UNIVERSITY OF SUSSEX

Local cancellation of IR singularities: an automated algorithm based on OpenLoops+LTD

Gloria Bertolotti
University of Sussex

HP2, Torino, 12/09/24

Based on ongoing work

in collaboration with N. Giraudo, F. Herren, J. Lindert, S. Pozzorini

Introduction

Following...

Towards an automated generator based on OpenLoops + LTD

Stefano Pozzorini

based on ongoing work with

Gloria Bertolotti, Nicolò Giraudo, Florian Herren and Jonas Lindert

OpenLoops+LTD project [Bertolotti, Giraudo, Herren, Lindert, SP]

Long term goal: NNLO automation (not discussed in this talk)

Based on "combination" of LTD with

- NLO automation in OpenLoops 2 [Buccioni, Maierhöfer, Lang, Lindert, SP, Zhang, Zoller '18]
- OpenLoops automation of 2-loop integrands [SP, Schär, Zoller '22]

First objective: NLO automation (first preliminary results today)

Main features of envisaged tool

- MC generator of real and "virtual" events with local IR cancellations
- flexible predictions for arbitrary observables (e.g. via Rivet)

Key motivation: assess potential of LTD vs established NLO generators

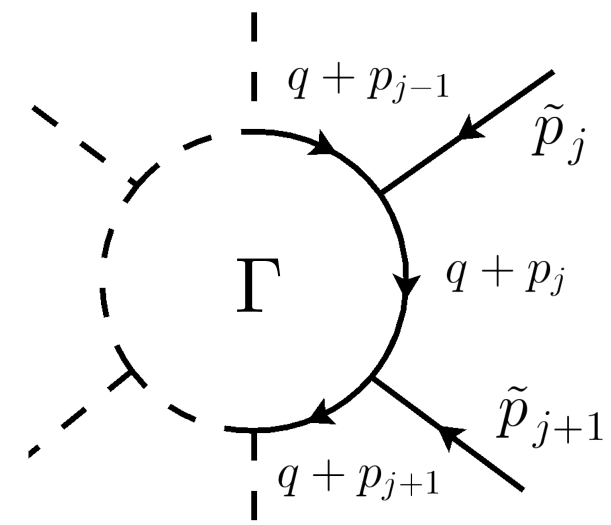
Virtual correction in LTD representation

[Catani, Rodrigo, et al]

$\mathrm{D}^{D-1}\mathbf{q} = \mu^{2\epsilon}\frac{\mathrm{d}^{D-1}\mathbf{q}}{(2\pi)^{D-1}}$

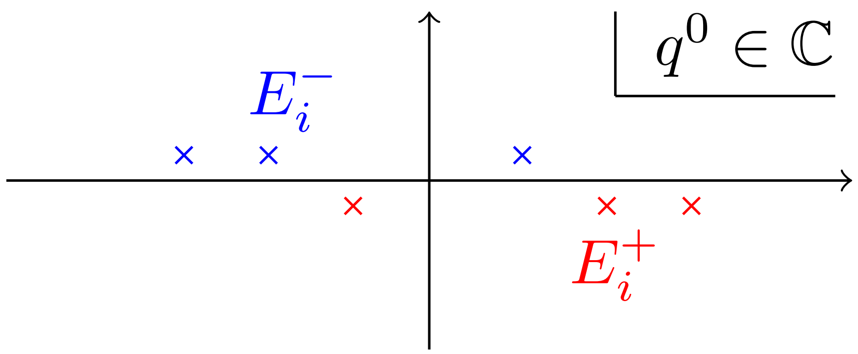
- Perform the **energy-integration** with Cauchy theorem

$$\mathcal{W}_\Gamma = -i\int\frac{\mathrm{D}^{D-1}\mathbf{q}}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}q^0\;N_\Gamma(q)\prod_{j\in S_\Gamma}\frac{1}{D_{\Gamma,j}(q)}$$



$$\begin{aligned}D_{\Gamma,j}(q) &= (q+p_j)^2 - m_j^2 + i0\\ &= \left(q^0+p_j^0\right)^2 - |\vec{q}+\vec{p}_j|^2 - m_j^2 + i0\\ &= \left(q^0 - E_j^+(\vec{q})\right)\left(q^0 - E_j^-(\vec{q})\right)\end{aligned}$$

$$\begin{aligned}E_j^\pm(\vec{q}) &= -p_j^0 \pm \sqrt{|\vec{q}+\vec{p}_j|^2 + m_j^2 - i0}\\ &= -p_j^0 \pm \epsilon_j(\vec{q})\end{aligned}$$



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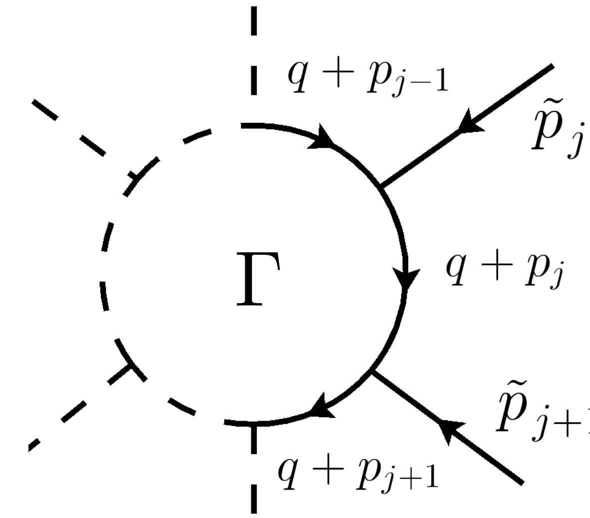

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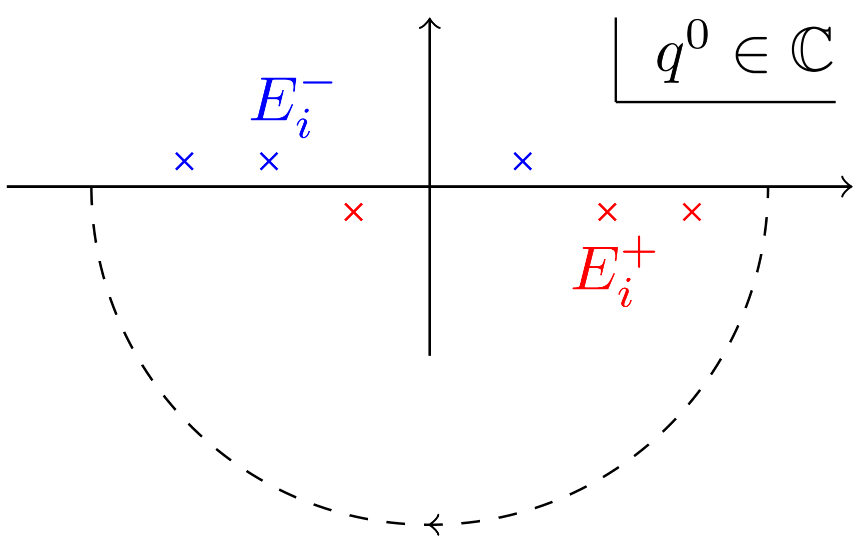

Closing the contour in the lower half plane yields **the LTD formula**

$$\mathcal{W}_\Gamma = -\int\mathrm{D}^{D-1}\mathbf{q}\sum_{j\in S_\Gamma}\left[\frac{N_\Gamma(E_j^+,\mathbf{q})}{2\epsilon_j(\vec{q})}\prod_{\substack{k\in S_\Gamma\\ k\neq j}}\frac{1}{(E_j^+(\vec{q})-E_k^+(\vec{q}))(E_j^+(\vec{q})-E_k^-(\vec{q}))}\right] = \int\mathrm{D}^{D-1}\mathbf{q}\; I_\Gamma(\mathbf{q})$$

Virtual correction in LTD representation

[Catani, Rodrigo, et al]

- Perform the **energy-integration** with Cauchy theorem
- Regularise integrand in **q**-space with auxiliary subtraction terms

$$\mathcal{W}_{\mathrm{V}}(\Phi_{\mathrm{B}}) = \int \mathrm{D}^{D-1}\mathbf{q}\; I_{\mathrm{V}}(\Phi_{\mathrm{B}},\mathbf{q}) = \int \mathrm{D}^{D-1}\mathbf{q}\; I_{\mathrm{V}}^{(\mathrm{bare})}(\Phi_{\mathrm{B}},\mathbf{q}) + \mathcal{W}_{\mathrm{CT}}^{(\mathrm{ren})}(\Phi_{\mathrm{B}})$$

Virtual correction in LTD representation

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Tangent **q**-parametrisation

$$|\mathbf{q}| = \mu \tan\left(\frac{\pi}{2}\, x_q\right)$$

[Becker, et al 1010.4187]

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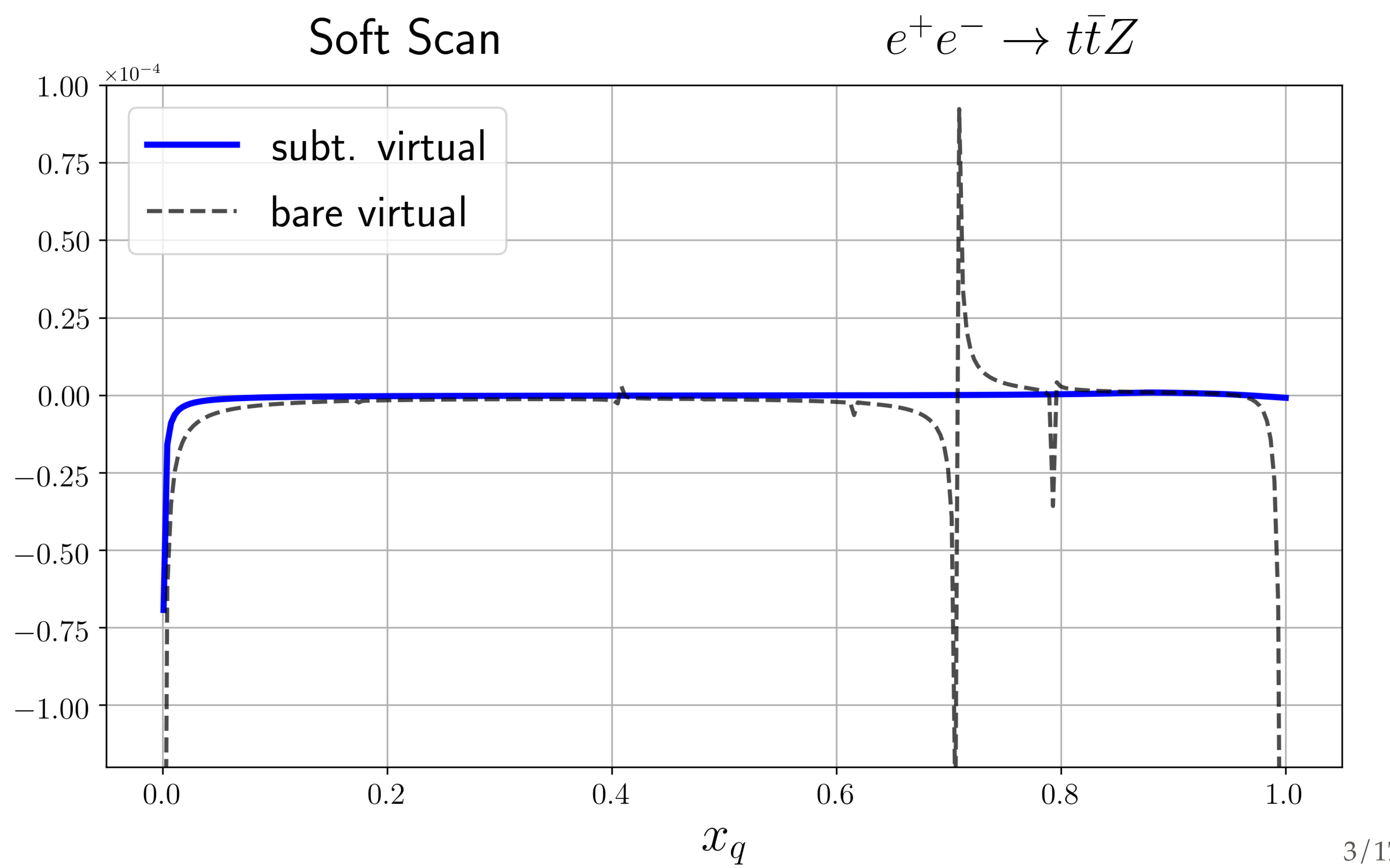
$$= \int \mathrm{D}^{D-1}\mathbf{q}\left[I_{\mathrm{V}}^{(\mathrm{bare})}(\Phi_{\mathrm{B}},\mathbf{q}) - I_{\mathrm{V}}^{(\mathrm{tad})}(\Phi_{\mathrm{B}},\mathbf{q}) + I_{\mathrm{V}}^{(\mathrm{frc})}(\Phi_{\mathrm{B}},\mathbf{q}) - I_{\mathrm{V}}^{(\mathrm{thr})}(\Phi_{\mathrm{B}},\mathbf{q})\right]$$

$$+\mathcal{W}_{\mathrm{CT}}^{(\mathrm{ren})}(\Phi_{\mathrm{B}}) + \mathcal{W}_{\mathrm{CT}}^{(\mathrm{tad})}(\Phi_{\mathrm{B}}) - \mathcal{W}_{\mathrm{CT}}^{(\mathrm{frc})}(\Phi_{\mathrm{B}}) + \mathcal{W}_{\mathrm{CT}}^{(\mathrm{thr})}(\Phi_{\mathrm{B}})$$



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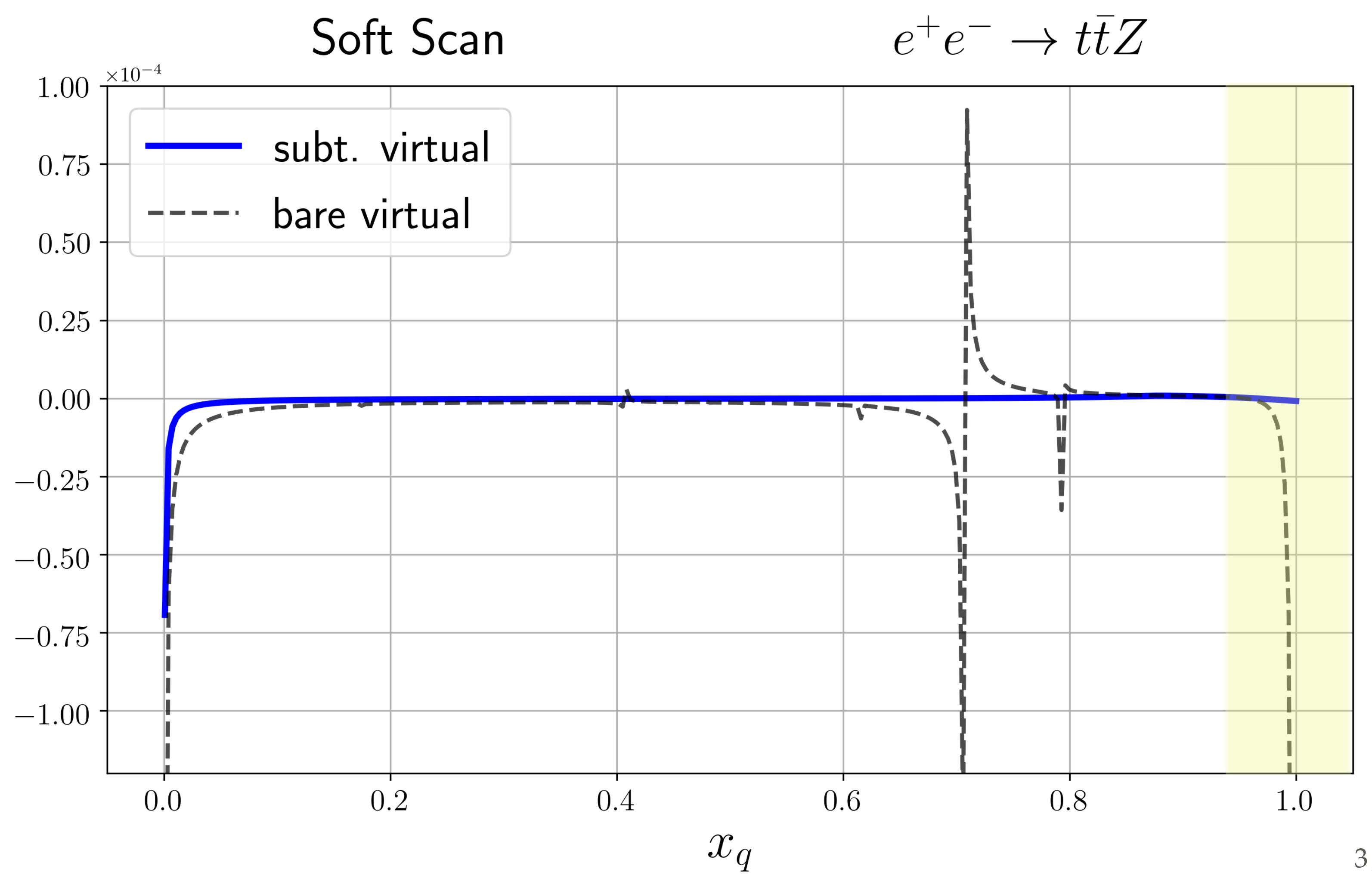
UV singularities



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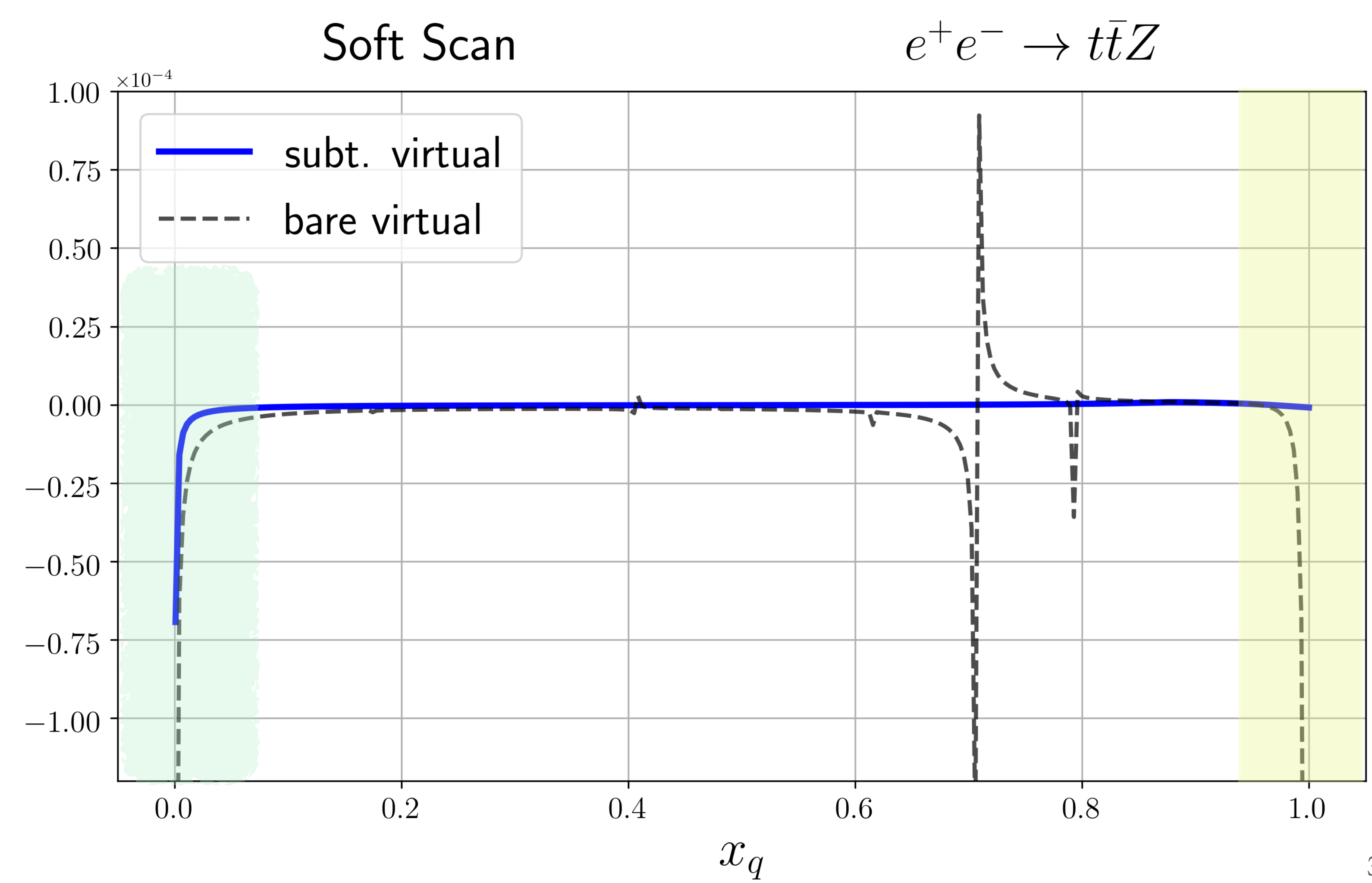
$$+\mathcal{W}_{\mathrm{CT}}^{(\mathrm{ren})}(\Phi_{\mathrm{B}}) + \mathcal{W}_{\mathrm{CT}}^{(\mathrm{tad})}(\Phi_{\mathrm{B}}) - \mathcal{W}_{\mathrm{CT}}^{(\mathrm{frc})}(\Phi_{\mathrm{B}}) + \mathcal{W}_{\mathrm{CT}}^{(\mathrm{thr})}(\Phi_{\mathrm{B}})$$

UV singularities

IR singularities from FRCs



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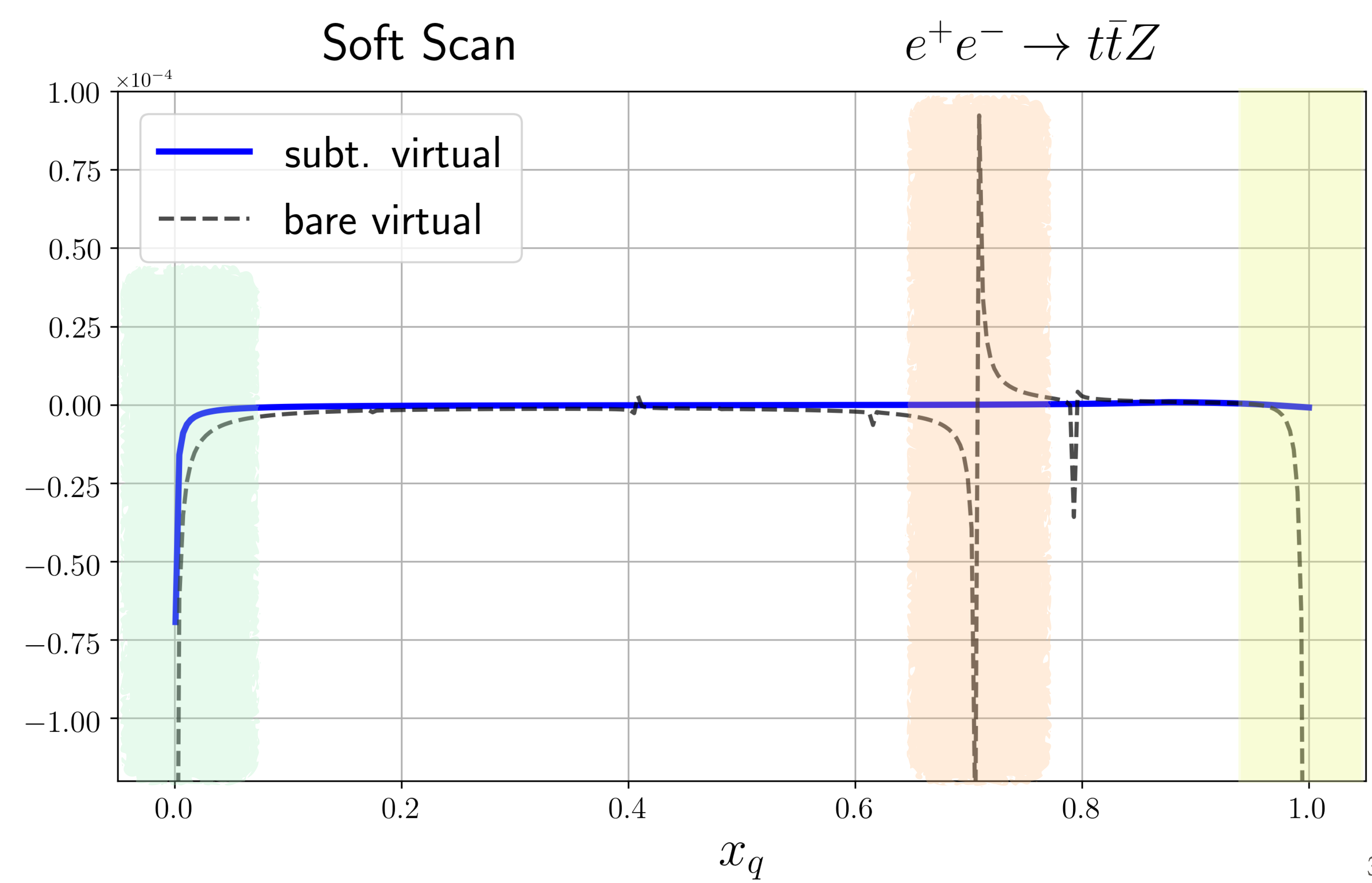
$$+\,\mathcal{W}_{\rm CT}^{\rm (ren)}(\Phi_{\rm B}) + \mathcal{W}_{\rm CT}^{\rm (tad)}(\Phi_{\rm B}) - \mathcal{W}_{\rm CT}^{\rm (frc)}(\Phi_{\rm B}) + \mathcal{W}_{\rm CT}^{\rm (thr)}(\Phi_{\rm B})$$

- UV singularities
- IR singularities from FRCs
- Threshold singularities



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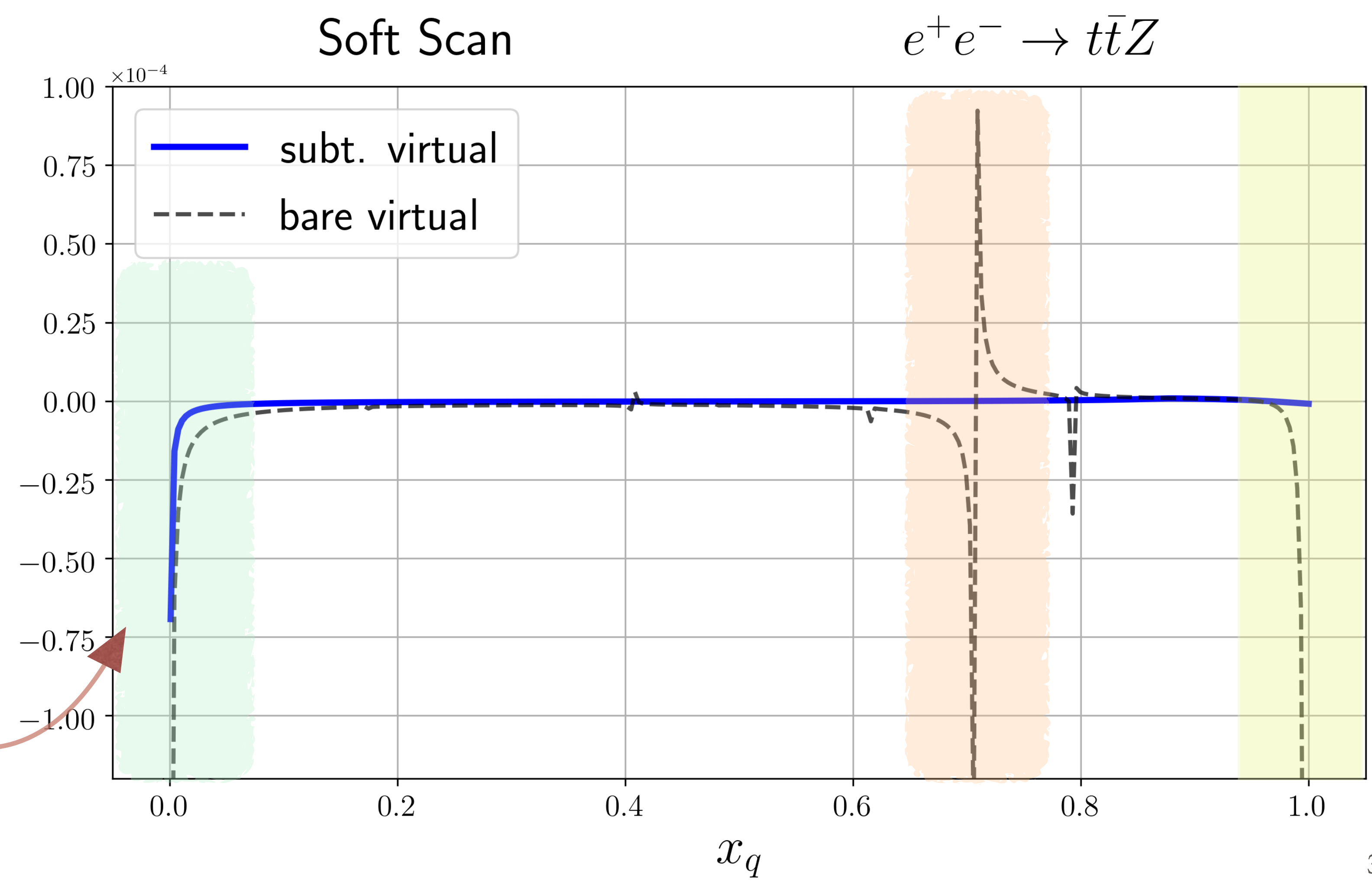

How to deal with **IR singularities?**

Subtraction vs LTD approach

Subtraction algorithm

$$\delta\sigma_{\mathrm{NLO}} = \int \mathrm{d}\Phi_{\mathrm{B}}\, \mathcal{W}_{\mathrm{V}}(\Phi_{\mathrm{B}}) + \int \mathrm{d}\Phi_{\mathrm{R}}\, \mathcal{W}_{\mathrm{R}}(\Phi_{\mathrm{R}})$$

LTD approach

$$\delta\sigma_{\mathrm{NLO}} = \int \mathrm{d}\Phi_{\mathrm{B}}\, \mathcal{W}_{\mathrm{V}}(\Phi_{\mathrm{B}}) + \int \mathrm{d}\Phi_{\mathrm{R}}\, \mathcal{W}_{\mathrm{R}}(\Phi_{\mathrm{R}})$$

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Subtraction algorithm

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$$= \int d\Phi_{\rm B} \left[\mathcal{W}_{\rm V}(\Phi_{\rm B}) + \mathcal{I}_{\rm IR}\right] + \int d\Phi_{\rm R} \left[\mathcal{W}_{\rm R}(\Phi_{\rm R}) - \mathcal{K}_{\rm IR}\right]$$

Analytic pole cancellation — *Numerically in D=4*

$$\int d\Phi_{\rm R}\, \mathcal{K}_{\rm IR} = \int d\Phi_{\rm B}\, \mathcal{I}_{\rm IR}$$

Analytic CT integration

LTD approach

$$\delta\sigma_{\rm NLO} = \int d\Phi_{\rm B}\, \mathcal{W}_{\rm V}(\Phi_{\rm B}) + \int d\Phi_{\rm R}\, \mathcal{W}_{\rm R}(\Phi_{\rm R})$$

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$$= \int d\Phi_{\mathrm{B}} \int D^{D-1}\mathbf{q}\, I_{\mathrm{V}}(\Phi_{\mathrm{B}}, \mathbf{q}) + \int d\Phi_{\mathrm{B}} \int D^{D-1}\mathbf{q}\, J_{\mathrm{R}}(\Phi_{\mathrm{B}}, \mathbf{q})\, \mathcal{W}_{\mathrm{R}}\left(\Phi_{\mathrm{R}}(\Phi_{\mathrm{B}}, \mathbf{q})\right)$$

Subtraction vs LTD approach

Subtraction algorithm

$$\delta\sigma_{\rm NLO} = \int d\Phi_B\, \mathcal{W}_V(\Phi_B) + \int d\Phi_R\, \mathcal{W}_R(\Phi_R)$$

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$$= \int d\Phi_B \int D^{D-1}\mathbf{q}\, I_V(\Phi_B, \mathbf{q}) + \int d\Phi_B \int D^{D-1}\mathbf{q}\, J_R(\Phi_B, \mathbf{q})\, \mathcal{W}_R\left(\Phi_R(\Phi_B, \mathbf{q})\right)$$

$$= \int d\Phi_B \int D^{D-1}\mathbf{q} \left[I_V(\Phi_B, \mathbf{q}) + I_R(\Phi_B, \mathbf{q})\right]$$

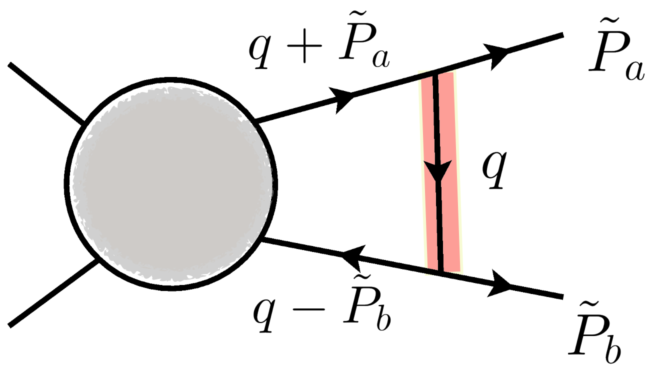
Numerically integrable in D=4

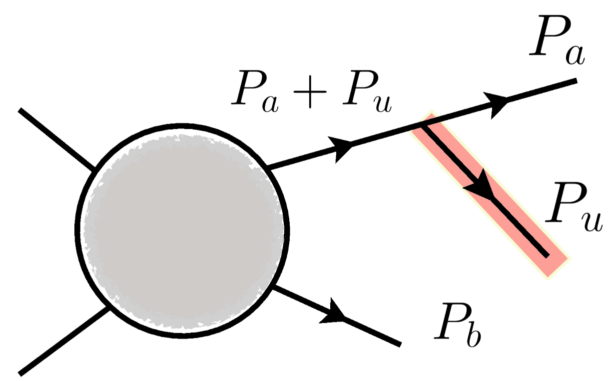
Direct cancellation of IR singularities!

Soft singularities

- Analyse the scaling behaviour in the soft limit

$$\delta\sigma_{\rm NLO} = \int \mathrm{d}\Phi_{\rm B} \int D^{D-1}\mathbf{q}\, I_{\rm V}(\Phi_{\rm B},\mathbf{q}) + \int \mathrm{d}\Phi_{\rm R}\, \mathcal{W}_{\rm R}(\Phi_{\rm R})$$





$$\sim \mathrm{d}|\mathbf{q}|\, |\mathbf{q}|^2 \times \frac{1}{|\mathbf{q}|^3}$$

$$\sim \mathrm{d}P_u^0\, P_u^0 \times \frac{1}{(P_u^0)^2}$$

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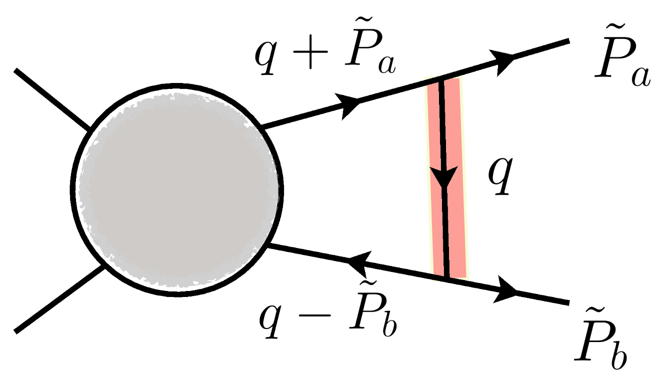
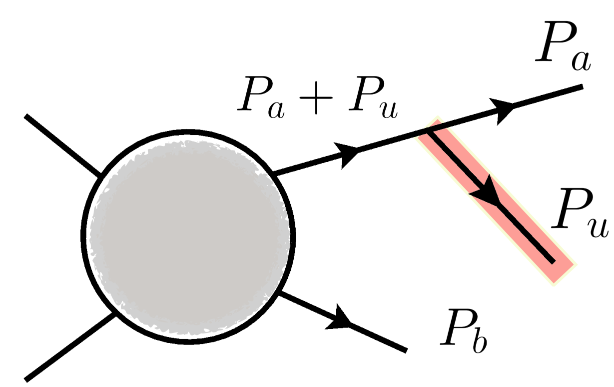
$$\delta\sigma_{\text{NLO}} = \int \mathrm{d}\Phi_{\text{B}} \int D^{D-1}\mathbf{q}\, I_{\text{V}}(\Phi_{\text{B}}, \mathbf{q}) + \int \mathrm{d}\Phi_{\text{R}}\, \mathcal{W}_{\text{R}}(\Phi_{\text{R}})$$



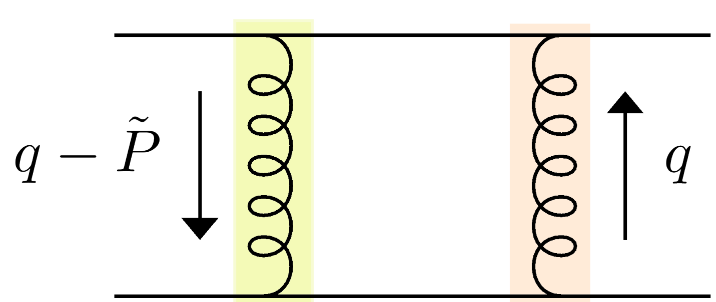


$$\sim \mathrm{d}|\mathbf{q}|\, |\mathbf{q}|^2 \times \frac{1}{|\mathbf{q}|^3} \qquad\qquad \sim \mathrm{d}P_u^0\, P_u^0 \times \frac{1}{(P_u^0)^2}$$

However, in a given default parametrisation not all soft singularities are located at $|\vec{q}| \to 0$!

$|\mathbf{q} - \tilde{\mathbf{P}}| \to 0$

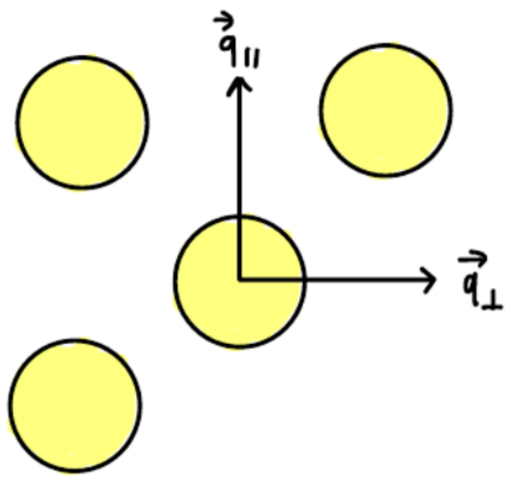
Reparametrise!



$|\mathbf{q}| \to 0$

Soft singularities

- Analyse scaling behaviour in the soft limit
- Bring IR singularities in standard form: **Soft Loop Sectors**



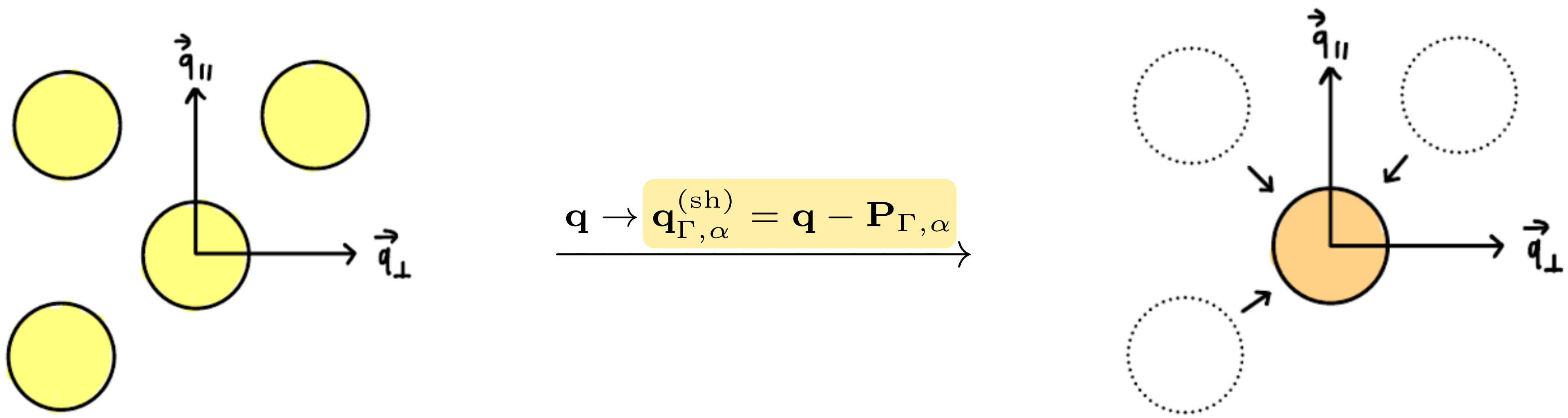
$$I_{\mathrm{V}}(\mathbf{q}) = \sum_{\Gamma} \underbrace{\sum_{\alpha\in\mathcal{S}_\Gamma} \varepsilon_{\Gamma,\alpha}(\mathbf{q})}_{=1} I_{\Gamma}(\mathbf{q})$$

$$\varepsilon_{\Gamma,\alpha}(\mathbf{q}) = \frac{\left|\rho_{\Gamma,\alpha}(\mathbf{q})\right|}{\sum_{\beta\in\mathcal{S}_\Gamma}\left|\rho_{\Gamma,\beta}(\mathbf{q})\right|}$$

Soft projectors: $\rho_{\Gamma,\alpha}(\mathbf{q}) \propto \frac{1}{|\mathbf{q}|^3}$

Soft singularities

- Analyse scaling behaviour in the soft limit
- Bring IR singularities in standard form: **Soft Loop Sectors**



$$I_{\mathrm{V}}(\mathbf{q}) = \sum_{\Gamma} \sum_{\alpha \in \mathcal{S}_\Gamma} \varepsilon_{\Gamma,\alpha}(\mathbf{q})\, I_\Gamma(\mathbf{q}) \quad \to \quad \hat{I}_{\mathrm{V}}(\mathbf{q}) = \sum_{\Gamma} \sum_{\alpha \in \mathcal{S}_\Gamma} \varepsilon_{\Gamma,\alpha}\big(\mathbf{q}^{(\mathrm{sh})}_{\Gamma,\alpha}\big)\, I_\Gamma\big(\mathbf{q}^{(\mathrm{sh})}_{\Gamma,\alpha}\big)$$

$$\varepsilon_{\Gamma,\alpha}(\mathbf{q}) = \frac{|\rho_{\Gamma,\alpha}(\mathbf{q})|}{\sum_{\beta \in \mathcal{S}_\Gamma} |\rho_{\Gamma,\beta}(\mathbf{q})|}$$

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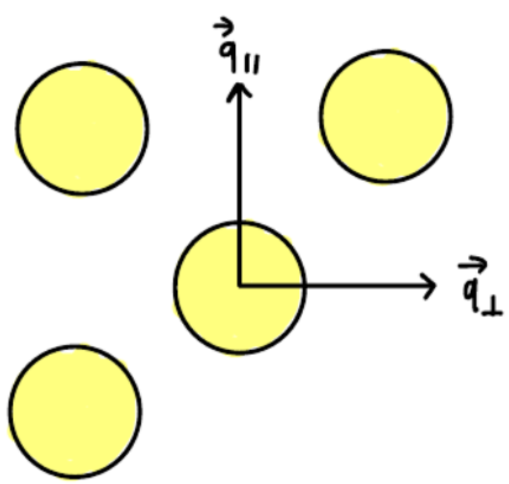
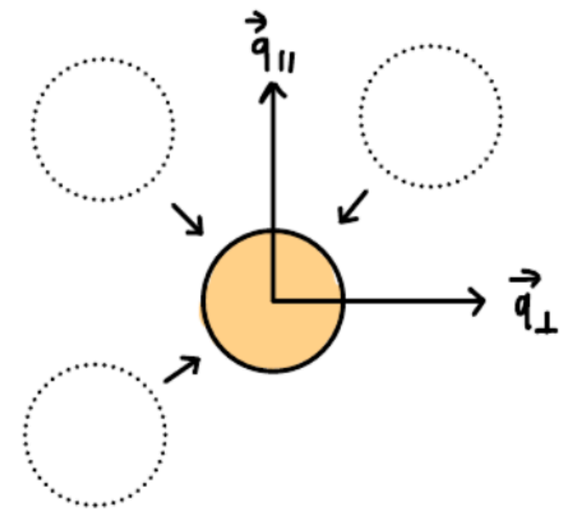
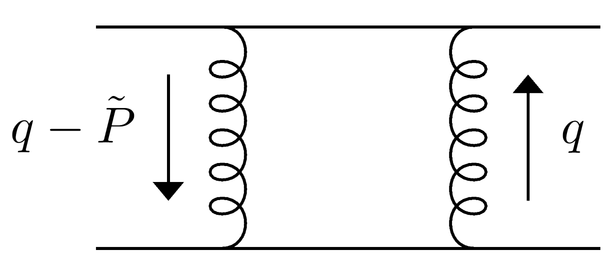


$$\mathbf{q} \to \mathbf{q}^{(\mathrm{sh})}_{\Gamma,\alpha} = \mathbf{q} - \mathbf{P}_{\Gamma,\alpha}$$



$$I_{\mathrm{V}}(\mathbf{q}) = \sum_{\Gamma} \sum_{\alpha \in \mathcal{S}_\Gamma} \varepsilon_{\Gamma,\alpha}(\mathbf{q})\, I_\Gamma(\mathbf{q}) \quad \to \quad \hat{I}_{\mathrm{V}}(\mathbf{q}) = \sum_{\Gamma} \sum_{\alpha \in \mathcal{S}_\Gamma} \varepsilon_{\Gamma,\alpha}\left(\mathbf{q}^{(\mathrm{sh})}_{\Gamma,\alpha}\right) I_\Gamma\left(\mathbf{q}^{(\mathrm{sh})}_{\Gamma,\alpha}\right)$$

More explicitly…



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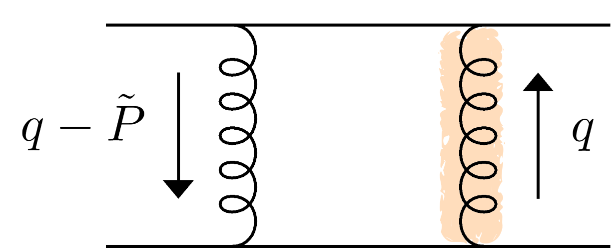

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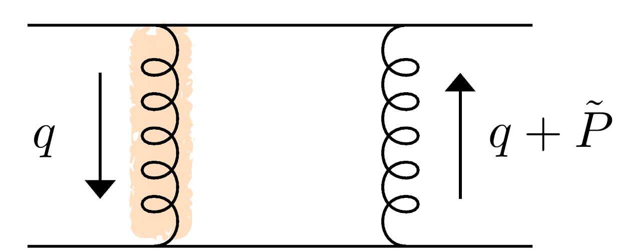
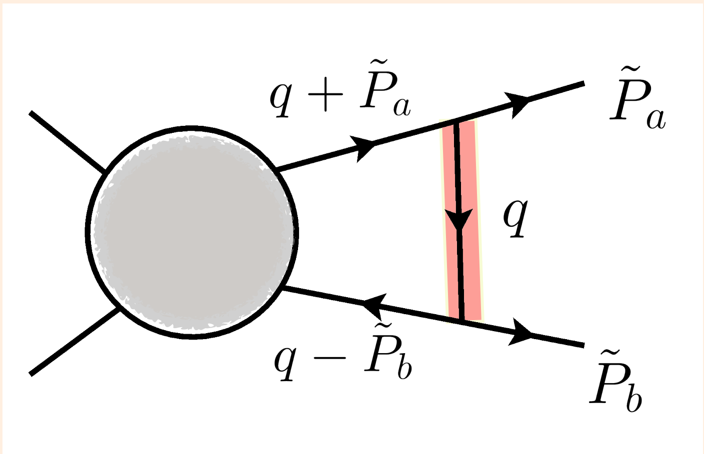
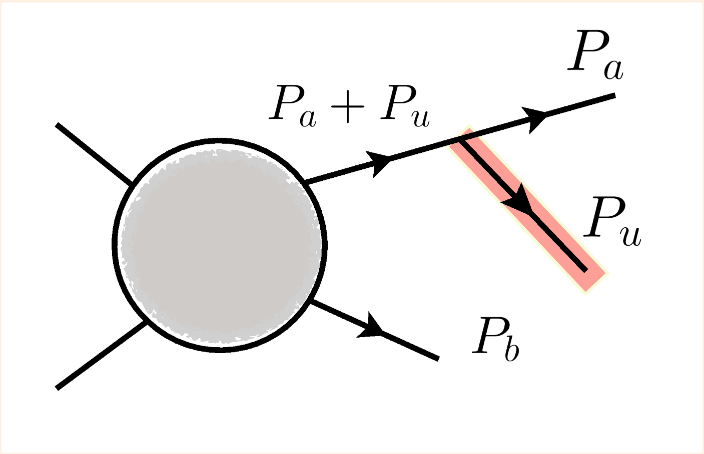


Soft singularities

- Analyse scaling behaviour in the soft limit
- Bring IR singularities in standard form: **Soft Loop Sectors**
- Introduce a proper $\Phi_{\rm V} \to \Phi_{\rm R}$ **mapping** to align the phase-space measure

$$\delta\sigma_{\rm NLO} = \int d\Phi_{\rm B} \int D^{D-1}\mathbf{q}\, \widehat{I}_{\rm V}\,(\Phi_{\rm B}, \mathbf{q}) + \int d\Phi_{\rm R}\, \mathcal{W}_{\rm R}(\Phi_{\rm R})$$





Soft singularities

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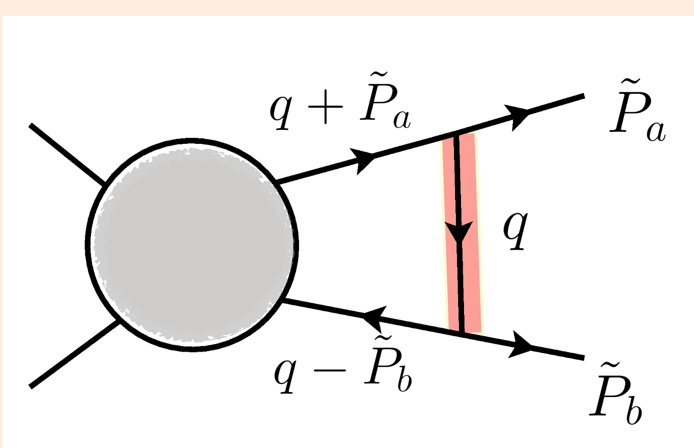
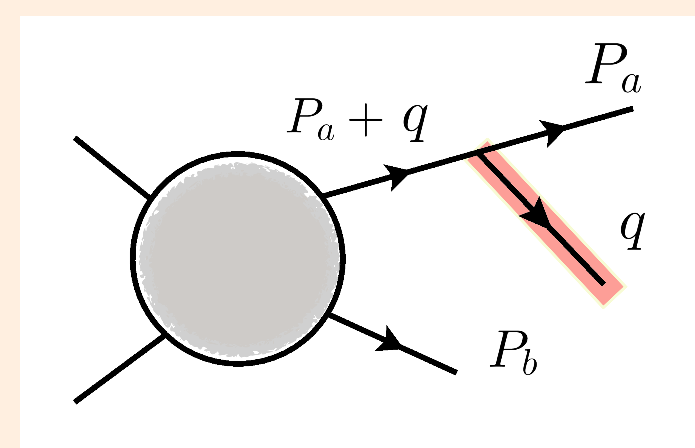
$$\begin{aligned}\delta\sigma_{\mathrm{NLO}} &= \int \mathrm{d}\Phi_{\mathrm{B}} \int D^{D-1}\mathbf{q}\, \widehat{I}_{\mathrm{V}}\left(\Phi_{\mathrm{B}}, \mathbf{q}\right) + \int \mathrm{d}\Phi_{\mathrm{R}}\, \mathcal{W}_{\mathrm{R}}(\Phi_{\mathrm{R}}) \\ &= \int \mathrm{d}\Phi_{\mathrm{B}} \int D^{D-1}\mathbf{q}\, \widehat{I}_{\mathrm{V}}\left(\Phi_{\mathrm{B}}, \mathbf{q}\right) + \int \mathrm{d}\Phi_{\mathrm{B}} \int D^{D-1}\mathbf{q}\, J_{\mathrm{R}}(\Phi_{\mathrm{B}}, \mathbf{q})\, \mathcal{W}_{\mathrm{R}}\left(\Phi_{\mathrm{R}}(\Phi_{\mathrm{B}}, \mathbf{q})\right)\end{aligned}$$



$$\begin{cases} P_u = q \\ P_a = f(q, \tilde{P}_a, \tilde{P}_b) \\ P_b = g(q, \tilde{P}_a, \tilde{P}_b) \end{cases} \xrightarrow[\Phi_{\mathrm{V}} \to \Phi_{\mathrm{R}} \text{ mapping}]{}$$

[Catani, et al. 0201036]



Soft singularities

- Analyse scaling behaviour in the soft limit
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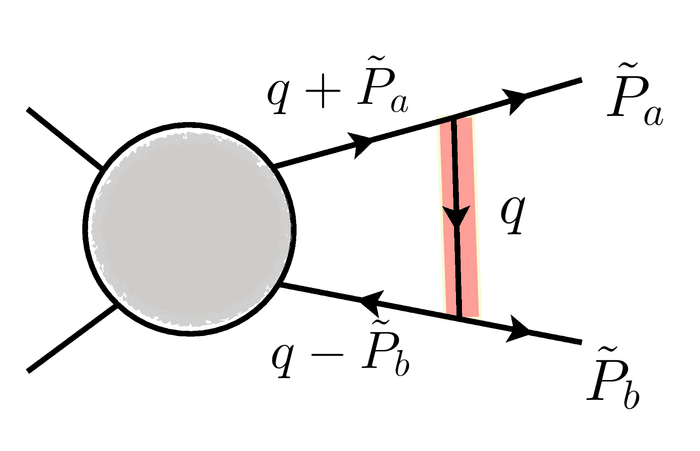
$$
\begin{aligned}
\delta\sigma_{\mathrm{NLO}} &= \int \mathrm{d}\Phi_{\mathrm{B}} \int D^{D-1}\mathbf{q}\, \widehat{I}_{\mathrm{V}}\left(\Phi_{\mathrm{B}}, \mathbf{q}\right) + \int \mathrm{d}\Phi_{\mathrm{R}}\, \mathcal{W}_{\mathrm{R}}(\Phi_{\mathrm{R}}) \\
&= \int \mathrm{d}\Phi_{\mathrm{B}} \int D^{D-1}\mathbf{q}\, \widehat{I}_{\mathrm{V}}\left(\Phi_{\mathrm{B}}, \mathbf{q}\right) + \int \mathrm{d}\Phi_{\mathrm{B}} \int D^{D-1}\mathbf{q}\, J_{\mathrm{R}}(\Phi_{\mathrm{B}}, \mathbf{q})\, \mathcal{W}_{\mathrm{R}}\left(\Phi_{\mathrm{R}}(\Phi_{\mathrm{B}}, \mathbf{q})\right) \\
&= \int \mathrm{d}\Phi_{\mathrm{B}} \int D^{D-1}\mathbf{q} \left[\widehat{I}_{\mathrm{V}}\left(\Phi_{\mathrm{B}}, \mathbf{q}\right) + I_{\mathrm{R}}\left(\Phi_{\mathrm{B}}, \mathbf{q}\right)\right]
\end{aligned}
$$

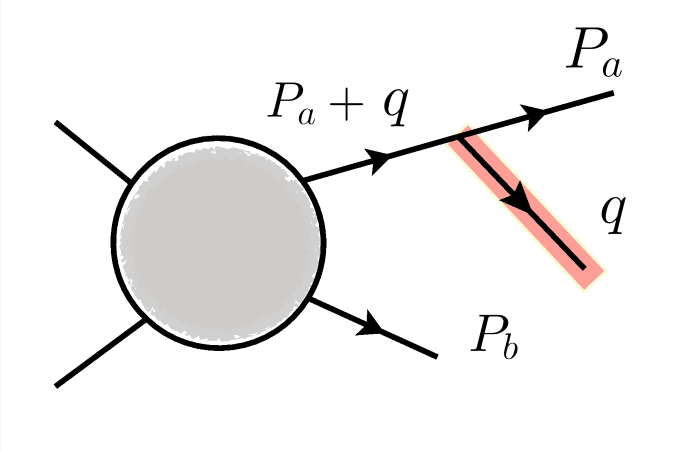
Locally finite in soft limit!



$$
\begin{cases} P_u = q \\ P_a = f(q, \tilde{P}_a, \tilde{P}_b) \\ P_b = g(q, \tilde{P}_a, \tilde{P}_b) \end{cases}
$$

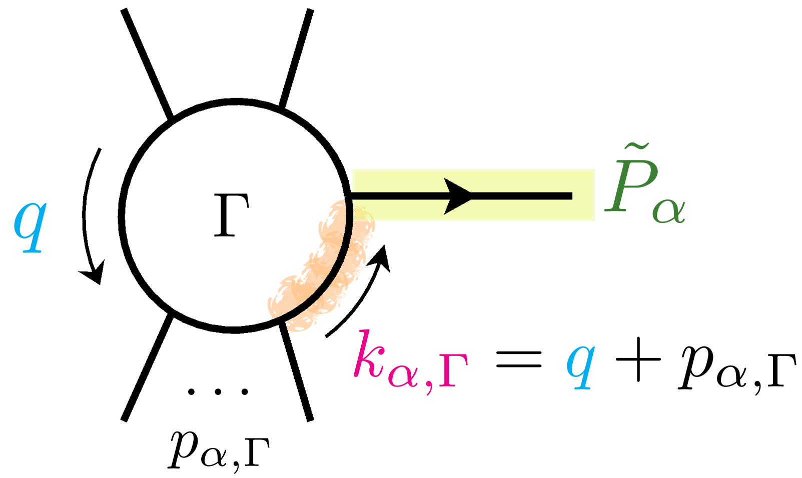
$\xrightarrow{\hspace{3em}}$ $\Phi_{\mathrm{V}} \to \Phi_{\mathrm{R}}$ mapping

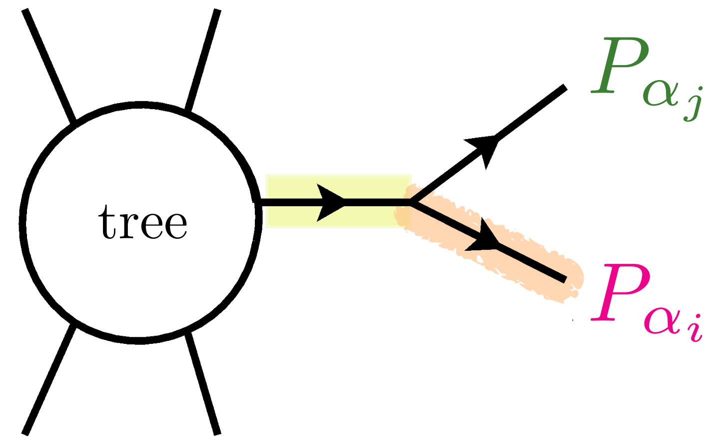
[Catani, et al. 0201036]



Collinear singularities

- Analyse the collinear singularity associated to a given **final-state** splitting $\alpha \to \alpha_i + \alpha_j$





$$I_{\mathrm{V}}(\Phi_{\mathrm{B}}, \mathbf{q}) \sim \frac{1}{2\, k_{\alpha,\Gamma} \cdot \tilde{P}_\alpha}\bigg|_{k^0_{\alpha,\Gamma} = |\vec{k}_{\alpha,\Gamma}|}$$

Depends on parametrisation
of individual loop diagrams

$$\mathcal{W}_{\mathrm{R}}(\Phi_{\mathrm{R}}) \sim \frac{1}{2\, P_{\alpha_i} \cdot P_{\alpha_j}}$$

Well-defined
external real momenta

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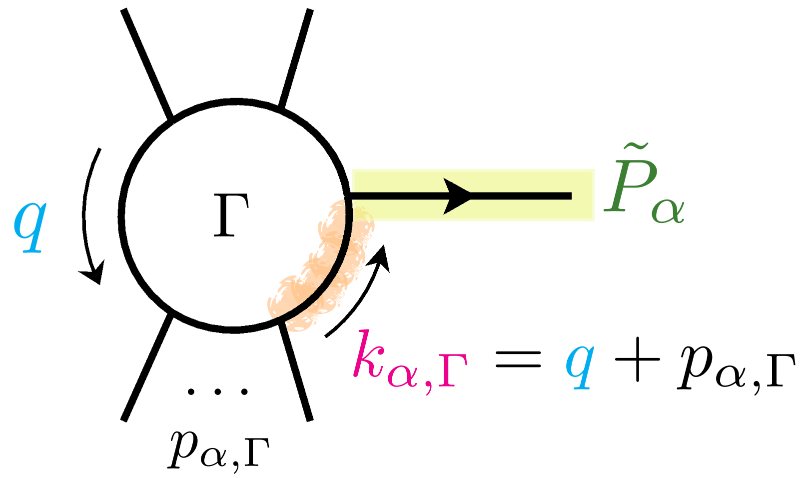
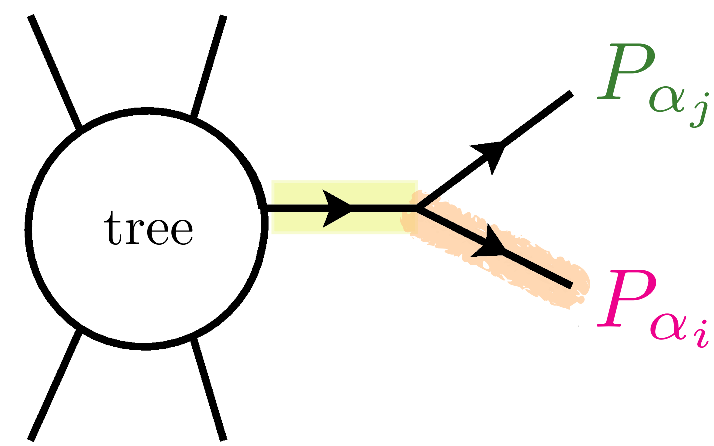




$$I_{\mathrm{V}}(\Phi_{\mathrm{B}}, \mathbf{q}) \sim \frac{1}{2\, k_{\alpha,\Gamma} \cdot \tilde{P}_\alpha}\Bigg|_{k^0_{\alpha,\Gamma} = |\vec{k}_{\alpha,\Gamma}|}$$

Depends on parametrisation of individual loop diagrams

$$\mathcal{W}_{\mathrm{R}}(\Phi_{\mathrm{R}}) \sim \frac{1}{2\, P_{\alpha_i} \cdot P_{\alpha_j}}$$

Well-defined external real momenta

↓

Loop-momentum **reparametrisation** is needed to recast singularity in standard form

$$k_{\alpha,\Gamma} \to q$$

↓

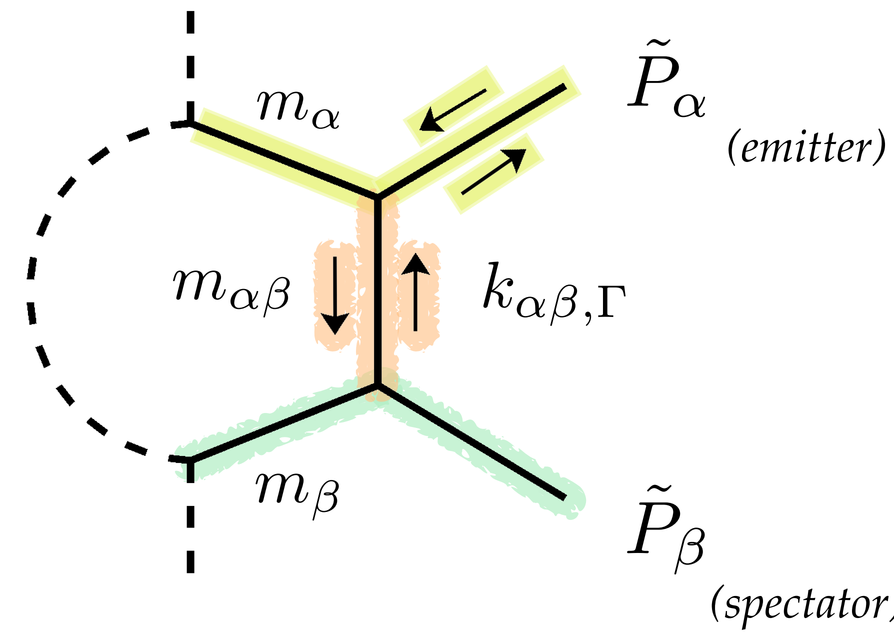
A momentum mapping from V to R phase-space is necessary to align singularities

$$P_{\alpha_i} = q$$

Collinear singularities

- Analyse the collinear singularity associated to a given **final-state** splitting $\alpha \to \alpha_i + \alpha_j$
- Add direction information in improved **Collinear Loop Sectors**

$$I_{\mathrm{V}}(\Phi_{\mathrm{B}}, \mathbf{q}) = \sum_{\Gamma} \underbrace{\sum_{\alpha\beta \in \mathcal{S}_\Gamma} \varepsilon_{\alpha\beta,\Gamma}(\Phi_{\mathrm{B}}, \mathbf{q})}_{=1} I_\Gamma(\Phi_{\mathrm{B}}, \mathbf{q})$$



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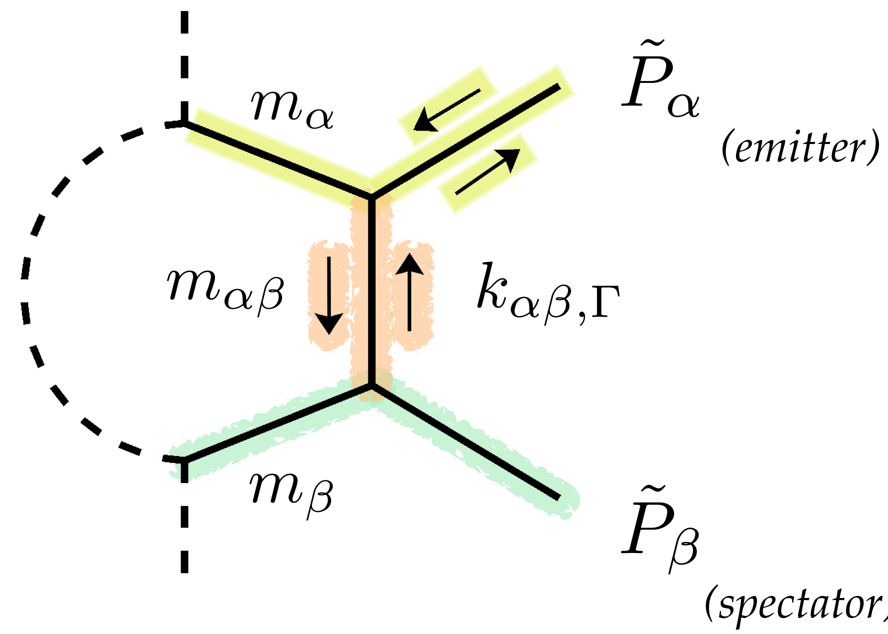

Conditions:

Collinear singularity: $\tilde{P}_\alpha^2 = m_\alpha^2 = m_{\alpha\beta}^2 = 0$ Soft singularity: $\tilde{P}_\alpha^2 - m_\alpha^2 = \tilde{P}_\beta^2 - m_\beta^2 = m_{\alpha\beta}^2 = 0$

Structure:

$$\varepsilon_{\alpha\beta,\Gamma}(\Phi_{\rm V}) = \frac{\rho_{\alpha\beta,\Gamma}(\Phi_{\rm V})}{\sum_{\alpha'\beta'\in S_\Gamma} \rho_{\alpha'\beta',\Gamma}(\Phi_{\rm V})} \qquad \text{with} \qquad \rho_{\alpha\beta}(\Phi_{\rm V}) \propto \frac{\tilde{P}_\alpha\cdot\tilde{P}_\beta}{|\mathbf{q}|\,(\mathbf{q}\cdot\tilde{P}_\alpha)\,(\mathbf{q}\cdot\tilde{P}_\alpha+\mathbf{q}\cdot\tilde{P}_\beta)}$$

Collinear singularities

- Analyse the collinear singularity associated to a given **final-state** splitting $\alpha \to \alpha_i + \alpha_j$
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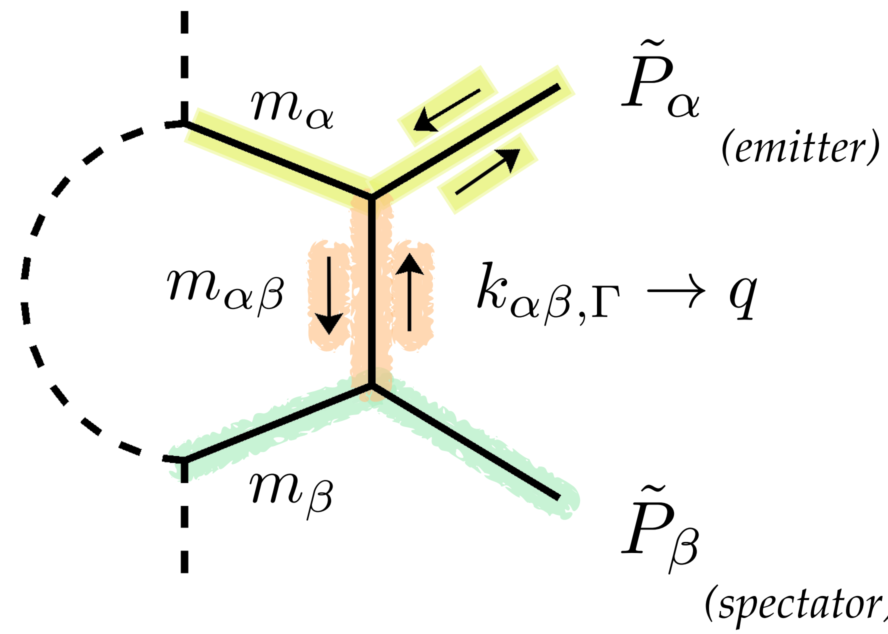
$$I_{\rm V}(\Phi_{\rm B},\mathbf{q}) = \sum_{\Gamma} \sum_{\alpha\beta\in\mathcal{S}_\Gamma} \varepsilon_{\alpha\beta,\Gamma}(\Phi_{\rm B},\mathbf{q})\, I_\Gamma(\Phi_{\rm B},\mathbf{q})$$

$$\downarrow \qquad \mathbf{q} \to \mathbf{q}^{(\rm sh)}_{\alpha\beta,\Gamma} = \mathbf{q} - \mathbf{p}_{\alpha\beta,\Gamma}$$

$$\hat{I}_{\rm V}(\Phi_{\rm B},\mathbf{q}) = \sum_{\Gamma} \sum_{\alpha\beta\in\mathcal{S}_\Gamma} \varepsilon_{\alpha\beta,\Gamma}(\Phi_{\rm B},\mathbf{q}^{(\rm sh)}_{\alpha\beta,\Gamma})\, I_\Gamma(\Phi_{\rm B},\mathbf{q}^{(\rm sh)}_{\alpha\beta,\Gamma})$$



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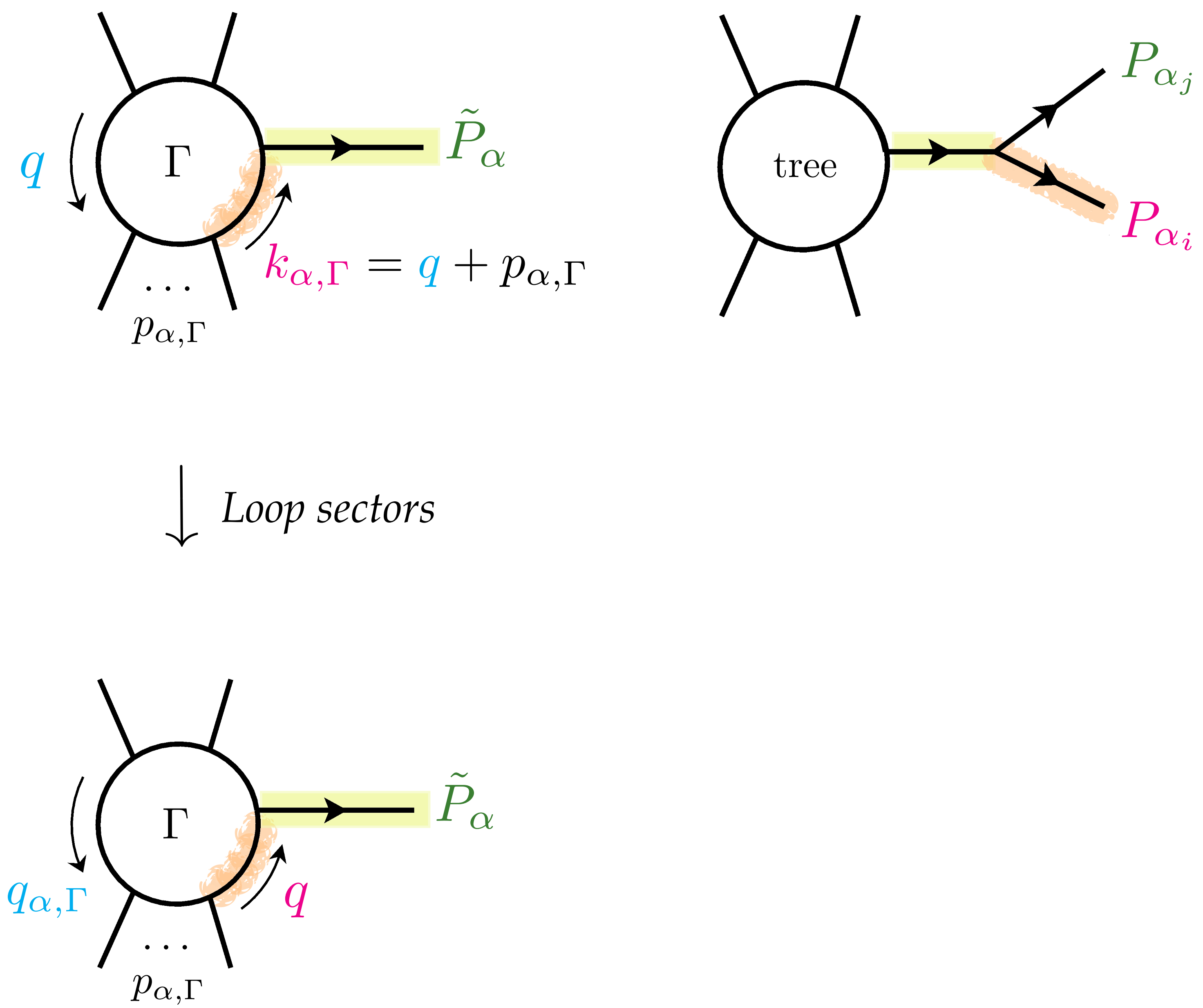
Collinear singularities

- Analyse the collinear singularity associated to a given **final-state** splitting
- Add direction information in improved **Collinear Loop Sectors**
- Introduce appropriate $\Phi_{\mathrm{V}} \to \Phi_{\mathrm{R}}$ **mapping**



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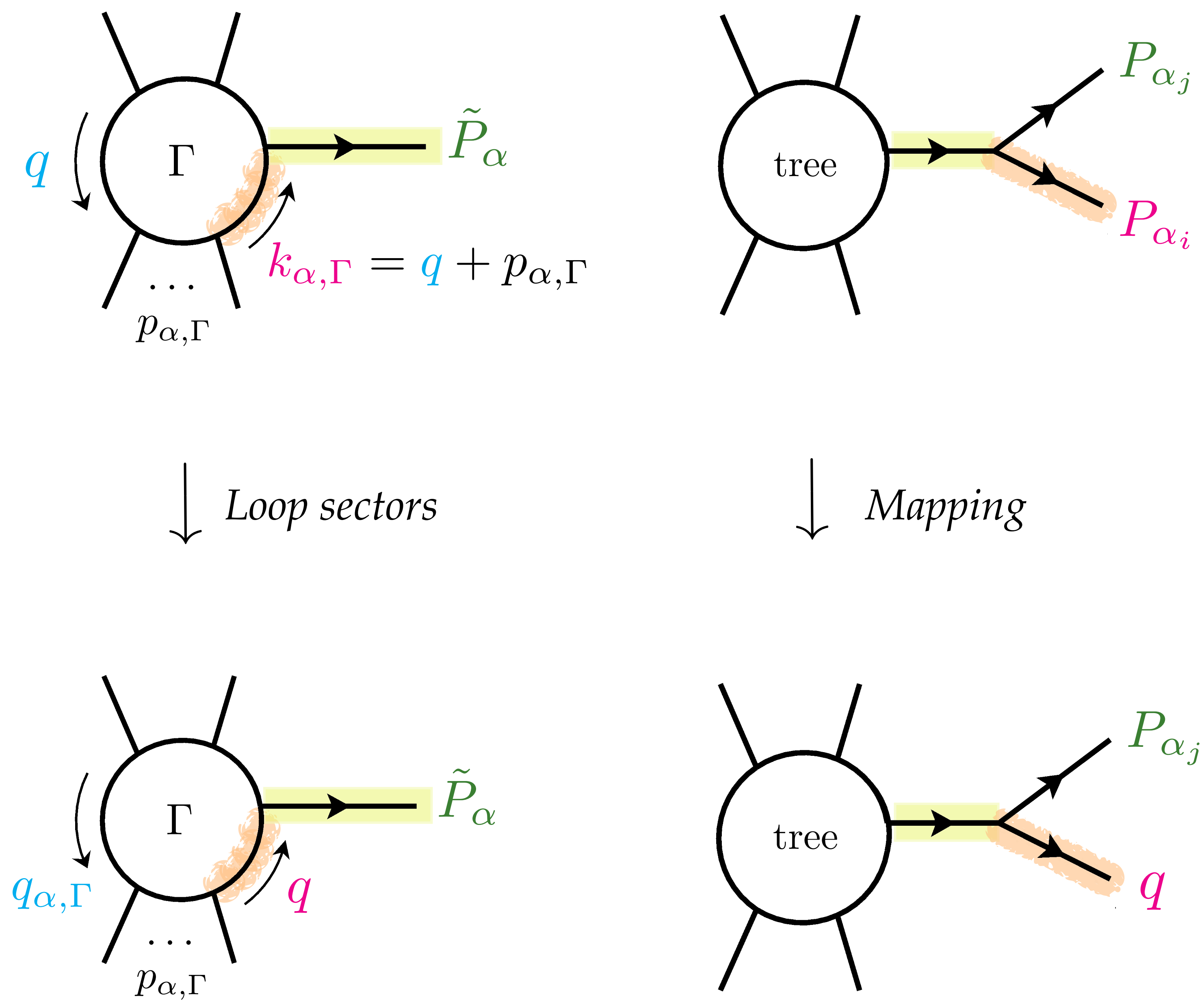

✱ Match **q** with collinear leg

$$\begin{cases} P_{\alpha_i} = q \\ P_{\alpha_j} = f(q, \tilde{P}_\alpha, \tilde{P}_\beta) \\ P_\beta = g(q, \tilde{P}_\alpha, \tilde{P}_\beta) \end{cases}$$

✱ Factorise **Dq** measure

[Catani, Seymour 9605323]

Collinear singularities

- Analyse the collinear singularity associated to a given **final-state** splitting
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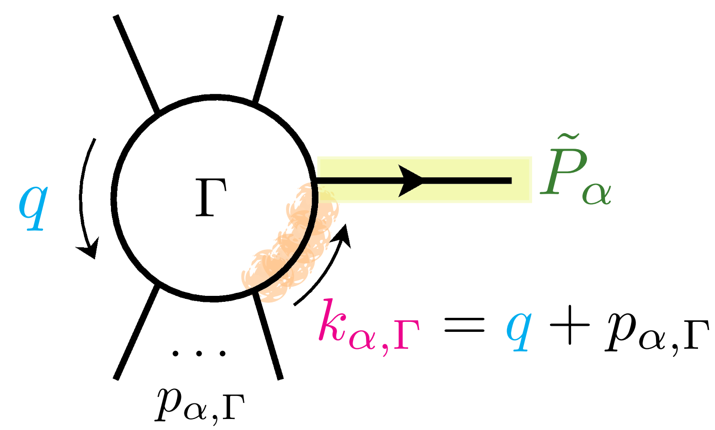

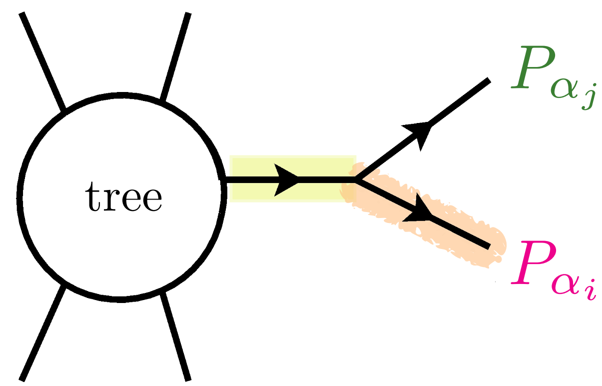

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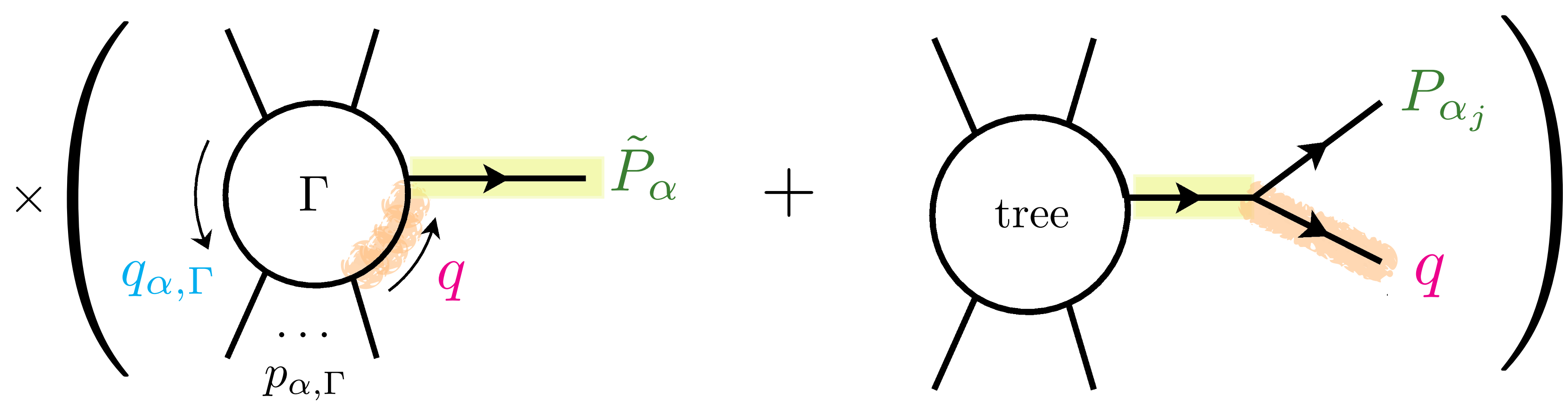
✱ Factorise **Dq** measure

[Catani, Seymour 9605323]

$\downarrow$ *Loop sectors* $\qquad\qquad$ $\downarrow$ *Mapping*

$$\lim_{\substack{\mathbf{q} \| \vec{\tilde{P}}_\alpha \\ \mathbf{q} \to 0}} \int \mathrm{D}^{D-1}\mathbf{q}$$



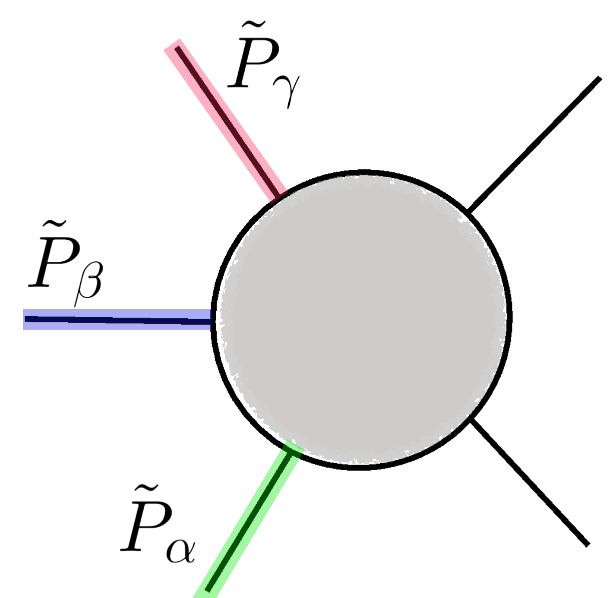
$=$

Locally free from IR singularities!

Multiple collinear singularities

- Separate different collinear directions with **Global Sectors**

$$\delta\sigma_{\mathrm{NLO}} = \int \mathrm{d}\Phi_{\mathrm{B}} \int D^{D-1}\mathbf{q} \left[\underbrace{\sum_{\alpha} \varepsilon^{\mathrm{V}}_{\alpha}(\Phi_{\mathrm{B}}, \mathbf{q})}_{=1} \hat{I}_{\mathrm{V}}(\Phi_{\mathrm{B}}, \mathbf{q}) + \underbrace{\sum_{\alpha} \varepsilon^{\mathrm{R}}_{\alpha}(\Phi_{\mathrm{R}}(\Phi_{\mathrm{V}}))}_{=1} I_{\mathrm{R}}(\Phi_{\mathrm{R}}(\Phi_{\mathrm{V}})) \right]$$



Multiple collinear singularities

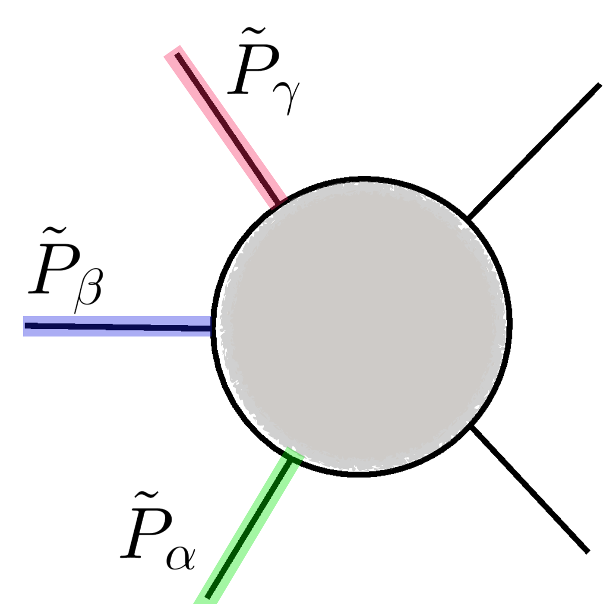
- Separate different collinear directions with **Global Sectors**
- Introduce **q**-parametrisation & mapping that **adapt** to each identified dipole

$$\delta\sigma_{\mathrm{NLO}} = \int \mathrm{d}\Phi_{\mathrm{B}} \int D^{D-1}\mathbf{q} \left[\sum_{\alpha} \varepsilon^{\mathrm{V}}_{\alpha}(\Phi_{\mathrm{B}},\mathbf{q})\, \hat{I}_{\mathrm{V}}(\Phi_{\mathrm{B}},\mathbf{q}) + \sum_{\alpha} \varepsilon^{\mathrm{R}}_{\alpha}(\Phi_{\mathrm{R}}(\Phi_{\mathrm{V}}))\, I_{\mathrm{R}}(\Phi_{\mathrm{R}}(\Phi_{\mathrm{V}})) \right]$$

$$= \sum_{\alpha} \int \mathrm{d}\Phi_{\mathrm{B}} \int D^{D-1}\mathbf{q}^{(\alpha)} \left[\varepsilon^{\mathrm{V}}_{\alpha}(\Phi_{\mathrm{B}},\mathbf{q}^{(\alpha)})\, \hat{I}_{\mathrm{V}}(\Phi_{\mathrm{B}},\mathbf{q}^{(\alpha)}) + \varepsilon^{\mathrm{R}}_{\alpha}(\Phi^{(\alpha)}_{\mathrm{R}}(\Phi_{\mathrm{V}}))\, I_{\mathrm{R}}(\Phi^{(\alpha)}_{\mathrm{R}}(\Phi_{\mathrm{V}})) \right]$$

- **Sector-dependent** parametrisation

$$\int \mathrm{D}^3\mathbf{q} \equiv \int \mathrm{d}^3\mathbf{x}\, J_{\mathbf{q}}(\mathbf{x}) \quad \longrightarrow \quad \lim_{\mathbf{q}\to 0} J_{\mathbf{q}}(\mathbf{x}) \propto |\mathbf{q}|^2 \qquad \lim_{\mathbf{q}\| \vec{\tilde{P}}_{\alpha}} J_{\mathbf{q}}(\mathbf{x}) \propto \theta(\mathbf{q}, \vec{\tilde{P}}_{\alpha})$$



- **Sector-dependent** mapping

$$\Phi_{\mathrm{V}} \to \Phi^{(\alpha)}_{\mathrm{R}}(\Phi_{\mathrm{V}})$$

Multiple collinear singularities

- Separate different collinear directions with **Global Sectors**

- Introduce **q**-parametrisation & mapping that **adapt** to each identified dipole

$$\delta\sigma_{\mathrm{NLO}} = \int \mathrm{d}\Phi_{\mathrm{B}} \int D^{D-1}\mathbf{q} \left[\sum_{\alpha} \varepsilon_{\alpha}^{\mathrm{V}}(\Phi_{\mathrm{B}},\mathbf{q})\, \hat{I}_{\mathrm{V}}(\Phi_{\mathrm{B}},\mathbf{q}) + \sum_{\alpha} \varepsilon_{\alpha}^{\mathrm{R}}(\Phi_{\mathrm{R}}(\Phi_{\mathrm{V}}))\, I_{\mathrm{R}}(\Phi_{\mathrm{R}}(\Phi_{\mathrm{V}})) \right]$$

$$= \sum_{\alpha} \int \mathrm{d}\Phi_{\mathrm{B}} \int D^{D-1}\mathbf{q}^{(\alpha)} \left[\varepsilon_{\alpha}^{\mathrm{V}}(\Phi_{\mathrm{B}},\mathbf{q}^{(\alpha)})\, \hat{I}_{\mathrm{V}}(\Phi_{\mathrm{B}},\mathbf{q}^{(\alpha)}) + \varepsilon_{\alpha}^{\mathrm{R}}(\Phi_{\mathrm{R}}^{(\alpha)}(\Phi_{\mathrm{V}}))\, I_{\mathrm{R}}(\Phi_{\mathrm{R}}^{(\alpha)}(\Phi_{\mathrm{V}})) \right]$$

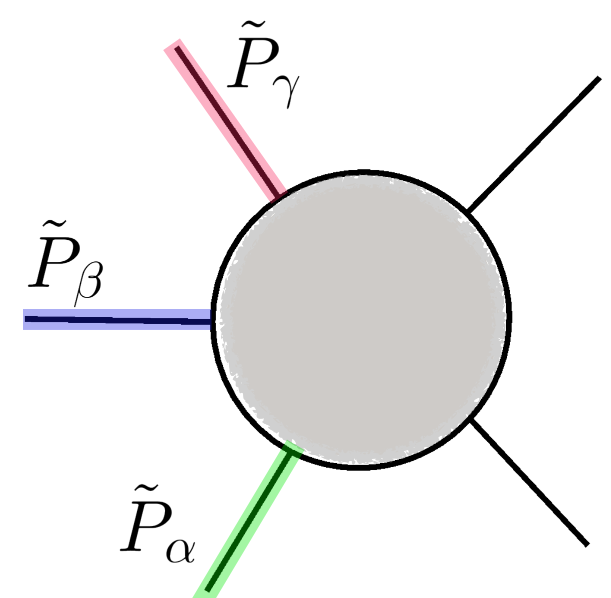
✱ **Structure**:

$$\varepsilon_{\alpha}^{\mathrm{R}} \propto \frac{P_{\alpha}\cdot P_{\beta}}{|\mathbf{q}|\,(\mathbf{q}\cdot P_{\alpha})\,(\mathbf{q}\cdot P_{\alpha}+\mathbf{q}\cdot P_{\beta})}$$

✱ **Requirement**: identical in the IR limits to allow V & R combination

$$\lim_{|\mathbf{q}|\to 0} \varepsilon_{\alpha}^{\mathrm{V}}(\Phi_{\mathrm{B}},\mathbf{q}^{(\alpha)}) = \lim_{|\mathbf{q}|\to 0} \varepsilon_{\alpha}^{\mathrm{R}}(\Phi_{\mathrm{R}}^{(\alpha)}(\Phi_{\mathrm{V}}))$$

$$\lim_{\mathbf{q}||\vec{\tilde{P}}_{\alpha}} \varepsilon_{\alpha}^{\mathrm{V}}(\Phi_{\mathrm{B}},\mathbf{q}^{(\alpha)}) = \lim_{\mathbf{q}||\vec{\tilde{P}}_{\alpha}} \varepsilon_{\alpha}^{\mathrm{R}}(\Phi_{\mathrm{R}}^{(\alpha)}(\Phi_{\mathrm{V}}))$$



OL+LTD at work

- Processes with **massive** external partons **soft singularities**

$$e^+e^- \to t\bar{t} + nZ \quad \text{with} \quad n = 0, 1, 2$$

OL+LTD at work

- Processes with **massive** external partons

soft singularities

$$e^+e^- \to t\bar{t} + nZ \quad \text{with} \quad n = 0, 1, 2$$

✱ Setup:

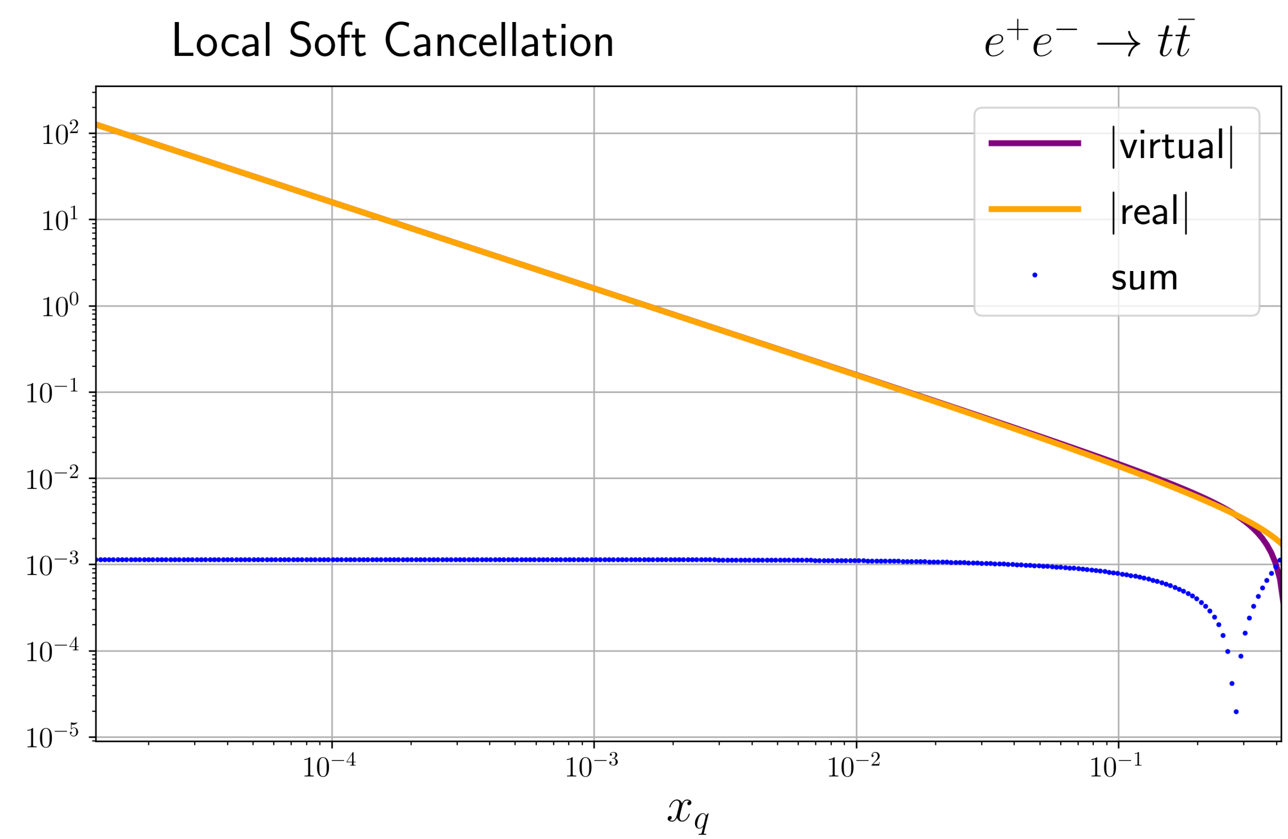
- Tangent **q**-parametrisation

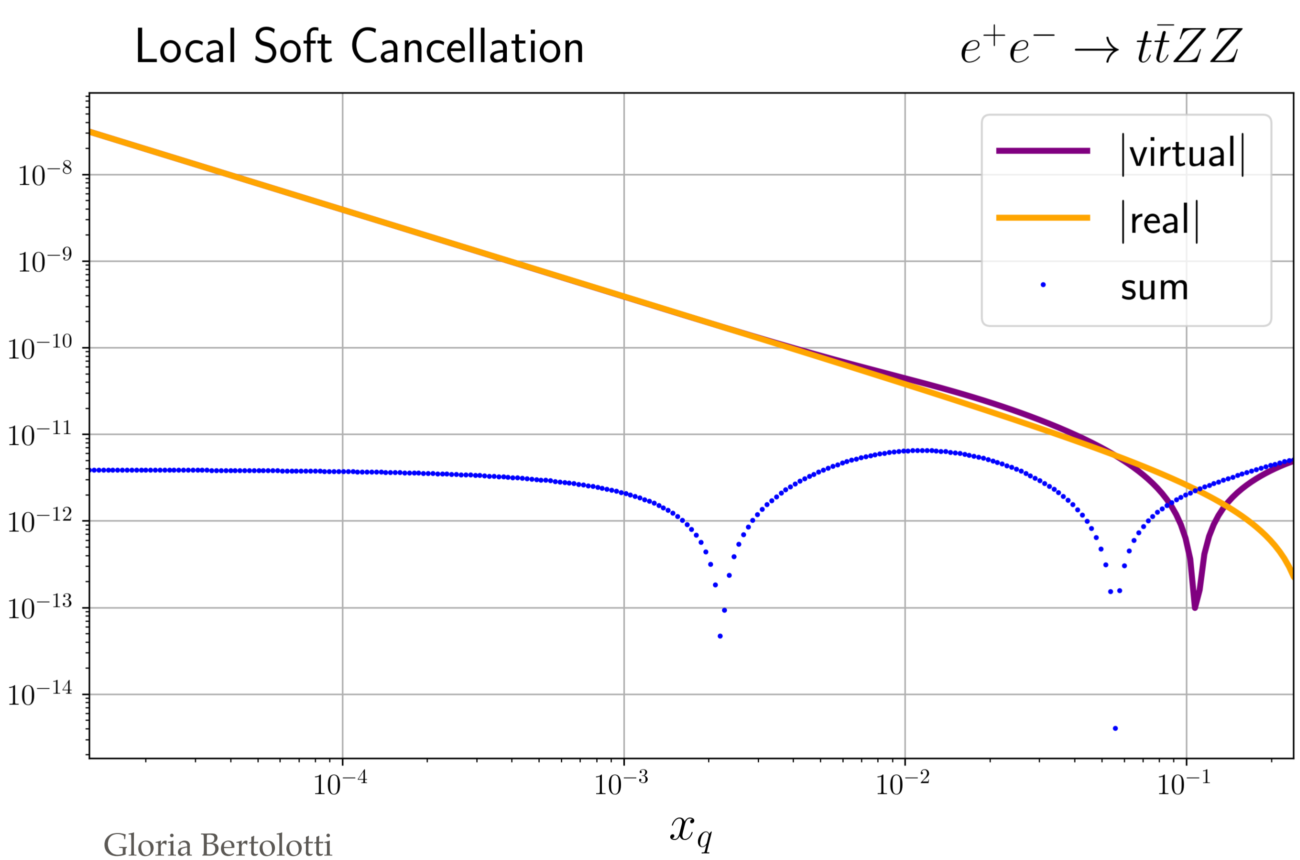
$$|\mathbf{q}| = \mu \tan\left(\frac{\pi}{2}\, x_q\right)$$

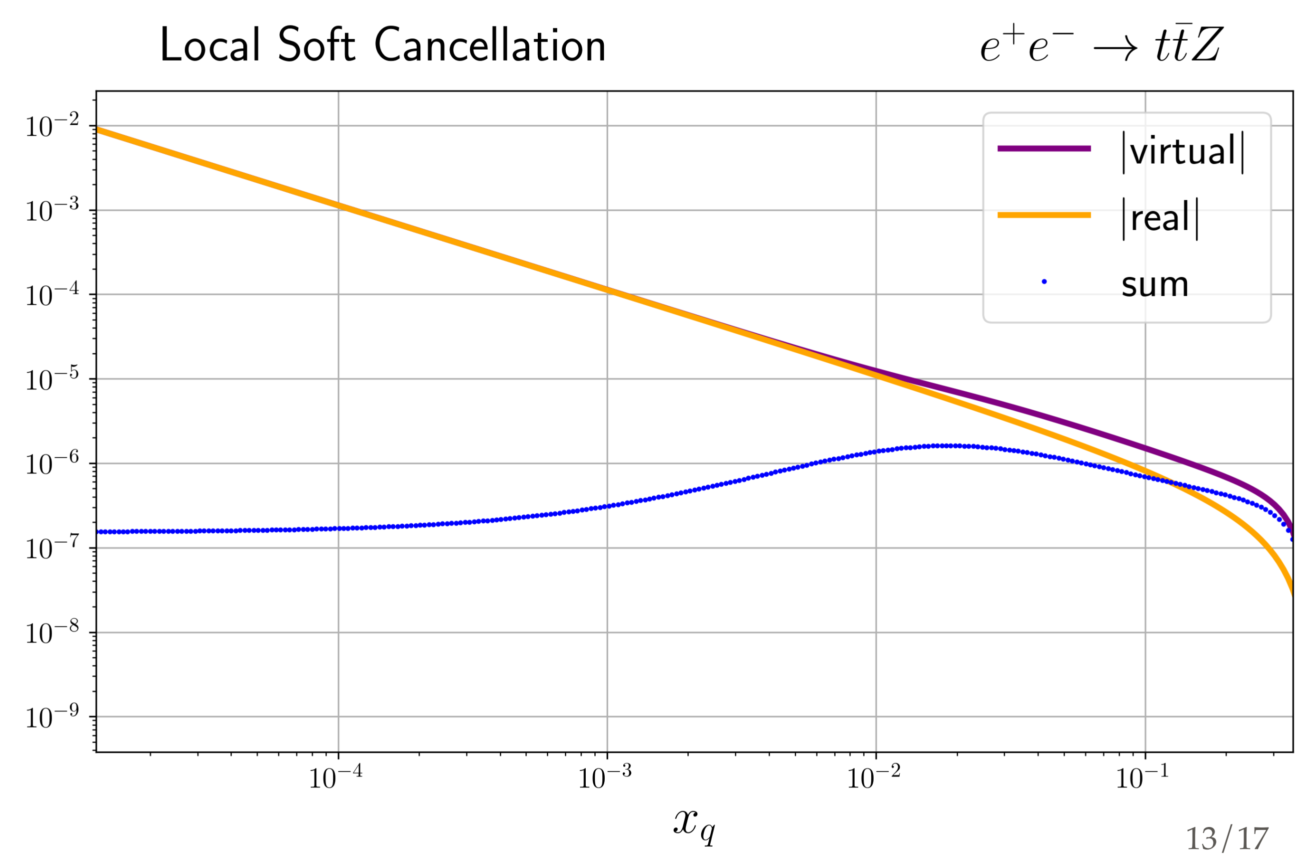
[Becker, et al 1010.4187]

- Massive dipole mapping

[Catani, et al. 0201036]





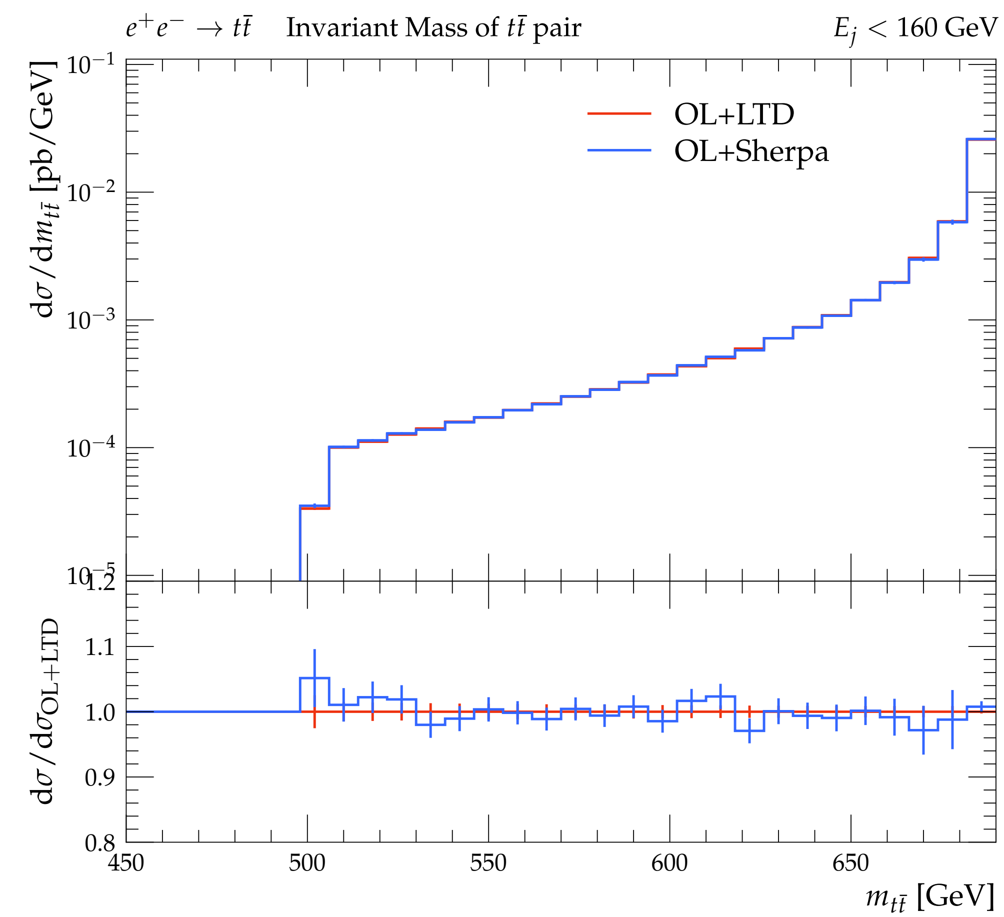


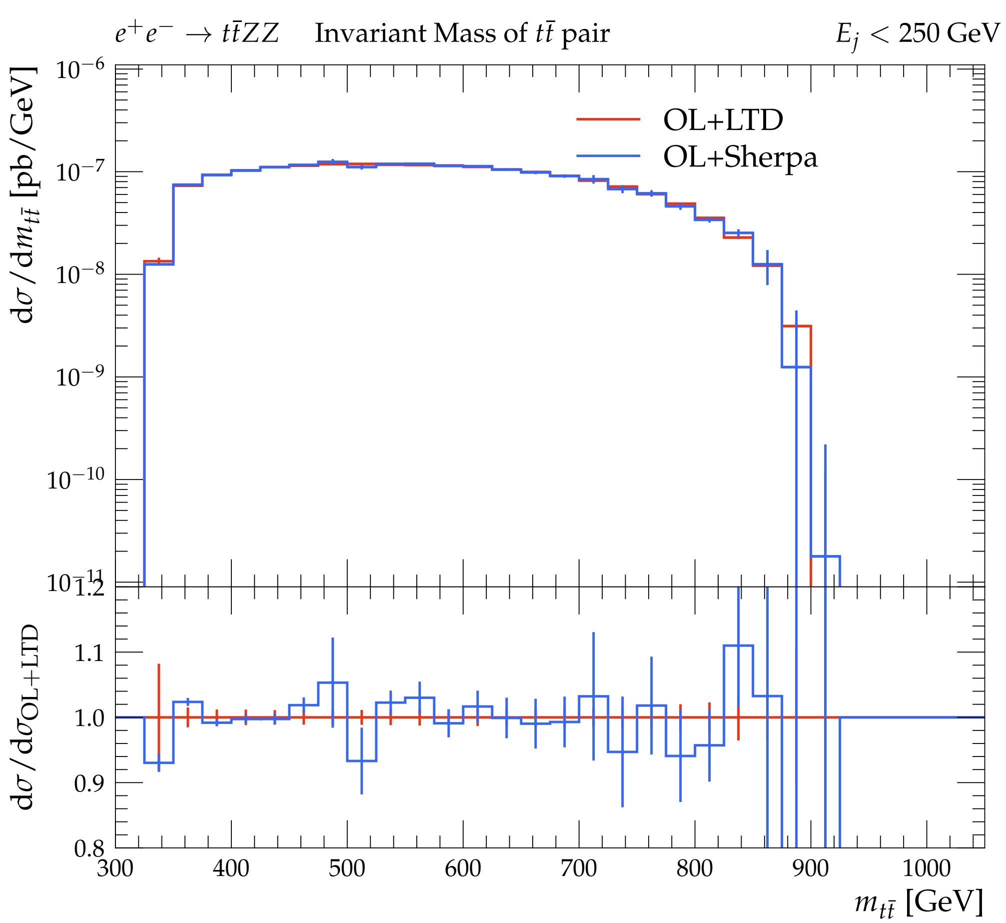
OL+LTD at work

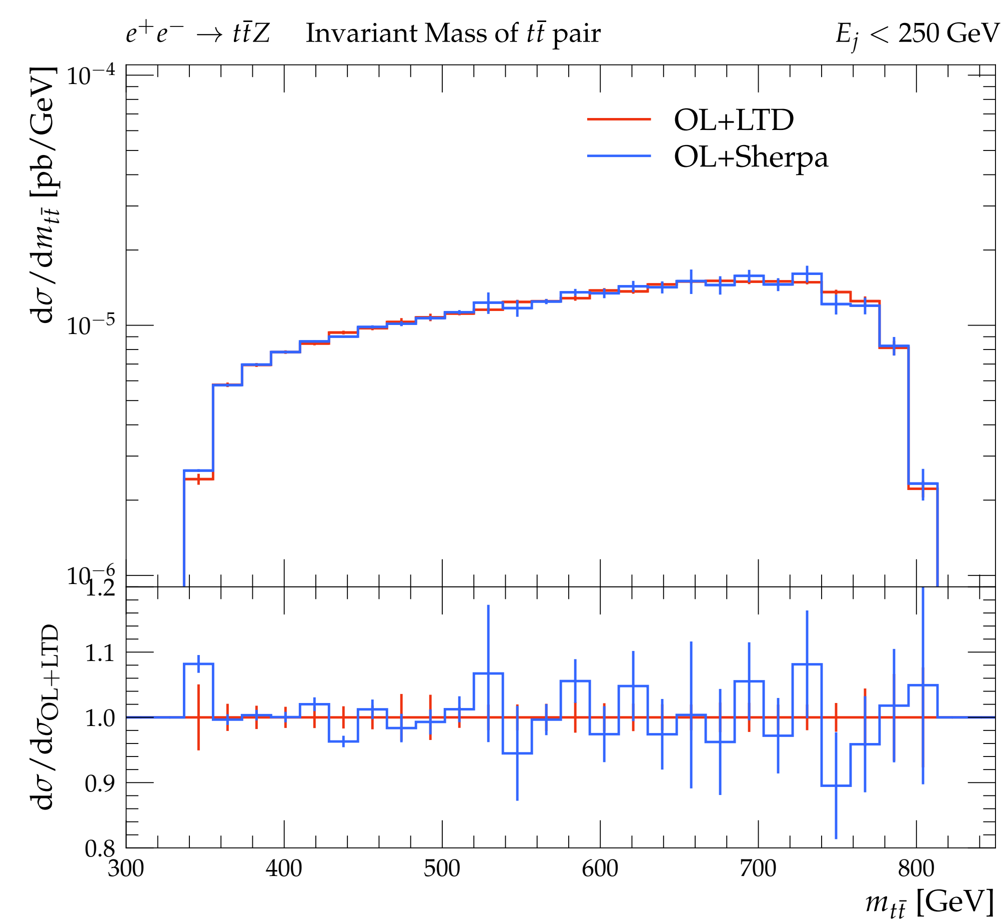
- Processes with **massive** external partons

$$e^+e^- \to t\bar{t} + nZ \quad \text{with} \quad n = 0, 1, 2$$

  - **OL+LTD**: 1M events with $n_q = 1$
  - **OL+Sherpa**: 1M events *[see details in SP's talk]*







OL+LTD at work

- Processes with **massive & massless** external partons

soft + one collinear singularity

$$e^+\nu_e \to t\bar{b}$$

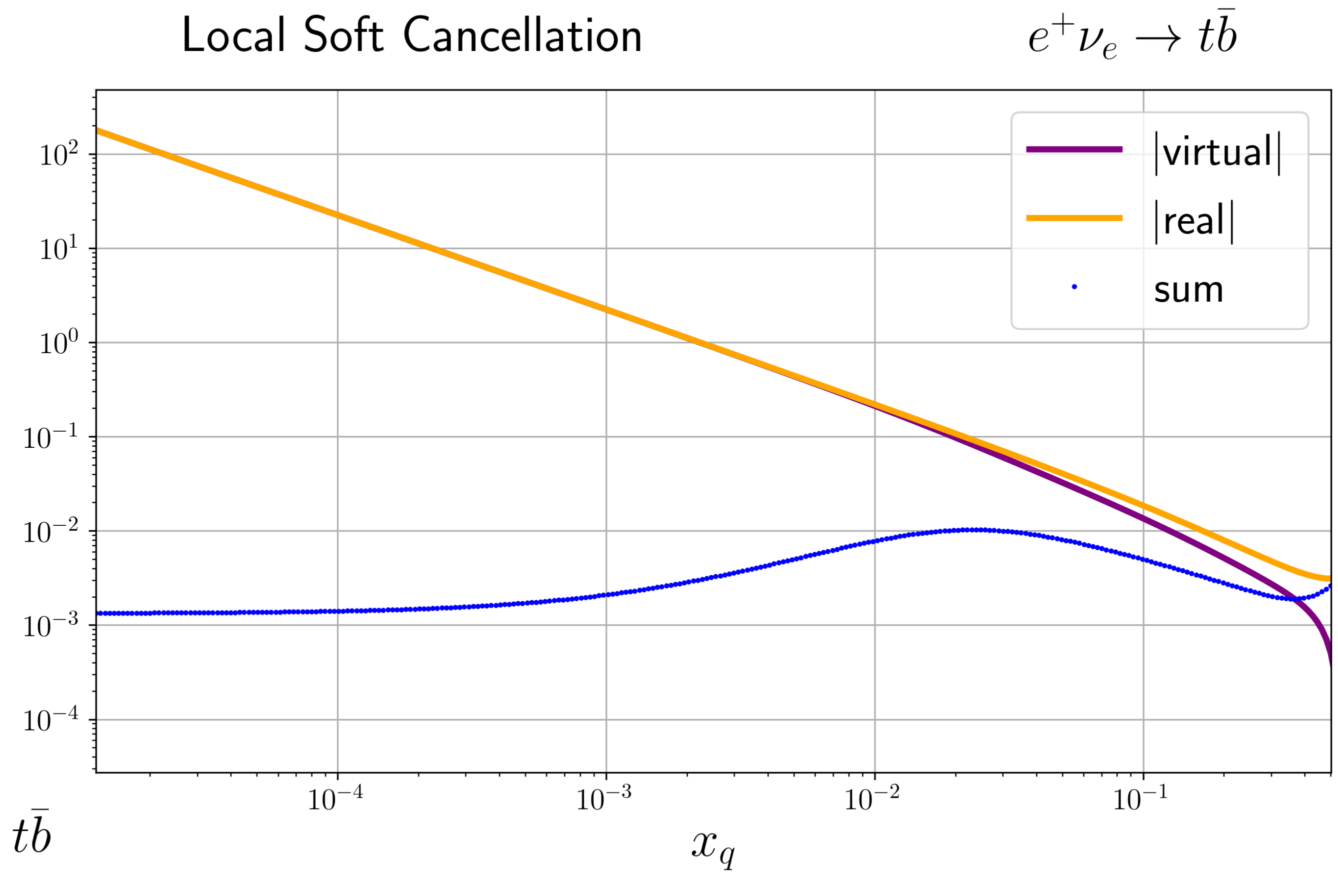
* Setup:
  - Tangent **q**-parametrisation

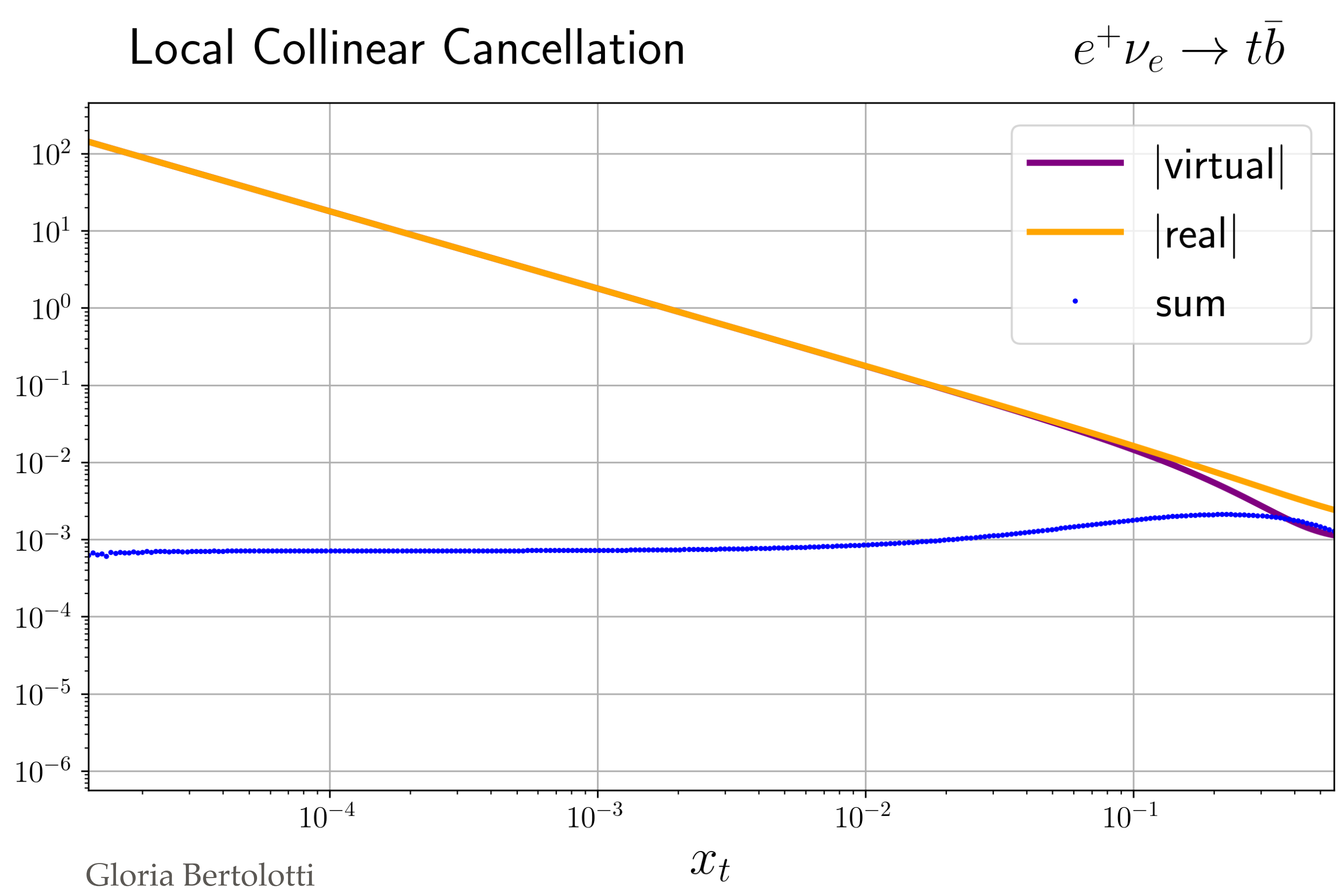
$$|\mathbf{q}| = \mu \tan\left(\frac{\pi}{2}\, x_q\right)$$

[Becker, et al 1010.4187]

  - Massive dipole mapping

[Catani, et al. 0201036]





- Angular parametrisation

$$x_t \sim \sqrt{1 - \cos(\theta_{g\bar{b}})}$$

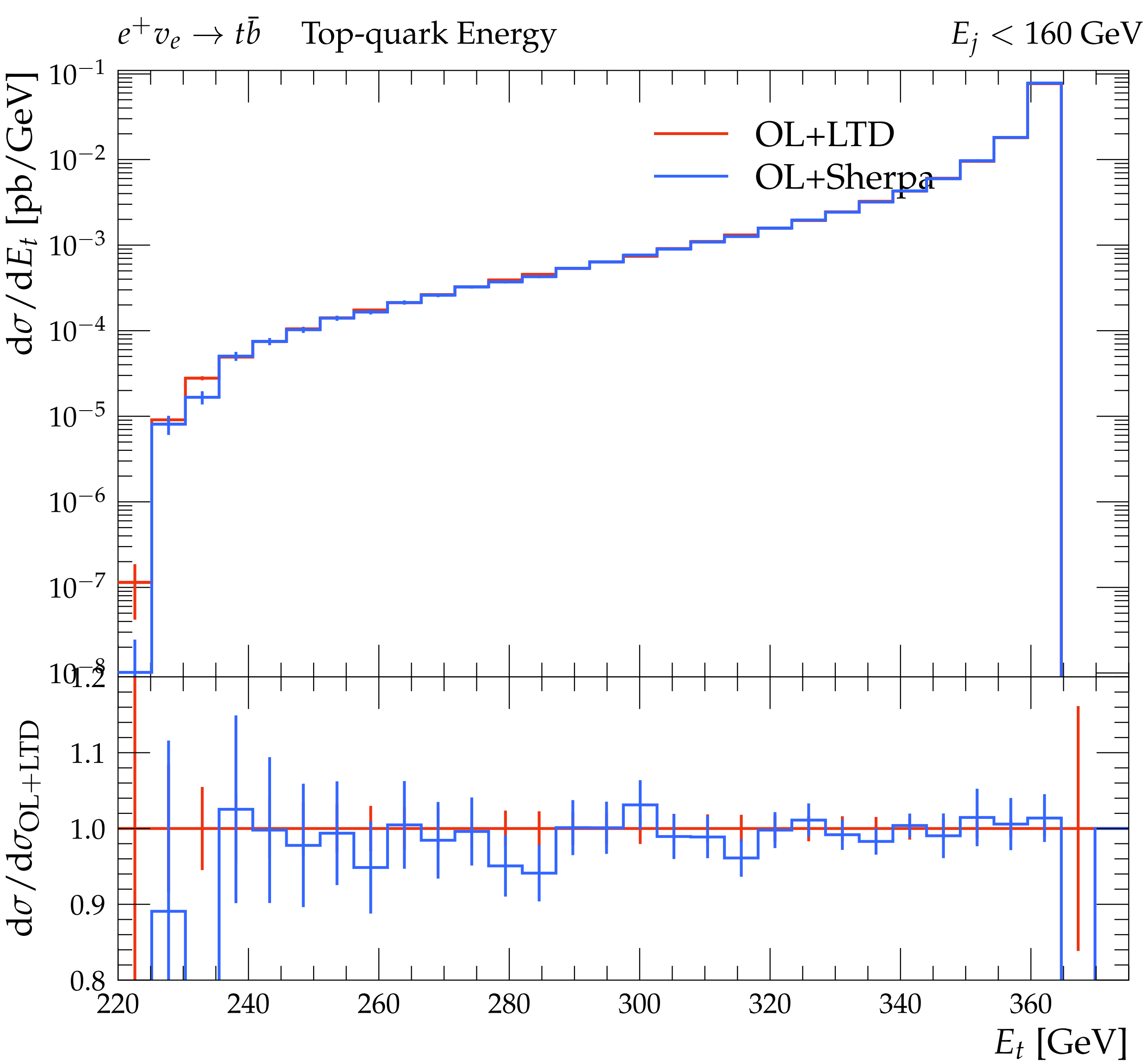
OL+LTD at work

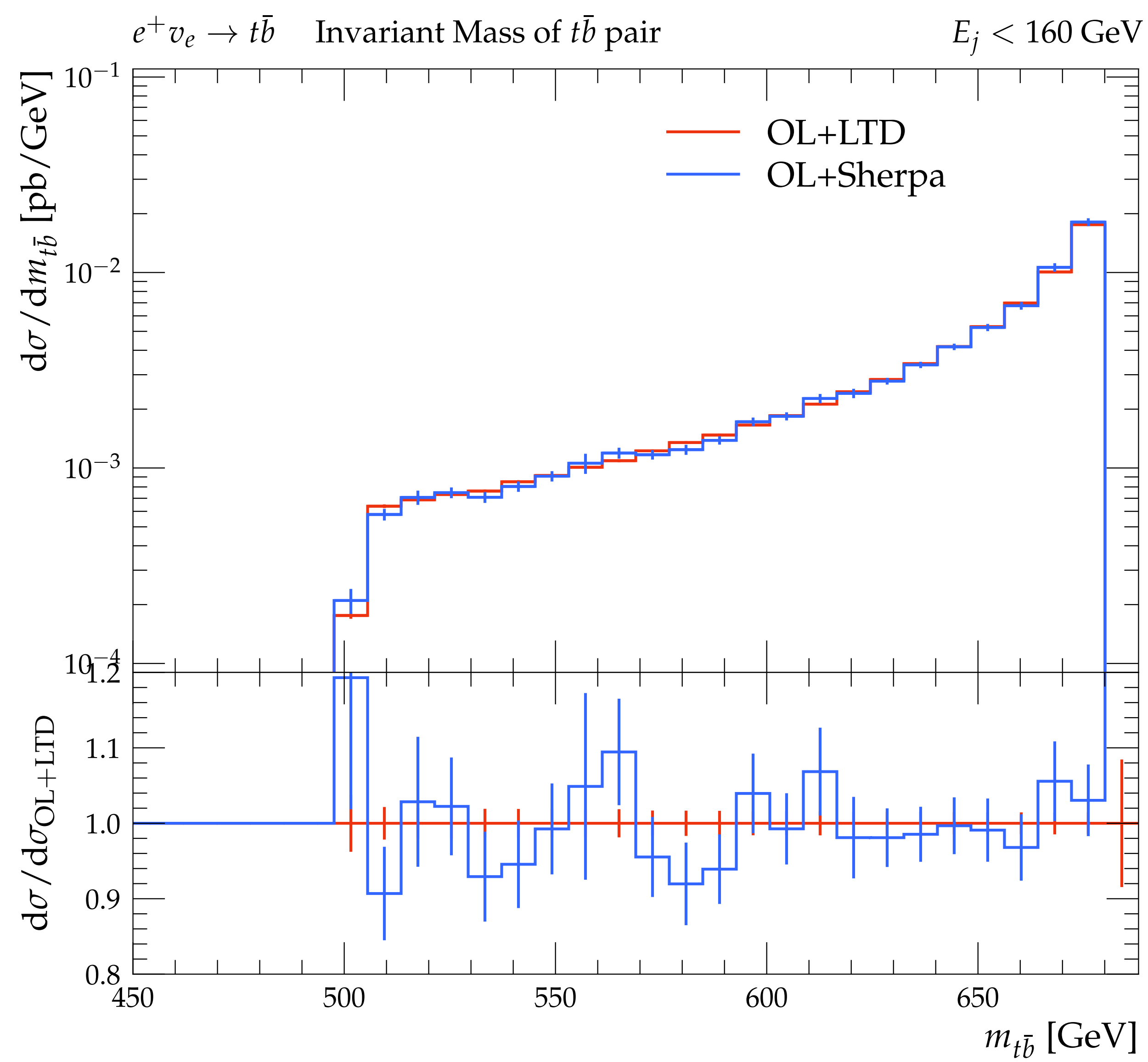
- Processes with **massive & massless** external partons

soft + one collinear singularity

$$e^+\nu_e \to t\bar{b}$$

- **OL+LTD**: 1M events with $n_q = 1$
- **OL+Sherpa**: 1M events *[see details in SP's talk]*





Conclusions & Outlook

- First steps towards a **new event generator** based on **OpenLoops+LTD**
- New **automated algorithm** for local cancellation of IR singularities (NLO FSR)
- **Promising preliminary results** at inclusive and differential level

What's next?

- Almost ready for application to processes with n massless final-state partons
- Include the treatment of initial-state collinear radiation
- Still wide room for optimisations at various stages of the procedure
- Eager to **uncover the full potentiality** of this approach!

Conclusions & Outlook

- First steps towards a **new event generator** based on **OpenLoops+LTD**
- New **automated algorithm** for local cancellation of IR singularities (NLO FSR)
- **Promising preliminary results** at inclusive and differential level

What's next?

- Almost ready for application to processes with n massless final-state partons
- Include the treatment of initial-state collinear radiation
- Still wide room for optimisations at various stages of the procedure
- Eager to **uncover the full potentiality** of this approach!

Thanks for your attention!