CDF Top Quark Mass Measurement with the Matrix Element Method

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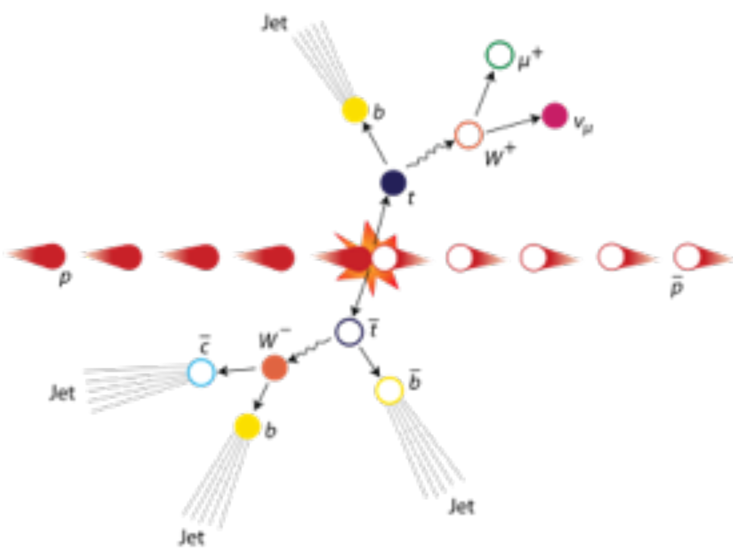
Fermilab Summer School - Final presentation

22 September 2016

Outline

- Top mass measurements
- Channel and Event Selection
- Matrix element method
- Study of the integration methods: pMC, qMC
- Analysis of the preliminary results with the new TF
- Study of the sensitivity to the Δ_{JES}
- Future development of the analysis

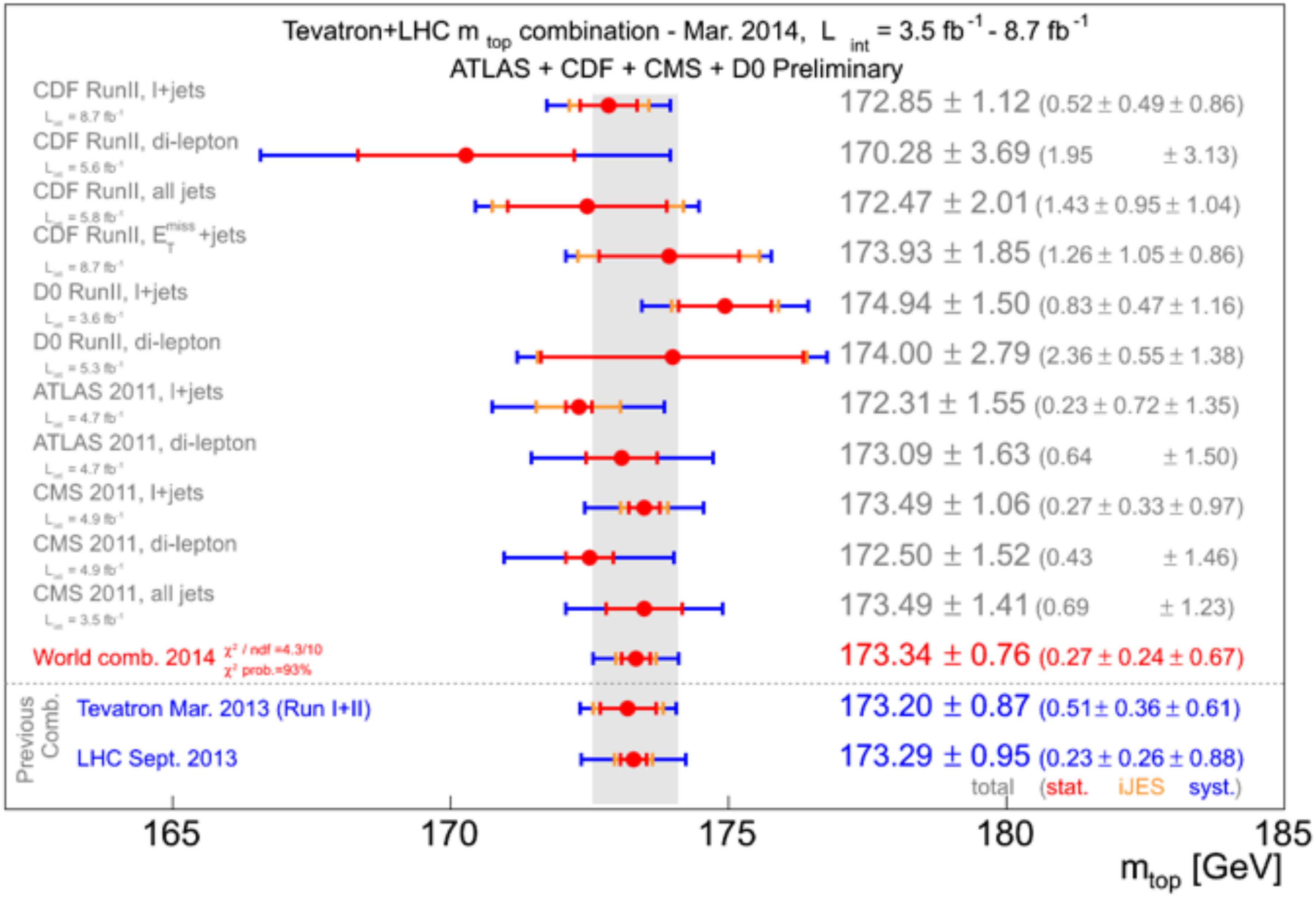
Tevatron Collider

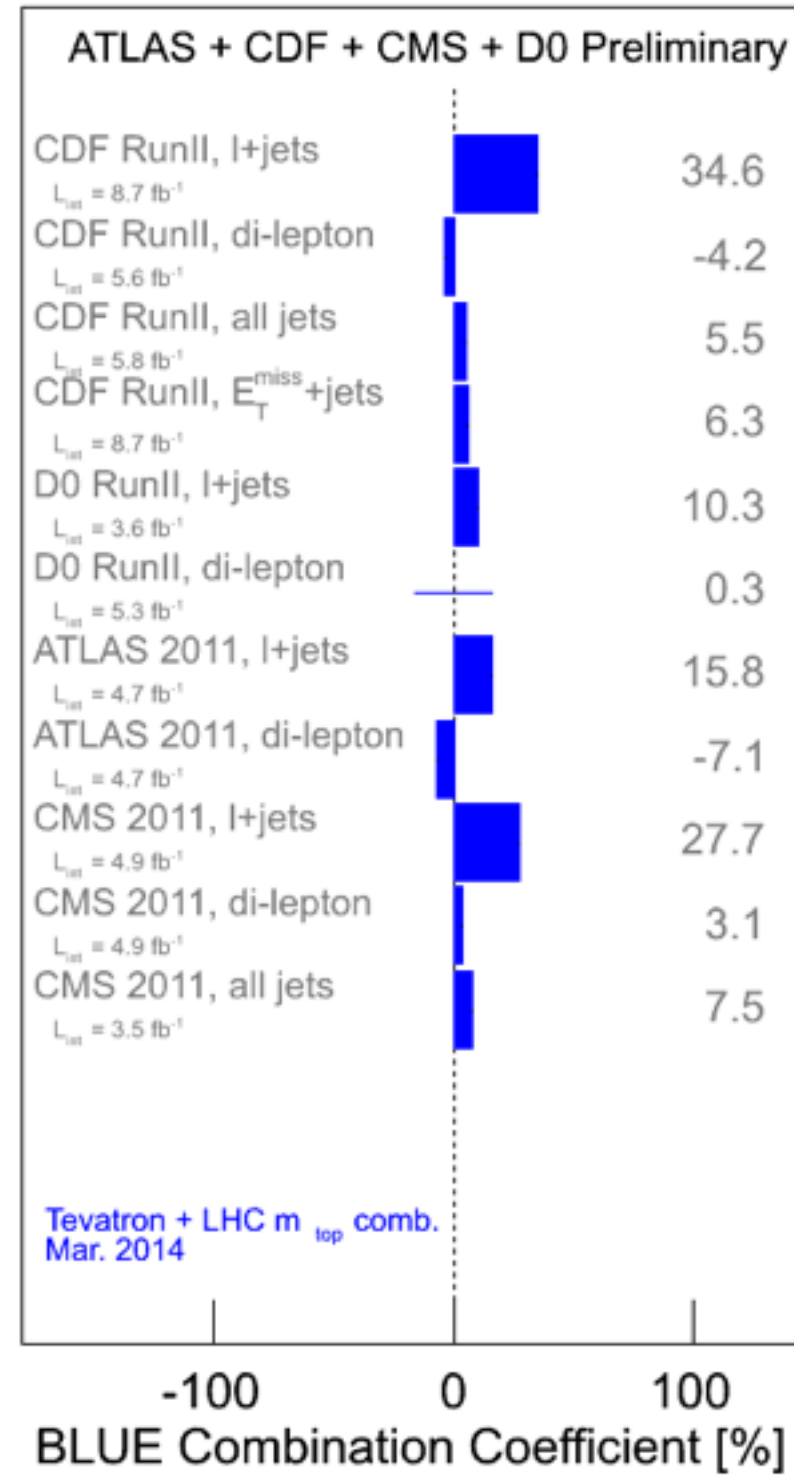


- **Proton - Antiproton collider**
- **Completed in 1983**
- **Main achievement: Top quark discovery, 1995**
- **Energy reached Run II:** $\sqrt{s} = 1.96\ TeV$

- **Lead in top physics:**
  - **Production properties (cross sections, A_{FB})**
  - **Decay properties (width, BR)**
  - **Intrinsic properties (mass, spin, charge)**
  - **Exotic searches involving top quarks**

Top Mass Measurement: Previous results





World comb. 2014 : 0.44% Precision

Top Mass measurements: Advances in Precision

- D0 final measurement in lepton+jets:

$$m_t = 174.98 \pm 0.58_{stat+JES} \pm 0.49_{syst}\ GeV/c^2 = 174.98 \pm 0.76\ GeV/c^2$$

$(0, 43\%$ precision)

PRL **113**, 032002 (2014); PRD **91**, 112003 (2015)

- CMS 7 + 8 TeV measurements in all channels: (Latest)

$$m_t = 172.44 \pm 0.13_{stat} \pm 0.44_{syst}\ GeV/c^2 = 172.44 \pm 0.48\ GeV/c^2$$

PRD **93**, 072004 (2016)

(0.28% precision!)

- CDF latest measurement aim:
  - Reach the highest possible precision From CDF Data.
  - Examine tension between LHC and Tevatron Results

CDF Top Mass Measurement: Channels

- Top decay Branching Ratio: $t \to Wb \sim 100\%$

- $t\bar{t}$ decay signatures:
  - *Dilepton* events: both W bosons decay into an $e\nu$ or $\mu\nu$ final state.
    - Lowest branching ratio: $\sim 7\%$ (including τ leptons)
    - Two undetected neutrinos: Unconstrained kinematics

  - *Hadronic* events: both *W* bosons decay hadronically (6 jets)
    - Highest branching ratio: $\sim 55\%$
    - Large QCD multi-jet background

  - Lepton + jets events: one *W* boson decay hadronically, the other into an $e\nu$ or $\mu\nu$.
    - Characterised by an isolated lepton, four jets, missing transverse energy.
    - Branching ratio: $\sim 38\%$ (τ events included)

    τ events: if τ decays into e or μ, it appears in the electron/muon+jets signal sample.

CDF Top Mass Meas.: L+jets event selection / Background

- Lepton + jets: event signature:
  - High transverse momentum p_T charged lepton;
  - Large missing transverse energy $\not{E}_T$ (escaping neutrino from *W* decay in the final state);
  - At least 4 jets;

- Tight or loose jets:
  - Tight jet: $E_T > 20GeV$, $|\eta| \leqslant 2.0$
  - Loose jet: $E_T > 12GeV$, $|\eta| \leqslant 2.4$

- 5 subsamples based on the number of identified (tagged) b-jets (T or L):
  - 0-tag, 1-tagL, 1-tagT, 2-tagL, 2-tagT.

- Background: non-$t\bar{t}$ events that mimics the L+jets signature:
  - W+jets (*W+* $b\bar{b}$, *W+* $c\bar{c}$, *W+* c , *W+LF*) Included in the Likelihood
  - QCD ("fake" electrons, secondary electron): reduced by selection cuts.
  - other: (*Single-top, Diboson (WW,WZ,ZZ), Z+jets)*

CDF Latest measurement: Improvement respect to past analysis

- Increase of Integrated Luminosity: Exploiting the full CDF Run II Dataset.
  - from *5.6* fb^{-1} to *9.0* fb^{-1}: ~ 60% more data;
- Inclusion of new sample categories:
  - untagged category: 0-tag;
  - loose categories: 1-tagL, 2tagL;
- For the first time in CDF analysis the Background Matrix Element modelling of the likelihood is included;
- Inclusion and refinement of the quasi-MC method in the Integration code;
- Smaller systematic uncertainties on the final measurement by introducing several new signal and background modelling;
- NLO signal MC: Reduction of uncertainty in Calibration Procedure

Matrix Element Technique

- Full kinematic and topological information in any event is considered.
- Calculation of the Matrix element to find the probability for the event:
  - Integration over phase space $d\Phi(\mathbf{x})$ $[t\bar{t} \to b(l\nu)b(qq')]$
    - 32 variables of integration: $\xrightarrow{constraints}$ 19 variables of integration:
- Signal and Background: $\longrightarrow$ for different (m_t, Δ_{JES})

$$L_{ev}(\mathbf{y}|m_t, \Delta_{JES}) = a(f_{sig})L_{sig}(\mathbf{y}|m_t, \Delta_{JES}) + b(f_{bkg})L_{bkg}(\mathbf{y}|\Delta_{JES})$$

- Likelihood:

$$L_{tot}(\mathbf{y}|m_t, \Delta_{JES}) = \prod_{i=1}^{N} L_{ev}(\mathbf{y}|m_t, \Delta_{JES})$$

- Calibration of Method:
  - Pseudo-experiment to correct biases and missing background modelling
- A fast and reliable integration algorithm is essential

Validation of the integration method : Pull Distribution pMC

- pMC integration: Random sequence of points with importance sampling;
- Relative error behaviour: $\Delta I/I \sim \mathcal{O}(N^{-1/2})$
- Pull distribution:

Mean: $$< W_{k,ij} >= \frac{1}{N}\sum_{l=1}^{N} W_{k,ij,l}$$

Standard dev: $$\sigma_{k,ij} = \sqrt{\frac{1}{N-1}\sum_{l=1}^{N}(W_{k,ij,l}- < W_{k,ij} >)^2}$$

Pull variable: $$\delta_{k,ij,l} = \frac{W_{k,ij,l}- < W_{k,ij} >}{\sigma_{k,ij}}$$

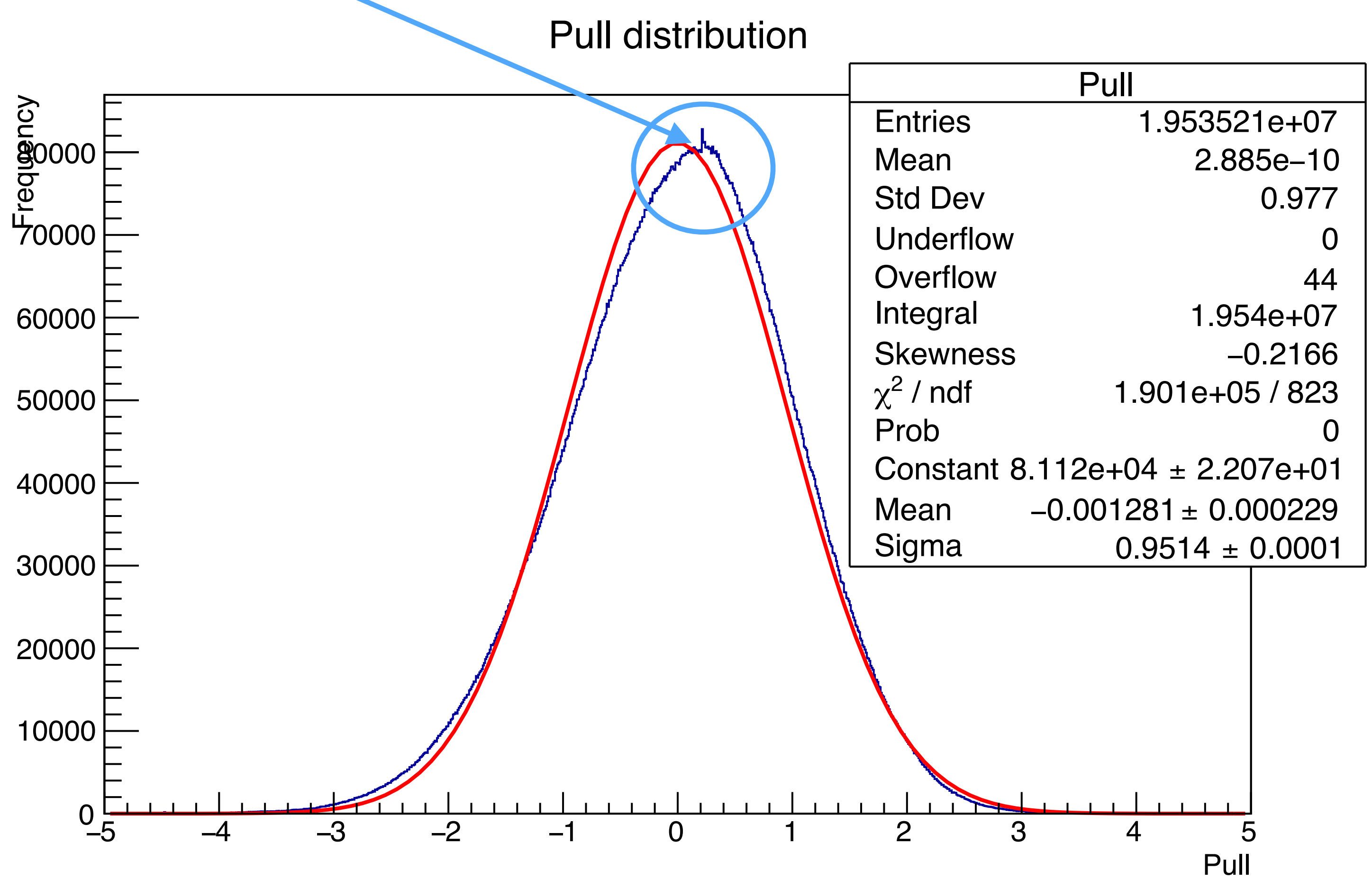
$k = event$

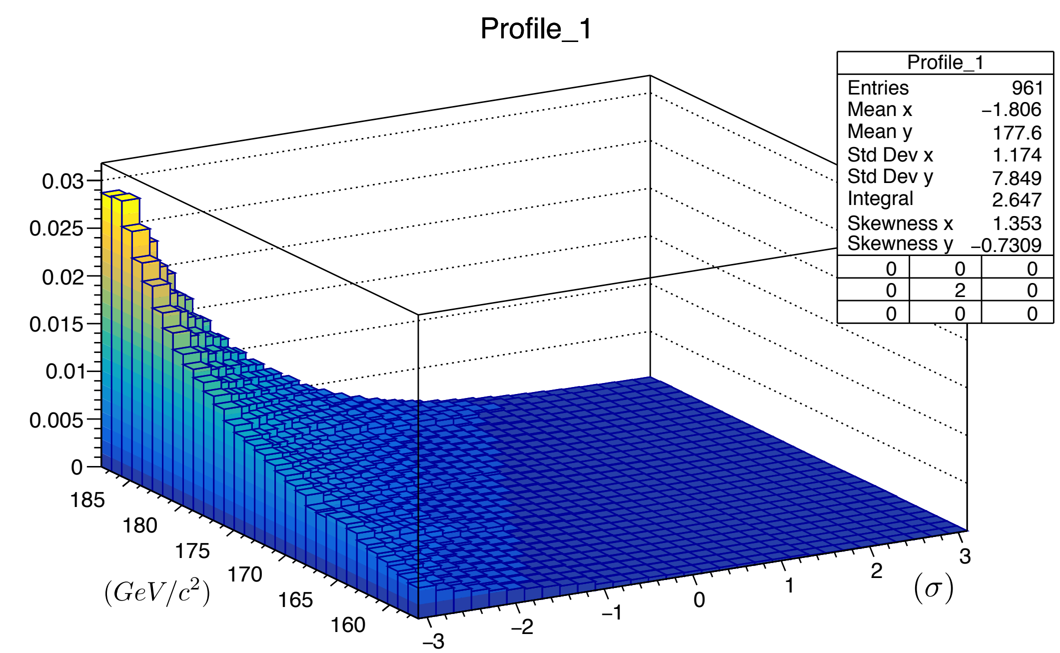
$i, j = (\Delta_{JES}, M_t)\ bins$

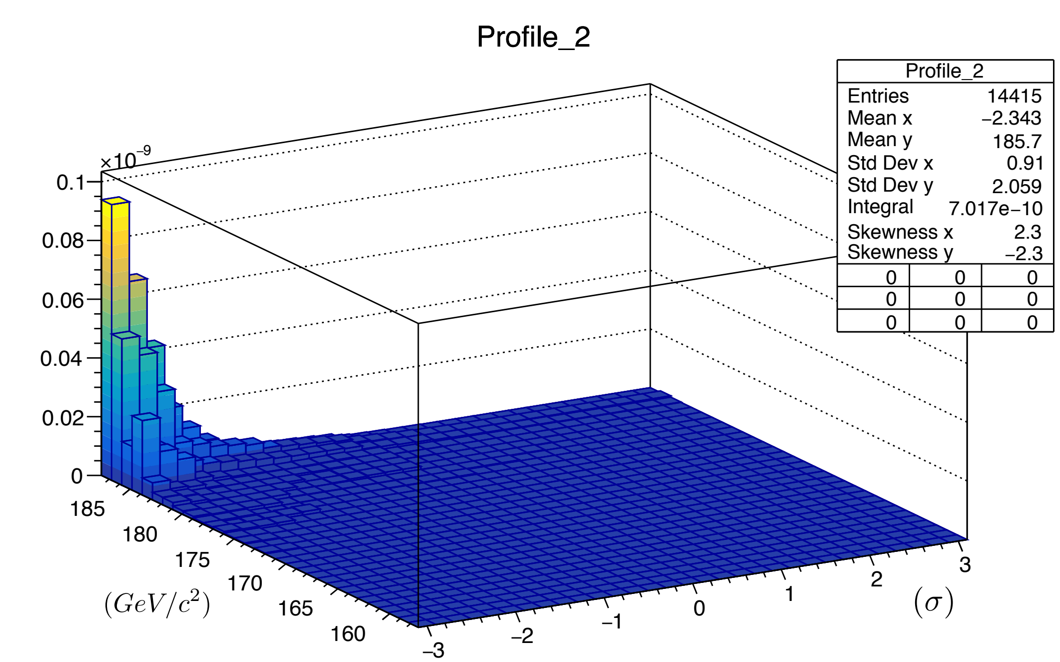
$l = integration$

Validation of the integration method : Pull Distribution pMC

- Problem in the pull distribution: 2 events gave a spike





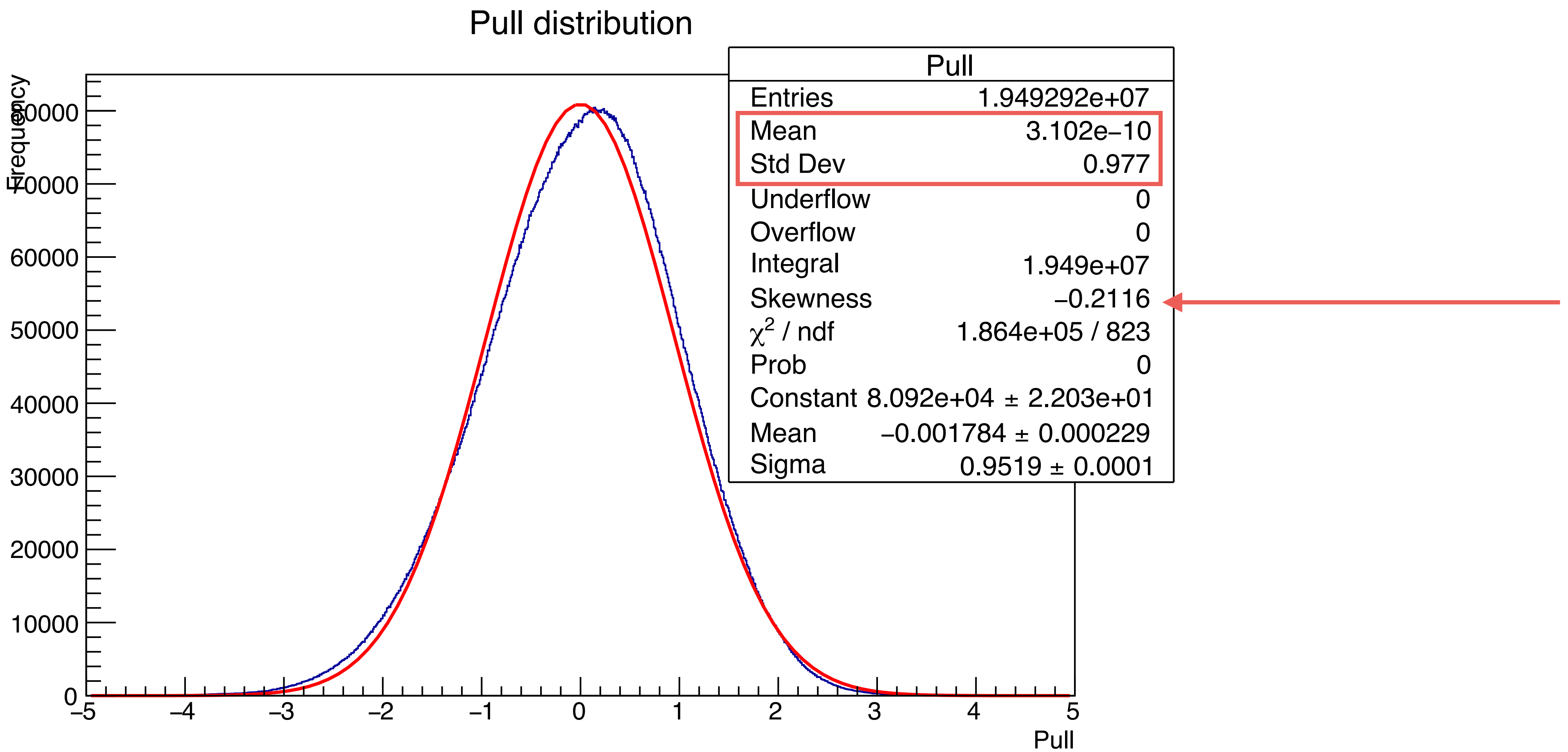


$$\delta_{k,ij,l}\ \forall(k,i,j,l)\ (l \in [1,22])$$

Validation of the integration method : Pull Distribution pMC

- 22 integrations with random seed of ~1000 events

(MC, $m_t = 173\ GeV/c^2$, $\Delta_{JES} = 0\ \sigma$, 1TagT)



$$\delta_{k,ij,l}\ \forall(k,i,j,l)\ \{l \in [1,22] \wedge k \neq (58,330)\}$$

Validation of the integration method : Pull Distribution qMC

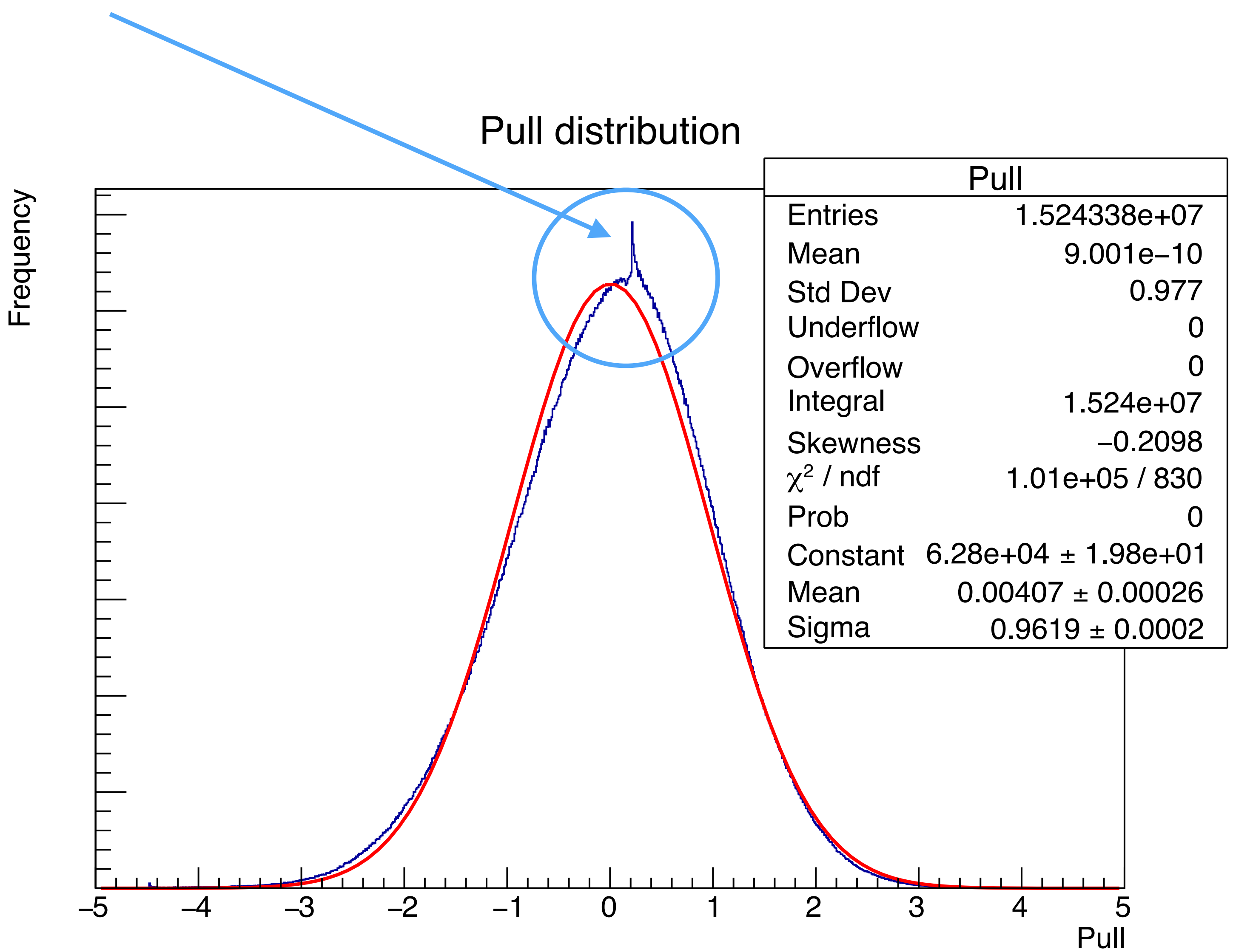
- qMC integration: sobol sequence of points instead of random sequence;
- Sobol sequence: LDS (Low-discrepancy sequence)

  (uniformly spread across the integration domain)
- Expected relative error: $\Delta I/I \sim \mathcal{O}(N^{-1+\varepsilon})$
- Introduction of random scrambling:

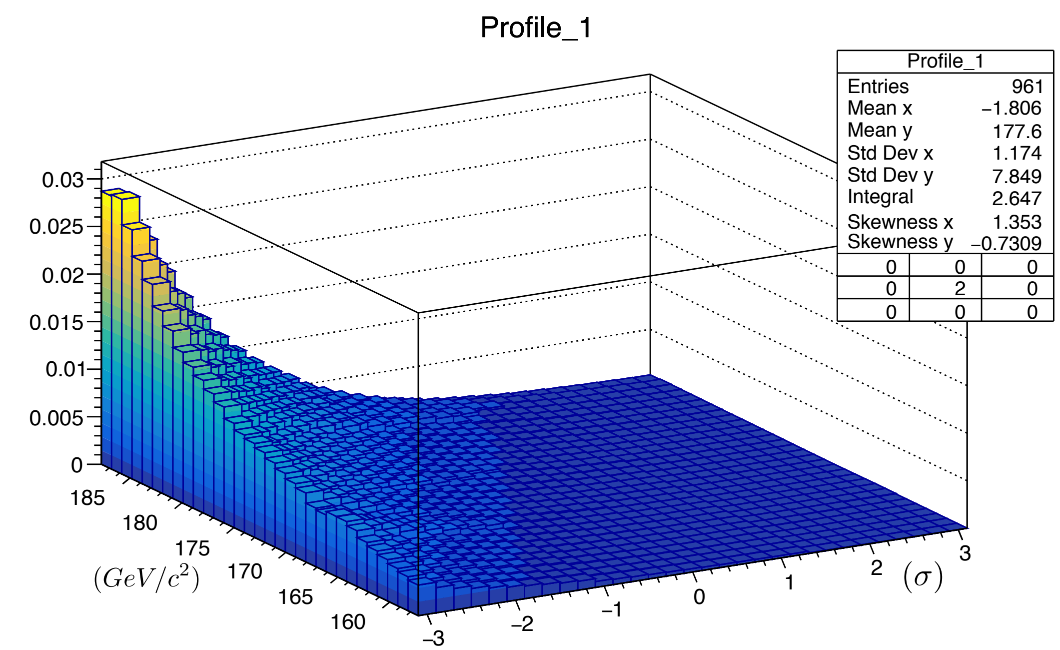
  Owen + Faure-Tezuka: random scrambling preserving LDS
- Importance sapling embedded in the code;
- Expected faster convergence of the integral.

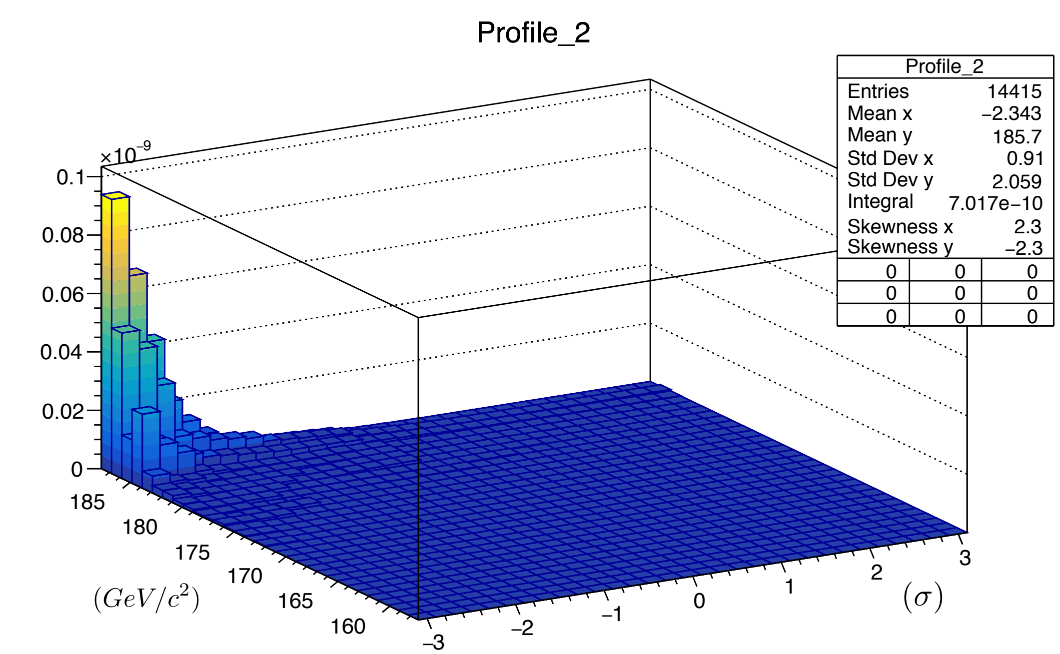
Validation of the integration method : Pull Distribution qMC

- Problem in the pull distribution: the same problematic events.



$$\delta_{k,ij,l} \ \forall(k,i,j,l) \ (l \in [1,22])$$

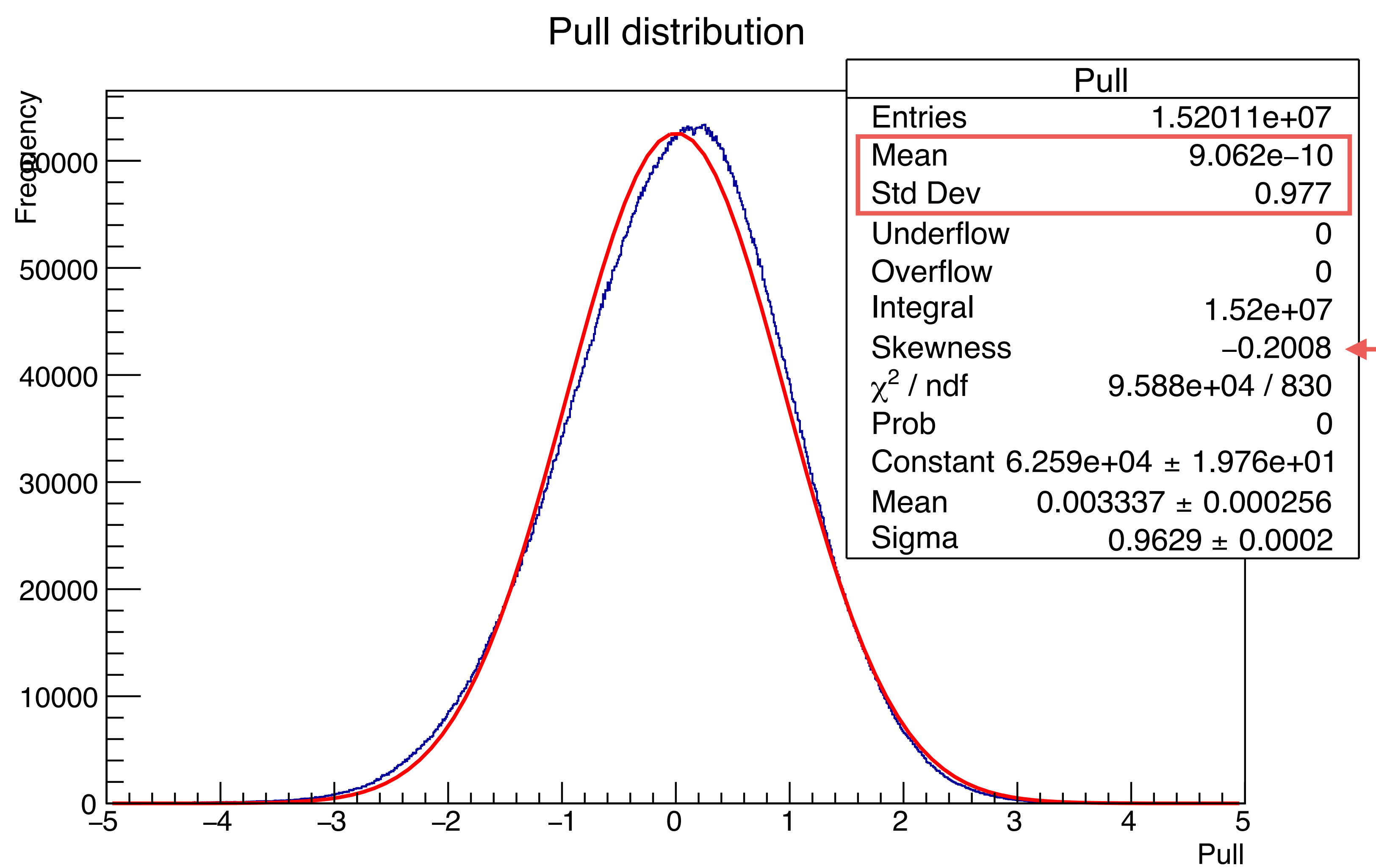




Validation of the integration method : Pull Distribution qMC

- 22 integrations with random seed of ~1000 events

(MC, $m_t = 173\ GeV/c^2$, $\Delta_{JES} = 0\ \sigma$, 1TagT)



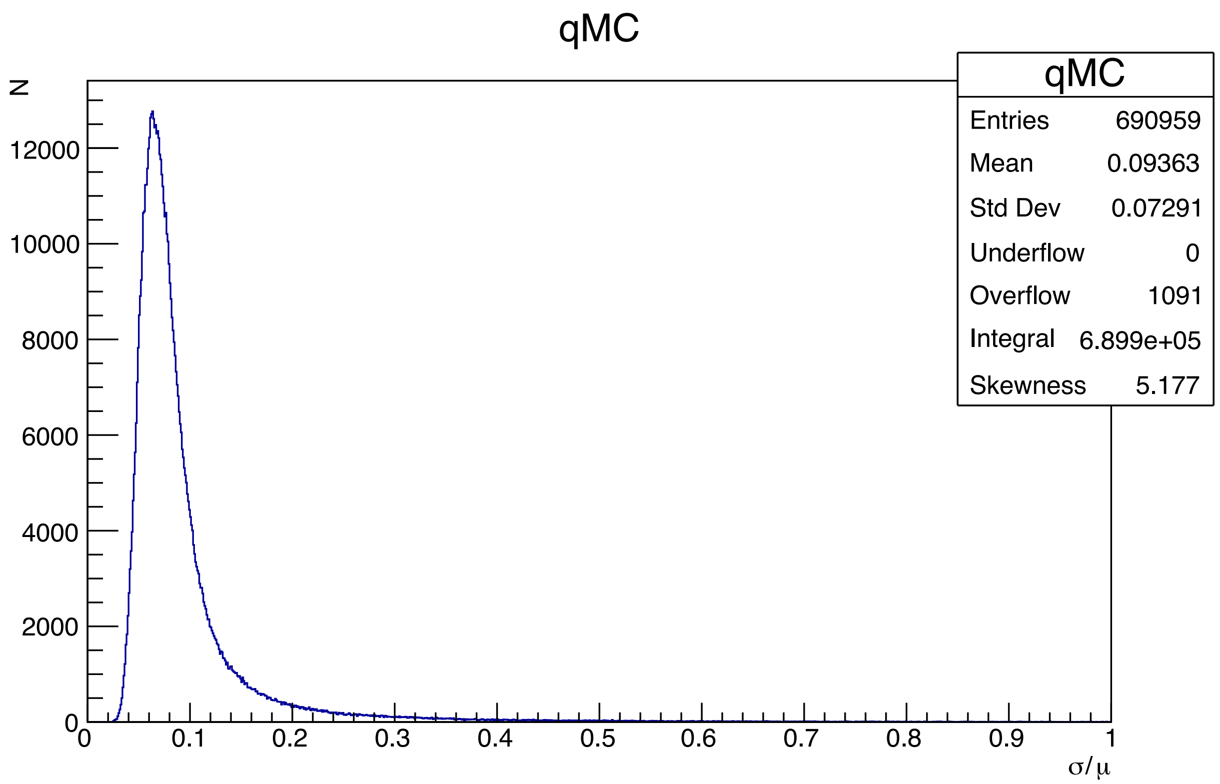
$$\delta_{k,ij,l}\ \forall(k,i,j,l)\ \{l \in [1,22] \wedge k \neq (58,330)\}$$

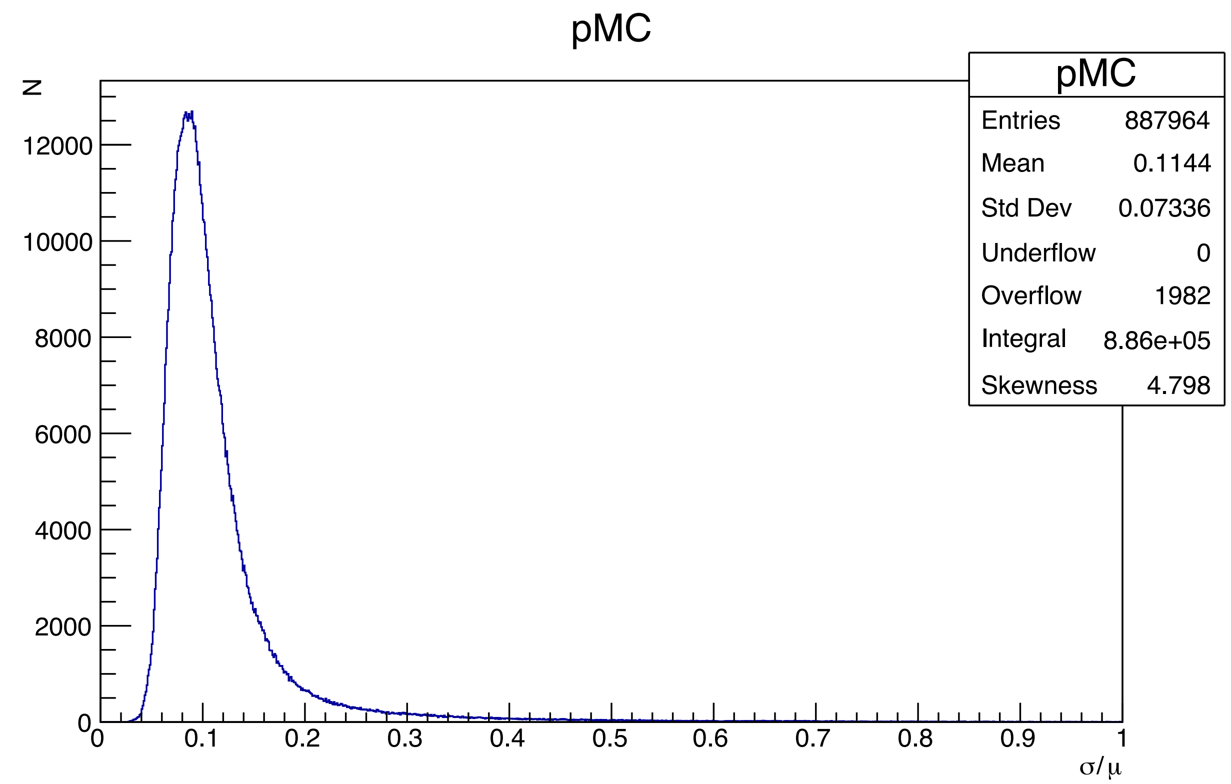
q-MC vs p-MC: estimation of precision

Given same termination parameters: (time, max n points, precision)

Compare the histograms of:

$$r_{k,ij} = \frac{\sigma_{k,ij}}{< W_{k,ij} >} \quad \forall \; k, i, j$$





$$< r_{k,ij} >= 0.094$$

$$< r_{k,ij} >= 0.11$$

More quantitative testing needed: time and convergence.

New Transfer Functions

- Transfer Functions. Probability density $T(\mathbf{x}|\mathbf{y}, \Delta_{JES})$ relating:

measured quantities $\eta_{jet}, \phi_{jet}, p_{t,jet}$ (observed in detectors)

parton-level quantities $\eta_{part}, \phi_{part}, p_{t,part}$ (used for ME calculation)

- TF can be factorised in:

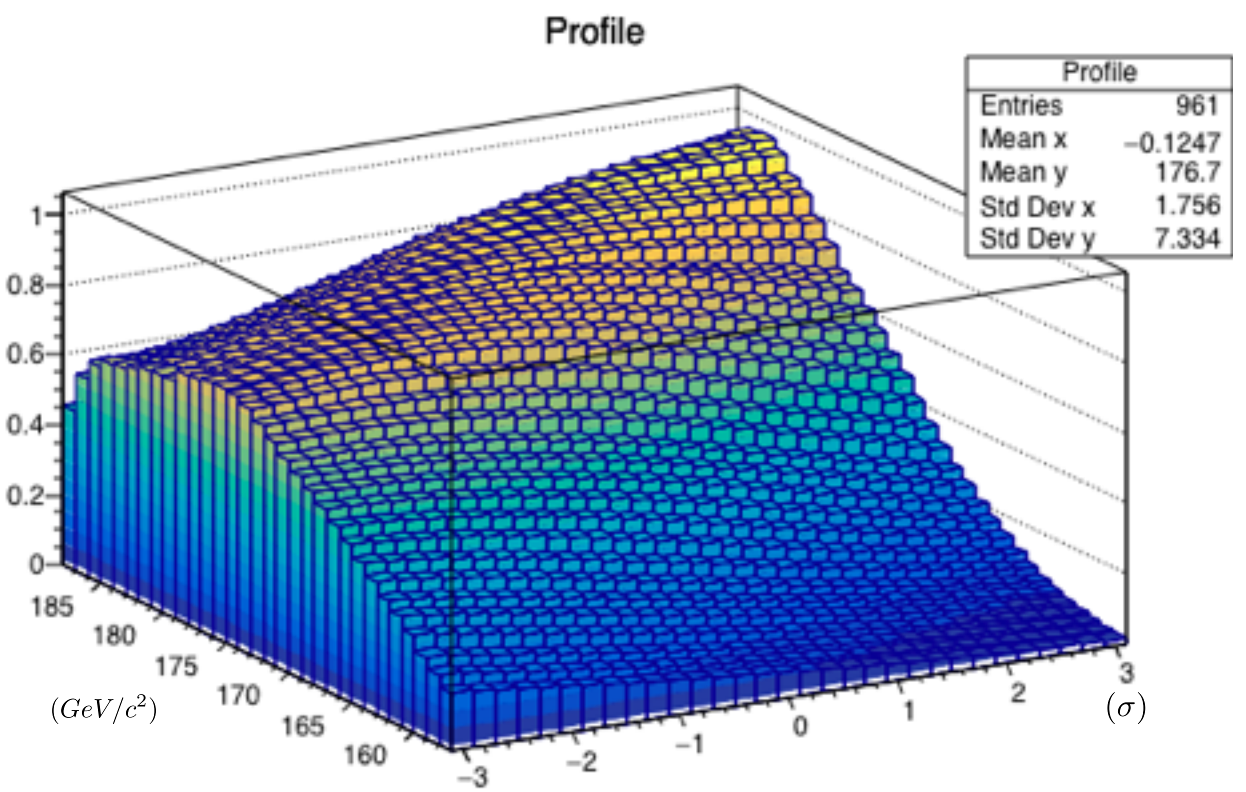
$$T(\mathbf{x}|\mathbf{y}, \Delta_{JES}) = T_a(\eta_{jet}, \phi_{jet}|\eta_{part}, \phi_{part})T_m(p_{t,jet}|p_{t,part})$$

- New TF derived from MC simulations including loose categories ($E_t > 12GeV$):
  - 1TagL
  - 2TagL

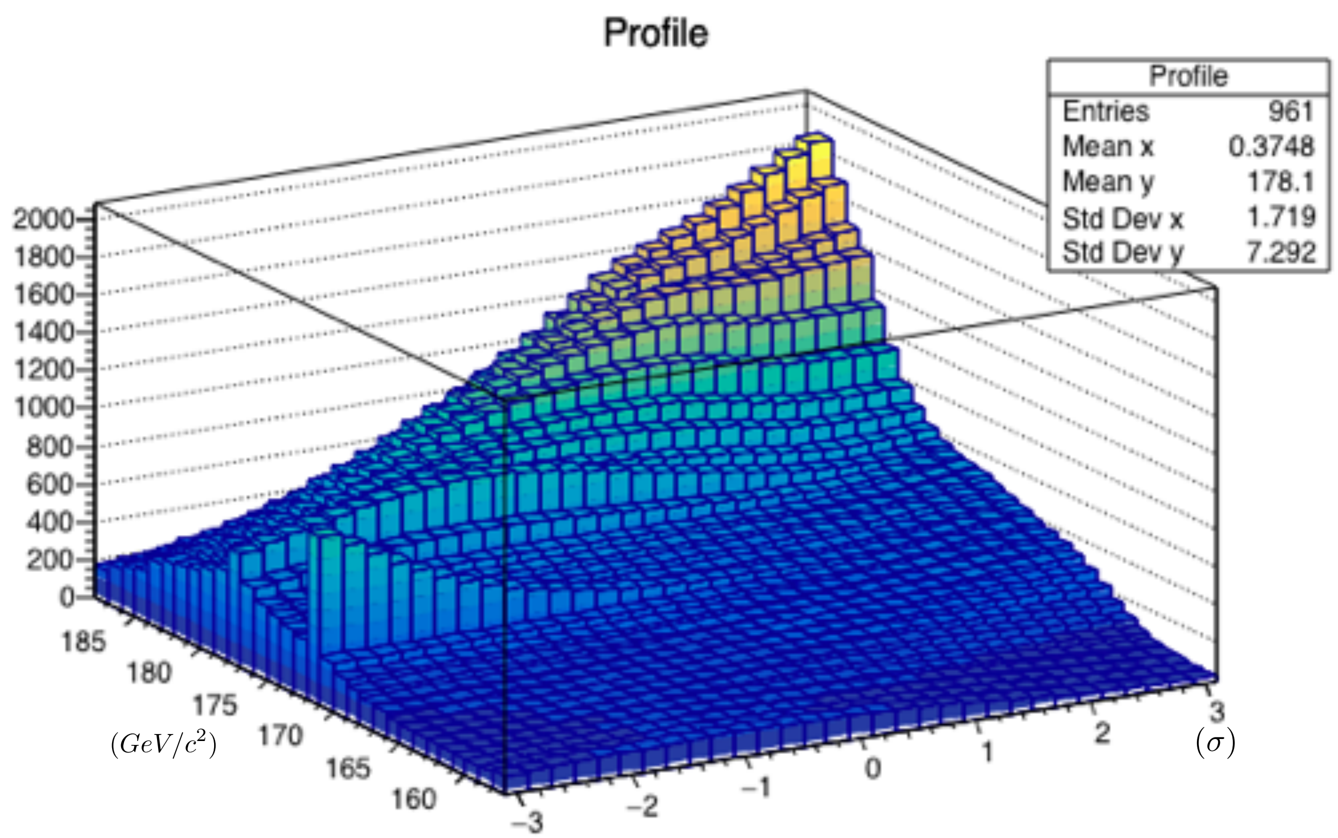
- For event with only tight categories old/new should produce the same results

Comparison between New/Old TF: Single event

MC Generated: PYTHIA, $M_t = 170 GeV/c^2$, $\Delta_{JES} = 0\sigma$, 0Tag category.



Old TF

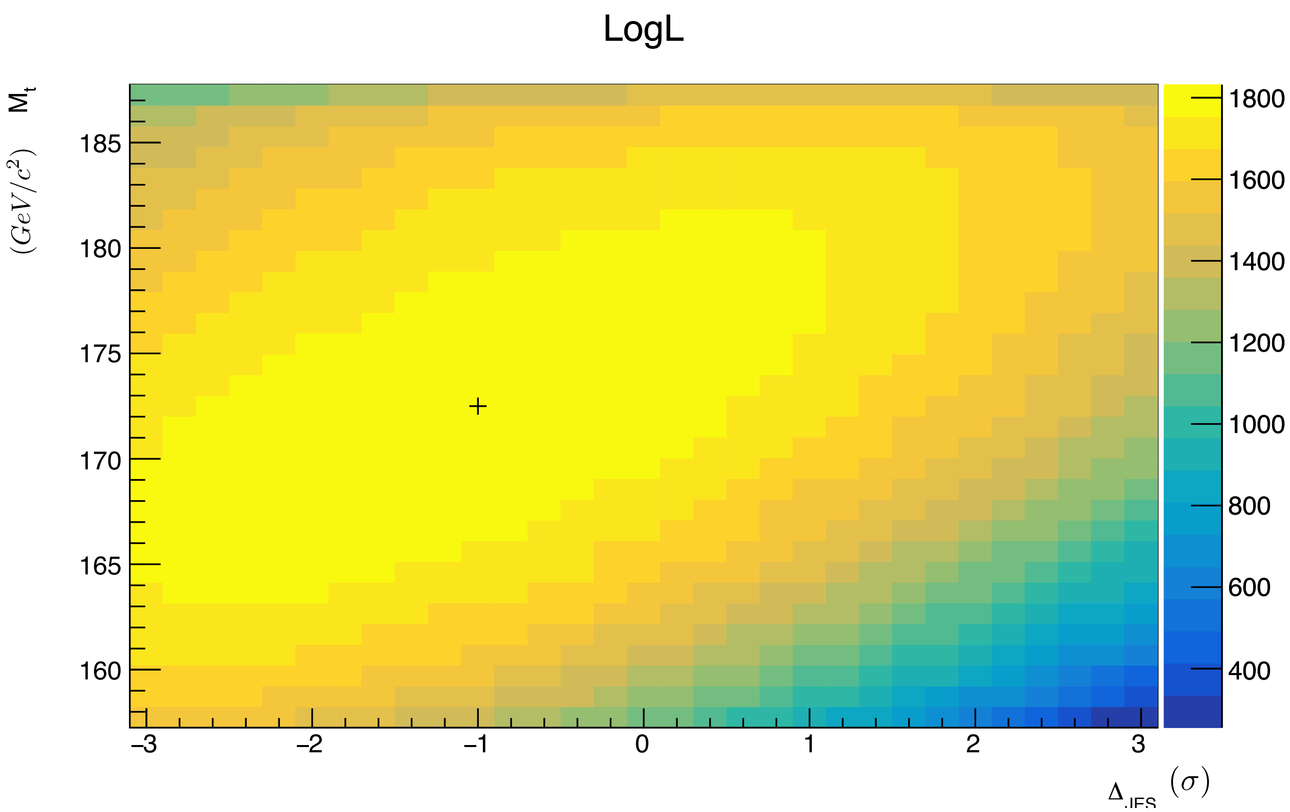


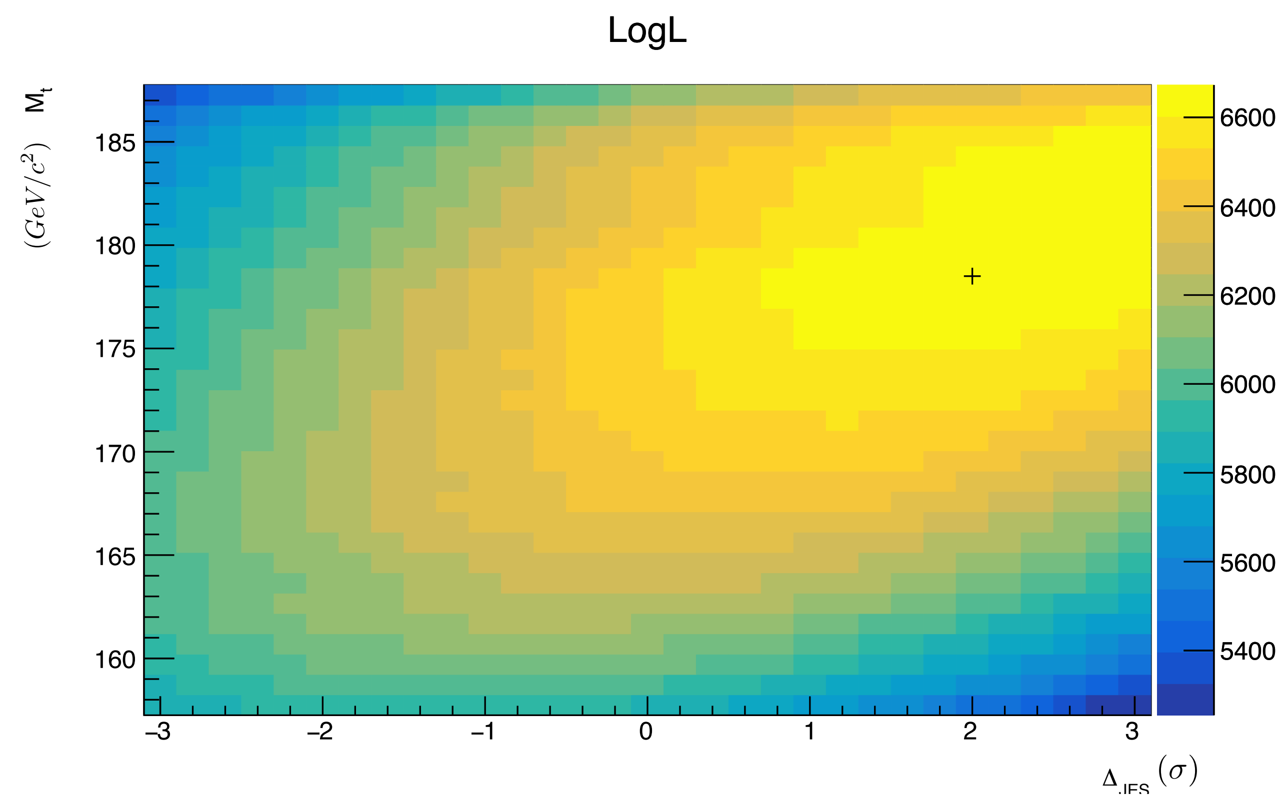
New TF

Event	Ntight	NLoose	nbtags	missingEt	ev.lpt	jet1pt	jet2pt	jet3pt	jet4pt
7	4	0	0	49.4882	52.4504	64.0641	70.1314	40.4148	28.5606

Comparison between New/Old TF

1000 ev MC Generated: PYTHIA, $M_t = 170 GeV/c^2$, $\Delta_{JES} = 0\sigma$,1TagT.





Sample	
Top Mass	$173 GeV/c^2$
Δ_{JES}	$0\ \sigma$
Results	
$M_t(2DPeak)$	$172.5 GeV$
$\Delta_{JES}(2DPeak)$	-1σ

Sample	
Top Mass	$173 GeV/c^2$
Δ_{JES}	$0\ \sigma$
Results	
$M_t(2DPeak)$	$178.5 GeV$
$\Delta_{JES}(2DPeak)$	2σ

Fermilab

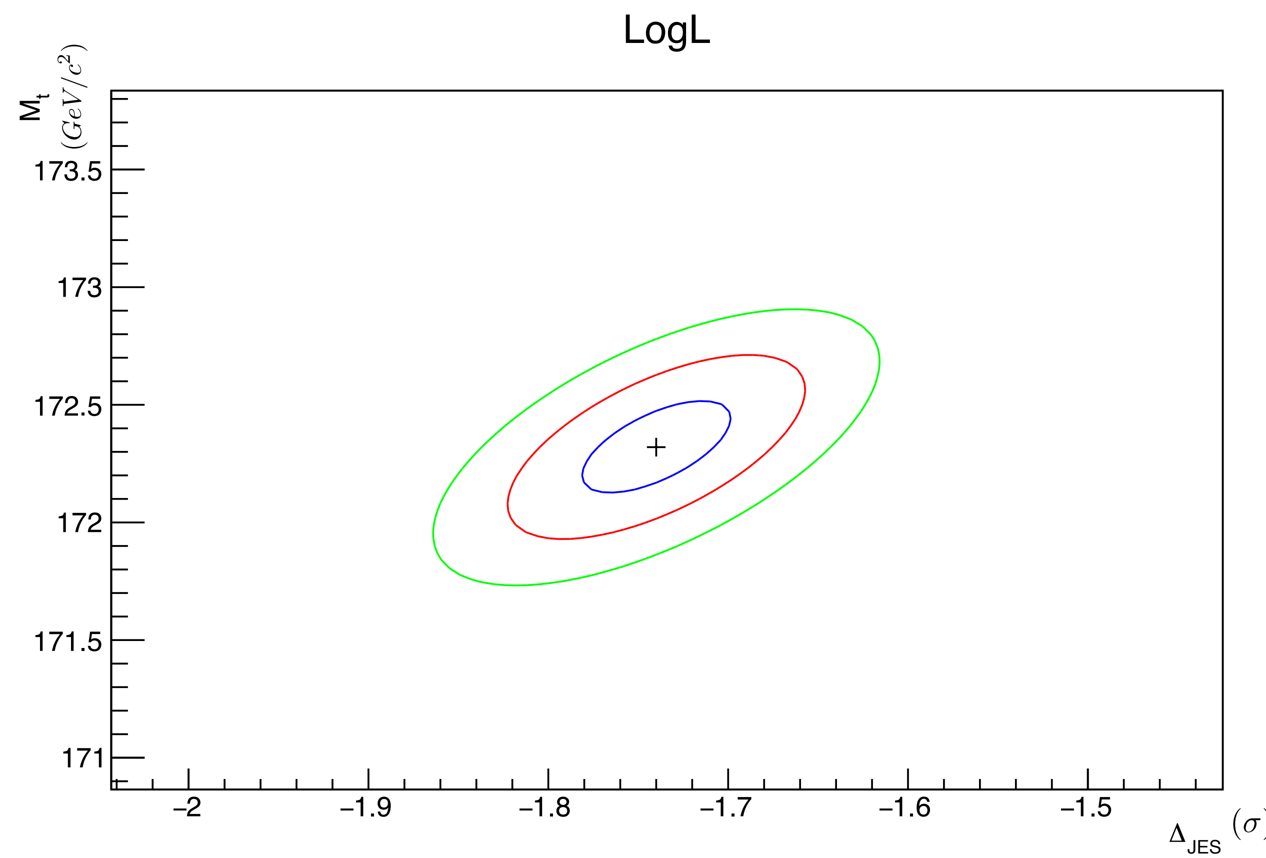
Study of the sensitivity on Djes

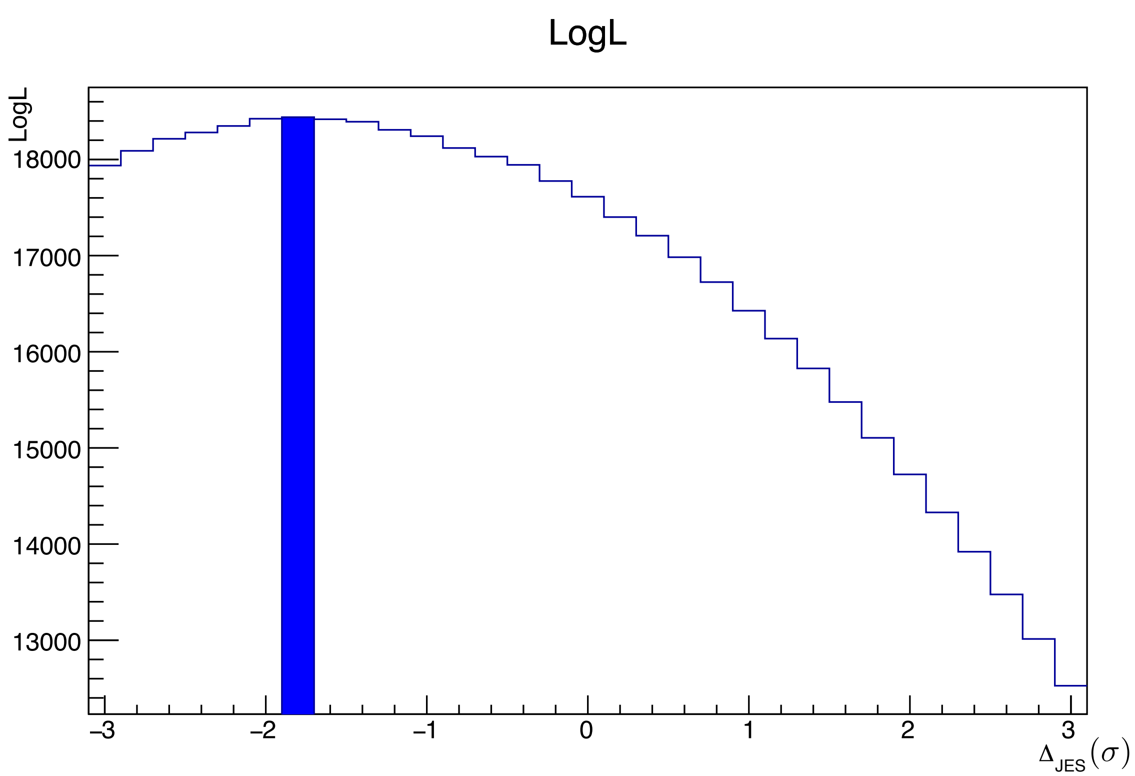
- Analysis of 3 MC samples with different parameters:
  - signal events, $M_t = 172.5 GeV/c^2$, $\Delta_{JES,MC} = \{-1, 0, +1\}$, 1TagT & 2TagT categories;
  - 10000 events for every sample.
- Calculate total $log(L)$: $$log(L)_{ij} = \sum_{k=1}^{10000} ln\left(W_{k,ij}\right) \; ;$$
- Create 1D histogram of profiled likelihood: $logL(M_t)$, $logL(\Delta_{JES})$;
  (using the profiled likelihood method)
- Extract M_t , Δ_{JES} (assuming gaussian behaviour of the likelihood in the limit of large statistics);
- Plot the dependency M_t , Δ_{JES} to the input $\Delta_{JES,MC}$;
- Expected linear dependency to be corrected with calibration.

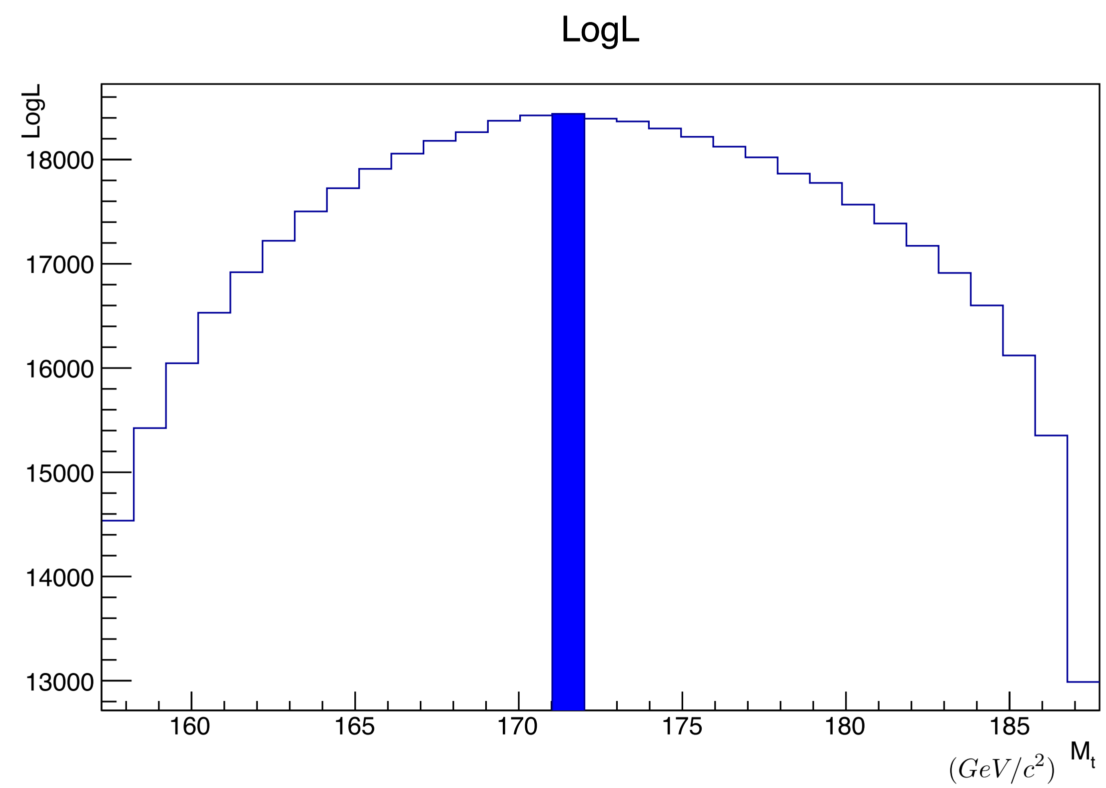
Study of the sensitivity on Djes

$$\Delta_{JES,MC} = -1\sigma$$

Sample	ttop25_mc_jes-1_btag_jpt20_.hs
$M_{t,MC}$	172.5 GeV/c^2
$\Delta_{JES,MC}$	-1 σ
	Results
$M_t(fit)$	$172.3 \pm 0.2\ GeV/c^2$
$\Delta_{JES}(fit)$	$-1.74 \pm 0.04\ \sigma$



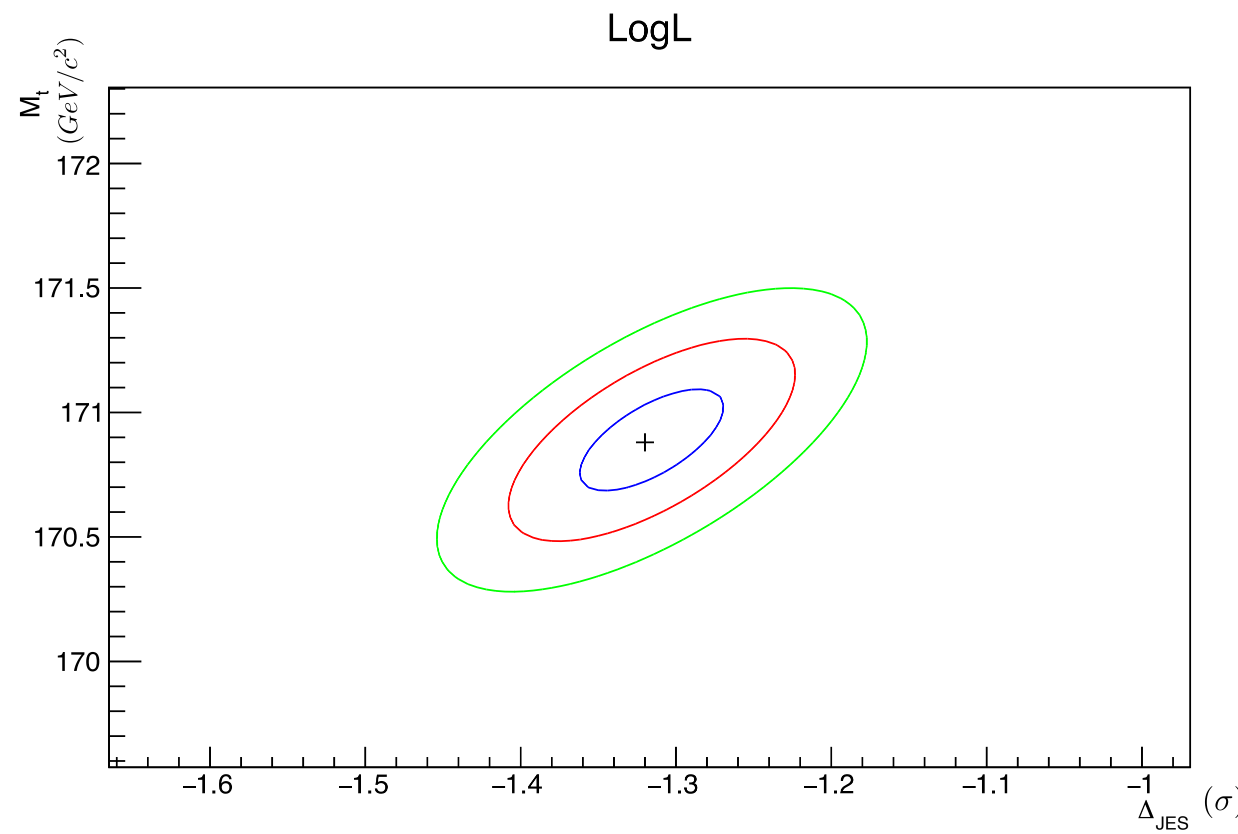


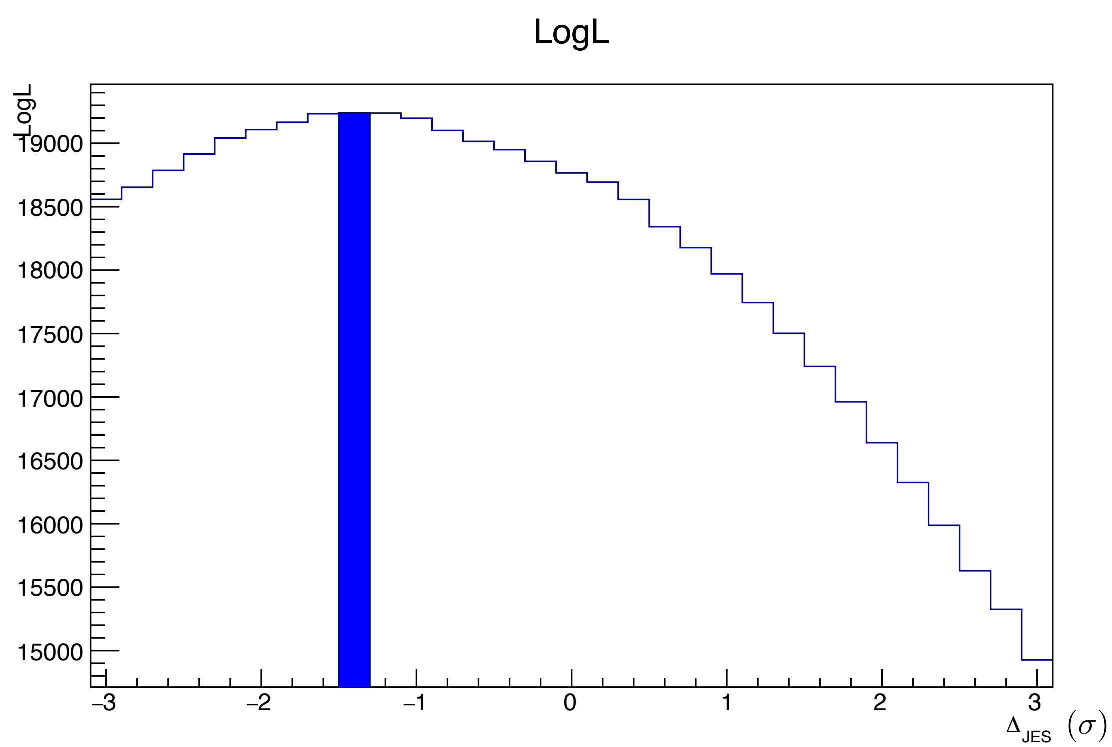


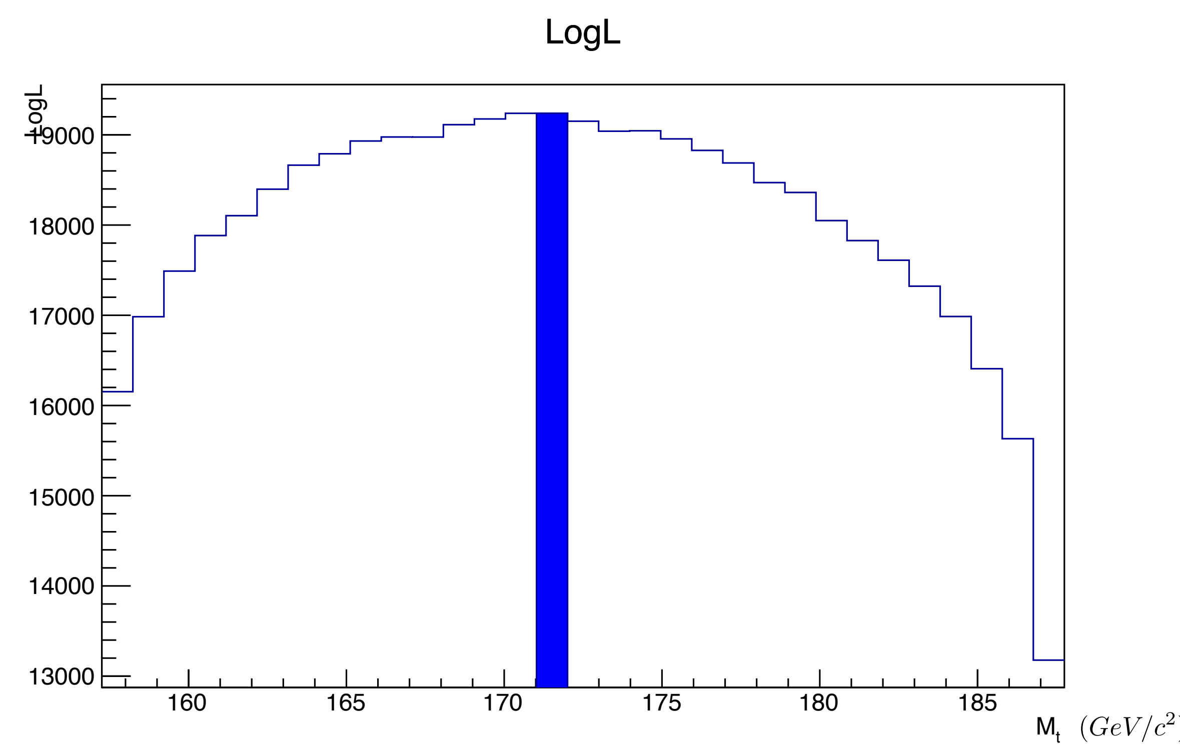
Study of the sensitivity on Djes

$\Delta_{JES,MC} = 0\sigma$

Sample	ttop25_mc_jes0_btag_jpt20_.hs
$M_{t,MC}$	$172.5 GeV/c^2$
$\Delta_{JES,MC}$	$0\ \sigma$
Results	
$M_t(fit)$	$170.9 \pm 0.2\ GeV/c^2$
$\Delta_{JES}(fit)$	$-1.32 \pm 0.05\ \sigma$



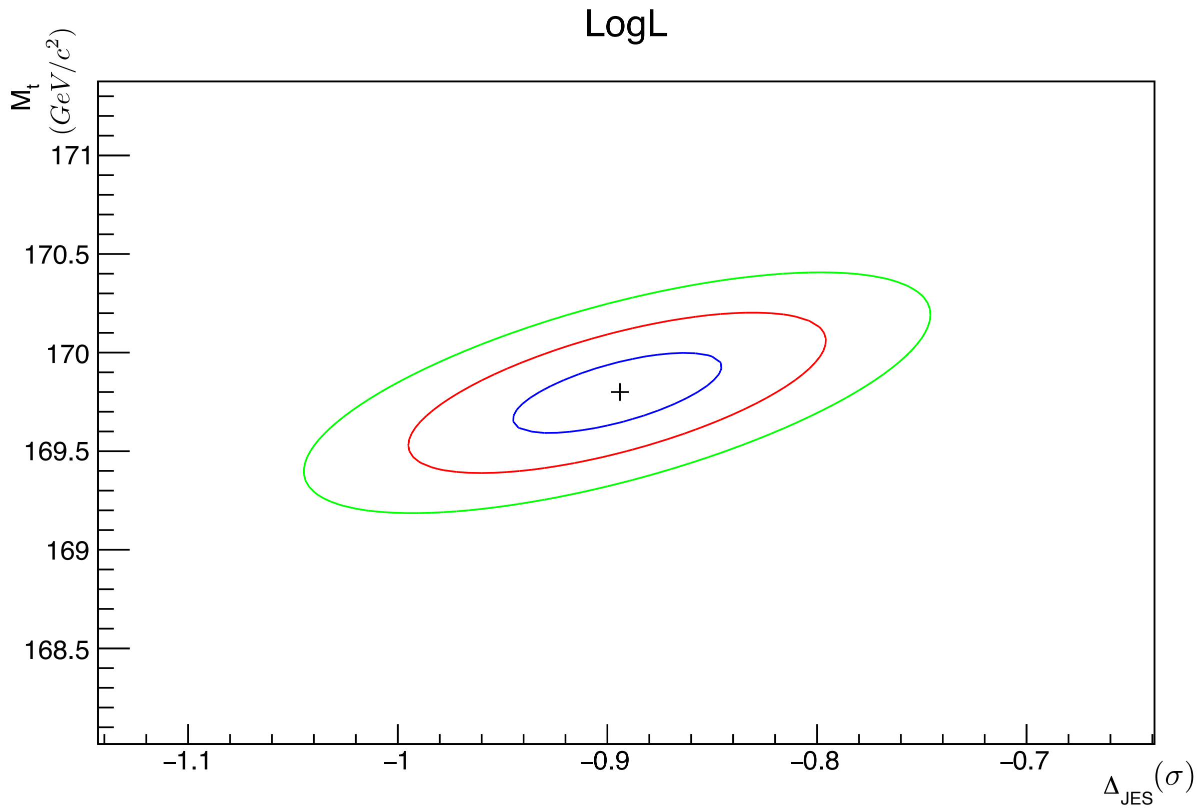


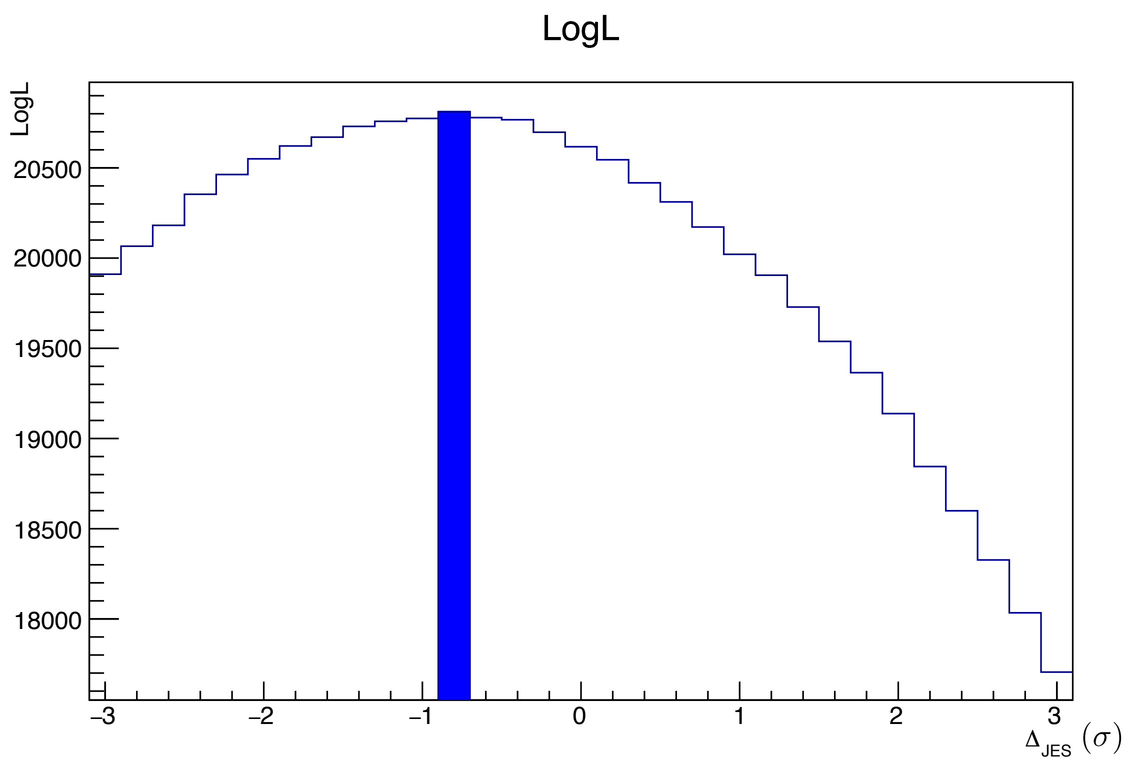


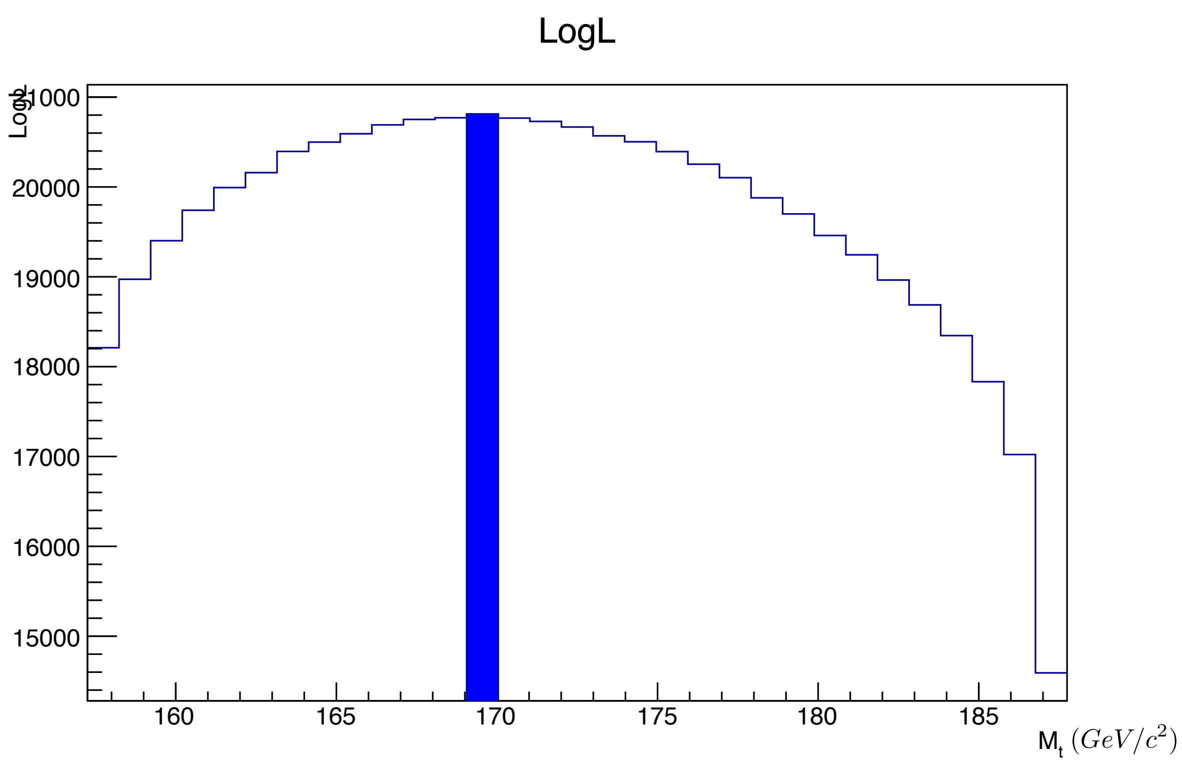
Study of the sensitivity on Djes

$$\Delta_{JES,MC} = 1\sigma$$

Sample	ttop25_mc_jes+1_btag_jpt20_.hs
$M_{t,MC}$	$172.5 GeV/c^2$
$\Delta_{JES,MC}$	$1\ \sigma$
Results	
$M_t(fit)$	$169.8 \pm 0.2\ GeV/c^2$
$\Delta_{JES}(fit)$	$-0.90 \pm 0.05\ \sigma$

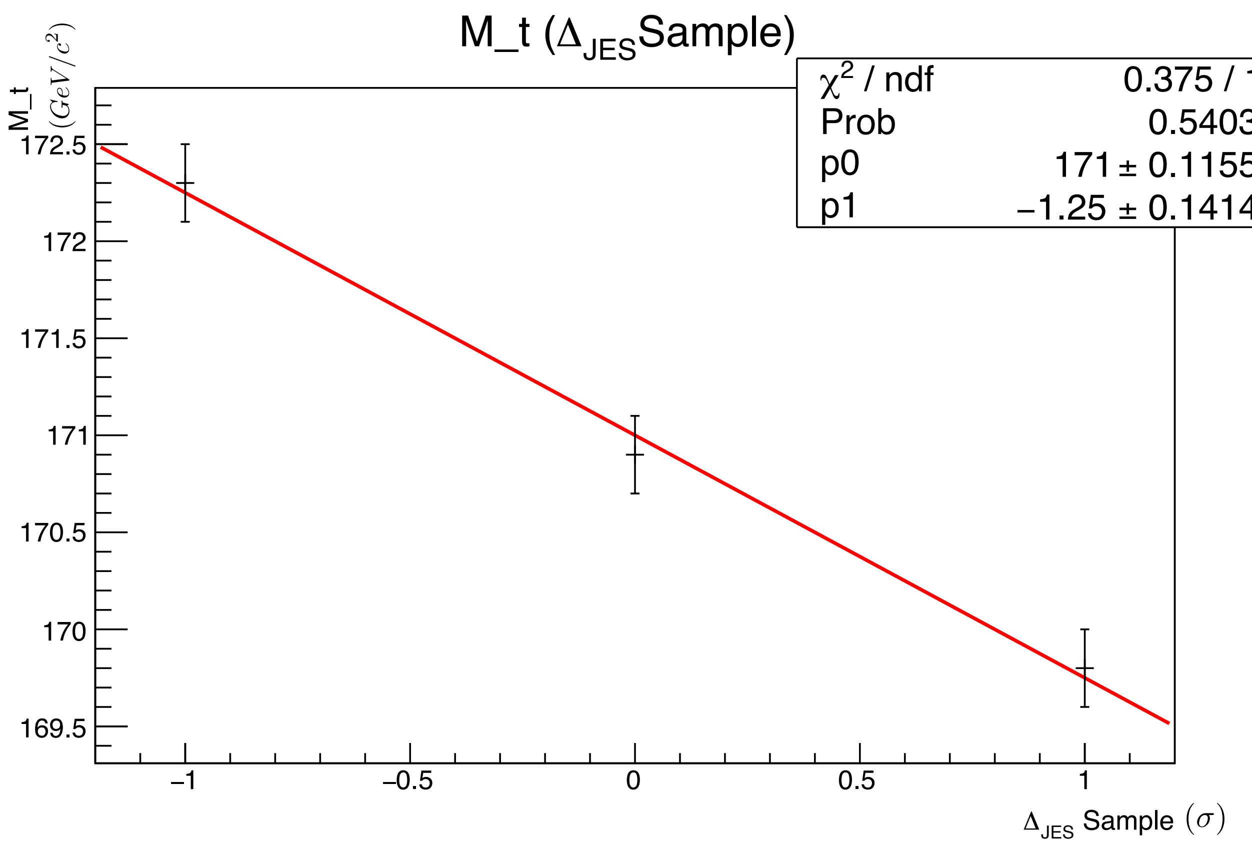


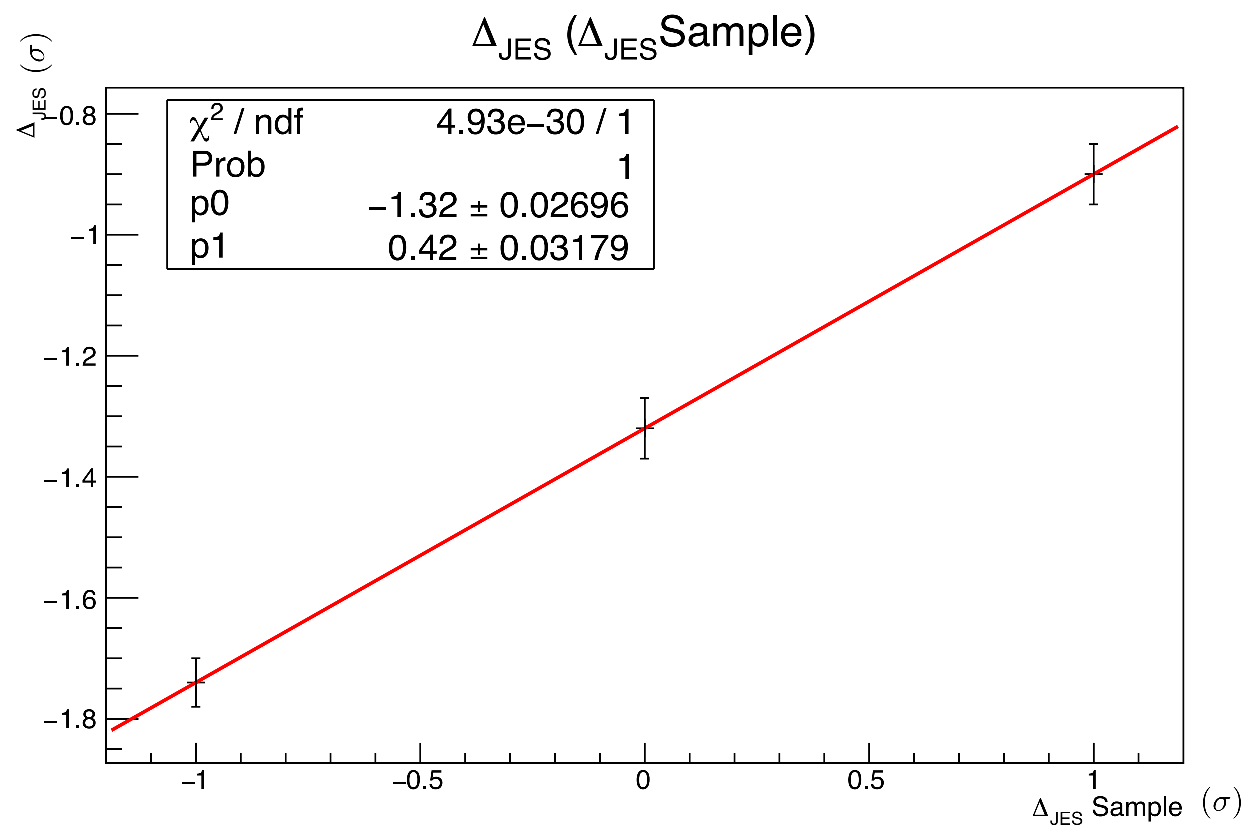




Study of the sensitivity on Djes

$\Delta_{JES,MC}$	-1σ	0σ	1σ
$M_t(fit)$	$172.3 \pm 0.2\ GeV/c^2$	$170.9 \pm 0.2\ GeV/c^2$	$169.8 \pm 0.2\ GeV/c^2$
$\Delta_{JES}(fit)$	$-1.74 \pm 0.04\ \sigma$	$-1.32 \pm 0.05\ \sigma$	$-0.90 \pm 0.05\ \sigma$





Linear dependence: expected to be corrected with calibration!

Future development of the analysis

- Further understand of qMC integration and implement error estimation;
- Include total cross section and acceptance as normalisation of the weights;
- Study the sensitivity on Δ_{JES} and M_t with the complete weight definition;
- Debug the new TF;
- Test the methods for loose categories;
- Combine signal and background likelihood;
- Final calibration (pseudo-experiments);

Backup Slides

Matrix Element Method: Motivation

- Provides superior Statistical sensitivity in the extraction of SM parameters;

- Completeness of information exploited in each event:
  - The superior sensitivity is achieved by taking into account the full topological and kinematic information in a given event;

- Can be used to determine several parameters:
  - Theoretical parameters describing the physics of the processes measured;
  - Experimental parameters: describing the detector response;

- Theoretical assumption about the process under study (PDF, ME, TF) are used in the most efficient manner:
  - In the limit which all the event probabilities are known, by the Neyman-Pearson Lemma, the likelihood is an optimal test statistic.

Validation of the integration method : Pull Distribution qMC

- qMC integration: sobol sequence of points instead of random sequence;

- Sobol sequence: LDS (Low-discrepancy sequence)

$$D_N^*(P_N) = Sup_{a \in \mathcal{A}} \left| \frac{\#\{P_N \in A\}}{N} - \lambda(A) \right| \;,\quad \mathcal{A} = \prod_{i=1}^{d} [0, b_i) \; \forall b_i \;\; s.t. \; 0 \leq b_i < 1$$

- Koksma–Hlawka inequality:

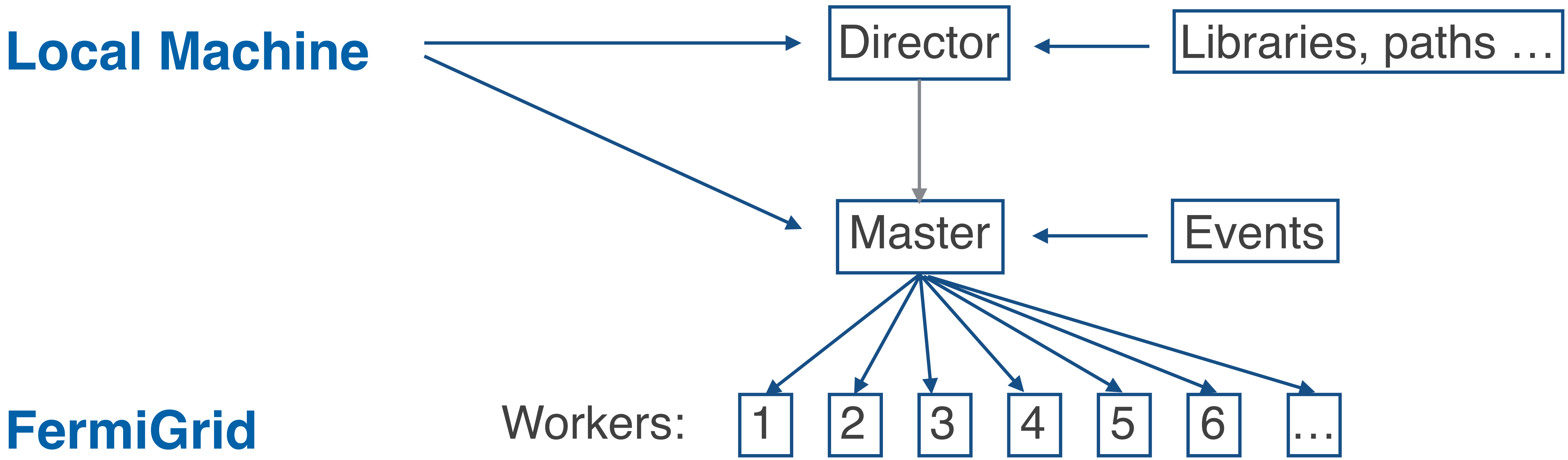
$$\left| \frac{1}{N} \sum_{i=1}^{N} f(x_i) - \int_{I^s} f(u) du \right| \leq V(f) D_N^*(x_1, ..., x_n)$$

- Expected relative error: $\Delta I / I \sim \mathcal{O}(N^{-1+\varepsilon})$ (faster convergence)

- Owen + Faure-Tezuka scrambling preserving LDS

- Importance sapling embedded in the code.

Integration Framework: Fermigrid



- Program installation;
- Learn job submission procedure;
- Learn program errors handling;

- Learn to analyse data format consistent with ME analysis code

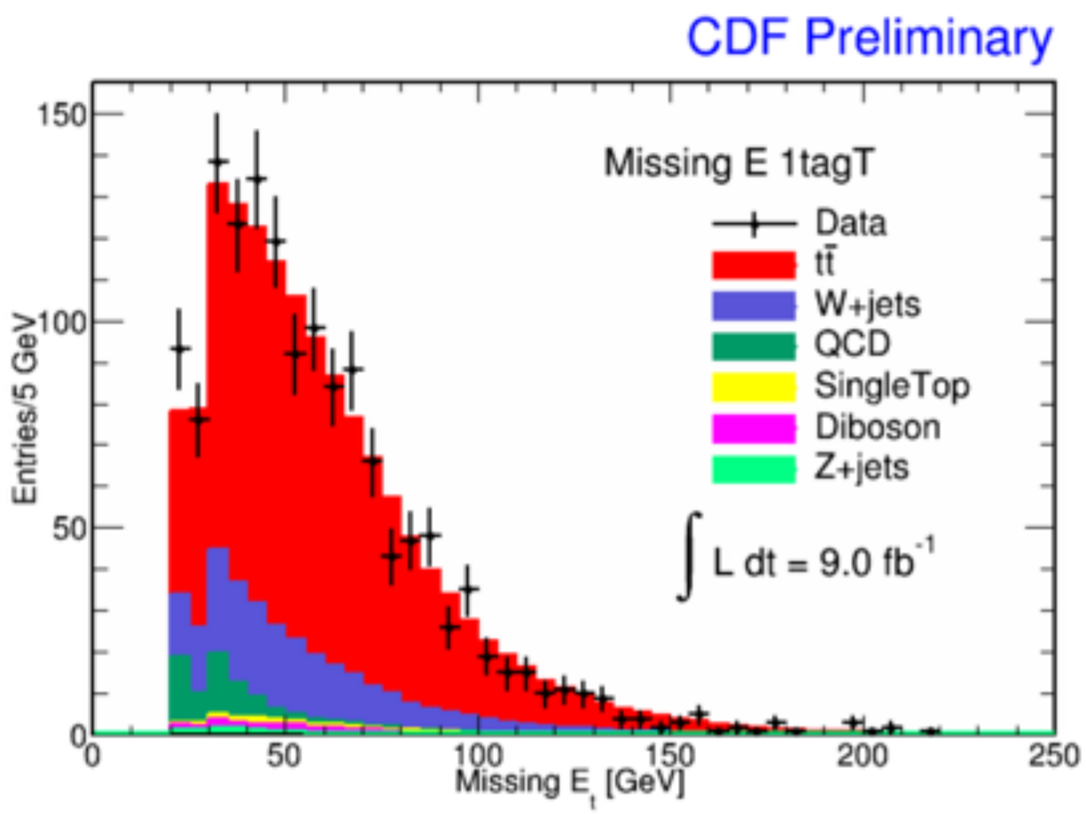
Signal and Background : Monte Carlo Samples / Validation

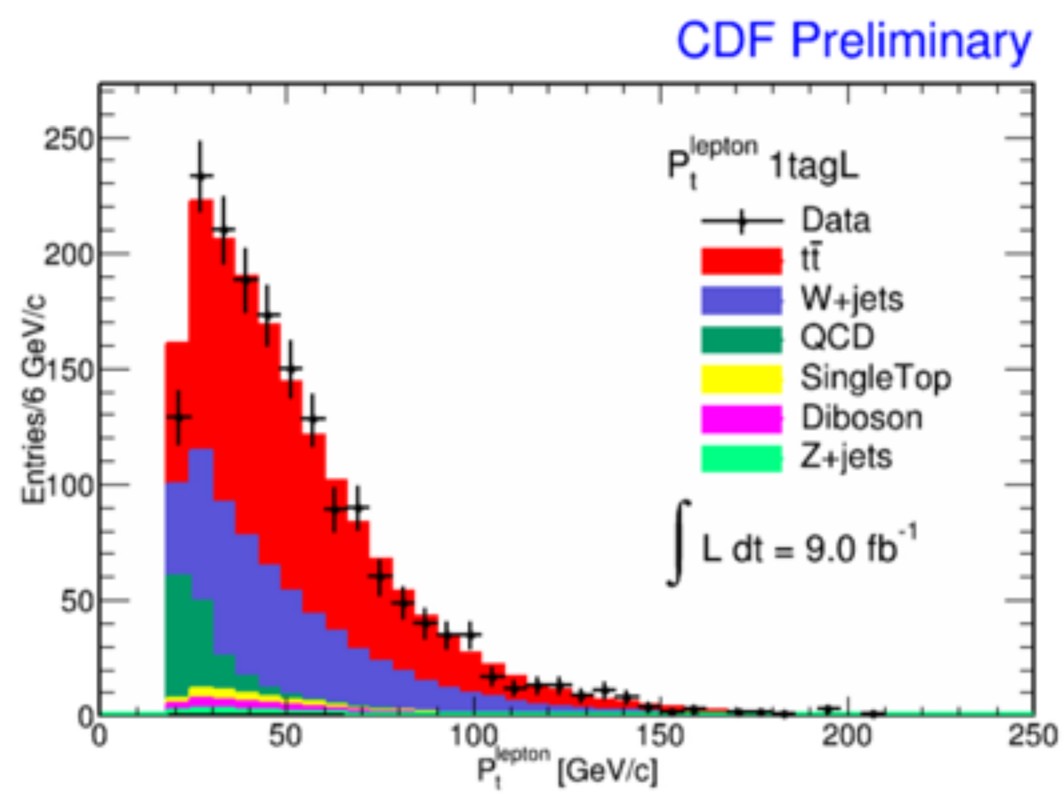
- Signal:
  - $t\bar{t}$ signal: Powheg + Phytia S.Frixione *et al.,* JHEP07 (2007)

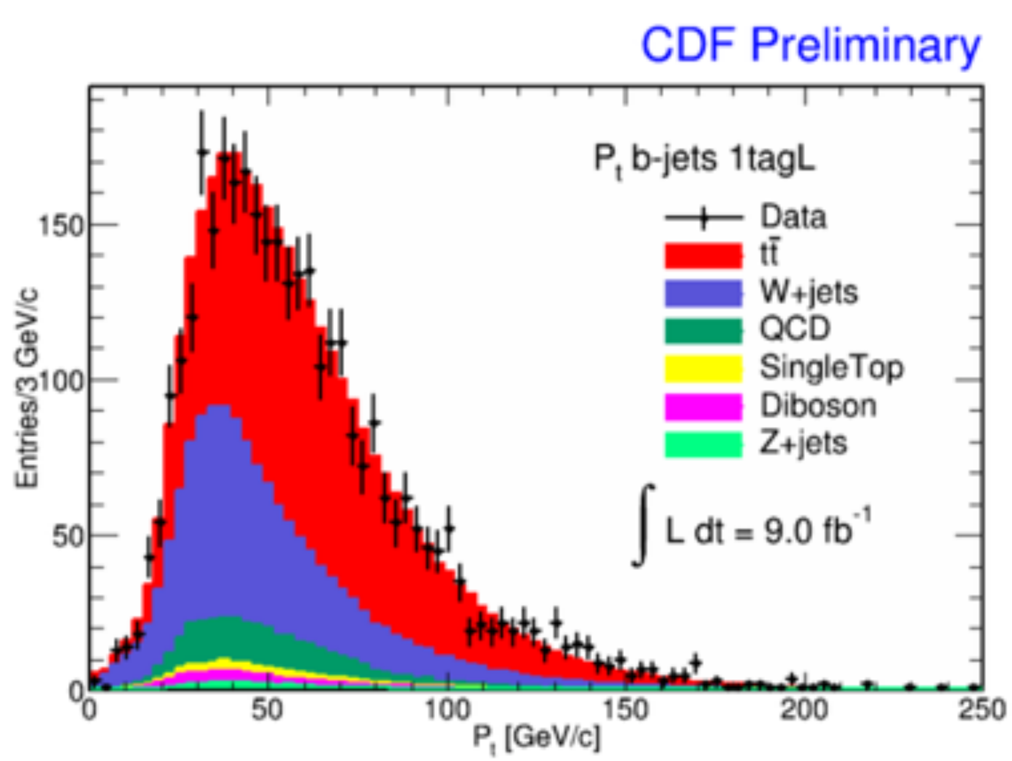
- Background:
  - W/Z + jets: Alpgen+Pythia M.L. Mangano *et al.* JHEP0307:001 (2003)
  - Diboson: Pythia 6 T. Sjöstrand *et al*., JHEP06, 026 (2006)
  - Single top: Madgraph 4 + Pythia J. Alwall *et al*., JHEP09, 028 (2007)
  - QCD: Data with lepton failing one of the “good lepton” criteria

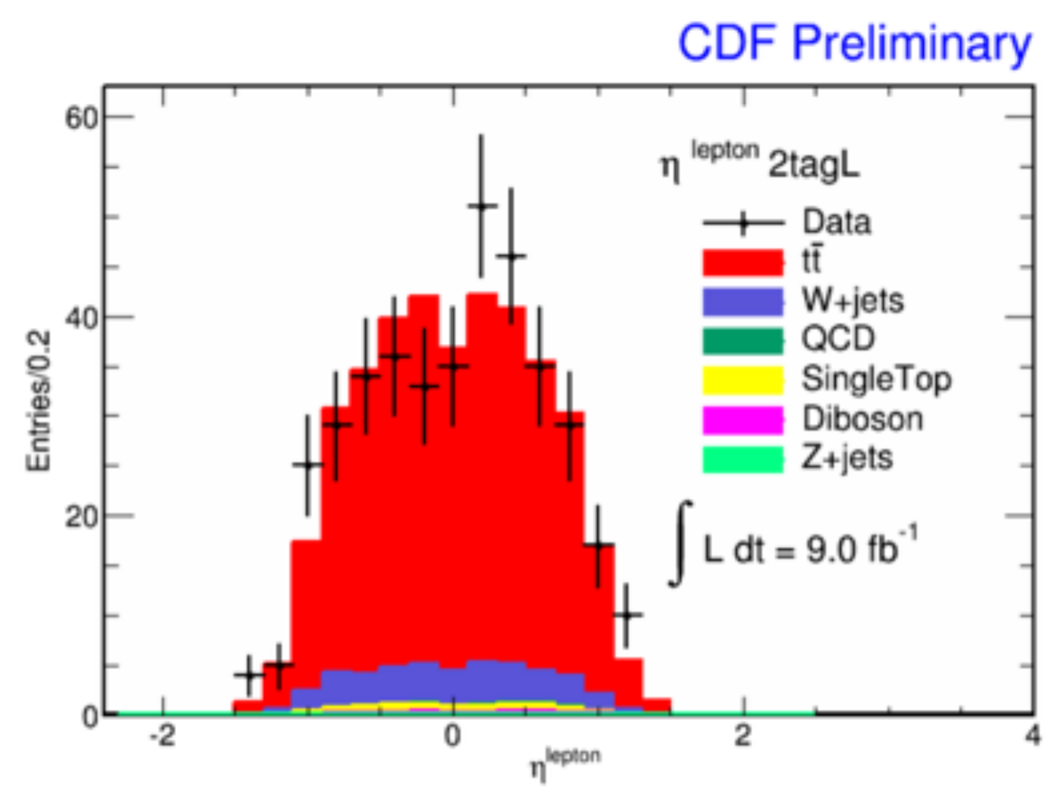
- Validation of Samples:
  - Validation plots : C. Tosciri-Laurea-University of Pisa
  - Compare quantity of interest between data and MC events of the samples

Signal and Background: MC samples - Validation Plots









Backup Slides: Systematic Uncertainties

List of systematic uncertainties on m_t.

Systematic Source	Uncertainty (GeV/c^2)
Calibration	0.10
Monte Carlo Generator	0.37
Initial State Radiation and Final State Radiation	0.15
Residual JES	0.49
b-JES	0.26
Lepton p_T	0.14
Multiple Hadron Interactions	0.10
PDFs	0.14
Background Modeling	0.33
Color Reconnection	0.37
Total	0.88

Remove overlap (Monte Carlo Generator, Residual JES)

Bkg in L_{ev} (Background Modeling)

New sgn MC (Color Reconnection)

Backup Slides: Expected and Observed Sample composition

Expected and Observed Sample composition

	0-tag	1-tagL	1-tagT	2-tagL	2-tagT
Wbbbar	125.6	177.1	82.2	27.3	17.0
Wccbar	384.8	112.9	52.5	4.1	2.6
Wc	186.7	66.8	25.9	2.3	1.3
W+light j	1580.9	170.7	77.2	2.9	1.9
Z+jets	169.4	25.2	13.7	2.0	1.3
single top	13.9	16.5	8.2	6.7	4.6
Diboson	166.3	31.0	17.9	2.7	1.8
QCD	623.2	119.9	60.3	60.3	6.3
Total Bkg	3250.8	720.1	337.9	49.0	36.8
ttbar	959.7	998.6	1086.3	331.3	425.5
Expected total	4210.5	1718.7	1424.2	380.3	462.3
Observed	4474	1711	1434	365	375

Backup slides: Selection requirements for event-category

Selection requirements for event-category

	0-tag	1-tagT	1-tagL	2-tagT	2-tagL
Lepton E_T [GeV]	> 20	> 20	> 20	> 20	> 20
Lepton $\|\eta\|$	$0-1$	$0-1$	$0-1$	$0-1$	$0-1$
$\not{E}_T$ [GeV]	20	20	20	20	20
Leading 3 jets E_T [GeV]	20	20	20	20	20
Leading 3 jets η	$0-2$	$0-2$	$0-2$	$0-2$	$0-2$
4^{th} jet E_T [GeV]	> 20	> 20	> 12	> 20	> 12
4^{th} jet η	$0-2$	$0-2$	$0-2.4$	$0-2$	$0-2.4$
Extra jets E_T [GeV]	< 20	Any loose	Any loose or ≥ 1 tight	Any loose	Any loose or ≥ 1 tight

Matrix Element Technique 2

- Determines $P_{ev}(\mathbf{y}|\mathbf{a})$ from:
  - Vector of observed data: $\mathbf{y} \longrightarrow$ event kinematics
  - Vector of model parameters: $\mathbf{a} \longrightarrow m_t$, Δ_{JES}

- Probability: Integration over all phase space $d\Phi(\mathbf{x})\ [t\bar{t} \rightarrow b(l\nu)b(qq')]$

  - 32 variables of integration:
    2 initial particles (8 var.)
    6 final particles (24 var.)

  → constraints →

  19 variables of integration:
  $M^2_{t,lep},\ M^2_{t,had},\ M^2_{W,lep},\ M^2_{W,had}$
  $\beta = \ln\frac{p_q}{p_{q'}},\ \vec{p_T}(t\bar{t}),\ m_{1...4},\ \eta_{1...4},\ \phi_{1...4}$

- Signal and Background → for different (m_t, Δ_{JES})

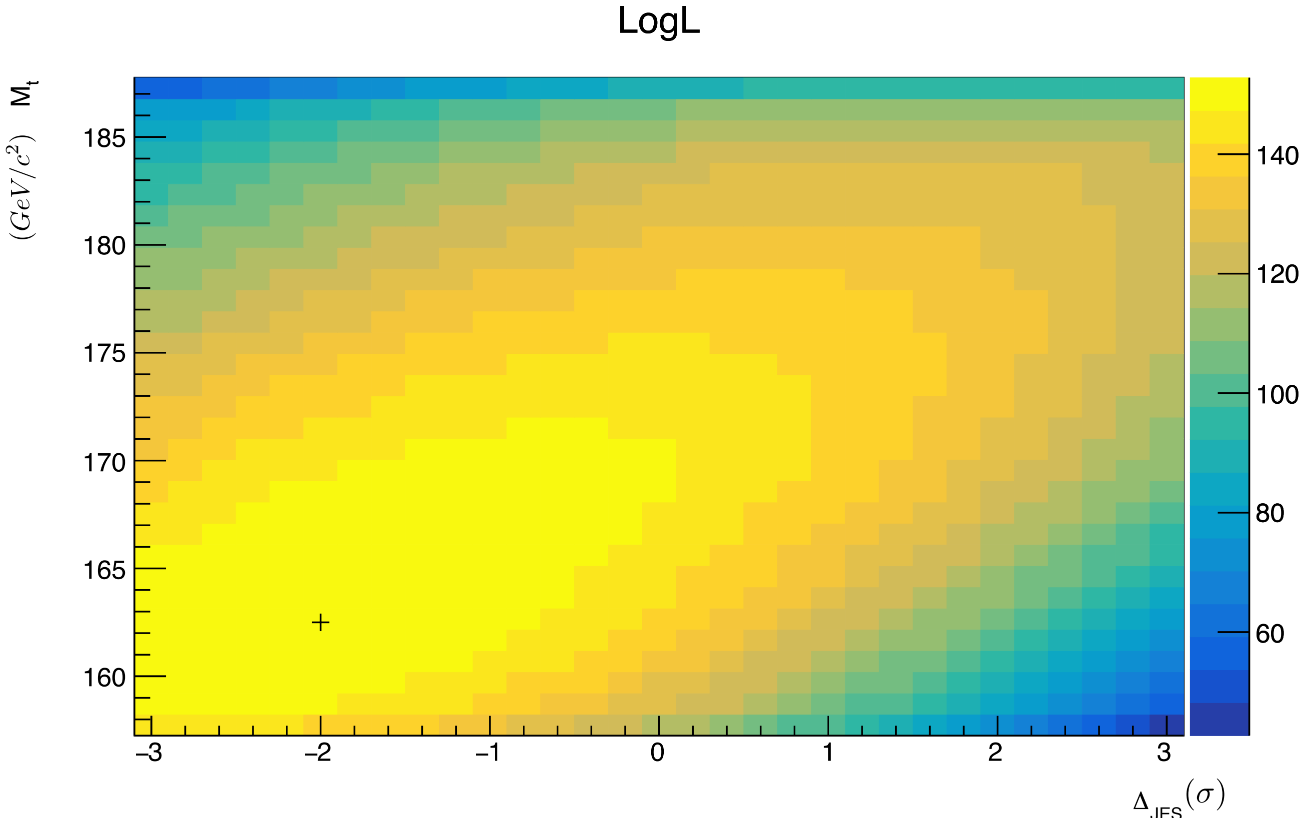
$$P_{ev}(\mathbf{y}|m_t, \Delta_{JES}) = A(\mathbf{y})[fP_{sig}(\mathbf{y}|m_t, \Delta_{JES}) + (1-f)P_{W+jets}(\mathbf{y}|\Delta_{JES})]$$

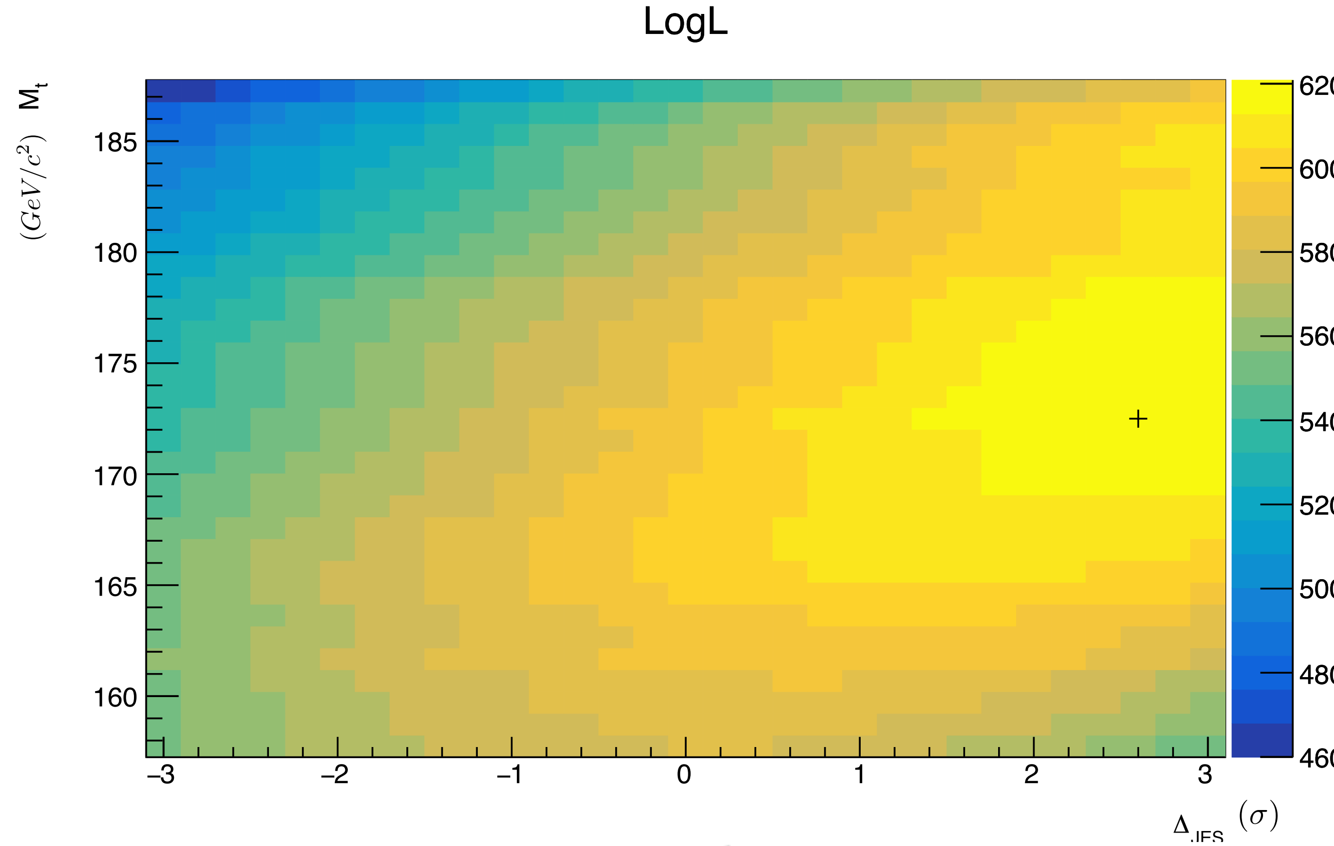
- Likelihood:

$$L(\mathbf{y}|m_t, \Delta_{JES}) = \prod_{i=1}^{N} P_{ev}(\mathbf{y}|m_t, \Delta_{JES})$$

Comparison between New/Old TF

100 ev MC Generated: PYTHIA, $M_t = 170 GeV/c^2$, $\Delta_{JES} = 0\sigma$, 0Tag.





Sample	
Top Mass	$170GeV/c^2$
Δ_{JES}	$0\ \sigma$
Results	
$M_t(2DPeak)$	$162.5GeV$
$\Delta_{JES}(2DPeak)$	-2σ

Sample	
Top Mass	$170GeV/c^2$
Δ_{JES}	$0\ \sigma$
Results	
$M_t(2DPeak)$	$172.5GeV$
$\Delta_{JES}(2DPeak)$	2.6σ