

# Towards a unified equation of state (not only!) for astrophysical simulations

Espresso Seminars

Laboratori Nazionali del Sud

Catania, 8<sup>th</sup> March, 2023



Author: S. Burrello

# Outline of the presentation

## 1 Introduction

- Equation of State of nuclear matter: general concepts
- Role in astrophysical simulations and nuclear studies
- Source of information and recent constraints

## 2 Theoretical models

- Mean-field approximation and phenomenological approaches
- Energy density functionals: nuclear structure and reactions

## 3 Recent developments and results

- Connection with ab-initio: improving description at low-density
- Beyond mean-field: many-body correlations and clustering phenomena

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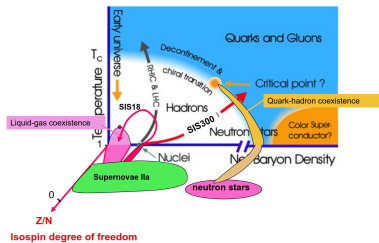
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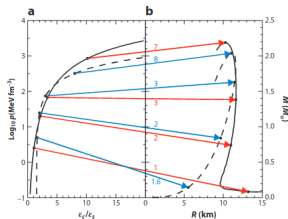
# Introduction: Equation of State of nuclear matter

- **Equation of State (EoS)** of nuclear matter (NM)
  - Extensive  $\xleftrightarrow{eq.}$  intensive variables ( $E = E(\rho, \delta, T)$ )
- Essential ingredient for compact star modelization
  - Mass-radius relation of neutron stars ( $M-R$ )
    - ⇒ General relativity hydrostatic equilibrium
  - Core-collapse supernovae, binary mergers, ...
- Links with nuclear structure and reaction properties
  - Compact stars: neutron stars, hyperons, ...



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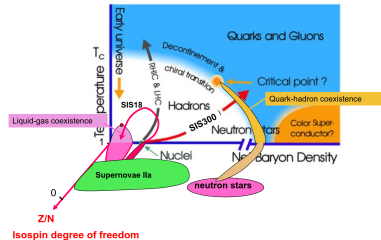


## TOV equation

$$\frac{dp}{dR} = - \frac{(\varepsilon + p)(M + 4\pi r^3 p)}{R(R - 2M)}$$

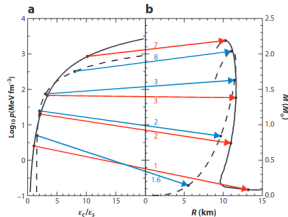
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- Links with nuclear **structure** and **reaction** properties
  - Collective excitations, neutron skins, clustering, ...



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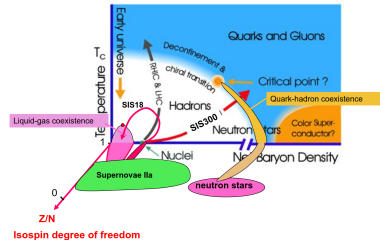
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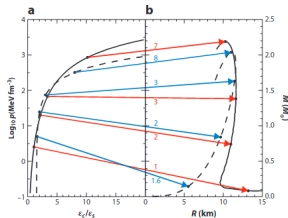
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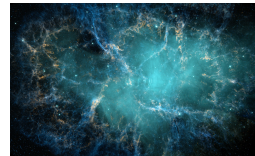
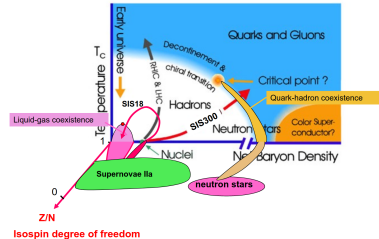


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# Multi-messenger astronomy and EoS constraints

- Extracting **information** on EoS  $\Rightarrow$  **Multi-disciplinary** approach

- **Theory** + astrophysical **observations** + nuclear **experiments**

- Compact stars probe interaction in **unexplored** regimes

- Improved estimations of ms pulsar:  $M > 2M_{\odot}$  (NICER coll.)

[M.C. Miller et al., ApJL 887, L24 (2019)] [T.E. Riley et al., ApJL 918, L27 (2021)]

[GW150914, GW151226, GW170817, GW190521, GW200105, GW200114, GW200210]

[GW170817: constraints on the tidal deformability from gravitational waves (LIGO/VIRGO)]

- Heavy-ion collisions (HIC) @  $\frac{E}{A} \approx (50 - 500) \frac{\text{MeV}}{A}$  with **RIB**

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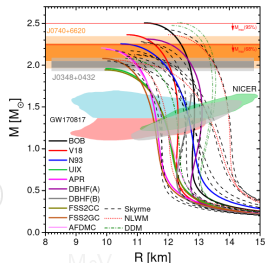
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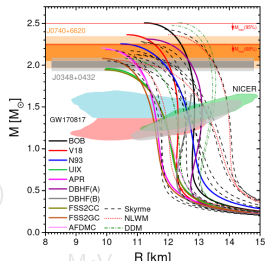
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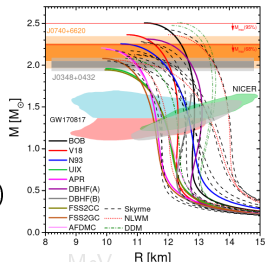
$$\Rightarrow S(\rho) = S(\rho_0) + S(\delta)$$

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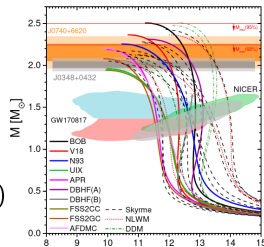
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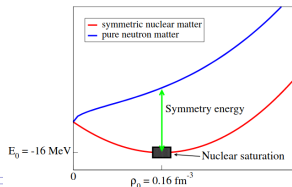
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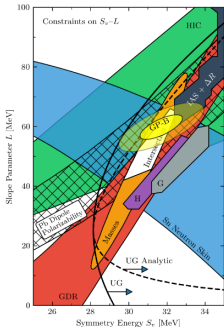
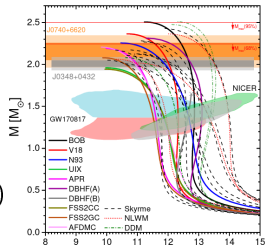
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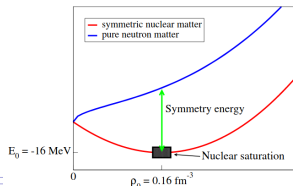


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 $\Rightarrow$  **symmetry energy**  $S(\rho)$  far from **saturation**  $\rho_0$

$$\frac{E}{A}(\rho, \delta) \approx \frac{E}{A}(\rho, \delta = 0) + S(\rho)\delta^2$$

$$S(\rho) = J + L \left( \frac{\rho - \rho_0}{3\rho_0} \right) + \dots$$



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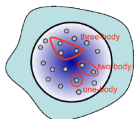
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- **Ab-initio** approaches based on **many-body** expansion
  - Realistic or **effective field theory** (EFT) interactions
    - ⇒ Diagrammatic hierarchy (**power counting**)

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- Phenomenological models with **effective** interaction
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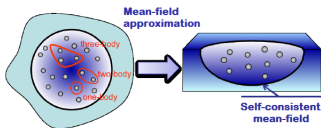
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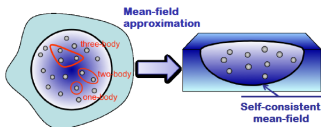


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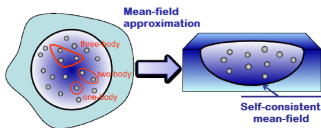
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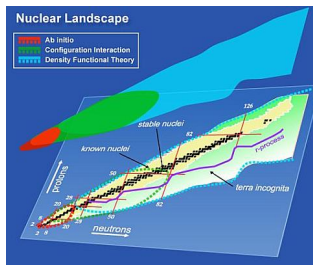
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⇒ Description of Hl **ground state** and **excitations**

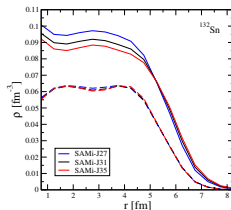


# Nuclear structure: symmetry energy and slope

- Structure of **neutron-rich** nuclei

[S. Burrello et al., PRC C99(5), 054314 (2019)]

- Neutron skin thickness  $\Delta r_{np} \Leftrightarrow L$



## 8 Time-Dependent-Hartree-Fock (TDHF)

Isovector dipole (collective) excitations.

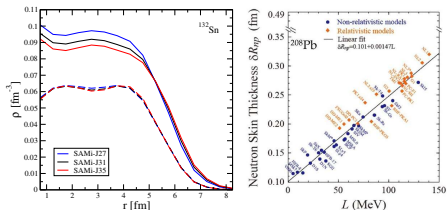
Quadrupole, Octupole, ...

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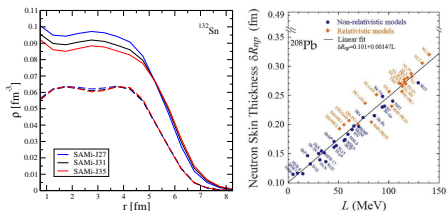
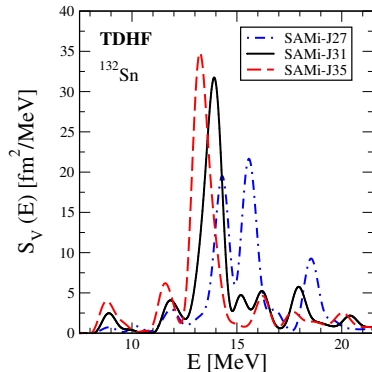
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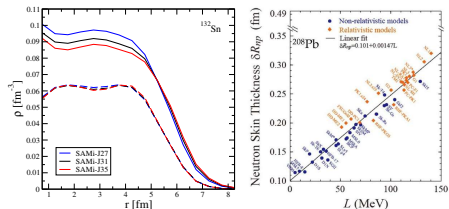
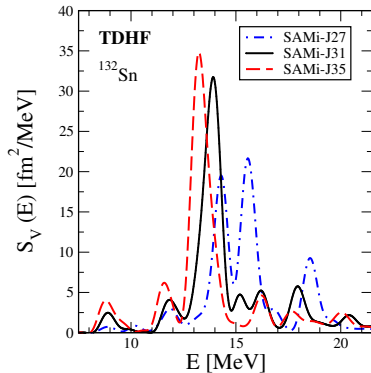
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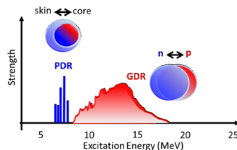
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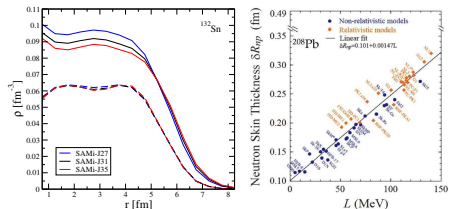
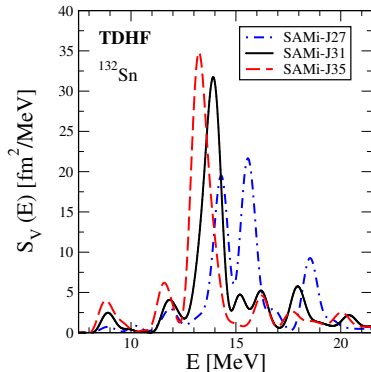


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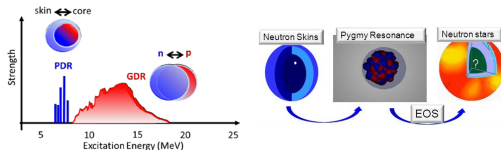


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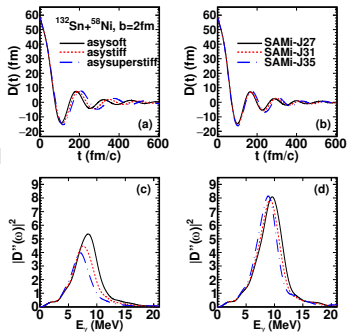
# HIC: surface and momentum dependence

- Pre-equilibrium in **charge-asymmetric reactions**

[H. Zheng, S. Burrello, M. Colonna, V. Baran, PLB 769 (2017)]

- Interplay between **fusion** and **quasi-fission** processes  
⇒ formation of **super-heavy elements**

[H. Zheng, S. Burrello, M. Colonna, D. Lacroix, G. Scamps, PRC 98 (2018)]



- Same **framework** as for nuclear **structure** ⇒ Merging with **reaction studies**
- Role of **different terms** of effective interaction (and EoS) on **final outcomes**
- HIC are a reliable tool to extract **information** of EoS!



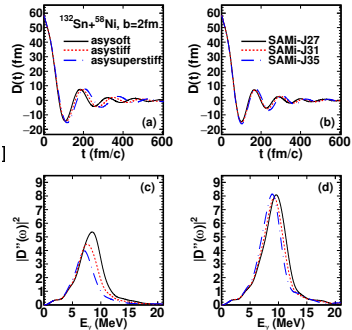
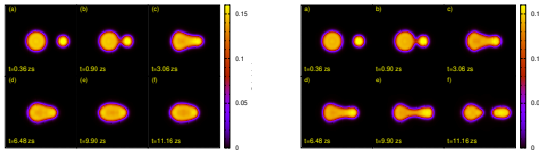
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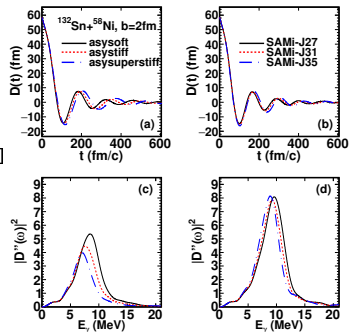
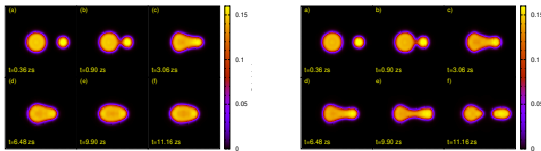
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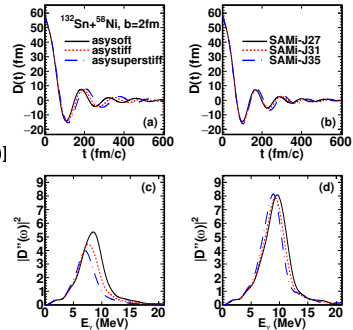
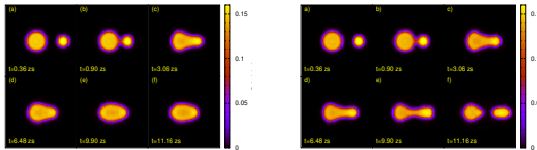
# HIC: surface and momentum dependence

- Pre-equilibrium in **charge-asymmetric reactions**

[H. Zheng, S. Burrello, M. Colonna, V. Baran, PLB 769 (2017)]

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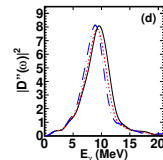
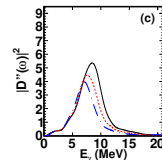
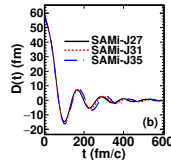
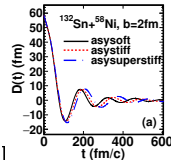
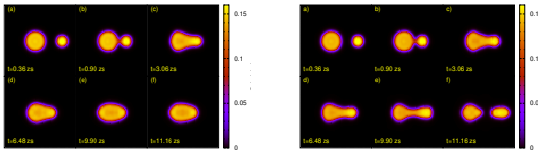
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- Equation of State of nuclear matter: general concepts
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- Source of information and recent constraints

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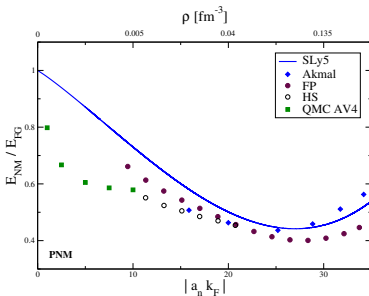
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# Low-density limit of pure neutron matter (PNM)

- **Dilute PNM** ( $a_s = -18.9$  fm)  $\Rightarrow$  close to **unitary** limit of interacting **Fermi** gas
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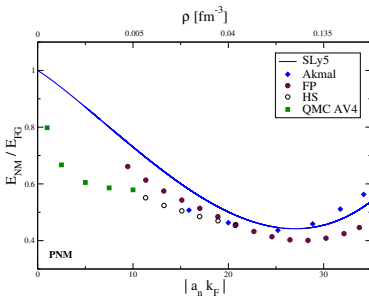
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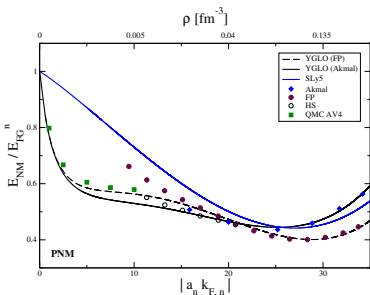


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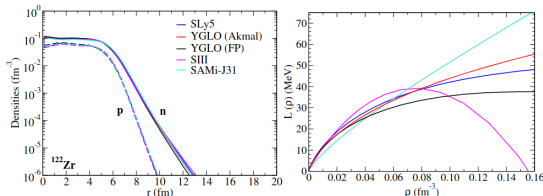
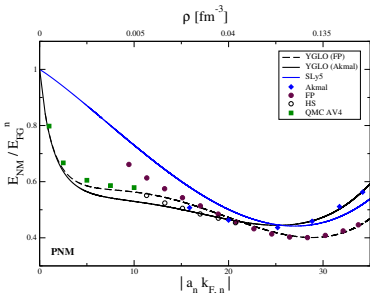
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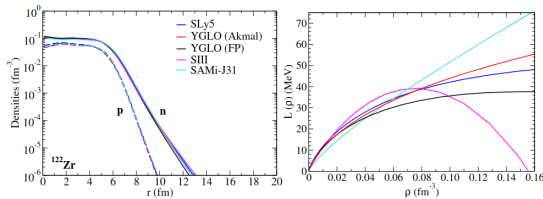
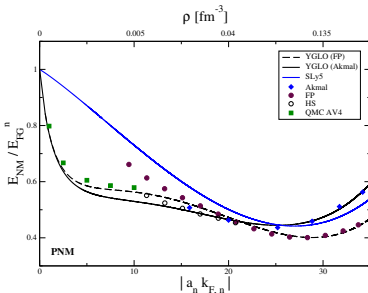
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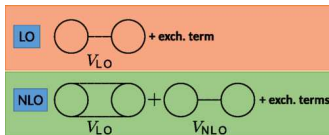
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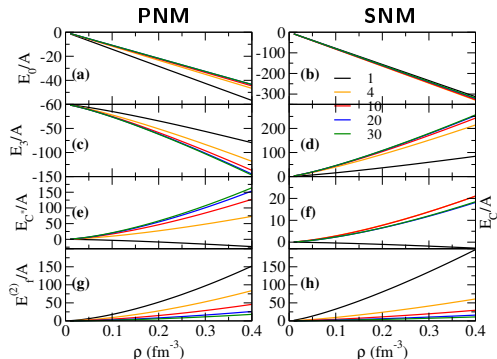
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# Beyond MF: towards a power counting in EDF

- **Beyond MF**  $\Rightarrow$  **correlations** explicitly taken into account (**double-counting**)
- **Hierarchy** of interaction (and EoS) contributions  $\Rightarrow$  **power counting** in EDF
- **EoSs** at next-to-leading order (**NLO**) for symmetric NM (SNM) and PNM



- **Renormalizability** analysis  $\Rightarrow$  **perturbative** scheme
- **Next-to-NLO** (in progress):
  - Expansion parameter
  - Breakdown scale



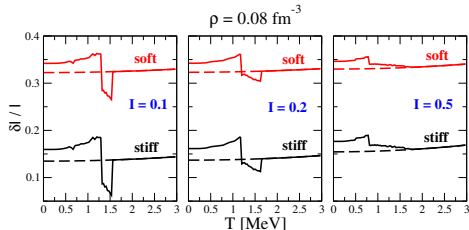
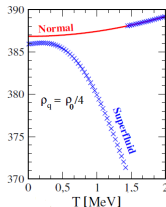
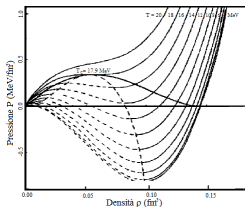
[S. Burrello, C.J. Yang, M. Grasso, PLB 811, 13593 (2020)]

- **BMF** study of **closed-shell** nuclei [C.J. Yang et al., PRC 106 (1), L011305 (2022)]

# Pairing correlations and nuclear superfluidity

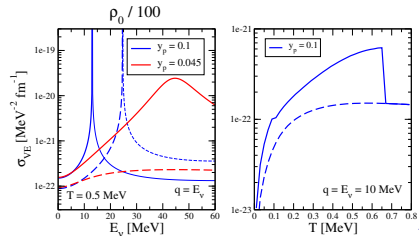
- Pairing effects on mechanical (**spinodal**) instability in low-density **nuclear matter**  
 $\Rightarrow$  variation on **compressibility** and **isotopic content** of the **clusterized** matter

[S. Burrello, M. Colonna, F. Matera, PRC 89 (2014)]



- Homogenous **stellar matter**:  
 impact of **superfluidity** on  $\nu$ -scattering  
 $\Rightarrow$  **cooling** process in **proto-NS** (PNS)  
 or pre-bounce of **supernova** explosions

[S. Burrello, M. Colonna, F. Matera, PRC 94 (2016)]



# Clustering phenomena and neutron star crust

- Many-body (**short-range**) correlations (SRCs) below  $\rho_0$ 
  - Formation of **bound** state of nucleons (**clustering**)

- Phenomenological models with **clusters**

- Describe matter as a mixture of nucleons and nuclei

- Nuclear statistical equilibrium (NSE) model

- [A. R. Raduta, F. Gulminelli, PRC 82, 065801 (2010)]

- Unified description of NS **crust-core** transition

- [S. Bazzani et al., PRC 92, 055804 (2015)]

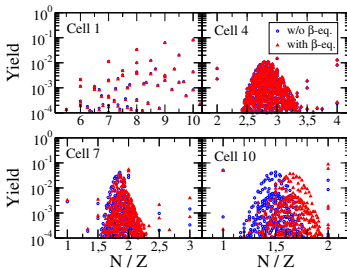
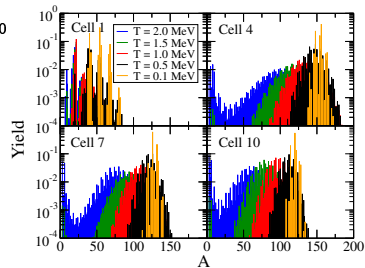
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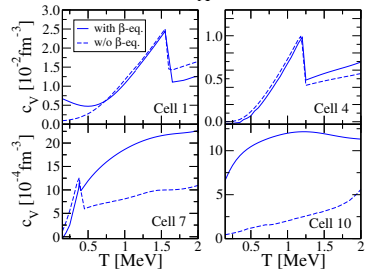
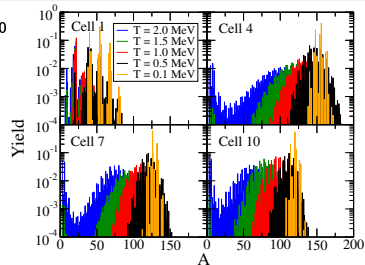
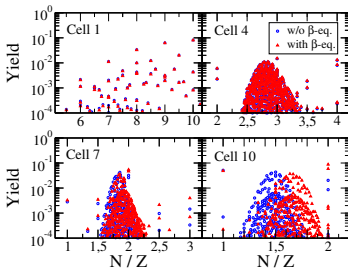
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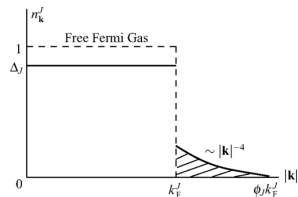
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- Nucleon knock-out in **inelastic electron scattering**  
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  - **Smearing** of Fermi surface (high- $k$  tail at  $T=0$ )
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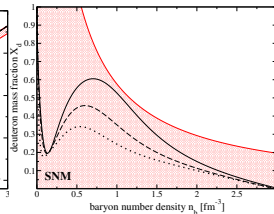
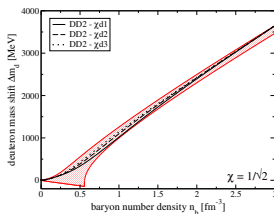
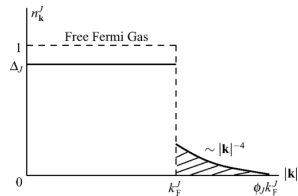
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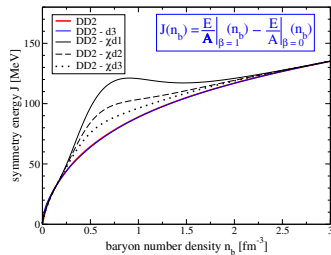
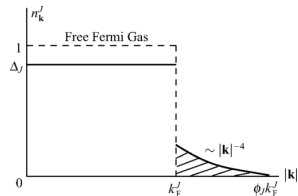
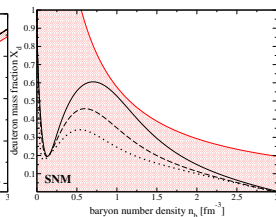
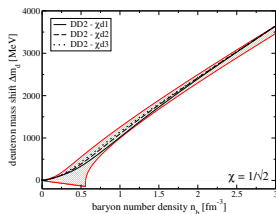
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**THANK YOU FOR YOUR ATTENTION!**

# Best wishes to all women!

## HAPPY WOMEN'S DAY!

