Locally finite two-loop amplitudes for multi Higgs production in gluon fusion

Julia Karlen, ETH Zurich

in collaboration with

Babis Anastasiou, George Sterman and Aniruddha Venkata

based on arXiv:2403.13712

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- Massive progress in computation of two-loop amplitudes in the last decades.
- Computational complexity grows fast with additional internal, external masses and legs.
- Can become accessible with numerical methods.

INTRODUCTION

- Universal numerical approach would permit to achieve high precision in relevant processes for LHC.
- Integrate numerically directly in momentum space: # of integrations per loop order is fixed.
- Major obstacle: removal of infrared and ultraviolet singularities at the integrand level.
- **GOAL:** universal method to create finite amplitude integrands in $D = 4 \longrightarrow$ integrable with Monte Carlo.
- Universal = IR factorization
- Build framework for factorization at the integrand level: local factorization.

PROGRESS TOWARDS A GENERAL FRAMEWORK

- Worked out previously for two examples at two loops

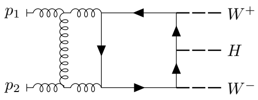
  $e^+e^- \to \gamma^* \dots \gamma^*$ (Anastasiou, Haindl, Sterman, Yang, and Zeng (2021))

  $q\bar{q} \to V_1 \dots V_n \quad \text{with } V_i \in \{\gamma^*, W, Z\}$ (Anastasiou and Sterman (2022))

- First demonstration of framework for two-loop processes with external gluons:

$$gg \to \text{N colorless particles.}$$

For example:

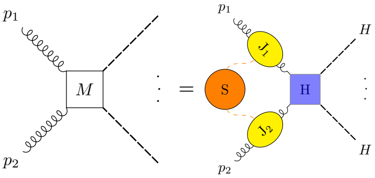


OUTLINE

- Infrared factorization
- What is the price to pay to make IR factorization local?
- Complexity of gluons: triple gluon vertex $\longrightarrow$ decomposition
- How does the decomposition of the triple gluon vertex help?
- Factorization at the integrand level
- More complicated gluonic processes

Wide-angle scattering amplitudes in gauge theories factorize to all orders:
(Ma (2020), Erdoğan and Sterman (2015), Dixon, Magnea, and Sterman (2008), Catani (1998), and Sen (1983))

$$\text{Amplitude} = \text{Hard} \cdot \text{Soft} \cdot \prod_i \text{Jet}_i,$$



- Soft and Jet functions S, J_i: contain all IR singularities, are universal functions.
- Hard function H: is process-dependent and IR finite.

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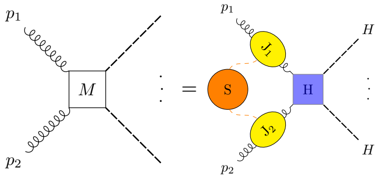


- Soft and Jet functions S, J_i: contain all IR singularities, are universal functions.
- Hard function H: is process-dependent and IR finite.

Expand up to two perturbative orders:

$$H^{(1)} = M^{(1)}$$
$$H^{(2)} = M^{(2)} - I^{(1)} \cdot M^{(1)} \,,$$

Goal: Make this manifestly local in momentum space! Generate **integrand for the hard function $\mathcal{H}$** free of singularities point-by-point in the integrand.

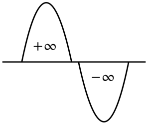
MAKE INFRARED FACTORIZATION LOCAL

Naively at first two perturbative orders at the integrand level:

$$\mathcal{H}^{(1)} = \mathcal{M}^{(1)}$$
$$\mathcal{H}^{(2)} = \mathcal{M}^{(2)} - \mathcal{F}^{(1)}\mathcal{M}^{(1)}$$

- Physical IR singularities factorize: subtracted by a universal one-loop form factor amplitude times the IR finite Born amplitude.

- Naive integrand construction has non-local cancellations $\rightarrow$ cannot be integrated numerically.



WHAT IS THE PRICE TO PAY?

Factorization at the integrand level:

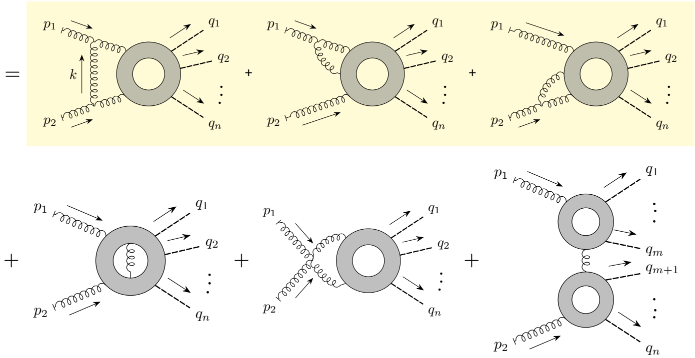
$$\mathcal{H}^{(2)}(k,l) = \mathcal{M}^{(2)}(k,l) - \mathcal{F}^{(1)}(k)\mathcal{M}^{(1)}(l) - \Delta\mathcal{M}^{(2)}(k,l),$$

- Additional counterterm $\Delta\mathcal{M}$. Serves a purpose locally but does not change integrated value of the finite amplitude:

$$\int \mathrm{d}l^D \Delta\mathcal{M}^{(2)}(k,l) = 0.$$

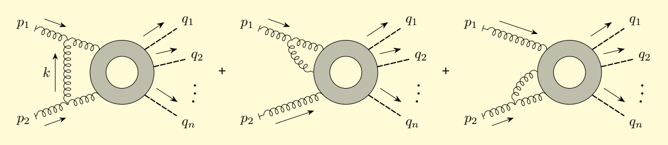
- Careful about the routing of loop momentum k, l in the diagrams $\rightarrow$ make gauge invariance apply locally.

$$\mathcal{M}_n^{(2)} =$$

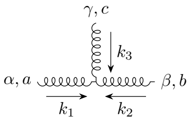


- Grey disk: heavy quark loop, gluons attach everywhere.
- Diagrams with triple gluon vertices are the origin of collinear singularities $k \parallel p_1$ and $k \parallel p_2$.
- Second line is IR finite.
- Interested in the first line of IR singular diagrams.

What is the issue with gluonic diagrams and handling their singularities?



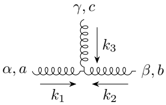
Too many terms!



$$= -g_s f_{abc}(k_1 - k_2)^{\gamma}\eta^{\alpha\beta} - g_s f_{bca}(k_2 - k_3)^{\alpha}\eta^{\beta\gamma} - g_s f_{cab}(k_3 - k_1)^{\beta}\eta^{\gamma\alpha}$$

- Each term exhibits a different behavior in collinear limits!
- As a single object the diagrams with a triple gluon vertex do not factorize in a local fashion.
- Analyze each contribution separately.

“SCALAR”-DECOMPOSITION



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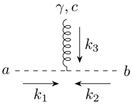
Note: vertex of color-octet scalars and a gluon is



$$= -g_s f_{abc}(k_1 - k_2)^\gamma$$

Appears in triple gluon vertex times a metric $\eta^{\alpha\beta}$!

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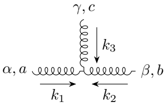

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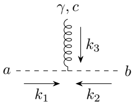

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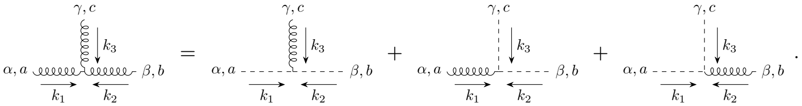
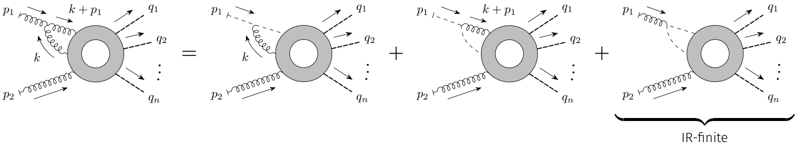
"Scalar"-decomposition



Note: Scalar lines are still gluons! Graphically only tells us which triple gluon vertex terms we consider.

Why is this useful?

Original momentum flow of diagram: does not lead to factorization at the integrand level.



Gluon must always have same momentum!

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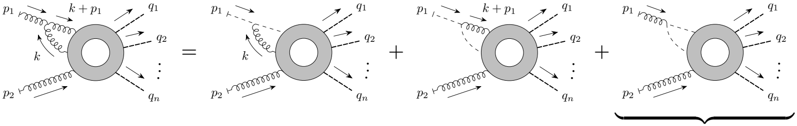
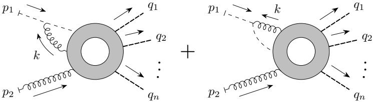
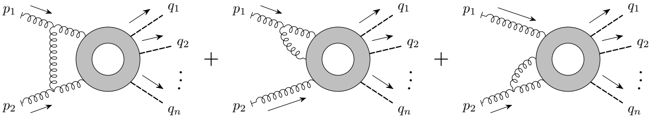


Gluon must always have same momentum! We can impose a different momentum routing for each decomposed diagram:

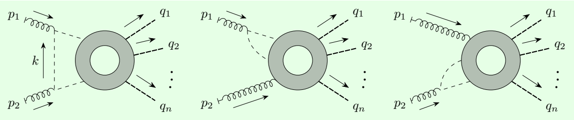


Apply “scalar” decomposition to all diagrams with triple-gluon vertices.

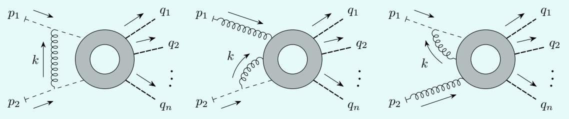
Analyze integrand $\rightarrow$ separates them in classes due to their behavior in the collinear limits.



“SCALAR” DECOMPOSITION OF IR SINGULAR DIAGRAMS

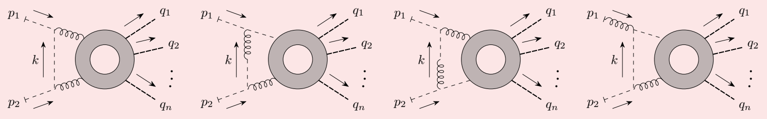


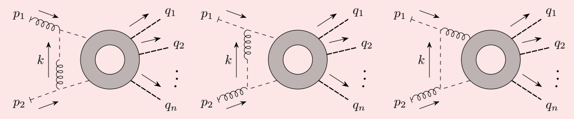
$= \mathcal{M}^{(2)}_{n,\text{IR-finite}}$ ✓



$= \mathcal{M}^{(2)\text{fact}}_{n,\text{IR}}$

physical singularities: factorize locally



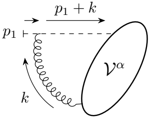


$= \mathcal{M}^{(2)\text{shift}}_{n,\text{IR}}$

non-local cancellations

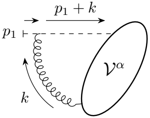
1. Factorizable diagrams $\mathcal{M}_{n,\mathsf{IR}}^{(2)\mathsf{fact}}$

Analyze collinear limit



$$= \frac{(2p_1 + k)_\alpha}{k^2(k+p_1)^2}\,\mathcal{V}^\alpha \xrightarrow{k=-xp_1} \frac{1}{k^2(k+p_1)^2}\,\frac{(2-x)}{x}\,(-k)_\alpha\,\mathcal{V}^\alpha$$

Analyze collinear limit



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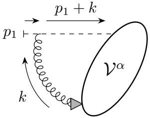
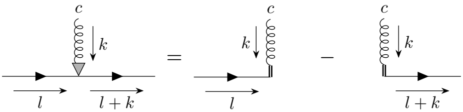
$$\equiv$$



The collinear gluon gets unphysical longitudinal polarization!

In diagrams with longitudinal gluons the Ward identity applies.

Tree level Ward identity (partial fraction decomposition)

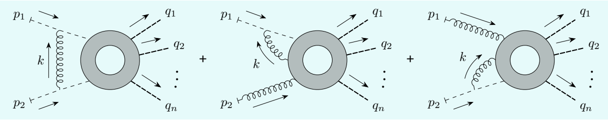


Ward identities lead to **cancellation between diagrams in the collinear limits**.

DIAGRAMS RELATED VIA GAUGE INVARIANCE: $\mathcal{M}^{(2)\mathsf{fact}}_{n,\mathsf{IR}}$

Factorizable diagrams

$$\mathcal{M}^{(2)\mathsf{fact}}_{n,\mathsf{IR}} =$$



Factorizable diagrams

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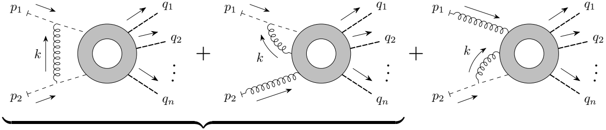

singular in $k \| p_1$

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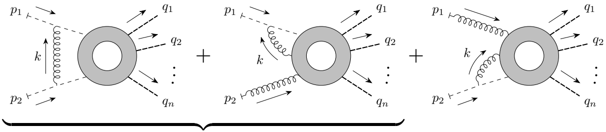


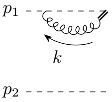
singular in $k \| p_1$

With

- chosen routing of gluon momentum k through decomposition,
- consistent treatment for the quark momentum routing,

cancellations through Ward identity leads to local factorization:

$\mathcal{M}_{n,\mathrm{IR}}^{(2)\mathrm{fact}} \xrightarrow{k=-xp_1}$



external leg correction

$$\times \Bigg(\underbrace{\mathcal{M}_n^{(1)}(l, p_1, p_2)}_{\text{Born amplitude}} + \mathcal{M}_n^{(1)}(l+k, p_1, p_2) \Bigg) .$$

FORM FACTOR COUNTERTERM

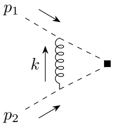
All IR limits ($k \parallel p_1$, $k \parallel p_2$, $k \sim 0$) factorize at the integrand level.

Removing IR singularities

IR singularities removed with a scalar-scalar form factor multiplied by an average over Born amplitudes.

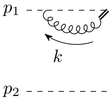
$$\mathcal{F}^{(1)}_{\text{scalar}}(k) \times \frac{1}{2}\Big(\mathcal{M}^{(1)}_n(l, p_1, p_2) + \mathcal{M}^{(1)}_n(l+k, p_1, p_2)\Big)$$

$$=$$



$$\times \frac{1}{2}\Big(\mathcal{M}^{(1)}_n(l, p_1, p_2) + \mathcal{M}^{(1)}_n(l+k, p_1, p_2)\Big)$$

$$\xrightarrow{k=-xp_1}$$



$$\times \Big(\mathcal{M}^{(1)}_n(l, p_1, p_2) + \mathcal{M}^{(1)}_n(l+k, p_1, p_2)\Big)\ .$$

Has same behavior as factorizable diagrams $\mathcal{M}^{(2)\text{fact}}_{n,\text{IR}}$ in all three IR limits.

HARD INTEGRAND FOR FACTORIZABLE DIAGRAMS

All IR singular behavior is removed locally by form factor times averaged Born amplitude.

$$\mathcal{H}_{n,\mathrm{IR}}^{(2)\mathrm{fact}} = \mathcal{M}_{n,\mathrm{IR}}^{(2)\mathrm{fact}} - \mathcal{F}_{\mathrm{scalar}}^{(1)}(k) \times \frac{1}{2}\left(\mathcal{M}_{n}^{(1)}(l, p_1, p_2) + \mathcal{M}_{n}^{(1)}(l+k, p_1, p_2)\right)$$

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By introducing a shift counterterm

$$\Delta\mathcal{M}_{n,\text{IR}}^{(2)\text{fact}} = \mathcal{F}_{\text{scalar}}^{(1)}(k) \times \frac{1}{2}\left(\mathcal{M}_n^{(1)}(l+k, p_1, p_2) - \mathcal{M}_n^{(1)}(l, p_1, p_2)\right)$$

we can rewrite

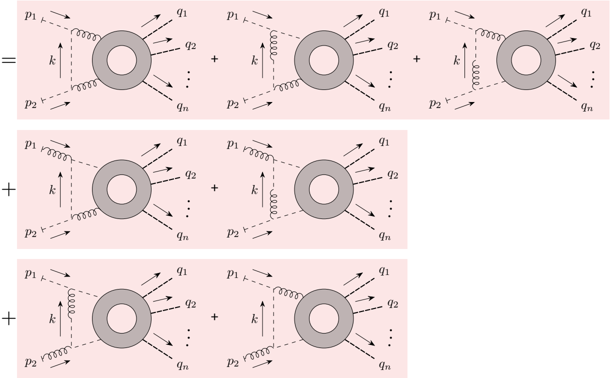
$$\mathcal{H}_{n,\text{IR}}^{(2)\text{fact}} = \mathcal{M}_{n,\text{IR}}^{(2)\text{fact}} - \mathcal{F}_{\text{scalar}}^{(1)}(k) \times \mathcal{M}_n^{(1)}(l, p_1, p_2) - \Delta\mathcal{M}_{n,\text{IR}}^{(2)\text{fact}} .$$

2. “Shift-integrable” diagrams $\mathcal{M}_{n,\mathsf{IR}}^{(2)\mathsf{shift}}$

"SHIFT-INTEGRABLE" DIAGRAMS: $\mathcal{M}_{n,\text{IR}}^{(2)\text{shift}}$

Remaining IR singular diagrams from "scalar" decomposition:

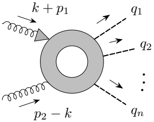
$$\mathcal{M}_{n,\text{IR}}^{(2)\text{shift}} =$$



- Diagrams are IR finite after integration.
- Have IR singularities at the integrand level.

In the collinear limit $k \parallel p_1$ the sum of all shift-integrable diagrams behave as:

$$\lim_{k=-xp_1} \left(\mathcal{M}^{(2)\text{shift}}_{n,\text{IR}}\right) \propto$$



$$= (k+p_1)^{\alpha}\mathcal{M}^{(1)}_{n,\alpha\beta}(k+p_1, p_2-k, l)$$

Longitudinally polarized gluon enters quark loop everywhere.

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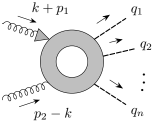

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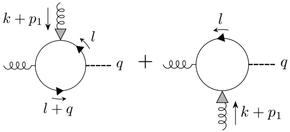
QED Ward identity applies



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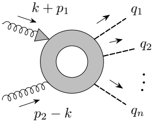

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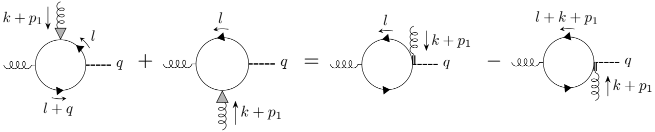


- Non-local cancellation: vanishes after integration over l.
- Remove this difference locally with counterterm before integration: **shift counterterm**

$$\Delta_1 \mathcal{M}^{(2)\text{shift}}_{n,\text{IR}} \propto (k+p_1)^{\alpha} \mathcal{M}^{(1)}_{n,\alpha\beta}(k+p_1, p_2 - k, l).$$

- Counterterm integrates to zero: $\int \mathrm{d}l^4 \Delta_1 \mathcal{M}^{(2)\text{shift}}_{n,\text{IR}} = 0.$

Local IR subtraction of amplitude

"Scalar" decomposition + specific loop momentum routing:
Removed all IR singularities locally with one **form factor counterterm** and **shift counterterms**:

$$\mathcal{H}_n^{(2)}(k,l) = \mathcal{M}_n^{(2)}(k,l) - \mathcal{F}_{\text{scalar}}^{(1)}(k) \times \mathcal{M}_n^{(1)}(l) - \Delta\mathcal{M}_{n,}^{(2)}(k,l) \; .$$

This is a general construction for an arbitrary number of external electroweak bosons in gluon fusion.

Fully finite amplitude

Remove UV singularities with local counterterms (local R-operator). Admits numerical integration in $D = 4$.

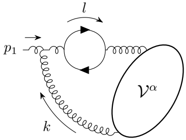
Towards a general framework

Next step towards general framework: NNLO for initial state gluons

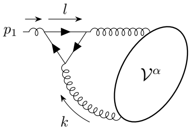
Additional issues in single collinear limits:

1. Power-like divergences in self energy correction diagrams.

   Solution: Tensor reduction to reduce to logarithmic divergences.



2. Loop polarizations in vertex correction diagrams: Collinear gluons are not longitudinally polarized.



$$\xrightarrow{k=-xp_1} l_\alpha \times \mathcal{V}_1^\alpha + (-k)_\alpha \times \mathcal{V}_2^\alpha$$

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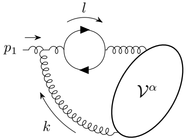
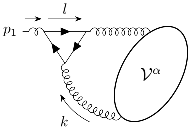
   Solution: Tensor reduction to reduce to logarithmic divergences.



2. Loop polarizations in vertex correction diagrams: Collinear gluons are not longitudinally polarized.



$$\xrightarrow{k=-xp_1} \underbrace{l_\alpha}_{\text{Loop polarization}} \times \mathcal{V}_1^\alpha + (-k)_\alpha \times \mathcal{V}_2^\alpha$$

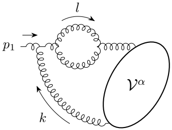
l_α can be a hard momentum pointing into **any** direction $\longrightarrow$ cannot apply Ward identity.
After integration over l : collinear gluons=longitudinal .

Solution: Symmetrization over $l_\perp \leftrightarrow -l_\perp$, partial Tensor reduction etc.

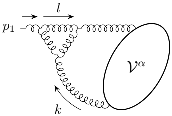
Challenge: not spoil other limits when solving one issue!

TOWARDS A GENERAL FRAMEWORK

- ✓ Fully local factorization for e^+e^- and $q\bar{q}$ annihilation at two loops in previous papers.
  (Anastasiou, Haindl, Sterman, Yang, and Zeng (2021)) (Anastasiou and Sterman (2022))
- Progress: initial state gluons at NNLO
  - ✓ All self energy correction are solved:



  - ✓ Loop polarization for gluon, ghost and fermion loop corrections removed.



  - Achieving full local factorization in all collinear limits: resolve shift mismatches.

CONCLUSION AND OUTLOOK

Conclusion

- Learned how to combine diagrams such that factorization is local.
- Established local factorization for loop induced colorless production at two loops with external gluons.
- Solved how to project loop polarizations onto longitudinal polarizations before integration at NNLO.

How to connect to phenomenology?

- Numerical integration via Monte Carlo: new problem $\rightarrow$ threshold singularities.
- Combine with real radiation to full cross section.
- Recent publication: 2-loop N_f contribution to $pp \rightarrow V_1 V_2 V_3$ with $V_i \in \{\gamma^*, W^+, W^-, Z\}$ (Kermanschah and Vicini (2024))
  $\longrightarrow$ Dario Kermanschah's talk tomorrow

Next steps

- Tackle factorization in all limits for NNLO gluon fusion processes.
- Expand framework to colorful final states.

Thanks for listening!