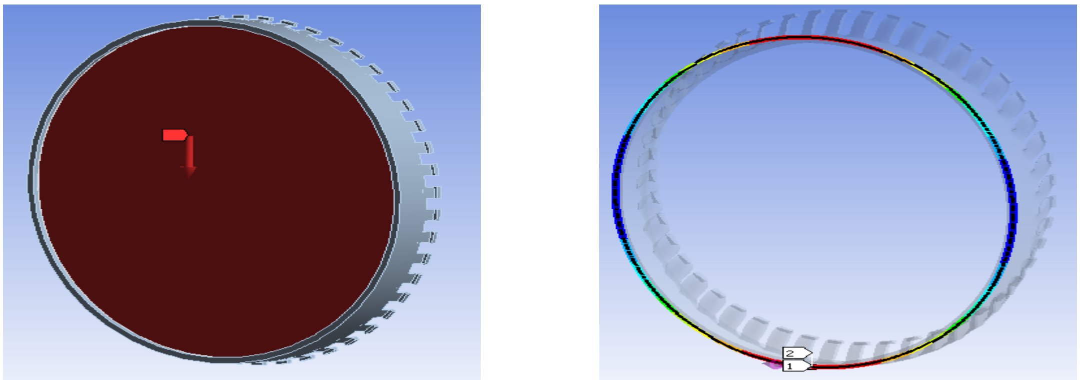


Figure 9: Load and Solution of FEM Analysis

In the first model it has been studied the displacements of the two cells. In this way it has been possible to change the geometry of the cell for the test in order to have a similar stiffness for the two cells. The model presents the cell and a disk which is loaded with a force of $4900N$. A constraint of *fixed support* is applied to the bases of the cells.

3.2 Second Model

The second model includes also the barrel. In this way we can see how the different geometry of the cells affects the displacements of the barrel when it is loaded. The loads is the same of the previous model while the constraints are applied to the barrel. In particular six points on the middle section can't move specific directions. This is due to the moviment system which is linked to the barrel. The contact between the cells and the barrel is *bonded* because for our purpose it is not necessary to include the bolts in the model. Also in the second models it is computed the displacements of an edge of the front section of the barrel.

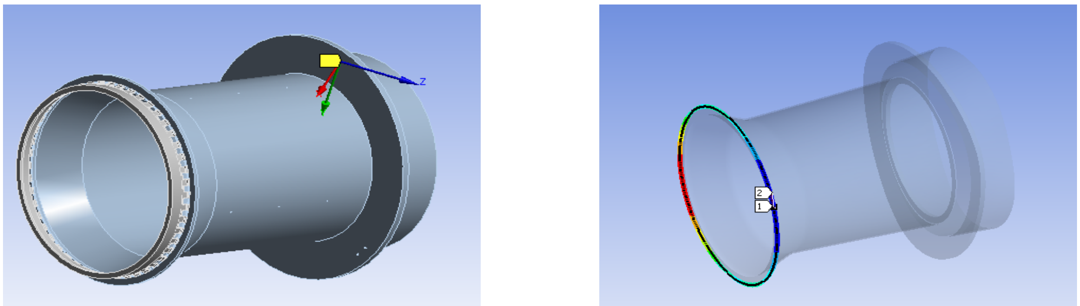


Figure 10: Constraints and Solution of FEM Analysis

Figures 11 show the results of this analysis. The difference in the displacements computed using the two cells are sufficiently low so the geometry of the cell shown in figures 12 is good and it will be used to manufacture the cell for the test.

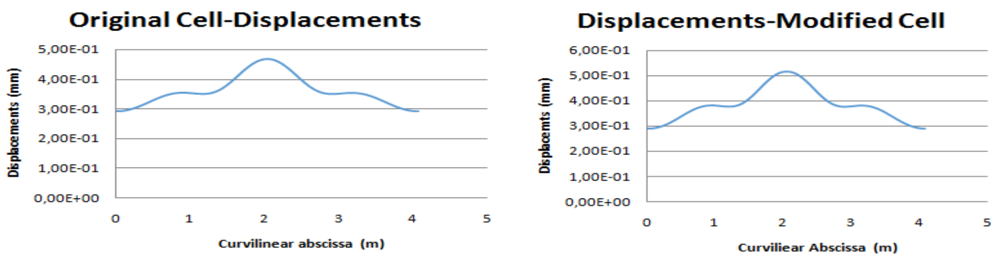


Figure 11: Displacements computed with different cells

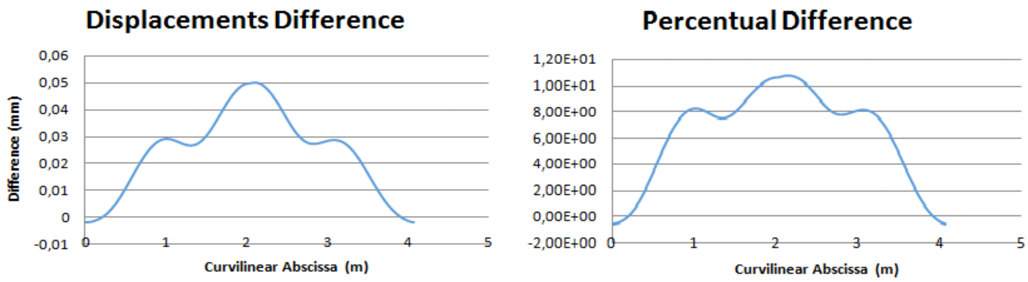


Figure 12: Difference between the displacements computed with different cells

4 Preliminary evaluation of stress due to Hertzian contact.

The goal of this analysis is to evaluate the stress due to the contact between the sphere and the flanges.

4.1 First Analysis

The figure 13 shows the model used for the analysis, where g is the distance between the flanges and s the displacement in the radial direction. The first computation is performed assuming that the flanges have no strain.

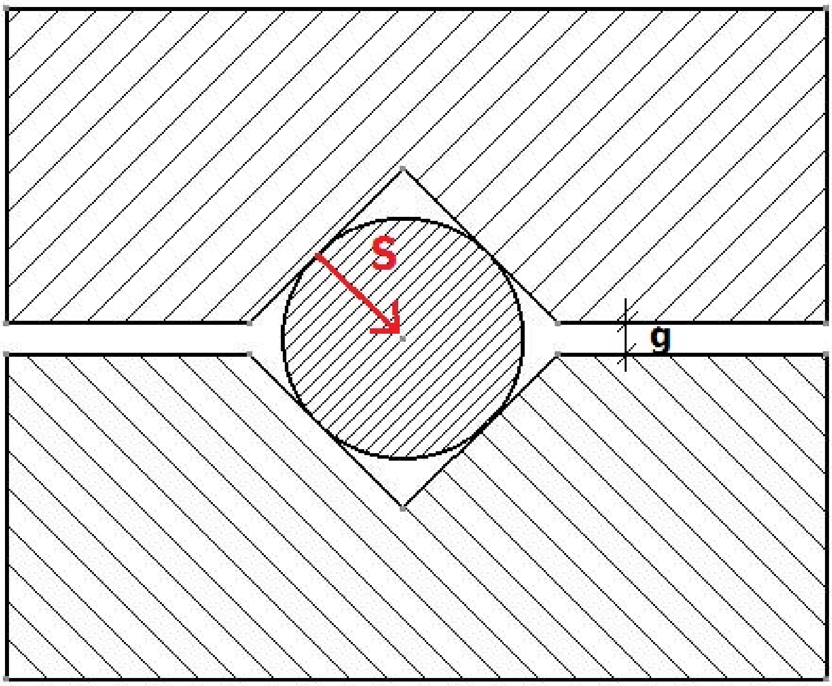


Figure 13: Gear shaft radius of the DESI proposal design

$$s = \frac{\sqrt{2}}{2} \cdot g \tag{21}$$

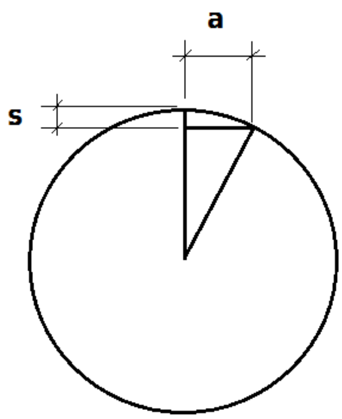


Figure 14: Gear shaft radius of the DESI proposal design

$$\Delta = \frac{2(1-\nu^2)}{E} \tag{22}$$

$$a = R \cdot \sqrt{1-\left(1-\frac{s}{R}\right)^2} \tag{23}$$

$$F = \frac{a^3}{0.908^3 \cdot \Delta \cdot R} \tag{24}$$

$$p = 0.578 \cdot \sqrt[3]{\frac{F}{R^2 \cdot \Delta^2}} \tag{25}$$

$$\tau_{max} \approx 0.3 \cdot p \tag{26}$$

With these equations is possible to compute the value of the force F necessary to put in contact the flanges and then the contact pressure p and τ_{max}

The data used for the analysis are:

- $g = 50 \cdot 10^{-3} mm$
- $R = 7.5 mm$
- $E = 210 \cdot 10^3 MPa$
- $\nu = 0.3$

And the results are:

- $F = 7900N$
- $p = 7300MPa$
- $\tau_{max} = 2200MPa$
- $z = 0.35mm$
- $A = 1.3mm^2$

4.2 Second Analysis

In this second model we make the hypothesis that the sphere has a very high stiffness compared to the flanges. Moreover a very simple model is used to evaluate the stress. The flanges are compared to beams with the length distance between the sphere and the bolt and it is evaluated the force necessary to close the gap between the flanges.

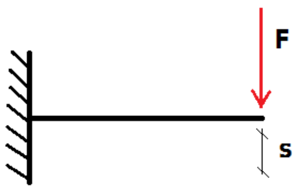


Figure 15: Gear shaft radius of the DESI proposal design

$$F = \frac{3EJg}{2L^3} \tag{27}$$

where $L = 85mm$, $J = 18900mm^4$.

The results are on this model are:

- $F = 490N$
- $p = 2900MPa$
- $\tau_{max} = 860MPa$
- $z = 0.14mm$
- $A = 0.2mm^2$

4.3 Conclusion

The first method gives a value of τ_{max} which is overestimated, while the second case gives a value of τ_{max} which is understimated.
In both cases the computed stress is greater than the yield strength of the material so, in the area of contact, there will be plasticity.

Thanks to these analysis it has been possible to understand the effect of the contact between the sphere and the flanges. At this point the analysis will be used to decide if this system is acceptable or if it needs to be changed.