Towards an automated generator based on OpenLoops + LTD

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based on ongoing work with

Gloria Bertolotti, Nicolò Giraudo, Florian Herren and Jonas Lindert

see also talk by Gloria Bertolotti today at 11 AM

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Loop Tree Duality (LTD)

Key ideas

- analytic loop-energy integration $\rightarrow$ 3D phase space for real and virtual d.o.f

$$\int \mathrm{d}q^0 \int \mathrm{d}\mathbf{q}^{D-1}\, \mathcal{I}_{\mathrm{V}}(q^0,\mathbf{q}) \quad = \int \mathrm{d}\mathbf{q}^{D-1}\, I_{\mathrm{V}}(\mathbf{q})$$

$\Rightarrow$ local IR cancellations w.o. subtraction terms

$\Rightarrow$ numerical N^{k}LO calculations (attractive e.g. for EW multi-loop problems)

Past and ongoing contributions to LTD

$\geq$ 1999 Seminal work [Soper]

$\geq$ 2008 General formulation [Catani, Rodrigo, Krauss, Gleisberg, Winter, Sborlini, Buchta, Chachamis, Draggiotis, Malamos Driencourt-Mangin, Hernandez-Pinto, . . .]

$\geq$ 2019 New groups and directions (e.g. Local Unitarity) [Capatti, Hirschi, Kermanschah, Pelloni, Ruijl, Vicini, Tramontano, Weinzierl, Runkel, Szor, Vesga, Kromin, Schwanemann, Ramirez-Uribe, Renteria-Olivo, Renteria-Estrada, Martinez de Lejarza, Torres Bobadilla, Dhani, Cieri, Aguilera-Verdugo, . . .]

Active & promising field. But potential for collider pheno still to be demonstrated.

OpenLoops+LTD project [Bertolotti, Giraudo, Herren, Lindert, SP]

Long term goal: NNLO automation (not discussed in this talk)

Based on "combination" of LTD with

- NLO automation in OpenLoops 2 [Buccioni, Maierhöfer, Lang, Lindert, SP, Zhang, Zoller '18]
- OpenLoops automation of 2-loop integrands [SP, Schär, Zoller '22]

First objective: NLO automation (first preliminary results today)

Main features of envisaged tool

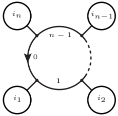
- MC generator of real and "virtual" events with local IR cancellations
- flexible predictions for arbitrary observables (e.g. via Rivet)

Key motivation: assess potential of LTD vs established NLO generators

LTD in a snapshot [Catani, Rodrigo et al.]

Generic n-point one-loop diagram

$$\mathrm{D}^D q \;=\; \mu^{2\varepsilon} \frac{\mathrm{d}^D q}{(2\pi)^D}$$



$$\mathcal{A} \;=\; -\mathrm{i} \int \mathrm{D}^D q \; \frac{N(q)}{\prod_{i=0}^{n-1} D_i(q)}$$

$$D_i(q) \;=\; (q+p_i)^2 - m_i^2 + \mathrm{i}0 \;=\; (q^0 - E_i^+)(q^0 - E_i^-)$$

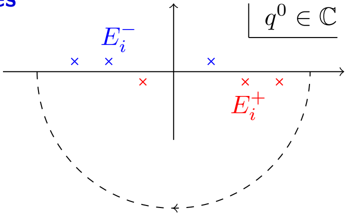
Positive and negative energy poles in q^0 space

$$E_i^{\pm}(\mathbf{q}) \;=\; -p_i^0 \pm \epsilon_i(\mathbf{q}) \,, \qquad \epsilon_i(\mathbf{q}) \;=\; \sqrt{|\mathbf{q}+\mathbf{p}_i|^2 + m_i^0 - \mathrm{i}0}$$

Analytic q^0 integration $\to$ sum over n residues

$$\mathcal{A} \;=\; \int \mathrm{D}^{D-1}\mathbf{q} \; I(\mathbf{q})$$

$$I(\mathbf{q}) \;=\; -\sum_{i=0}^{n-1} \frac{1}{2\epsilon_i} \, \frac{N(q)}{\prod_{j\neq i} D_j(q)} \Bigg|_{q^0 = E_i^+}$$



Automation of standard LTD integrand in OpenLoops

$$I(\mathbf{q}) = -\int \mathrm{D}^{D-1}\mathbf{q} \sum_{i=0}^{n-1} \frac{1}{2\epsilon_i} \frac{N(q)}{\prod_{j\neq i} D_j(q)} \Big|_{q^0=E_i^+} \qquad \mathcal{A} = \int \mathrm{D}^{D-1}\mathbf{q}\; I(\mathbf{q})$$

OpenLoops implementation of LTD integrand

- similar ingredients as standard 1-loop integrand

$\Rightarrow$ automated for any SM processes at NLO QCD+EW

Polynomial numerator in OpenLoops

$$N(q) = \sum_{r=0}^{R} \underbrace{N_{\mu_1\ldots\mu_r}(\{p_i\}, m_i)}_{\text{computed numerically}} q^{\mu_1}\ldots q^{\mu_r}$$

- full information about analytic q-dependence of intergand
- $I(\mathbf{q})$ evaluations for many $\mathbf{q}$'s very fast $\Rightarrow$ speeds up MC integration

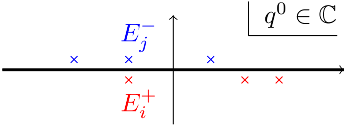
Challenges: UV+threshold+IR cancellation & small $I(\mathbf{q})$ variance (fast integ.)

Spurious singularities [Catani, Rodrigo et al.]

$$I(\mathbf{q}) \;=\; -\sum_{i=0}^{n-1} \frac{1}{2\epsilon_i} \frac{N(E_i^+, \mathbf{q})}{\prod_{j\neq i}(E_i^+ - E_j^+)(E_i^+ - E_j^-)}$$

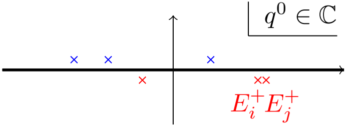
Physical singularities of $I(\mathbf{q})$ (see later)

- in $\mathbf{q}$ regions where $E_i^+(\mathbf{q}) \to E_j^-(\mathbf{q})$
- q^0-contour pinched $\Rightarrow$ threshold+IR sing.



Spurious singularities of LTD residues

- in $\mathbf{q}$ regions where $E_i^+(\mathbf{q}) \to E_j^+(\mathbf{q})$
- q^0 contour not pinched $\Rightarrow$ $I(\mathbf{q})$ finite ...



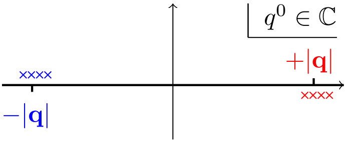
... but **dual cancellations** between singularities from $q^0 = E_i^+, E_j^+$ residues

We observe significant numerical instabilities only in the deep UV region ...

Dual cancellations in the UV region

UV scaling of LTD on-shell energies

$$E_i^{\pm}(\mathbf{q}) \underset{|\mathbf{q}|\to\infty}{\longrightarrow} \pm|\mathbf{q}| + \mathcal{O}(|\mathbf{q}|^0)$$



$$I(\mathbf{q}) = -\sum_{i=0}^{n-1} \frac{1}{2\epsilon_i} \frac{N(E_i^+, \mathbf{q})}{\prod_{j\neq i} \underbrace{(E_i^+ - E_j^+)}_{\mathcal{O}(|\mathbf{q}|^0)} \underbrace{(E_i^+ - E_j^-)}_{\mathcal{O}(|\mathbf{q}|^1)}} \quad \Rightarrow \quad \text{residues} \propto I(\mathbf{q})\times|\mathbf{q}|^{n-1}$$

Implications for $\mathbf{q}$-intergation in deep UV region $\quad |\mathbf{q}|/Q_{\mathrm{hard}} \geq 10^P$

- contribution of this region is at most $\mathcal{O}(10^{-P})$ of full integral
- however dual cancellations jeopardize $(n-1)\times P$ digits

$\Rightarrow$ possible fake orders-of-magnitude enhancements

Instabilities can be avoided through systematic expansion in $1/|\mathbf{q}|$

Expansion of LTD intergand at $\mathbf{q} \to \infty$

Expansion of D-dim integrand for $q^2 - \mu_{\rm tad}^2 \gg \Delta_i$ [Chetyrkin, Misiak, Münz]

$$(q+p_i)^2 - m_i^2 \;=\; \underbrace{q^2 - \mu_{\rm tad}^2}_{\mathcal{O}(q^2)} \;-\; \underbrace{(m_i^2 - p_i^2 - \mu_{\rm tad}^2 - 2q\cdot p_i)}_{\Delta_i \;=\; \mathcal{O}(q)}$$

$$\mathcal{I}(q) \;=\; \frac{N(q)}{\prod_{i=0}^{n-1} D_i(q)} \quad \xrightarrow[q^2\to\infty]{} \quad \mathcal{I}_{\rm tad}(q) \;=\; \sum_{k=n}^{n+p(X)} \frac{U_k(q)}{(q^2-\mu_{\rm tad}^2)^k} \;+\; \mathcal{O}\left(q^{-X}\right)$$

Expanded LTD integrand $\equiv$ residues of single poles at $q_0 = \epsilon_{\rm tad} = \sqrt{|\mathbf{q}|^2 + \mu_{\rm tad}^2}$

$$I_{\rm tad}(\mathbf{q}) \;=\; \int {\rm d}q^0\, \mathcal{I}_{\rm tad}(q) \;=\; \underbrace{\sum_{k=n}^{n+p} \frac{1}{(k-1)!} \frac{\partial^{k-1}}{\partial q_0^{k-1}} \frac{U_k(q^0,\mathbf{q})}{(q^0+\epsilon_{\rm tad})^k}\; \Bigg|_{q_0=\epsilon_{\rm tad}}}_{\text{automated in OL+LTD up to 2nd order in } 1/|\mathbf{q}|}$$

Similar technique used for UV renormalisation…

UV renormalisation

Standard renormalisation in D dimemsions with UV and rational CTs

$$\mathcal{A} = -\mathrm{i}\int \mathrm{D}^D q\, \mathcal{I}(q) \ \ni \epsilon_{\mathrm{UV}}^{-1} \quad \Longrightarrow \quad \mathbf{R}\,\mathcal{A} = \mathcal{A} + \delta Z + \delta R \ \not\ni \epsilon_{\mathrm{UV}}^{-1}$$

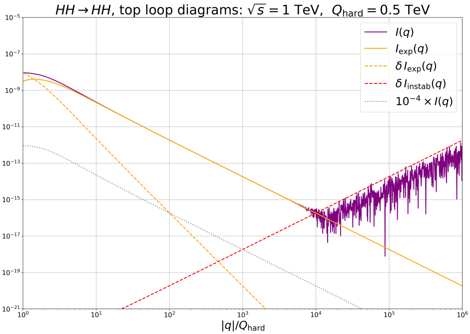
Local UV subtraction using divergent part of UV tadpole expansion

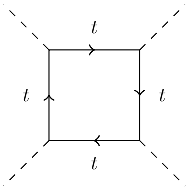
$$\mathbf{R}\,\mathcal{A} = \underbrace{-\mathrm{i}\int \mathrm{D}^D q\, \left[\mathcal{I}(q) - \mathcal{I}_{\mathrm{tad}}^{\mathrm{div}}(q)\right]}_{\overset{\mathrm{LTD}}{\longrightarrow} \int \mathrm{D}^3 q\, \left[I(\mathbf{q}) - I_{\mathrm{tad}}^{\mathrm{div}}(\mathbf{q})\right]} + \underbrace{\delta Z + \delta R - \mathrm{i}\int \mathrm{D}^D q\, \mathcal{I}_{\mathrm{tad}}^{\mathrm{div}}(q)}_{\text{analytic } \epsilon_{\mathrm{UV}}^{-1} \text{ pole cancellation}}$$

- subtracted LTD intergrand UV finite
- matching to correct δZ in dim reg via integrated tadpole integral

Expansion and renormalisation automated at NLO QCD+EW

Example of UV instability & expansion

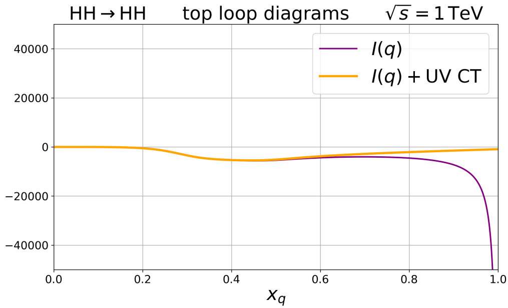




- UV subtracted LTD integrand features convergent $\mathrm{d}|\mathbf{q}|/|\mathbf{q}|^2$ scaling
- but becomes completely unstable at $|\mathbf{q}|/Q_{\text{hard}} > 10^4$
- expansion $\Rightarrow$ stability and better than 10^{-4} accuracy for $|\mathbf{q}|/Q_{\text{hard}} > 10^2$

Convenient parametrisation of $\mathbf{q}$ and UV subtraction

Tan-parametrisation: $|\mathbf{q}| = Q_{\text{hard}} \tan\left(\frac{\pi}{2}\, x_{\mathbf{q}}\right)$ [Becker et al. '10]



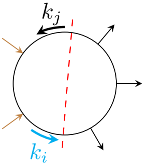
$$\int_0^\infty \mathrm{d}|\mathbf{q}|\, |\mathbf{q}|^2 \quad \longrightarrow \quad \int_0^1 \mathrm{d}x_{\mathbf{q}}$$

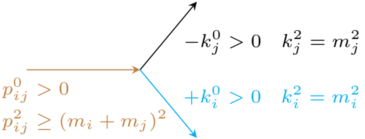
$$I(\mathbf{q}) \longrightarrow \underbrace{\frac{|\mathbf{q}|^2 \mathrm{d}|\mathbf{q}|}{\mathrm{d}x_{\mathbf{q}}}}_{\text{Jacobian}} I(\mathbf{q})$$

$\Rightarrow$ renormalised integrands $\propto \mathrm{d}|\mathbf{q}|/|\mathbf{q}|^2$ become flat at $x_{\mathbf{q}} \to 1$

$\Rightarrow$ deep UV region $|\mathbf{q}|/Q_{\text{hard}} > 10^4 \quad \equiv \quad x_{\mathbf{q}} \in [0.9999, 1]$

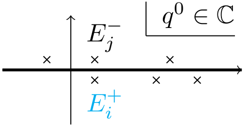
Threshold singularities from two-particle cuts





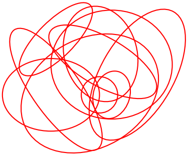
Threshold singularity from $q^0 = E_i^+$ LTD residue

$$\mathcal{A}_{ij}^{(\mathrm{thr})} = \int \frac{\mathrm{D}^{D-1}\mathbf{k}_i}{2\epsilon_i} \frac{g_{ij}(\mathbf{k}_i)}{(E_i^+ - E_j^+)\underbrace{(E_i^+ - E_j^- - \mathrm{i}0)}_{\text{threshold sing.}}}$$



Challenges for general solution

- multiple cuts per diagram and residue
- singular surface = multiple overlapping ellipsoids
- $\mathbf{q}$-contour deformation [Soper, Capatti et al.] challenging



Local subtraction in $\mathbf{q} \in \mathbb{R}^3$ space much more natural, simple and efficient

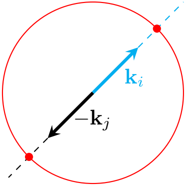
Local subtraction in cut-CM frame [Kermanschah et al.]

On-shell back-to-back 2-body decay

$$|\mathbf{k}_i|^2 \;\rightarrow\; |\mathbf{k}_{i,\mathrm{cut}}|^2 \;=\; \frac{\lambda(m_i^2, m_j^2, p_{ij}^2)}{4p_{ij}^2}$$

$$\epsilon_i^2 \;\rightarrow\; \epsilon_{i,\mathrm{cut}}^2 \;=\; |\mathbf{k}_{i,\mathrm{cut}}|^2 + m_i^2$$

Spherical singular surface



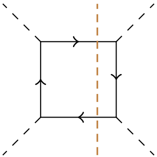
Threshold singularity and 1-dim local subtraction

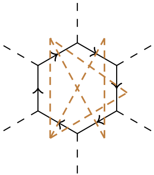
$$\mathcal{A}_{ij}^{(\mathrm{thr})} = \int \mathrm{d}\Omega_i \int_{m_i}^{\infty} \mathrm{d}\epsilon_i \; \underbrace{\frac{1}{\epsilon_i - \epsilon_{i,\mathrm{cut}} - \mathrm{i}0}}_{\mathrm{PV}\left(\frac{1}{\epsilon_i - \epsilon_{i,\mathrm{cut}}}\right) + \mathrm{i}\pi\delta\left(\epsilon_i - \epsilon_{i,\mathrm{cut}}\right)} \left[f_{ij}(\epsilon_i, \Omega_i) \;-\; \underbrace{f_{ij}(\epsilon_{i,\mathrm{cut}}, \Omega_i)}_{\text{local subtraction}} \right]$$

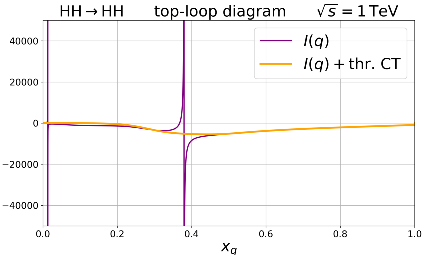
- subtraction defined in CM frame of each cut and “boosted” to LAB
- integrated subtraction terms added back $\Rightarrow$ $\mathrm{i}\pi$ absorbtive parts

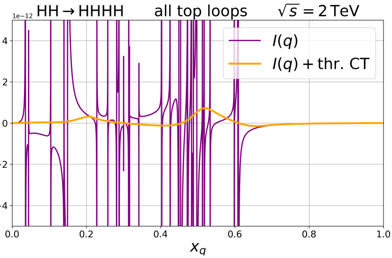
Entirely automated for one-loop diagrams with real masses

LTD integrands with threshold subtractions









Smooth subtracted integrands also for non-trivial multi-cut configurations!

MC integration & NLO predictions: virtual part

Integrand with UV+threshold subtractions in $\Phi_\mathrm{V} = (\Phi_\mathrm{B}, \mathbf{q})$ space

$$I_\mathrm{V}(\Phi_\mathrm{V}) = \sum_{\mathrm{diag/col/hel}} \left[I_\mathrm{LTD}(\Phi_\mathrm{V}) - I^{(\mathrm{subt})}_\mathrm{UV}(\Phi_\mathrm{V}) - I^{(\mathrm{subt})}_\mathrm{thr}(\Phi_\mathrm{V}) \right]$$

Integration with $B \times \frac{V}{B}$ factorisation to supress variance in q-space

$$\sigma^{(\mathrm{V})}_\mathrm{NLO} = \int \mathrm{d}\Phi_\mathrm{V}\, I_\mathrm{V}(\Phi_\mathrm{V}) = \int \mathrm{d}\Phi_\mathrm{B}\, I_\mathrm{B}(\Phi_\mathrm{B}) \int \mathrm{D}^3\mathbf{q} \underbrace{r_{\mathrm{V/B}}(\Phi_\mathrm{V})}_{I_\mathrm{V}(\Phi_\mathrm{V})/I_\mathrm{B}(\Phi_\mathrm{B})}$$

MC integration of V/B using unweighted Born sample

$$\sigma^{(\mathrm{V})}_\mathrm{NLO} = \frac{1}{N_\mathrm{B}} \sum_{i=1}^{N_\mathrm{B}} \underbrace{w_\mathrm{MC}(\mathbf{q}^{(i)})}_{\equiv \mathrm{D}^3\mathbf{q}/\mathrm{d}^3\mathbf{x}} r_{\mathrm{V/B}}(\Phi^{(i)}_\mathrm{V}), \qquad \Phi^{(i)}_\mathrm{V} = \underbrace{(\Phi^{(i)}_\mathrm{B}, \mathbf{q}^{(i)})}_{\text{one random } \mathbf{q}^{(i)} \text{ for each } \Phi^{(i)}_\mathrm{B}}$$

Boosting MC convergence by sampling over multiple $\mathrm{q}^{(ij)}$ at fixed $\Phi^{(i)}_\mathrm{B}$

- very fast for $N_\mathbf{q} \leq 10$–100 in OpenLoops
- can reduce statistical uncertainty by up to a factor $\sqrt{N_\mathbf{q}}$

Integration of Virtual and Real corrections

$$\sigma_{\mathrm{NLO}}^{(\mathrm{B+V+R})} = \int \mathrm{d}\Phi_{\mathrm{B}} \Bigg\{ I_{\mathrm{B}}(\Phi_{\mathrm{B}}) + I_{\mathrm{CT}}(\Phi_{\mathrm{B}}) + I_{\mathrm{subt}}^{(\mathrm{int})}(\Phi_{\mathrm{B}}) + \int \mathrm{D}^3\mathbf{q} \Big[I_{\mathrm{V}}(\Phi_{\mathrm{V}}) - I_{\mathrm{subt}}(\Phi_{\mathrm{V}}) + I_{\mathrm{R}}(\Phi_{\mathrm{R}}(\Phi_{\mathrm{V}})) \Big] \Bigg\}_{\Phi_{\mathrm{V}}=(\Phi_{\mathrm{B}},\mathbf{q})}$$

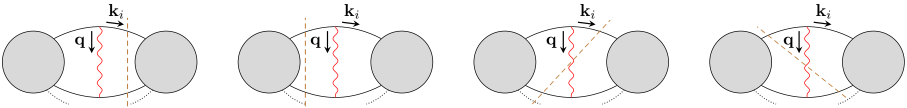
Ingredients and features of NLO master formula

- Φ_{B} space $\ni$ Born + renormalisation CTs
- $\Phi_{\mathrm{V/B}}$ space $\ni$ local and integrated UV + field-ren. + threshold subtr. terms
- $\mathbf{q}^3$ integration of V + R connected through $\Phi_{\mathrm{V}} \to \Phi_{\mathrm{R}}(\Phi_{\mathrm{V}})$ mapping
- $I_{\mathrm{R}}(\Phi_{\mathrm{R}})$ includes usual $|\mathcal{A}_{\mathrm{real}}|^2$ and $\mathrm{d}\Phi_{\mathrm{R}}/\mathrm{d}\Phi_{\mathrm{V}}$ Jacobian

Each real radiation event accompanied by a “virtual radiation” event

Main task: local cancellation of IR divergences

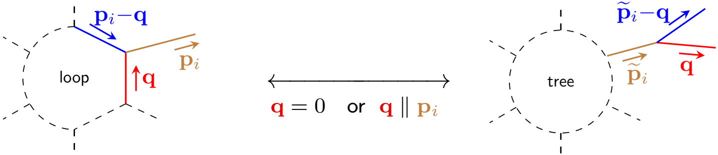
Matching real and virtual IR singularities



Cutkosky cuts of 2-loop forward scattering diagrams [Soper]

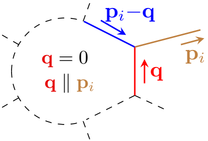
- fixed $\mathbf{q}$ and $\mathbf{k}_i$ $\Rightarrow$ IR finite combination of B–V and R–R interferences
- we use this as guideline to align virtual and real momenta
- but we assign $\mathbf{q}$ in a more flexible way while keeping fixed hard kinematics

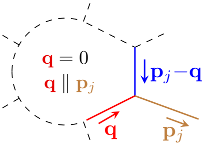
Example of $\mathbf{q}$ alignment at "amplitude" level



Same loop diagrams can require different parametrisations in different $\mathbf{q}$ regions!

Aligning virtual singularities through loop sectors





Requirement: all virtual IR singularities should be at $\mathbf{q} = 0$ and $\mathbf{q} \parallel \mathbf{p}_k$

- to achieve this we split loop diagrams into sectors with different singularities
- and in each loop sector we apply the required $\mathbf{q}$-shift and/or flip

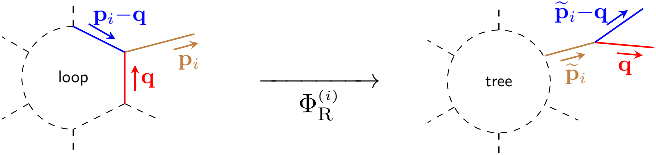
Projectors to separate the singular sectors of a loop diagram Γ

$$\sum_{\alpha \in \mathcal{S}_\Gamma} \epsilon_\alpha(\mathbf{q}) = 1 \,, \qquad \epsilon_\alpha(\mathbf{q}) \in [0,1] \,, \qquad \epsilon_\alpha(\mathbf{q}) \xrightarrow[\mathbf{q} \to \mathbf{q}_{\beta\,\mathrm{sing}}]{} \delta_{\alpha\beta}$$

Alignment via sector-dependent $\mathbf{q} \to \sigma_\alpha \mathbf{q} + \mathbf{k}_\alpha$ shifts and flips $(\sigma_\alpha = \pm)$

$$\mathcal{A}_\Gamma = \int \mathrm{D}^3\mathbf{q} \left(\sum_\alpha \epsilon_\alpha(\mathbf{q}) \right) I_\Gamma(\mathbf{q}) = \sum_\alpha \int \mathrm{D}^3\mathbf{q}\; \epsilon_\alpha(\sigma_\alpha \mathbf{q} + \mathbf{k}_\alpha)\, I_\Gamma(\sigma_\alpha \mathbf{q} + \mathbf{k}_\alpha)$$

Matching of R+V singularities through global sectors



Each collinear region requires a different mapping $(\Phi_{\mathrm{B}}, \mathbf{q}) \to \Phi_{\mathrm{R}}^{(i)}(\Phi_{\mathrm{B}}, \mathbf{q})$

- we split the Φ_{V} and Φ_{R} spaces into global collinear sectors
- we use Catani–Seymour mappings
- we identify the loop momentum $\mathbf{q}$ with the unresolved real momentum

Master formula for $V + R$ **integration with global sectors**

$$\sigma_{\mathrm{NLO}}^{(\mathrm{V+R})} = \int \mathrm{d}\Phi_{\mathrm{B}} \sum_{\alpha} \int \mathrm{D}^3\mathbf{q} \left[\epsilon_{\alpha}^{(\mathrm{V})}(\Phi_{\mathrm{V}})\, I_{\mathrm{V}}^{(\mathrm{subt})}(\Phi_{\mathrm{V}}) + \epsilon_{\alpha}^{(\mathrm{R})}(\Phi_{\mathrm{R}})\, I_{\mathrm{R}}(\Phi_{\mathrm{R}}) \right]_{\Phi_{\mathrm{R}} = \Phi_{\mathrm{R}}^{(i)}(\Phi_{\mathrm{V}})}$$

$\Rightarrow$ generation of MC events & finite predictions for any IR finite observables

Automation, implementation and validation

Status (work in progress)

- Generic algorithm applicable to arbitrary processes without initial-state collinear singularities (next step)
- Currently almost completely automated and validated against Sherpa+OpenLoops for $e^+e^- \to t\bar{t} + X$ processes

Cross section comparisons

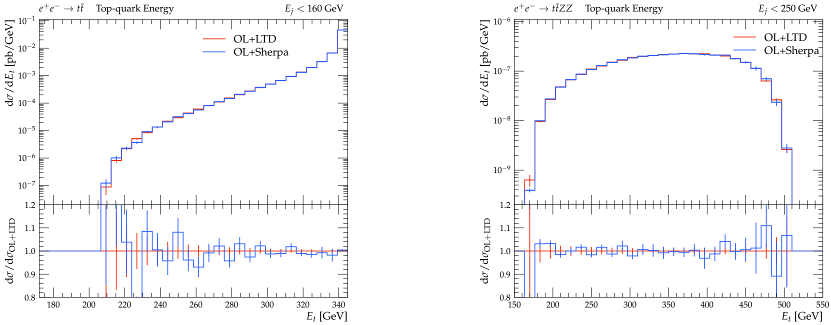
similar moderate stat. (1M evts, $N_{\mathbf{q}} = 1$) & runtimes (on a laptop)

	OL+Sherpa		OL+LTD	Comparison	
process	σ	$\Delta\sigma/\sigma$	σ	$\frac{\sigma_{\text{Sherpa}}-\sigma_{\text{LTD}}}{\sigma_{\text{Sherpa}}}$	$\frac{\sigma_{\text{Sherpa}}-\sigma_{\text{LTD}}}{\Delta\sigma_{\text{Sherpa}}}$
$e^+e^- \to t\bar{t}$	359.35	0.11%	359.27	0.02%	$0.2\,\sigma$
$e^+e^- \to t\bar{t}Z$	5.2406	0.11%	5.2366	0.08%	$0.7\,\sigma$
$e^+e^- \to t\bar{t}ZZ$	0.04595	0.12%	0.04591	0.10%	$0.9\,\sigma$
$e^+\nu_e \to t\bar{b}$	742.32	0.11%	3741.19	0.15%	$1.4\,\sigma$

$\mathcal{O}(10^{-3})$ agreement & similar or better accuracy than Sherpa+OL

Differential observables for $e^+e^- \to t\bar{t}$ and $t\bar{t}ZZ$

Results generated with OL+LTD & Rivet (automatically for any observable)



Very good consistency within statistical errors

- 1-2% per bin in the bulk (accuracy in the tails can be easily augmented)
- OL+LTD statistical uncertainties competitive with OL+Sherpa

More details and results in the talk by Gloria Bertolotti

Summary

Key features of OpenLoops+LTD at NLO

- local cancellation of threshold and UV singularities
- local IR cancellations via loop sectors and global sectors
- Automated MC generator based on Born events that emit real and "virtual" radiation

First results

- performance of first $e^+e^- \to t\bar{t}$ + multi-Z calculations very promising
- performance for more involved applications still to be investigated

Next steps

- validation for processes with final-state collinear singularities (in progress)
- local cancellation of initial-state collinear singularities

Backup slides

Example of local cancellation test