Spacecraft Dynamics Employing a General Multi-tank and Multi-thruster Mass Depletion Formulation

Paolo Panicucci

Supervized by:
Dr. Hanspeter Schaub

September 15, 2016

Plan

Notations

- $\boldsymbol{r}_{C/N}$ is the vector pointing from N to C
- ${}^{\mathcal{B}}\boldsymbol{r}$ is the vector $\boldsymbol{r}$ expressed in the $\mathcal{B}$ reference frame
- $\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}}$ is the angular velocity of the $\mathcal{B}$ reference frame about the $\mathcal{N}$ one
- $\boldsymbol{r}'$ denotes the derivate with respect to the time in the body fixed reference frame
- $\dot{\boldsymbol{r}}$ denotes the derivate with respect to the time in the $\mathcal{N}$ reference frame

Reference frames

- The inertial reference frame $\mathcal{N}$ centered in N and oriented freely in space
- The body fixed reference frame $\mathcal{B}$ with origin B and versors $\{\hat{b}_1, \hat{b}_2, \hat{b}_3\}$ oriented in any direction of the space
- The nozzle fixed reference frame $\mathcal{M}$ with origin N_j and whose axis $\hat{m}_1$ is oriented as the echausted particles' velocity

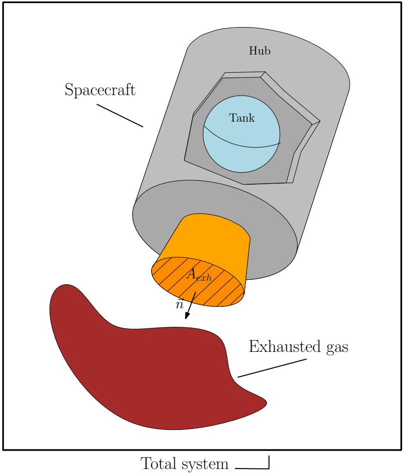
Objectives

The objectives can be divided in:

- Derivation of Equation of Motions (EOMs) without tracking the exhausted fuel
- Formulate tank's models for the inertia variation and the tank's barycenter movement
- Evaluation of relevant term in the EOMs
- Control of the moving spacecraft
- Simulation of concrete cases:
  1. Geostationary Hohmann maneuver
  2. Spin-stabilized rocket (validation case)

EOMs' derivation approach

1. Consider a constant mass system where equation Newton equation of motion are true
2. Use the Reynolds transport theorem and consider a moving control volume
3. Write the exhausted gas dependence using known nozzles' propriety
4. Tracking the movement of the spacecraft's barycenter from a fixed point B on the vehicle



Translational EOM

By considering a moving volume in an non-inertial and rotating frame and under the hypothesis of no whirling motion:

$$\ddot{\boldsymbol{r}}_{B/N} = \frac{\boldsymbol{F}_{\text{ext}}}{m_{\text{sc}}} + \frac{1}{m_{\text{sc}}} \sum_{j=1}^{N} \boldsymbol{F}_{\text{thr}_j} - 2\,\frac{\dot{m}_{\text{fuel}}}{m_{\text{sc}}} \left(\boldsymbol{c}' + \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{c}\right) - \boldsymbol{c}'' - 2\,\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{c}' +$$

$$- \dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{c} - \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \left(\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{c}\right) + \frac{2}{m_{\text{sc}}} \sum_{j=1}^{N} \dot{m}_{\text{noz}_j} \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{r}_{N_j/B}$$

where $\boldsymbol{c}$ is defined by the following expression and $\boldsymbol{c}'$ and $\boldsymbol{c}''$ can be deduced by its derivative in the $\mathcal{B}$ reference frame:

$$\boldsymbol{c} = \frac{m_{\text{hub}}\, \boldsymbol{r}_{Bc/B} + \sum_{i=1}^{M} m_{\text{fuel}_i}\, \boldsymbol{r}_{Fc_i/B}}{m_{\text{hub}} + \sum_{i=1}^{M} m_{\text{fuel}_i}}$$

Rotational EOM

The rotational dynamics can be represented by the following equation:

$$[I_{\text{sc, }B}]\,\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} + \left[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}\right][I_{\text{sc, }B}]\,\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + \sum_{i=1}^{M}\Big(m_{\text{fuel}_i}\left[\tilde{\boldsymbol{r}}_{Fc_i/B}\right]\boldsymbol{r}''_{Fc_i/B} +$$

$$+\dot{m}_{\text{fuel}_i}\left[\tilde{\boldsymbol{r}}_{Fc_i/B}\right]\boldsymbol{r}'_{Fc_i/B} + 2\,m_{\text{fuel}_i}\left[\tilde{\boldsymbol{r}}_{Fc_i/B}\right]\left[\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}}\right]\boldsymbol{r}'_{Fc_i/B}\Big) +$$

$$+[K]\,\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} = \boldsymbol{L}_{B_{\text{ext, sc}}} + \sum_{j=1}^{N}\boldsymbol{L}_{\text{thr}_j} + \ddot{\boldsymbol{r}}_{B/N}\times m_{\text{sc}}\,\boldsymbol{c}$$

where:

$$\sum_{j=1}^{N}\boldsymbol{L}_{\text{thr}_j} = \boldsymbol{L}_{\text{sc, exh}} + \sum_{j=1}^{N}\int_{\dot{m}_{\text{noz}_j}}\boldsymbol{r}_{M/B}\times\boldsymbol{v}_{\text{exh}_j} = \sum_{j=1}^{N}\left(\boldsymbol{L}_{\text{sc, noz}_j} + \int_{\dot{m}_{\text{noz}_j}}\boldsymbol{r}_{M/B}\times\boldsymbol{v}_{\text{exh}_j}\right)$$

$$[K] = \sum_{i=1}^{M}\left(\left[I_{\text{fuel}_i,\,Fc_i}\right]' + \dot{m}_{\text{fuel}_i}\left[\tilde{\boldsymbol{r}}_{Fc_i/B}\right]\left[\tilde{\boldsymbol{r}}_{Fc_i/B}\right]^T\right) - \sum_{j=1}^{N}\int_{\dot{m}_{\text{fuel}_i}}\left[\tilde{\boldsymbol{r}}_{M/B}\right]\left[\tilde{\boldsymbol{r}}_{M/B}\right]^T d\dot{m}$$

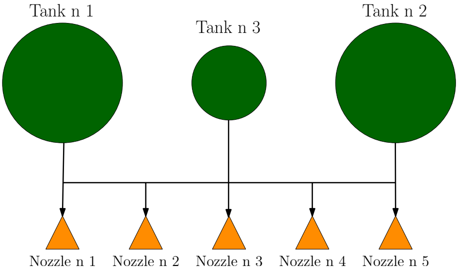
Fuel transport

If a matrix notation is considered:

$$\dot{\boldsymbol{m}}_{\text{fuel}} = [A] \; \dot{\boldsymbol{m}}_{\text{noz}}$$

Moreover, from mass flows' conservation:

$$\sum_{i=0}^{M} A_{ij} = 1 \quad \forall j \in (1, N)$$



Control feedback law

A Modified Rodrigues Parameter feedback control law has been chosen as it can always assure global asymptotic stability avoiding singularities.

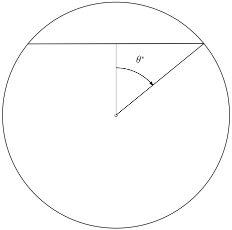
$$\boldsymbol{u} = -K\,\boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}} - P\,\boldsymbol{\omega}_{\mathcal{B}/\mathcal{R}}$$

where $\boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}}$ is the MRP defining the attitude from the $\mathcal{R}$ frame to the $\mathcal{B}$ one and $\boldsymbol{\omega}_{\mathcal{B}/\mathcal{R}}$ is the angular velocity of the $\mathcal{B}$ frame about the $\mathcal{R}$ one, K and P are control gains.

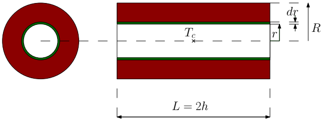
From the $\boldsymbol{u}$ control torque the forces providing that torque can be computed minimizing the applied force in the spacecraft and considering a general set of thrusters.

Tank Models

- The constant tank's volume model.
- The constant fuel's density model.
- The emptying tank model.
- The centrifugal burn cylinder.
- The uniform burn cylinder.



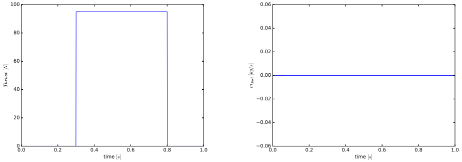
(a) The emptying tank model



(b) The centrifugal burn cylinder

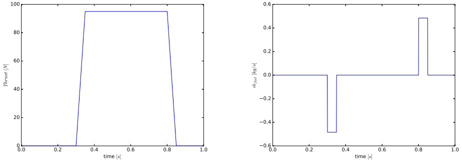
Thruster Models

- The impulsive thruster model where the thrust is immediately generated during the firing time.
- The ramping thruster model where, once the valve is opened to provide thrust, a time span of response Δt_{resp} is required to acquire the steady state.



(c) Thrust   (d) $\ddot{m}_{fuel}$

Figure: Impulsive thruster



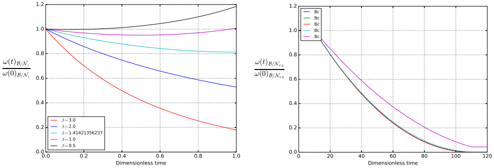
(a) Thrust   (b) $\ddot{m}_{fuel}$

Figure: Ramping thruster

Spin-stabilized rocket

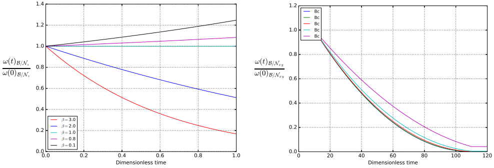
In order to validate the model, two cases from ”Dynamics of variable mass systems (F.O. Eke - 1998)” have been simulated.

In this case an axial symmetric rocket is analyzed changing the shape os the nozzle area (in the on-axis simulation) and the length of the tank (in the out-of-axis one).



(a) Spin rate (b) Traversal rate

Figure: Centrifugal Burn Cylinder



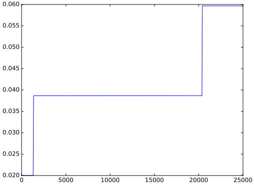
(a) Spin rate (b) Traversal rate

Figure: Constant Burn Cylinder

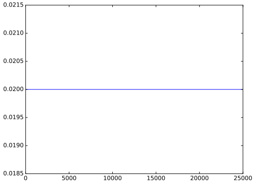
GEO Hohmann maneuver

Three cases have been compared for an Hohmann transfer from 200 *km* to 36000 *km*:

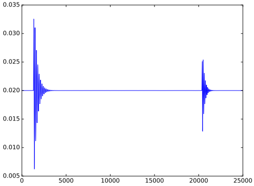
- Non-controlled case
- Update-parameters case
- MRP-controlled case



(a) Non-controlled simulation



(b) Update-only simultaion



(c) MRP-controlled simulation

Conclusion

The objectives obtained from this project are:

- Development and validation of the presented model
- Implementation of a control strategy
- Importance of considering mass depletion effects in high-fidelity simulation

The future work to be performed are:

- Analysis of the bugs os the simulation environment through tests
- Presentation of a paper in February at GNC conference