The QED MC@NLO method in SHERPA

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Based on my PhD thesis [2409.02203] and future publications

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Outline

Introduction

- Next-to-leading order electroweak corrections now needed for many observables at the LHC
- EW Sudakov high-energy resummation (of $\log(s/m_W^2)$) already used Denner, Pozzorini '00, Bothmann, Napoletano '20
- QED resummation effects also need to be included for any leptonic final state
- Either YFS matched to higher orders Yennie, Frautschi, Suura '61, Krauss, Schönherr '08, Krauss, Price, Schönherr '22, LF, Schönherr '22 or NLO-matched QED parton showers
- Use well-tested methods developed for QCD: MC@NLO Frixione, Webber '02, POWHEG Nason '04, Frixione, Nason, Oleari '07

NLO matching

What is NLO matching?

NLO matching: producing a prediction for an IR-safe observable $\langle O \rangle$ which contains the parton shower resummation but which gives the correct NLO value for the observable:

$$\langle O \rangle^{\text{Matched}} = \langle O \rangle^{\text{NLO}} + \mathcal{O}(\alpha^{m+2})$$

Crucially, we must avoid double counting of the first emission.

Starting point: interleaved Catani-Seymour dipole QCD+QED shower Schumann, Krauss '07, LF '24 and SHERPA's implementation of the QCD MC@NLO Höche et al. '12

The MC@NLO algorithm

1. Produce a seed event, which is either an S-event with Born kinematics, or an H-event with real-emission kinematics, according to their subtracted matrix elements.
2. If S-event: run a one-step Sudakov parton shower, using the MC@NLO subtraction terms as the splitting kernels, on the event, either generating an emission or not.
3. If no emission generated, leave event as-is.
4. Pass S-events with an emission and H-events to the usual parton shower. Generate further emissions from the appropriate starting scale.

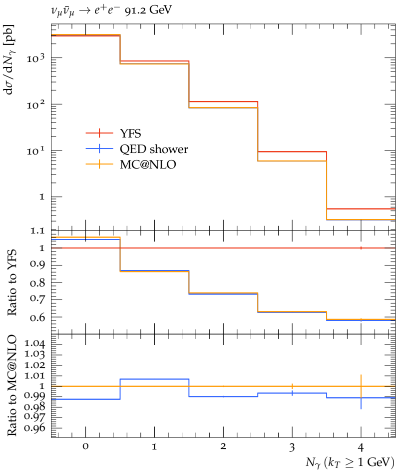
More details in backup slides

Advantages of MC@NLO

- YFS is more easily generalisable to higher orders, however, fermion masses are needed to regulate collinear singularities
- In our MC@NLO, masses can be included or not as convenient - collinear logs are resummed either way
- MC@NLO is well-suited to Higgs production since it avoids exponentiation beyond the logarithmically enhanced regions
- Exact NLO accuracy, including exact LO in differential distributions of the first emission
- Can match NLO to initial-state showers at lepton-lepton colliders - work in progress

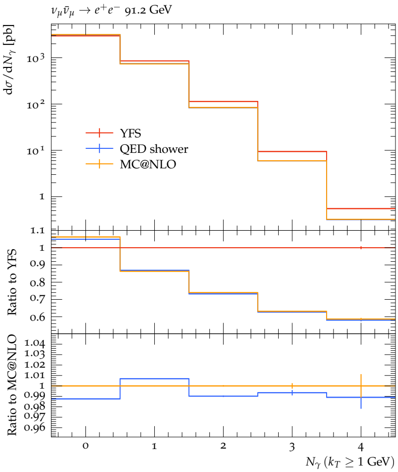
Results

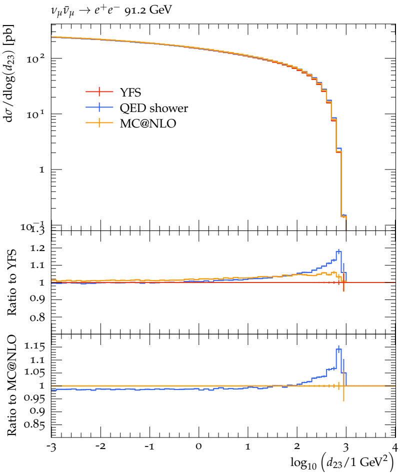
$\nu_\mu \bar{\nu}_\mu \to e^+ e^-$ at 91.2 GeV



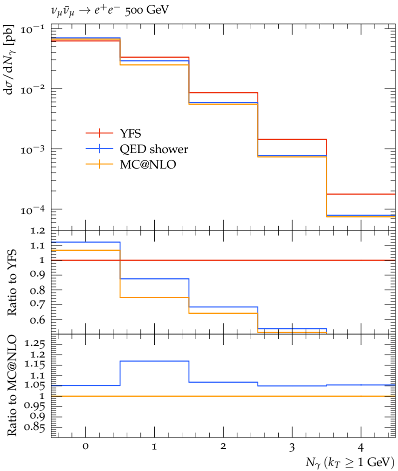
- YFS prediction contains exact NLO on-shell Z decay ME for radiation pattern – but no overall K-factor
- YFS produces significantly more photons
- QED shower is LO+LL
- MC@NLO contains exact NLO $\nu_\mu \bar{\nu}_\mu \to e^+ e^-$ ME (virtual from OpenLoops)
- Difference between shower and MC@NLO only in 0- and 1-photon bins

$\nu_\mu \bar{\nu}_\mu \to e^+ e^-$ at 91.2 GeV



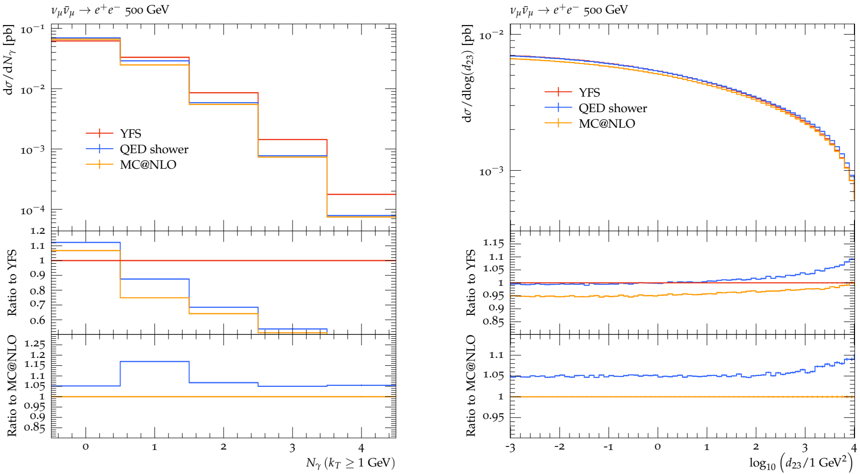


$\nu_\mu \bar{\nu}_\mu \to e^+ e^-$ at 500 GeV

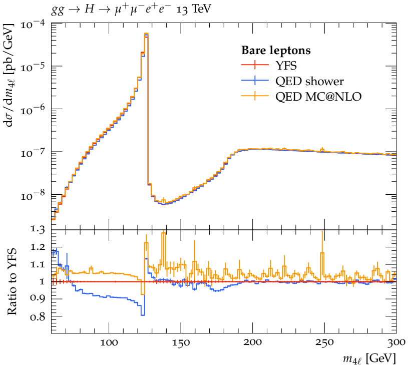


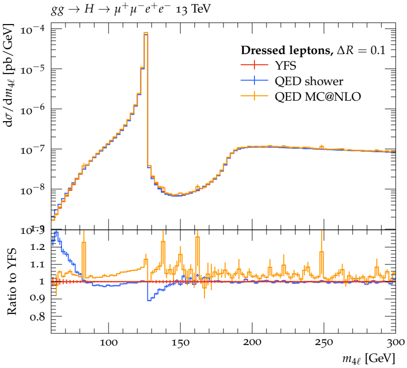
- YFS prediction contains exact on-shell Z decay ME
- YFS produces significantly more photons
- QED shower is LO+LL
- MC@NLO contains exact $\nu_\mu \bar{\nu}_\mu \to e^+ e^-$ ME from OpenLoops

$\nu_\mu \bar{\nu}_\mu \to e^+ e^-$ at 500 GeV



$gg \to H \to \mu^+\mu^-e^+e^-$





Conclusions

- We introduced an automated method to match QCD+EW NLO with a final-state QCD+QED parton shower
- We demonstrated the matching for QED in two different scenarios
- Next step: QCD+QED MC@NLO
- Needs a few process handling changes
- These developments will be released in a future version of SHERPA (3.x)
- See Peter Meinzinger's talk for the latest in SHERPA 3.0

Backup

Backup: Anatomy of an NLO calculation

A general NLO calculation of an infrared-safe observable O in a $2 \to n$ process can be written schematically in the form

$$\langle O \rangle^{\text{NLO}} = \int \mathrm{d}\Phi_n \left[B + \tilde{V} \right] O\left(\{p_n\}\right) + \int \mathrm{d}\Phi_{n+1}\, R_{n+1} O\left(\{p_{n+1}\}\right)$$

$$= \int \mathrm{d}\Phi_n \left[B + \tilde{V} + \sum_{\widetilde{ij},\tilde{k}} I^S_{\widetilde{ij},\tilde{k}} \right] O\left(\{p_n\}\right)$$

$$+ \int \mathrm{d}\Phi_{n+1} \left[R_{n+1} O\left(\{p_{n+1}\}\right) - \sum_{ij,k} D^S_{ij,k}\, O\left(\{p_n\}\right) \right]$$

where $D^S_{ij,k}$ are a set of process-independent subtraction terms and $I^S_{\widetilde{ij},\tilde{k}}$ are their analytic d-dimensional integrals.

Backup: Introducing the parton shower

A leading-order plus parton shower calculation has the form

$$\langle O \rangle^{\mathrm{PS}} = \int \mathrm{d}\Phi_n \, B \, \mathcal{F}_n(\Phi_n, O)$$

where $\mathcal{F}_n(\Phi_n, O)$ is the unitary parton shower factor, defined recursively as

$$\mathcal{F}_n(\Phi_n, O) = \Delta_n(\mu_Q^2, t_c) \; O\left(\{p_n\}\right) + \int \mathrm{d}\Phi_1 \; \Delta_n(\mu_Q^2, t) \; \mathcal{F}_{n+1}(\Phi_{n+1}, O) \; \mathcal{K}$$

where

$$\Delta_n(\mu_Q^2, t) = \exp\left(- \int_t^{\mu_Q^2} \mathrm{d}\Phi_1 \; \mathcal{K} \right)$$

is the Sudakov form factor which describes the no-emission probability, and $\mathcal{K}$ is the parton shower splitting kernel.

Backup: What is NLO matching?

NLO matching: producing a prediction for $\langle O \rangle$ which contains the parton shower factor $\mathcal{F}_n(\Phi_n, O)$ but which gives the correct NLO value for the observable:

$$\langle O \rangle^{\text{Matched}} = \langle O \rangle^{\text{NLO}} + \mathcal{O}(\alpha^{m+2})$$

Crucially, we must avoid double counting of the first emission.

Starting point: interleaved dipole QCD+QED shower (to be published) and SHERPA's implementation of the QCD MC@NLO Höche et al. '12

Adding and subtracting an extra set of subtraction terms D^A:

$$\langle O\rangle^{\text{NLOPS}} = \int \mathrm{d}\Phi_n \bar{B}\, O(\Phi_n) + \int \mathrm{d}\Phi_{n+1} \left[R - \sum_{ij,k} D^A_{ij,k}\right] O(\Phi_{n+1})$$
$$+ \int \mathrm{d}\Phi_{n+1} \sum_{ij,k} D^A_{ij,k} \left[O(\Phi_{n+1}) - O(\Phi_n)\right]$$

where we have introduced the shorthand (suppressing sums and indices)

$$\bar{B} = B + \tilde{V} + I^S + \int d\Phi_1 \left[D^A - D^S\right].$$

Then applying the parton shower:

$$\langle O\rangle^{\text{NLOPS}} = \int \mathrm{d}\Phi_n \bar{B}\, \mathcal{F}_n(\Phi_n, O) + \int \mathrm{d}\Phi_{n+1} \left[R - \sum_{ij,k} D^A_{ij,k}\right] \mathcal{F}_{n+1}(\Phi_{n+1}, O)$$

Expanding in α, we can see this is $\langle O\rangle^{\text{NLO}}$ to $\mathcal{O}(\alpha)$.

The MC@NLO matching method can be written as

$$\langle O \rangle^{\text{MC@NLO}} = \int \mathrm{d}\Phi_n \, \bar{B} \left[\bar{\Delta} \, O(\{p_n\}) + \sum_{ij,k} \int \mathrm{d}\Phi_1 \, O(\{p_{n+1}\}) \frac{D^A_{ij,k}}{B} \bar{\Delta} \right] + \int \mathrm{d}\Phi_{n+1} \left[R - \sum_{ij,k} D^A_{ij,k} \right] O(\{p_{n+1}\})$$

where the modified Sudakov factor is

$$\bar{\Delta}(\mu_Q^2, t) = \exp\left(- \int_t^{\mu_Q^2} \mathrm{d}\Phi_1 \sum_{ij,k} \frac{D^A_{ij,k}}{B} \right)$$

Many different methods, but here we use the S–MC@NLO method Höche et al. '12

$$B\,\mathcal{K}_{ij,k} = D^{A}_{ij,k} = D^{S}_{ij,k}\Theta(\mu_Q^2 - t)$$

where μ_Q^2 is the shower starting scale.
This means that the $\bar{B}$ function does not (usually) depend on the radiative phase space, so the evaluation is simpler.

Backup: QCD+QED shower for $gg \to H \to \mu^+\mu^-$