



FERMI NATIONAL ACCELERATOR
LABORATORY

TECHNICAL DIVISION

SUPERCONDUCTING MAGNETS DEPARTMENT

**Sub-modeling of Nb₃Sn 10-stacks
compression in elasto-plastic
regime, and How to apply results
to coils**

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Abstract

Mechanical design of Nb₃Sn cables still remains a challenging task even after years of researches. This two summer school work is inserted in a set of studies carried on by previous students focused in the modelling of each step of the manufacturing phase and the operative phase, and tries to use simple and efficient ideas, as the submodeling (1) and the simulation of a test instead of directly the electro-mechanical problem (2), to understand the stress and strain distribution inside the superconductor, in order to define a critical zone and some parameters helpful for the design and for stress and strain management techniques (3).

Five finite elements models, created with the software ANSYS[®], are proposed to simulate a compressive test, than a micro mechanical model of a single cell of a cable is used to obtain the anisotropic elastic parameters of the macro composite material and to have a order of magnitude of how more stressed are the cables with respect to the homogeneous material. In parallel to finite elements analysis, some tensile tests were performed on reacted Nb₃Sn + Cu wires, unreacted Nb₃Sn + Cu wires, heat treated Cu wires and standard Cu wires.

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1 Introduction

The application of *Nb₃Sn* in superconducting magnets still remains a challenging task even after twenty years of studies.

The combination of the strain sensitive and brittle structure makes the Nb₃Sn an incredibly difficult material to manage, indeed it has never been reached an higher field of 13.8 T on an accelerator magnet. Nowadays the method used for the mechanical design of a coil is to model it as an homogeneous material. In this way the outputs are medium stress and strain state that can be useful for the design of all the mechanical structure. However the information about local state inside the coil are lost.

The idea is to model the coil with all the components in order to understand how stresses and strains are distributed between them and to see which are the critical parts and the critical parameters.

Of course this grade of detailing is quite difficult to reach, both because it is quite complex to know all the residual and actual stress acting on the conductor, both because in this way the computational time and power increase a lot. A possible solution to the second problem is to use a Finite Element Analysis called *Submodeling*. This work tries to give a better understanding of how stresses and strains are distributed into the conductor using this kind of technique, applying it to a compressive test on a ten stack specimen.

1.0.1 The Sub-modeling technique

Sub-modeling is a finite element technique used to get more accurate results in a region of the model. Often in finite element analysis, the finite element mesh may be too coarse to produce satisfactory results in a region of interest, such as a stress concentration region in a stress analysis. The results away from this region, however, may be adequate (4) (5). To obtain more accurate results, there are two options:

- re-analyze the entire model with greater mesh refinement
- generate an independent, more finely meshed model of only the region of interest and analyze it

The first option is, in a magnet design, too time-consuming and costly. It has to be noticed that in this case the sub-model is not only in terms of mesh refinement, but also in terms of mechanical detail, indeed the model switches to one with homogeneous properties to one with inhomogeneous ones, and this is the only way to understand local stress state. The type of analysis done is so a combination of

Finite Element Techniques with the composite materials theory, in particular the mechanical theory of a laminated section (6).

1.0.2 The compressive test

Since the technique is innovative, it has been applied to something easier and more manageable than a dipole finite element simulation. It was decided to reproduce a compressive test on a so called *Nb₃Sn ten stacks*, made by members of the Technical Division (7). Moreover the test would be useful to compare various mechanical model, in terms of computational time and accuracy, and to define an operative procedure to create an analytical model of the homogeneous material, giving its mechanical properties (E , ν and G) in all the three directions. Finally this test, whose experimental apparatus is shown in Figure 1.2 even if it is made compressing the specimen one direction at the time, is a good way to understand how the coil is loaded during the assembly phase, given the correspondence between the loading directions (Figure 1.1).

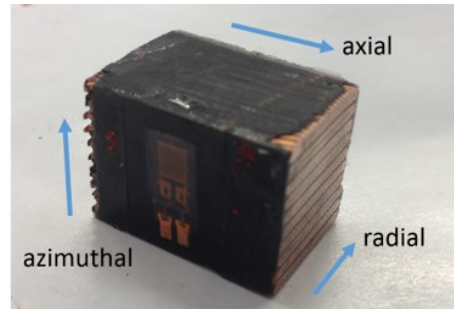


Figure 1.1: 10-stacks with loading directions



(a) Hydraulic press



(b) Fixed and movable jaws

Figure 1.2: Experimental setup

2 Modelling

In this section are discussed the assumptions made for the analysis, from the values of the mechanical properties to the geometrical features. The specimen in exam is to all effects a composite material, its components are:

- Nb₃Sn strands
- epoxy
- insulation

It can be modelled as ten rectangular plates one above the other. Each one is composed by 40 strands, with an insulating tape on the outer borders and the epoxy to impregnate the remaining parts, as can be seen from Figures 2.1 and 2.2.

It has to be noticed that in this composite the fibres, i.e. the strands, are not



Figure 2.1: Strands and insulation

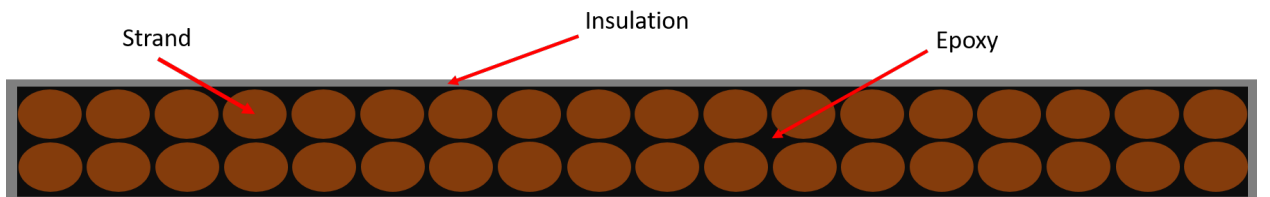


Figure 2.2: Model of one layer

much more rigid than the matrix, i.e. epoxy plus insulation, as usually is in fibrous materials.

2.1 Material Properties

The set of materials used is the following, each material, except for copper and the homogeneous strand, is modelled as linear elastic, since from literature both Nb₃Sn both epoxy and insulation, which is also a kind of epoxy, are considered as brittle materials (8)(9). For the copper and the strand a bi-linear model is used, with the

plastic region described by an horizontal line (elastic perfectly plastic behaviour). For the values of Young's modulus, Poisson's ratio and yielding of each materials, data were collected in literature and in data sheets. Finally all of the components, except for the homogeneous model, are considered as isotropic.

2.2 Elements and connections between components

For all the models a plane problem was analysed, in order to reduce the number of elements generated by the meshing. All of the components were created through the element PLANE 182, with the generalized plain strain option activated, since the specimen is free to deform during the compressive test. It has to be noticed that this element requires as a material parameters three Young's's modulus, three Poisson's ratio and the shear modulus on the plane. All of the models were meshed with less than 10% of triangular elements to have a results independent of mesh size.

All the different components are connected through a linear contact, generated by the *AGLUE* command: it has the advantage to make the simulation faster and less oscillating, while in this way it is possible only to see the action exchanged at the interface, instead of considering also the possibility of detachment.

2.3 Model assumptions

The purpose of these analysis is to give an idea of the order of magnitude of the stress acting on the Nb₃Sn + Cu wire and to have a good approximation of the behaviour of the composite macro material.

In order to make a simple and rapid analysis some assumptions had to be made. It should be stressed that the resistance results are heavily influenced by geometrical details, so the study will give a range of how much the strand is more stressed than the homogeneous material.

Below are listed all the approximation of the five models.

- Geometrical model is undeformed
- No residual stresses coming from the manufacturing process are considered, for instance winding or folding of the wires
- No stainless steel sheet is inserted between the two layer of the cable
- The two levels of strands are in contact, through a thin layer of epoxy, in a point contact
- The dimensions of the single strand are adapted to the dimensions of the cable

- All the materials are considered isotropic
- Plastic behaviour of the copper is considered as perfect, no work hardening is introduced
- Copper is the only material that can bear plastic deformation
- Inner core of the wire is modelled as copper and not as bronze
- Nb₃Sn core is considered as an homogeneous mixture of niobium and tin instead as circular rings inserted in a copper matrix as shown in Figure 5
- The behaviour of each material in traction and compression is the same

2.4 Observations about uncertainty of parameters

All the properties used for these simulations have a great grade of uncertainty, since no many works could be found in literature about the definition of elastic properties of Nb₃Sn (10) (11), besides also epoxy and insulation can vary their properties a lot if their composition or heat treatment is different.

2.5 Observations about symmetries

Looking both at the geometry of the specimen both at loading and constrains, it is possible to assume that good results can be obtained by only modelling a single cell of the specimen instead of the entire ten-stack. Anyway studies were made to confirm these assumptions. The model used is the one shown in Figure 2.3.

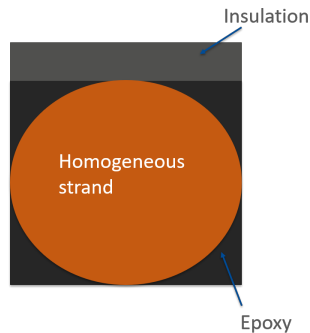


Figure 2.3: FE model with homogeneous strand

3 Finite Element Analysis, step 1

Through the last model presented two parallel task were accomplished: a sensitive analysis on the critical parameters and the obtaining of the global properties of the composite material. Finally a detailed analysis was ran with the Nb₃Sn core model trying to include the more realist properties and geometrical conditions.

3.1 Obtaining of the homogeneous properties

Using the assumption of orthotropic transversally isotropic behaviour of the composite, two parameters can be obtained through a FE simulation of the cell, E_{\perp} and ν_{\perp} , and with the analytical correlation G_{\perp} .

3.1.1 FE procedure

Here is described the steps to be followed to extrapolate the values of Young's modulus and Poisson's ratio (Figure 3.1).

- Load the model with a datum displacement
- Isolate the upper contour reaction forces
- Integrate those along the perpendicular direction to the loading (*FSUM* command)
- Extrapolate the displacement of the free to move face along the perpendicular direction to the loading

The results of the integration, in a plane problem, has the dimension of $\frac{N}{mm}$, to obtain the average stress it has be divide to the width of the cell. Normalizing imposed displacement with the height of the cell and X displacement with le cell width the deformation along Y and X are obtained. At this point the values of the two parameters are extracted thanks to their definition.

$$E_{\perp} = \frac{\sigma_{av}}{\epsilon_y} \quad (3.1)$$

$$\nu_{\perp} = -\frac{\epsilon_x}{\epsilon_y} \quad (3.2)$$

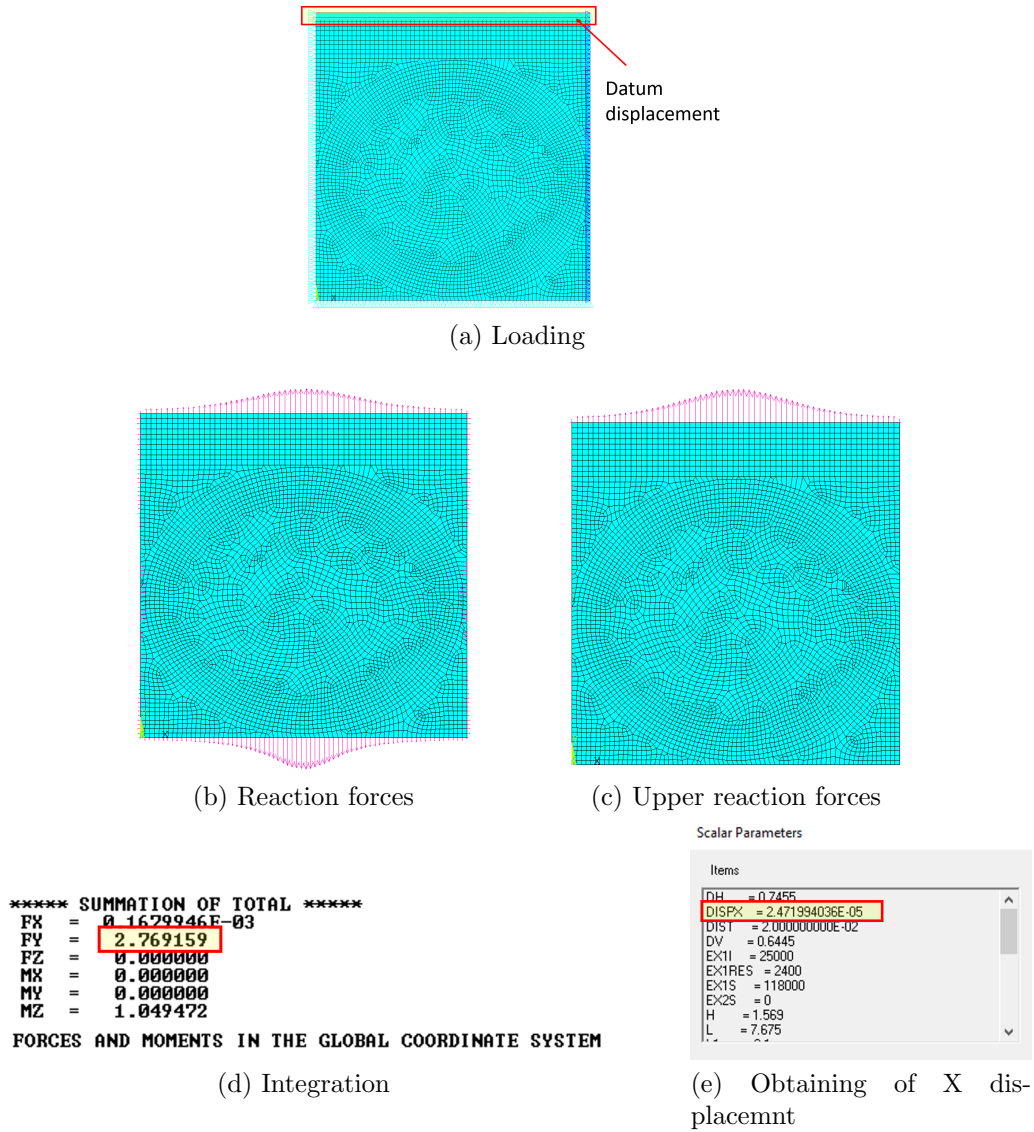
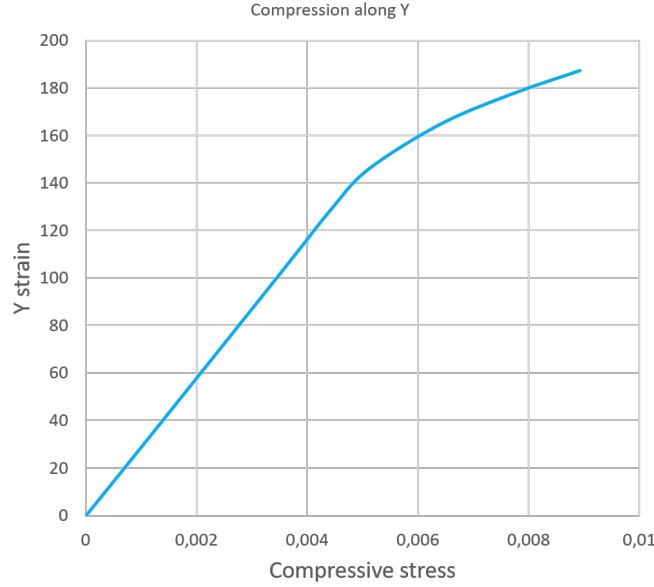


Figure 3.1: FE procedure

3.2 First set of results

Using the mechanical properties listed in section 3.3, the results for the two parameters and the σ - ϵ plot are the following, meaning with E_{\perp} the Young modulus on the plane of the model (equal in horizontal and vertical direction), with ν_{\perp} the Poisson's ratio on the plane and with G_{\perp} the shear modulus on the plane, linked with the other two parameters with the relation $G_{\perp} = \frac{E_{\perp}}{2(1+\nu_{\perp})}$.

- $E_{\perp} = 29 \text{ GPa}$
- $\nu_{\perp} = 0.26$
- $G_{\perp} = 11.5 \text{ GPa}$

Figure 3.2: σ - ϵ diagram

As it is possible to see also the plastic regime can be considered, in this way it is introduced another uncertain parameter, i.e. the yielding point of the material. In parallel to the creation of the plot, also the ratio between maximum internal stress and average external stress is evaluated.

$$k = \frac{\sigma_{int,max}}{\sigma_{av}} \quad (3.3)$$

For this set of parameters the ratio $k = 2.03$. As a confirmation of the mechanical model, also a test with an horizontal loading was made.

- $E_{\perp} = 30.5 \text{ GPa}$
- $\nu_{\perp} = 0.27$
- $G_{\perp} = 12 \text{ GPa}$

The difference in the Young moduli is under 5%, so the model, with this grade of detailing, is widely confirmed.

Since the properties of each component are the same in traction and compression, so will be the properties of the global composite.

3.2.1 Comparison with the other model

Using the same procedure in the quarter of a cable model and in the entire model, the following results are obtained.

- $E_{\perp,model4} = 29.5$ GPa
- $E_{\perp,1/4cable} = 28.5$ GPa

The percentage error is really contained, with the great advantage of two order of magnitude less of computational time.

3.3 Sensitivity analysis

Both geometrical and mechanical features are quite uncertain. Shape of deformed strand, thickness of epoxy between two wires or between wire and stainless steel sheet is really variable. Even the way the strands are bundle is variable, making really complex to make an accurate study.

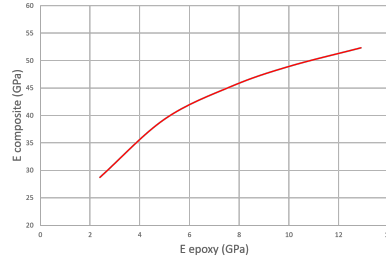
The following parameters are varied in a reasonable range, and the trend of Young's modulus, Poisson's ratio and peak pressure ratio is studied.

- Young's modulus of epoxy
- Young's modulus of insulation
- Young's modulus of strand
- Thickness of epoxy between two layer
- Shape of the cable, from elliptical to hexagonal
- Presence or not of SS sheet
- Anisotropic of epoxy plus insulation

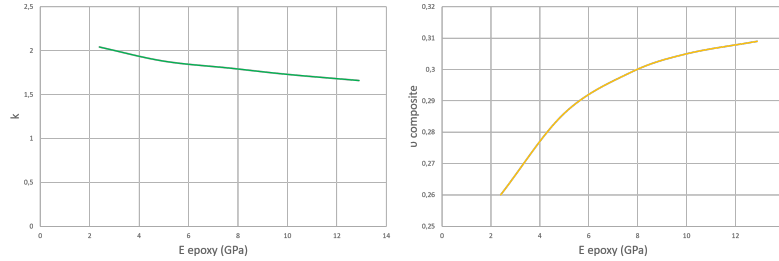
The range of mechanical parameters of epoxy and insulation was chosen thanks to data sheet of different kind of resins, as for the anisotropic model (12)(13)(14)(15)(16)(17). The big uncertainty still remains in elasticity modulus of the strand. The thickness of epoxy, and so of the SS, was chosen varying it between 0 and the thickness of underformed layer of steel(18). It has to be noticed that one parameter at a time is varied. Finally four plot of σ - ϵ diagram are made with four combination of component's Young's modulus. Is it possible to see that the intensification stress factor is more affected by geometrical features, even if it does not vary so much, it always has a quite high value, from 2 to 3. Young's modulus of the overall composite material is instead more affected by the mechanical properties of the components. Varying

the Young's modulus of epoxy from 2 to 13 GPa (Figure 3.3) the variation of ν is of 20 %, for E of 82 %, for k of 23%, as the overall stiffness is the most affected by the variation of mechanical characteristic. The rigidity values of insulation and strand are respectively 25 GPa and 118 GPa.

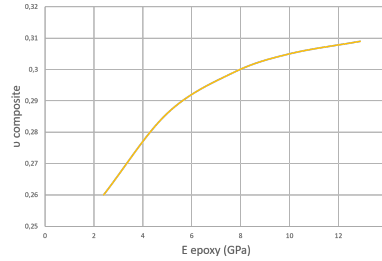
Varying the Young's modulus of epoxy from 40 to 120 GPa (Figure 3.4) the varia-



(a) Young's modulus of composite



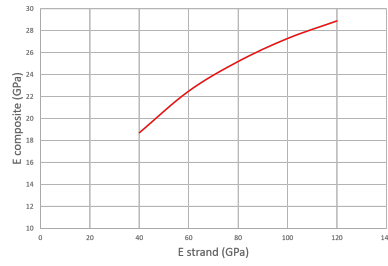
(b) Poisson's ratio



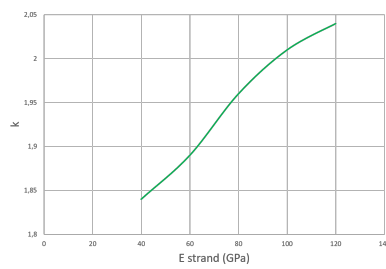
(c) Stress intensification factor

Figure 3.3: Variation of Young's modulus of epoxy

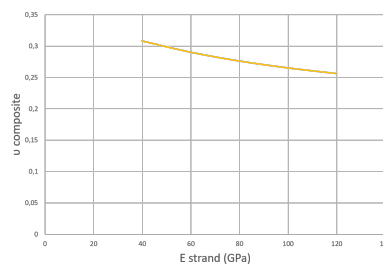
tion of ν is of 20 %, for E of 55 %, for k of 11%, as the overall stiffness is the most affected by the variation of mechanical characteristic. The rigidity values of insulation and epoxy are respectively 25 GPa and 2.4 GPa. With the variation of this geometrical features (Figure 3.5), from 0 to 20 μm , mechanical properties vary in a 20% range while the intensification factor varies of 40% reaching a value near to 3. Inserting the stainless steel sheet between two layers and varying its thickness from 20 to 25 μm (25%) mechanical properties and the intensification factor don't vary, obviously both of them increase, since the SS sheet is a more rigid material than epoxy (Figure 3.6). Finally some curves are extrapolated with different combination of stiffness represented in Figure 3.7, the yielding point of the homogeneous strand was set to 282 MPa, obtained by using the rules of mixture between niobium tin and copper. Since this is a problem similar to a contact analysis, the critical zone is located near the contact zone (Figure 3.8), here this situation is simulated through the use of symmetries, moreover the presence of the epoxy between two layers of cables cushions the stress intensity.



(a) Young's modulus of composite

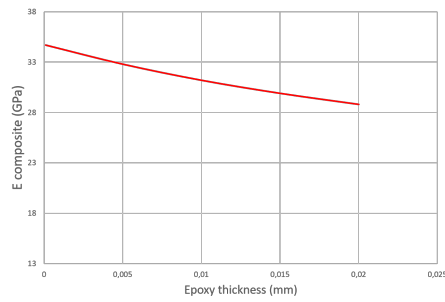


(b) Poisson's ratio

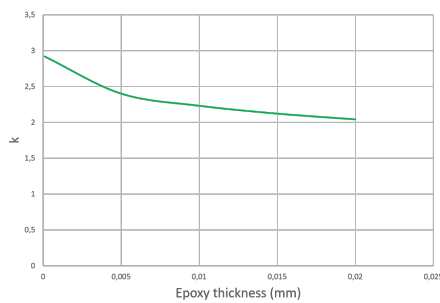


(c) Stress intensification factor

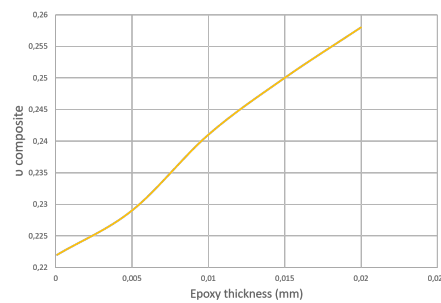
Figure 3.4: Variation of Young's modulus of homogeneous strand



(a) Young's modulus of composite

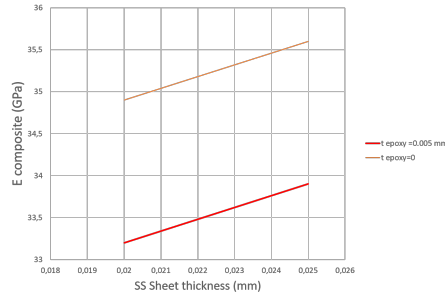


(b) Poisson's ratio

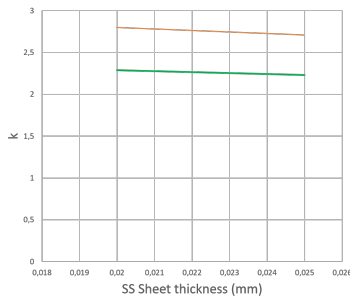


(c) Stress intensification factor

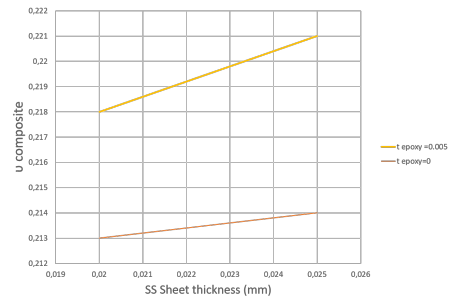
Figure 3.5: Variation of epoxy thickness



(a) Young's modulus of composite



(b) Poisson's ratio



(c) Stress intensification factor

Figure 3.6: Variation of epoxy thickness

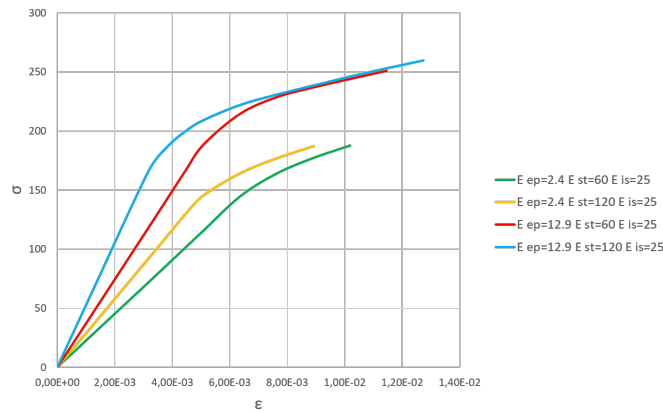


Figure 3.7: σ - ϵ diagram for various combination of mechanical properties

4 Modelling of the cell in more detail

A second set of results is obtained by modelling the cell with more detail, as it is possible to see in Figure 4.1, adding the stainless steel sheet, including the Nb₃Sn core and switching the shape of the strand from elliptical to hexagonal, more realistic.

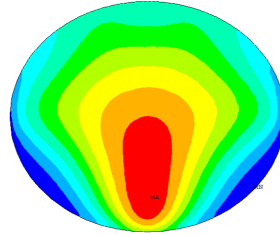


Figure 3.8: Critical point on the strand

Six curves are obtained (Figure 4.2), they are stopped when the internal stress

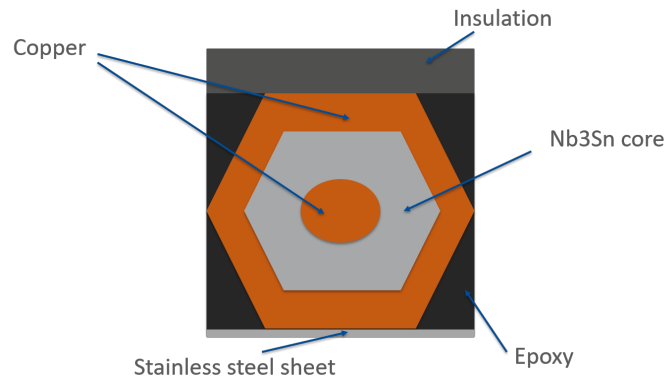


Figure 4.1: Detailed model of the strand

on Nb₃Sn reaches about 220 MPa, fracture point whose value is taken by previous summer intern's simulation. As it is notable, the behaviour of the composite is always elastic, no work-hardening is present and the failure seems to be a brittle breaking. Finally, also in this case are located the critical zones. With this grade of detailing they moves in the central part of the strand, where geometrical features act as stress intensification factors, as it is notable from Figure 4.

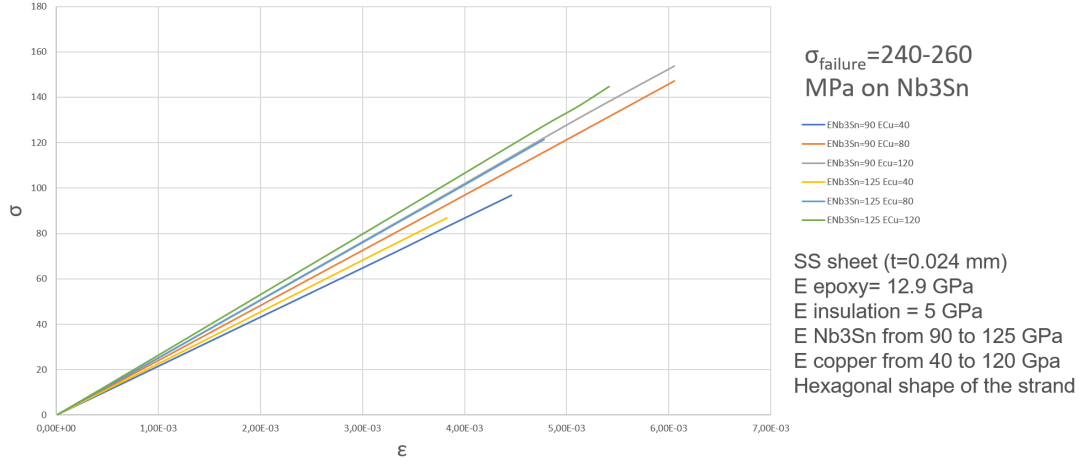
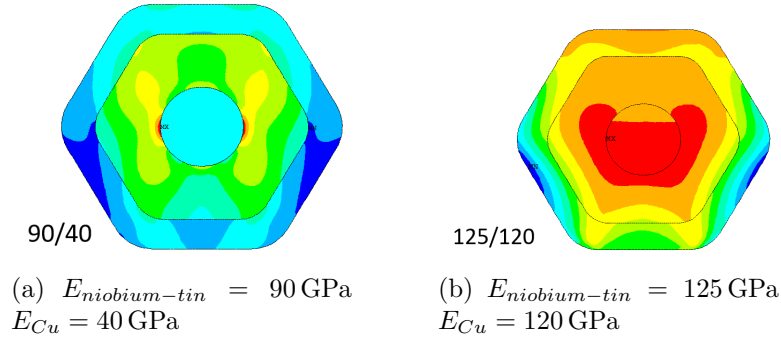
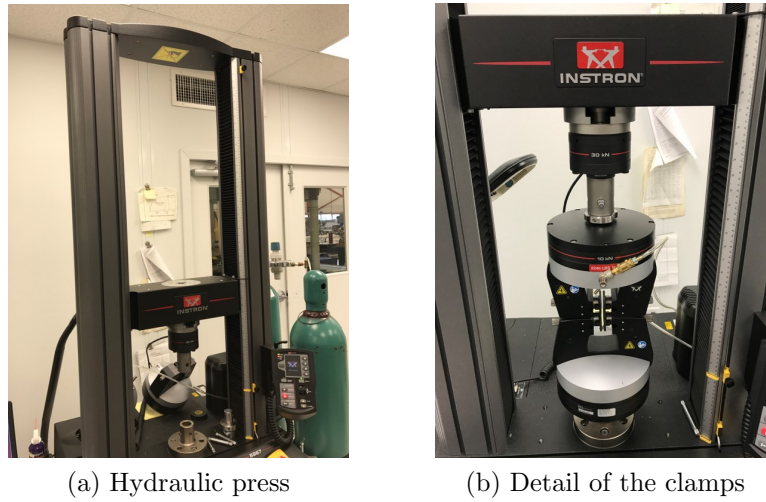
Figure 4.2: σ - ϵ diagram for various combination of mechanical properties

Figure 4.3: Critical zones

5 Tensile tests on wires

Due to the big uncertainty about mechanical properties of the various components, it was decided to perform tensile, see Figure 5.1 test on wires, in order to have a better idea of the stiffness and the yielding point of Nb₃Sn and copper and using the rule of mixtures, once known copper properties, obtain niobium tin characteristics.

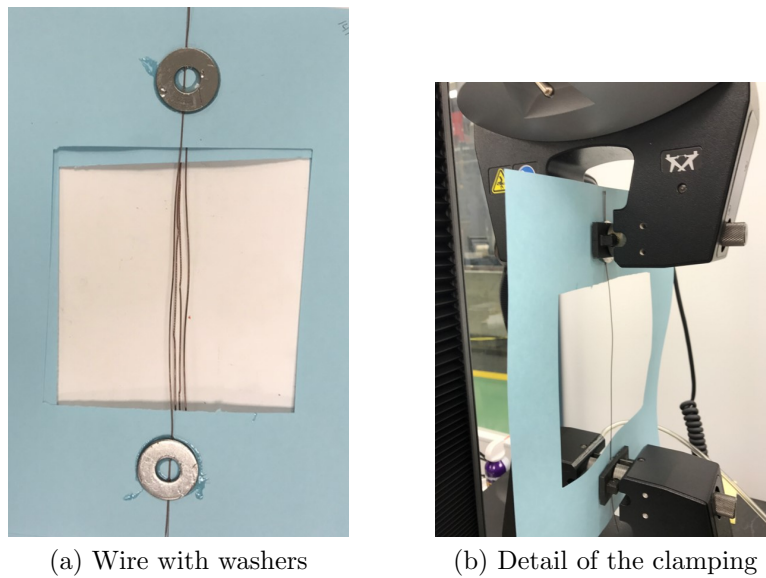
It was decided to test reacted and unreacted wires of the conductor and also of just copper. The single wire is glued to two washers used as support for the clamping, as in Figure 5.2. About 15 valid samples of unreacted conductor, 10 of reacted, and 8 of reacted and unreacted copper were tested. Unfortunately there was no time left to make a detailed post processing of the data obtained, even if the rigidity of the conductor seems to be about 50 GPa while the fracture point, it is clear from Figure 5.3 b that the wire has a brittle behaviour, 200 MPa. It is possible to see a first zone



(a) Hydraulic press

(b) Detail of the clamps

Figure 5.1: Experimental setup



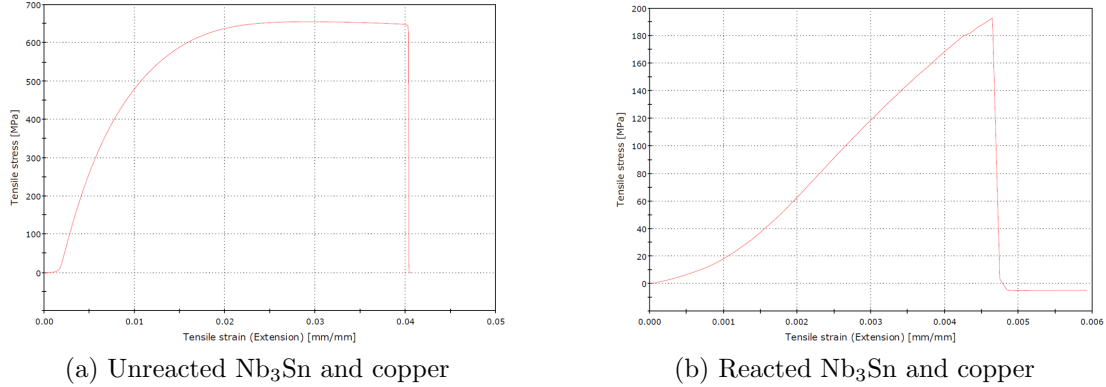
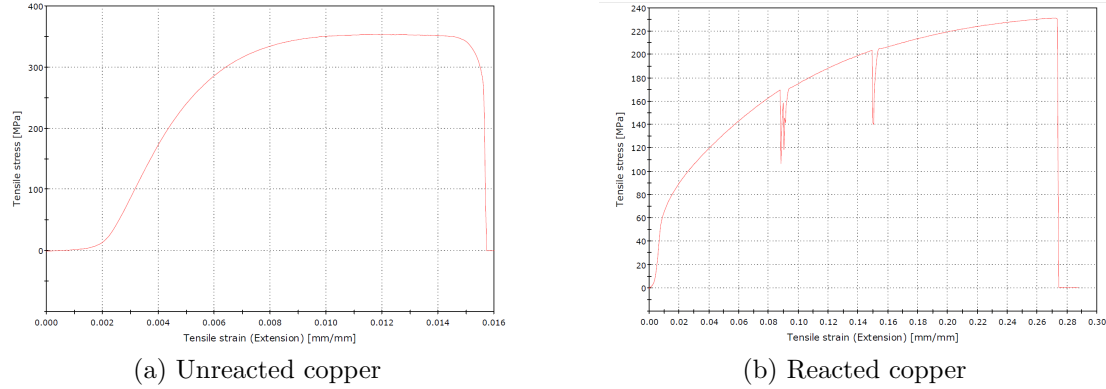
(a) Wire with washers

(b) Detail of the clamping

Figure 5.2: Specimen

were the curve as an opposite concavity, this is due to the fact that the wire is not perfectly straight and the press firstly has to stretch it. As the graphics show, some problems were encountered during the test of reacted copper wires, this is probably due to an incorrect and incomplete gluing between washer and wire.

From this first results, using data taken from literature for properties of copper and niobium tin and using the rules of mixtures, takes to a value of the Young's modulus of the homogeneous strand about twice than the experimental one.

Figure 5.3: σ - ϵ curves obtainedFigure 5.4: σ - ϵ curves obtained

6 Conclusions and further developments

6.1 Achievements

During these two months of summer internship a new way of mechanical analysis of the coil was proposed, through the study of a compressive test, representing the main load acting on a coil during the room temperature phase. Moreover critical areas were determined, where a peak of stress is present.

A procedure to obtain more accurate homogeneous properties is presented, preferably performed in parallel to experimental tests, together with an extensive sensitivity analysis to numerous parameters.

Tensile tests were performed to define more precise characteristic of the strand and of the copper, accurate post processing is required.

6.2 Further developments

The present study is the beginning of what could be a new way to analyse mechanical stress on the coil and to understand and overcome the current limits of the conductor. The models studied are a first approximation, nevertheless is evident that the strand, in particular the niobium tin core, is much more stressed than the homogeneous material. In order to improve the model, it could be interesting to perform analysis on the shape of the cable after the manufacturing process besides inserting residual stresses. It should be also considered to make detailed analysis on the way the coil actually fails, since the failure mode can occur through rupture of one component or also through the detachment of two components, as strand and epoxy. Finally mechanical characterization of each components is necessary to reduce the range of variability of the parameters.

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