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Factorization Restoration Through Glauber Gluons

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2408.10308 with Patrick Hager, Sebastian Jaskiewicz, Matthias Neubert, and Dominik Schwienbacher

High Precision for Hard Processes (HP^2 2024)
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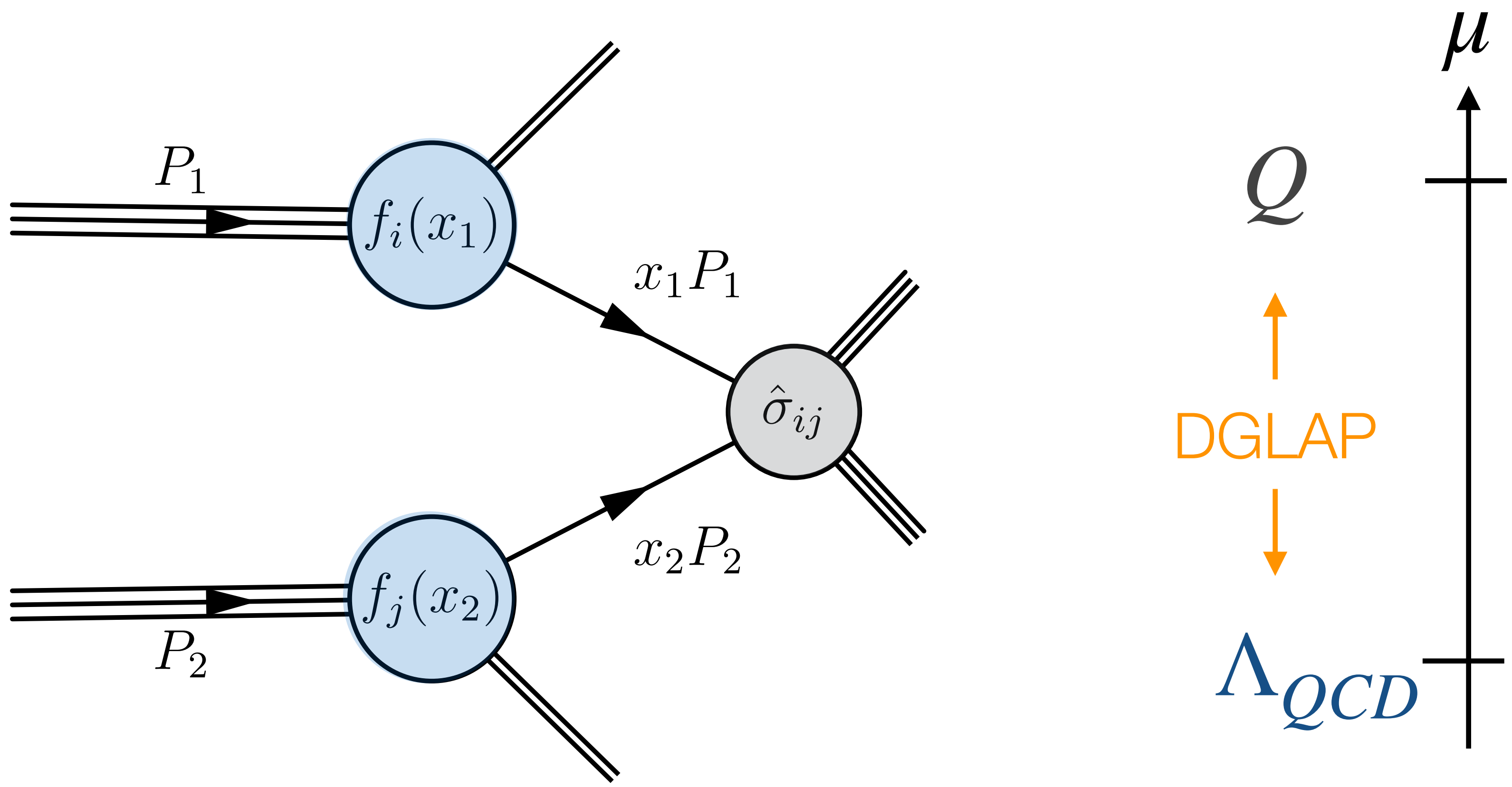
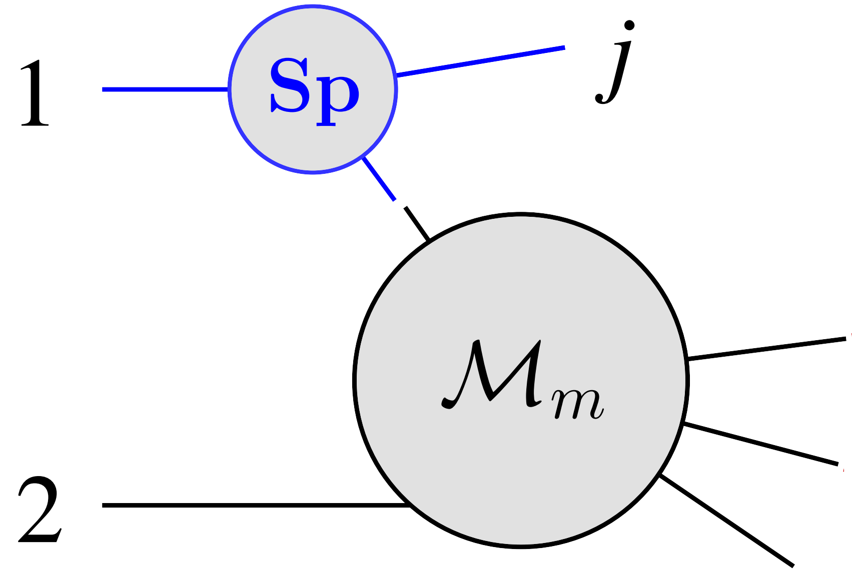
PDF Factorization



- Scale separation
  - perturbative hard-scattering $\hat{\sigma}_{ij}$ at scale Q
  - non-perturbative PDFs $f_i(x)$ at scale Λ_{QCD}
- No low-energy interactions between incoming hadrons
  - cancellation of soft and Glauber physics CSS '85 (for DY)
- Purely collinear, single logarithmic DGLAP evolution

Collinear Factorization Violation

Catani, de Florian, Rodrigo '11; Forshaw, Seymour, Siodmok '12;
→ talk by Prasanna Kumar Dhani on Friday



New results for **Sp**
Henn, Ma,Xu, Yan, Zhang, Zhu '24
Guan, Herzog, Ma, Mistlberger, Suresh '24

For space-like collinear limit $1 \parallel j$ the splitting amplitude **Sp** depends on the colors and directions of the partons not involved in the splitting!

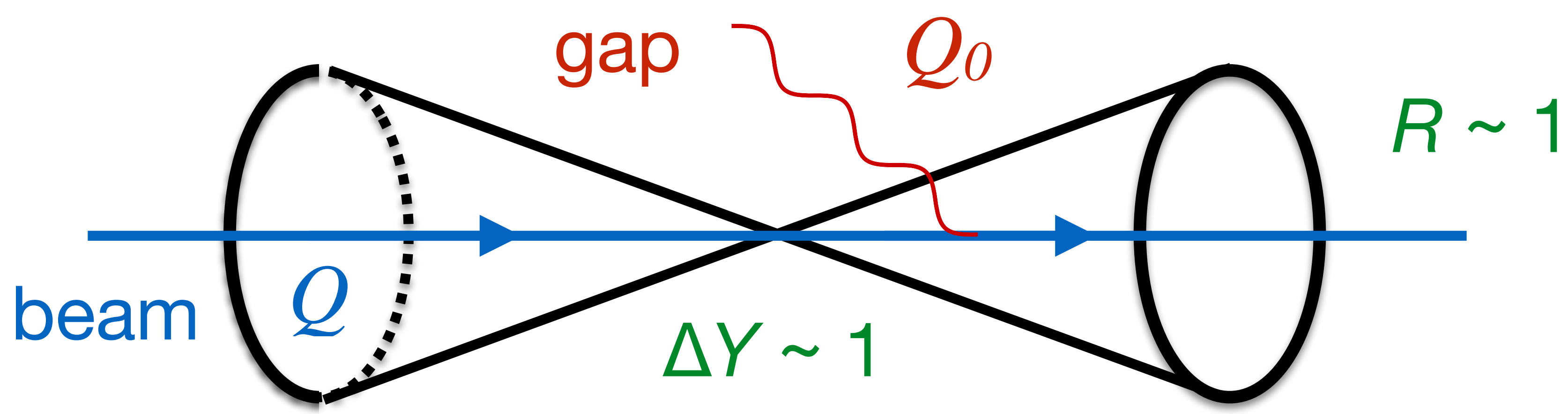
- Related to non-cancellation of soft phases

Implications for PDF factorization?

Super-Leading Logs (SLLs)

Forshaw, Kyrieleis, Seymour ’06 ’08

Consider gap between jets at hadron collider, cone around beam direction



Large logarithms $\alpha_s^n L^m$ with $L = \ln(Q/Q_0)$

- e^+e^- : $m \leq n$, leading logs $m = n$
- pp : $\alpha_s L,\ \alpha_s^2 L^2,\ \alpha_s^3 L^3\ ,\alpha_s^4 L^5 \ldots\ ,\alpha_s^{3+n} L^{3+2n}$

Super-Leading Logs (SLLs)

Forshaw, Kyrieleis, Seymour ’06 ’08

Double logarithms due to Glauber phases in amplitudes which spoil cancellations of soft+collinear terms in cross section

- directly related to collinear factorization breaking

Effect first arises at four-loop order; need

- two phase-factors, a collinear emission, and an emission into gap

Soft+collinear double logarithms vs. single-log evolution of PDFs?

Have modern EFT framework to analyze and resum observables with SLLs. TB, Neubert, Shao ’21; + Stillger ’23; Böer, Hager, Neubert, Stillger, Xu ’24 → talk by Philipp Böer

Questions about PDF factorization on previous slides can be formulated concisely and answered using the RG and the method of regions.

Will present an analysis of 4-loop SLLs and demonstrate that Glauber gluon exchanges at the scale Q_0 restore the single-logarithmic evolution relevant for PDFs TB, Hager, Jaskiewicz, Neubert, Schwienbacher 2408.10308

Upshot of the talk

Collinear factorization breaking at $\mu = Q$

X

soft-collinear factorization breaking by Glaubers modes at $\mu = Q_0$

=

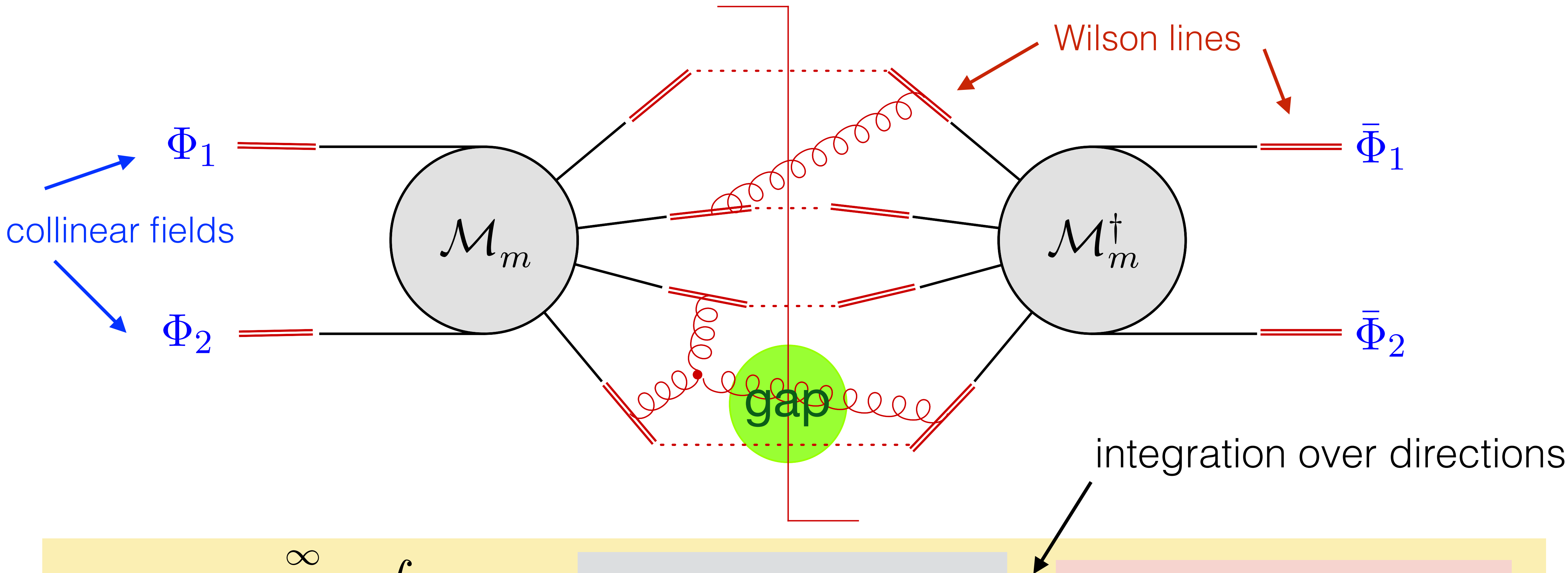
PDF factorization for $\mu < Q_0$

"factorization restoration"

Outline

- EFT framework for SLLs
  - Factorization theorem
  - SLLs from RG evolution
- Low-energy analysis
  - Renormalization consistency conditions
- Region analysis of pentagon integrals
  - Glauber contribution to low-E matrix elements
- Consistency with PDF factorization

Factorization for gaps between jets



$$\sigma(Q_0) = \sum_{m=m_0}^{\infty} \int d\xi_1 d\xi_2 \langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\}, Q, \xi_1, \xi_2, \mu) \otimes \boldsymbol{\mathcal{W}}_m(\{\underline{n}\}, Q_0, \xi_1, \xi_2, \mu) \rangle$$

Hard functions
m hard partons along
fixed directions $\{n_1, \ldots, n_m\}$
$\boldsymbol{\mathcal{H}}_m \propto |\mathcal{M}_m\rangle\langle\mathcal{M}_m|$

Soft + collinear function
squared amplitude
for *m* Wilson lines
+collinear fields

RG evolution

Renormalized hard functions fulfill RG equation

$$\frac{d}{d\ln\mu}\boldsymbol{\mathcal{H}}_m = -\sum_{l=m_0}^{m}\boldsymbol{\mathcal{H}}_l\,\boldsymbol{\Gamma}^H_{lm}$$

matrix in multiplicity and color space

One-loop hard anomalous dimension:

cusp-piece soft+collinear

purely soft

$$\boldsymbol{\Gamma}^H = \gamma_{\rm cusp}(\alpha_s)\left(\boldsymbol{\Gamma}^c \ln\frac{\mu^2}{Q^2} + \boldsymbol{V}^G\right) + \frac{\alpha_s}{4\pi}\overline{\boldsymbol{\Gamma}} + \boldsymbol{\Gamma}^C$$

generates SLLs

$\propto i\pi$ Glauber

generates NGLs
→ talk by Jürg Haag

purely collinear

SLLs from RG evolution

Evolve hard function from $\mu_h \sim Q$ to $\mu_s \sim Q_0$

$$\boldsymbol{U}(\mu_h,\mu_s) = \mathbf{P}\exp\left[\int_{\mu_s}^{\mu_h}\frac{d\mu}{\mu}\,\boldsymbol{\Gamma}^H(\mu)\right] = \mathbf{1} + \int_{\mu_s}^{\mu_h}\frac{d\mu_1}{\mu_1}\,\boldsymbol{\Gamma}^H(\mu_1) + \int_{\mu_s}^{\mu_h}\frac{d\mu_1}{\mu_1}\int_{\mu_1}^{\mu_h}\frac{d\mu_2}{\mu_2}\,\boldsymbol{\Gamma}^H(\mu_2)\boldsymbol{\Gamma}^H(\mu_1) + \dots$$

Resummed cross section

$$\sigma(Q_0) = \sum_{m,l=m_0}^{\infty}\int d\xi_1 d\xi_2 \big\langle \boldsymbol{\mathcal{H}}_m(Q,\mu_h)\,\boldsymbol{U}_{ml}(\mu_h,\mu_s)\otimes \boldsymbol{\mathcal{W}}_l(Q_0,\mu_s)\big\rangle$$

- Independence of $\sigma(Q_0)$ from μ_s leads to consistency conditions for $\mathscr{W}_l(\mu_s)$

Leading SLLs

Anomalous dimension fulfills simple identities

$$[\boldsymbol{\Gamma}^c, \overline{\boldsymbol{\Gamma}}] = 0\,, \quad \langle \ldots \boldsymbol{\Gamma}^c \otimes \mathbf{1} \rangle = 0\,, \quad \big\langle \ldots \boldsymbol{V}^G \otimes \mathbf{1} \big\rangle = 0\,.$$

Only very specific combinations contribute to leading SLLs. At four loops

$$C_{01} = \big\langle \boldsymbol{\mathcal{H}}_{m_0}^{(0)} \boldsymbol{V}^G \boldsymbol{\Gamma}^c \boldsymbol{V}^G \, \overline{\boldsymbol{\Gamma}} \otimes \mathbf{1} \big\rangle$$

$$C_{11} = \big\langle \boldsymbol{\mathcal{H}}_{m_0}^{(0)} \boldsymbol{\Gamma}^c \boldsymbol{V}^G \, \boldsymbol{V}^G \, \overline{\boldsymbol{\Gamma}} \otimes \mathbf{1} \big\rangle$$

Born-level $\mathscr{H}$

$\boldsymbol{\mathcal{W}}_m^{(0)} \propto \mathbf{1}$

RG invariance imposes a constraint on the form of the bare low-E matrix element $\boldsymbol{\mathcal{W}}_m(\mu_s) = \boldsymbol{Z}\boldsymbol{\mathcal{W}}_m^{\text{bare}}$

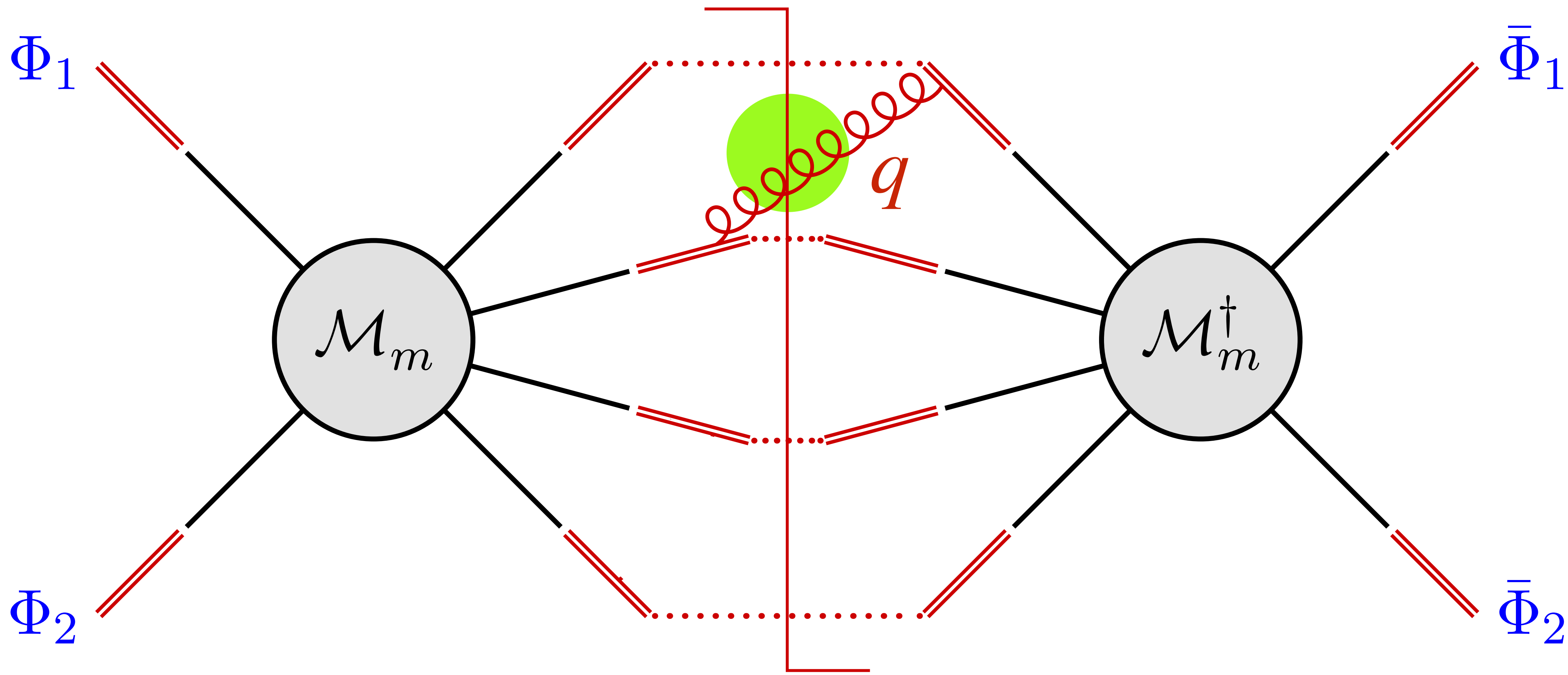
Leading poles in Z-factor from one-loop Γ

$$\boldsymbol{\mathcal{W}}_m^{\text{bare}} = \boldsymbol{1} + \frac{\alpha_s}{4\pi}\,\frac{\overline{\boldsymbol{\Gamma}}}{2\varepsilon} + \left(\frac{\alpha_s}{4\pi}\right)^2 \left(\frac{\boldsymbol{V}^G\,\overline{\boldsymbol{\Gamma}}}{2\varepsilon^2} + \dots\right) + \left(\frac{\alpha_s}{4\pi}\right)^3 \left(\frac{\boldsymbol{V}^G\boldsymbol{V}^G\,\overline{\boldsymbol{\Gamma}}}{3\varepsilon^3} - \frac{\boldsymbol{\Gamma}^c\,\boldsymbol{V}^G\,\overline{\boldsymbol{\Gamma}}}{3\varepsilon^3}\ln\frac{Q^2}{\mu_s^2} + \dots\right) + \mathcal{O}(\alpha_s^4)$$

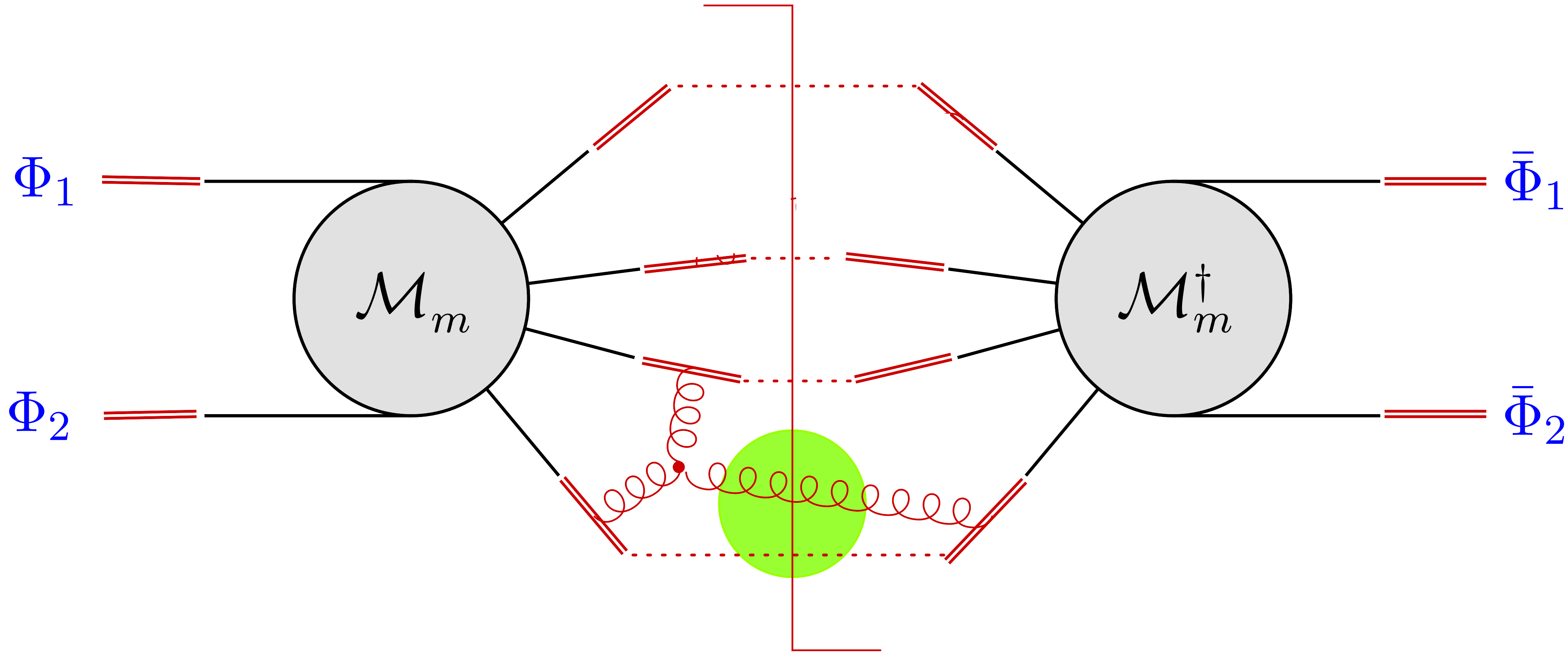
(only show terms for SLLs)

dependence on hard scale !!

Now verify this structure order by order…



$\mathscr{W}_m$ at NLO is obtained by computing soft Wilson line matrix element or, equivalently, from product of soft currents $J_\mu^a(q)$.



- $\mathbf{V}^G\,\overline{\mathbf{\Gamma}}$ in $\mathscr{W}_m^{(2)}$ from real-virtual soft corrections, from imaginary part of one-loop soft current Catani, Grazzini '00.
- $\mathbf{V}^G\,\mathbf{V}^G\,\overline{\mathbf{\Gamma}}$ in $\mathscr{W}_m^{(3)}$ from two-loop soft current (including tripole terms!) comparing space-like and time-like case Duhr, Gehrmann '13; Dixon, Herrmann,Yan, Zhu '19

$$\mathcal{W}_m^{\text{bare}} = \mathbf{1} + \frac{\alpha_s}{4\pi}\,\frac{\overline{\mathbf{\Gamma}}}{2\varepsilon} + \left(\frac{\alpha_s}{4\pi}\right)^2 \left(\frac{\boldsymbol{V}^G\,\overline{\mathbf{\Gamma}}}{2\varepsilon^2} + \dots\right) + \left(\frac{\alpha_s}{4\pi}\right)^3 \left(\frac{\boldsymbol{V}^G\boldsymbol{V}^G\,\overline{\mathbf{\Gamma}}}{3\varepsilon^3} - \frac{\mathbf{\Gamma}^c\,\boldsymbol{V}^G\,\overline{\mathbf{\Gamma}}}{3\varepsilon^3}\ln\frac{Q^2}{\mu_s^2} + \dots\right) + \mathcal{O}(\alpha_s^4)$$

✓ ✓ ✓ ?

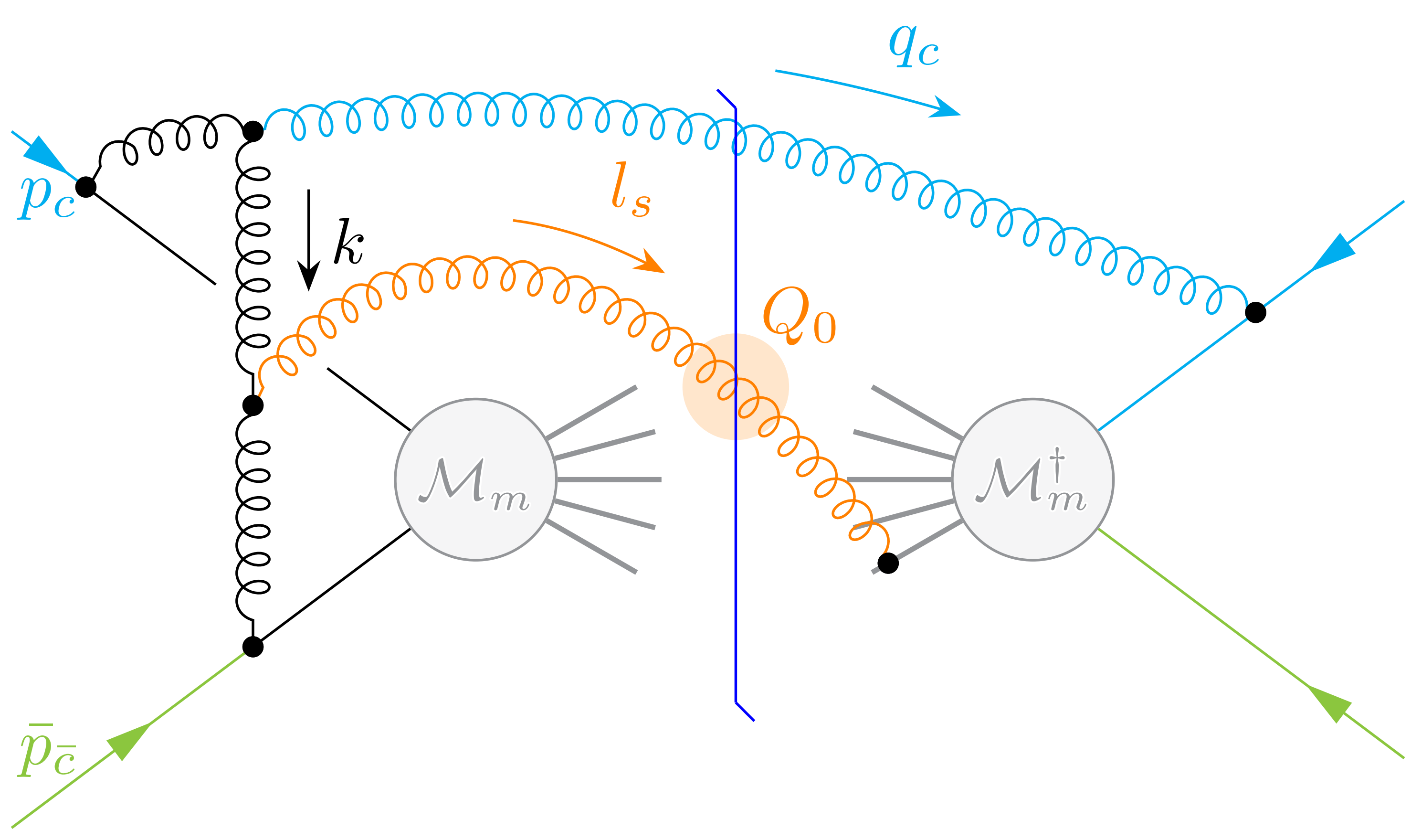
- All terms except last are reproduced by soft matrix elements.
  - purely soft matrix elements are Q-independent
- Purely collinear matrix elements are scaleless
  - unrestricted phase-space integrals since collinear emissions never enter gap

➡ Term with Q-dependence can only arise through soft-collinear interactions!

Soft-collinear interactions

Three options for $\ln(Q)$-dependent term

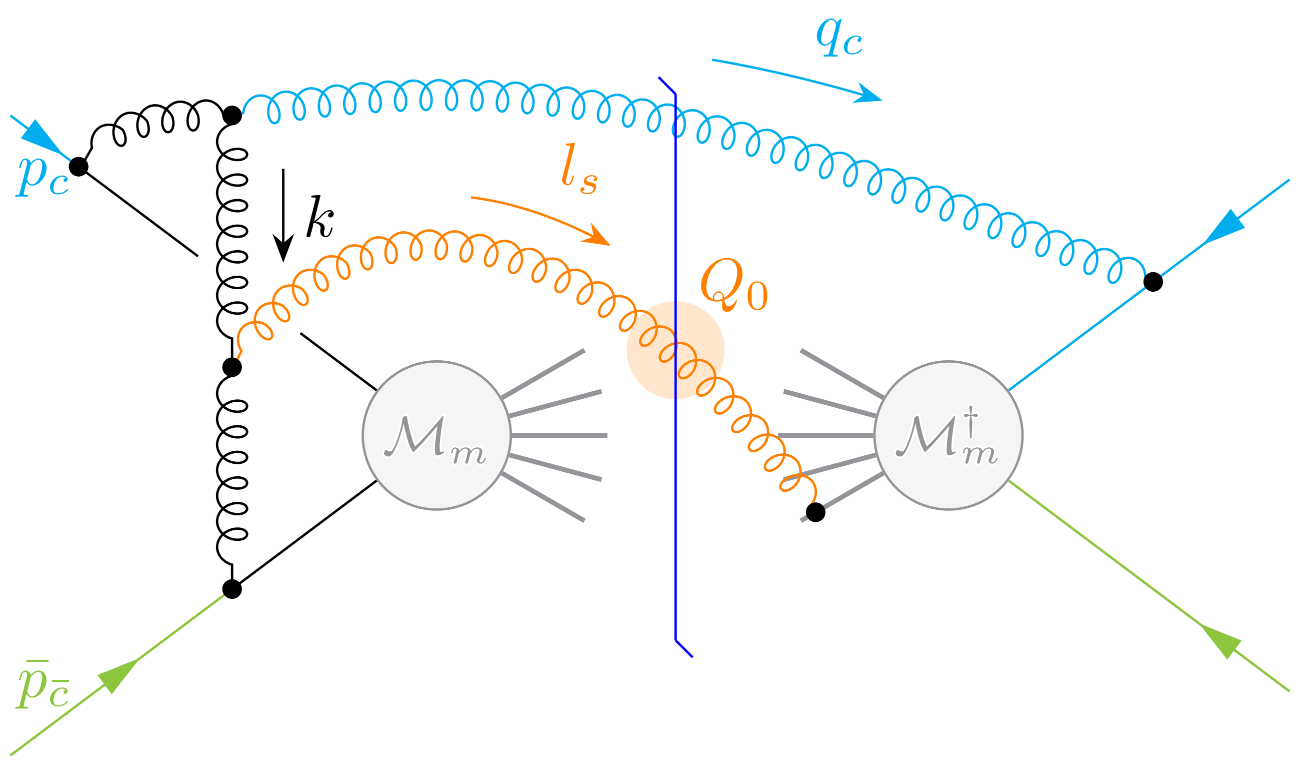
1. Perturbative on-shell modes with virtuality below soft scale Q_0 mediating soft-collinear interactions
    - e.g. ultra-soft mode in $\mathrm{SCET_I}$, soft-collinear mode in $\mathrm{SCET_{II}}$
2. Collinear anomaly inducing rapidity logs + off-shell modes (e.g. Glaubers) at scale Q_0
3. Non-perturbative low-energy interactions among incoming hadrons
    - Breaks PDF factorization! Non-perturbative two-nucleon matrix elements.



Consider diagrams with soft and collinear emission and exchange between soft and collinear.

Perform method-of-region analysis to find possible scalings of loop momentum k.

Perturbative analysis can uncover scenarios 1.) and 2.) at scale Q_0. If it fails, this would imply scenario 3.).

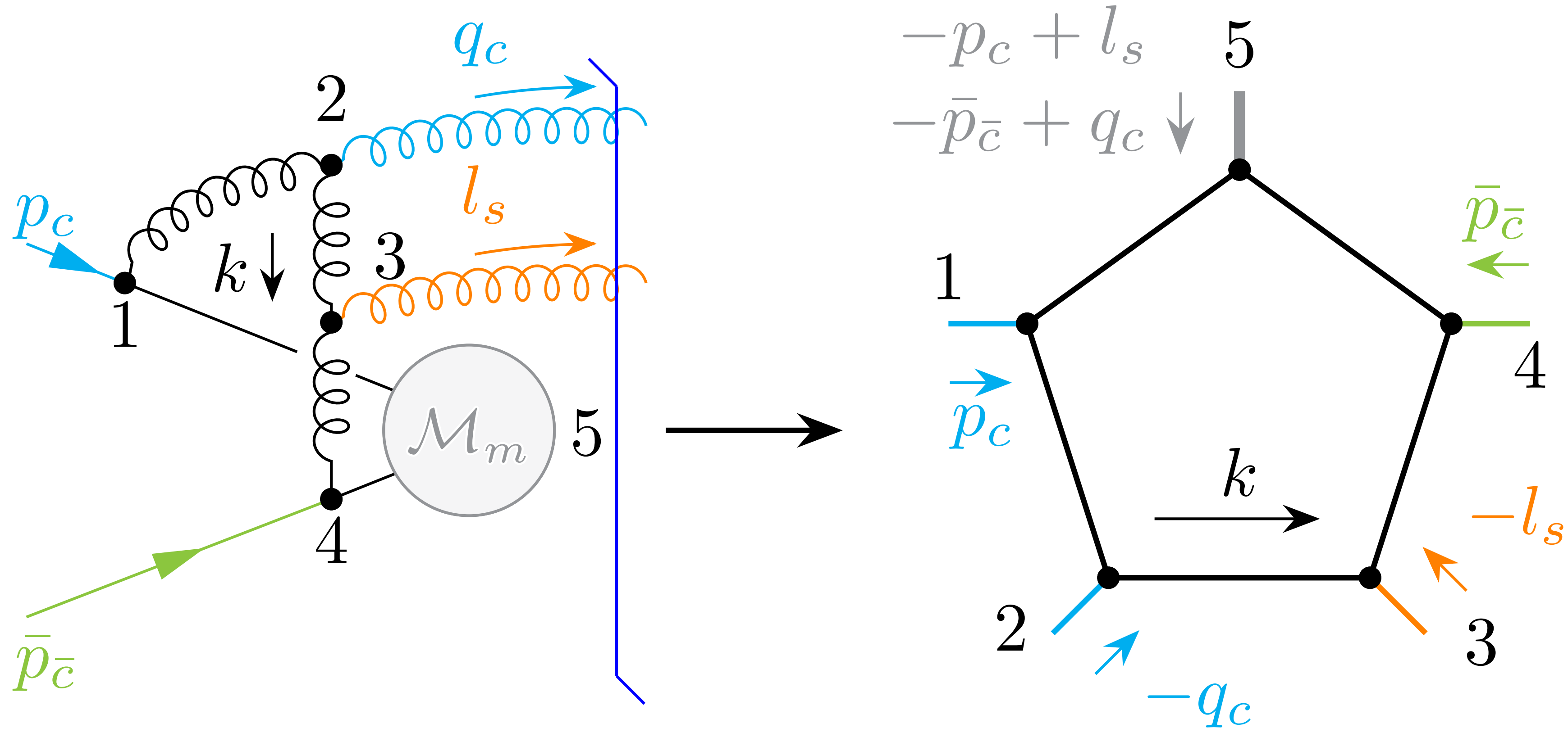


Light cone reference vectors n_μ, $\bar{n}_\mu$ and expansion parameter $\lambda = Q_0/Q$. External momenta

$$(n \cdot p_c, \bar{n} \cdot p_c, p_{c\perp}) \equiv (p_c^+, p_c^-, p_{c\perp}) \sim Q(\lambda^2, 1, \lambda)$$

$$q_c \sim Q(\lambda^2, 1, \lambda) \qquad \bar{p}_{\bar{c}} \sim Q(1, \lambda^2, \lambda) \qquad l_s \sim Q(\lambda, \lambda, \lambda)$$

Consider all possible scalings for loop momentum k



Reduces to analysis of scalar pentagon integrals. Can check against full result Bern, Dixon, Kosover '93.

Region finder in `Asy2.1`, `pySecDec` identify single region:

$$k \sim k_{sc} \sim (\lambda^2, \lambda, \lambda^{3/2})$$ "soft-collinear"

On-shell mode with small transverse momentum, compatible with soft and collinear scaling (scenario 1).

In Euclidean kinematics $s_{ij} = (p_i + p_j)^2 < 0, p_5^2 < 0$ soft-collinear mode fully reproduces pentagon integral, but for physical kinematics

$$s_{12} = -p_c^- q_c^+ \,, \quad s_{23} = q_c^- l_s^+ \,, \qquad s_{45} = -(p_c^- - q_c^-) l_s^+ \,,$$
$$s_{34} = -\bar{p}_{\bar{c}}^+ l_s^- \,, \quad s_{51} = -q_c^- \bar{p}_{\bar{c}}^+ \,, \quad p_5^2 = (p_c^- - q_c^-) \bar{p}_{\bar{c}}^+$$

extra terms are present.

Related to cancellation

$$\underbrace{s_{45}\, s_{51}}_{\lambda} - \underbrace{p_5^2 s_{23}}_{\lambda} = \underbrace{p_c^- \bar{p}_{\bar{c}}^+ \left(q_{cT} + l_{sT}\right)^2}_{\lambda^2} > 0$$

Extra terms power suppressed in Euclidean kinematics and proportional to a prefactor

$$P = \underbrace{\frac{s_{45}\, s_{51}}{s_{45}\, s_{51} - p_5^2 s_{23}}}_{\lambda^{-1}} \left[1 - e^{i\pi\varepsilon\,\Theta} \left(1 + \underbrace{\frac{p_5^2\, s_{23} - s_{45}\, s_{51}}{s_{45}\, s_{51}}}_{\lambda} \right)^{-\varepsilon} \right]$$

with $\Theta \equiv \theta(p_5^2) + \theta(s_{23}) - \theta(s_{45}) - \theta(s_{51})$

$$P \sim \begin{cases} 1 & \text{for } \Theta = 0 \\ \lambda^{-1} & \text{for } \Theta \neq 0 \end{cases}$$

Power enhancement due to complex phases!

Glauber contribution

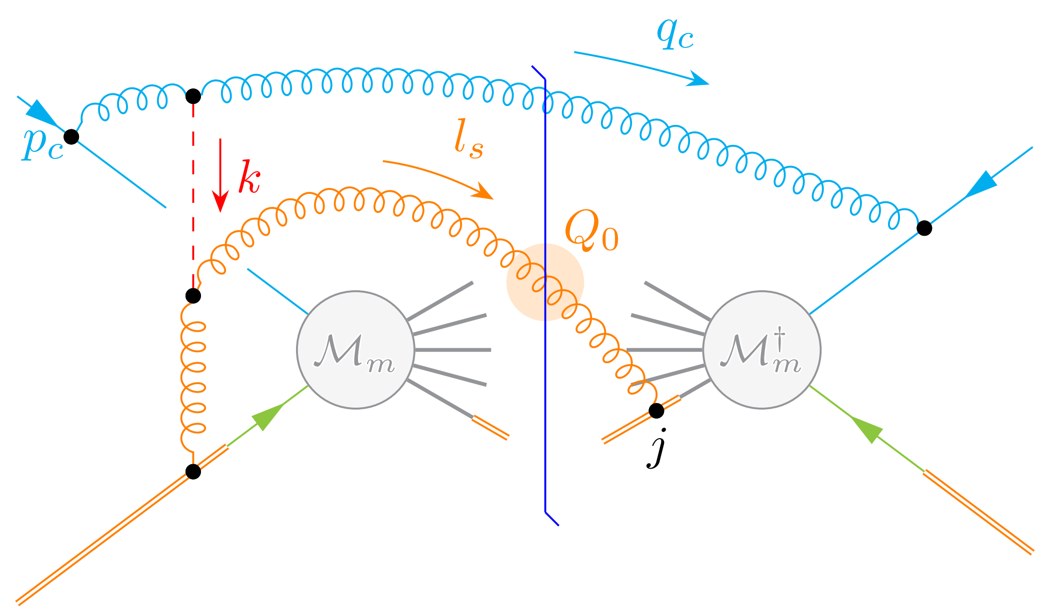
Hidden region (see backup) not present in Euclidean case:

$k \sim (\lambda^2, \lambda, \lambda)$ "Glauber"

Expanded loop integral

$$I^{\mathrm{g}} = i(4\pi)^{2-\varepsilon} \int \frac{d^d k}{(2\pi)^d} \frac{1}{-k_T^2} \frac{1}{k^+ q_c^- - k_T^2 - 2k_T \cdot q_{cT}} \times \frac{1}{\left[-k^+(p_c^- - q_c^-) - q_c^+ p_c^- - k_T^2 - 2k_T \cdot q_{cT}\right]} \times \frac{1}{\bar{p}_{\bar{c}}^+ (k^- - l_s^-)} \frac{1}{-l_s^+ k^- - k_T^2 + 2k_T \cdot l_{sT}}$$

well-defined in dim.reg. Perform k_+ and k_- integrals using residues. What remains is Euclidean off-shell triangle in $d = 2 - 2\varepsilon$.



- Soft-collinear + Glauber modes correctly reproduce leading power result for scalar pentagon in physical region.
- Soft-collinear part has scaleless integration over collinear emission. No contribution to cross section.
- Glauber contribution only from diagram shown above + mirrored counterparts!
  - Result is also obtained using Glauber-SCET Rothstein, Stewart '16 (additional diagrams due to $|k_z|^{\eta'}$ regulator cancel against zero-bin subtractions)

Compute QCD diagram + mirrored counterparts

$$\mathcal{W}_m^{\text{bare}} \ni \frac{i\alpha_s^3}{12\pi^2\varepsilon^3} f^{abc} f^{ade} \sum_{j>2} J_j \times \left[\boldsymbol{T}_{1L}^d \boldsymbol{T}_{1R}^e \boldsymbol{T}_{2L}^b \boldsymbol{T}_{jR}^c \left(-\frac{1}{2\eta} - \ln \frac{\nu}{p_c^-} \right) + \boldsymbol{T}_{2L}^d \boldsymbol{T}_{2R}^e \boldsymbol{T}_{1L}^b \boldsymbol{T}_{jR}^c \left(\frac{1}{2\eta} + \ln \frac{\nu \bar{p}_{\bar{c}}^+}{Q_0^2} \right) \right]$$
$$- (L \leftrightarrow R) \,,$$

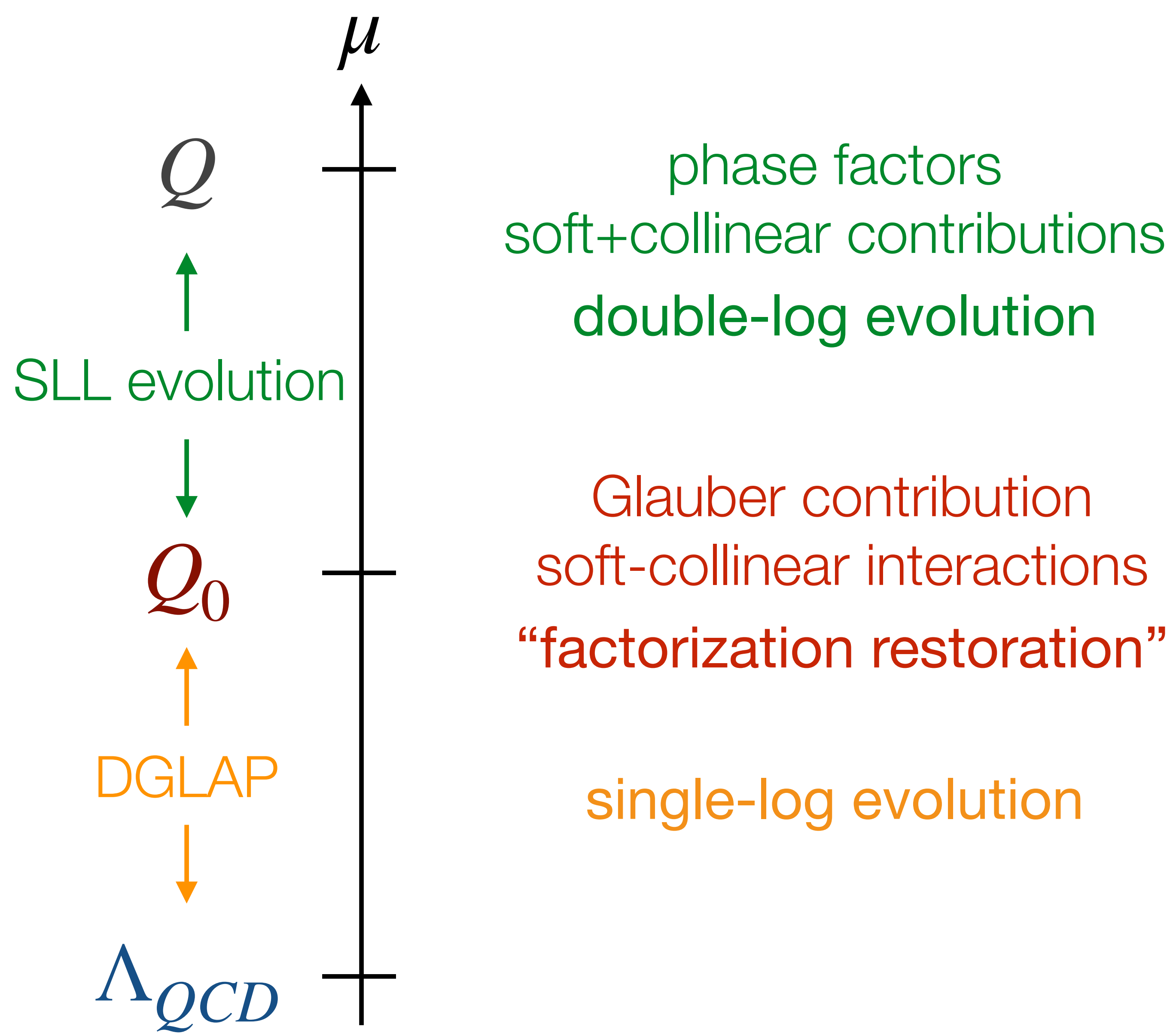
where J_i is angular integral over gap. Need regulator $(k_+/\nu)^\eta$ for collinear phase space integral. Divergences drop out, leave logarithm of $Q^2 = p_c^- \bar{p}_{\bar{c}}^+$.

Under color trace expression simplifies to

$$\mathcal{W}_m^{\text{bare}} \ni -\frac{iN_c\alpha_s^3}{12\pi^2\varepsilon^3}\, i f^{abc} \sum_{j>2} J_j\, \boldsymbol{T}_1^a \boldsymbol{T}_2^b \boldsymbol{T}_j^c \ln \frac{p_c^- \bar{p}_{\bar{c}}^+}{Q_0^2} = -\frac{\boldsymbol{\Gamma}^c \boldsymbol{V}^G \overline{\boldsymbol{\Gamma}}}{3\varepsilon^3} \ln \frac{Q^2}{\mu_s^2} + \dots \quad ✓$$

Perturbative Glauber contribution yields $\ln(Q)$ term!
Matrix element of $\mathscr{W}_m^{(3)}$ consistent with DGLAP and SLL evolution.

Conclusion



Long live PDF factorization!

Outlook

- Demonstrated consistency of 4-loop SLL with PDF evolution
    - Remarkable since all ingredients for the breaking of PDF factorization are present at this order.
- To do list
    - Compute non-$\log(Q)$ collinear pieces, show that they indeed reproduce DGLAP
    - Consistency for matrix element $\mathscr{W}_m^{(4)}$?
    - All order structure of Glauber terms?
    - Factorization proof?

Extra slides

Glauber region in parameter space

Can perform region analysis in Schwinger or Lee-Pomeransky parameter space (like **`Asy`** and **`PySecDec`**)

$$(x_1, x_2, x_3, x_4, x_5) \sim (\lambda^{-2}, \lambda^{-2}, \lambda^{-2}, \lambda^{-1}, \lambda^{-2})$$

$$\mathcal{F} = -\underbrace{x_1 x_3\, s_{23}}_{\lambda^{-3}} - \underbrace{x_1 x_4\, s_{51}}_{\lambda^{-3}} - \underbrace{x_3 x_5\, s_{45}}_{\lambda^{-3}}$$
$$- \underbrace{x_4 x_5\, m^2}_{\lambda^{-3}} - \underbrace{x_2 x_4\, s_{34}}_{\lambda^{-2}} - \underbrace{x_2 x_5\, s_{12}}_{\lambda^{-2}}$$

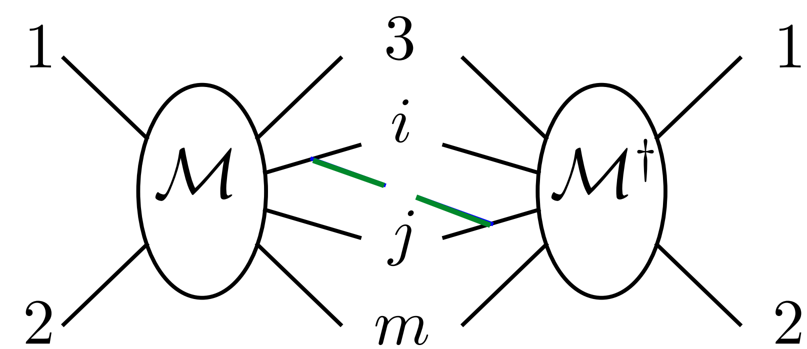
The Glauber region corresponds to a pinch due to cancellations in the $\mathscr{F}$ polynomial

$$\mathcal{F} = \underbrace{\left(-q_c^- x_1 + (p_c^- - q_c^-)\, x_5\right)}_{\lambda^{-2}} \underbrace{\left(l_s^+ x_3 - \bar{p}_{\bar{c}}^+ x_4\right)}_{\lambda^{-1}}$$

Action of anomalous dimensions on $\mathscr{H}_m$

Soft wide-angle emissions $\overline{\boldsymbol{\Gamma}}$

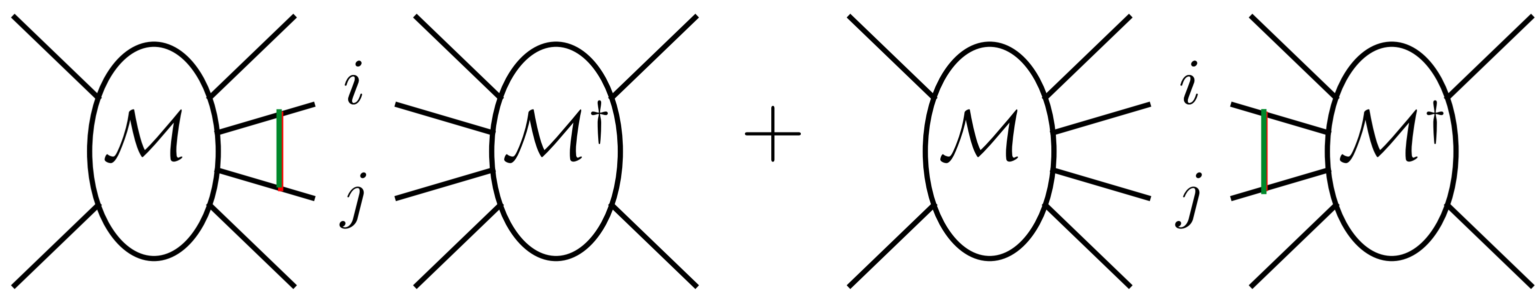
$$\boldsymbol{\mathcal{H}}_m \overline{\boldsymbol{R}}_m = \sum_{(ij)}$$



$$\overline{\boldsymbol{R}}_m = -4 \sum_{(ij)} \boldsymbol{T}_{i,L} \circ \boldsymbol{T}_{j,R}\, \overline{W}_{ij}^{m+1}\, \Theta_{\text{hard}}(n_{m+1})$$

extra hard parton!

$$\boldsymbol{\mathcal{H}}_m \overline{\boldsymbol{V}}_m = \sum_{(ij)}$$



$$\overline{\boldsymbol{V}}_m = 2 \sum_{(ij)} \left(\boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,L} + \boldsymbol{T}_{i,R} \cdot \boldsymbol{T}_{j,R}\right) \int \frac{d\Omega(n_k)}{4\pi}\, \overline{W}_{ij}^{k}$$

soft dipole

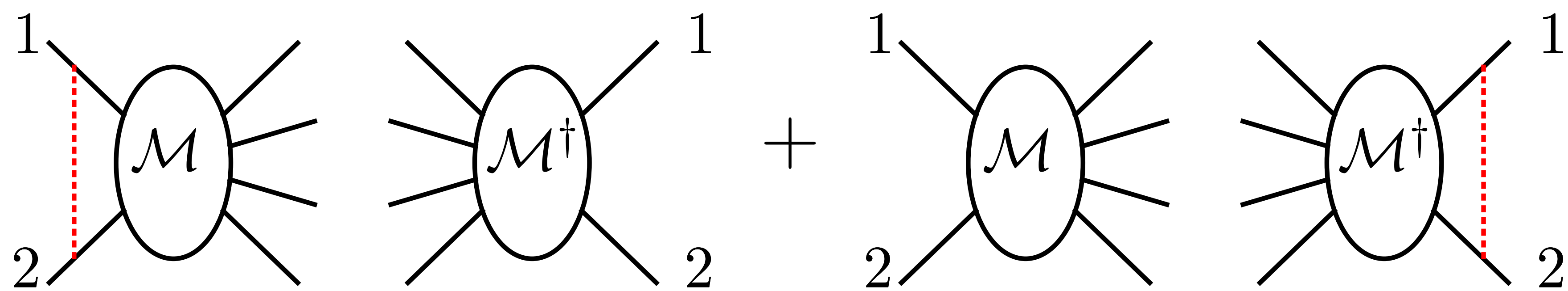
$$W_{ij}^{q} = \frac{n_i \cdot n_j}{n_i \cdot n_q\, n_j \cdot n_q}$$

soft dipole with collinear subtraction

$$\overline{W}_{ij}^{q} = W_{ij}^{q} - \frac{1}{n_i \cdot n_q}\delta(n_i - n_q) - \frac{1}{n_j \cdot n_q}\delta(n_j - n_q)$$

Glauber term V^G

$$\mathcal{H}_m\, V^G =$$



$$V^G = -8i\pi \left(\boldsymbol{T}_{1,L} \cdot \boldsymbol{T}_{2,L} - \boldsymbol{T}_{1,R} \cdot \boldsymbol{T}_{2,R} \right)$$

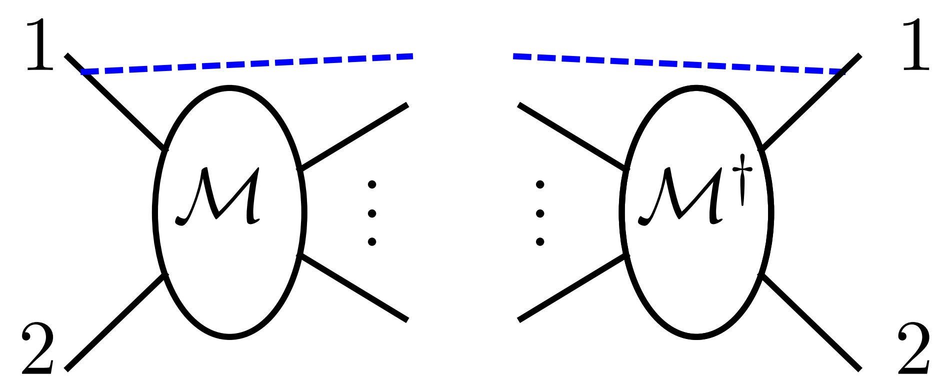
Used color conservation $\sum_i \boldsymbol{T}_i = 0$ to simplify Glauber terms in 1 + 2 → 3 + … + *m*

Π_{ij} = 1 if both inc./out.

$$\sum_{(ij)} \left(\boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,L} - \boldsymbol{T}_{i,R} \cdot \boldsymbol{T}_{j,R} \right) \Pi_{ij} = 4 \left(\boldsymbol{T}_{1,L} \cdot \boldsymbol{T}_{2,L} - \boldsymbol{T}_{1,R} \cdot \boldsymbol{T}_{2,R} \right)$$

(Soft+)Collinear Cusp Term $\mathbf{\Gamma}^c$

$$\boldsymbol{\mathcal{H}}_m\, \boldsymbol{R}_1^c \;=$$



$$\boldsymbol{R}_i^c = -4\boldsymbol{T}_{i,L} \circ \boldsymbol{T}_{i,R}\, \delta(n_{m+1} - n_i)$$
$$\boldsymbol{V}_i^c = 4C_i\, \mathbf{1}$$

- Only present for initial-state partons i=1,2. Final state terms cancel!
- Multiplied by $\ln \frac{\mu^2}{\hat{s}}$ → double logarithms!

$\mathcal{N}$=4 SYM space-like *Sp*

From 2406.14604 by Henn, Ma, Xu, Yan, Zhang, Zhu

for process $\mathcal{A}_5(p_a,p_b,p_i,p_j,p_k) \xrightarrow{a\|b} \mathbf{Sp} \times \mathcal{A}_4(P,p_i,p_j,p_k)$

$$\mathbf{Sp}^{(1)} = \left[\frac{\mu^2\, z}{s_{ab}\,(1-z)}\right]^{\epsilon} \left\{2N_c\, \overline{r}_S^{(1)}(z+i0) + \mathbf{T}_a \cdot \mathbf{T}_{\text{in}}\; (2\pi i)\, c_1(\epsilon)\frac{1}{\epsilon}\right\} \mathbf{Sp}^{(0)}\,,$$

$$\begin{aligned}\mathbf{Sp}^{(2)} = \left[\frac{\mu^2\, z}{s_{ab}\,(1-z)}\right]^{2\epsilon} &\left\{4N_c^2\, \overline{r}_S^{(2)}(z+i0)\right.\\
&+ N_c\, \mathbf{T}_a \cdot \mathbf{T}_{\text{in}}\; (2\pi i)\left[c_2(\epsilon)\,\frac{1}{\epsilon^3} + c_1^2(\epsilon)\left(-\frac{2}{\epsilon^2}\ln z + \frac{2}{\epsilon}\ln z \ln\left(\frac{z}{z-1}\right) - 2\,\mathrm{Li}_3\left(1-\frac{1}{z}\right) - \ln(z)\ln^2\left(\frac{z}{z-1}\right)\right)\right]\\
&+ \sum_{I\in\text{outgoing}} [\mathbf{T}_a \cdot \mathbf{T}_{\text{in}}, \mathbf{T}_a \cdot \mathbf{T}_I]\,(2\pi i)\left[\left(\frac{1}{2\epsilon^2} - \frac{1}{2}\zeta_2\right)(\ln|z_I|^2 + i\pi) + \frac{1}{6}\left(\ln^2\frac{z_I}{\bar{z}_I} + 4\pi^2\right)\ln\frac{z_I}{\bar{z}_I} + 2\zeta_3\right]\\
&\left.+ \sum_{I\in\text{outgoing}} \{\mathbf{T}_a \cdot \mathbf{T}_{\text{in}}, \mathbf{T}_a \cdot \mathbf{T}_I\}\,(2\pi^2)\left[\frac{1}{2\epsilon^2} - \frac{1}{2}\zeta_2\right]\right\}\mathbf{Sp}^{(0)}\,.\end{aligned}$$

b: incoming, a: outgoing