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Factorization Restoration Through Glauber Gluons

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2408.10308 with Patrick Hager, Sebastian Jaskiewicz, Matthias Neubert, and Dominik Schwienbacher

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PDF Factorization

- Scale separation
	- perturbative hard-scattering $\hat{\sigma}_{ij}$ at scale Q ̂
- non-perturbative PDFs $f_i(x)$ at scale Λ_{QCD}
	- No low-energy interactions between incoming hadrons (*P*1*, P*2) = ^X ac d*x*1d*x*² *fi*(*x*1*, µ*)*f^j* (*x*2*, µ*) ˆ*ij* (*p*1*, p*2*,* ↵*s*(*µ*)*, µ*)*,* (2.10)
		- cancellation of soft and Glauber physics CSS '85 (for DY)
- Purely collinear, single logarithmic DGLAP evolution

Collinear Factorization Violation

Catani, de Florian, Rodrigo '11; Forshaw, Seymour, Siodmok '12; \rightarrow talk by Prasanna Kumar Dhani on Friday

New results for **Sp** Henn, Ma,Xu, Yan, Zhang, Zhu '24 Guan, Herzog, Ma, Mistlberger, Suresh '24

For space-like collinear limit $1 \parallel j$ the splitting amplitude Sp depends on the colors and directions of the partons not involved in the splitting! s on the colors and directions of the partons not.
S on the eplitting!

• Related to non-cancellation of soft phases

Implications for PDF factorization?

Super-Leading Logs (SLLs)

Forshaw, Kyrieleis, Seymour '06 '08

Consider gap between jets at hadron collider, cone around beam direction

Large logarithms $\alpha_s^n L^m$ with $L = \ln(Q/Q_0)$

- *e+e[−]* : *m* ≤ *n*, leading logs *m = n*
- $pp: \alpha_s L, \alpha_s^2 L^2, \alpha_s^3 L^3, \alpha_s^4 L^5 \ldots, \alpha_s^{3+n} L^{3+2n}$

Super-Leading Logs (SLLs)

Forshaw, Kyrieleis, Seymour '06 '08

Double logarithms due to Glauber phases in amplitudes which spoil cancellations of soft+collinear terms in cross section

• directly related to collinear factorization breaking

Effect first arises at four-loop order; need

• two phase-factors, a collinear emission, and an emission into gap

> Soft+collinear double logarithms vs. single-log evolution of PDFs?

Have modern EFT framework to analyze and resum observables with SLLs. TB, Neubert, Shao '21; + Stillger '23; Böer, Hager, Neubert, Stillger, Xu '24 → talk by Philipp Böer

Questions about PDF factorization on previous slides can be formulated concisely and answered using the RG and the method of regions.

Will present an analysis of 4-loop SLLs and demonstrate that Glauber gluon exchanges at the scale *Q0* restore the single-logarithmic evolution relevant for PDFs TB, Hager, Jaskiewicz, Neubert, Schwienbacher 2408.10308

Upshot of the talk

x

Collinear factorization breaking at *μ* = *Q*

=

soft-collinear factorization breaking by Glaubers modes at $\mu = Q_0$

PDF factorization for $\mu < Q_0$

"factorization restoration"

Outline

- EFT framework for SLLs
	- Factorization theorem
	- SLLs from RG evolution
- Low-energy analysis
	- Renormalization consistency conditions
- Region analysis of pentagon integrals
	- Glauber contribution to low-*E* matrix elements
- Consistency with PDF factorization

Factorization for gaps between jets

Mm Special Contract of the Contract of **Hard functions** *m* hard partons along $fixed$ directions $\{n_1, \ldots, n_m\}$ \mathcal{H}_m \mathcal{R} \mathcal{M}_m $+$ collinear *Q*⁰ *qc k ls*

Soft + collinear function squared amplitude for *m* Wilson lines +collinear fields for m Wilcon lines corresponds to the gap, and *C*_{responding to the gap, and α} denotes purely collinear contributions. The anomalous

dimension is an operator in color space and a matrix

9

\bigcap \bigcap dynamics associated with the perturbative scale *Q*0, as RG evolution = cusp(↵*s*) ⇣ $2e$ *AVI ^Q*² ⁺ *^V ^G* \overline{a} r ↵*s* \mathbf{L}

Renormalized hard functions fulfill RG equation Renormalized hard functions fulfill RG equation

$$
\frac{d}{d\ln\mu}\mathcal{H}_m=-\sum_{l=m_0}^m\mathcal{H}_l\Gamma_{lm}^H
$$

matrix in multiplicity and color space

One-loop hard anomalous dimension: from and anomalous dimension: The loop ridity with large of the bit iteration

cusp-piece
\nsoft+collinear
\n
$$
\Gamma^{H} = \gamma_{\text{cusp}}(\alpha_s) \left(\frac{\Gamma^{c} \ln \frac{\mu^2}{Q^2}}{\Gamma^{2}} + \frac{\Gamma^{G}}{Q^2} \right) + \frac{\alpha_s}{4\pi} \overline{\Gamma} + \Gamma^{C}
$$
\n
$$
\gamma \propto i\pi \qquad \text{generates SLLs} \qquad \text{Glauber} \qquad \text{collinear} \qquad \text{generales NGLs} \qquad \text{Joucl.}
$$

SLLs from RG evolution *dµ*¹ *µ*1 *µ*¹ Z *^µ^h µ*2 $\frac{1}{2}$

Evolve hard function from $\mu_h \sim Q$ to $\mu_s \sim Q_0$ Γ *µ^s µ* O H *µ^s µ*1 *^H*(*µ*1) + ^Z *^µ^h* \mathbf{h} *µ*1 *µ*¹ *µ*2 *^H*(*µ*2)*^H*(*µ*1) + *... ,* (2)

$$
\mathbf{U}(\mu_h, \mu_s) = \mathbf{P} \exp \left[\int_{\mu_s}^{\mu_h} \frac{d\mu}{\mu} \mathbf{\Gamma}^H(\mu) \right]
$$

= $\mathbf{1} + \int_{\mu_s}^{\mu_h} \frac{d\mu_1}{\mu_1} \mathbf{\Gamma}^H(\mu_1) + \int_{\mu_s}^{\mu_h} \frac{d\mu_1}{\mu_1} \int_{\mu_1}^{\mu_h} \frac{d\mu_2}{\mu_2} \mathbf{\Gamma}^H(\mu_2) \mathbf{\Gamma}^H(\mu_1) + \dots$

Resummed cross section *m*=*m*⁰

Leading SLLs sponding to a real emission, or leave the main \mathcal{S} of *^H* can either increase the number of partons, corresponding to a real emission, or leave them unchanged for virtual terms. The SLLs originate from *^c*. Using simple *C*¹¹ = ⌦ *^H*(0) *c ^V ^G ^V ^G* ⌦ ¹ ↵ *.* (8)

Anomalous dimension fulfills simple identities virtual terms. The SLLs originate from *^c*. Using simple identities among the various terms in (??) [?], one finds Anomalous dimension fulfills simple identities

$$
[\mathbf{\Gamma}^c,\overline{\mathbf{\Gamma}}]=0\,,\quad \langle \dots\,\mathbf{\Gamma}^c\otimes\mathbf{1}\rangle=0\,,\quad \langle \dots\,\boldsymbol{V}^G\otimes\mathbf{1}\rangle=0\,.
$$

Only very specific combinations contribute to leading SLLs. At four loops \overline{O} *ns* contribute to **...** *Crn* = ⌦ *^H*(0) *^m*⁰ (*^c*) *^r V ^G* (*^c*) \overline{M} *Contr*ibute \overline{a} In the vertorming the combinations contribute to the very specific combinations contribute to from *Q* down to the scale *q* down to the search α *q* down to the search α *q* and α *q* down to the search α *q down to the search of a galaxy single search of a galaxy single search* α *q down to the sea* rithms for *V ^G* and , but double logarithms for *^c*. The

$$
C_{01}=\left\langle \boldsymbol{\mathcal{H}}_{m_{0}}^{(0)}\boldsymbol{V}^{G}\boldsymbol{\Gamma}^{c}\boldsymbol{V}^{G}\overline{\boldsymbol{\Gamma}}\otimes \boldsymbol{1}\right\rangle
$$

$$
C_{11}=\left\langle \boldsymbol{\mathcal{H}}_{m_{0}}^{(0)}\boldsymbol{\Gamma}^{c}\boldsymbol{V}^{G}\,\boldsymbol{V}^{G}\,\overline{\boldsymbol{\Gamma}}\otimes \boldsymbol{1}\right\rangle
$$

Born-level \mathscr{H}

^m / 1 at lowest order.

RG invariance imposes a constraint on the form of the bare low-E matrix element r ia invariar
*n*f the hare **100 111100000 a 001**
IOM-F matrix elen $\boldsymbol{\mathcal{W}}_m(\mu_s) = \boldsymbol{Z}\boldsymbol{\mathcal{W}}_m^{\text{bare}}$ of the bare low-E mat t loop expression one finds that the leading UV poles in ement $\bm{\mathcal{W}}_m(\mu_s) = \bm{Z}\bm{\mathcal{W}}_m^{\text{bare}}$

Leading poles in *Z*-factor from one-loop *Γ* **Leading poles in Z-tactor trong**

$$
\mathbf{W}_{m}^{\text{bare}} = \mathbf{1} + \frac{\alpha_{s}}{4\pi} \frac{\overline{\mathbf{\Gamma}}}{2\varepsilon} + \left(\frac{\alpha_{s}}{4\pi}\right)^{2} \left(\frac{\mathbf{V}^{G}\overline{\mathbf{\Gamma}}}{2\varepsilon^{2}} + \dots\right) \n+ \left(\frac{\alpha_{s}}{4\pi}\right)^{3} \left(\frac{\mathbf{V}^{G}\mathbf{V}^{G}\overline{\mathbf{\Gamma}}}{3\varepsilon^{3}} - \frac{\mathbf{\Gamma}^{c}\mathbf{V}^{G}\overline{\mathbf{\Gamma}}}{3\varepsilon^{3}} \ln \frac{Q^{2}}{\mu_{s}^{2}} + \dots\right) + \mathcal{O}(\alpha_{s}^{4})
$$
\n(only show terms for SLLs) dependence on hard scale!!

Now verify this structure order by order... $W = \frac{1}{2} \int_{0}^{2\pi} \frac{1}{2} \, dx$ conclude that all terms in (9) other than the structure than the structure than the structure than the structure

 at NLO is obtained by computing soft Wilson line matrix element or, equivalently, from product of soft currents $J^a_\mu(q)$. *m* W at NIM is obtained by computing soft Wilson \mathbb{Z}_m at induction by softh indical particular parti V^{c}

- $\mathbf{V}^G \overline{\mathbf{\Gamma}}$ in $\mathcal{W}_m^{(2)}$ from real-virtual soft corrections, from imaginary part of one-loop soft current Catani, Grazzini '00. 6
- $V^G V^G \overline{\Gamma}$ in $\mathcal{W}_m^{(3)}$ from two-loop soft current (including tripole terms!) comparing space-like and time-like case Duhr, Gehrmann '13; Dixon, Herrmann,Yan, Zhu '19

$$
\mathbf{W}_{m}^{\text{bare}} = 1 + \frac{\alpha_{s}}{4\pi} \frac{\overline{\mathbf{\Gamma}}}{2\varepsilon} + \left(\frac{\alpha_{s}}{4\pi}\right)^{2} \left(\frac{\mathbf{V}^{G}\overline{\mathbf{\Gamma}}}{2\varepsilon^{2}} + \dots\right) + \left(\frac{\alpha_{s}}{4\pi}\right)^{3} \left(\frac{\mathbf{V}^{G}\mathbf{V}^{G}\overline{\mathbf{\Gamma}}}{3\varepsilon^{3}} - \frac{\Gamma^{c}\mathbf{V}^{G}\overline{\mathbf{\Gamma}}}{3\varepsilon^{3}} \ln \frac{Q^{2}}{\mu_{s}^{2}} + \dots\right) + \mathcal{O}(\alpha_{s}^{4})
$$

- All terms except last are reproduced by soft matrix the elements. beyond. Under the color trace, the color trace, the above expressions can be above expressions can be above exp
In the above expressions can be above expressions can be above expressions can be above expressions of the abo alone. This final term involves a logarithm of the hard α logarithm of the hard α
	- purely soft matrix elements are Q-independent
	- · Purely collinear matrix elements are scaleless A *Q* dependence in the low-energy theory can arise i)
- unrestricted phase-space integrals since collinear emissions never enter gap *j>*2 *Q*² ⁰ (ultra-soft modes in SCET^I and soft-collinear modes incograp of the collinear
	- \rightarrow Term with Q -dependence can only arise through softcollinear interactions! divergences can cell and divergences cancel sectors but leaves in the divergence sectors but leaves in the sectors but

Soft-collinear interactions

Three options for ln(*Q*)-dependent term

- 1. Perturbative on-shell modes with virtuality below soft scale *Q*⁰ mediating soft-collinear interactions
	- e.g. ultra-soft mode in SCET₁, soft-collinear mode in SCET_{II}
- 2. Collinear anomaly inducing rapidity logs + offshell modes (e.g. Glaubers) at scale *Q*⁰
- 3. Non-perturbative low-energy interactions among incoming hadrons
	- Breaks PDF factorization! Non-perturbative two-nucleon matrix elements.

Consider diagrams with soft and collinear emission and exchange between soft and collinear. the low-energy matrix \mathcal{L} on dynamics associated with the perturbative scale *Q*0, as Fig. 1. Sample perturbative contribution to the gap-betweendi diagrams with suit and collineal diffission \overline{a} orange gluon is soft and enters the veto region, the blue and enters the blue and \overline{b}

Perform method-of-region analysis to find possible scalings of loop momentum *k*. fects. The main Part to three-loop order, the perturbative part of *W^m* is conthe virtual gluon momentum *k* will be analyzed below.

Perturbative analysis can uncover scenarios 1.) and 2.) at scale Q_0 . If it fails, this would imply scenario 3.). **The SLL and SLL** renormalization of the second vertex $\mathop{\mathrm{scal}}$ *m ^H^l ^H lm .* (5)

Light cone reference vectors n_{μ} , \bar{n}_{μ} and expansion parameter $\lambda = Q_0/Q$. External momenta the low-energy matrix elements *Wm*, which contain the para n ie reference vectors n_{μ} , n_{μ} and ex μ μ $\partial \Omega = O_0/O$. External momenta $\mathcal{L} \mathcal{U} \cong \mathcal{L} \mathcal{U}' \mathcal{L}$. External indirionted

$$
(n \cdot p_c, \bar{n} \cdot p_c, p_{c\perp}) \equiv (p_c^+, p_c^-, p_{c\perp}) \sim Q(\lambda^2, 1, \lambda)
$$

$$
q_c \sim Q(\lambda^2, 1, \lambda) \qquad \bar{p}_{\bar{c}} \sim Q(1, \lambda^2, \lambda) \qquad l_s \sim Q(\lambda, \lambda, \lambda)
$$

Consider all possible scalings for loop momentum *k* \sim leading logarithms were obtained by iterating the set of \sim where cusp = ↵*s/*⇡ + *...* is the light-like cusp anoma-

Reduces to analysis of scalar pentagon integrals. Can check against full result Bern, Dixon, Kosover '93. diated o↵ a virtual gluon connecting the collinear and anti-collinear sectors, and the sector \mathbf{c} *pc a* and *s* calar pentagon inter ≠*p*¯*c*¯ + *q^c ls*

Region finder in Asy2.1, pySecDec identify single region: relevant diagrams are obtained by attaching the virtual gluon to the upper participation of the upper \mathcal{F}_1 \overline{a} $\ddot{}$

$$
k \sim k_{sc} \sim (\lambda^2, \lambda, \lambda^{3/2})
$$
 "soft-collinear"

On-shell mode with small transverse momentum, compatible with soft and collinear scaling (scenario 1). \overline{a} structure of the numerators and considering the regions decomposition of the dimensionally regulated scalar in*p*¯*c*¯ 4 momentum, $\frac{1}{2}$

In Euclidean kinematics $s_{ij} = (p_i + p_j)^2 < 0$, $p_5^2 < 0$ soft-collinear mode fully reproduces pentagon integral, but for physical kinematics $\begin{array}{ccc} \hline \end{array}$ In Euclidean kinematics $s_{ij} = (p_i + p_j)^2 < 0$, $p_5^2 < 0$ \mathbf{f}_c

$$
s_{12} = -p_c^- q_c^+, \quad s_{23} = q_c^- l_s^+, \quad s_{45} = -(p_c^- - q_c^-)l_s^+, s_{34} = -\bar{p}_{\bar{c}}^+ l_s^-, \quad s_{51} = -q_c^- \bar{p}_{\bar{c}}^+, \quad p_5^2 = (p_c^- - q_c^-) \bar{p}_{\bar{c}}^+
$$

extra terms are present. cancellation of two *^O*() terms, resulting in an *^O*(²) contribution, e.g. for the kinematic extension of the kinematic contribution of the kinematic contribution of the contribu

Related to cancellation Fig. 2. For the upper graph in Fig. 2. For the upper graph in Fig. 2. For the lower graph in Fig. 2. For the lower graph in Fig. 2. For the lower graph in Fig. 2. For the upper graph in Fig. 2. For the upper graph in Fig. Related to cancellation

$$
\underbrace{s_{45} s_{51}}_{\lambda} - \underbrace{p_5^2 s_{23}}_{\lambda} = \underbrace{p_c^- \bar{p}_c^+} \left(q_{cT} + l_{sT} \right)^2 > 0
$$

Extra terms power suppressed in Euclidean
kinematics. and prepartienal to a prefector kinematics and proportional to a prefactor this factor of the full result for the full result for the pentagon inkinematics and proportional to a prefactor r suppres: d in F₁ ⁵ *s*²³ *s*⁴⁵ *s*⁵¹ lidean Eytro torme nower cunnreceed in Euclidean LAUA (UITTO PUWU UUPPROUUU III LUURUUI

$$
P = \frac{s_{45}s_{51}}{s_{45}s_{51} - p_5^2s_{23}} \left[1 - e^{i\pi\varepsilon} \Theta \left(1 + \frac{p_5^2s_{23} - s_{45}s_{51}}{s_{45}s_{51}} \right)^{-\varepsilon} \right]
$$

with
$$
\Theta \equiv \theta(p_5^2) + \theta(s_{23}) - \theta(s_{45}) - \theta(s_{51})
$$

$$
P \sim \begin{cases} 1 & \text{for } \Theta = 0 \\ \lambda^{-1} & \text{for } \Theta \neq 0 \end{cases}
$$

Power enhancement due to complex phases! *e*2221 **ending to a power to a power procession** Power enhancement due to complex phases! 1 for $\frac{1}{2}$ for $\frac{1}{2}$ for $\frac{1}{2}$ for $\frac{1}{2}$ for $\frac{1}{2}$ This quantity vanishes for the kinematics of the lower

Glauber contribution IGUUU CUHINUUU

Hidden region (see backup) not present in Euclidean case: $k \sim (\lambda^2, \lambda, \lambda)$ "Glauber" derive the extra terms in the physical scattering kinematiegion (see packup) not present in Eu $\overline{}$ $k \sim (2^2, 2, 3)$ "Glauhar" $\alpha \in (N, N, N)$ diagool \sim 10 \pm 0.10 \pm J Galar example, we context of the scalar example, we contribute J

Expanded loop integral expanded in the manufacturer the form (with implicit +*i*0 prescriptions)

$$
I^{\mathbf{g}} = i(4\pi)^{2-\varepsilon} \int \frac{d^d k}{(2\pi)^d} \frac{1}{-k_T^2} \frac{1}{k^+ q_c^- - k_T^2 - 2k_T \cdot q_{cT}}
$$

$$
\times \frac{1}{\left[-k^+ (p_c^- - q_c^-) - q_c^+ p_c^- - k_T^2 - 2k_T \cdot q_{cT}\right]}
$$

$$
\times \frac{1}{\bar{p}_c^+(k^- - l_s^-)} \frac{1}{-l_s^+ k^- - k_T^2 + 2k_T \cdot l_{sT}}
$$

well-defined in dim.reg. Perform *k*+ and *k*− integrals using residues. What remains is Euclidean off-shell triangle in $d = 2 - 2\varepsilon$. and is well-defined in dimensional regularization. The ined in dim.reg. Perform κ_{\pm} and κ_{-} inte angle intervention intervention in the second contribution in the second of the second intervention in the second als integrals and therefore does not contribute t_c sociated low-energy scale *Q*²

- Soft-collinear + Glauber modes correctly reproduce leading power result for scalar pentagon in physical region.
- Soft-collinear part has scaleless integration over collinear emission. No contribution to cross section.
- Glauber contribution only from diagram shown above + mirrored counterparts!
	- Result is also obtained using Glauber-SCET Rothstein, Stewart '16 (additional diagrams due to $|k_z|^{\eta}$ regulator cancel against zero-bin subtractions) *η*′

Compute QCD diagram + mirrored counterparts

$$
\mathbf{\mathcal{W}}_m^{\text{bare}} \ni \frac{i\alpha_s^3}{12\pi^2 \varepsilon^3} f^{abc} f^{ade} \sum_{j>2} J_j \times \left[T_{1L}^d T_{1R}^e T_{2L}^b T_{jR}^c \left(-\frac{1}{2\eta} - \ln \frac{\nu}{p_c^+} \right) + T_{2L}^d T_{2R}^e T_{1L}^b T_{jR}^c \left(\frac{1}{2\eta} + \ln \frac{\nu \bar{p}_c^+}{Q_0^2} \right) \right] - (L \leftrightarrow R),
$$

where J_i is angular integral over gap. Need regulator $(k_+/\nu)^\eta$ for collinear phase space integral. Divergences drop out, leave logarithm of $Q^2 = p_c^-\bar{p}_{\bar{c}}^+$. where the terms with \sim 2 have canceled out in the terms with \sim collinear phase space integral. L \sim in \sim Ω \sim \sim \sim \sim right logarithm of $Q^2 = p_c^2 p_c^2$. Research Council (ERC) under the European Union's ergences arop out, leave where the terms with $\frac{1}{2}$ and $\frac{1}{2}$ in the canceled out in the color of $\frac{1}{2}$

> Under color trace expression simplifies to \overline{a} *s* \overline{b} *color tra J^j T ^a* ¹ *T ^b* ² *T ^c ^j* ln *^p ^c p*¯ *c*¯ SI(.
n cin of Matter, EXC 2118/1) funded by the German Research $F_{\rm G}$ under $F_{\rm G}$ under Germany's Eq. () under Germany's Eq. () is equal to $F_{\rm G}$ and $F_{\rm G}$ is eq. (

work, one additionally encounters diagrams with Glauber

$$
\mathbf{W}_{m}^{\text{bare}} \ni -\frac{iN_{c}\alpha_{s}^{3}}{12\pi^{2}\varepsilon^{3}} i f^{abc} \sum_{j>2} J_{j} T_{1}^{a} T_{2}^{b} T_{j}^{c} \ln \frac{p_{c}^{-} \bar{p}_{\bar{c}}^{+}}{Q_{0}^{2}} = -\frac{\Gamma^{c} \mathbf{V}^{G} \overline{\mathbf{\Gamma}}}{3\varepsilon^{3}} \ln \frac{Q^{2}}{\mu_{s}^{2}} + \dots
$$

in all production production of the soft-momentum particles in and production of the soft-momentum particles in production yields ln(*Q***) term!** emission vertex. After regularizing the contribution of $\langle 2 \rangle$ Matrix element of $\mathcal{W}_m^{(3)}$ consiste rapidity regulator, and performing the regulator, and performing the required \boldsymbol{m} [2] J. Collins and J.-W. Qiu, *k^T* factorization is violated $\sum_{i=1}^{n}$ hadron collisions, $\sum_{i=1}^{n}$ h DG | ΔP and S | ΔP with Berlin, and SEE overation. Matrix element of $\mathcal{U}^{(3)}$ consiste m denote a structure required m μ ith DGI AP and SLL evolution was supported by the Swiss National Science \mathbf{r} Matrix element of $\mathcal{W}_m^{(3)}$ consistent with DGLAP and SLL evolution. *m*

actions, JHEP 07, 110, arXiv:1405.2080 [hep-ph].

Conclusion

Long live PDF factorization!

Outlook

- Demonstrated consistency of 4-loop SLL with PDF evolution
	- Remarkable since all ingredients for the breaking of PDF factorization are present at this order.
- To do list
	- Compute non-log(Q) collinear pieces, show that they indeed reproduce DGLAP
	- Consistency for matrix element $\mathcal{W}_m^{(4)}$? (4) *m*
	- All order structure of Glauber terms?
	- Factorization proof?

Extra slides

Glauber region in parameter space kinematics, a subtlety arises upon performing the anawith *s^T* = *lsT* + *qcT* [43]. It is interesting to understand the appearance of the appearance of the appearance of the appearance of this i
It is interesting to understand the appearance of the appearance of the appearance of the appearance of the ap Γ uhar ragion in paramatar er precisly the closely related Lee-Pomeransky space [44]), ilauber region in parameter spac propagators, e.g. *x*

Can perform region analysis in Schwinger or Lee-Pomeransky parameter space (like Asy and PySecDec) \mathcal{L} $t_{\rm eff}$ and $t_{\rm eff}$ nerform region analysis in Schwinger or Leepentaming given analysis in Seminager of 200 $\frac{p}{2}$ erform region analysis in Schwinger or Leetween vertices 1 and 2, and the ensuing *xⁱ* follow in m eransky parameter space (like ${\bf ASy}$ and ${\bf FysecDe}$ tween vertices 1 and 2, and the ensuing *xⁱ* follow in in penominggion diraysis in Schwinger or Lee-

$$
(\overline{x}_1, x_2, x_3, x_4, x_5) \sim (\lambda^{-2}, \lambda^{-2}, \lambda^{-2}, \lambda^{-1}, \lambda^{-2})
$$

$$
\mathcal{F} = -\underbrace{x_1 x_3 s_{23}}_{\lambda^{-3}} - \underbrace{x_1 x_4 s_{51}}_{\lambda^{-3}} - \underbrace{x_3 x_5 s_{45}}_{\lambda^{-2}}
$$

$$
-\underbrace{x_4 x_5 m^2}_{\lambda^{-3}} - \underbrace{x_2 x_4 s_{34}}_{\lambda^{-2}} - \underbrace{x_2 x_5 s_{12}}_{\lambda^{-2}}
$$

The Glauber region corresponds to a pinch due to cancellations in the $\mathscr F$ polynomial associated *F* polynomial factorizes to leading power, The Glauber region corresponds to a pinch due to and *s*45*, s*⁵¹ *<* 0. This generates a non-trivial phase **e**2*i*ⁱ car $\frac{1}{2}$ $\frac{1}{2}$ *c* , contractions of α , contract and α $b \in \mathbb{Z}$ can be factorized in the $\mathbb Z$ can be formed in the formulation c

$$
\mathcal{F} = \underbrace{\left(-q_c^- x_1 + (p_c^- - q_c^-) x_5\right)}_{\lambda^{-2}} \underbrace{\left(l_s^+ x_3 - \bar{p}_c^+ x_4\right)}_{\lambda^{-1}}
$$

Action of anomalous dimensions on ℋ*m*

Soft wide-angle emissions $\overline{\mathbf{\Gamma}}$ real-emission piece $R_{\rm eff}$ on the simplification. After the simplification. After the simplifications distributed by $\frac{1}{2}$ cul wicle-angle emissions F tions HmR^C involve one additional hard gluon (dashed blue line) which is de-angle en en . . 22 Ω amic OUIL VVIL 1e-angie emissi \bigcap_{α} finished in the text, the text, the real correction $\overline{}$. tions House-andie emissions Line collinear to one of the income of the in $\overline{\Gamma}$

$$
\mathcal{H}_{m}\overline{R}_{m} = \sum_{(ij)} \underbrace{\left\{\mathcal{M}\right\}}_{2} \underbrace{\left\{\mathcal{H}_{m}\overline{R}_{m} = -4 \sum_{(ij)} \mathbf{T}_{i,L} \circ \mathbf{T}_{j,R} \overline{W}_{i,j}^{T+1} \Theta_{\text{hard}} \overline{W}_{i,k+1}\right\}}_{\text{extra hard parton!}} \text{extra hard parton!}
$$
\n
$$
\mathcal{H}_{m}\overline{V}_{m} = \sum_{(ij)} \underbrace{\left\{\mathcal{M}\right\}}_{(ij)} \underbrace{\left\{\mathcal{H}_{i,L} \circ \mathbf{T}_{j,L} + \mathbf{T}_{i,R} \cdot \mathbf{T}_{j,R}\right\}}_{(ij)} \underbrace{\left\{\mathcal{M}\right\}}_{(ij)} + \underbrace{\left\{\mathcal{M}\right\}}_{4\pi} \underbrace{\left\{\mathcal{M}\right\}}_{W_{ij}}.
$$

 $\eta_i \cdot n_j$ are given by η_i n_j is $n_i \cdot n_q$ if $n_j \cdot n_q$ $\overline{\omega}$ soft dipole **interest in soft dipole with collinear subtraction** $\overline{W}_{ij}^q = W_{ij}^q - \frac{1}{n_i}$ $n_i \cdot n_q$ $\delta(n_i - n_q) - \frac{1}{n_i \, .}$ $n_j \cdot n_q$ $\int -\frac{1}{n_i \cdot n_a} \delta(n_j - n_q)$ $f(x) = \frac{f(x)}{x}$ pulu $W_{ij}^q =$ $n_i \cdot n_j$ $n_i\cdot n_q\, n_j\cdot n_q$ the virtual correction of the virtual correction B and B and B

Glauber term
$$
\overrightarrow{w}
$$

\n
$$
u_m v^G = \frac{1}{2} \left(\frac{1}{2} \right) \left(\frac{1}{2} \right)^{-1} \left(\frac{1}{2} \right
$$

$$
\boldsymbol{V}^G = -8\,i\pi\left(\boldsymbol{T}_{1,L}\cdot \boldsymbol{T}_{2,L}-\boldsymbol{T}_{1,R}\cdot \boldsymbol{T}_{2,R}\right)
$$

Used color conservation $\sum_i T_i = 0$ to simplify Glauber $\frac{1}{2}$ terms in 1 + 2 \rightarrow 3 + ... + *m* $\mathcal{L} = \frac{1}{\sqrt{2}}$ $+ m \qquad \pi_{ij} = 1$ if both inc./out. .lon \sum_i $\text{terms in } 1 + 2 \rightarrow 3 + ... + n.$ $\frac{c}{\sqrt{2}}$ $= 0$ to simplify (Γ Γ + ... Used color conservation $\sum_i \bm{T}_i = 0$ to simplifi

$$
\sum_{(ij)}\left(\boldsymbol{T}_{i,L}\cdot\boldsymbol{T}_{j,L}-\boldsymbol{T}_{i,R}\cdot\boldsymbol{T}_{j,R}\right)\Pi_{ij}=4\left(\boldsymbol{T}_{1,L}\cdot\boldsymbol{T}_{2,L}-\boldsymbol{T}_{1,R}\cdot\boldsymbol{T}_{2,R}\right)
$$

(Soft+)Collinear Cusp Term . of the anomalous dimension on the hard function \sim \sim 1 ...m \sim 1 ... indices indices indices indices indices indices induced gluon (blue), i.e. \sim σ $\sqrt{ }$ S ij de la constantino de la constantino
La constantino de la $\mathbf{\Gamma}^c$

$$
\mathbf{R}_i^c = -4\mathbf{T}_{i,L} \circ \mathbf{T}_{i,R} \,\delta(n_{m+1} - n_i)
$$

$$
\mathbf{V}_i^c = 4C_i \,\mathbf{1}
$$

• Only present for initial-state partons i=1,2. Final state terms cancel! Sing proporte for imaginary partonon ingles. rila state temis canceliant control piece and the simplification of simplification. After the simplification of α $\overline{}$

• Multiplied by
$$
\ln \frac{\mu^2}{\hat{s}} \rightarrow
$$
 double logarithms!

N=4 SYM space-like *Sp* us to determine the two-loop factorization breaking terms \sim two-loop amplitudes have the following generalized generaliz $=$ 4 SYM space-like S*p z*)*P*, *p^b* ⇠ = *zP*, $\mathcal{L} = \mathcal{L} \times \mathcal{L}$ and the explicit computation of two-putation of two-p $N=1$ SYM engra-lika S_{n}

From 2406.14604 by Henn, Ma, Xu, Yan, Zhang, Zhu for process $\mathcal{A}_5(p_a,p_b,p_i,p_j,p_k)\stackrel{a||b}{\longrightarrow} \mathbf{Sp}\times \mathcal{A}_4(P,p_i,p_j,p_k)$ S_z \Box \Box Ω \overline{A} \overline{A} \overline{C} \overline{A} by Catalogue \overline{A} A result of the analysis in the analysis in the previous sections, we have previous sections, we have \mathcal{A} for process $A_5(p_a, p_b, p_i, p_i, p_b)$ via, xu, Yan, Zhang, Zhu \mathbf{b} \rightarrow Sp \times $A_4(P, p_i)$.

$$
\mathbf{Sp}^{(1)} = \left[\frac{\mu^2 z}{s_{ab}(1-z)}\right]^{\epsilon} \left\{2N_c \overline{r}_S^{(1)}(z+i0) + \mathbf{T}_a \cdot \mathbf{T}_{\text{in}}(2\pi i) c_1(\epsilon) \frac{1}{\epsilon}\right\} \mathbf{Sp}^{(0)},
$$

$$
\mathbf{Sp}^{(2)} = \left[\frac{\mu^2 z}{s_{ab}(1-z)}\right]^{2\epsilon} \left\{4N_c^2 \overline{r}_S^{(2)}(z+i0) + N_c \mathbf{T}_a \cdot \mathbf{T}_{\text{in}}(2\pi i) \left[c_2(\epsilon) \frac{1}{\epsilon^3} + c_1^2(\epsilon) \left(-\frac{2}{\epsilon^2} \ln z + \frac{2}{\epsilon} \ln z \ln\left(\frac{z}{z-1}\right) - 2 \text{Li}_3(1-\frac{1}{z}) - \ln(z) \ln^2\left(\frac{z}{z-1}\right)\right)\right] + \sum_{I \in \text{outgoing}} \left[\mathbf{T}_a \cdot \mathbf{T}_{\text{in}}, \mathbf{T}_a \cdot \mathbf{T}_I\right] (2\pi i) \left[\left(\frac{1}{2\epsilon^2} - \frac{1}{2}\zeta_2\right) (\ln |z_I|^2 + i\pi) + \frac{1}{6} \left(\ln^2 \frac{z_I}{\overline{z}_I} + 4\pi^2\right) \ln \frac{z_I}{\overline{z}_I} + 2\zeta_3\right] + \sum_{I \in \text{outgoing}} \left\{\mathbf{T}_a \cdot \mathbf{T}_{\text{in}}, \mathbf{T}_a \cdot \mathbf{T}_I\right\} (2\pi^2) \left[\frac{1}{2\epsilon^2} - \frac{1}{2}\zeta_2\right] \right\} \mathbf{Sp}^{(0)}.
$$

\overline{a} by incoming a outgoing b: incoming, a: outgoing