Systematic errors caused by imperfections of Mu2e beam line magnetic lattice

Fabio Spagliardi
Università di Pisa

Supervisor: Eric Prebys
Summer student 2016

October 10, 2016

ABSTRACT

The aim of this project is to understand how systematic errors of the Mu2e M4 beam line could affect the level of out-of-time beam in the proton's bunch structure. This will be done carrying out simulations with specific tools. In particular, misallignements of collimators and errors in the magnetic fields will be taken into account.

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1 THE MU2E EXPERIMENT

[1]

Mu2e experiment will search for the conversion of a muon to an electron in the field of an aluminum nucleus:

$$\mu^- + N(Z, A) \to e^- + N(Z, A).$$

The experiment will measure the ratio of conversion to the usual muon capture:

$$R_{\mu e} = \frac{\Gamma\left(\mu^- N(Z, A) \to e^- N(Z, A)\right)}{\Gamma\left(\mu^- N(Z, A) \to \nu_\mu N'(Z-1, A)\right)} \qquad (1)$$

The experimental signal is a mono-energetic electron with an energy similar to the muon rest mass.

1.1 Physics motivation

This conversion is an example of Charged Lepton Flavour Violation (CLFV),a process which has never been observed experimentally. The rate at which lepton flavor-violating (LFV) processes occur in the neutrino sector is constrained by the neutrino mixing parameters, but the rate of CLFV processes is very model-dependent. This rate can vary over many orders of magnitude. The most stringent limits come from the muon sector, as there exist intense sources for them.

The are three rare muon processes:

- $\mu^+ \to e^+\gamma$
- $\mu^+ \to e^+e^+e^-$
- $\mu^- N \to e^- N$

In Table 1 are listed upper experimental limits of CLFV processes, all at 90% CL.

$BR(\mu^+ \to e^+\gamma)$	$< 5.7 \cdot 10^{-13}$
$BR(\mu^+ \to e^+e^+e^-)$	$< 1.0 \cdot 10^{-12}$
$R(Au)(\mu^- + N \to e^- + N)$	$< 7 \cdot 10^{-13}$

Table 1: Data from current experimental limits at 90% CL

$\mu^- + N \to e^- + N$ offers the greatest potential sensitivity. Mu2e aims at sensitivities of $10^{-16} - 10^{-17}$ on $R_{\mu e}(Al)$. Since this process is allowed within the SM but at the level of 10^{-54}, any signal would therefore be evidence of the existence of CLFV far beyond what is expected from standard theory.

1.2 Mu2e set up

In Fig.1 is presented a picture of the Mu2e solenoids.

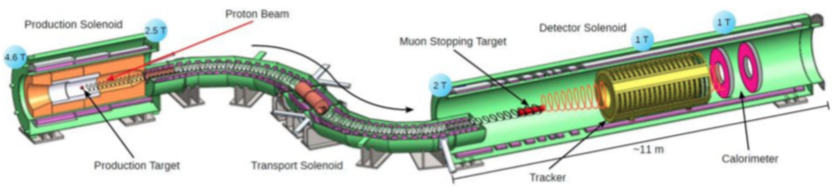

Figure 1: Mu2e setup

An integrated array of superconducting solenoids forms a graded magnetic system that includes the Production Solenoid, the Transport Solenoid and the Detector Solenoid. This system has some fundamental functions:

- Capture of pions from the production target
- Formation of the muon beam
- Background rejection by shifting the pitch of high energy particles in the muon beamline before they reach the target

Production Solenoid
This solenoid is a high field magnet with a graded solenoidal field varying from 4.6 Tesla to 2.5 Tesla. This is designed to capture pions and muons and guide them downstream to the Transport Solenoid. This process starts with 8 GeV protons hitting a production target near the center of the Production Solenoid.
Transport Solenoid
The Transport Solenoid constits of a set of superconducting solenoids and toroids that form a magnetic channel that efficiently transmits low energy negatively charged muons from the Production Solenoid to the Detector Solenoid. Negatively charged particles with high energy, positively charged particles and line-of-sight neutral particles are nearly all eliminated by absorbers and collimators before reaching the Detector Solenoid.
Detector Solenoid
It is a large, low field magnet that houses the muon stopping target and the detectors. The muon stopping target resides in a graded field that varies from 2 Tesla to 1 Tesla. This field captures conversion electrons that are emitted in the direction

opposite the detector components causing them to reflect back towards the detector.

The Production Target in the Production Solenoid intercepts an 8 GeV kinetic energy, high intensity, pulsed proton beam. Then, the S-shaped Transport Solenoid transports low energy μ^- from the Production Solenoid to the Detector Solenoid and it is long enough for a large fraction of the pions to decay to muons.

The tracking detector is made from low mass straw tubes oriented transverse to the solenoid axis, while the calorimeter consists of about 1900 crystals arranged in two disks oriented transverse to the solenoid axis.

1.3 Signal and Predominant background

Mu2e experiment will search for process $\mu^- + N \rightarrow e^- + N$, where N is a nucleus of Aluminum. The conversion is called coherent: the muon recoil off the entire nucleus with the same kinimatics of a two-body decay. Given that, the outgoing electron is monoenergetic with his energy slightly less than the muon rest mass. The muon mass of 105.6 MeV is sufficiently above the maximum energy of the electron from muon decay at 52.8 MeV. Thus, most of the muons decays do not contribute background.

When a negatively charged muon stops in a target, it cascades down to the 1S state. Capure, decay or conversion takes place with a lifetime that in various materials range from 100 ns to over 2 μs. Neutrinoless conversion will produce an electron with the energy:

$$E_e = m_\mu c^2 - E_b - \frac{E_\mu^2}{2m_N} \tag{2}$$

where m_μ is the muon mass, E_b is the atomic binding energy of the muon and last term is the nuclear recoil energy. In the case of muonic aluminum the conversion electron has energy $E_e = 104.97$ MeV and the muon lifetime is 864 ns.

Mu2e goal is to reach sensitivity of $10^{-16} - 10^{-17}$ on $R_{\mu e}(Al)$ which is more than four order of magnitude beyond the current limit as shown in Fig 2.

With this sensitivity, some processes can fake a muon-to-electron conversion signal. Controlling this backgrounds drives the de-

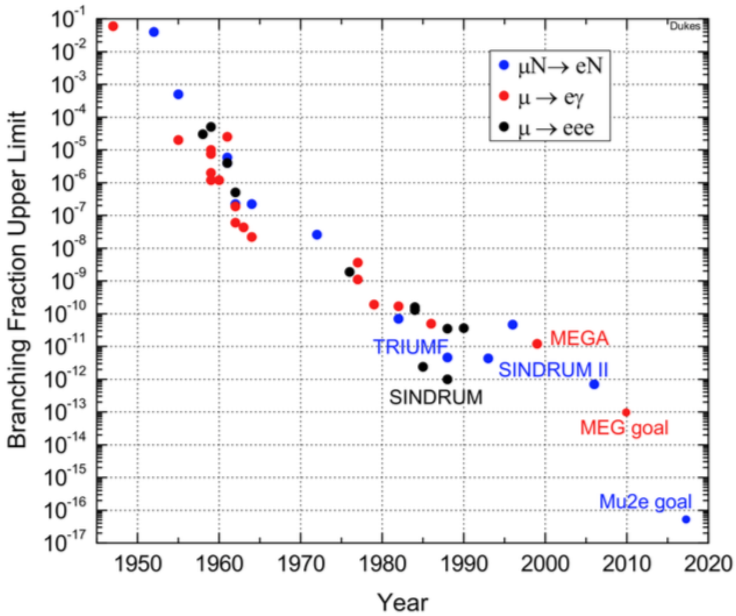

Figure 2: Limits for the muon conversions.

sign of Mu2e and my project as well. The main sources of background are:

- **Muon Decay-In-Orbit (DIO)**: if the muon is bound in atomic orbit, it can undergoes the standard muon decay. The electron produced can exchange momentum with the nucleus. There exist a small probability to have an electron with a maximum possible energy. At the kinematic limit of the bound decay, the neutrinos carry away no momentum and the electron recoils simulating the two-body final state of muon to electron conversion.

- **Radiative Muon Capture (RMC)** $\mu^- Al \to \gamma\nu Mg$: this process is the source of high energy photons that can convert to an electron-positron pair in the stopping target or other surrounding material, producing an electron near the conversion electron energy. In order to reduce this background the stopping target is chosen so that the minimum masses of daughter nuclei are all at least a couple of MeV/c^2 above the rest mass of the stopping target nucleus, in order to push the RMC photon energy below the conversion electron energy.

- **Presence of antiprotons**: they can be coincident in time with a conversion electron, simulating the energy of a conversion electron signal. The products of their interaction with the matter can be also a source of background.

- **Radiative Pion Capture (RPC)**: pions that contaminate the muon beam can produce background being captured by the nucleus: $\pi^- N \rightarrow \gamma N^*$. The photon coming from this process can undergo pair production $\gamma \rightarrow e^+e^-$ and the e^- could fake the signal. RPC background suppression will be discussed later on.

- **Cosmic Rays (electrons, photons, muons)**: source of electrons near the conversion electron energy. If such electrons have trajectories that appear to originate in the stopping target, they can fake a muon conversion. In order to suppress this background passive shielding and veto counters will be used.

1.4 Proton Beam requirements and Extinction

Mu2e requires a pulsed proton beam to produce an intense beam of low energy muons. Protons are acquired from the Booster: two proton batches, each containing $4.0 \cdot 10^{12}$ protons with a kinetic energy of 8 GeV, are injected into the Recycler Ring. In the Recycler Ring each batch is divided in four bunches that will be transfered to the Delivery Ring. A resonant extraction system injects $3 \cdot 10^7$ protons into the external beamline every 1.7 μs. An *extinction system*, composed by a high frequency *AC dipole* and collimators, is required to suppress out-of-time beam (between two bunches) that could bring to experimental background. We call "extinction" the fraction of out-of-time beam with respect to the number of protons that hit the production target.

Studying the background, it has been extablished that an extinction of approximately 10^{-10} is required. This level will be achieved in two steps:

- Generating the required bunch structure in the Recycler Ring will lead to a high level of extinction. Taking into account that some beam will leak out of the RF bucket in the Delivery Ring during the extraction, extinction of 10^{-4} or better is expected when beam is extracted to the beamline.

- The beamline incorporates a set of oscillating dipoles ("AC dipoles") that will be able to kick out-of-time beam into a system of collimators. With this system, an additional level of 10^{-7} will be achieved.

1.5 How Extinction reduces RPC Backgound

The pion lifetime is 26 ns while the muon lifetime in Aluminum is 864 ns. In order to reduce the RPC background it is sufficient to wait long enough for the pions to be decayed. For this reason Mu2e needs very stringent requirements for the bunch structure of the proton beam. In Fig.3 is showed the buch structures and the time intervals for the signal window and the time distribution of pions and muons decays.

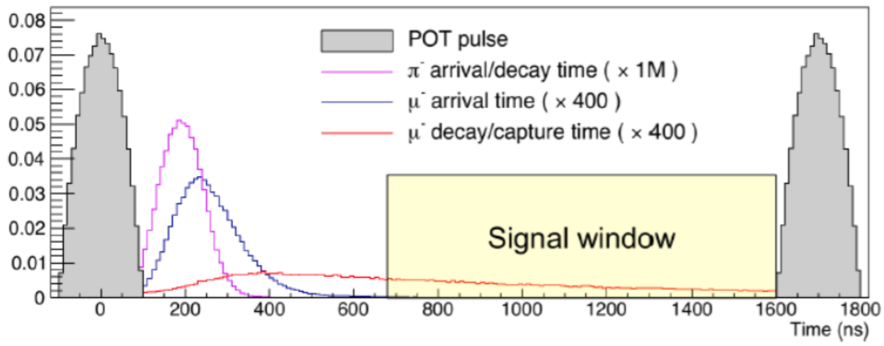

Figure 3: Proton bunches distribution and other time intervals.

Since no protons must be present just before and during the signal window, it is important to have a very low extincion level. In fact a out-of-time proton could hit the Production Target emitting pions that could lead to fake signal.

2 ACCELERATOR PHYSICS

A brief introduction to accelerator physics is needed in order to understand some parameters that will be used in the simulations.

A strongly focused particle motion can be described in terms of initial conditions and a "beta functions" $\beta(s)$, where s is the coordinate along the ideal trajectory of the particle in the beamline. The beta functions is only a function of position along this nominal path and is a periodic function with the same period of the machine structure. Using this functions and working out the algebra, one finds out that particles undergo a pseudo-harmonic motion about the nominal trajectory. With refer to the formalism presented in Fig.4 one finds that the displacement in

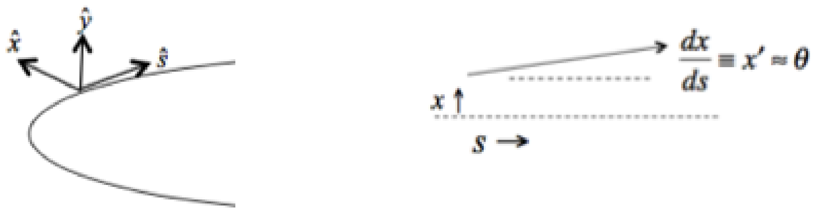

Figure 4: Coordinate system.

the x (and similarly in the y) direction is:

$$x(s) = A\sqrt{\beta(s)}\cos(\Psi(s) + \delta) \quad \Psi(s) = \int_0^s \frac{ds}{\beta(s)} \tag{3}$$

where $\Psi(s)$ is the phase advance. The "beta function" $\beta(s)$ is effectively the local wavenumber.

Two useful quantities are also defined:

- **Emittance**: roughly the area in the phase space (x', x) of the beam;
- **Admittance**: the largest value of the emittance which the machine can transport without loss.

With these definitions and formalism a normalised angle δ is defined:

$$\delta \equiv \frac{\theta}{\theta_0} \tag{4}$$

where θ is the resulting bending angle of the dipole system and

$$\theta_0 = \sqrt{\frac{A}{\beta_D \beta\gamma}}. \tag{5}$$

A is the admittance of the collimator just downstream the AC dipole, β_D is the beta functions evaluated at the s coordinate of the AC dipole and $\beta\gamma$ is the relativistic factor of the beam. The idea is to use the AC dipole to kick the out-of time beam againg the jaws of the downstream collimator (DS).

With such a definition, if δ will be set equal to 1 in the simulation the center of the beam is expected to hit the edge of the collimator so that just 50% of the beam will be transmitted. If $\delta = 2$ will be set, no transmission is expected. In Fig.5 a graphical rappresentation is showed. Mathematically, an angular deflection at the AC dipole cause a position displacement 90° later in phase advace (where the downstream collimator is set).

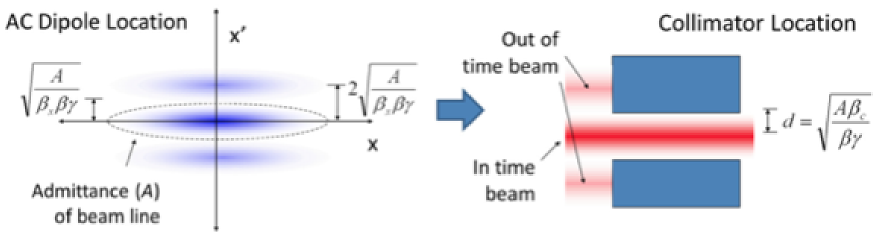

Figure 5: The blue clouds represent the beam in the phase space where the upper e lower halos are the out-of-time beam and the darker blue bunch is the in-time beam. On the right, a scheme of the downstream collimator where the red halos are the out-of-time beam having been deflected against the jaws.

2.1 AC dipole design

[1] The magnetic design of the AC dipole is based on three harmonics: a 300 kHz harmonic is phased in such a way that in-time beam will pass through the collimator at the nods. A 4.5 MHz harmonic reduces the slewing during transmission and is optimized to minimize loss of in-time beam. A 900 kHz harmonic is designed to reduce the maximum amplitude to prevente beam scraping upstream of the collimator.

The final waveform is showed in Fig.6.

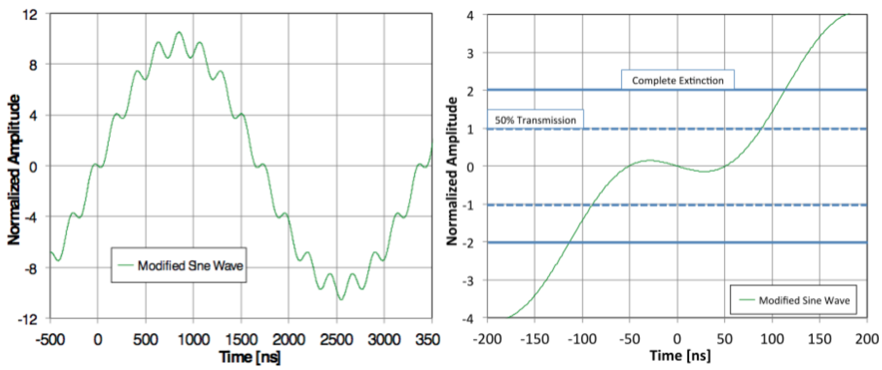

Figure 6: Final AC dipole waveform. On the left is the waveform over a complet cycle and on the right is the waveform over the transmission window.

3 SIMULATIONS DESCRIPTION

In this section will be presented the simulations that have been carried on in order to verify how systematic errors in the beamline could affect the extinction level.

In the first place the code of the simulations has been checked. This includes the magnetic description of the beam line and the proper use of the δ normalised angle. After that, the effect on the extinction of misallignement of the downstream collimator and systematic errors in the magnetic lattice have been studied.

3.1 Tools for the simulations

These are the main tools exploited for the simulations:

- **MADX**: MADX is a tool developed at CERN and it has been used to design the optic of the beam line. The MADX code contains all the informations about dipoles and quadrupoles field and their position along the beam line.
- **G4beamline**[2]: this is a GEANT4 scripting tool developed by Muons, Inc. The G4beamline code includes the description of magnets, collimators and beam pipes. The version used is 2.16.
- **Python**: a python script has been used to convert the MADX optic description in G4beamline scripts.

3.2 Simulation procedure

Starting from the MADX scripts, a description of the entire beam line is obtained using G4beamline. This description includes dipoles and quadrupoles (fields and position), beam pipes and collimators starting from the end of the Delivery Ring Enclosures. Since in this case there is only the need to see how the extinction is affected by systematic errors, in order to save computing power simulations are done starting just upstream of the AC dipole. For this reason a mathematical model is used for the "core" of the beam. This mathematical model, in practice a ROOT file, has been used as input for the simulations.

In Fig.7 the mathematical distributions in the phase space of the beam just upstream of the AC dipole is shown. On the left side is a full normalized emittance (x direction) and on the right side is a normalized Gaussian emmitance (y direction, i.e. non-bend plane).

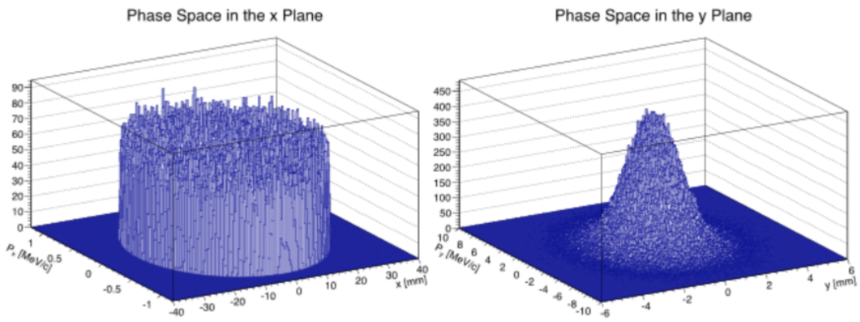

Figure 7: Mathematical distributions in the phase space of the beam just upstream of the AC dipole.

Starting from those mathematical models just upstream of the AC dipole, particles are propagated through the beam line and are defined to be "transmitted" if they fall within 5 mm of the target. The actual radius of the target is in fact 3 mm, so this is an overestimate: the goal is to minimize the number of transmitted particles as now the out-of-time beam is being simulated (particles that miss the target do not produce experimental backgrounds).

Transmission results are tabulated as function of the normalised deflection angle $\delta \equiv \sqrt{\frac{A}{\beta_D \beta \gamma}}$. At a later time, these tables will be combined with the waveform of the AC dipole to determine the relations between transmission and time. This relation will be convoluted with the simulated bunch shape to determine the out-of-time transmission.

To give an idea of the elements simulated and of the beam line description, in Fig.8 is shown the grafic model of the simulations obtained with G4beamline.

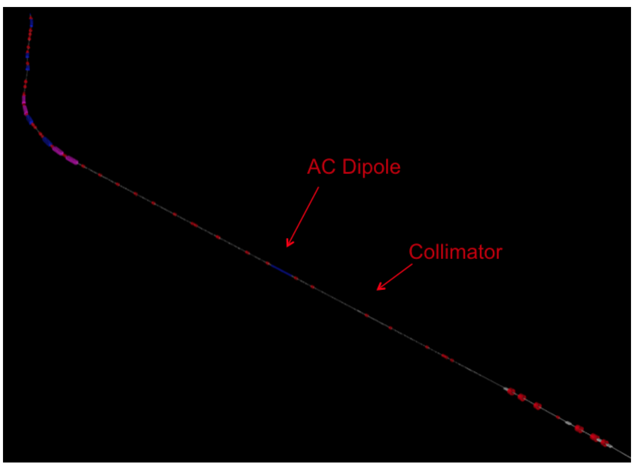

Figure 8: G4beamline graphic model.

4 CHECK OF THE CODE

The first thing that was done was to check if the code worked properly. Full simulation has been run on varying of the angle δ. G4beamline code produces as output a ROOT file for each specified location s containing information about x, y distribution of the particles at the given s coordinate, and also their momentum components distribution. This could be used to check the beam behavior when crossing each element of the beamline. As an example, some distributions right at the beginning of the beam line are showed in Fig.9.

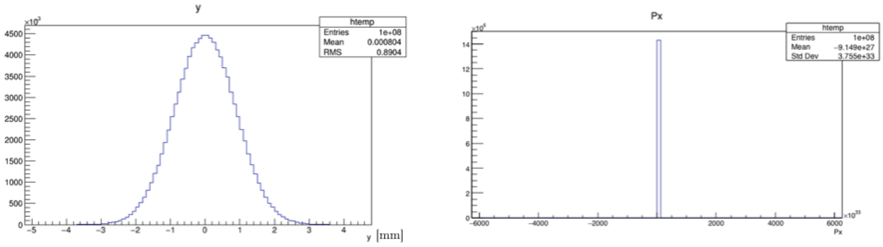

Figure 9: On the left: displacement distribution in the y direction. On the right: distribution of the x component of the momentum.

In particular, simulations have been run in the Mu2e cluster with 10^8 events (10^6 on 100 processors) for each value of δ going from 1.8 to 2.5 with .1 increments.

A ROOT script takes then the output distributions at the s position of the downstream collimator and computes the number of particles that fall within the target. This is easily done having the information about the x and the y coordinate. Table 2 summaries the results of the simulations.

δ	End	Hit
1.8	177890	5309
1.9	35755	667
2.0	2841	1
2.1	869	0
2.2	359	0
2.3	147	0
2.4	76	0
2.5	33	0

Table 2: Values of transmission as a function of δ. In the "End" column is the fraction of simulated particle that reach the s coordinate of the target while in the "Hit" column is the fraction that actually hit the particle.

The code work as expected: no transmission has been found for $\delta \gtrsim 2$.

5 MISALLIGNEMENT OF THE DOWNSTREAM COLLIMATOR

Among others systematic errors that could affect the extinction, is the misallignement of the Downstream Collimator. In order to understand if this could be a real problem, full simulations have been run again modifying the orientation of both the collimator's jaws in the G4beamline code. In Fig.10 is shown how the two jaws have been tilted.

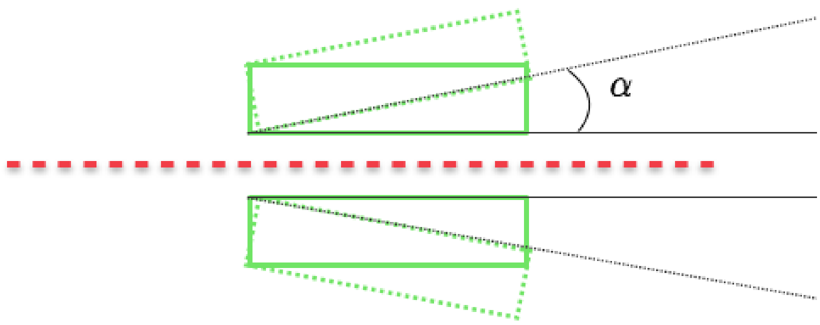

Figure 10: Graphic model that shows how the jaws orientation has been modified.

This has been done for values of δ going from 1.8 to 2.5 with 0.1 increments for $\alpha = 1$ mr and $\alpha = 2$ mr. As shown in Fig.11, extinction still meets the requirement as a level of $10^{-5} - 10^{-4}$ is achieved with the Delivery Ring.

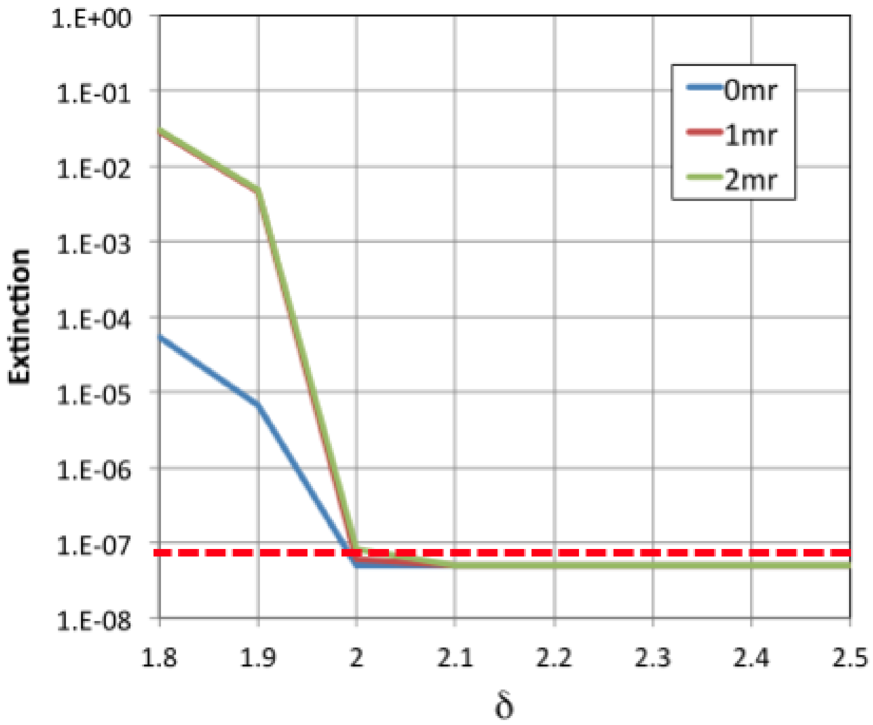

Figure 11: Extinction level as a function of the δ and for $\alpha = 0$ mr, 1 mr and 2 mr. The dotted line marks the level required.

6 OPTICAL ERRORS IN BEAM OPTICS

The goal of this is to understand how errors on the quadrupoles fields could affect the extinction level. This analysis begins modifying the MADX optic file. A random error has been added to the quadrupole field with the following features: the added error has a Gaussian distribution with a cut at 2.5 sigma and a rms value of 10^{-3} for the field relative error. This value has been chosen in order to get an idea of the simulations while waiting for more realistic errors that have to be established.

In order to have a statistical distribution, the MADX script has been run five times[1]. As an example, Fig.12 shows the little variation of the $\beta(s)$ function due to the errors added to the quadrupole fields as calculated with MADX. For each run, the MADX output file with the optic description of the beam line has been converted with the Python script in a script for G4beamline. In order to use the G4beamline script and compute transmission for each run, the mathematical distribution

1 Only five, because simulations take time and a lot of other things needed to be adapted for each MADX output as stated later on.

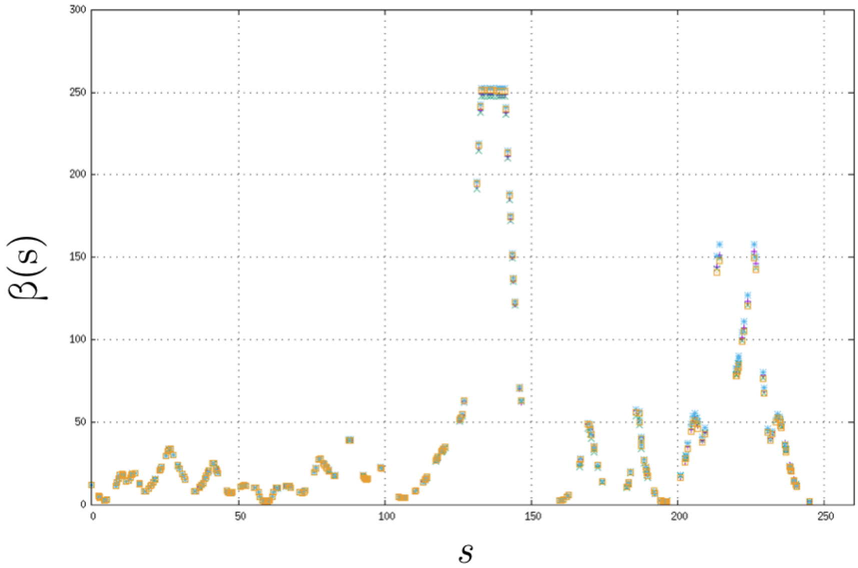

Figure 12: Modification of the $\beta(s)$ function with quadrupole's field random errors. Each color represents a different run.

of the beam just upstream of the AC dipole had to be fixed in order to match the new optic. Remember that simulations run from the AC dipole using a mathematical model of the beam but MADX contains the decription of the whole beam line. Therefore random errors affect also the upstream part of the beam line (with respect to the AC dipole). The "optic-rematching" has been done for each of the five simulations. So far the random optic has been generated five times and each configuration has been translated in G4 beamline script using the corresponding mathematical beam distribution.

Since this whole process has taken a lot of time, extinction level has been evaluated for δ going from 1.8 to 2.2 with 0.1 increments. Fig.13 summarizes the obtained results. The conclusion is that extinction is not sensible to 0.1% of the quadrupole magnetic fields.

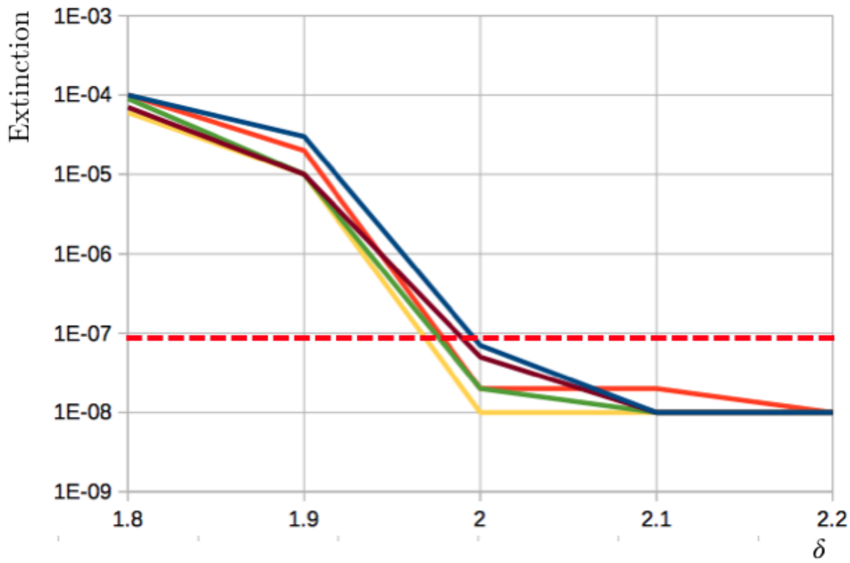

Figure 13: Extinction level as a function of δ for quadrupole fields random errors. The different colors represent the five runs. The dotted line marks the level required.

7 CONCLUSION

The effect of systematic imperfections on extinction has been evalueted. In particular, two main effects have been taken into account: misallignement of the downstream collimator and errors in the quadrupole magnetic fields. After ensuring that the code of the simulations work as expected, it is possible to conclude that at the level of the uncertainty considered ($\alpha = 1$mr and 2 mr for the misallignement of the DS collimator and relative 0.1% error on magnetic fields) the extinction level is not affected. The level of extinction obtained as a function of δ will be combined in the future with the AC dipole waveform to determine transmission vs. time.

REFERENCES

[1] Mu2e Collaboration. Mu2e Technical Design Report. *Fermi National Accelerator Laboratory*, 2014

[2] http://public.muonsinc.com/Projects/G4beamline.aspx