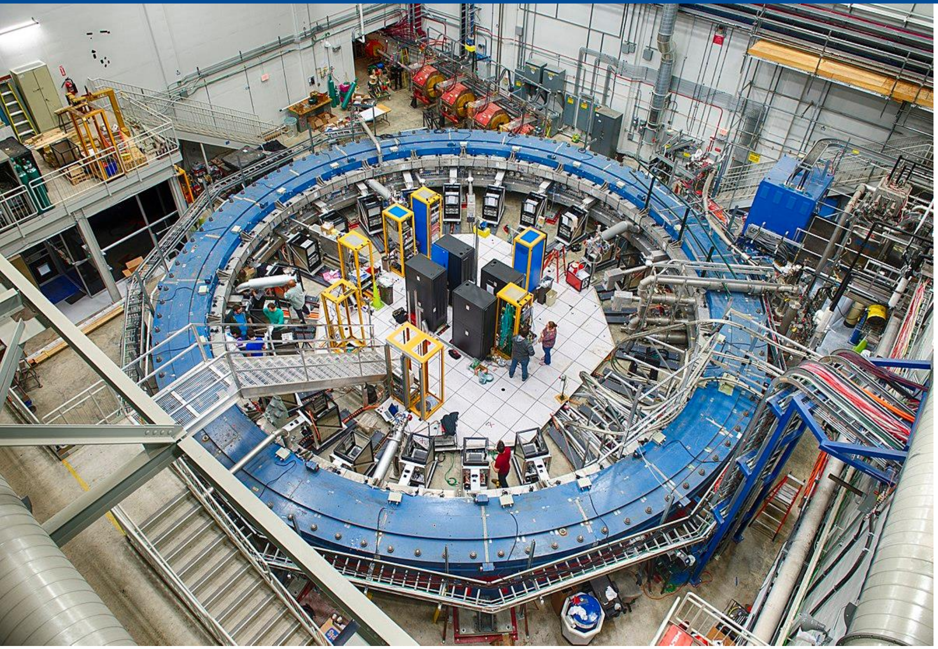
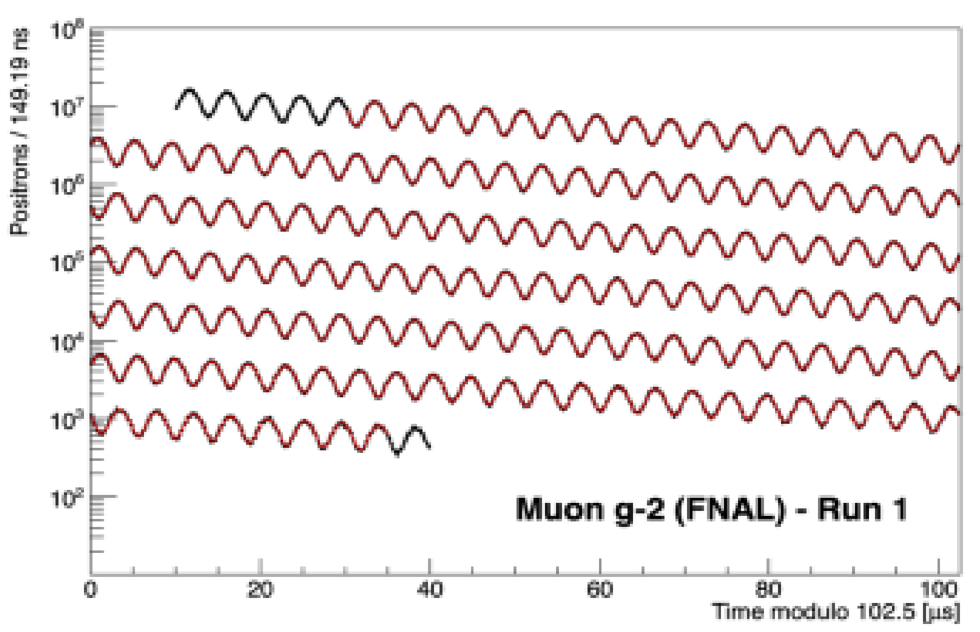
Fermilab U.S. DEPARTMENT OF ENERGY Office of Science





Analysis of the muon’s spin anomalous precession frequency

Final Report

Daniele Boccanfuso
Supervisors: Brendan Casey, Marco Incagli, Matteo Sorbara
28/09/2022

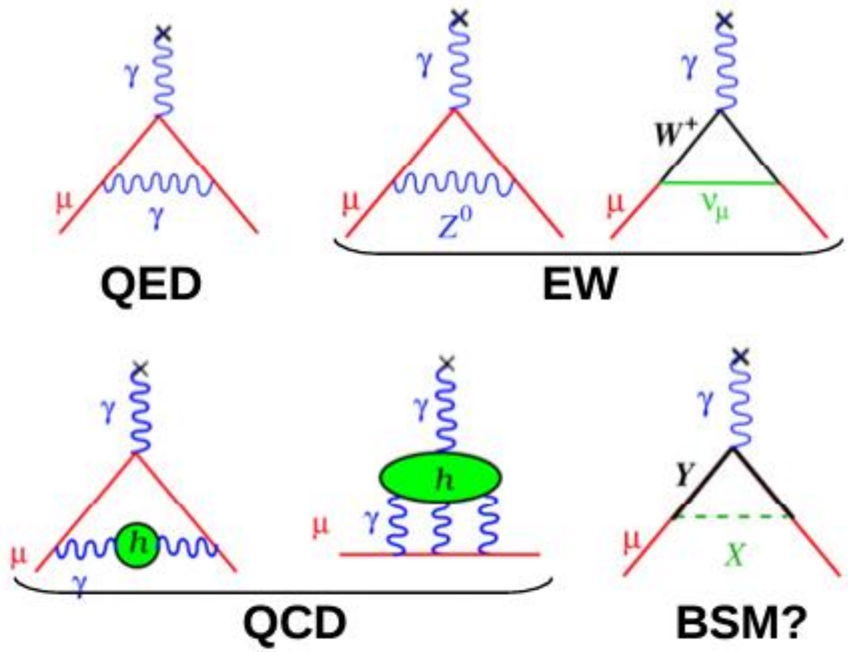
The muon magnetic moment

The muon's magnetic moment is given by $\vec{\mu} = g\frac{q}{2m}\vec{S}$

The g-factor of an elementary particle is predicted to be 2 at the first order. Higher order corrections shift this value by $\sim 10^{-3}$.

We define the muon anomaly as:

$$a_\mu = \frac{g-2}{2}$$



All particles interacting with the muon (high order loops) contribute to a_μ, even the ones we haven't discovered yet! This makes any discrepancy between the theoretical and experimental value of a_μ hint of new physics.

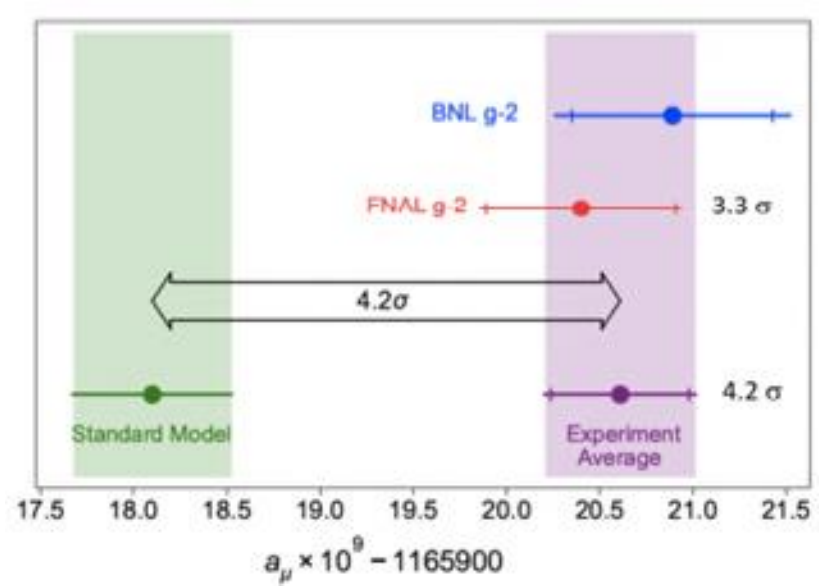
Muon precession

A magnetic field induces a precession motion of the particle's spin.

The precession frequency is given by:

$$\vec{\omega}_a = a_\mu \frac{e\vec{B}}{m}$$

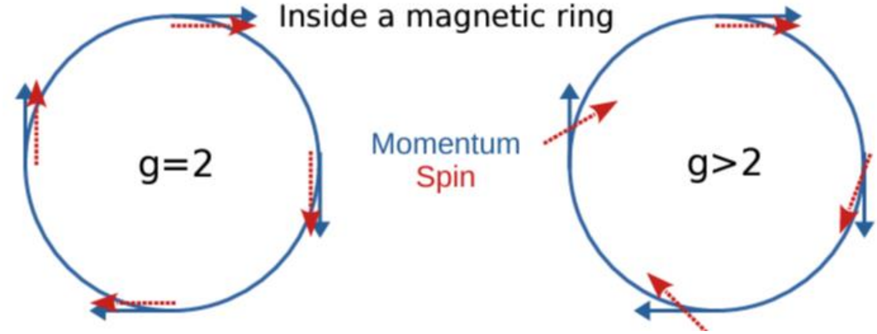
The Muon g-2 collaboration aims to measure a_μ with a 140 ppb uncertainty, by measuring with high precision both the spin precession frequency and the magnetic filed.



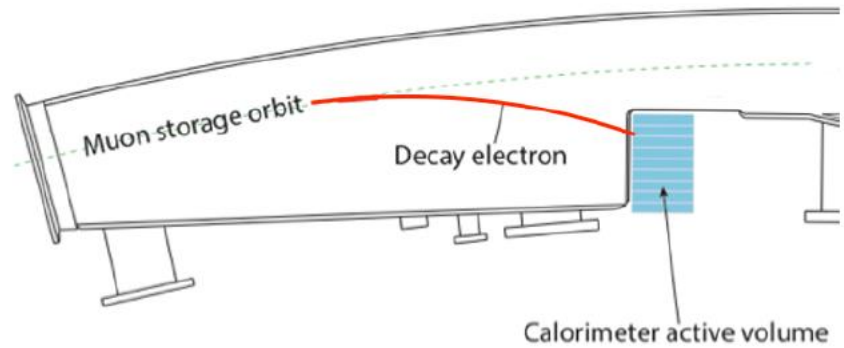
The a_μ value from Run1 analysis shows a 4.2σ tension with the theoretical value.

Measuring precession frequency

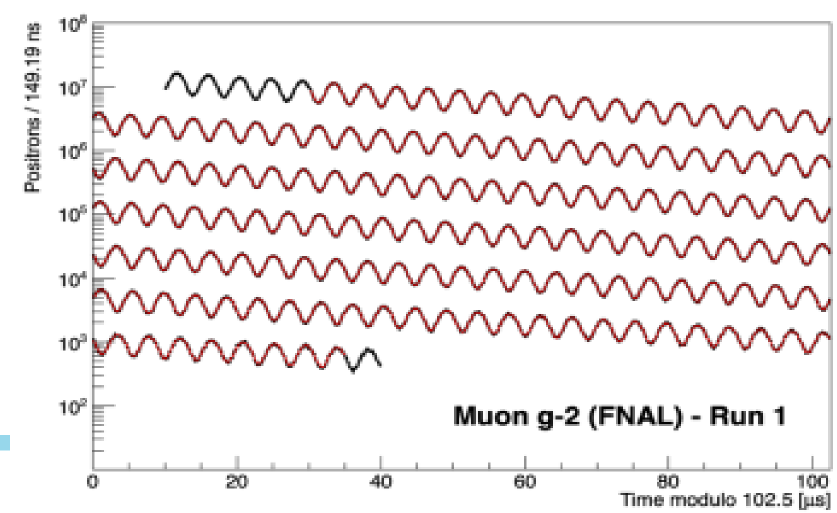
To measure the spin precession a beam of polarized muons is injected into a superconducting storage ring, where muons circulate for roughly 750 μs.



The spin rotates around the magnetic field direction.



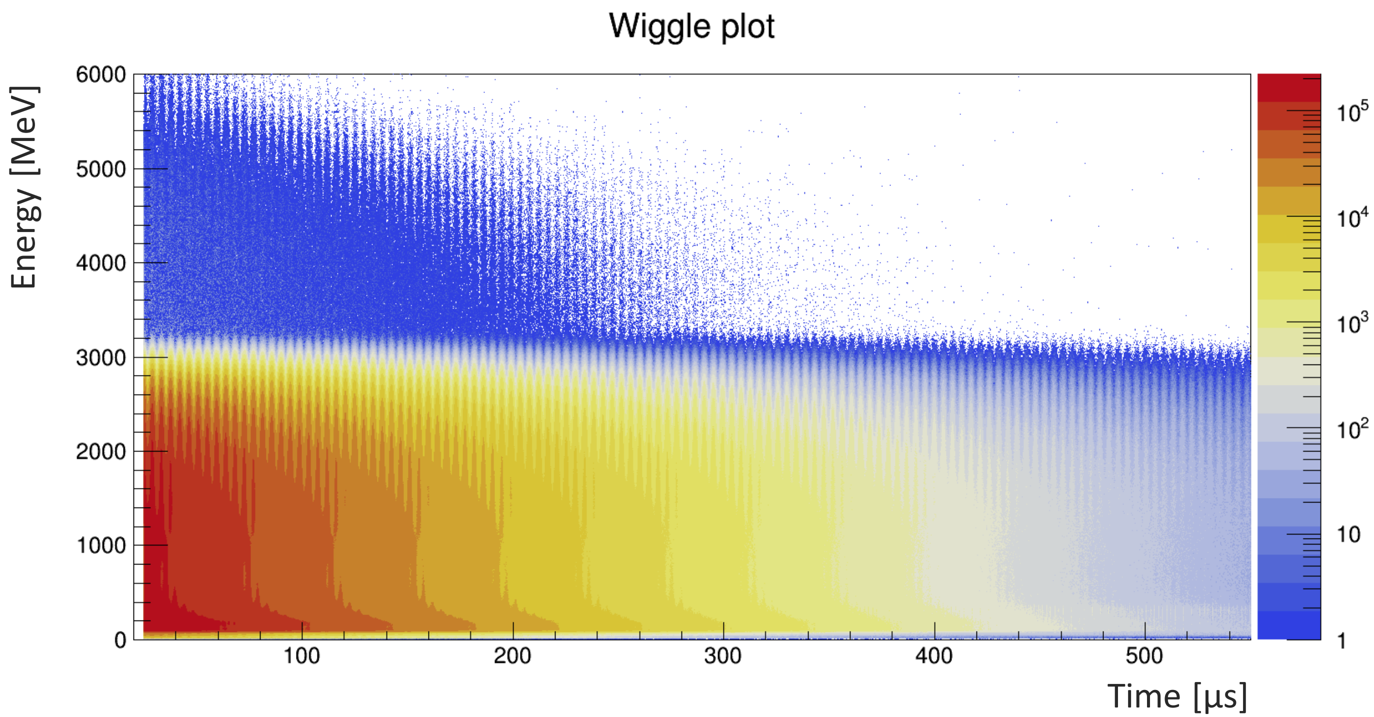
Muons decay into positrons. High energy positrons have higher probability to be emitted in the muon spin direction: we detect more positrons when the spin faces towards the calorimeters and less when it faces away. The frequency of this oscillation is the signal we want to measure: $\vec{\omega}_a$



Positrons distribution

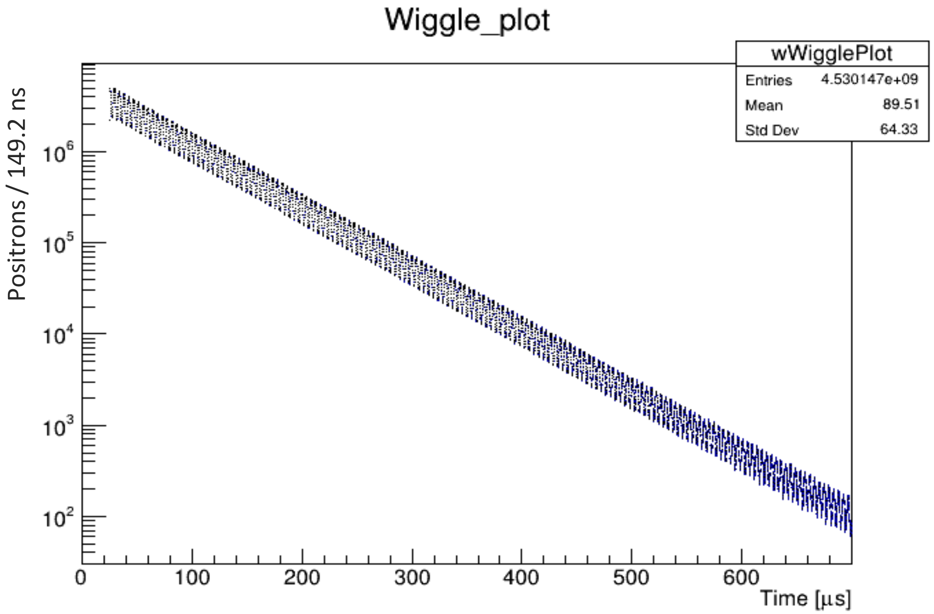
In the past weeks I have analyzed data from Run-2 acquisition period (2019). This plot shows the positron time of arrival (X axis) and its energy (Y axis) as measured by the calorimeters.

Events above 3100 MeV are due to pile-up.

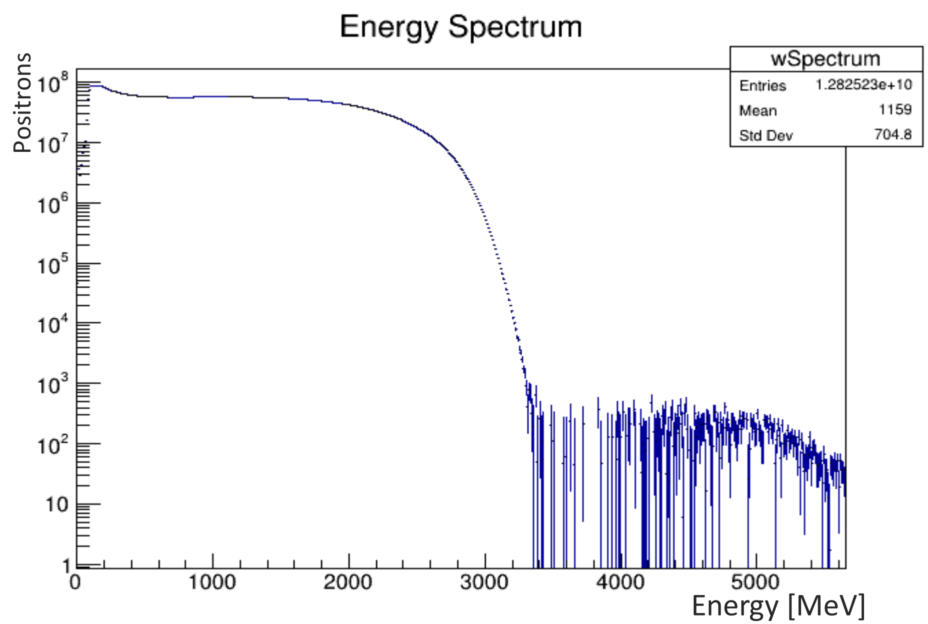


Wiggle Plot and Energy Spectrum

Projecting on the Y axis we get the 1-D Wiggle Plot, this figure is obtained integrating events from 1700 MeV.



Projecting on the X axis, i.e. integrating from 30 μs to 650 μs, we get the energy spectrum.



5 parameters fit function

The simplest equation that describes the number of positrons detected by the calorimeters is the following:

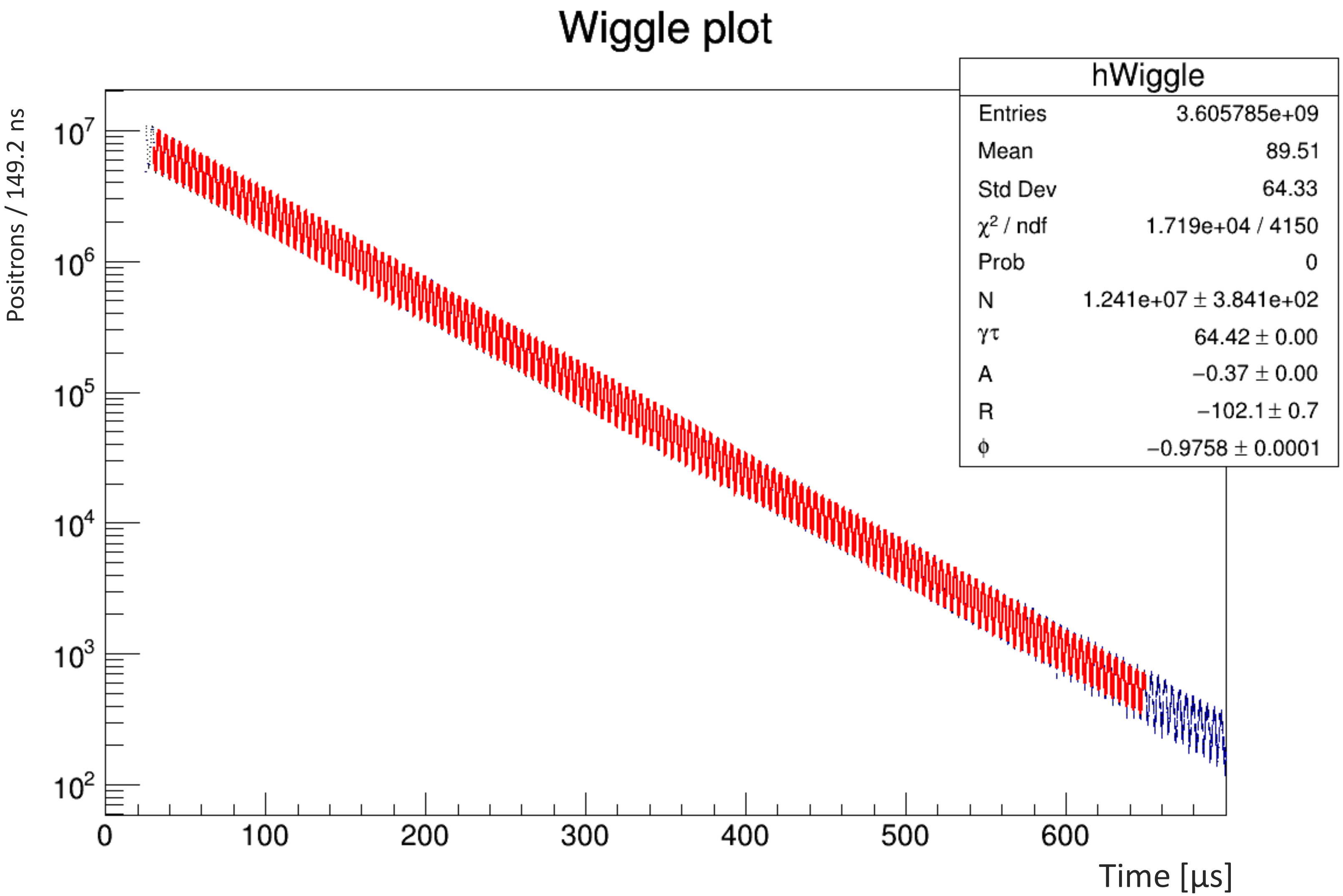
$$\mathrm{N(t)} = \mathrm{N}_0 e^{-\frac{t}{\gamma\tau}}(1 + A\cos(\omega_a t + \phi))$$

Where:

- N_0 is the number of muons at t=0
- $\gamma\tau$ is the muon lifetime in the lab frame of reference
- A is the asymmetry, related to the probability that a positron is emitted in the same direction of the spin
- ω_a is the precession frequency we want to measure
- φ is the phase at t=0

To avoid cognitive bias, ω_a is blinded by a dimensionless parameter R, defined as the unknown offset in ppm from a reference value.

5 parameter fit results



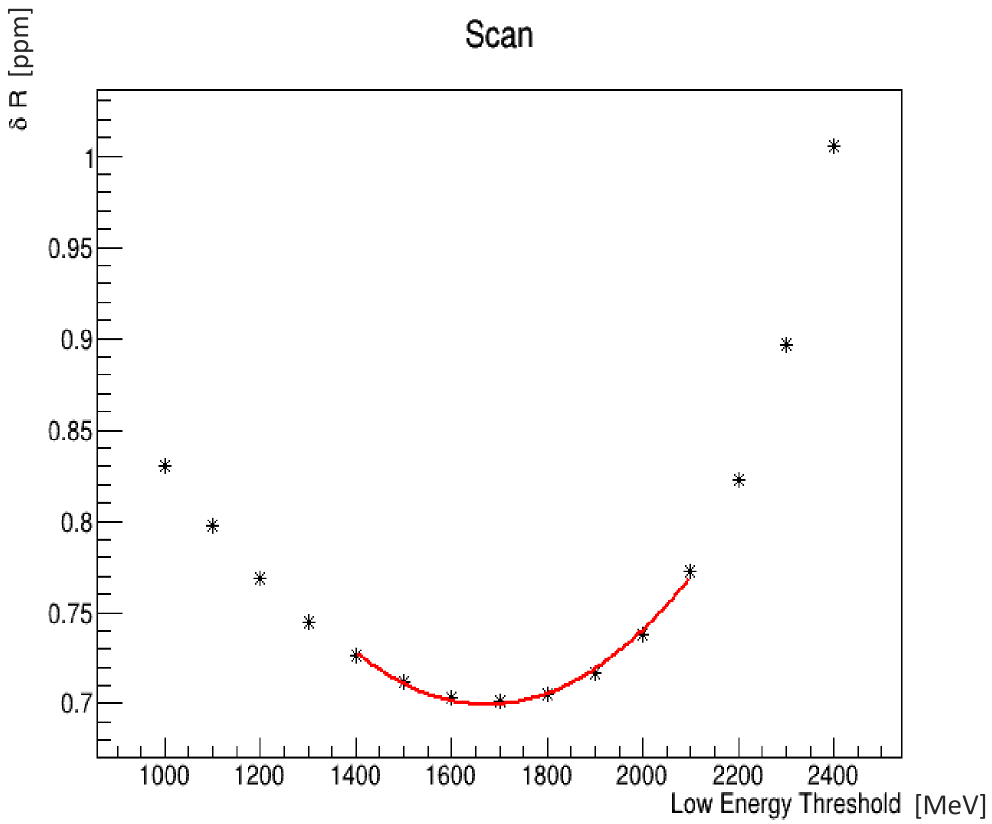
T-Method

The T-Method consist in building the wiggle plot summing all positrons above an energy threshold:

- At high energy positrons have higher asymmetry, which is better for the fit, but there’s low statistics
- At low energy positrons have lower asymmetry, but there is more statistics
- It’s necessary to find a compromise between the two cases: we have to determine the ideal energy threshold that minimizes the error on R
- We build different wiggle plots by changing the lower energy threshold
- We fit every one of them and the figure of merit will be the smallest uncertainty on R

T-Method

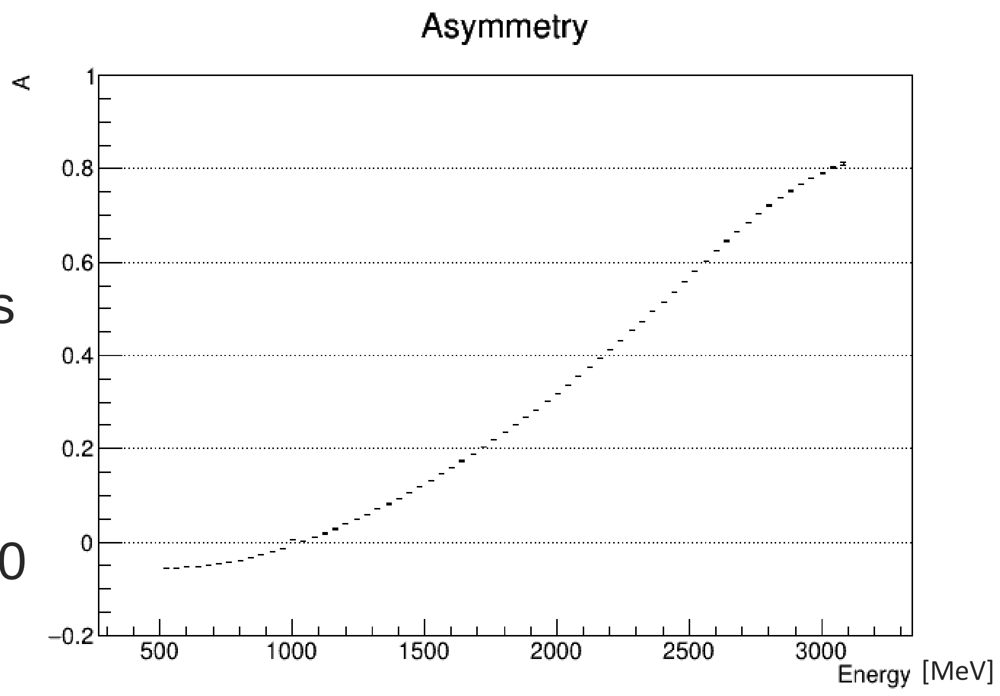
- The plot shows the uncertainty on the R parameter as a function of the energy threshold
- The distribution is fitted with a quadratic function and shows a minimum at the optimal point
- The minimum found for this dataset is at 1673 MeV



A-Method

The wiggle plot can be also built by weighting the positrons with their asymmetry (function of the energy). High energy positrons weight more, increasing the sensitivity to the precession frequency signal. This also allows to lower the energy threshold, hence the statistics increases.

- To obtain the asymmetry the region from 500 MeV to 3100 MeV is sliced into bins of 40 MeV
- From each slice a wiggle plot is produced
- Each wiggle plot is fitted to extract the asymmetry
- The fit is less precise near 1000 MeV because A is zero in that region.

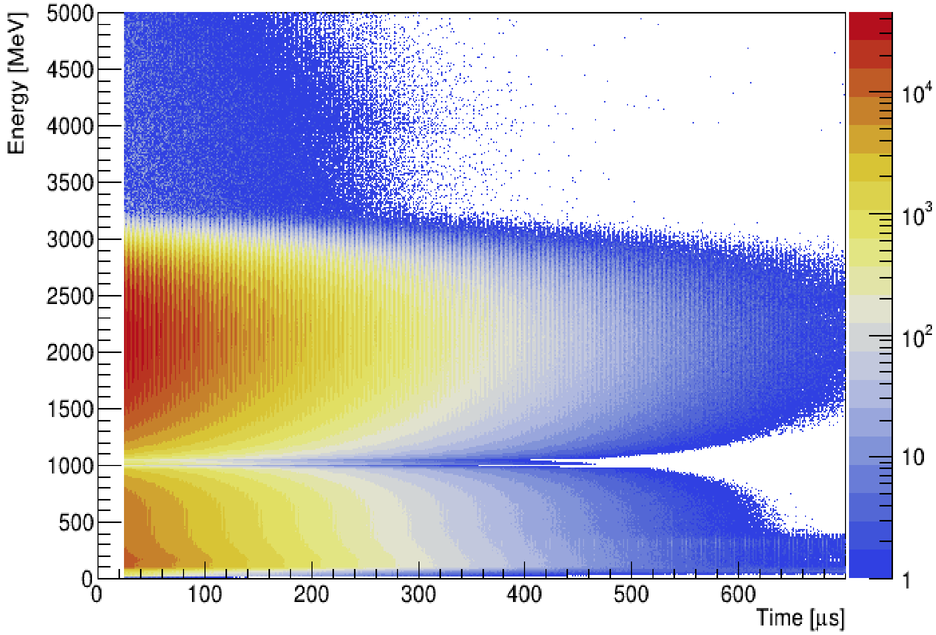


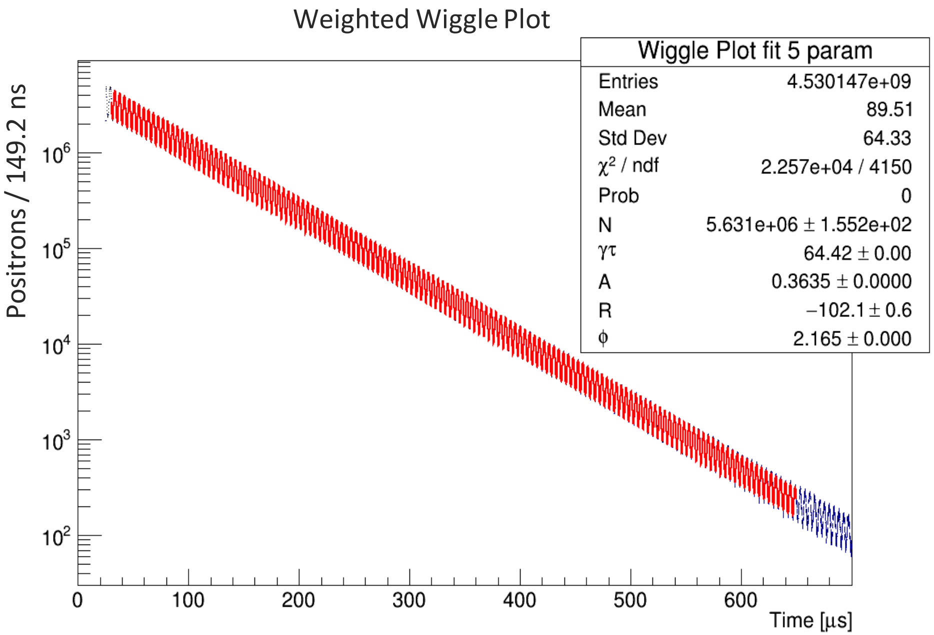
Weighted Wiggle Plot

Positron distribution where each entry has been multiplied by its asymmetry.

Note: the asymmetry below 1000 MeV is negative, here the absolute value is plotted for better visualization.

The wiggle plot is built integrating from 1100 MeV to 3000 MeV. The number of entries increased from 3.6×10^9 to 4.5×10^9





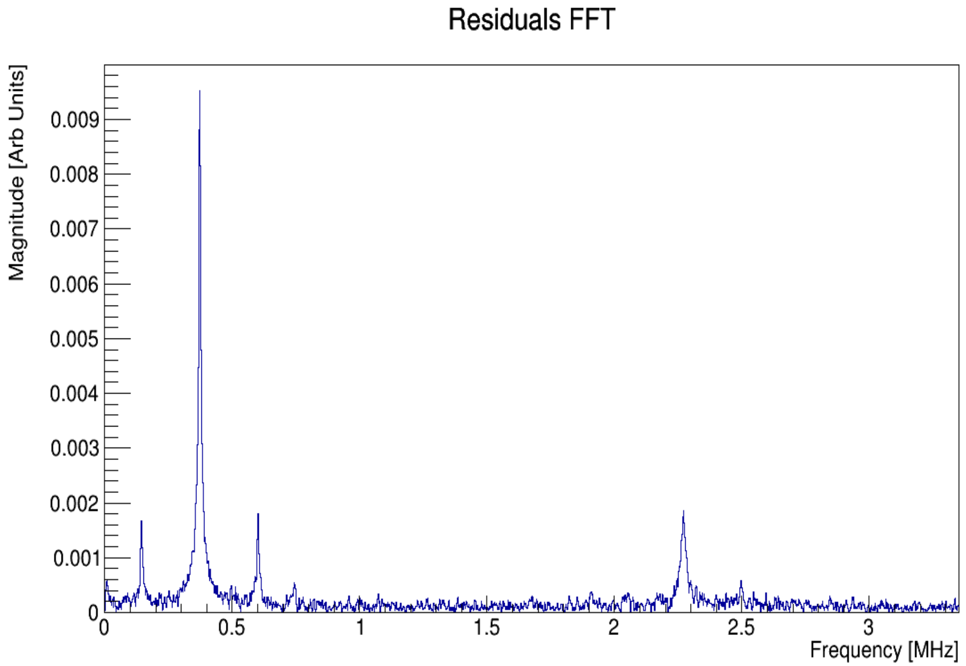
Beam motion effects

In this experiment, effects of beam dynamics have direct impact on the result. The most important one is the effect of coherent betatron oscillations (CBO): the oscillation of the beam along the horizontal plane.

The fit is also sensitive to oscillations in the width of the beam in the vertical plane (vertical waist) and of its mean.

The Fourier transform of the residuals of the fit highlights the frequencies of the beam dynamics oscillations:

- $f_{CBO} \sim 0.37\ MHz$
- $f_{CBO} \pm f_a$
- $f_{VW} \sim 2.3\ MHz$
- $f_y \sim 2.5\ MHz$
- Low frequency peak due to lost muons



Fermilab

Beam motion effects

Since the FFT residuals show peaks at beam dynamics frequencies, the fitting function must be modified to include their contribution. This will also improve the fit χ^2. The final function has 22 parameters. The effects considered are the following, from the most to the least important:

- CBO: the oscillation of the beam mean in the horizontal axis
- Vertical waist: the oscillation of the beam width in the vertical axis
- Lost muons: muons lost from the storage ring before they decay into positrons.
- Oscillation of the beam mean along the vertical axis
- Second harmonic of the CBO
- Oscillation in the value of g-2 asymmetry and phase due to CBO

22 parameters fit function

The final function is the following:

$$N_0 e^{-\frac{t}{\gamma\tau}}(1 + A_a \cdot A_{BO}(t)\cos(\omega_a t + \phi + \phi_{BO}(t))) \cdot N_{CBO}(t) \cdot N_{VW}(t) \cdot N_y(t) \cdot N_{2CBO}(t) \cdot \Lambda(t)$$

$$N_{CBO}(t) = 1 + A_{CBO}\cos(\omega_{CBO}(t) + \phi_{CBO})\, e^{-\frac{t}{\gamma\tau}}$$

$$N_{2CBO}(t) = 1 + A_{2CBO}\cos(2\omega_{CBO}(t) + \phi_{2CBO})e^{-\frac{t}{2\tau_{CBO}}}$$

$$N_{VW}(t) = 1 + A_{VW}\cos(\omega_{VW}(t)t + \phi_{VW})e^{-\frac{t}{\tau_{VW}}}$$

$$N_y(t) = 1 + A_y\cos(\omega_y(t)t + \phi_y)e^{-\frac{t}{\tau_y}}$$

$$\Lambda(t) = 1 - k_{LM}\int_{t_0}^{t} e^{\frac{t'}{\gamma\tau}} L(t')dt'$$

These terms introduce more than 22 parameters, but some of them are not independent: e.g. $\tau_y = 2\tau_{VW}$, thus the number of independent parameters ends up being 22.

Fit procedure

The fit procedure uses several steps to include all the beam dynamics parameters. This allows to use the result from each step to initialize the fit parameters in the following one.

The procedure starts with the 5 parameters function, then includes:

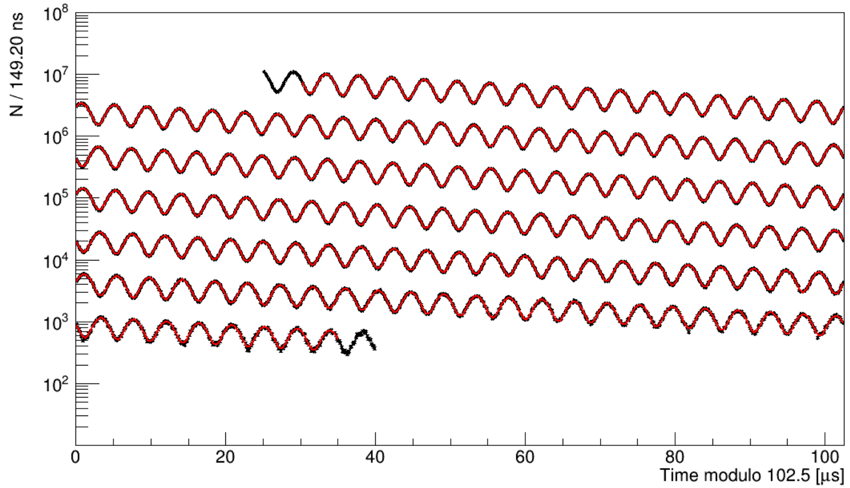
- CBO terms (9 parameters)
- Vertical waist (12 parameters)
- Lost muons (13 parameters)
- Variable CBO terms (20 parameters)
- Higher order oscillations (22 parameters)

The fit is done using the χ^2 minimization using the TMinuit2 package in ROOT.

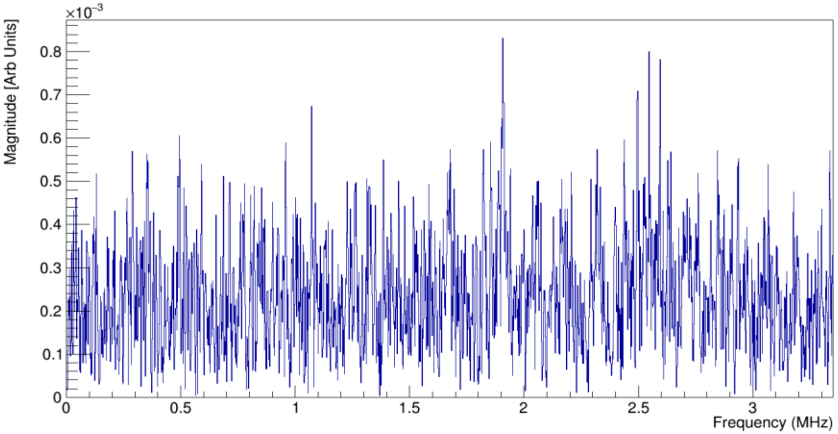
22 parameter fit results

With every beam motion effect taken into account, the Fourier Transform of the residuals shows no peaks.
On the right a fitted Wiggle Plot where the time axis is "wrapped up" to help see the oscillation.

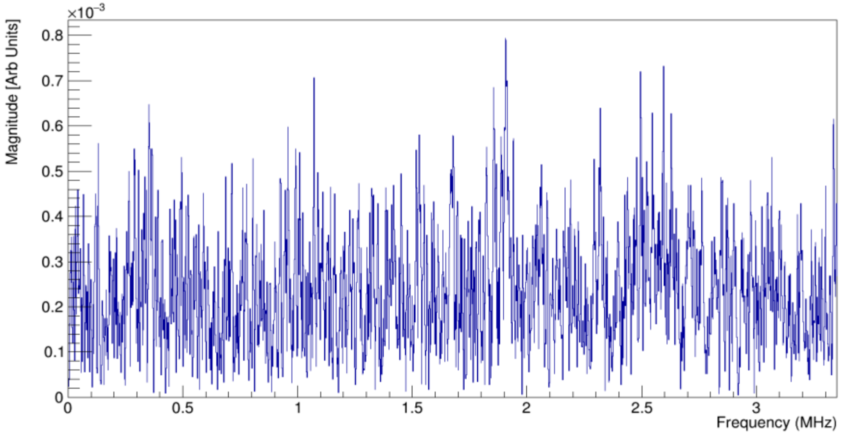
Folded Wiggle Plot



T-Method



A-Method



Fermilab

T and A methods comparison

The two methods give us these results:

	R (ppm)	δR (ppm)	X^2/ndf	X^2/ndf
T-method	-81.709	0.701	4297.19/4133	1,039
A-method	-81.429	0.631	4364.88/4133	1,056

Agreement between the two methods is checked calculating the difference

$$|R_T - R_A| = 0.280$$

Which should be lower than the allowed statistical deviation (the 1σ difference due to different amount of statistic used by the two methods)

$$\sqrt{\delta R_T^2 - \delta R_A^2} = 0.305$$

T and A methods correlation

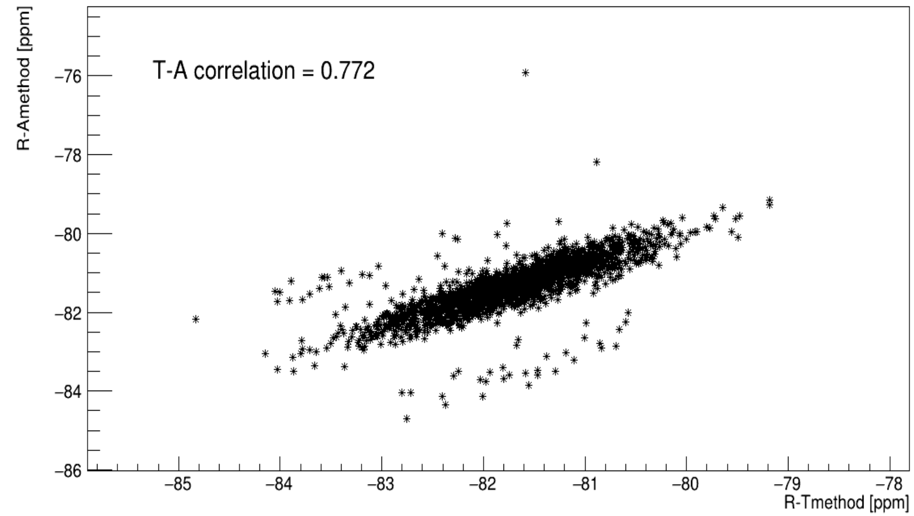
The two methods are not independent because part of the data is shared. The correlation factor between the two methods is needed in order to combine the results. To estimate it we generate many independent measurements of R using the bootstrap method, summarized as it follows:

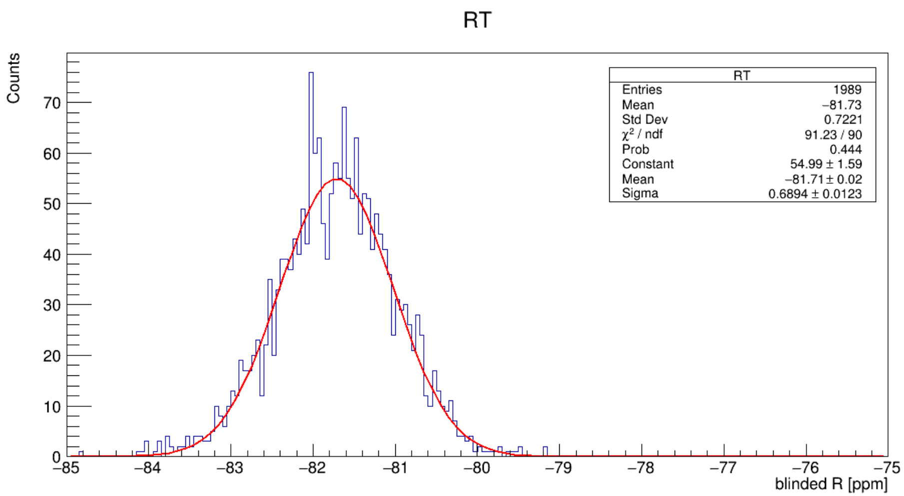
- Starting from the pile-up corrected energy-time 2D histograms we generate 2000 pseudoexperiments
- For each pseudoexperiment we build the wiggle plot using the T method and the A method
- Each wiggle plot is fitted with the 22 parameters function
- The values of R obtained from the fit are used to calculate the correlation factor

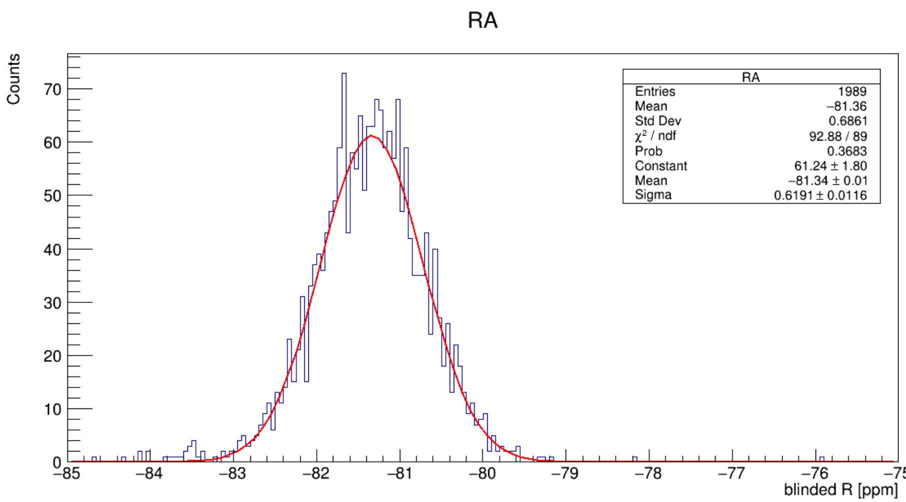
In some cases the fit didn’t properly converge, those points are excluded from the calculation

T and A methods correlation

The correlation between the two methods is r = 0.772
This is lower than the one obtained in Run1, this suggests that this method may not be accurate enough for this case.







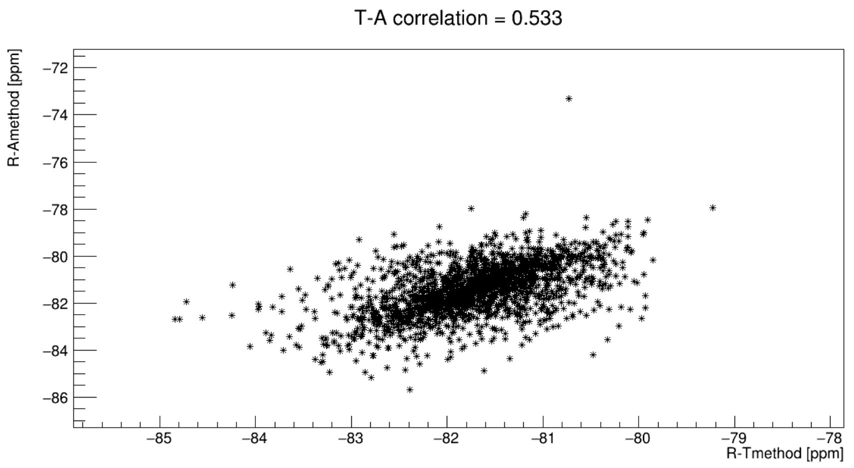
T and A methods correlation

Instead of building the pseudoexperiments from the energy-time 2D histogram, we generate them from the 1D wiggle plot:

- We fill every bin with a number of entries that is randomly extracted from a Poisson distribution with mean equal to the number of entries of the original wiggle plot.
- The error is calculated as:

$$error_{OG} \times \frac{entries_{bootsrap}}{entries_{OG}}$$

- This method also has the advantage of greatly reduce the computational time



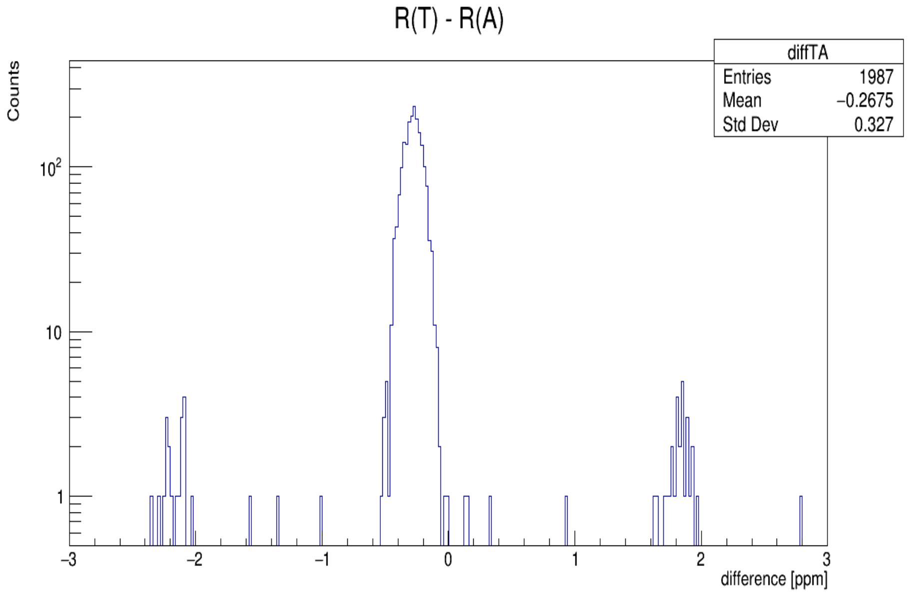
With this method the correlation factor is estimated to be r = 0.533, even lower than before

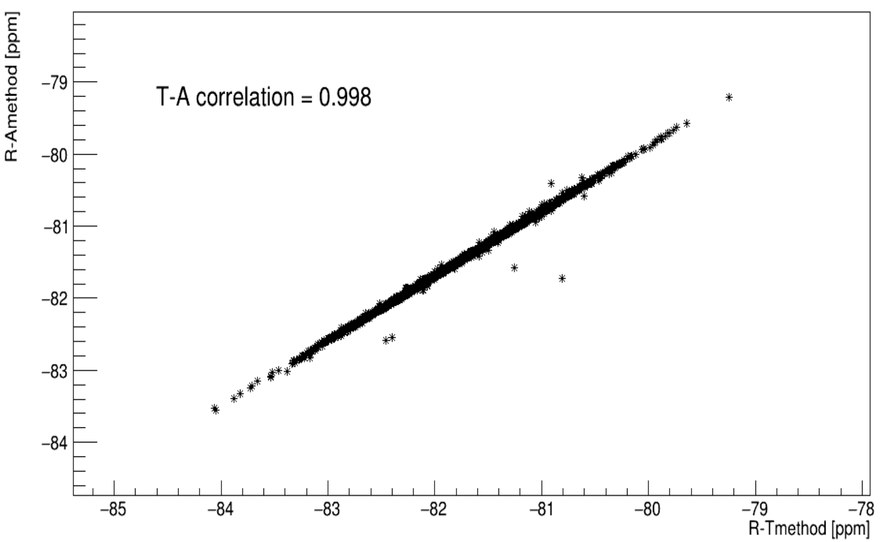
T and A method correlation

Another approach is to extract the entries from a Gaussian. The distribution of the difference R(T) – R(A) highlights that some points have almost a fixed difference. This effect could be due to the random number generator algorithm. When points with a difference greater than 1 are excluded, the correlation factor is r = 0.998.

The allowed statistical deviation is:

$$\sqrt{\delta R(T)^2 + \delta R(A)^2 \ - 2r\delta R(T)\delta R(A)} = 0.081$$





Summary

During my Internship at Fermilab:

1. Learned about the goal and the specifics of the g-2 experiment.
2. Built a Wiggle Plot from a time-energy histogram.
3. Analyzed the data from Run2 using the T and A methods.
4. Learned about beam dynamics effects and their impact on the measurement.
5. Studied the correlation between the T and A methods with the bootstrap method.

THANKS FOR YOUR ATTENTION

Fermilab

References

- B. Abi et al. (Muon g -2 Collaboration) – Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46 ppm
Phys. Rev. Lett. **126**, 141801 (April 2021)
- T. Albahri et al. (Muon g-2 Collaboration) – Measurement of the anomalous precession frequency of the muon in the Fermilab Muon g-2 Experiment
Phys. Rev. D **103**, 072002 (April 2021)
- M. Sorbara – Measurement of the Anomalous Precession Frequency in the Muon g-2 Experiment at Fermilab
PhD. Thesis (March 2022)