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1 Introduction

2 Previous years work

During previous years, the basic idea behind the study of Nb3Sn superconducting magnet has been to go from an homogeoneous, isotropic model for Nb3Sn cables to a more detailed one, in order to understand the real stress and strain distribution inside the supercondoctor. To achieve this goal, the *sub-modeling* technique has been used on a *10-stack* specimen instead of coils; this makes the analysis simpler because the only external loads are pressures applied to the faces of the specimen. In fact, each cable is subject to external pressure both during assembly and functioning, due to precompression, cool-down process and Lorentz forces that act on the wires.

2.1 FEM model

The 10-stack specimen used for the compressive test is made of 10 cables with 40 strands each and has been studied with increasing level of detail:

- Model 1: homogeoneous cables with non-isotropic properties

- Model 2: cables made of strands + epoxy + insulator; strands have homogeneous, isotropic properties (rule of mixture used to obtain the material properties of the strands)

- Model 3: cables made of strands + epoxy + insulator; strands contain 3 different homogeneous regions (internal copper, Nb3Sn and external copper)

Because of the high level of simmetry of the problem, it has been noted that it would be better to perform FEM simulations on a single strand using appropriate simmetry constraints, instead of modeling the entire 10-stack model that takes a lot of time to compute. In fact, the test results are the same when a "single cell" model is used. The following constraints are necessary to reproduce the test conditions taking into account the symmetry of the specimen:

- fixed y-displacement of the lower surface of a cell

- planarity of the upper surface of a cell, where the external pressure is applied (y symmetry)

- fixed x-displacement of the left surface of a cell

- planarity of the right surface of a cell, where an external pressure can be applied in case of bi-axial loading (x symmetry)

If only a single cell is modeled, some geometrical features can not be taken into account, for example irregularities in the geometry of the cable, presence of a keystone angle, variations in the relative position of the strands; however, the single cell model gives a good approximation of the stress and stiffness of the entire cable.

3   Sensitivity analysys

The model used for the sensitivity analysys has been the same as...; the influence of some geometrical parameters and material properties on the maximum equivalent stress in niobium-tin has been studied in order to see the effect that the uncertainty about this data has on the simulation results, but also to understand how to improve cables properties by modifying these parameters.

Geometrical parameters:

- thickness of the epoxy layer between two strands in a cable

- niobiom-tin region dimensions (diameter of both the inner and outer hexagon of this region)

Material properties:

- copper yield strength (in order to evaluate the benefit from using higher-strenght copper alloys such as Glidcop, which has a much higher yield strength after annealing compared to oxigen-free high-conductivity copper)

- insulator Young modulus

- epoxy resin Young modulus

- niobium-tin Young modulus

4   New FEM model: strand with subelements

In order to perform a more detailed analysis of the structural behavior of Nb3Sn cable, a new, more refined cable model has been introduced. Instead of modeling the superconducting region of the strand as homogeoneous, Nb3Sn-bronze sub-elements have been implemented in the geometry; while this change does not affect too much the mechanical behavior when thermal contraction is not calculated, it leads to dramatic changes in the stress distribution due to differential thermal contraction.

In this model, each "sub-element" is made of a superconducting Nb3Sn hexagon, with a circular, porous bronze rod in its center, surrounded by a thin Cu layer. For this analysis a xxx-xxx configuration has been used.

For the structural analysis of the cell, plane183 (3 nodes, quadratic) elements have been used, with generalized plane strain behaviour in order to take into account the fact that each section of the wire remains planar.

5   Structural FEM analysys of tensile and transverse pressure tests

A simulation of tensile and transverse pressure tests has been set up in Ansys mechanical APDL. Each run consists of 3 steps:

- evaluation of the residual stress in the strand, due to the manifacturing process

- thermal contraction during cool-down from ambient temperature to 4.2K

- external load (ramped axial force or transverse pressure) application

5.1 Cable manifacturing and residual stress

The manifacturing process of a Nb3Sn cable consists of different phases:

- Each subelement is drawn from a billet, then restacked (RRP) with the others and drawn again, forming a single wire

- The strands are placed in the cabling machine, which produces a cable with the desired shape, thickness, width and keystone angle made of up to 42 strands. During this process the strands undergo plastic deformation, that can have effects on the residual stress in the cable. An insulator tape (made of glass fiber) is wrapped around the strands.

- The cables are placed in position in order to obtain the required coil geometry and then cured at 150° C

- Reaction: the coils are heat-treated at temperatures up to 650° C for 2 weeks in order to get the superconducting properties. Brittle Nb3Sn forms; the shape of the cables can not be changed without cracking Nb3Sn.

- Impregnation: the strands and the insulator layer are impregnated with epoxy resin that gives them the required strength for the assembling process

The residual stress in Nb3Sn is very difficult to obtain because of the complexity of the phenomena that take place inside the strand during assembly and reaction, which include plastic deformation during cabling and phase changes during reaction. However, the residual stress distribution can be approximately evaluated considering the differential thermal contraction of copper, niobium-tin and bronze after the reaction at 650°, during the cool-down to ambient temperature (because of the slow rate of cool-down, the temperature inside the strand has been assumed uniform during this phase).

- Copper/bronze: $\alpha = 17 \cdot 10^{-6} \rightarrow \epsilon_{th} = 0.46\%$

- Nb3Sn: $\alpha = 7 \cdot 10^{-6} \rightarrow \epsilon_{th} = 1.11\%$

The process has been simulated in Ansys APDL, and the resulting strain distribution has been saved on a file and used as an initial condition for following tests. Since the calculated value of pre-stress are very high (equivalent stress exceed the yield strength both of Niobium-tin and copper) and have a big impact on the results, the FEM analysis of tensile and transverse pressure tests has been carried on also without the initial pre-stress state condition.

5.2 Tensile test

Tensile tests are usually performed on a single strand using a helical Walters' spring device, where the strand is wrapped to form a coil and tested by appling torque to one of the cylinders of the device; however, the results would be equal if a traditional uniaxial tensile test had been done. In the tensile test FEM analysis the strand is cooled from 300K down to 4.2K, and then a ramped axial force from 0 to 100N is applied, which produces a maximum axial strain (ϵ_{appl}) of approx. 0.7%.
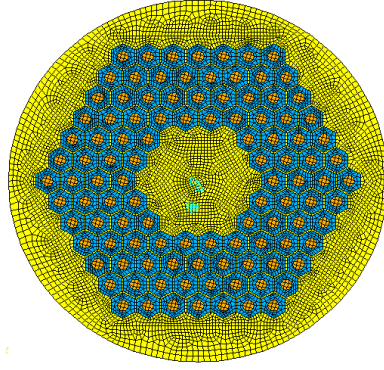
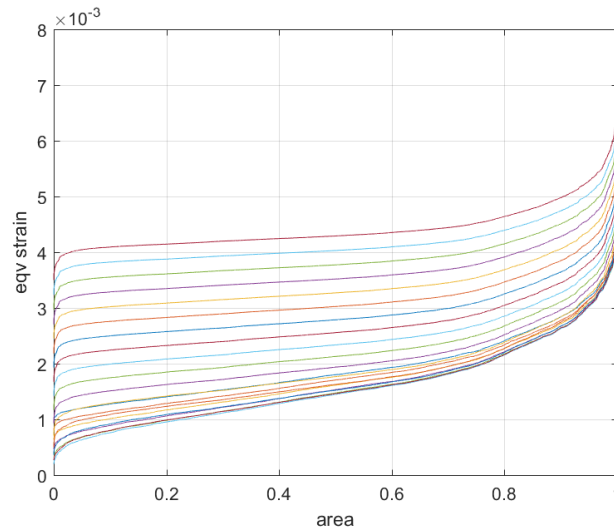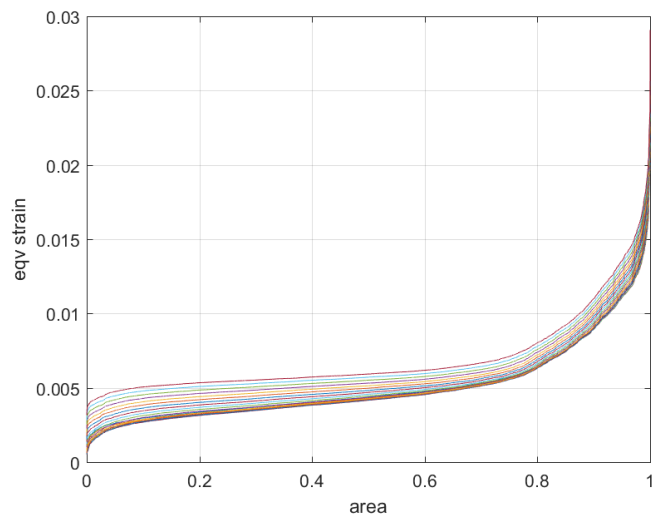Figure 1: Ansys model used for tensile test simulation

Figure 2: equivalent strain (function of area) in the strand cross section with increasing values of applied axial strain without the effect of residual stress

5.2.1 Results

- After the initial cool down phase, Niobium-tin is axially compressed (it has negative σ_{zz})

- The cooling to 4.2K causes very high values of stress in all the components of the strand

- When an axial strain is applied to the wire, σ_{zz} in Niobium-tin increases linearly and becomes positive.

- During this phase, equivalent stress and equivalent strain (averaged on the niobium-tin region of the strand) reach a minimum when the strain applied to the strand has a value of $1 \div 1.5 \cdot 10^{-3}$

5.3 Transverse pressure test

Transverse pressure tests have been done both on the ten stack specimen, without measuring the critical current loss, and on a specimen made of 2-3 stacked cables, with all the strands made of copper only except for one copper-Nb3Sn strand, used for the critical current measurement.
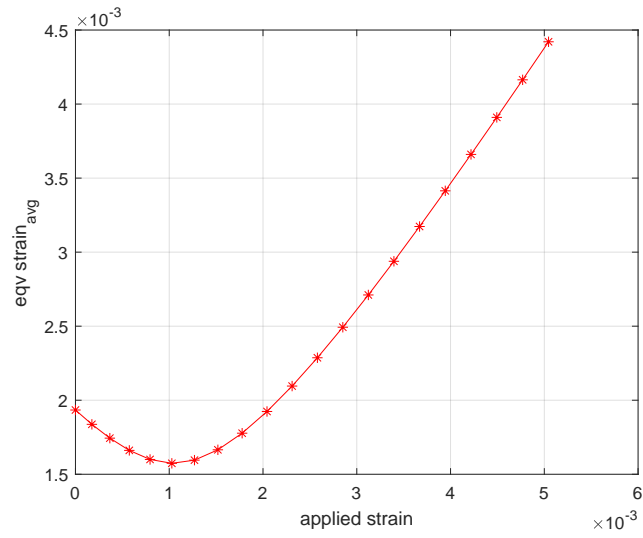
Figure 3: average equivalent strain in the strand's cross section as a function of the applied axial strain value without the effect of residual stress
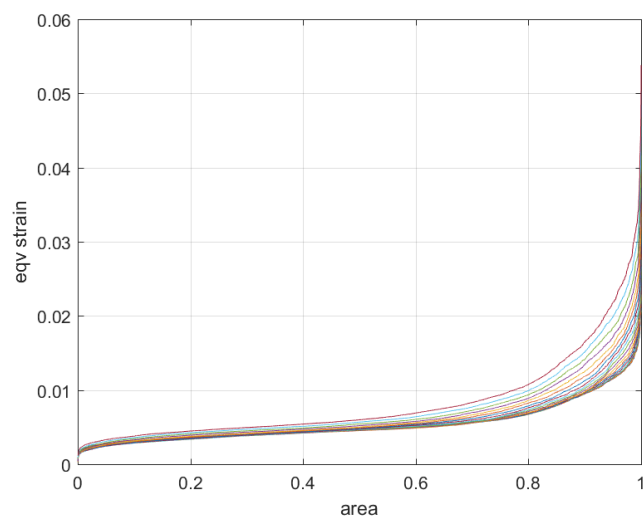

Figure 4: equivalent strain (function of area) in the strand cross section with increasing values of applied axial strain with the effect of residual stress

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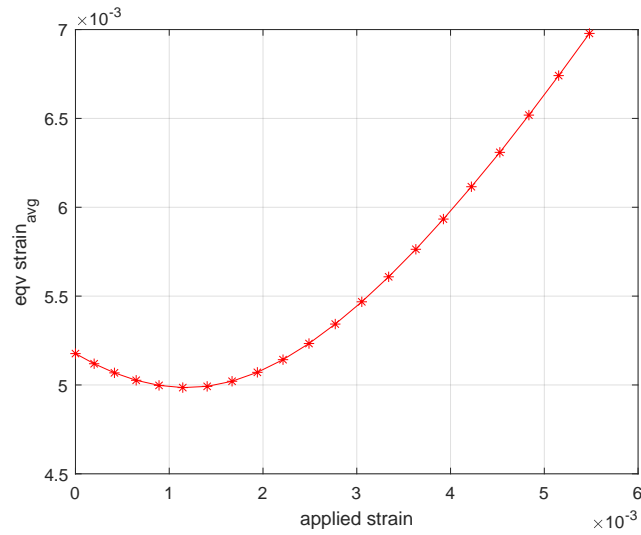
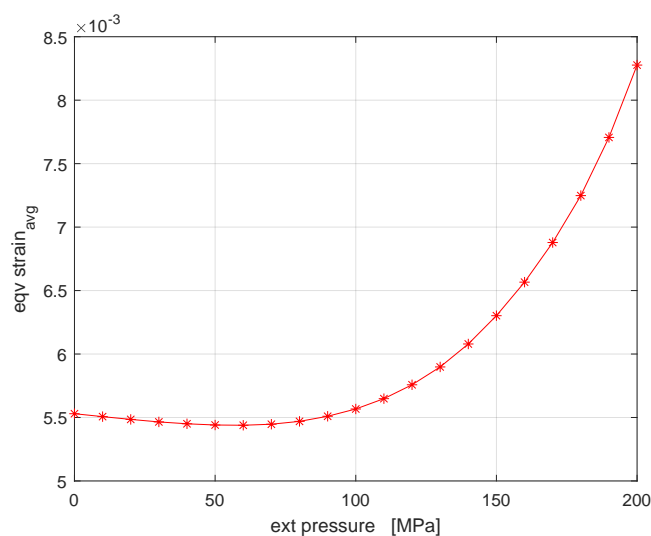Figure 5: average equivalent strain in the strand's cross section as a function of the applied axial strain value with the effect of residual stress
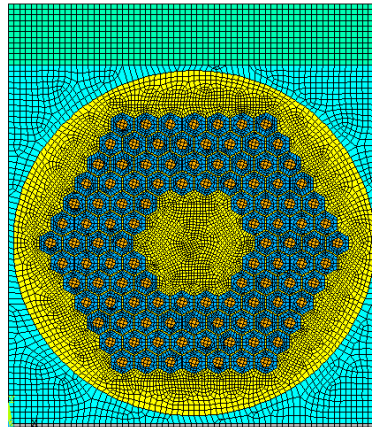
Figure 6: Ansys model used for transverse pressure test simulation

In the transverse pressure FEM analysis the cable cell is cooled down from ambient temperature (300K) to 4.2, and then external pressure is increased from 0 MPa up to a maximum of 200 MPa. In this test the presence of resin, insulator and stainless steel in the region surrounding the strand has to be considered.

5.3.1 Results

- Because
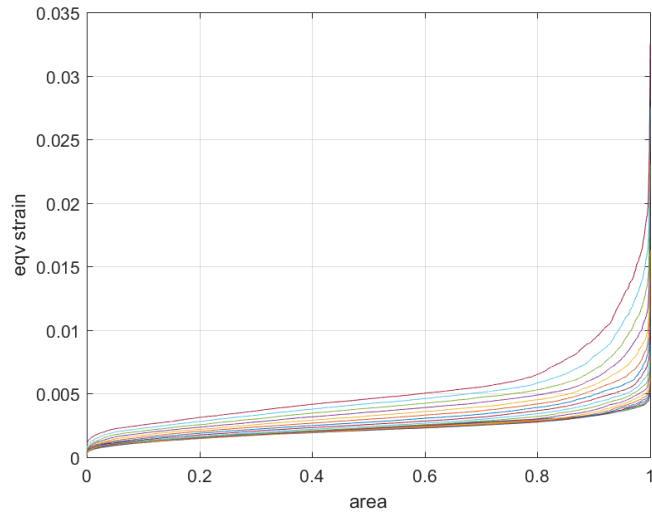
Figure 7: equivalent strain (function of area) in the strand's cross section with increasing values of external transverse pressure without the effect of residual stress
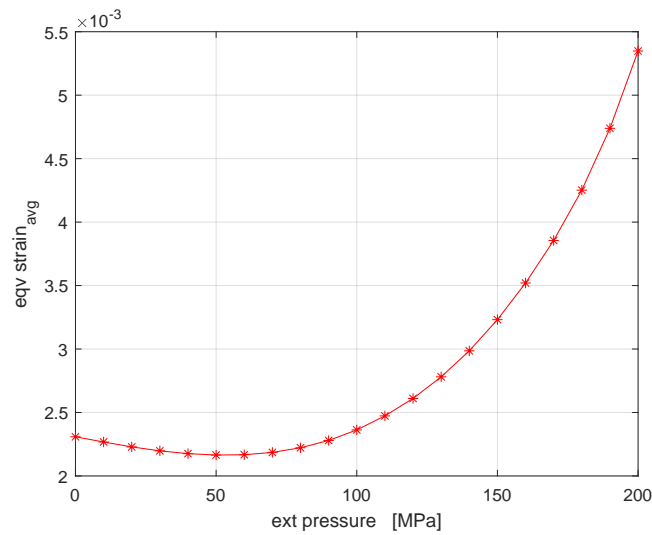
Figure 8: average equivalent strain in the strand's cross section as a function of the external transverse pressure without the effect of residual stress

Figure 9: equivalent strain (function of area) in the strand's cross section with increasing values of external transverse pressure with the effect of residual stress
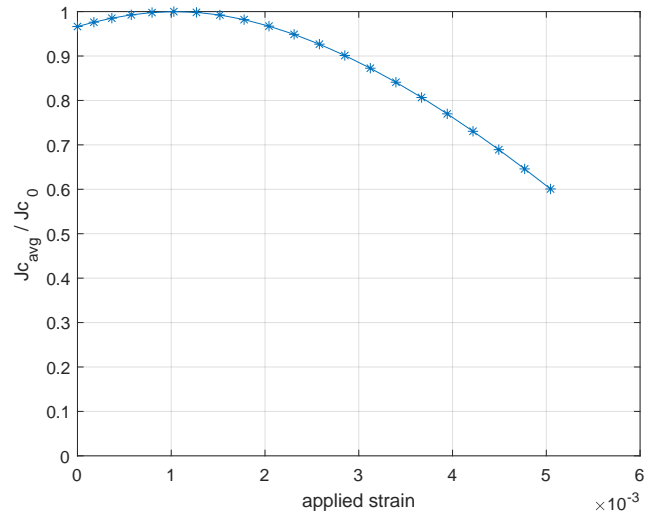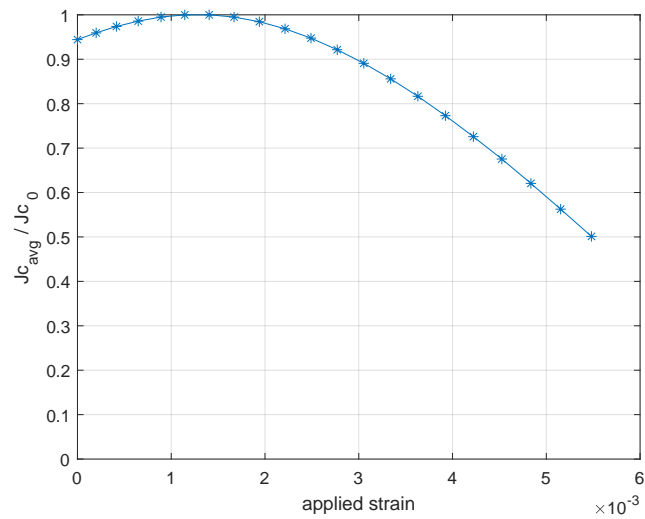

Figure 10: average equivalent strain in the strand's cross section as a function of the external transverse pressure with the effect of residual stress

6 Critical current dependence on strain

During the last decades, the critical current degradation of Nb3Sn cables and strands during tensile and transverse pressure tests have been studied.

To study this phenomenon, it is better to start from a tensile test, since less variables (geometrical parameters and materials properties) are involved. The results of critical current measurement during tensile test on RRP Nb3Sn strands show that the I_c - *axial strain* curve has a maximum for positive values of axial strain; these results have been used during the last decades to obtain a strain-critical current law.

6.1 Ekin's law

Using the hypothesis of uniaxial strain in Nb3Sn, the instrinsic equivalent strain in Nb3Sn can be evaluated as the difference between the total axial strain of the tested wire and a constant value, determined by the difference between the thermal expansion coefficients of copper and Nb3Sn:

$$\epsilon_{intr} = \epsilon_{wire} - \epsilon_0 \tag{1}$$

Using this simple equation, the dependence of critical current on intrinsic strain can be obtained by shifting the results of the measurement of critical current in a tensile test of a quantity ϵ_0 (neglecting the effect of strain components in the strand's cross section plane).

6.2 Strain energy

7 Critical current loss

??? The equivalent total (mechanical) strain in each element of Nb3Sn is the main output of the analysys. The output is calculated for increasing values of the external load in 5% increments and exported to *.txt* files as a vector ???

??? for each value of the external load the resulting strain in the superconductor elements have been imported in MatLab and used to evaluate the critical current degradation. In order to calculate the critical current in each point, Ekin's law has been used (discussed later in this report); by integrating the effects on the strand's cross section, the critical current loss for the entire cable is obtained. ???

Using the results of the structural analysys, an attempt has been made to reproduce the experimental results regarding the crititcal current degradation during both kinds of test; to do so, it has been hypothesized that the critical current degradation depends only on the equivalent strain in the superconductor with a non-linear decreasing law. This approach has some limitations:

- The maximum value of the critical current in Niobium-tin, that is to say the value of critical current with zero equivalent strain, can not be known nor measured, because the manifacturing process of superconducting cables and the cooling to cryogenic temperatures leads to values of equivalent strain that are comparable with those producted by the external loads. The only thing that can be measured is the value of critical current of a complete wire, that contains inevitably these effects. For this reason, it makes sense to try to predict the trend of the critical current - load curve only in terms of relative values, that is to say fixing to 1 the maximum of each curve.

- There is a huge uncertainty on the values of strain in the superconductor, both because of the complexity of the various processes and because of the uncertainty on the material properties and behaviour in a wide range of temperatures.

- There is no evidence that the equivalent strain is the correct parameter to use (the critical current could be a function of the axial strain in the superconductor, or of an other scalar function of the strain tensor, such as elastic energy)

Regarding the trend of the critical current - equivalent strain curve to use:

- Linear: the obtained critical current curve would be the same as the equivalent strain - load curve (flipped upside-down)

- Parabolic, with value zero when the supercondctor breaks

- As suggested by Ekin's law, a non linear relation in the form of $(1 - a|\epsilon|^{1.7})^{1/2}$ can be used. This approach is not completely correct, because when this formula has been obtained it has not be considered the presence of strain components other than ϵ_{zz}. These components leads to an equivalent strain value that is much higher than zero even when the zz-component of the strain tensor reaches a value of zero (in a tensile test)

The following formulas have been used in order to evaluate the critical current degradation in the superconductor:

$$Jc(B, T, \epsilon) = \frac{C(\epsilon)}{\sqrt{B}}[1 - \frac{B}{B_{c2}(T,\epsilon)}]^2[1 - \frac{T}{T_{c0}(\epsilon)}]^2$$

$$\frac{B_{c2}(T,\epsilon)}{B_{c20}(\epsilon)} = [1 - \frac{T}{T_{c0}(\epsilon)}]^2$$

$$B_{c02}(T, \epsilon) = B_{c02m}(T, \epsilon)(1 - a|\epsilon|^{1.7})$$

$$T_{c02}(T, \epsilon) = T_{c0m}(T, \epsilon)(1 - a|\epsilon|^{1.7})^{1/3}$$

$$C(\epsilon) = C_0(1 - a|\epsilon|^{1.7})^{1/2}$$

Using this method, the critical current - external load curves have been plotted, by integrating the effects on the area of niobium-tin inside the strand for each 5% load increment.

7.1 Tensile test results

- critical current has a maximum for 0.10 to 0.15 % applied axial strain (while experimental results show that usually this maximum is at higher values such as 0.30 %)

7.2 Transverse pressure test results

[1]

- The curves in the transverse pressure test start at a value that is 85 to 92 % of the maximum, while the ones in the tensile test start at 95 to 96 %; this 5-10 % difference at zero external load is due to the differential thermal contraction of the strands and all the other cable components that surround them (resin, insulator...)

- Critical current is almost constant before 100MPa, and then starts to decrease noticeably

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[1]The value "1" corresponds to the maximum of critical current in a tensile test executed in the same conditions

Figure 11: average equivalent strain in the strand's cross section as a function of the applied axial strain value without the effect of residual stress (B=15T, T=4.2K)

Figure 12: equivalent strain (function of area) in the strand cross section with increasing values of applied axial strain with the effect of residual stress (B=15T, T=4.2K)
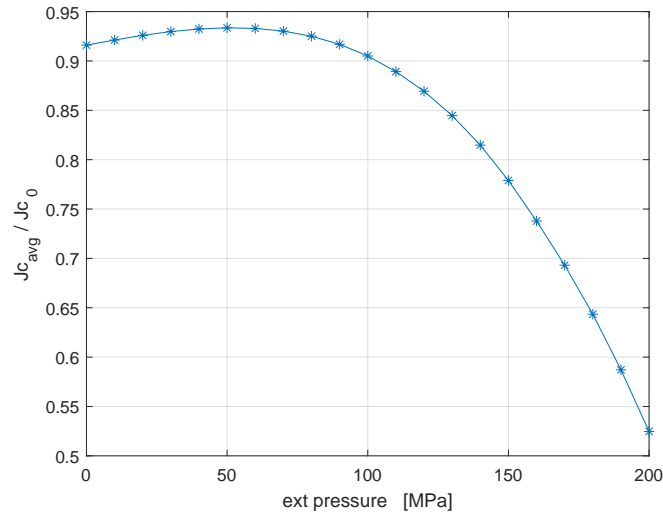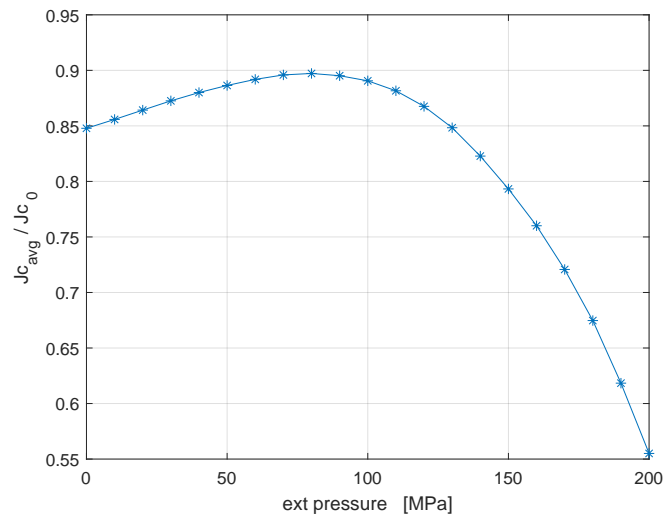
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Figure 13: equivalent strain (function of area) in the strand's cross section with increasing values of external transverse pressure without the effect of residual stress (B=15T, T=4.2K)

Figure 14: equivalent strain (function of area) in the strand's cross section with increasing values of external transverse pressure with the effect of residual stress (B=15T, T=4.2K)

7.3 Future developments

8 Appendix

8.1 Material Properties

The following values have been used as "default values" for the material properties:

Materials	E [GPa]	Sy [MPa]	ν	$\alpha \ [1/K \cdot 10^{(-6)}]$
Epoxy resin (CTD 101K)	2.4	150	0.4	50
Insulator (CTD 101K + S-2 Glass)	19	320	0.3	36
Stainless steel	206	800	0.3	12
Nb3Sn	125	800	0.365	7
OHFC copper (annealed)	118	100	0.36	17
Glidcop	118	450	0.36	17
α - bronze	118	110	0.36	17

Notes:

- Copper and Nb3Sn have temperature-dependent material properties

- The bronze rods are porous, so it is not completely correct to use Von Mises yield criterion (because in a porous material also the idrostatic component of the stress tensor has an impact on that)

8.2 Experimental data

8.3 Ansys plots

8.3.1 Stress and strain distribution in tensile tests

8.3.2 Stress and strain distribution in transverse pressure tests
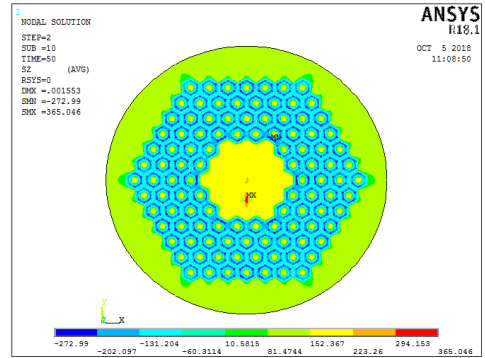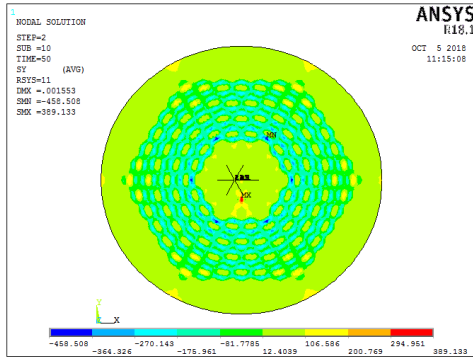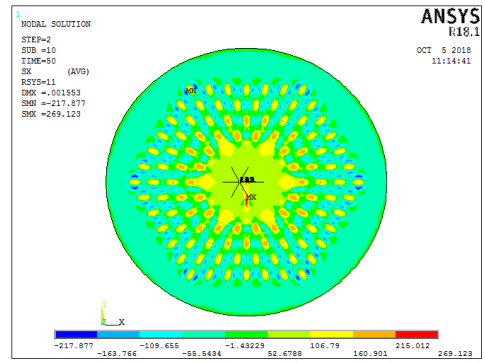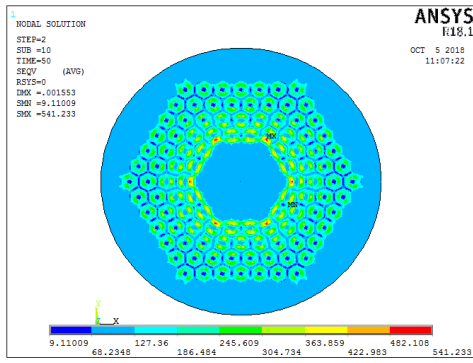
Figure 15: tensile test - σ_{eqv}, σ_{rr}, $\sigma_{\theta\theta}$, σ_{zz} at 0% load and 4.2K without residual stress
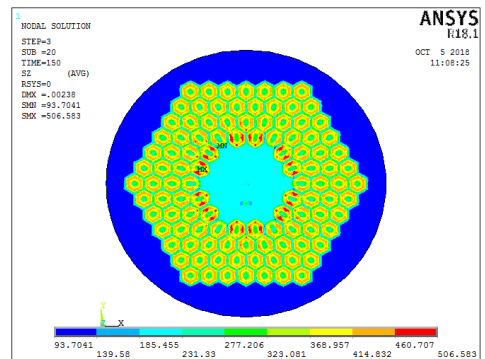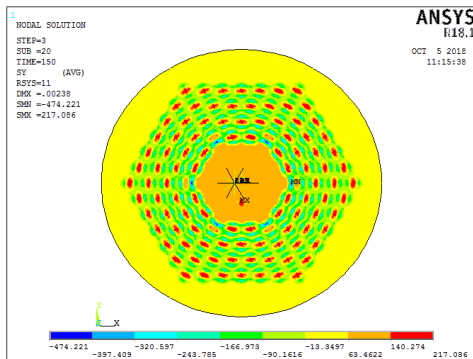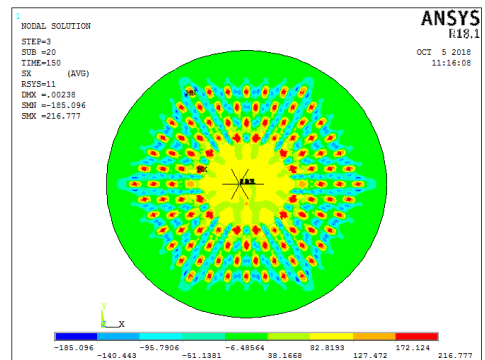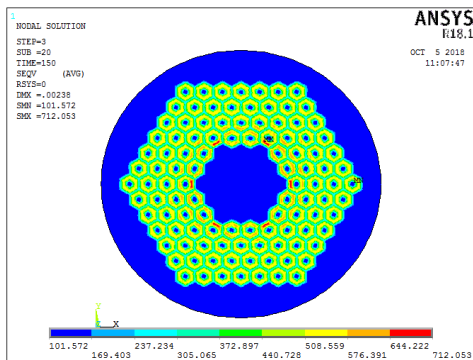
Figure 16: tensile test - σ_{eqv}, σ_{rr}, $\sigma_{\theta\theta}$, σ_{zz} at 100% load and 4.2K without residual stress
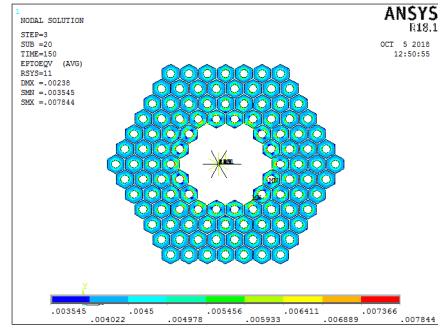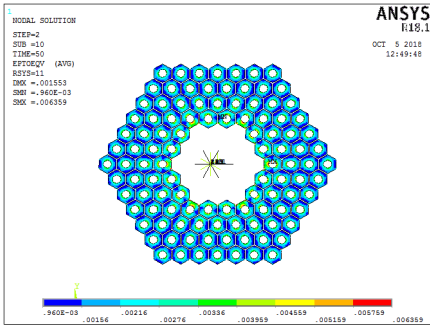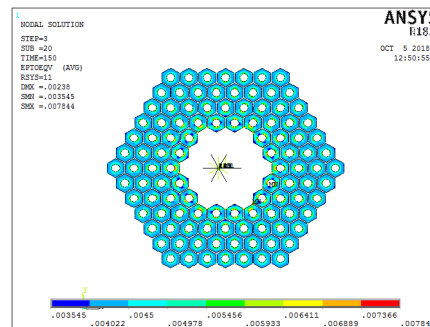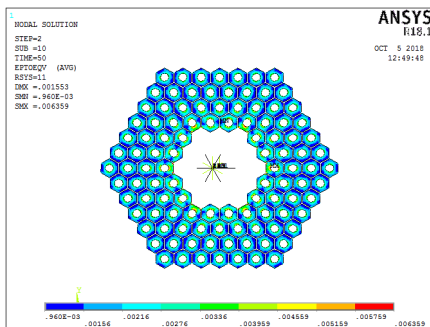
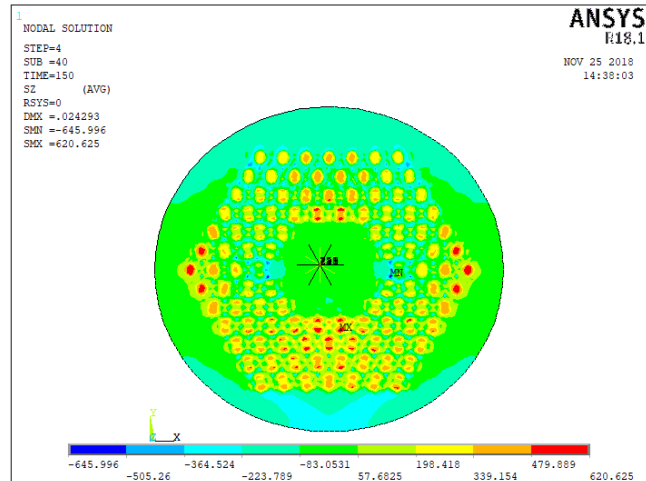Figure 17: tensile test - equivalent strain in Nb3Sn at 0% and 100% load and 4.2K with residual stress
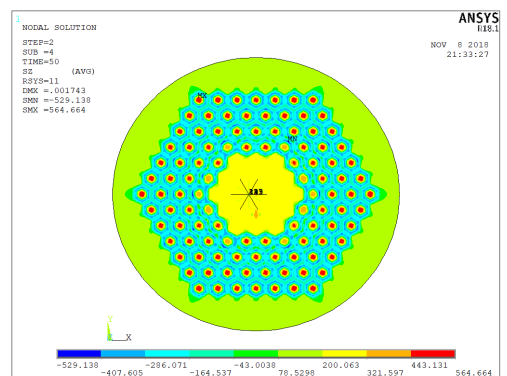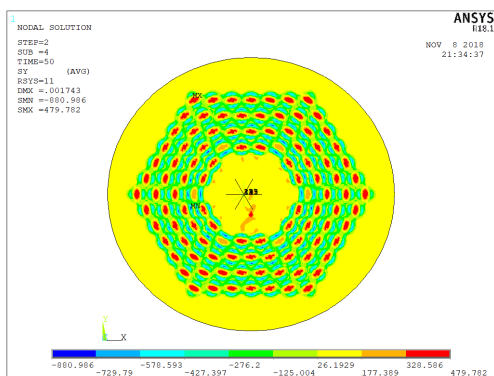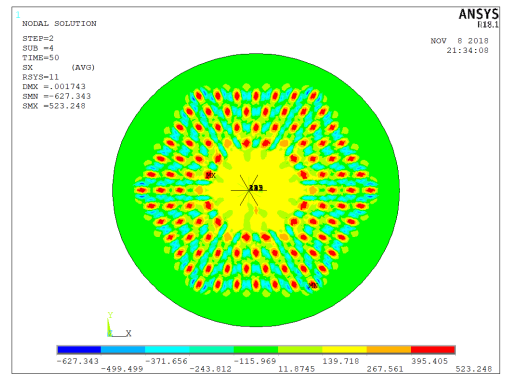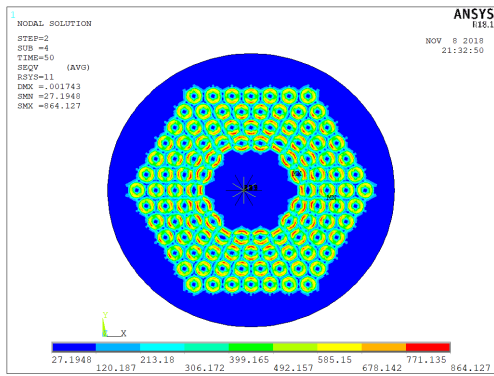
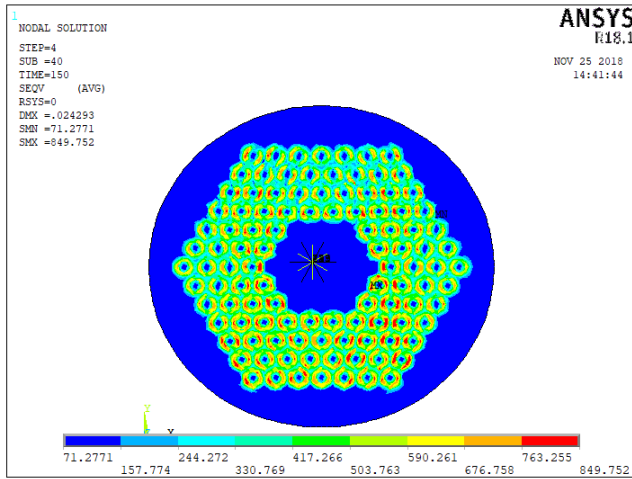Figure 18: tensile test - σ_{eqv}, σ_{rr}, $\sigma_{\theta\theta}$, σ_{zz} at 0% load and 4.2K with residual stress
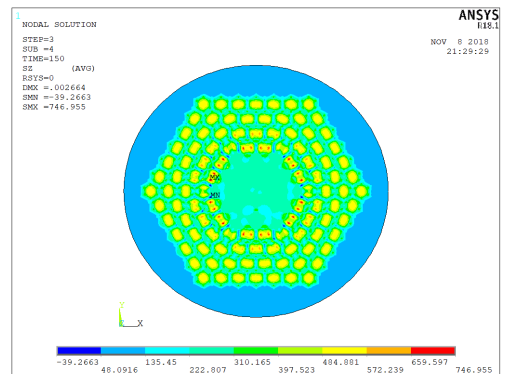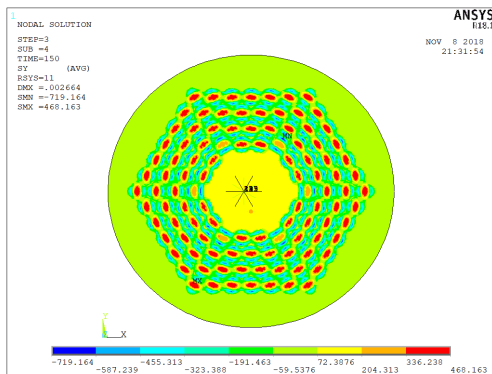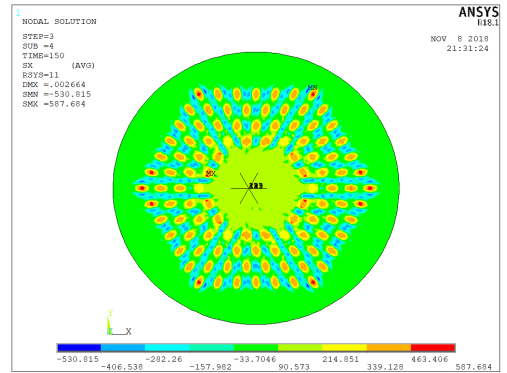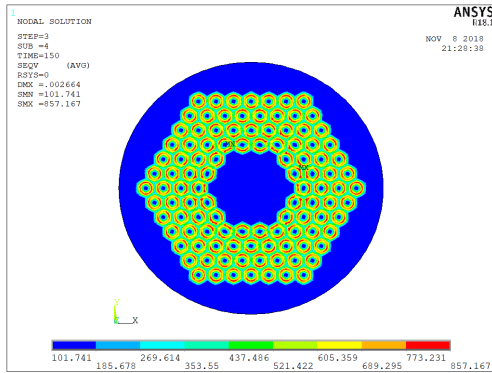
Figure 19: tensile test - σ_{eqv}, σ_{rr}, $\sigma_{\theta\theta}$, σ_{zz} at 100% load and 4.2K with residual stress

Figure 20: tensile test - equivalent strain in Nb3Sn at 0% and 100% load and 4.2K with residual stress
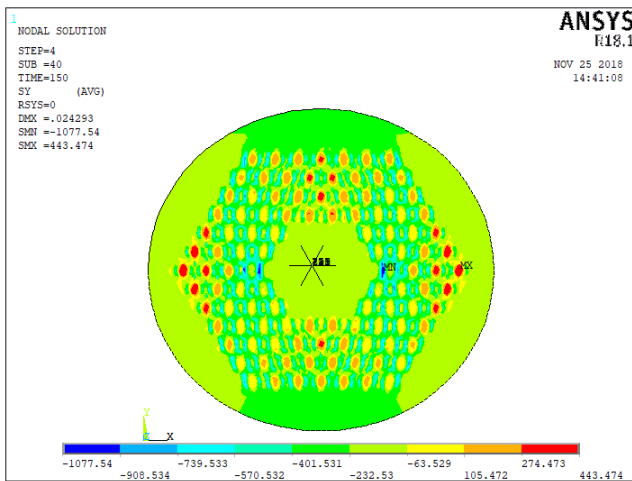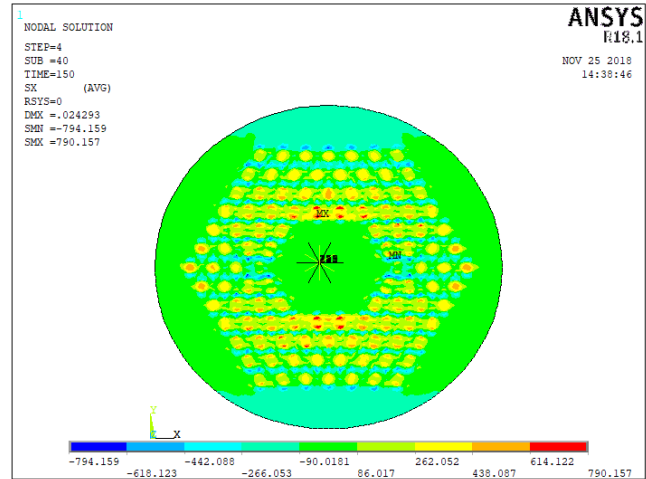
Figure 21: tensile test - σ_{eqv}, σ_{xx}, σ_{yy}, σ_{zz} at 0% load and 4.2K with residual stress
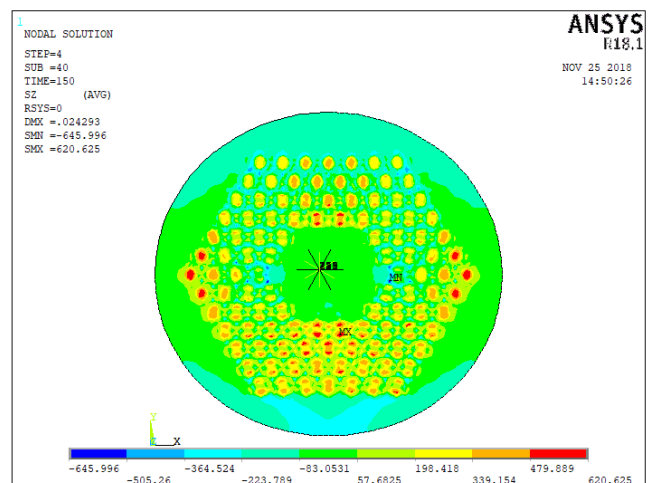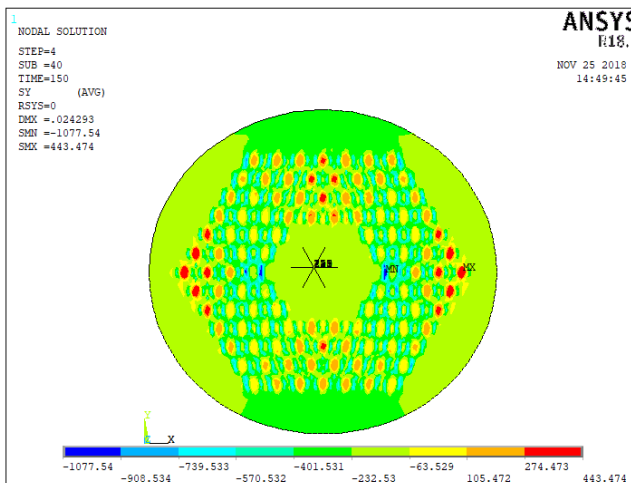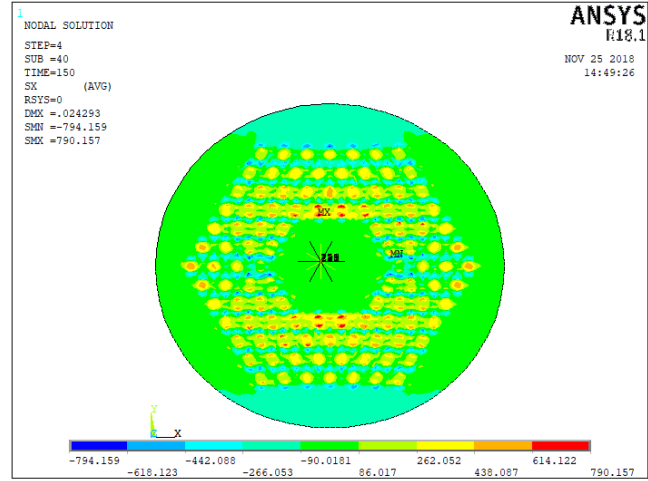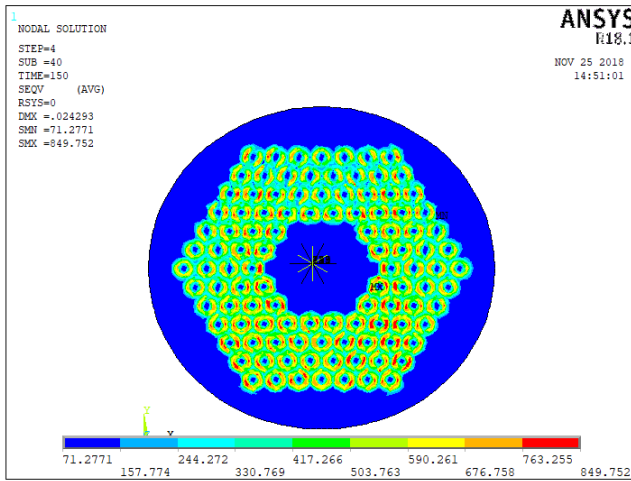
Figure 22: tensile test - σ_{eqv}, σ_{xx}, σ_{yy}, σ_{zz} at 100% load (external pressure = 200Mpa) and 4.2K with residual stress