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NNLO corrections with local subtractions using NNLOCAL

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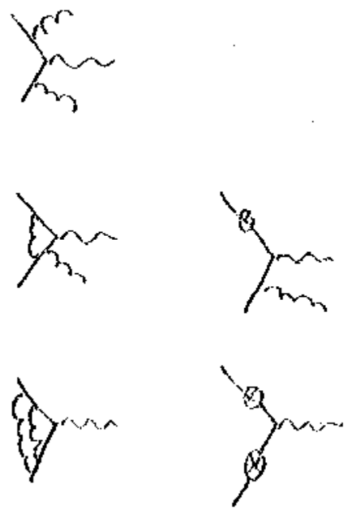
- NNLO cross section formula

$$\sigma(p_A, p_B) = \sum_{a,b} \int_0^1 \mathrm{d}\eta_1 f_{a/A}(\eta_1, \mu_F^2) \int_0^1 \mathrm{d}\eta_2 f_{b/B}(\eta_2, \mu_F^2) \Big[\sigma_{ab}^{\mathrm{LO}}(p_1, p_2; \mu_F^2) + \sigma_{ab}^{\mathrm{NLO}}(p_1, p_2; \mu_F^2) + \sigma_{ab}^{\mathrm{NNLO}}(p_1, p_2; \mu_F^2) + \ldots \Big],$$

- NNLO cross section formula (for an explicit observable *J*, removing subs *ab*)

$$\sigma^{NNLO}[J] = \int_{m+2} d\sigma^{RR}_{m+2} J_{m+2} + \int_{m+1} [d\sigma^{RV}_{m+1} + d\sigma^{C_1}] J_{m+1} + \int_{m+2} [d\sigma^{VV}_{m} + d\sigma^{C_2}] J_m$$



	Div: Loop	Div: PS	
$d\sigma^{RR}_{m+2}$	0	2	∞
$d\sigma^{RV}_{m+1} + d\sigma^{C_1}$	1	1	∞
$d\sigma^{VV}_{m} + d\sigma^{C_2}$	2	0	∞

- Problem conceptually solvable with local subtraction method (Ellis, Ross, Terrano 1980)

- NNLO cross section formula

$$\sigma(p_A,p_B) = \sum_{a,b}\int_0^1 \mathrm{d}\eta_1 f_{a/A}(\eta_1,\mu_F^2)\int_0^1 \mathrm{d}\eta_2 f_{b/B}(\eta_2,\mu_F^2)\Big[\sigma_{ab}^{\mathrm{LO}}(p_1,p_2;\mu_F^2)+\sigma_{ab}^{\mathrm{NLO}}(p_1,p_2;\mu_F^2) + \sigma_{ab}^{\mathrm{NNLO}}(p_1,p_2;\mu_F^2)+\ldots\Big],$$

- NNLO cross section formula with subtractions
  (for a given observable J and removing subs *ab*)

$$\sigma^{NNLO}[J] = \int_{m+2}\Big\{\Big[d\sigma_{m+2}^{RR}J_{m+2} - d\sigma_{m+2}^{RR,A_2}J_m\Big] - \Big[d\sigma_{m+2}^{RR,A_1}J_{m+1} - d\sigma_{m+2}^{RR,A_{12}}J_m\Big]\Big\}$$

$$+\int_{m+1}\Bigg\{\Big[d\sigma_{m+1}^{RV} + d\sigma_{m+1}^{C_1} + \int_1 d\sigma_{m+2}^{RR,A_1}\Big]J_{m+1} - \Bigg[d\sigma_{m+1}^{RV,A_1} + d\sigma_{m+1}^{C_1,A_1} + \Big(\int_1 d\sigma_{m+2}^{RR,A_1}\Big)^{A_1}\Bigg]J_m\Bigg\}$$

$$+\int_m\Bigg\{d\sigma_m^{VV} + d\sigma_m^{C_2} + \int_2\Big[d\sigma_{m+2}^{RR,A_2} - d\sigma_{m+2}^{RR,A_{12}}\Big] + \int_1\Bigg[d\sigma_{m+1}^{RV,A_1} + d\sigma_{m+1}^{C_1,A_1} + \Big(\int_1 d\sigma_{m+2}^{RR,A_1}\Big)^{A_1}\Bigg]\Bigg\}J_m$$

Role of NNLO counterterms

$$\left\{\begin{array}{c} d\sigma^{RR,A_2} \\[2em] d\sigma^{RR,A_1} \\[2em] d\sigma^{RR,A_{12}} \\[2em] d\sigma^{RV,A_1} \\[2em] d\sigma^{C_1,A_1} \\[2em] \left(\int_1 d\sigma^{RR,A_1}\right)^{A_1} \end{array}\right\} \begin{array}{c}\text{capture}\\ \text{singular}\\ \text{behavior of}\end{array} \left\{\begin{array}{c} d\sigma^{RR} \\[2em] d\sigma^{RR} \\[2em] d\sigma^{RR,A_2},\ d\sigma^{RR,A_1} \\[2em] d\sigma^{RV} \\[2em] d\sigma^{C_1} \\[2em] \int_1 d\sigma^{RR,A_1} \end{array}\right\} \text{as} \left\{\begin{array}{c} 2 \\[2em] 1 \\[2em] 1,\ 2 \\[2em] 1 \\[2em] 1 \\[2em] 1 \end{array}\right\} \begin{array}{c}\text{partons}\\ \text{become}\\ \text{unresolved}\end{array}$$

- Basic idea (Del Duca, Somogyi, Trocsanyi 2005, 2007): build the auxiliary cross sections at NNLO using well known limiting behaviors of the amplitudes

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- Catani, Grazzini 1999

$$
\begin{aligned}
\hat{P}^{\mu\nu}_{g_1 g_2 g_3} = \; & C_A^2 \left\{ \frac{(1-\epsilon)}{4 s_{12}^2} \left[-g^{\mu\nu} t_{12,3}^2 + 16 s_{123} \frac{z_1^2 z_2^2}{z_3(1-z_3)} \left(\frac{\widetilde{k}_2}{z_2} - \frac{\widetilde{k}_1}{z_1} \right)^{\mu} \left(\frac{\widetilde{k}_2}{z_2} - \frac{\widetilde{k}_1}{z_1} \right)^{\nu} \right] \right. \\
& - \frac{3}{4}(1-\epsilon) g^{\mu\nu} + \frac{s_{123}}{s_{12}} g^{\mu\nu} \frac{1}{z_3} \left[\frac{2(1-z_3) + 4z_3^2}{1-z_3} - \frac{1 - 2z_3(1-z_3)}{z_1(1-z_1)} \right] \\
& + \frac{s_{123}(1-\epsilon)}{s_{12} s_{13}} \left[2 z_1 \left(\widetilde{k}_2^{\mu} \widetilde{k}_2^{\nu} \frac{1-2z_3}{z_3(1-z_3)} + \widetilde{k}_3^{\mu} \widetilde{k}_3^{\nu} \frac{1-2z_2}{z_2(1-z_2)} \right) \right. \\
& + \frac{s_{123}}{2(1-\epsilon)} g^{\mu\nu} \left(\frac{4 z_2 z_3 + 2 z_1 (1-z_1) - 1}{(1-z_2)(1-z_3)} - \frac{1 - 2z_1(1-z_1)}{z_2 z_3} \right) \\
& \left. \left. + \left(\widetilde{k}_2^{\mu} \widetilde{k}_3^{\nu} + \widetilde{k}_3^{\mu} \widetilde{k}_2^{\nu} \right) \left(\frac{2 z_2 (1-z_2)}{z_3(1-z_3)} - 3 \right) \right] \right\} + (5 \text{ permutations}) \; .
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{S}_{ij}(q_1,q_2) = \; & \frac{(1-\epsilon)}{(q_1 \cdot q_2)^2} \frac{p_i \cdot q_1 \, p_j \cdot q_2 + p_i \cdot q_2 \, p_j \cdot q_1}{p_i \cdot (q_1+q_2) \, p_j \cdot (q_1+q_2)} \\
& - \frac{(p_i \cdot p_j)^2}{2 p_i \cdot q_1 \, p_j \cdot q_2 \, p_i \cdot q_2 \, p_j \cdot q_1} \left[2 - \frac{p_i \cdot q_1 \, p_j \cdot q_2 + p_i \cdot q_2 \, p_j \cdot q_1}{p_i \cdot (q_1+q_2) \, p_j \cdot (q_1+q_2)} \right] \\
& + \frac{p_i \cdot p_j}{2 q_1 \cdot q_2} \left[\frac{2}{p_i \cdot q_1 \, p_j \cdot q_2} + \frac{2}{p_j \cdot q_1 \, p_i \cdot q_2} \right. \\
& \left. - \frac{1}{p_i \cdot (q_1+q_2) \, p_j \cdot (q_1+q_2)} \left(4 + \frac{(p_i \cdot q_1 \, p_j \cdot q_2 + p_i \cdot q_2 \, p_j \cdot q_1)^2}{p_i \cdot q_1 \, p_j \cdot q_2 \, p_i \cdot q_2 \, p_j \cdot q_1} \right) \right]
\end{aligned}
$$

$$
\begin{aligned}
& |\mathcal{M}_{g,g,a_1,\ldots,a_n}(q_1,q_2,p_1,\ldots,p_n)|^2 \simeq \left(4\pi\alpha_S \mu^{2\epsilon}\right)^2 \\
& \cdot \left[\frac{1}{2} \sum_{i,j,k,l=1}^{n} \mathcal{S}_{ij}(q_1) \, \mathcal{S}_{kl}(q_2) \, |\mathcal{M}^{(i,j)(k,l)}_{a_1,\ldots,a_n}(p_1,\ldots,p_n)|^2 - C_A \sum_{i,j=1}^{n} \mathcal{S}_{ij}(q_1,q_2) |\mathcal{M}^{(i,j)}_{a_1,\ldots,a_n}|^2 \right]
\end{aligned}
$$

- Basic idea (Del Duca, Somogyi, Tricsanyi 2005, 2007): build the auxiliary cross sections at NNLO using well known limiting behaviors of the amplitudes

- Catani, Grazzini 1999

- By using general (universal) formulae to build a set of subtractions there are two caveats:

1. internal overlap in A1 and in A2 (for A1 that is the single unresolved: C + S - CS).
   By construction taking the soft limits first provides a consistent picture

2. external overlap among A1 and A2 singularities. Cured by the A12 terms
   General rule: start from the most unresolved (A2) limit to obtain iterated limits (A12)

- Internal and external cancellations impose constraints that simplify the construction, but still a lot of freedom remains.

- It is possible to exploit this freedom to push as much as possible the analytic integration of the counterterms.
  Having an analytic formula could in principle provides some advantages like an easy way to check pole cancellation

- Freedom in the choice of the counterterms:

  1. Convenient choice of momentum fractions can be made upon examination of the explicit form of the collinear integral without affecting the structure of singularities

  2. We multiply the counterterms by appropriate functions (that go to 1 in the unresolved limit) chosen in such a way that they cancel regular factors

- Reverse unitarity and IBP (very fast) used to reduce the most complicated counterterms (double and triple collinear)

- $gg \to Hgg$ as RR for $gg \to H$ representative of the complexity

Cancellation of singularities in RR

	Types	Terms	Counter events	Master integrals
A1	5	13	7	11
A2	5(11)	9(29)	5	42
A12	13(23)	39(81)	17	104
Total	**23**	**61**	**29**	**157**

- $gg \to Hg$ as RV for $gg \to H$ representative of the complexity

Cancellation of singularities in RV

	Types	Terms	Counter events	Master integrals
A1A1	5	13	2	65
A1RV	2	3	2	24
A1C1	3	6	2	10
Total	**10**	**22**	~~**6**~~ **2**	**99**

For a total of 256 integrals

- All integrals solved analytically up to the required order in the dimensional parameter

  - ✓ Most integrals solved by directly integrating a suitable phase space parametrization

  - ✓ A few were already available in the literature (found in papers on the Antenna method)

  - ✓ Result is at most weight two GPL (some square root, and also sqrt of sqrt coming from quartic polynomials)

- Distributional expansion challenging because of overlap: 1/[(1-xa)(1-xb)(1-xa*xb)] solution:
  - go back to the integral version and use expansion by regions (asy.m) to compute parametric limiting behaviour of the integral when one or both variables go to 1
  - build subtractions in pretty much the same way we built A1, A2 and A12
  - A12 built from A2, but asy.m needed again because of xa/xb in the leftover integration
  - integrate all the terms and build the distribution

- All analytic formulas inserted in a fortran code called NNLOCAL
Infrastructure taken from MCFM (We have ensured that it is tolerated by the authors!)

- Support to check pole cancellation both kinematically and order by order in epsilon

- Support for running in parallel if needed (execution via a shell script, new vegas grids built combining the output from the previous run) and to visualize vegas grids

- There are regions in the middle of phase space (in the xa and xb square) that are not especially privileged but nevertheless require higher precision because of numerical cancellation

- As a an example of application of our code we considered gluon fusion Higgs production in EFT with only gluons (n_f is non zero only in the running of α_s)

- Comparison at inclusive level with N3LOXS (by Baglio, Duhr, Mistlberger, Szafron) (not the default N3LOXS, we exported our definition of α_s and extracted the all glue results)

❖ Table for the inclusive result for different values of the Higgs boson mass (~2 CPU hours)

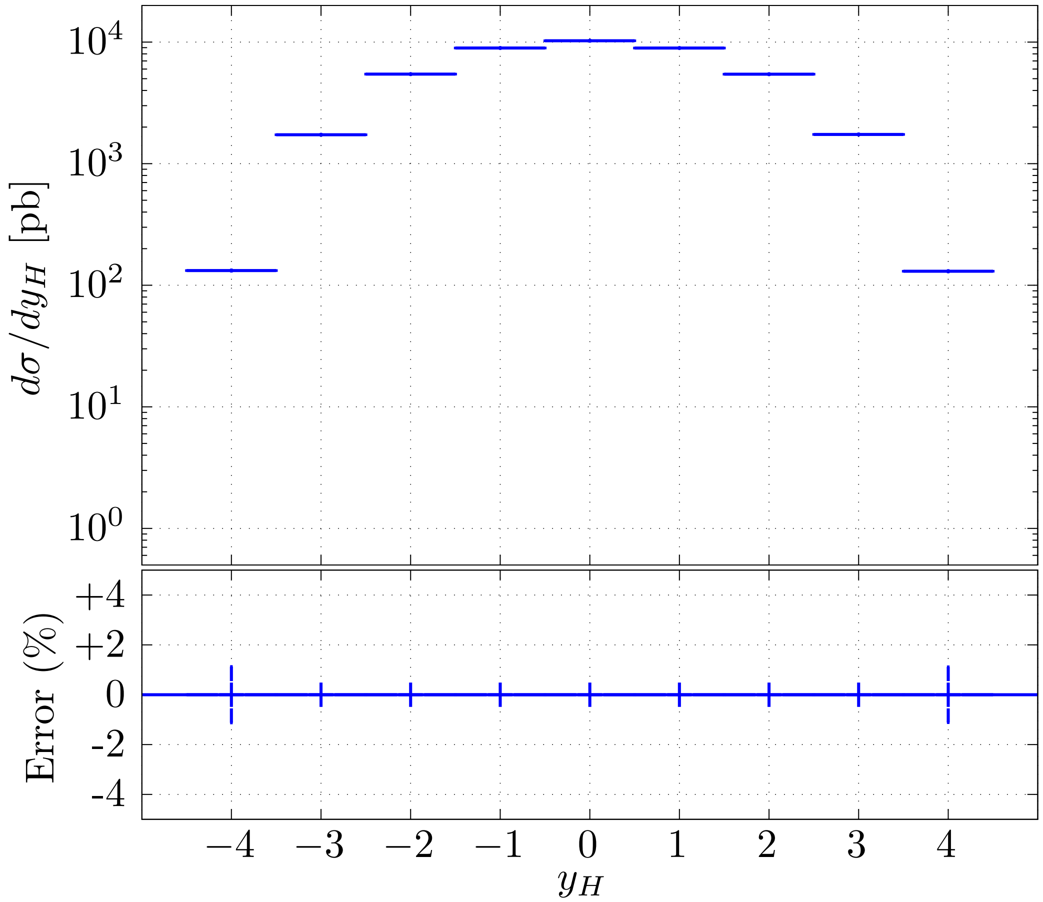
$gg \to H$ LHC@13TeV Preliminary

M_H (GeV)	N3LOXS(gg)	NNLOCAL(gg)
125	**42.934pb**	**42.84 +/- 0.08pb**
250	9.7290pb	9.717 +/- 0.017pb
500	1.6253pb	1.622 +/- 0.003pb
1000	173.59fb	173.5 +/- 0.3fb
2000	8.7835fb	8.781 +/- 0.017fb

❖ Rapidity distribution of the Higgs boson ($m_H = 125\,\text{GeV}$, ~200 CPU hours)

$$\Delta y_H = 1.0$$

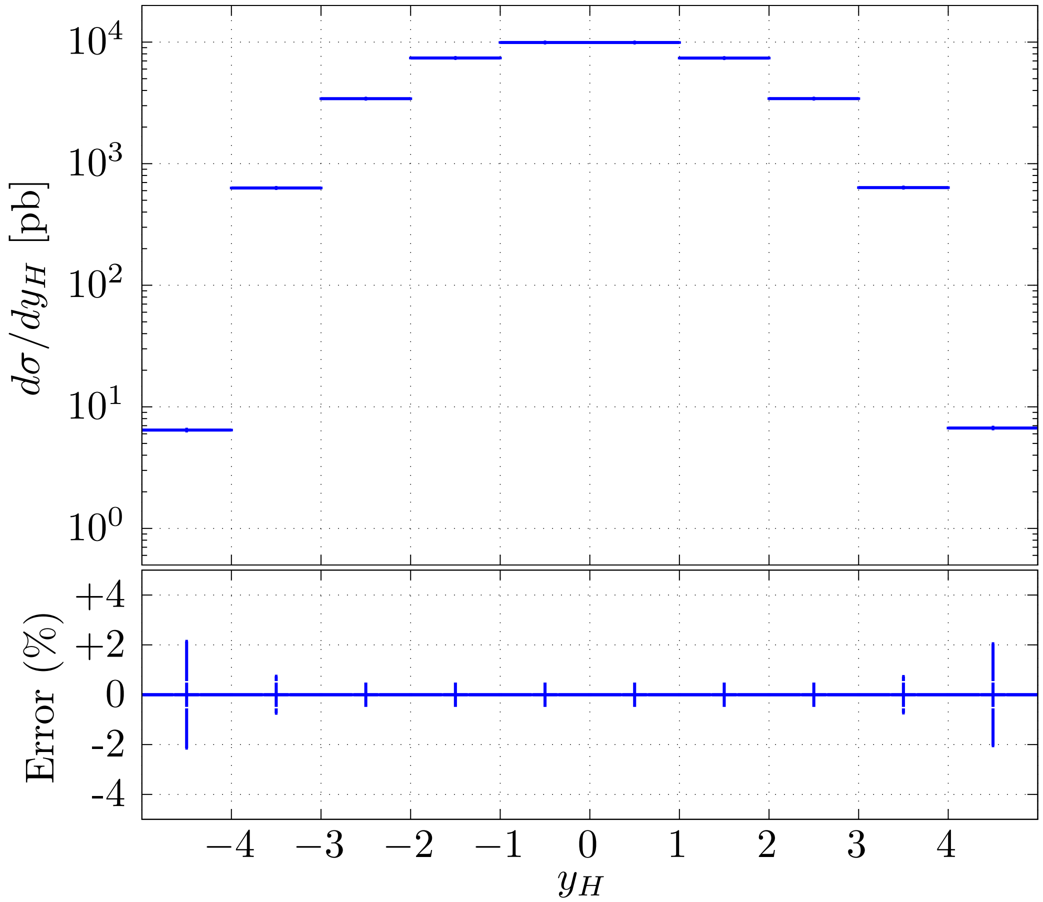
Preliminary



❖ Rapidity distribution of the Higgs boson (m_H = 125 GeV, ~200 CPU hours)

$\Delta y_H = 1.0$

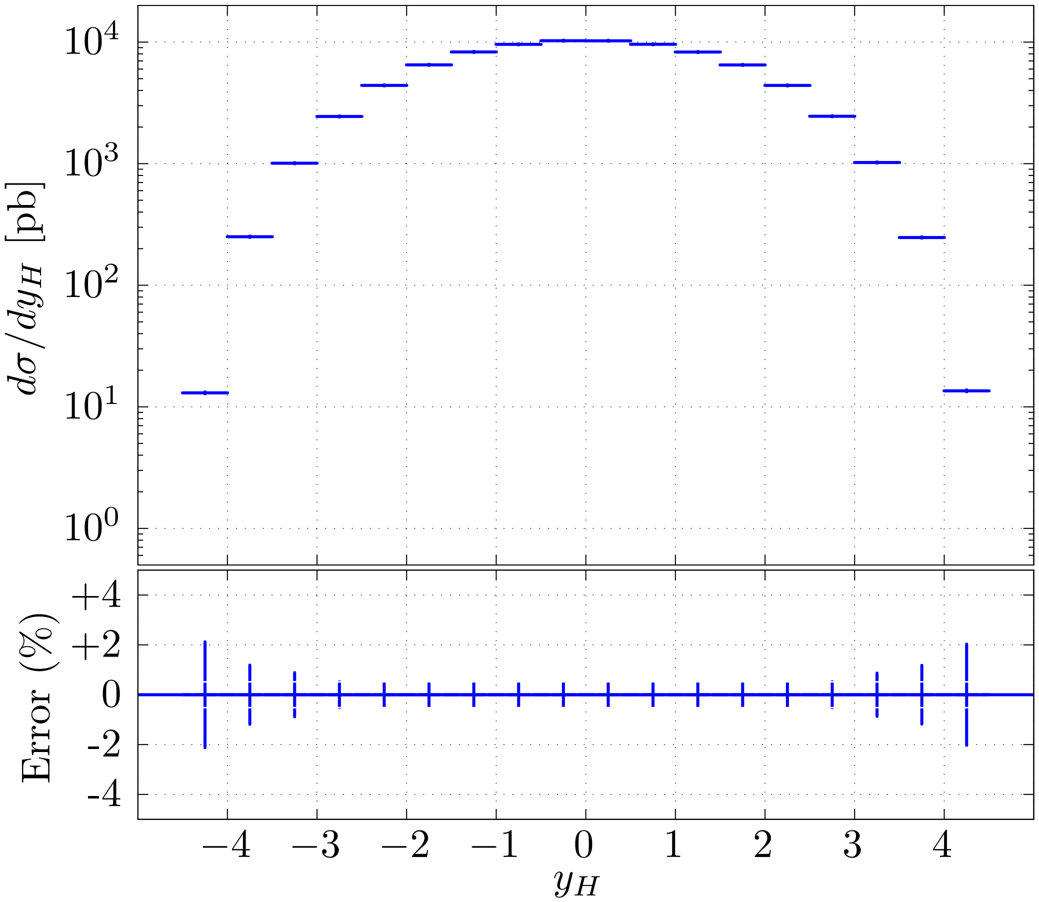
Preliminary



❖ Rapidity distribution of the Higgs boson ($m_H = 125$ GeV, ~200 CPU hours)

$$\Delta y_H = 0.5$$

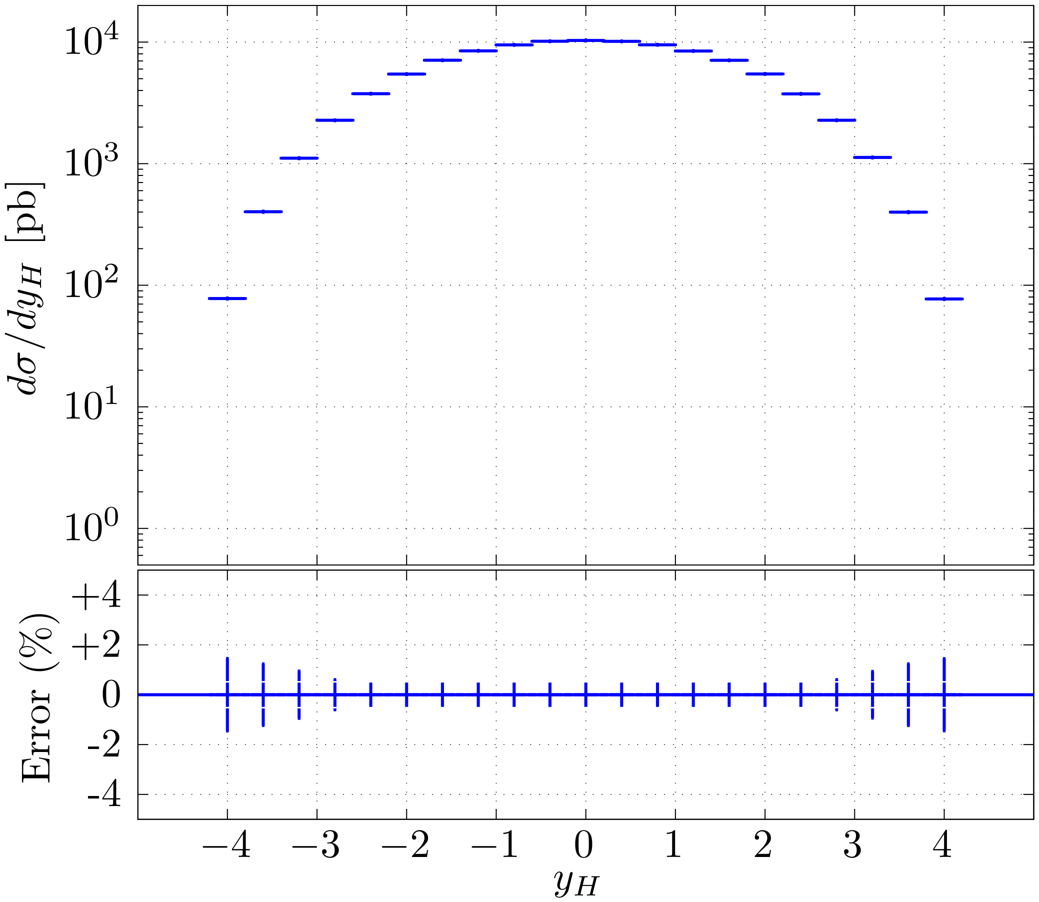
Preliminary



❖ Rapidity distribution of the Higgs boson ($m_H = 125\,\mathrm{GeV}$, ~200 CPU hours)

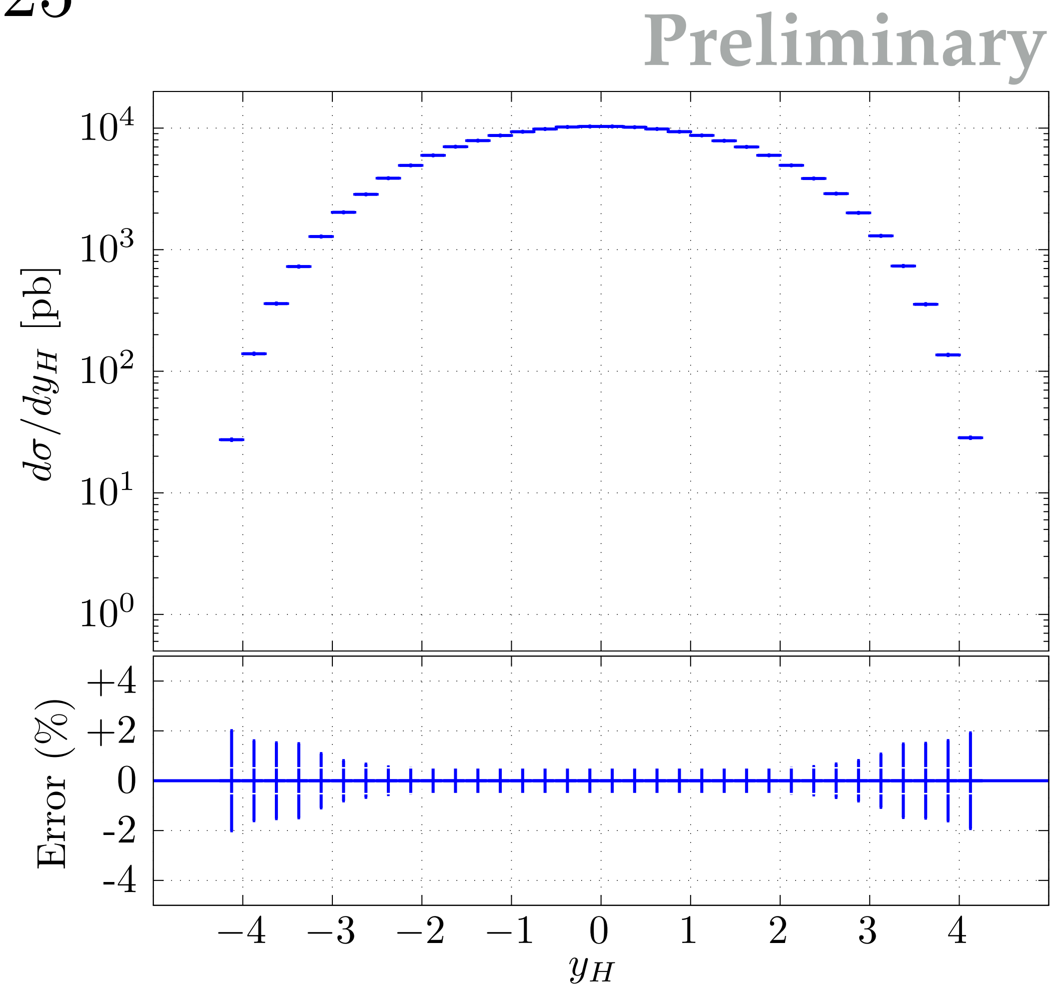
$$\Delta y_H = 0.4$$

Preliminary



- Rapidity distribution of the Higgs boson ($m_H = 125\,\mathrm{GeV}$, ~200 CPU hours)

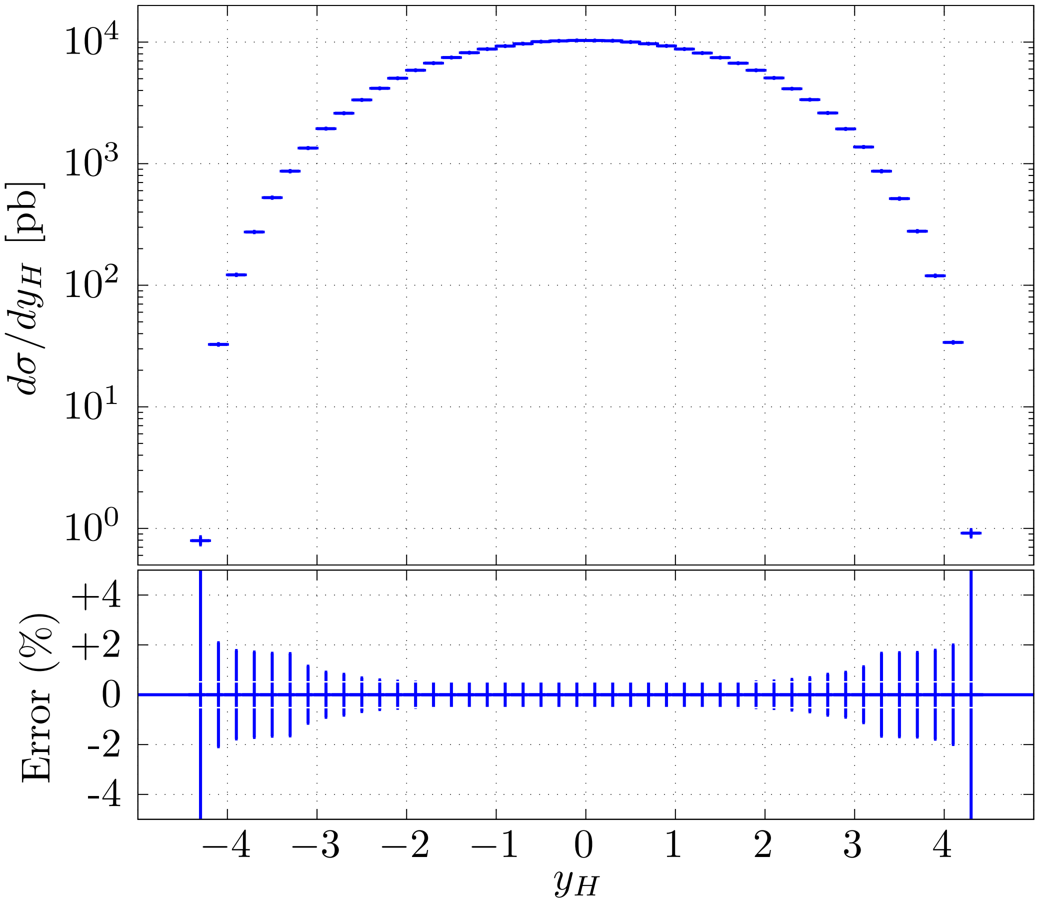
$$\Delta y_H = 0.25$$



- Rapidity distribution of the Higgs boson ($m_H = 125\,\mathrm{GeV}$, ~200 CPU hours)

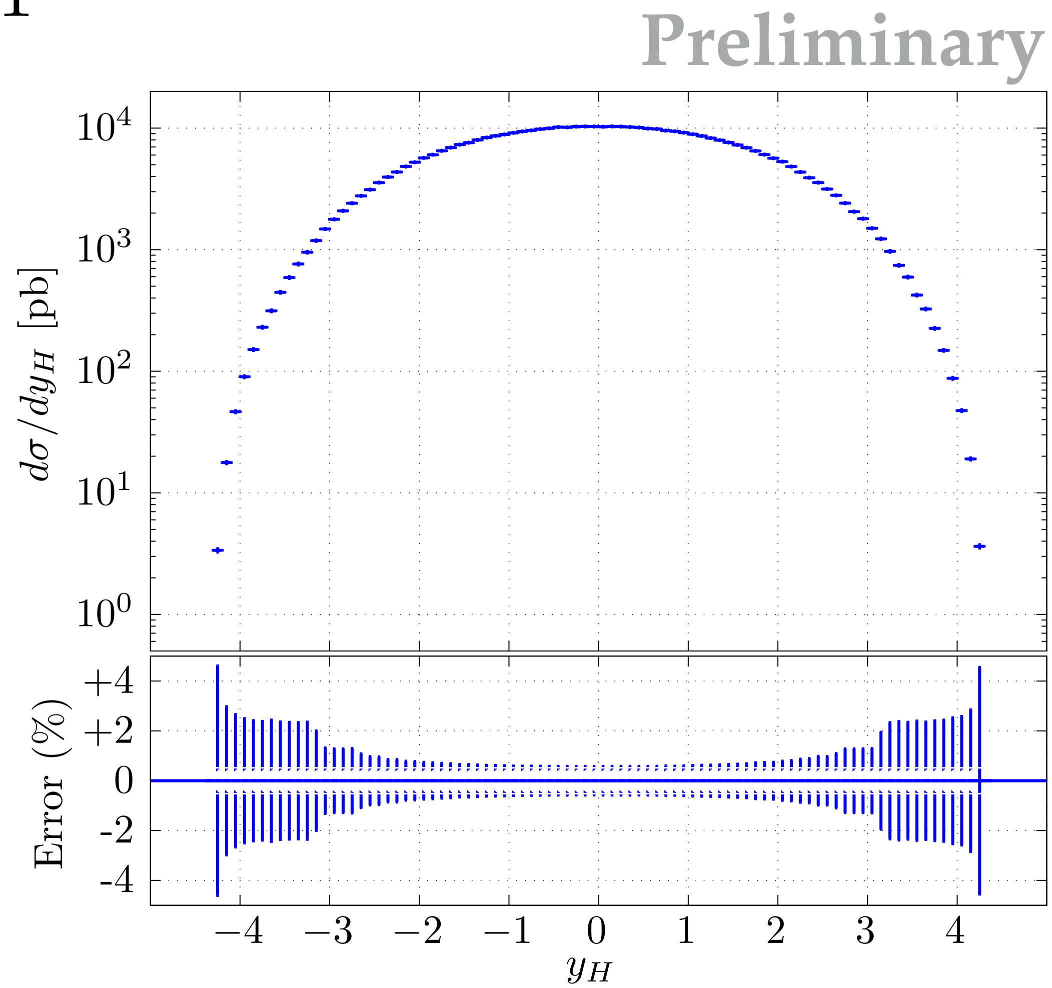
$$\Delta y_H = 0.2$$

Preliminary



❖ Rapidity distribution of the Higgs boson ($m_H = 125$ GeV, ~200 CPU hours)

$$\Delta y_H = 0.1$$



- Suitable procedures also for GPU
  (for high precision on GPU see for example https://github.com/lumianph/gpuprec)

- Extensions to the next order conceivable

$$d\sigma^{RRR} - A_1^{RRR} - A_2^{RRR} - A_3^{RRR} + A_{12}^{RRR} + A_{13}^{RRR} + A_{23}^{RRR} - A_{123}^{RRR}$$

$$+d\sigma^{RRV} - A_1^{RRV} - A_2^{RRV} + A_{12}^{RRV} + \int_1 A_1^{RRR} - \left(\int_1 A_1^{RRR}\right)^{A_1} - \left(\int_1 A_1^{RRR}\right)^{A_2} + \left(\int_1 A_1^{RRR}\right)^{A_{12}}$$

$$+d\sigma^{RVV} - A_1^{RVV} + \int_1 A_1^{RRV} + \int_2 A_2^{RRR} - \int_2 A_{12}^{RRR} - \left(\int_1 A_1^{RRV}\right)^{A_1}$$

$$- \left(\int_2 A_2^{RRR}\right)^{A_1} + \left(\int_2 A_{12}^{RRR}\right)^{A_1} + \int_1 \left(\int_1 A_1^{RRR}\right)^{A_1} - \left(\int_1 \left(\int_1 A_1^{RRR}\right)^{A_1}\right)^{A_1}$$

$$+d\sigma^{VVV} + \int_1 A_1^{RVV} + \int_2 A_2^{RRV} - \int_2 A_{12}^{RRV} + \int_3 A_3^{RRR} - \int_3 A_{13}^{RRR} - \int_3 A_{23}^{RRR}$$

$$+ \int_3 A_{123}^{RRR} + \int_1 \left(\int_1 A_1^{RRV}\right)^{A_1} + \int_1 \left(\int_2 A_2^{RRR}\right)^{A_1} - \int_1 \left(\int_2 A_{12}^{RRR}\right)^{A_1}$$

$$+ \int_2 \left(\int_1 A_1^{RRR}\right)^{A_2} - \int_2 \left(\int_1 A_1^{RRR}\right)^{A_{12}} + \int_1 \left(\int_1 \left(\int_1 A_1^{RRR}\right)^{A_1}\right)^{A_1}$$

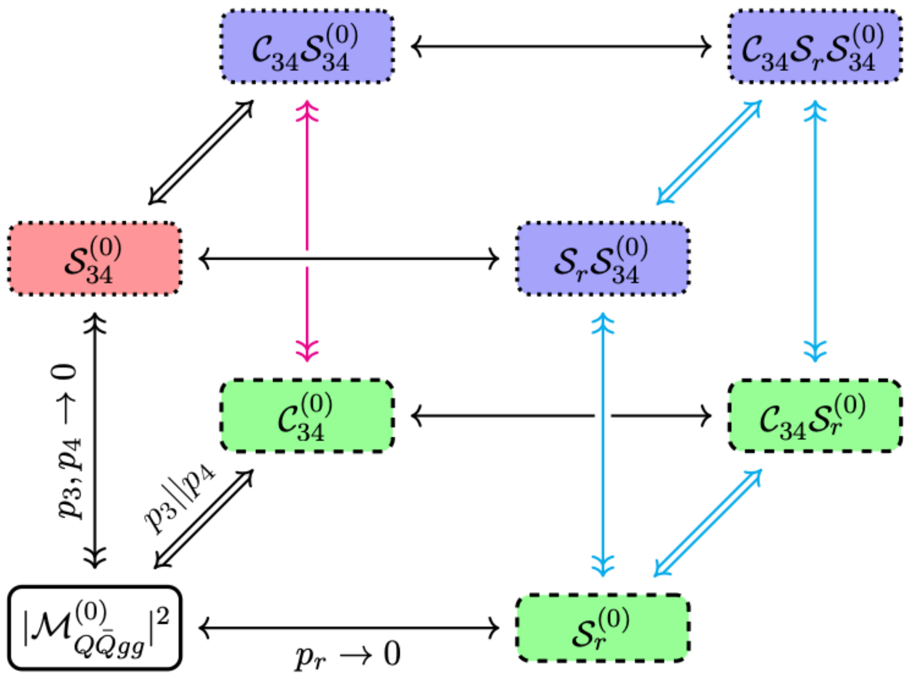
❖ valid approach also when there are massive particles in the final state

✓ As an example we consider the decay of a colour singlet initial state into a pair of heavy quarks (Somogyi, FT 2020)

✓ Double soft radiation is the only NNLO singular limit

✓ 2 counter events in total

RR



- Freedom in the choice of the counterterms

1. Convenient choice of momentum fractions can be made upon examination of the explicit form of the collinear integral without affecting the structure of singularities

Map:

$$\widehat{p}^{\,\mu}_{34} = 1/\beta(p_3^\mu + p_4^\mu - \alpha P^\mu)\,, \qquad \widehat{p}^{\,\mu}_{n} = \Lambda^\mu_\nu(\widehat{K}, K) p_n^\nu\,; \quad n = 1, 2$$

$$\alpha = \frac{1}{2}\left[y_{(34)P} - \sqrt{y^2_{(34)P} - 4y_{34}}\right] \quad \text{and} \quad \beta = \frac{\sqrt{y^2_{(34)P} - 4y_{34}}}{y_{(34)P} - y_{34}}$$

PS factorization:

$$[\mathrm{d}p] = x^{-1+2\epsilon}\frac{P^2}{2\pi}\int_{\alpha_{\min}}^{\alpha_{\max}} \mathrm{d}\alpha\,(1-\alpha)^{-3+2\epsilon}(x - 2\alpha + \alpha^2)^{2-2\epsilon}\mathrm{d}\phi_2(p_3, p_4; \alpha P + \beta\widehat{p}_{34})$$

$$\mathrm{d}\phi_2(p_3, p_4; \alpha P + \beta\widehat{p}_{34}) = \frac{1}{8\pi}\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}(P^2)^{-\epsilon}\mathrm{d}v\,\alpha^{-\epsilon}(\alpha + \beta x)^{-\epsilon}v^{-\epsilon}(1-v)^{-\epsilon}\Theta(v)\Theta(1-v)$$

$$x \equiv y_{\widehat{34}P} = \frac{2\widehat{p}_{34}\cdot P}{P^2} \qquad \beta = \frac{x - 2\alpha + \alpha^2}{x(1-\alpha)} \qquad \alpha_{\min} = 0 \quad \text{and} \quad \alpha_{\max} = 1 - \sqrt{1-x}$$

CT:

$$|\mathcal{M}^{(0)}_{Q\bar{Q}f_3f_4}(p_1, p_2, p_3, p_4)|^2 \simeq 8\pi\frac{\alpha_s\mu_R^{2\epsilon}}{C(\epsilon)}\frac{1}{s_{34}}\hat{P}_{f_3f_4}(z_3, k_\perp; \epsilon) \otimes |\mathcal{M}^{(0)}_{Q\bar{Q}g}(p_1, p_2, p_3 + p_4)|^2$$

Mom.Frac.:

$$z_3 = \frac{p_3\cdot P}{(p_3 + p_4)\cdot P} \quad \text{and} \quad 1 - z_3 = \frac{p_4\cdot P}{(p_3 + p_4)\cdot P}$$

$$z_3 = \frac{\alpha(1-\alpha) + (x - 2\alpha + \alpha^2)v}{x - \alpha^2} \quad \Rightarrow \quad \hat{z}_3 = \frac{\alpha + xv}{2\alpha + x}$$

❖ Freedom in the choice of the counterterms

1. Convenient choice of momentum fractions can be made upon examination of the explicit form of the collinear integral without affecting the structure of singularities

2. We redefine the counterterm multiplying by an appropriate function (that goes to 1 in the unresolved limit) chosen in such a way that it cancels regular factors

$$[\mathrm{d}p]\frac{1}{s_{34}} =$$
$$= \frac{x^{-1+2\epsilon}}{(4\pi)^2}\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}(P^2)^{-\epsilon}\mathrm{d}v\,\alpha^{-1-\epsilon}v^{-\epsilon}(1-v)^{-\epsilon}(1-\alpha)^{-2+3\epsilon}(x-\alpha)^{-1-\epsilon}(x-2\alpha+\alpha^2)^{2-2\epsilon}\,.$$

$$\mathcal{G}(\alpha,x;\epsilon) \equiv (1-\alpha)^{-2+3\epsilon}(x-\alpha)^{-1-\epsilon}(x-2\alpha+\alpha^2)^{2-2\epsilon}$$

$$\frac{\lim\limits_{\alpha\to 0}\mathcal{G}(\alpha,x;\epsilon)}{\mathcal{G}(\alpha,x;\epsilon)} = x^{1-3\epsilon}(1-\alpha)^{2-3\epsilon}(x-\alpha)^{1+\epsilon}(x-2\alpha+\alpha^2)^{-2+2\epsilon}$$

❖ Freedom in the choice of the counterterms

1. Convenient choice of momentum fractions can be made upon examination of the explicit form of the collinear integral without affecting the structure of singularities

2. We multiply the soft integral by an appropriate function (that goes to 1 in the unresolved limit) chosen in such a way that it cancels regular factors

3. Restrict the phase space of the subtraction to avoid complicated boundary (sqrt)

$$\alpha_{\text{min}} = 0 \qquad \text{and} \qquad \alpha_{\text{max}} = 1 - \sqrt{1-x}$$

$$\alpha_0(x) = K \cdot \frac{x}{2} < 1 - \sqrt{1-x}\,, \qquad K, x \in (0,1]$$

K can be used to check the implementation (as with the α parameters)

"Good" factor: $\mathcal{F}_{34}^{C} \equiv x^{1-3\epsilon}(1-\alpha)^{2-3\epsilon}(x-\alpha)^{1+\epsilon}(x-2\alpha+\alpha^2)^{-2+2\epsilon}\Theta[\alpha_0(x)-\alpha]$

$$\boldsymbol{I}_2(p_1,p_2;\epsilon)=\left[\frac{\alpha_s}{2\pi}\left(\frac{\mu_R^2}{P^2}\right)^{\epsilon}\right]^2\left\{C_F^2\left(\frac{b_{-2}}{\epsilon^2}+\frac{b_{-1}}{\epsilon}+b_0\right)+C_F C_A\left(\frac{c_{-2}}{\epsilon^2}+\frac{c_{-1}}{\epsilon}+c_0\right)+C_F T_R n_l\left(\frac{d_{-2}}{\epsilon^2}+\frac{d_{-1}}{\epsilon}+d_0\right)+\mathrm{O}(\epsilon^1)\right\}$$

$$b_{-2}=2+\frac{4(1+y^2)}{1-y^2}G_0+\frac{4(1+y^2)^2}{(1-y^2)^2}G_{0,0}$$

$$\begin{aligned}b_{-1}=\;&8-16G_1+\frac{4(1-3y)}{1-y}G_0+\frac{8(1+y^2)}{1-y^2}(2G_{-1,0}-4G_{0,1}-G_{1,0}-2\zeta_2)-\frac{4(1+y^2)}{(1-y^2)^2}\\&\times\Big[(3+4y+9y^2)G_{0,0}-(1+y^2)(8G_{-1,0,0}+4G_{0,-1,0}-3G_{0,0,0}-8G_{0,0,1}-2G_{0,1,0}\\&+4G_{1,0,0}-4G_0\zeta_2)\Big]\end{aligned}$$

$$\begin{aligned}b_0=\;&-156-64(G_1+\ln2)+128G_{1,1}-\frac{32(1-3y)}{1-y}G_{0,1}-\frac{4}{1-y^2}\Big[2(11+20y+19y^2)G_0\\&+(11-23y-25y^2)\zeta_2+12(1+y^2)G_0\ln2+2(19-3y+19y^2)G_{-1,0}+2(17+2y+y^2)G_{1,0}\\&+4(1+y^2)(8G_{-1,0,1}-2G_{-1,1,0}-16G_{0,1,1}-2G_{1,-1,0}-4G_{1,0,1}-7G_{1,1,0}+2G_1\zeta_2\\&+2G_{-1,0}\ln2+2G_{1,0}\ln2-\zeta_2\ln2)+4(8-y+8y^2)G_{-1}\zeta_2-8(2+y+2y^2)G_{-1,-1,0}\Big]\\&-\frac{8(1-9y+17y^2-25y^3)}{(1-y)(1-y^2)}G_{1,0,0}-\frac{8(23+27y)(1+y^2)}{(1+y)(1-y^2)}G_{0,1,0}-\frac{4}{5(1-y^2)^2}\\&\times\Big[5(16-y-20y^2-19y^3+64y^4)G_{0,0}-20(2+3y+4y^2+5y^3+6y^4)\zeta_3+40(1+3y^4)G_{0,0}\ln2\\&-10(1+10y+24y^2+12y^3+17y^4)G_0\zeta_2+20(9+3y+24y^2+y^3+11y^4)G_{-1,0,0}\\&+40(8+5y+10y^2+4y^3+5y^4)G_{0,-1,0}-5(3+8y+44y^2-4y^3+13y^4)G_{0,0,0}\\&-40(1+y^2)(3+4y+9y^2)G_{0,0,1}-(1+y^2)(101+132y^2)\zeta_2^2-10(1+y^2)^2(40G_{-1,-1,0,0}\\&+28G_{-1,0,-1,0}-32G_{-1,0,0,1}-8G_{-1,0,1,0}+10G_{0,-1,-1,0}-16G_{0,-1,0,1}+4G_{0,-1,1,0}\\&+12G_{0,0,0,1}-23G_{0,0,1,0}+32G_{0,0,1,1}+4G_{0,1,-1,0}+8G_{0,1,0,1}+14G_{0,1,1,0}+24G_{1,0,-1,0}\\&+6G_{1,0,0,0}-16G_{1,0,0,1}+4G_{1,0,1,0}-24G_{1,1,0,0}-10G_{-1,0}\zeta_2-15G_{0,-1}\zeta_2-4G_{0,1}\zeta_2\\&-8G_{1,0}\zeta_2-4G_{-1}\zeta_3-12G_1\zeta_3+8G_{-1,0,0}\ln2-4G_{0,-1,0}\ln2-4G_{0,1,0}\ln2+8G_{1,0,0}\ln2\\&+2G_0\zeta_2\ln2+4\zeta_3\ln2)-10(1+y^2)(1+5y^2)G_0\zeta_3-10(2-y^2)(1+y^2)G_{0,0}\zeta_2\\&+80y^2(1+y^2)G_{0,0,0}\ln2+20(1+y^2)(3+11y^2)G_{-1,0,0,0}+10(1+y^2)(21+17y^2)G_{0,-1,0,0}\\&+60(1+y^2)(5+6y^2)G_{0,0,-1,0}-5(1+y^2)(7+9y^2)G_{0,0,0,0}+10(9-7y^2)(1+y^2)G_{0,1,0,0}\Big]\end{aligned}$$

$$c_{-2}=\frac{11}{6}+\frac{11(1+y^2)}{6(1-y^2)}G_0$$

$$y\equiv\frac{\sqrt{P^2}-\sqrt{P^2-4m_Q^2}}{\sqrt{P^2}+\sqrt{P^2-4m_Q^2}}$$

$$G_{a_1,\ldots,a_n}(y)=(-1)^p H_{a_1,\ldots,a_n}(y)\,,\qquad a_i\in\{0,\pm1\}$$

$$\begin{aligned}c_{-1}=\;&\frac{181}{18}-\frac{44}{3}G_1+\frac{1}{18(1-y^2)}\Big[(1-132y-263y^2)G_0-6(47+71y^2)\zeta_2\\&+12(1+y^2)(19G_{-1,0}-22G_{0,1}+8G_{1,0})-6(11-y^2)G_{0,0}\Big]\\&+\frac{(1+y^2)}{(1-y^2)^2}\Big[(1+y^2)(-2G_{0,-1,0}-2G_{0,1,0}+\zeta_3)-(1+9y^2)G_0\zeta_2+4y^2G_{0,0,0}\Big]\end{aligned}$$

$$\begin{aligned}c_0=\;&\frac{3277}{27}-\frac{724}{9}G_1+\frac{121}{3}\ln2-24\ln3+\frac{352}{3}G_{1,1}+\frac{1}{27(1-y^2)}\Big[(541+642y-1631y^2)G_0\\&-216(1-y^2+G_0+y^2G_0)(\mathrm{Li}_2(-2)+\zeta_2)-3(370-471y-746y^2)\zeta_2-648(1+y^2)G_0\ln3\\&+9(73+48y+73y^2)G_0\ln2-6(88+381y+88y^2)G_{-1,0}-12(1-132y-263y^2)G_{0,1}\\&-12(97+81y-167y^2)G_{1,0}-108(16-y+16y^2)G_{-1}\zeta_2-72(7-17y^2)G_1\zeta_2\\&-36(1+y^2)(76G_{-1,0,1}-12G_{-1,1,0}-88G_{0,1,1}-12G_{1,-1,0}+32G_{1,0,1}+9G_{1,1,0}\\&-24G_{-1,0}\ln2-12G_{1,0}\ln2)+72(49+3y+49y^2)G_{-1,-1,0}+72(11-y^2)G_{0,0,1}\\&+648(1+y^2)G_{0,-1,0,0}\Big]+\frac{1}{90(1-y^2)^2}\Big[10(77-99y-534y^2-657y^3+565y^4)G_{0,0}\\&-60(163+54y+12y^2+42y^3-55y^4)\zeta_3-120(2+12y-18y^2-15y^3-107y^4)G_0\zeta_2\\&+720(1+7y^4)G_{0,0}\ln2-120(35+45y+39y^3+y^4)G_{-1,0,0}-240(17-12y-30y^2-15y^3\\&-38y^4)G_{0,-1,0}+60(23-6y-48y^2+48y^3+91y^4)G_{0,0,0}-60(91-60y-72y^2-60y^3\\&-91y^4)G_{0,1,0}-60(115+72y+48y^2+72y^3+173y^4)G_{1,0,0}-9(109+136y-54y^2-136y^3\\&-163y^4)\zeta_2^2-180(1+y^2)^2(16G_{-1,-1,0,0}+28G_{-1,0,-1,0}+4G_{-1,0,1,0}-32G_{-1,1,0,0}\\&+10G_{0,-1,-1,0}-8G_{0,-1,0,1}+14G_{0,-1,1,0}+14G_{0,1,-1,0}-8G_{0,1,0,1}+6G_{0,1,1,0}-32G_{1,-1,0,0}\\&+28G_{1,0,-1,0}+12G_{1,0,1,0}-64G_{1,1,0,0}-11G_{0,-1}\zeta_2-10G_{-1}\zeta_3-22G_1\zeta_3+16G_{-1,0,0}\ln2\\&-8G_{0,-1,0}\ln2-8G_{0,1,0}\ln2+16G_{1,0,0}\ln2+4G_0\zeta_2\ln2+8\zeta_3\ln2)-90(21+8y+66y^2-8y^3\\&+45y^4)G_0\zeta_3-360(1+y^2)(11+23y^2)G_{-1,0}\zeta_2+180(1-4y+4y^2+4y^3+3y^4)G_{0,0}\zeta_2\\&+180(1+y^2)(9+41y^2)G_{0,1}\zeta_2-360(1+y^2)(5+9y^2)G_{1,0}\zeta_2+360y^2(1+y^2)(-3G_{0,0,0,0}\\&-8G_{0,0,0,1}+8G_{0,0,0}\ln2)-360(9+8y-8y^2-8y^3-17y^4)G_{-1,0,0,0}+360(9+4y+40y^2\\&-4y^3+31y^4)G_{0,0,-1,0}+360(5+4y+18y^2-4y^3+13y^4)G_{0,0,1,0}\\&-360(1+y^2)(5+37y^2)G_{0,1,0,0}-360(1+y^2)(15+13y^2)G_{1,0,0,0}\Big]\end{aligned}$$

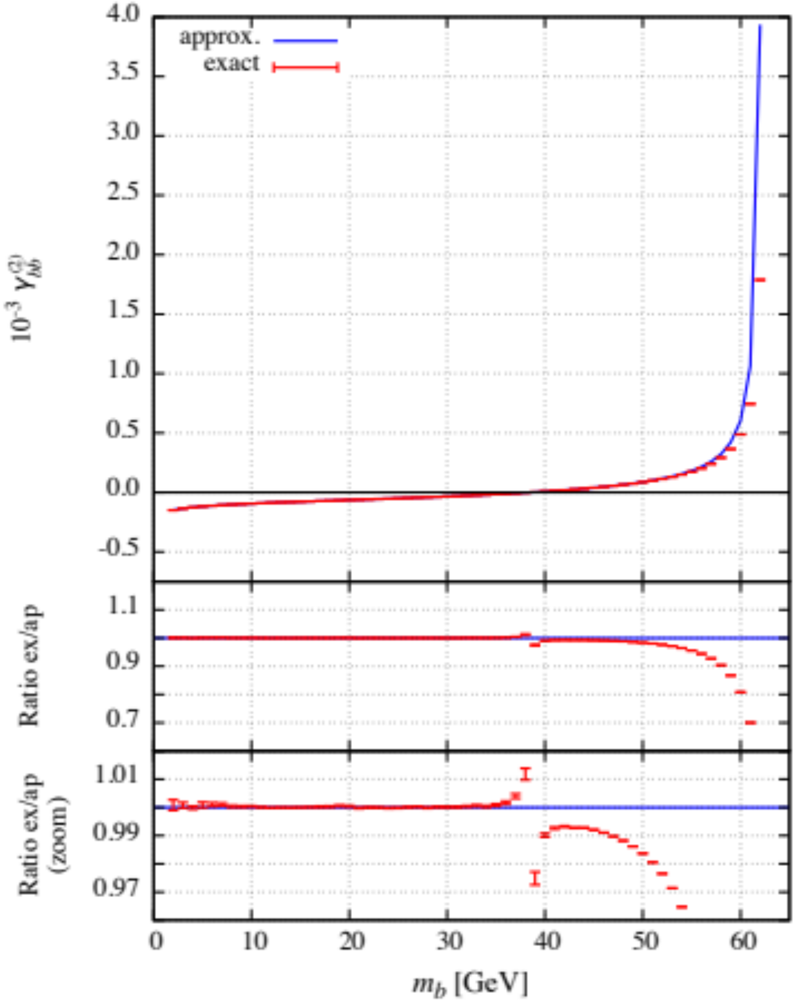
$$d_{-2}=-\frac{2}{3}-\frac{2(1+y^2)}{3(1-y^2)}G_0$$

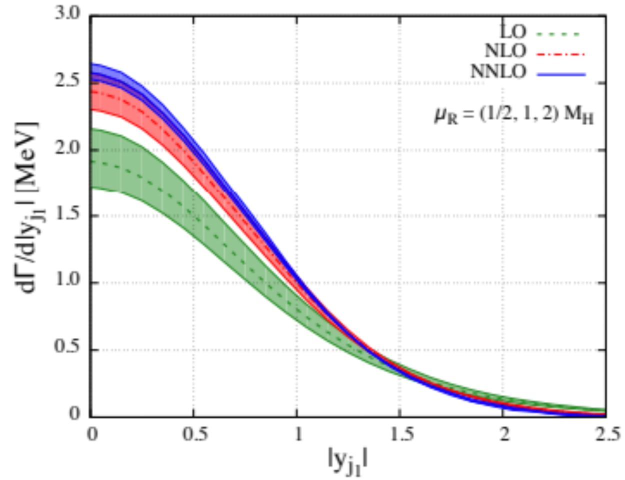
$$\begin{aligned}d_{-1}=\;&-\frac{34}{9}+\frac{16}{3}G_1+\frac{2(1+12y+25y^2)}{9(1-y^2)}G_0\\&-\frac{4(1+y^2)}{3(1-y^2)}(4G_{-1,0}-G_{0,0}-4G_{0,1}+2G_{1,0}-4\zeta_2)\end{aligned}$$

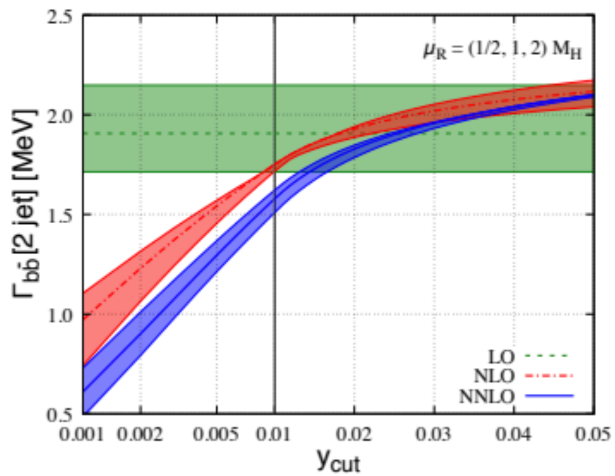
$$\begin{aligned}d_0=\;&-\frac{284}{27}+\frac{10}{3}\ln2+\frac{272}{9}G_1-\frac{128}{3}G_{1,1}+\frac{2}{27(1-y^2)}\Big[2(55+129y+259y^2)G_0\\&+3(43-93y-173y^2)\zeta_2+45(1+y^2)G_0\ln2+6(49+39y+49y^2)G_{-1,0}\\&-24(1+12y+25y^2)G_{0,1}+6(53+18y-43y^2)G_{1,0}-18(1+y^2)(28G_{-1,-1,0}-4G_{-1,0,0}\\&-32G_{-1,0,1}+12G_{-1,1,0}-20G_{0,-1,0}+2G_{0,0,0}+8G_{0,0,1}-22G_{0,1,0}+32G_{0,1,1}+12G_{1,-1,0}\\&+2G_{1,0,0}-16G_{1,0,1}-30G_{-1}\zeta_2+2G_0\zeta_2-10G_1\zeta_2-13\zeta_3)\Big]+\frac{2(10-37y-23y^2+14y^3)}{9(1-y)(1-y^2)}G_{0,0}.\end{aligned}$$

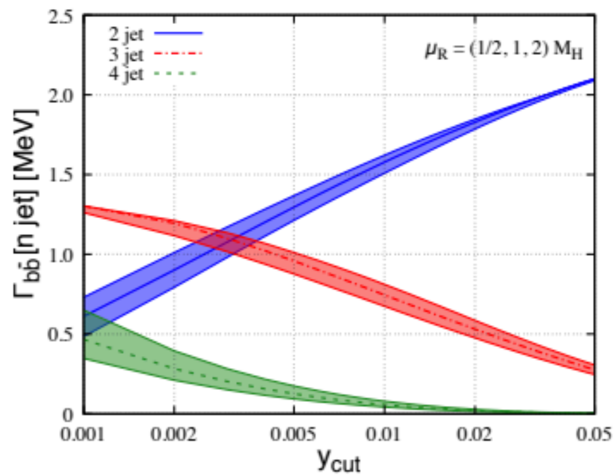
❖ Some numerical results for $H \to b\bar{b}$ @ NNLO

approx.: R. Harlander and M. Steinhauser 1997









Conclusion

- NNLOCAL at work also for the cancellation of initial state singularities
- EFT Higgs boson production in gluon fusion has very simple matrix elements but otherwise all the essential features
  - ✓ NNLOCAL for ggH (EFT) @NNLO (public soon)
  - ✓ quark channels, and full mass dependence will be available in the near future
- Massive final state counterterms (and their integrated version) all in 2007.15015

Outlook

- The inclusion of final state jets and heavy quarks in hadron collisions appears feasible with our methodology
- Extension to the next order for colour singlet appears conceivable

Backup

❖ Normal average