



ASI & CAIF SUMMER INTERNSHIP FINAL REPORT

LOW-ENERGY TOUR OF THE GALILEAN MOONS

Andrea Viale

Master student at University of Padova

Visiting Scholar at Purdue University – Aeronautics and Astronautics

OUTLINE

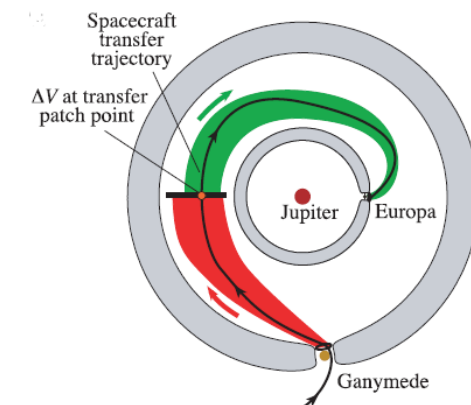
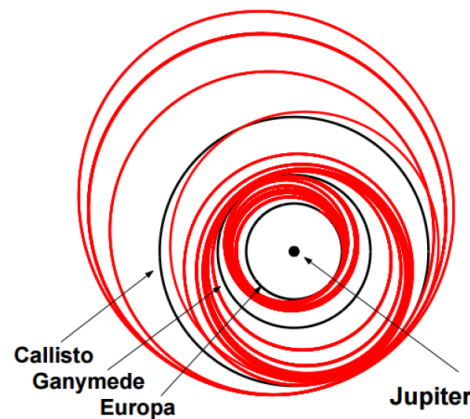
1. Objective & background
2. The method
3. Applications
4. Tour of the Galilean Moons
5. The rephasing problem

OBJECTIVE & BACKGROUND

- To construct a low-energy tour of the Galilean system based on direct transfers between moons.
- The method makes use of the transit orbits (TOs) associated to the invariant manifolds (IMs) of Lyapunov orbits of the several Jupiter-moon CR3BPs
- Exploration of the planetary systems of the giant planets, Jupiter in particular, and the consequent need for efficient trajectories enabling the execution of transfers among the moons (see e.g. GTOC6)

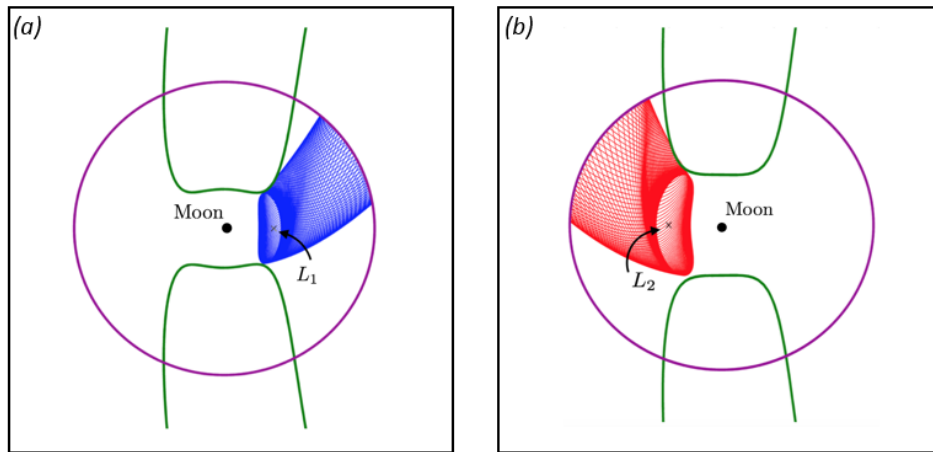
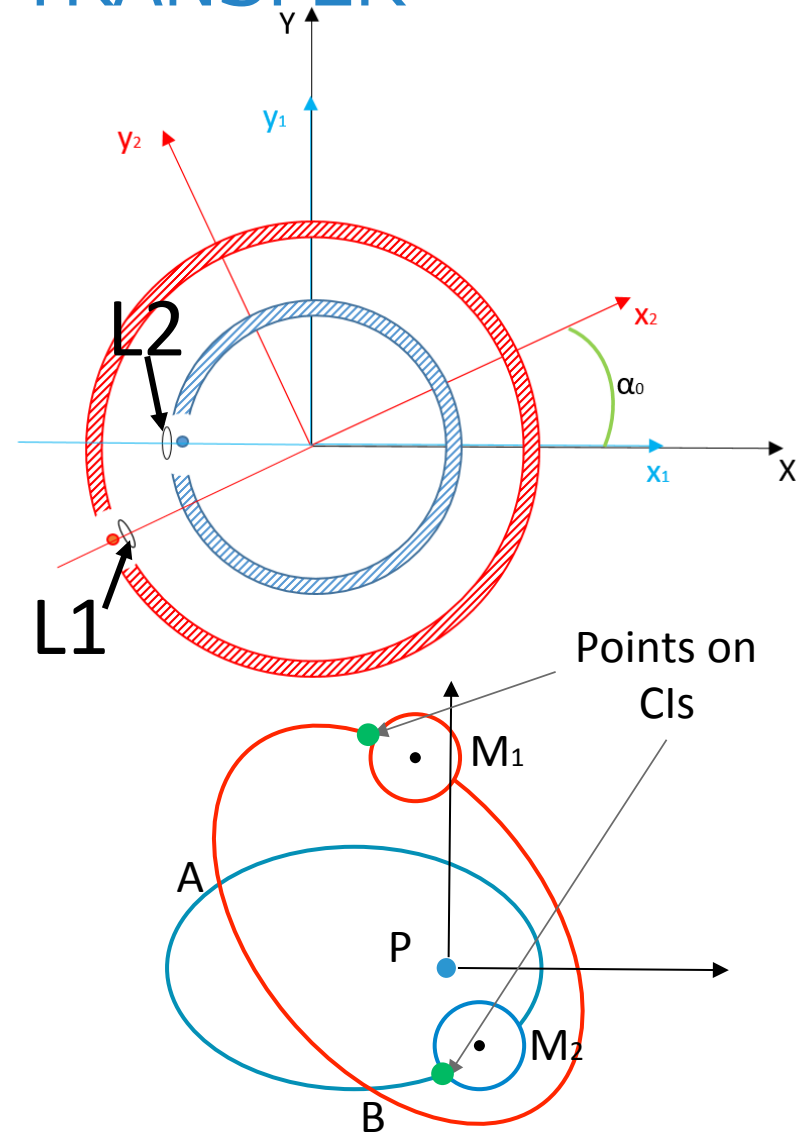
Example missions:

- Juice (ESA)
- Europa (NASA)
- JIMO (NASA)

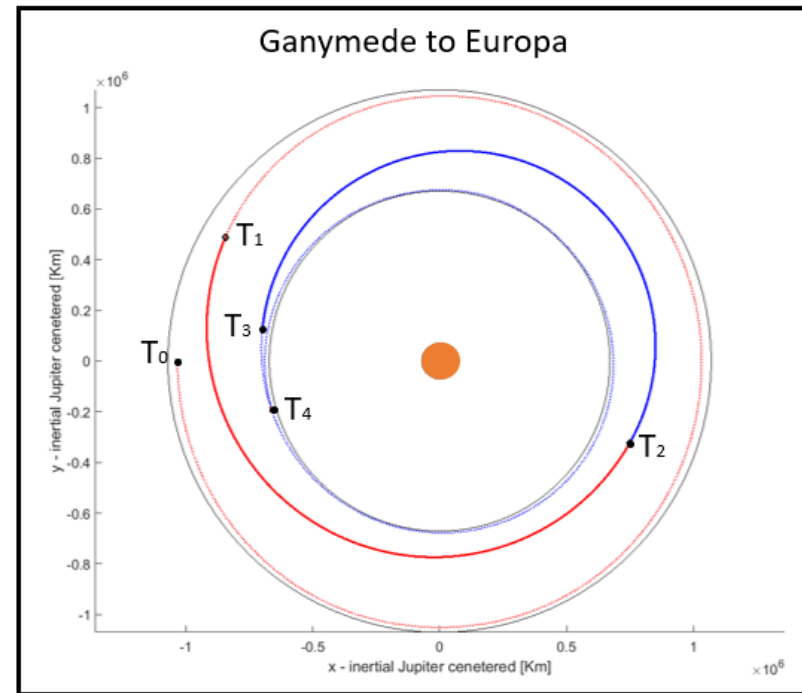
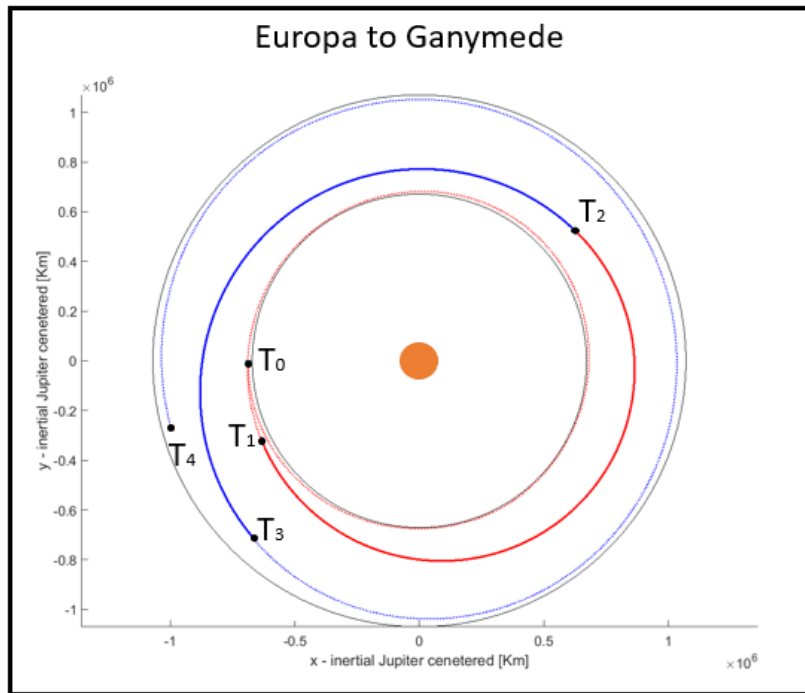


THE METHOD: INTER-MOON TRANSFER

1. PLO database around relevant Li.
2. Stable/Unstable IM to an appropriate distance from the moon (CI).
3. Transformation from Synodical Barycentric to inertial Jupiter-centered \rightarrow osculating ellipses.
4. Repeat the same for the other moon.
5. Look for intersection between ellipses
 - Different α_0 gives rotated ellipses



APPLICATION: FROM EUROPA TO GANYMEDE



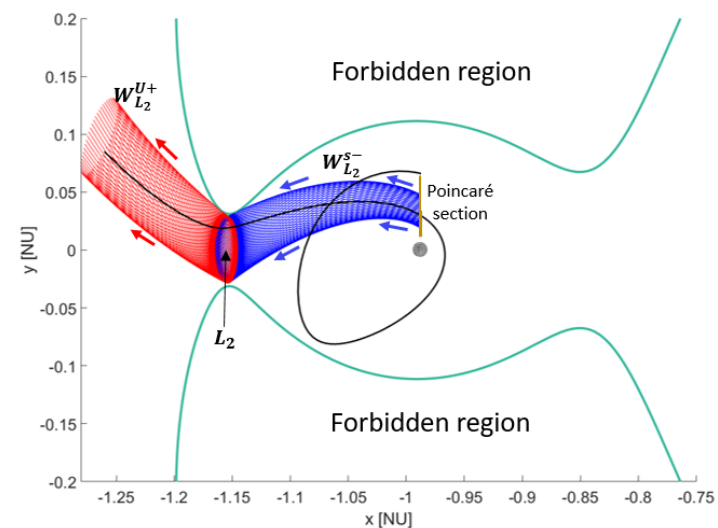
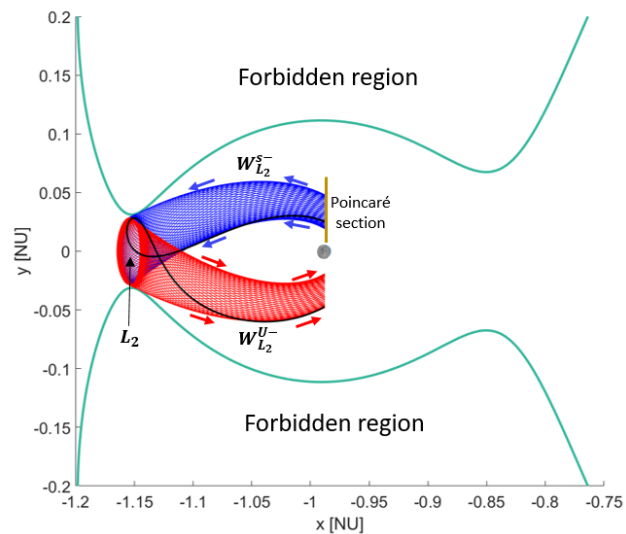
$\Delta V = 0,88 \text{ km/s}$ TOF = 12,38 days

How to reduce TOF ?

METHOD ENHANCEMENT: TRANSIT ORBITS

IMs act as separatrices for the flow:

Transit and **non-transit** orbits can be classified.



All **transit orbits** (TO) travel inside the planar Lyapunov orbit (PLO) with the same energy.

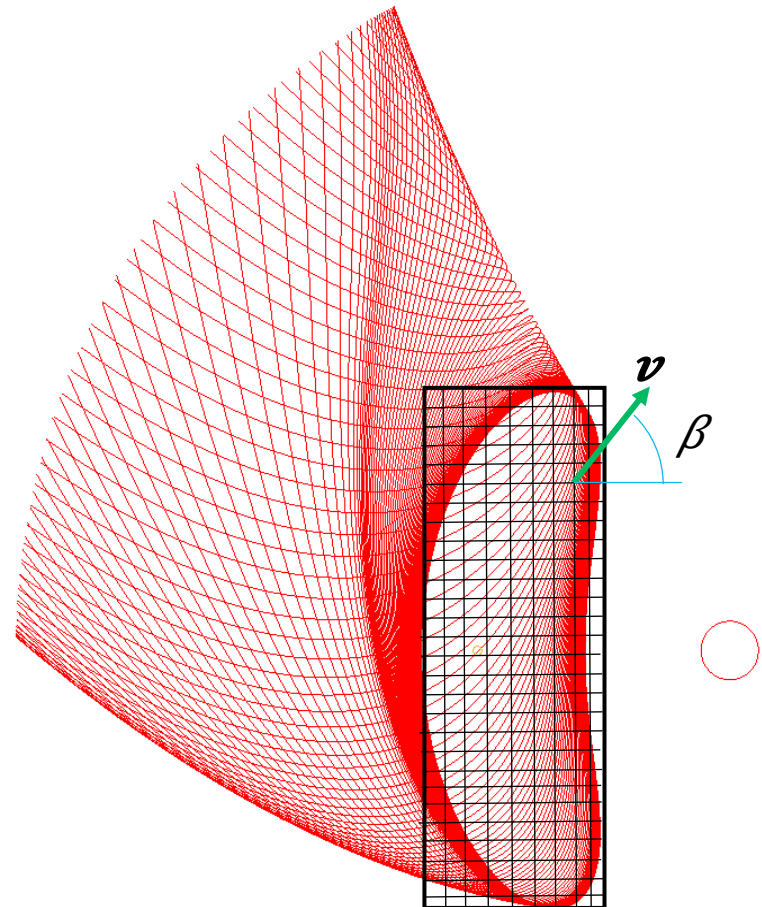
Initial conditions for TO are chosen inside the PLO.

METHOD ENHANCEMENT: TRANSIT ORBITS

- For a given energy level, a grid is computed around the planar Lyapunov orbit (PLO).
- Points inside the PLO are selected as initial conditions.
- Given the velocity orientation angle β , the components of the velocity vector are given:

$$(v_x, v_y) = f(x_0, y_0, J, \beta)$$

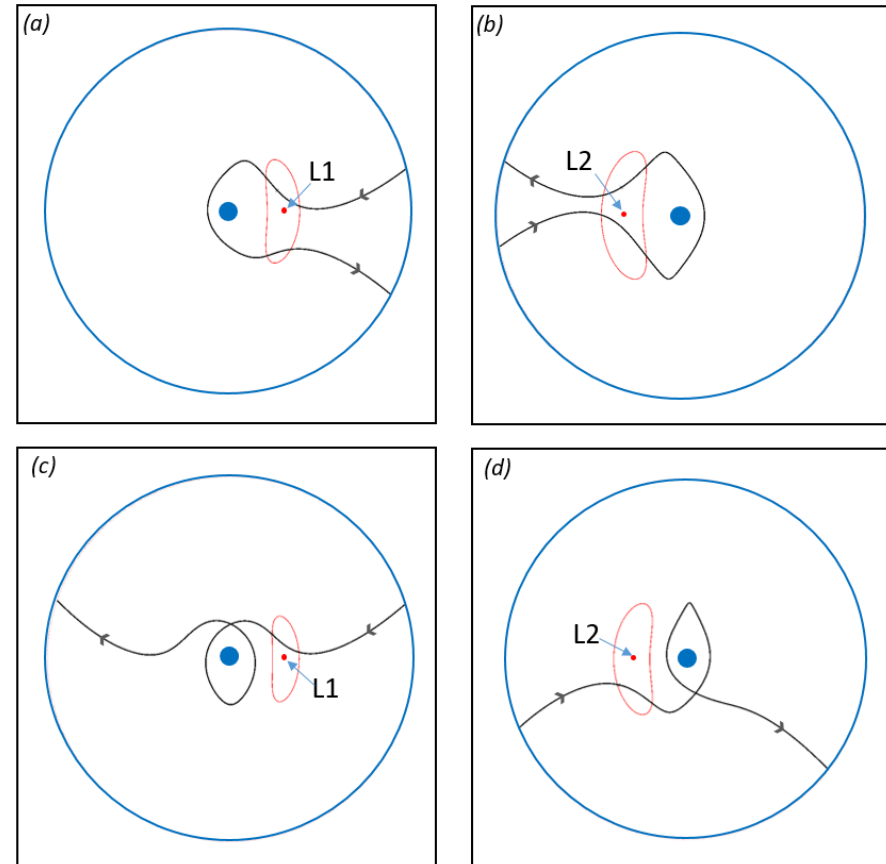
- TOs are selected according with specific criteria
 - Approaching/leaving gateway
 - Number of revs around the moon
 - Min approaching distance



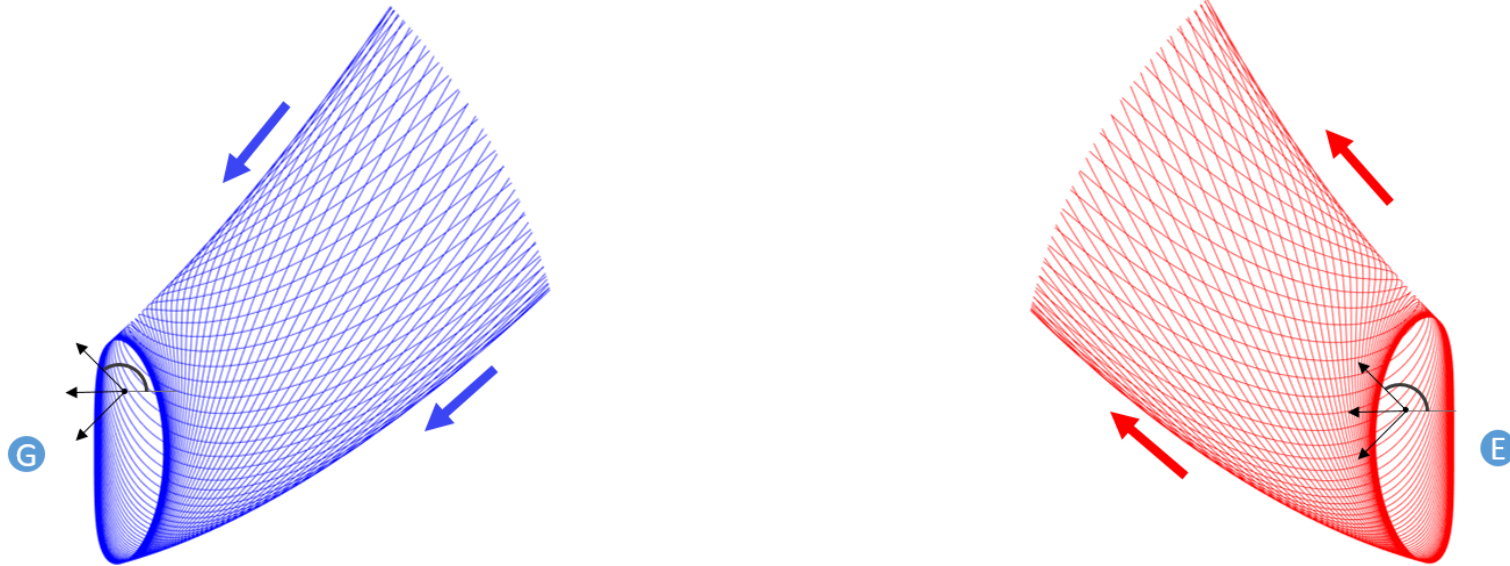
CHARACTERIZATION OF TRANSIT ORBITS

- Different morphologies exist:

1. TOs turning the moon and leaving from the same gateway (figures (a) and (b)).
2. TOs turning the moon by an angle of 2π (or even multiples) and then leaving from the opposite gateway they approached (figures (c) and (d)).
3. TOs approaching the moon without turning it.
4. TOs impacting the moon. Of course these trajectories are rejected.



APPLICATION: FROM EUROPA TO GANYMEDE



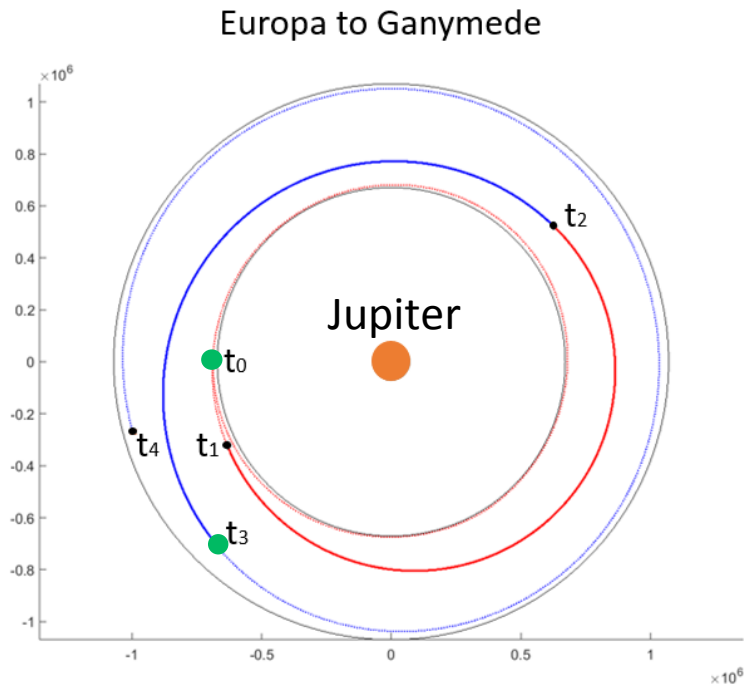
- Method applied between Europa and Ganymede, to compare results between IMs and TOs
- 3 initial angles are selected for each grid points, trying to follow the general motion of the corresponding IM.

APPLICATION: FROM EUROPA TO GANYMEDE

STABLE/UNSTABLE MANIFOLDS

$$\Delta V_{\text{MIN}} = 0,88 \text{ Km/s}$$

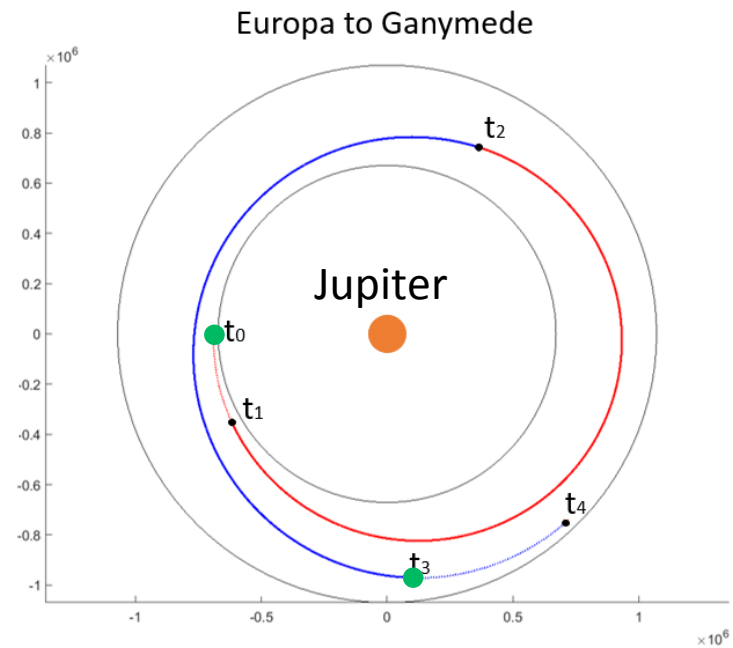
$$\text{TOF} = 12,38 \text{ days}$$



TRANSIT ORBITS

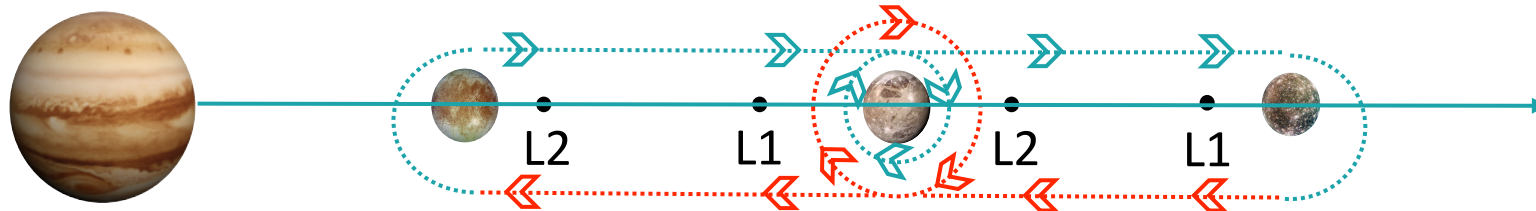
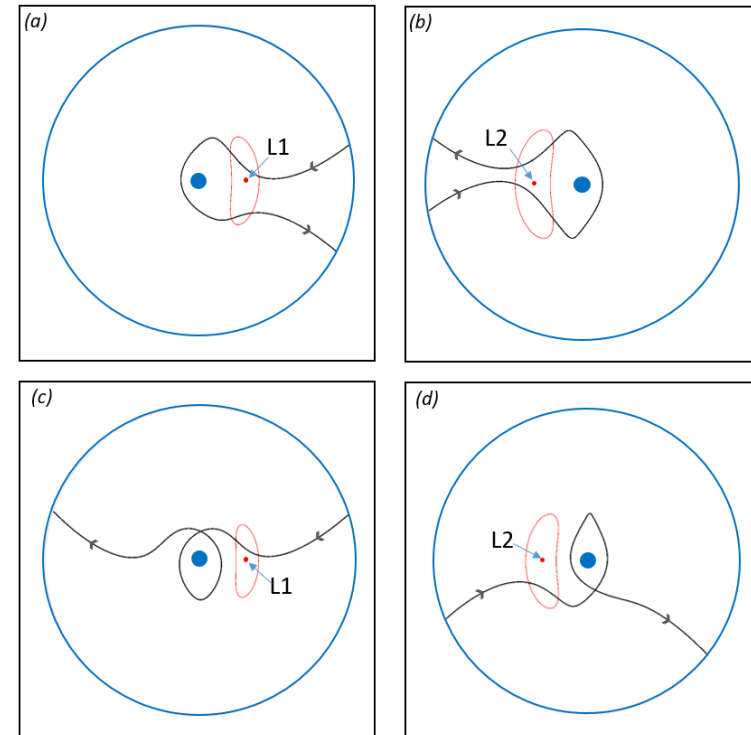
$$\Delta V_{\text{MIN}} = 0,52 \text{ Km/s}$$

$$\text{TOF} = 2,78 \text{ days}$$

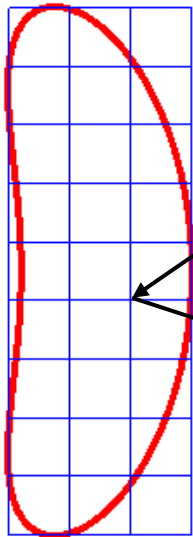


CLOSED TOUR OF EUROPA GANYMEDE AND CALLISTO

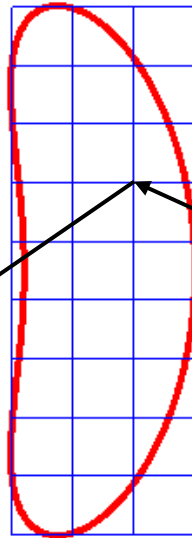
- From the results of the previous application, one could repeat the same computation between 3 consecutive moons: Europa, Ganymede and Callisto.
- Optimal couplings are sought independently for E-G, G-C, C-G and G-E and then connected together



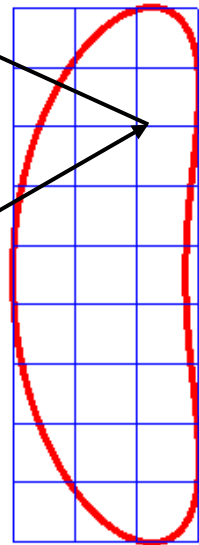
C



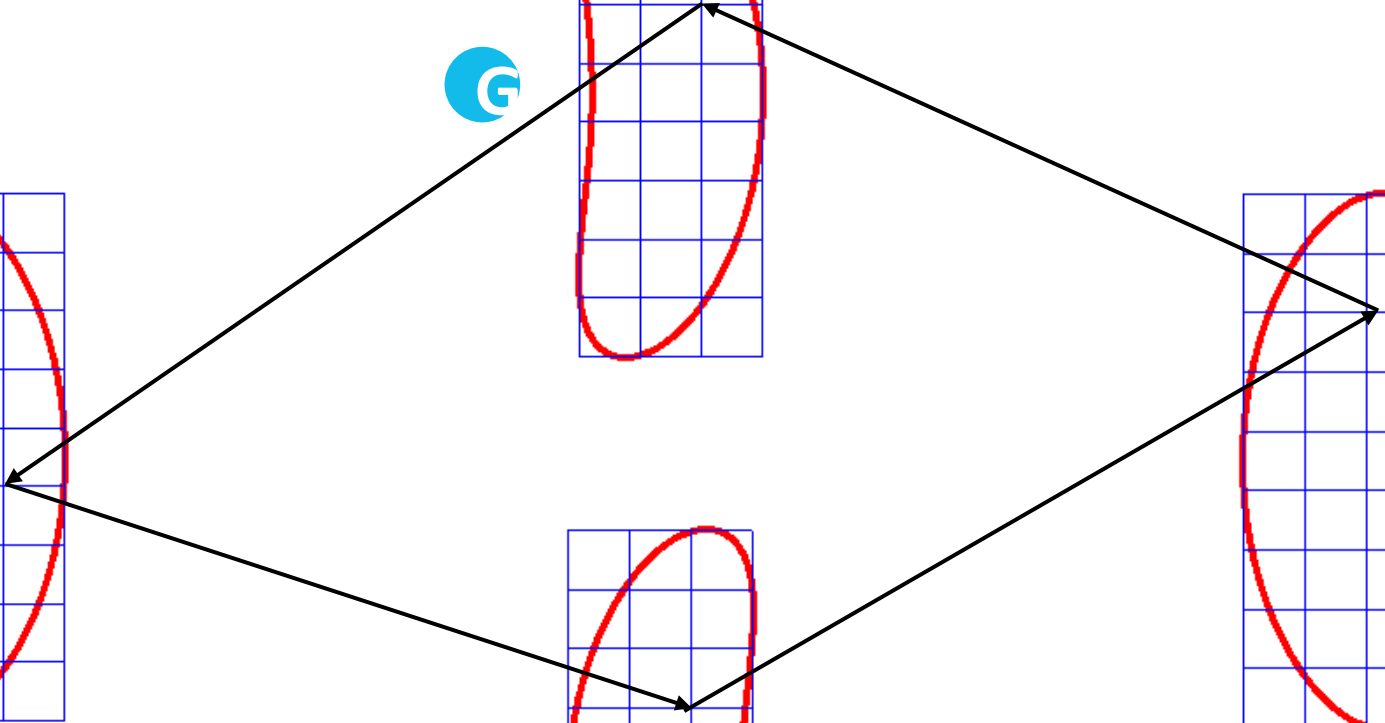
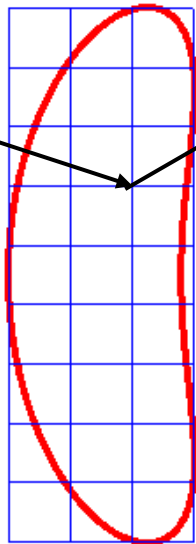
G



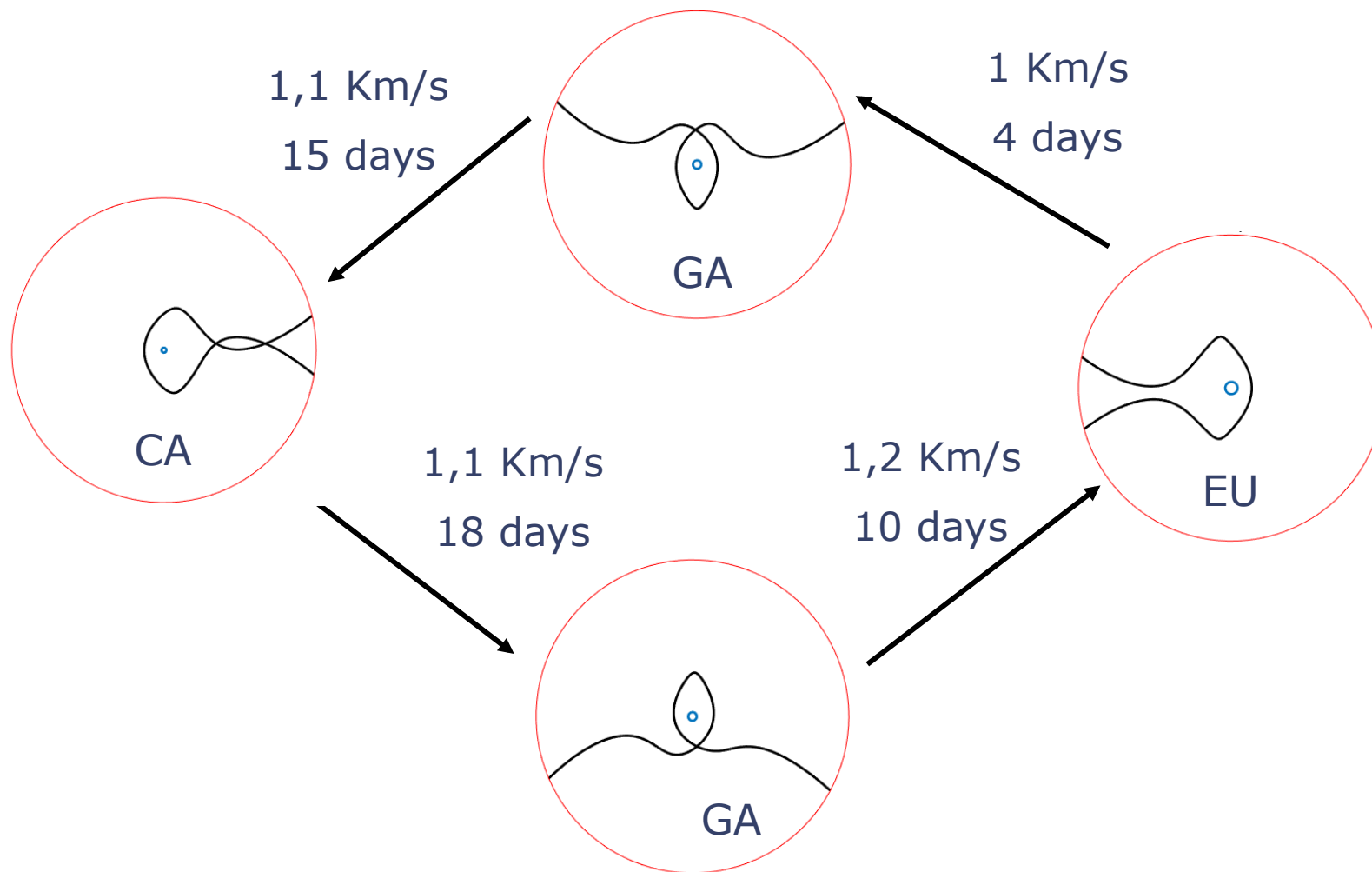
E



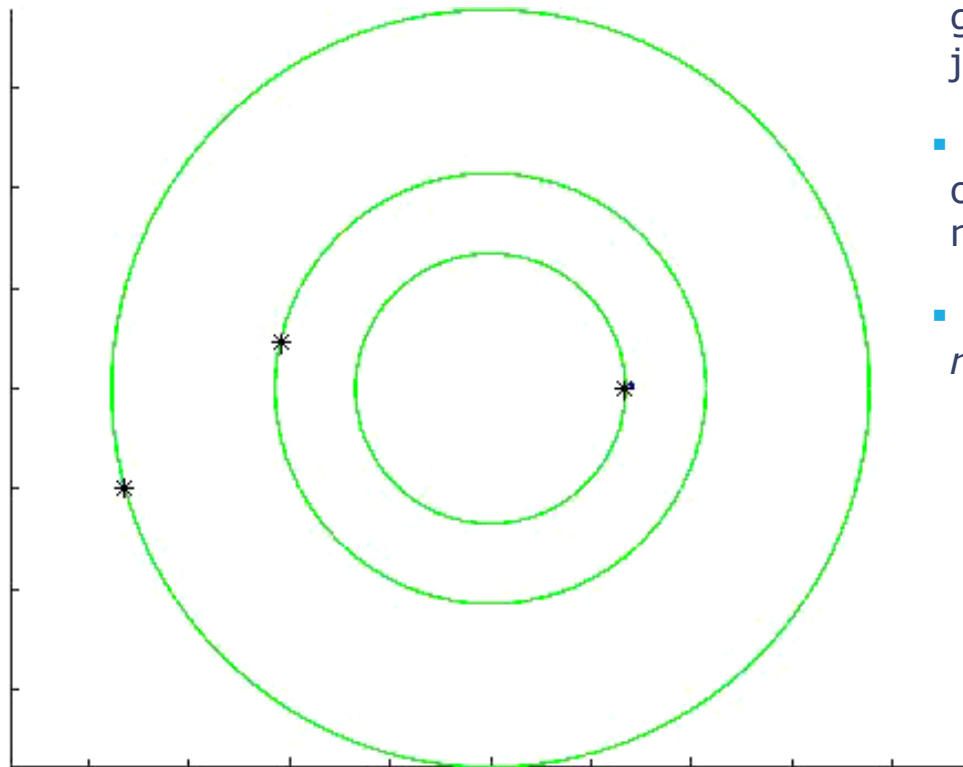
G



TOUR SELECTION



REPHASING PROBLEM

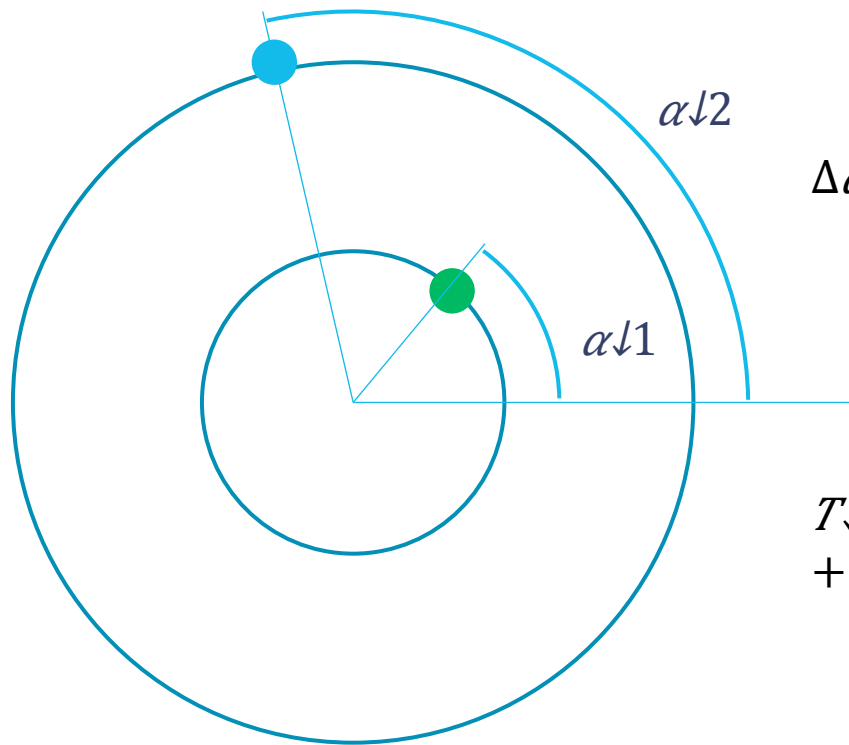


- There exist a combination of initial angular positions of the moons that guarantee the applicability of the outward journey.
- Once reached Callisto, Ganymede is not on the correct position to guarantee the next coupling.
- It is necessary to find a *parking* (or *rephasing*) orbit.

$$T_{\downarrow REPH}(n) = T_{\uparrow*} + n T_{\downarrow SYN}$$

$\underbrace{\hspace{1.5cm}}$
Synodic
period

REPHASING TIME



$$\alpha_1 = \alpha_{10} + \omega_1 t$$

$$\alpha_2 = \alpha_{20} + \omega_2 t$$

$$\Delta\alpha_{21}^t = (\alpha_{20} - \alpha_{10})^t + (\omega_2 - \omega_1) t$$

$$\Delta\alpha_{21}^{t'} = (\alpha_{20} - \alpha_{10})^{t'} + (\omega_2 - \omega_1) t$$

$$\begin{aligned} \Delta\alpha_{21}^{t'} - \Delta\alpha_{21}^t &= (\alpha_{20} - \alpha_{10})^{t'} - (\alpha_{20} - \alpha_{10})^t \\ &= \cos t \\ &= \Delta\alpha_{REPH} \end{aligned}$$

$$T_{REPH} = \Delta\alpha_{REPH} / \omega_2 - \omega_1 + n 2\pi / \omega_2 - \omega_1$$

$$\underbrace{\hspace{10em}}_{T^*}$$

For Callisto:

$$T_{REPH}^{CA}(n) = 2,68 + n 15,21 \text{ [days]}$$

Suitable orbits for rephasing: $\frac{T_{SY}}{N}$

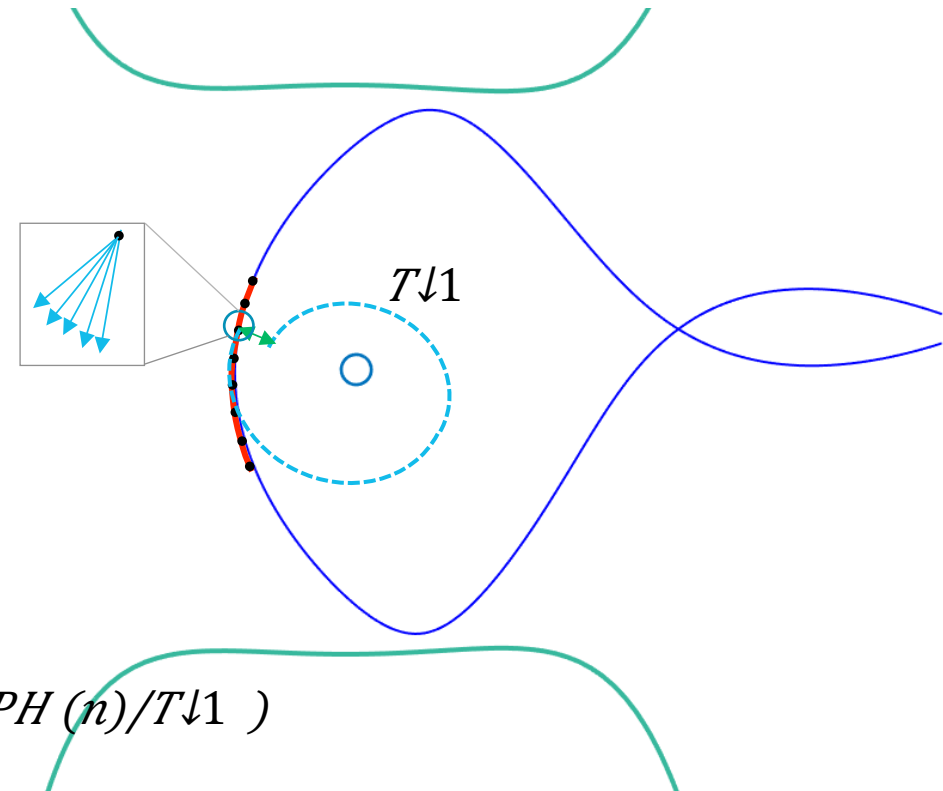
- Should be stable.
- Should minimize the required ΔV .

SELECTION PROCESS - 1

- Select a neighbourhood of the transit orbit periaapse
- Discretize this region [$P \downarrow i$]
- For each point, define:
 1. An angular range [$\alpha \downarrow i$]
 2. An energy range [$J \downarrow i$]
- $\forall P \downarrow i, \alpha \downarrow i, J \downarrow i$ propagate for 3 revs and check stability
- Evaluate $T \downarrow 1$ after 1 rev
- Define the *time performance parameter*:

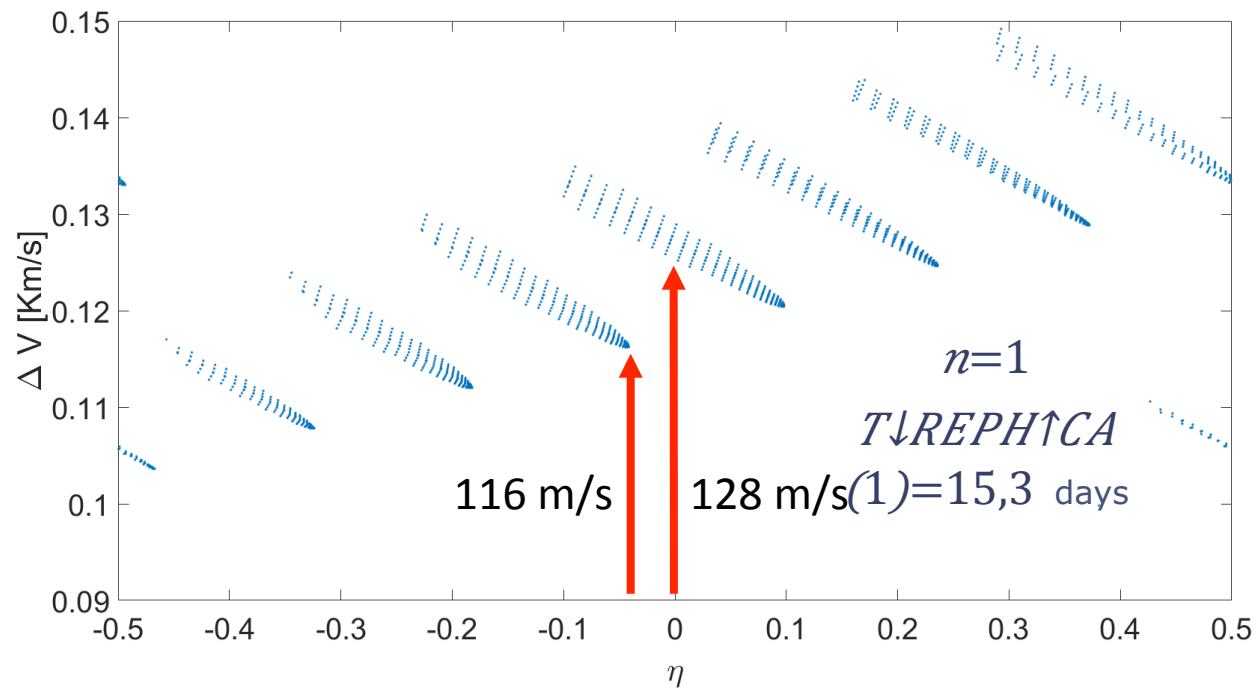
$$\eta = T \downarrow REPH(n) / T \downarrow 1 - \text{round}(T \downarrow REPH(n) / T \downarrow 1)$$

$$= \begin{cases} \text{mod}(T \downarrow REPH(n) / T \downarrow 1) & \text{if} \\ \text{mod}(T \downarrow REPH(n) / T \downarrow 1) < 0,5 & \\ 1 - \text{mod}(T \downarrow REPH(n) / T \downarrow 1) & \text{if} \\ \text{mod}(T \downarrow REPH(n) / T \downarrow 1) > 0,5 & \end{cases}$$



SELECTION PROCESS - 2

- The value η relates the «period» of the stable orbit with the required rephasing time.
- $\eta \approx 0$ defines a good matching.

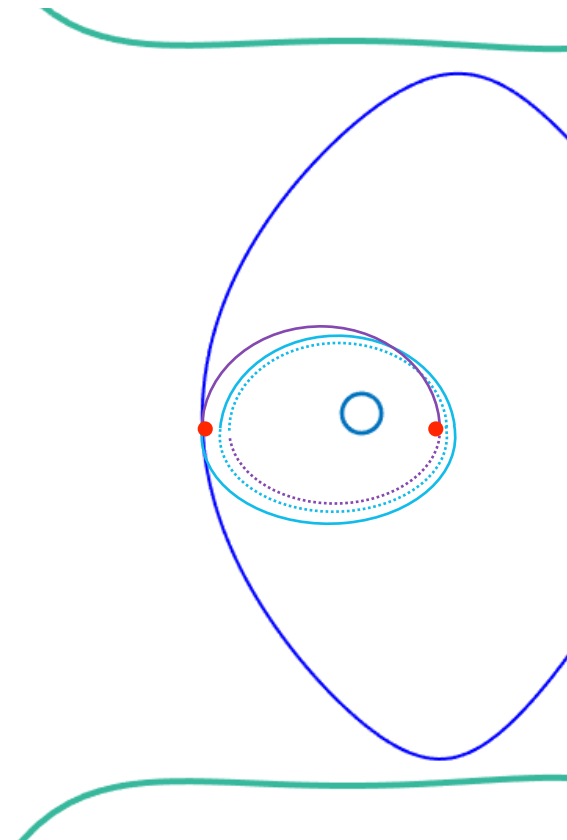


SELECTION PROCESS - 3

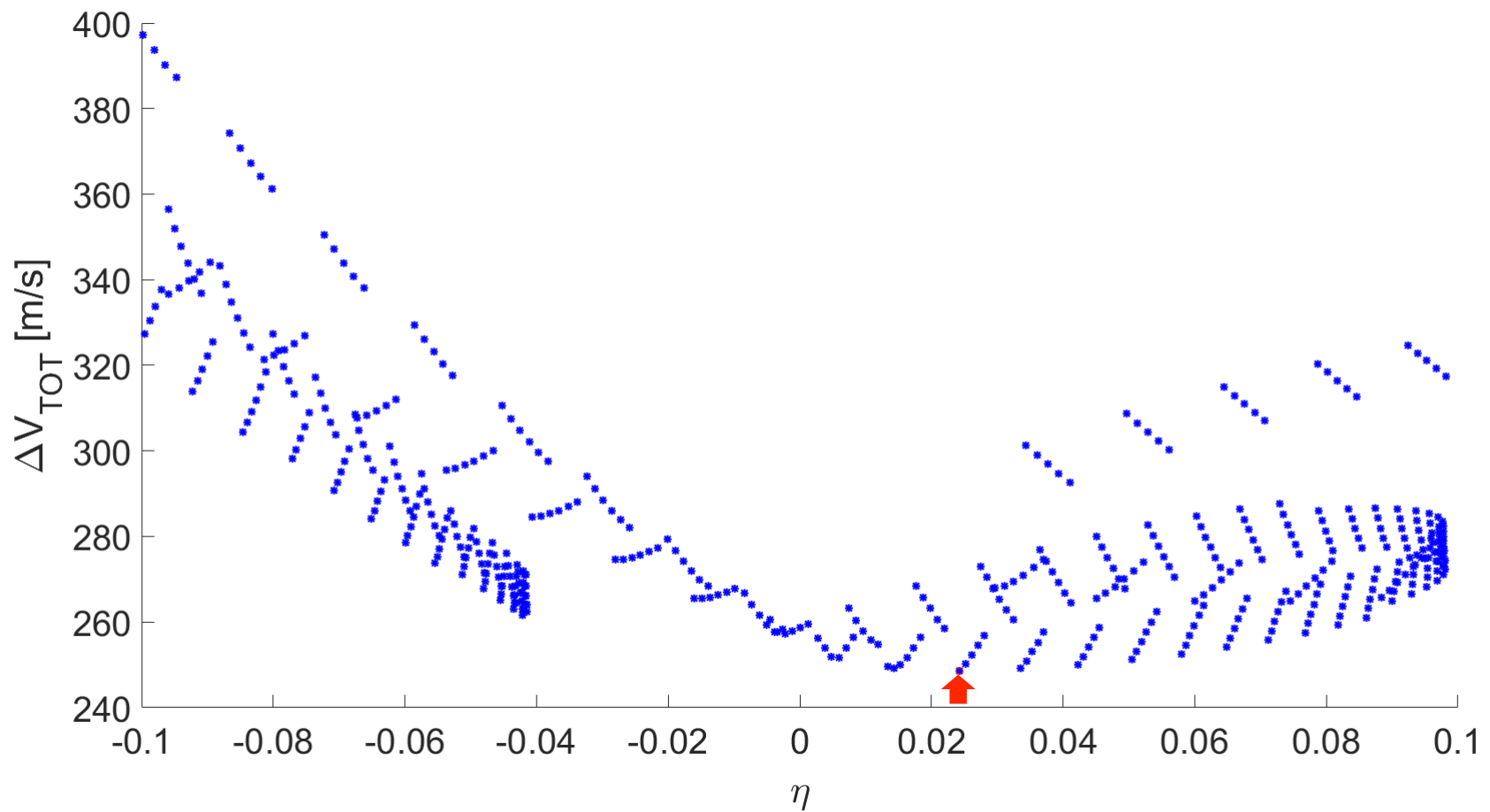
- Select a range of initial conditions in a limited range $[\eta^-, \eta^+]$
- Propagate each i.c. for k revs, such that:

$$T_{\text{REPH}} - \sum_{i=1}^k T_i \approx T_1$$

- Propagate the (k+1)-th rev until intersection with x axis
- Use single shooting technique to target the original position with constraints on **initial and final position** and **velocity** as free variable.
- Evaluate the total $\Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3$



SELECTION PROCESS - 4



MINIMUM ΔV REPHASING ORBIT

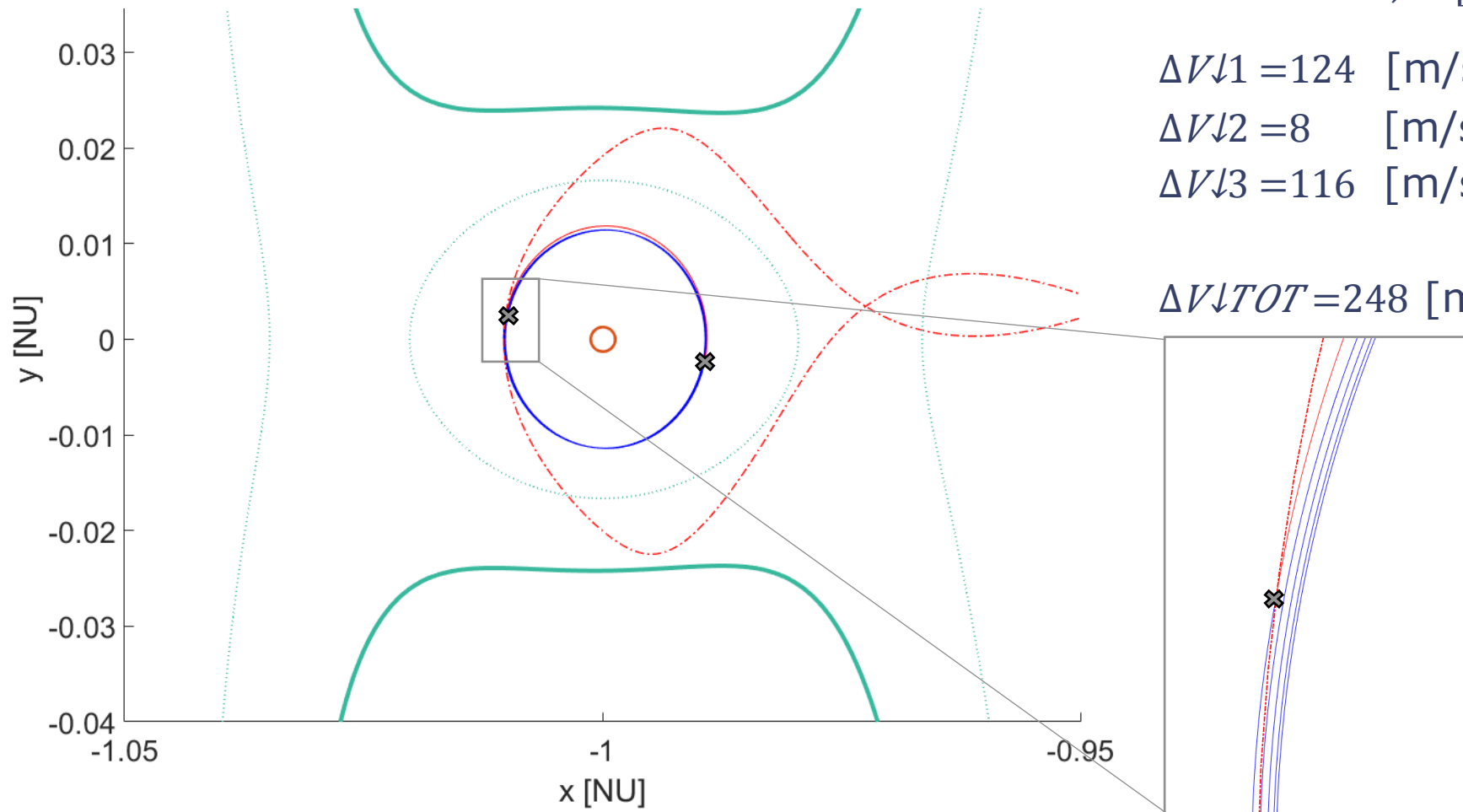
$T_{\downarrow REPH} = 15,21$ [days]

$\Delta V_{\downarrow 1} = 124$ [m/s]

$\Delta V_{\downarrow 2} = 8$ [m/s]

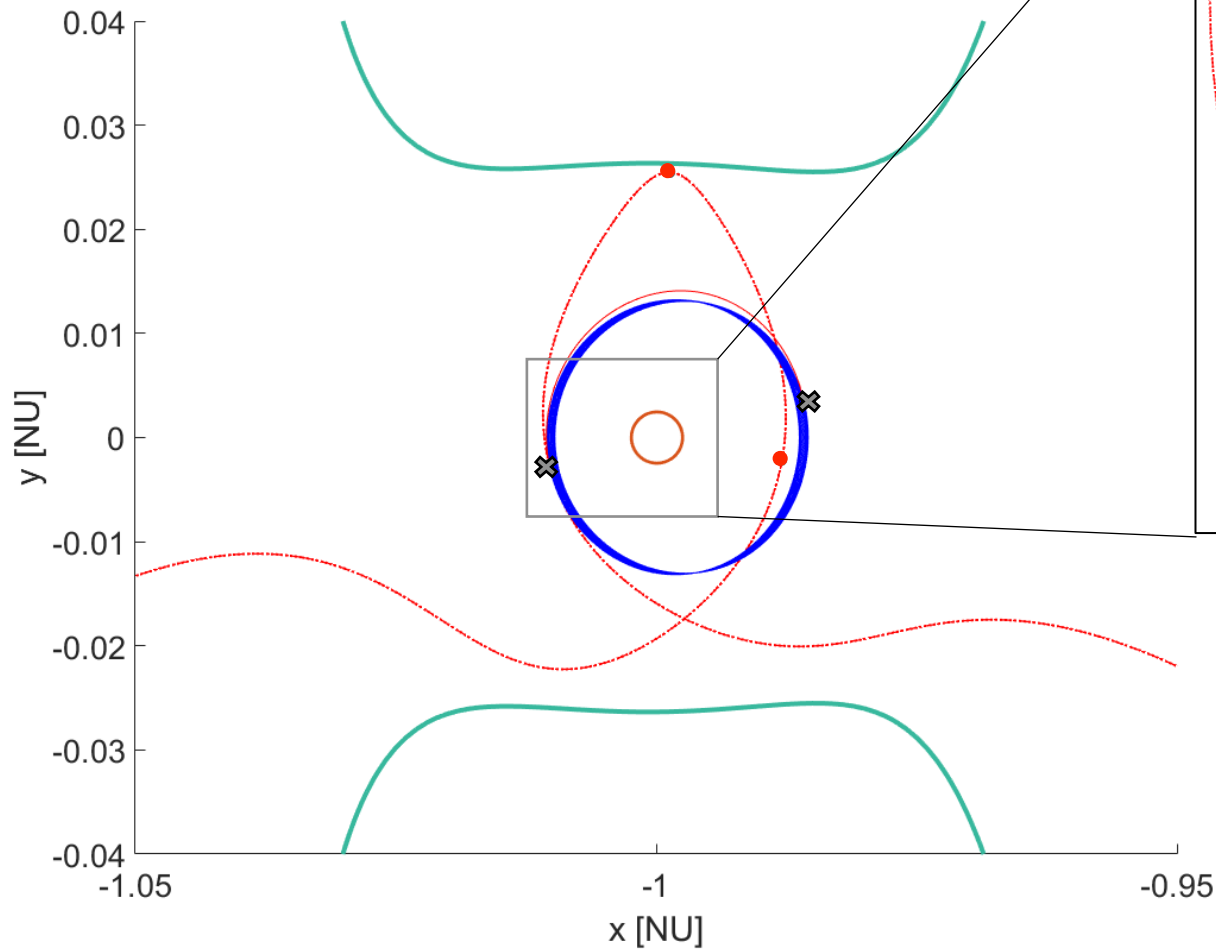
$\Delta V_{\downarrow 3} = 116$ [m/s]

$\Delta V_{\downarrow TOT} = 248$ [m/s]



REPHASING - GANYMEDE

$$T_{\downarrow REPH} = 5,65 + n7,08 \text{ [days]}$$



$$T_{\downarrow REPH} = 12,70 \text{ [days]}$$

$$\Delta V_{\downarrow 1} = 150 \text{ [m/s]}$$

$$\Delta V_{\downarrow 2} = 29 \text{ [m/s]}$$

$$\Delta V_{\downarrow 3} = 133 \text{ [m/s]}$$

$$\Delta V_{\downarrow TOT} = 312 \text{ [m/s]}$$

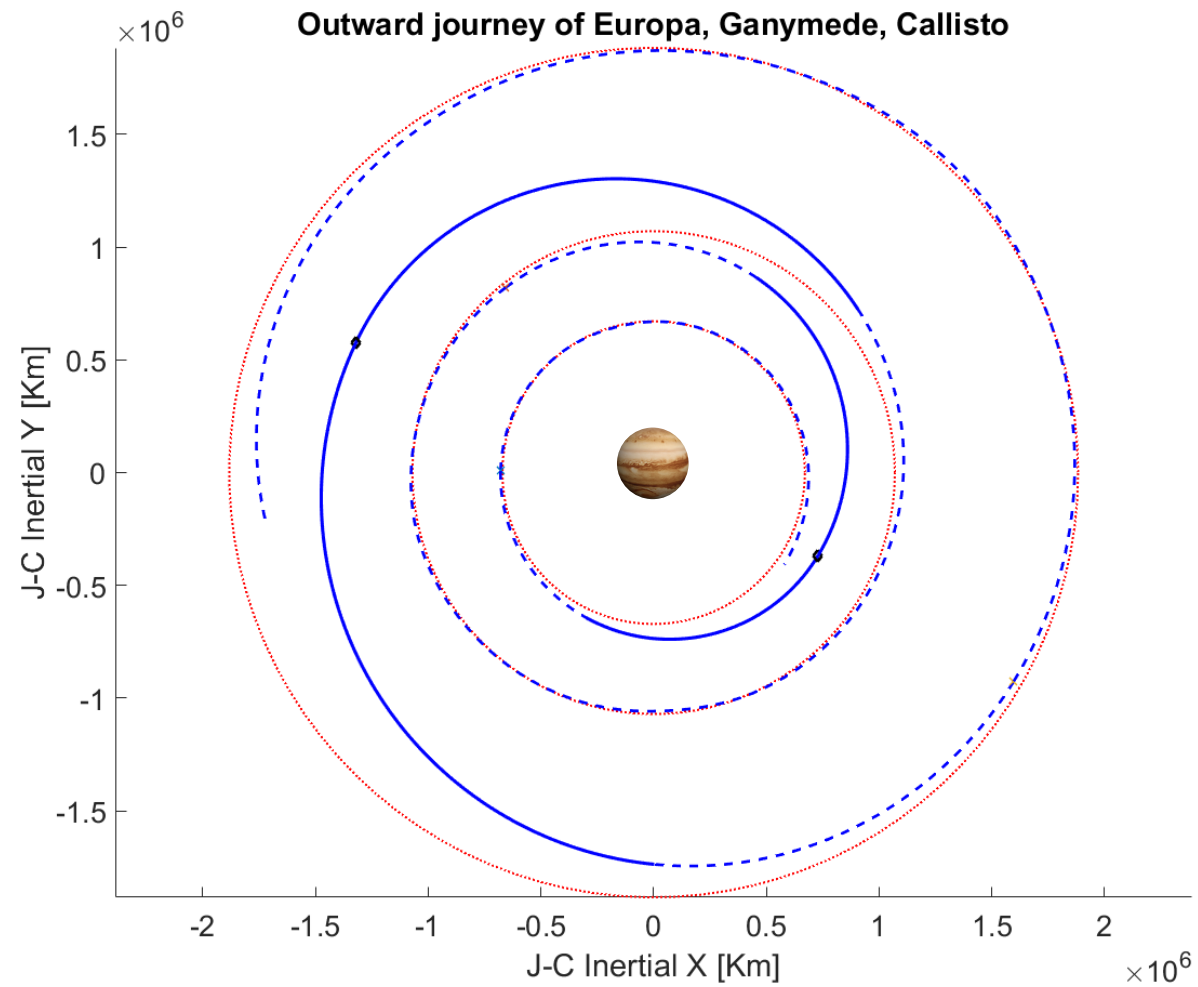
CONCLUSIONS

- The method combines efficiency and accuracy. It is a natural evolution of PGT.
- Parking orbits has 2 pros: more time spent on the moon, more flexibility on the method.
- Our relatively high cost is due to the «direct» transfer (different from «indirect» transfer on MMO): it is a consequence of the planetary system dynamics.
- ΔV s are high \rightarrow to be affordable electrical motors must be used.
High I_{sp} of e.m. \rightarrow less fuel.

	ΔV [Km/s]	TOF
Hohmann	~11	~ 17 days
Present work	4,59 + 0,56	77 days
MMO	0,022	4 years

Thank you

LOW - THRUST



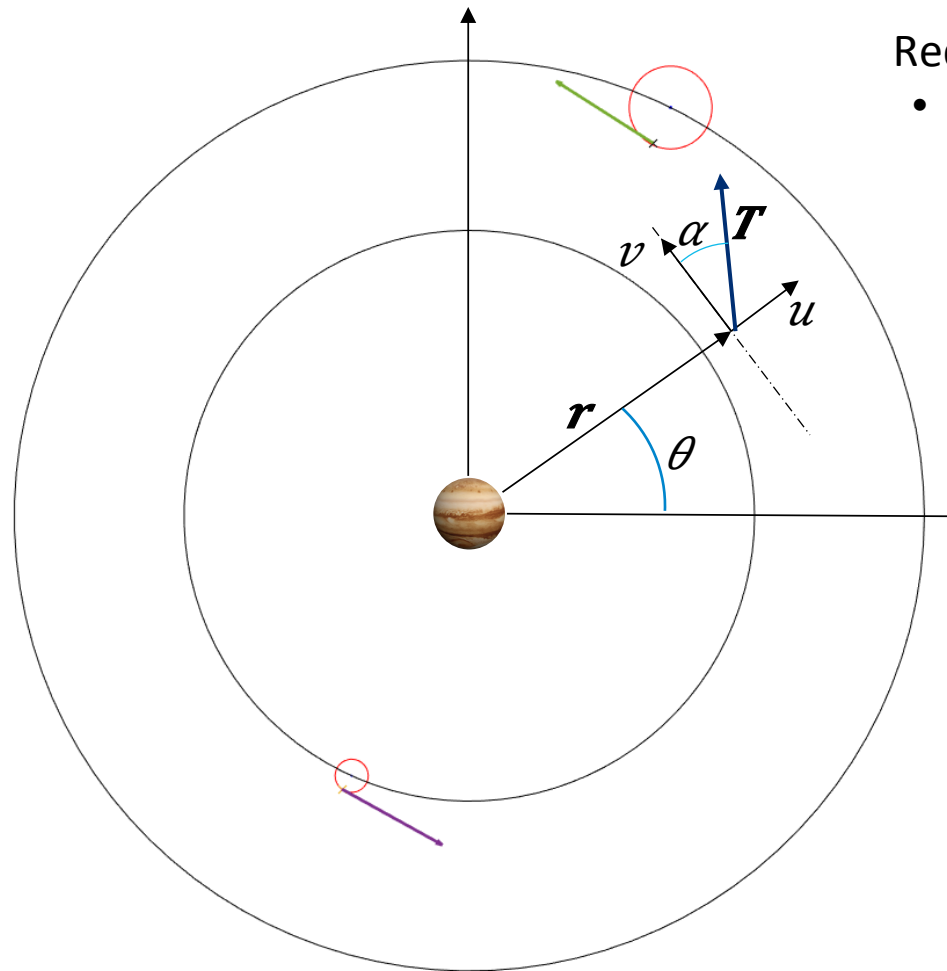
LOW-THRUST

CSI
engine

$I_{sp} = 3500 \text{ s}$
 $m_{\dot{0}} = 500 \text{ Kg}$

As a first guess

- Initial state
- Final orbit altitude



Required constraints:

- Initial/final state
- TOF

LOW-THRUST

Maximize:

$$J = r(t_f)$$

subjected to:

$$\{ \ddot{r}, \dot{u}, \dot{v}, \dot{\theta} \} = f(r, u, v, \theta) = \{ \ddot{u} = v^2/r - \mu/r + T \sin \alpha / m, \dot{v} = -u v / r + T \cos \alpha / m, \dot{\theta} = -v / r \}$$

Euler-Lagrange eqs

$$\begin{aligned} \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}} \right) - \frac{\partial L}{\partial x} &= \lambda \left(-\frac{\partial f}{\partial x} \right) \\ \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{u}} \right) - \frac{\partial L}{\partial u} &= \lambda \left(\frac{\partial f}{\partial u} \right) \\ \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{v}} \right) - \frac{\partial L}{\partial v} &= \lambda \left(\frac{\partial f}{\partial v} \right) \\ \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}} \right) - \frac{\partial L}{\partial \theta} &= \lambda \left(\frac{\partial f}{\partial \theta} \right) \end{aligned}$$



$$\{ x(t_f) = x(t_0), \lambda(t_f) = (\partial \Phi / \partial x) \}$$

Control variable

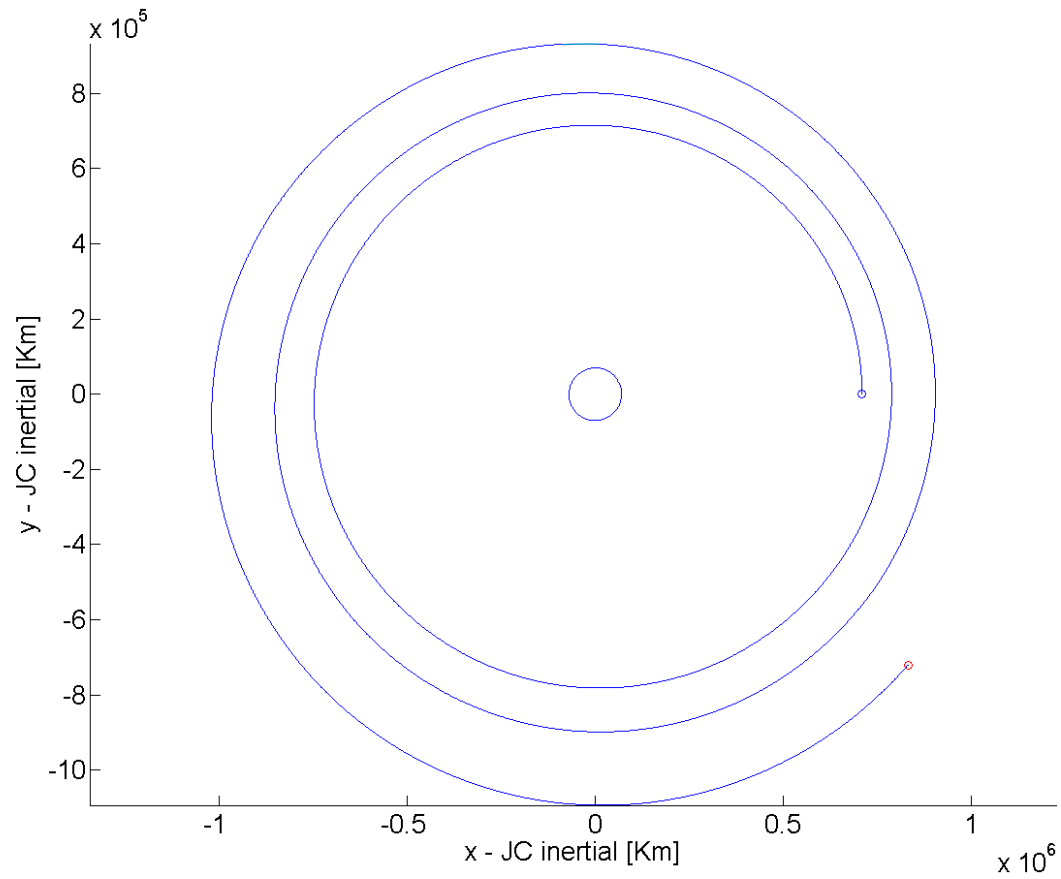
$$u = \alpha$$

State variables

$$x = \{ r, u, v, \theta \}$$

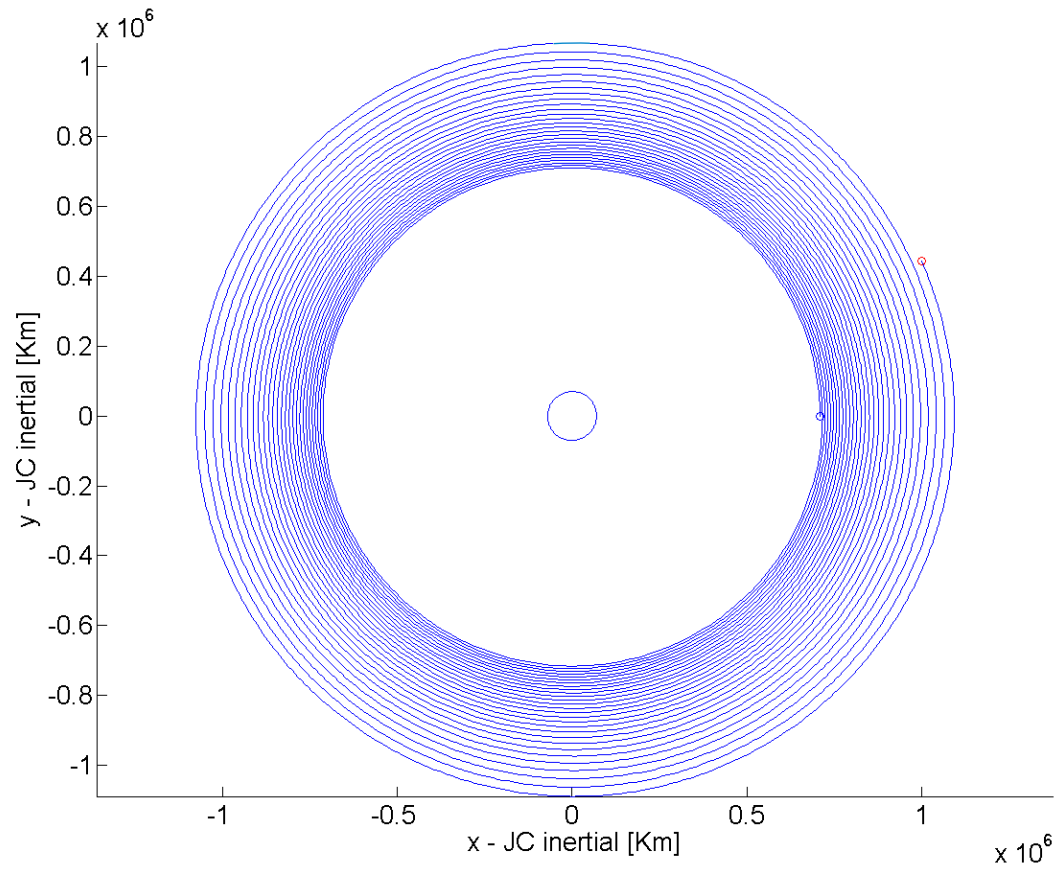
LOW-THRUST

$T=1\text{ N}$
TOF = 15 days



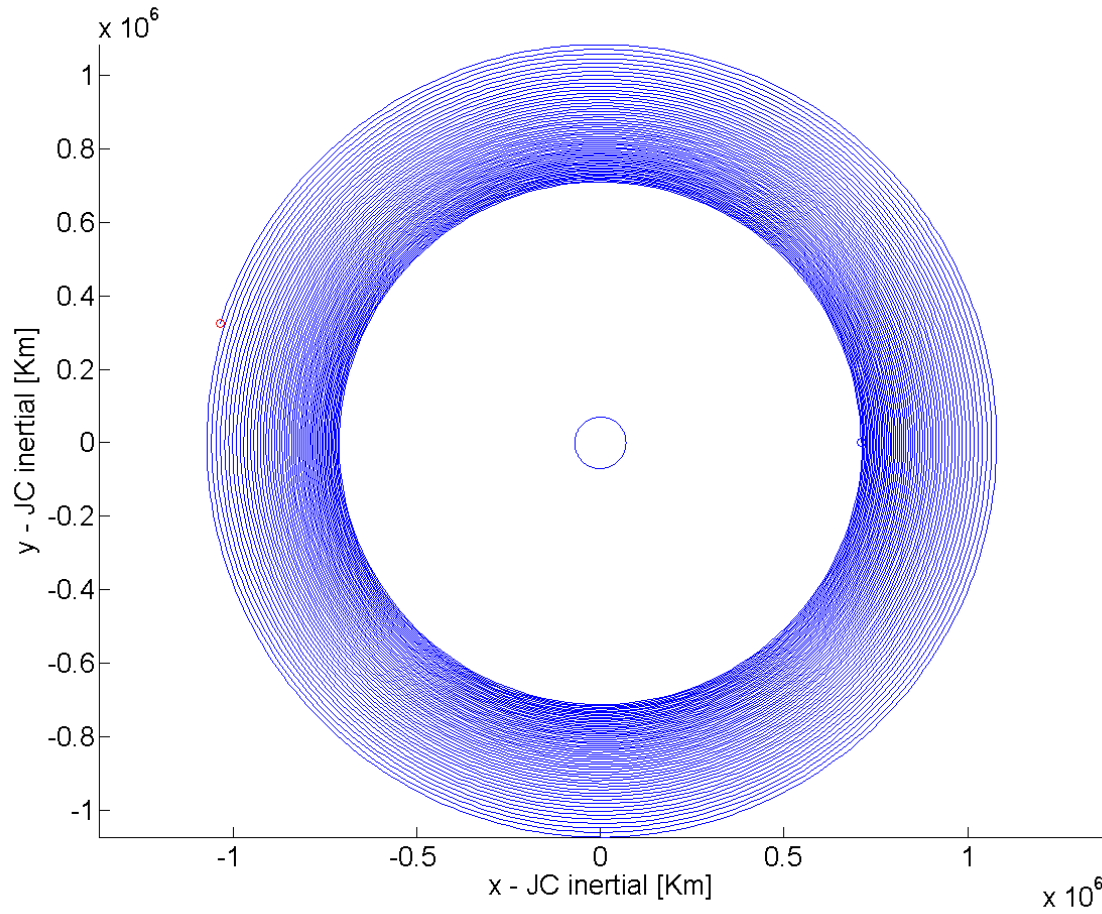
LOW-THRUST

$T=200 \text{ mN}$
TOF = 75 days

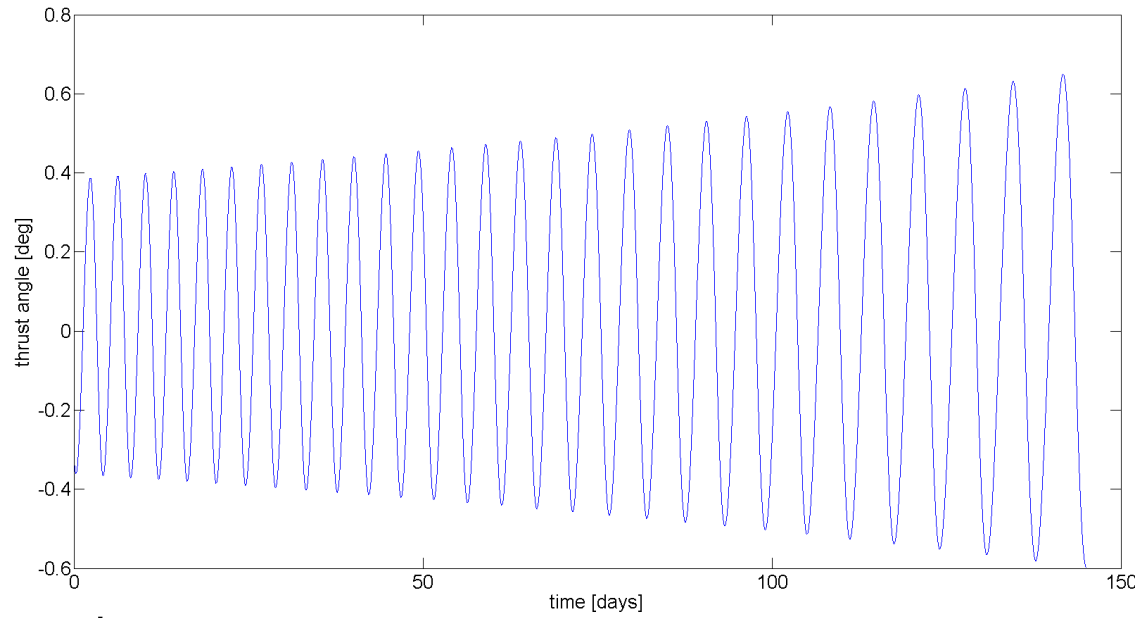


LOW-THRUST

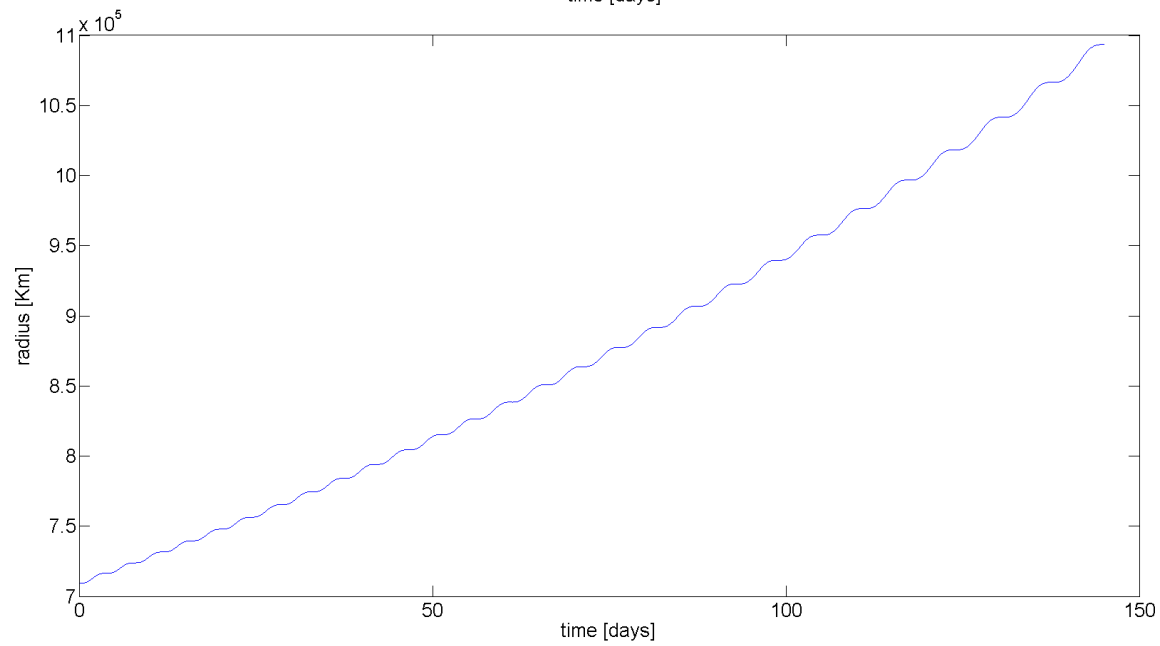
$T=100 \text{ mN}$
TOF = 145 days



Thrust
Angle



Altitude



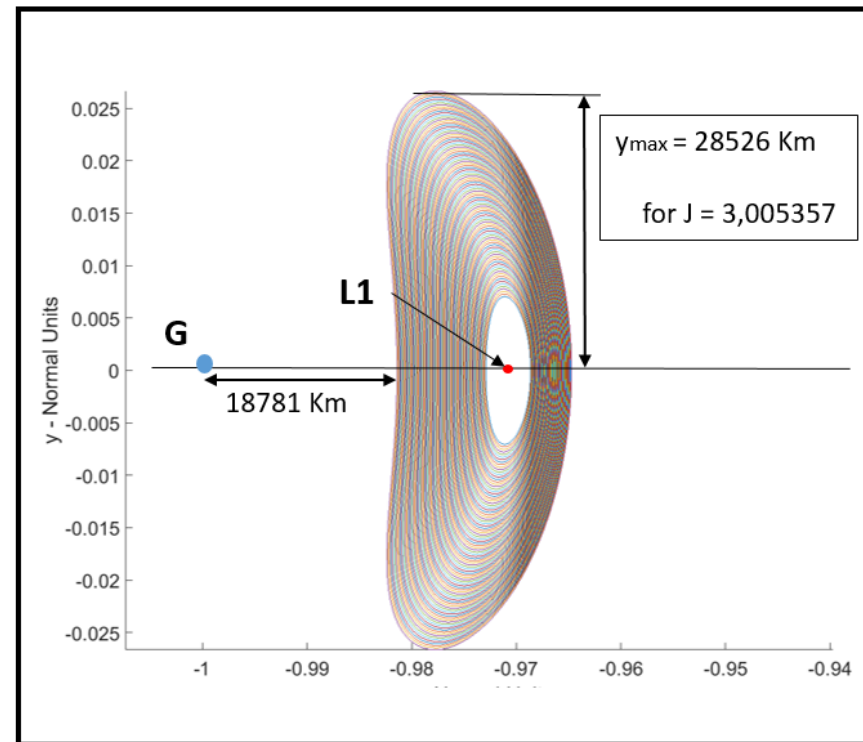
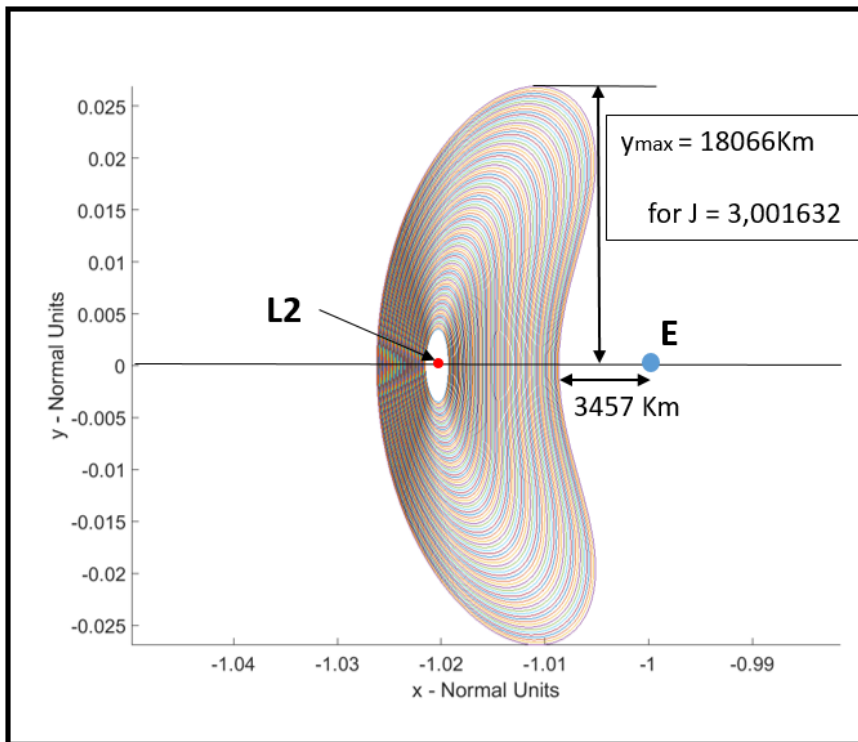
$T=100 \text{ mN}$

NEXT STEPS

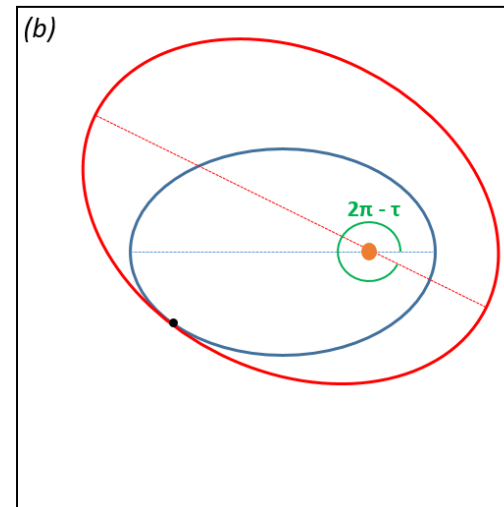
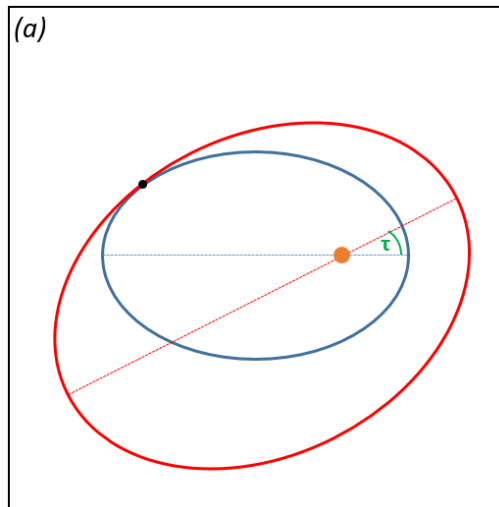
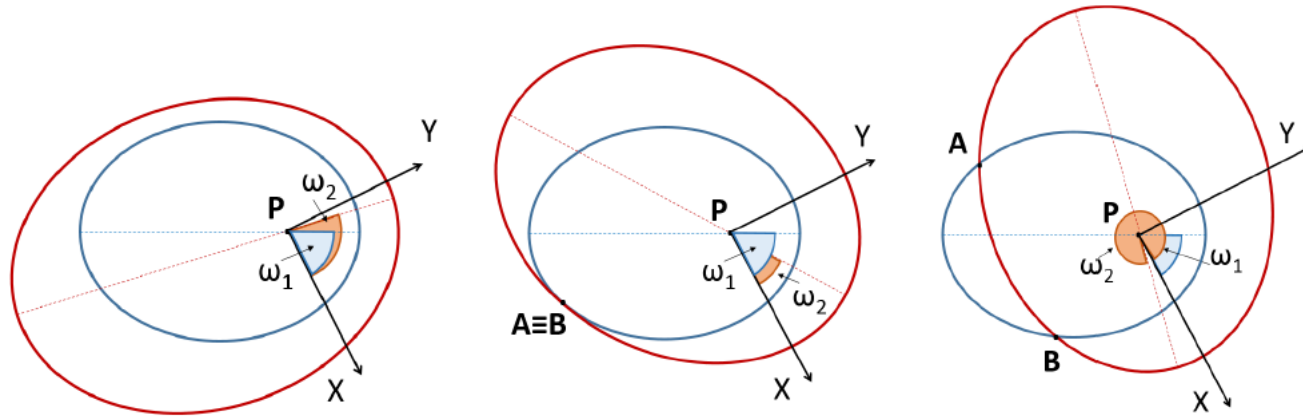
- Use low-thrust initial guess to add initial/final state constraints.
- Find impulsive ΔV s using ephemeris model.

APPLICATION: FROM EUROPA TO GANYMEDE

PLO database around Europa L2 and Ganymede L1.



ELLIPSES INTERSECTION



TRANSIT COUPLINGS

