Two-loop amplitudes for W$\gamma\gamma$ production at the LHC

based on work with Simon Badger, Zihao Wu, Yang Zhang and Simone Zoia
(arXiv:2409.XXXXX)

Heribertus Bayu Hartanto
Asia Pacific Center for Theoretical Physics (APCTP) Pohang, South Korea

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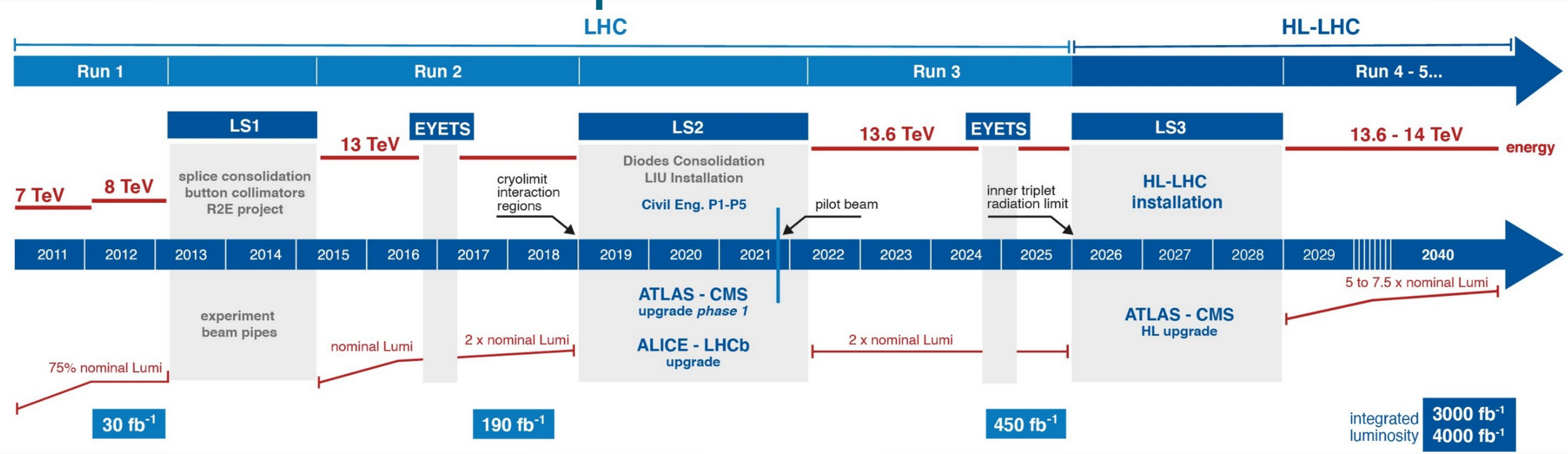
LHC as a precision machine



!!! High-precision theoretical predictions are mandatory !!!

$$d\hat{\sigma} = \underbrace{d\hat{\sigma}^{(0)}}_{\text{LO}} + \underbrace{\alpha\, d\hat{\sigma}^{(1)}}_{\delta\text{NLO}} + \underbrace{\alpha^2\, d\hat{\sigma}^{(2)}}_{\delta\text{NNLO}} + \ \ldots$$

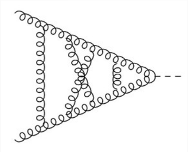
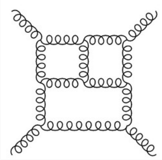
Th. uncertainties on :

LO > 50%

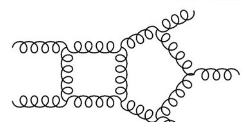
NLO QCD ~20-30%

NNLO QCD ~1-10%

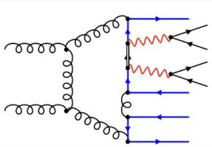
Loop Frontiers

N4LO 2→1  N3LO 2→2 

Multiplicity Frontiers

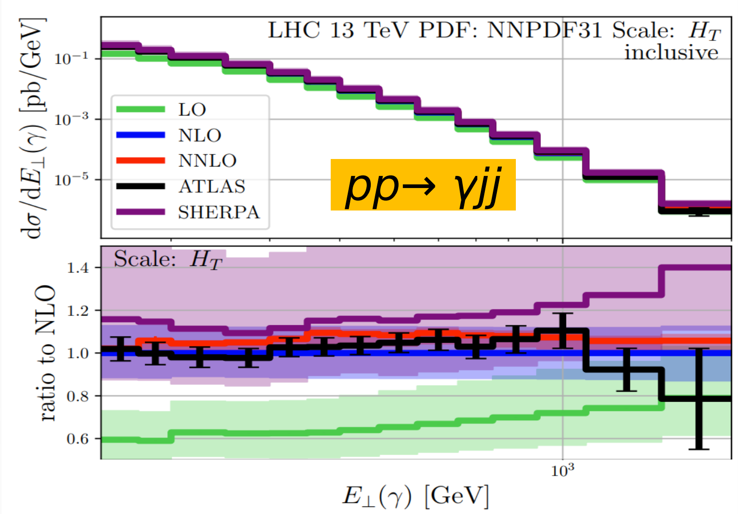
NNLO 2→3 

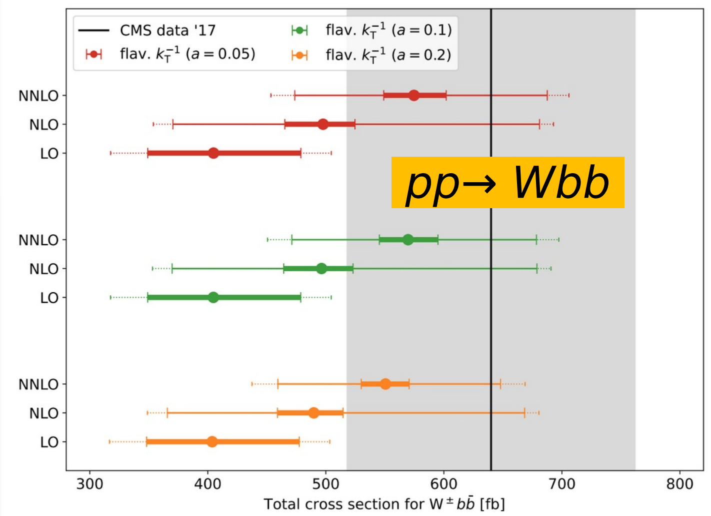
this talk

NLO 2→6 

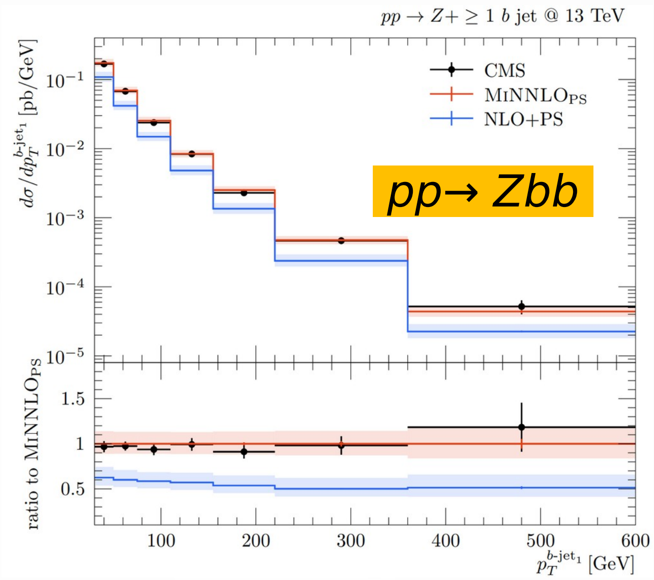
[Badger,Czakon,**HBH**,Moodie,Peraro,Poncelet,Zoia(2022)]

NNLO QCD calculation for 2→3 scattering processes

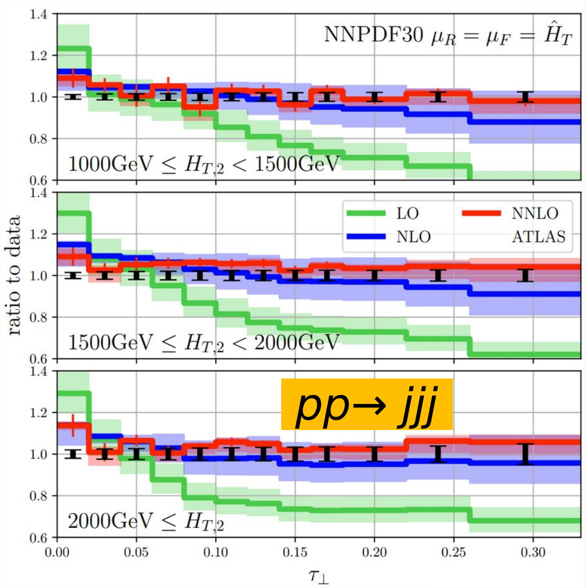




[**HBH**,Poncelet,Popescu,Zoia(2022)]



[Mazzitelli,Sotnikov,Wiesemann(2024)]



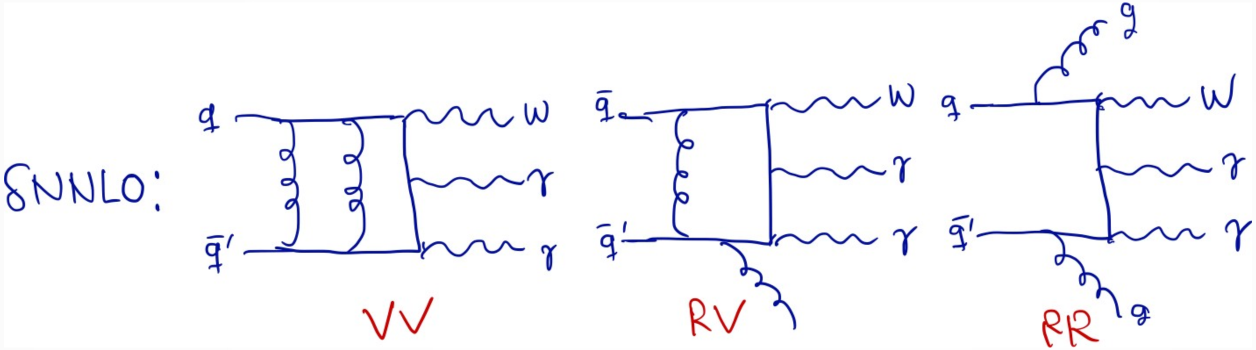
[Czakon,Mitov,Poncelet(2021)][Chen,Gehrmann,Glover,Huss,Marcoli(2022)]Alvarez,Cantero,Czakon,Llorente,Mitov,Poncelet(2023)]

Talk by V.Sotnikov, P.Garbarino, C.Savoini

also: *pp→γγγ, pp→γγj, pp→Wbb, pp→ttW, pp→ttH*

[Chawdhry,Czakon,Mitov,Poncelet(2019)][Kallweit,Sotnikov,Wiesemann(2020)]
[Czakon,Mitov,Poncelet(2020)][Buonocore,Devoto,Kallweit,Mazzitelli,Rottoli,Savoini(2022)][Buonocore,Devoto,Grazzini,Kallweit,Mazzitelli,Rottoli,Savoini(2022)]
[Catani,Devoto,Grazzini,Kallweit,Mazzitelli,Savoini(2020)]

NNLO QCD calculation: ingredients



NNLO subtraction schemes to cancel IR singularities (applied to 2→3 process):
STRIPPER[Czakon(2010)], Antenna Subtraction[Gehrmann,Gehrmann-de Ridder,Glover(2005)],
q_T-subtraction[Catani,Grazzini(2007)]

Talks by G.Bertolloti, M.Marcolli, A.Kardos, E.Fox, L.Bonino, M.Lochner, F. Tramontano, C.Signorile-Signorile

Remaining bottleneck: two-loop five-point amplitudes

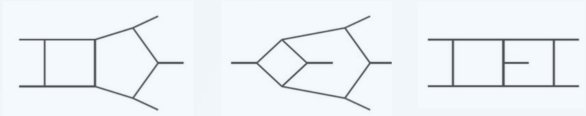
$$\text{loop amplitude} = \sum (\text{rational coefficients}) \times (\text{integral/special functions})$$

highly non-trivial for multi-scale process!!

Talks by K.Schonwald, R.Grober, A.H.Ajjath, T.Armadillo, B.Agarwal, H.Zhang, F.Buccioni, J.Lim, D.Kermanschach, G.Wang, A.Olson

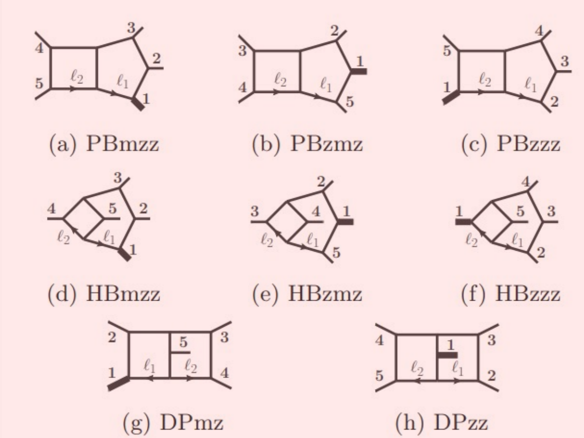
2-loop 5-point master integrals

massless



[Abreu,Chicherin,Dixon,Gehrmann,Henn, Herrmann,LoPresti,Papadopoulos,Page, Sotnikov,Tomassini,Wasser,Wever,Zeng, Zhang,Zoia(2015-2020)]

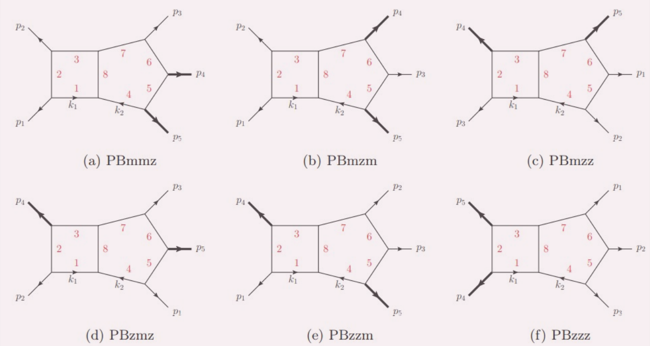
one external mass



(a) PBmzz (b) PBzmz (c) PBzzz
(d) HBmzz (e) HBzmz (f) HBzzz
(g) DPmz (h) DPzz

[Abreu,Canko,Chicherin,Ita,Kardos,Moriello, Page,Papadopoulos,Smirnov,Sotnikov,Syrrakos, Tomassini,Tschernow,Wever,Zeng,Zoia (2015-2023)]

two external masses (planar)



(a) PBmmz (b) PBmzm (c) PBmzz
(d) PBzmz (e) PBzzm (f) PBzzz

[Jiang,Liu,Xu,Yang(2024)]
[Abreu,Chicherin,Sotnikov,Zoia(2024)]

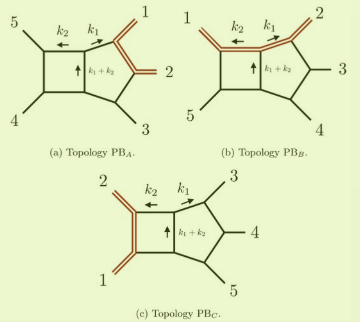
pentagon functions for massless and one external mass

[Chicherin,Sotnikov(2020)][Chicherin, Sotnikov,Zoia(2021)][Abreu,Chicherin,Ita, Page,Sotnikov,Tschernow,Zoia(2023)]

pp→ttj: leading colour

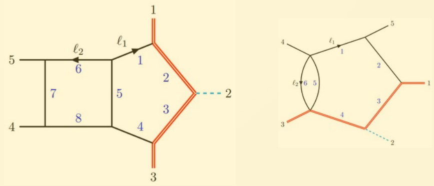
[Badger,Becchetti,Chaubey, Marzucca(2022)]
[Badger,Becchetti,Giraudo, Zoia(2024)]

$PB_B \rightarrow$ elliptic



(a) Topology PB_A. (b) Topology PB_B. (c) Topology PB_C.

pp→ttH: leading colour, N_f part



[Febres Cordero,Figueiredo,Kraus,Page,Reina(2023)]

2-loop 5-point amplitudes

massless

- $pp \to \gamma\gamma\gamma$ [Abreu,Page,Pascual,Sotnikov(2020)][Chawdhry,Czakon,Mitov,Poncelet(2021)] [Abreu,De Laurentis,Ita,Klinkert,Page,Sotnikov(2023)]
- $pp \to \gamma\gamma j$ [Agarwal,Buccioni,von Manteuffel,Tancredi(2021)][Chawdhry,Czakon,Mitov,Poncelet(2021)] [Badger,Brönnum-Hansen,Chicherin,Gehrmann,**HBH**,Henn,Marcoli,Moodie,Peraro,Zoia(2021)]
- $pp \to \gamma jj$ [Badger,Czakon,**HBH**,Moodie,Peraro,Poncelet,Zoia(2023)]
- $pp \to jjj$ [Abreu,Febres Cordero,Ita,Page,Sotnikov(2021)][De Laurentis,Ita,Klinkert,Sotnikov(2023)] [Agarwal,Buccioni,Devoto,Gambuti,von Manteuffel,Tancredi(2023)][De Laurentis,Ita,Sotnikov(2023)]

Talk by F. Buccioni

1 external mass

- $pp \to Wbb$ (planar, massless b) [Badger,**HBH**,Zoia(2021)][**HBH**,Poncelet,Popescu,Zoia(2022)]
- $pp \to Wjj$ (planar) [Abreu,Febres Cordero,Ita,Klinkert,Page,Sotnikov(2022)]
- $pp \to Hbb$ (planar, massless b) [Badger,**HBH**,Krys,Zoia(2021)]
- $pp \to W\gamma j$ (planar) [Badger,**HBH**,Krys,Zoia(2022)]
- $pp \to W/Z+bb$ (planar, massive b) [Buonocore,Devoto,KallweitMazzitelli,Rottoli,Savoini(2022)] [Mazzitelli,Sotnikov,Wiesemann(2024)]

Talk by V. Sotnikov

full colour result still missing!!

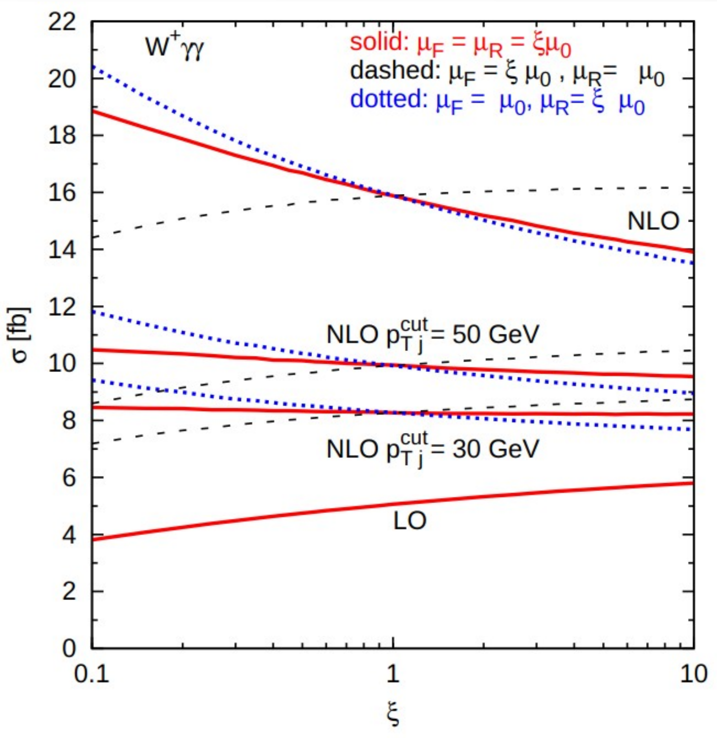
$$\mathcal{M}_2^m = \mathcal{M}_2^{m=0} + Z^1_{[q]}\mathcal{M}_1^{m=0} + Z^2_{[q]}\mathcal{M}_0^{m=0}$$

$$Z^l_{[q]} = f\left(\epsilon, \log m_b^2/Q^2\right)$$

W(→lν)γγ production at the LHC

Large NLO K-factor and NLO scale dependence (only qq in the initial state at LO)

LHC ($\sqrt{s} = 14$ TeV)	LO [fb]	NLO [fb]	K-factor
$\sigma(\text{"}W^+\gamma\gamma\text{"} \to e^+\nu_e\gamma\gamma)$			
$p_{T\gamma(\ell)} > 20(20)$ GeV	2.529	7.940	3.14
$p_{T\gamma(\ell)} > 30(20)$ GeV	0.979	3.172	3.24
$\sigma(\text{"}W^-\gamma\gamma\text{"} \to e^-\bar{\nu}_e\gamma\gamma)$			
$p_{T\gamma(\ell)} > 20(20)$ GeV	1.946	6.759	3.47
$p_{T\gamma(\ell)} > 30(20)$ GeV	0.686	2.583	3.77

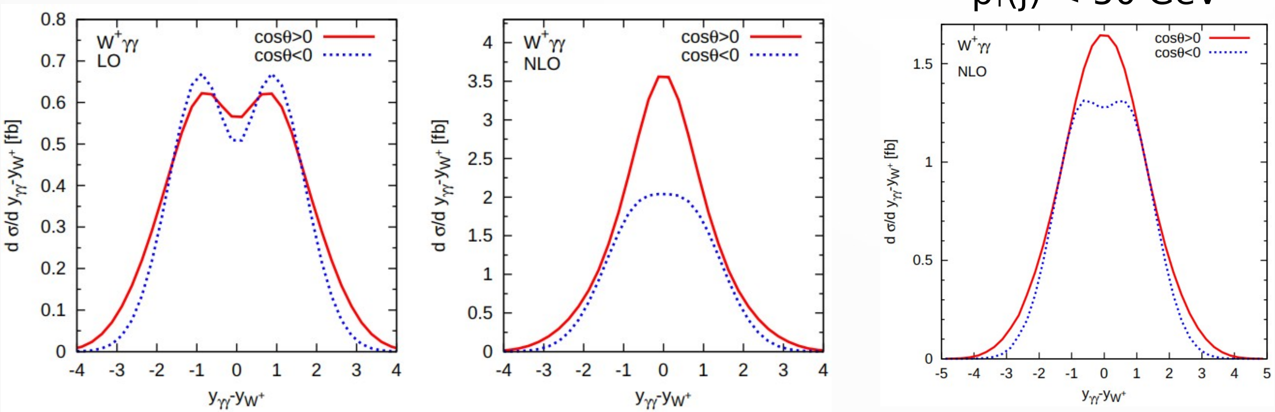


[Bozzi,Campanario,Rauch,Zeppenfeld(2011)]

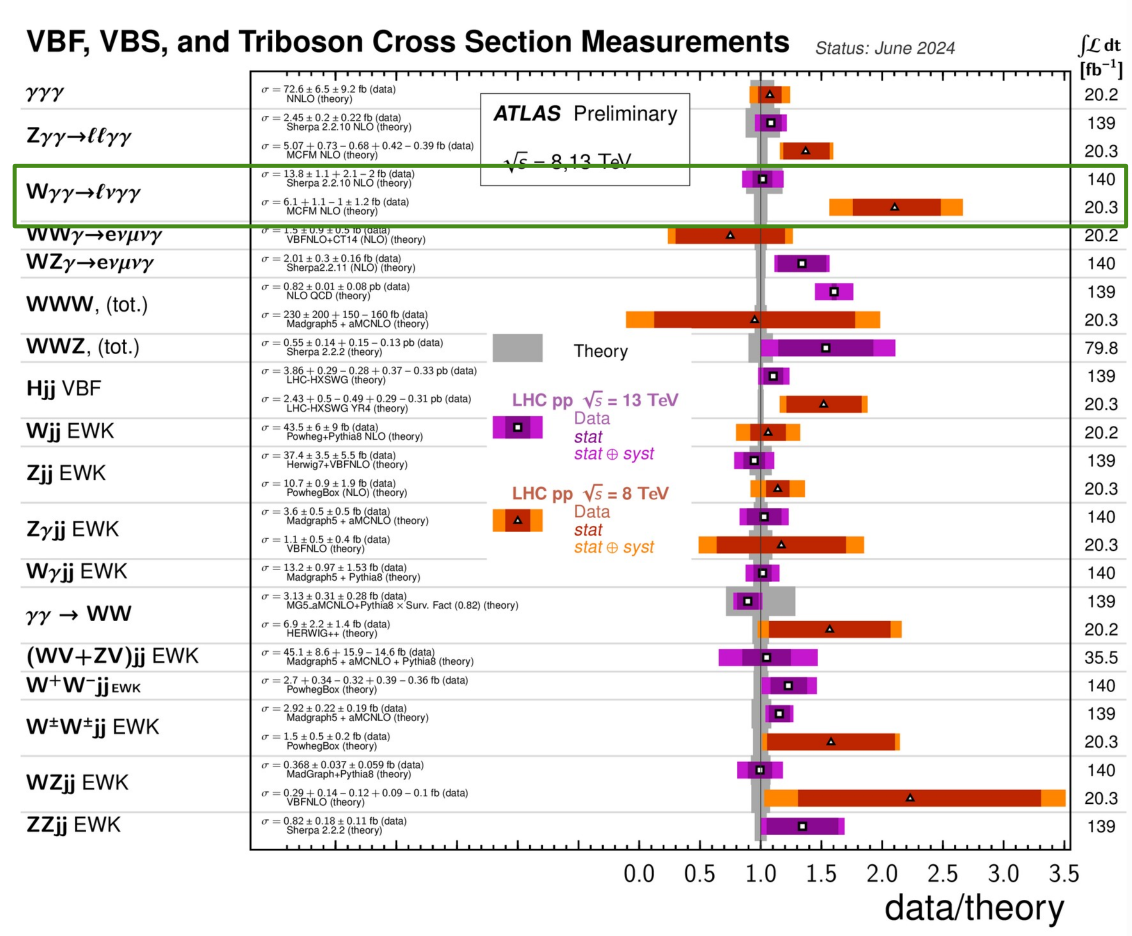
Radiation amplitude zero

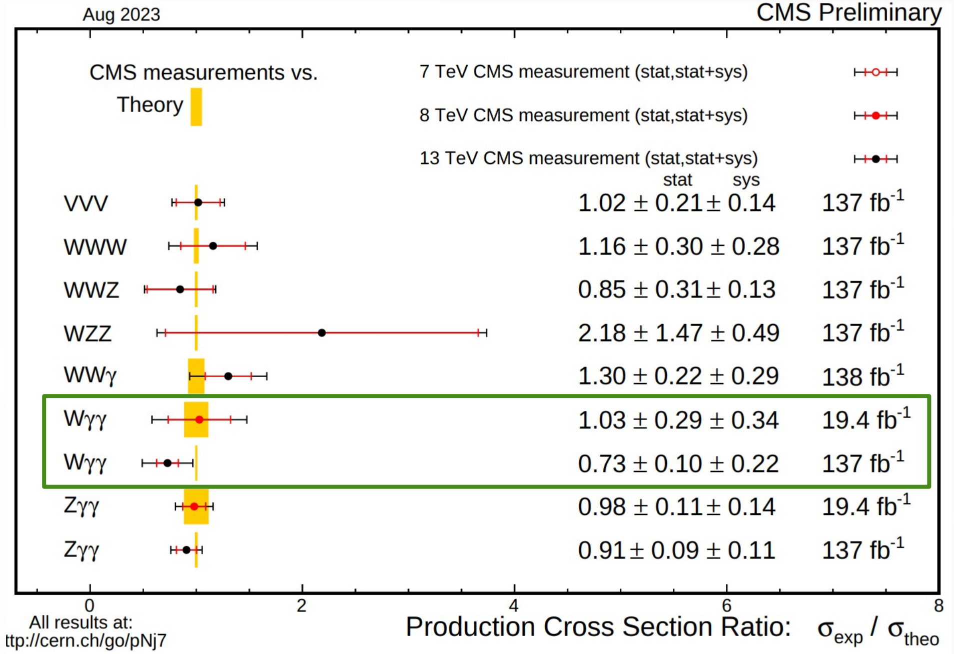
[Baur,Han,Kauer,Sobey,Zeppenfeld(1997)][Bell(2009)]

$p_T(j) < 50$ GeV



W(→lν)γγ measurements at the LHC



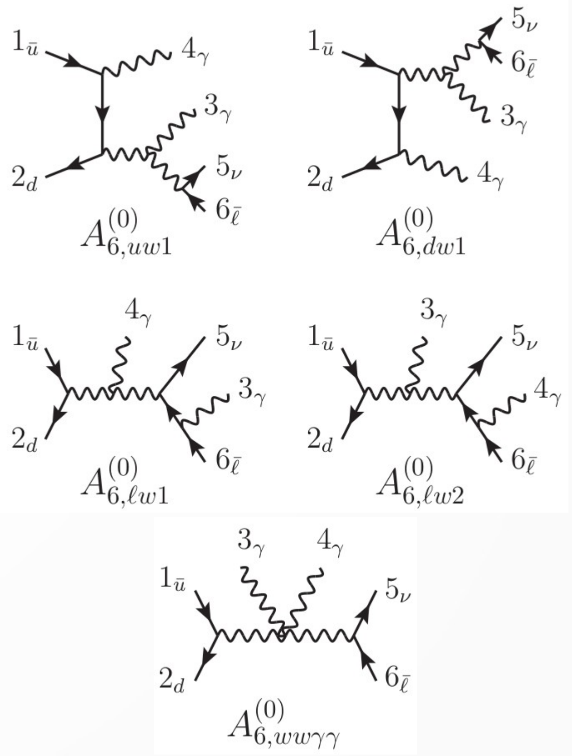


NNLO QCD predictions?

W(→lν)γγ production at the LHC

Rich structure: γ can be emitted from initial quark line, W-boson and charged lepton sensitive to triple- and quartic-gauge couplings

$$
\begin{aligned}
A_6^{(L)} = & \left[Q_u^2 A_{6,uu}^{(L)} + Q_u Q_d A_{6,ud}^{(L)} + Q_d^2 A_{6,dd}^{(L)} + \left(\sum_{q=1}^{n_f} Q_q^2\right) A_{6,q}^{(L)}\right] P(s_{56}) \\
& + \left[Q_u Q_\ell A_{6,u\ell 1}^{(L)} + Q_d Q_\ell A_{6,d\ell 1}^{(L)}\right] P(s_{356}) + \left[Q_u Q_\ell A_{6,u\ell 2}^{(L)} + Q_d Q_\ell A_{6,d\ell 2}^{(L)}\right] P(s_{456}) \\
& + \left[Q_u Q_w A_{6,uw1}^{(L)} + Q_d Q_w A_{6,dw1}^{(L)}\right] P(s_{356}) P(s_{56}) \\
& + \left[Q_u Q_w A_{6,uw2}^{(L)} + Q_d Q_w A_{6,dw2}^{(L)}\right] P(s_{456}) P(s_{56}) \\
& + Q_\ell Q_w A_{6,\ell w1}^{(L)} P(s_{356}) P(s_{3456}) + Q_\ell Q_w A_{6,\ell w2}^{(L)} P(s_{456}) P(s_{3456}) \\
& + Q_w^2 A_{6,ww1}^{(L)} P(s_{56}) P(s_{356}) P(s_{3456}) + Q_w^2 A_{6,ww2}^{(L)} P(s_{56}) P(s_{456}) P(s_{3456}) \\
& + Q_\ell^2 A_{6,\ell\ell}^{(L)} P(s_{3456}) + A_{6,ww\gamma\gamma}^{(L)} P(s_{56}) P(s_{3456}),
\end{aligned}
$$



Sub-amplitudes are not separately gauge invariant

Off-shell W boson → use complex mass scheme to ensure gauge invariance

Two-loop amplitudes for W(→lν)γγ production

[Badger,**HBH**,Krys,Wu,Zhang,Zoia(to appear)]

remove the decays, compute 5-, 4- and 3-particle currents

5pt

$$A^{(L)}_{6,i}(1_{\bar{u}},2_d,3_\gamma,4_\gamma,5_\nu,6_{\bar{\ell}}) = A^{(L)\mu}_{5,i}(1_{\bar{u}},2_d,3_\gamma,4_\gamma,p_{56})\,L_\mu(5_\nu,6_{\bar{\ell}})$$

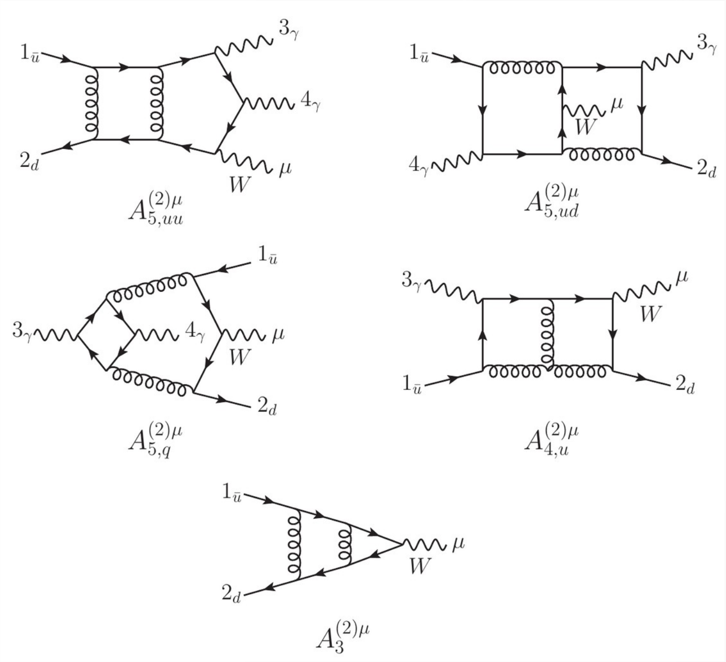
4pt

$$A^{(L)}_{6,i}(1_{\bar{u}},2_d,3_\gamma,4_\gamma,5_\nu,6_{\bar{\ell}}) = A^{(L)\mu}_{4,u/d}(1_{\bar{u}},2_d,3_\gamma,p_{456})\,L^i_\mu(4_\gamma,5_\nu,6_{\bar{\ell}})$$

$$A^{(L)}_{6,i}(1_{\bar{u}},2_d,3_\gamma,4_\gamma,5_\nu,6_{\bar{\ell}}) = A^{(L)\mu}_{4,u/d}(1_{\bar{u}},2_d,4_\gamma,p_{356})\,L^i_\mu(3_\gamma,5_\nu,6_{\bar{\ell}})$$

3pt

$$A^{(L)}_{6,i}(1_{\bar{u}},2_d,3_\gamma,4_\gamma,5_\nu,6_{\bar{\ell}}) = A^{(L)\mu}_{3}(1_{\bar{u}},2_d,p_{3456})\,L^i_\mu(3_\gamma,4_\gamma,5_\nu,6_{\bar{\ell}})$$



$$A^{(L)\mu}_{5,uu/ud} = \sum_{i=1}^{4} \Omega^{(L)}_{uu/ud;i}\, u^\mu_i\,, \qquad \Omega^{(L)}_{uu/ud;i} = \sum_j \left(\Delta_5^{-1}\right)_{ij} \tilde{A}^{(L)}_{5,uu/ud;j}\,, \qquad \tilde{A}^{(L)}_{5,uu/ud;i} = u_{i\mu}\, A^{(L)\mu}_{5,uu/ud;i}$$

$$A^{(L)\mu}_{4,u} = \sum_{i=1}^{4} \Omega^{(L)}_{u;i}\, v^\mu_i\,, \qquad \Omega^{(L)}_{u;i} = \sum_j \left(\Delta_4^{-1}\right)_{ij} \tilde{A}^{(L)}_{4,u;j}\,, \qquad \tilde{A}^{(L)}_{4,u;i} = v_{i\mu}\, A^{(L)\mu}_{4,u;i}$$

$$A^{(L)\mu}_{3} = \sum_{i=1}^{4} \Omega^{(L)}_{3;i}\, w^\mu_i\,, \qquad \Omega^{(L)}_{3;i} = \sum_j \left(\Delta_3^{-1}\right)_{ij} \tilde{A}^{(L)}_{3;j}\,. \qquad \tilde{A}^{(L)}_{3;i} = w_{i\mu}\, A^{(L)\mu}_{3;i}$$

(N_c,n_f) decomposition

$$A^{(2)}_{6,i} = N_c^2 A^{(2),N_c^2}_{6,i} - \left(A^{(2),N_c^2}_{6,i} + A^{(2),1/N_c^2}_{6,i}\right) + \frac{1}{N_c^2} A^{(2),1/N_c^2}_{6,i} + \left(N_c - \frac{1}{N_c}\right) n_f A^{(2),N_c n_f}_{6,i}$$

$$A^{(2)}_{6,q} = \left(N_c - \frac{1}{N_c}\right) A^{(2),N_c}_{6,q}\,.$$

Leading colour limit →

$$A^{(2),\mathrm{lc}}_{6,i} = N_c^2 A^{(2),N_c^2}_{6,i} + N_c n_f A^{(2),N_c n_f}_{6,i}$$

$$A^{(2),\mathrm{lc}}_{6,q} = N_c A^{(2),N_c}_{6,q}\,.$$

Amplitude computation

$$A^{(2)}(\{p\},\epsilon) = \sum_i (\text{Feynman diagram})_i$$

$\downarrow$ map loop numerators to $\mathcal{I}$

$$A^{(2),h}(\{p\},\epsilon) = \sum_i c_i^h(\{p\},\epsilon)\, \mathcal{I}_i(\{p\},\epsilon)$$

$\downarrow$ IBP reduction

$$A^{(2),h}(\{p\},\epsilon) = \sum_i d_i^h(\{p\},\epsilon)\, \mathrm{MI}_i(\{p\},\epsilon)$$

$\downarrow$ map to pentagon functions

$\downarrow$ subtract UV/IR poles

$\downarrow$ ϵ expansion

$$F^{(2),h}(\{p\}) = \sum e_i^h(\{p\})\, \mathrm{mon}_i(f) + \mathcal{O}(\epsilon)$$

- Compute $e_i^h(\{p\})$ numerically over finite fields
- Use momentum twistor parametrisation
- Analytic reconstruction from numerical samples

Reconstructed in $\dfrac{\#\text{ points} \times \text{eval. time}}{\#\text{ cores}}$

guess denominators, linear relations among coeffs, univariate partial fraction decomposition

- Optimised IBP relations from NeatIBP [Wu,etal(2023)]
  → solved numerically over finite fields
- for LC amplitude: ~8x speed up and ~3x RAM improvement compared to Laporta IBPs
- One-mass pentagon functions [Abreu,Chicherin,Ita,Page,Sotnikov,Tschernow,Zoia(2023)]

QGRAF[Nogueira],FORM[Vermaseren,etal],FiniteFlow[Peraro(2019)]
Spinney[Cullen,etal], LiteRed[Lee], NeatIBP[Wu,etal(2023)] Talk by Zihao Wu

Two-loop amplitudes for W(→lν)+γγ production

$$A^{(2)}_{6,i} = N_c^2 A^{(2),N_c^2}_{6,i} - \left(A^{(2),N_c^2}_{6,i} + A^{(2),1/N_c^2}_{6,i}\right) + \frac{1}{N_c^2} A^{(2),1/N_c^2}_{6,i} + \left(N_c - \frac{1}{N_c}\right) n_f A^{(2),N_c n_f}_{6,i}$$

	original	stage 1	stage 2	stage 3	stage 4	# points (# primes)	analytic expression
$\tilde{A}^{(2),N_c^2}_{5,uu;1}$	159/155	159/155	159/0	33/31	33/0	27728 (2)	✓
$\tilde{A}^{(2),N_c^2}_{5,uu;2}$	147/143	147/143	147/0	33/31	33/0	37132 (2)	✓
$\tilde{A}^{(2),N_c^2}_{5,uu;3}$	157/153	157/153	157/0	31/29	31/0	31610 (2)	✓
$\tilde{A}^{(2),N_c^2}_{5,uu;4}$	143/139	143/139	143/0	35/33	35/0	38710 (2)	✓
$\tilde{A}^{(2),1/N_c^2}_{5,uu;1}$	223/219	223/219	223/0	50/48	50/0	134551 (?)	✗
$\tilde{A}^{(2),1/N_c^2}_{5,uu;2}$	208/204	208/204	208/0	41/42	41/0	81973 (?)	✗
$\tilde{A}^{(2),1/N_c^2}_{5,uu;3}$	219/215	219/215	219/0	49/46	49/0	130146 (?)	✗
$\tilde{A}^{(2),1/N_c^2}_{5,uu;4}$	202/199	202/199	202/0	48/49	48/0	143320 (?)	✗

derived analytic form of the finite remainders, except for 5-pt sub-leading colour (SLC) amplitude

5-pt SLC vs LC
evaluation time → 10x slower
RAM usage → 3x larger

multivariate partial fraction decomposition of the analytic results are using Pfd-Parallel
→ a factor of ~2 compression

How important are SLC contributions?
→ other processes: 2-10% of σ
→ resort to numerical evaluation

Talk by P. Garbarino

[Bendle,Boehm,Heymann,Ma,Rahn,Ristau,Wittman,Wu.Zhang(2021)]

Numerical evaluation of 5-pt SLC amplitudes

Use finite-field based framework: rationalised 5-pt Mandelstam invariants (s_{ij})
(some of the mom. twistor variables are complex for physical PS point)

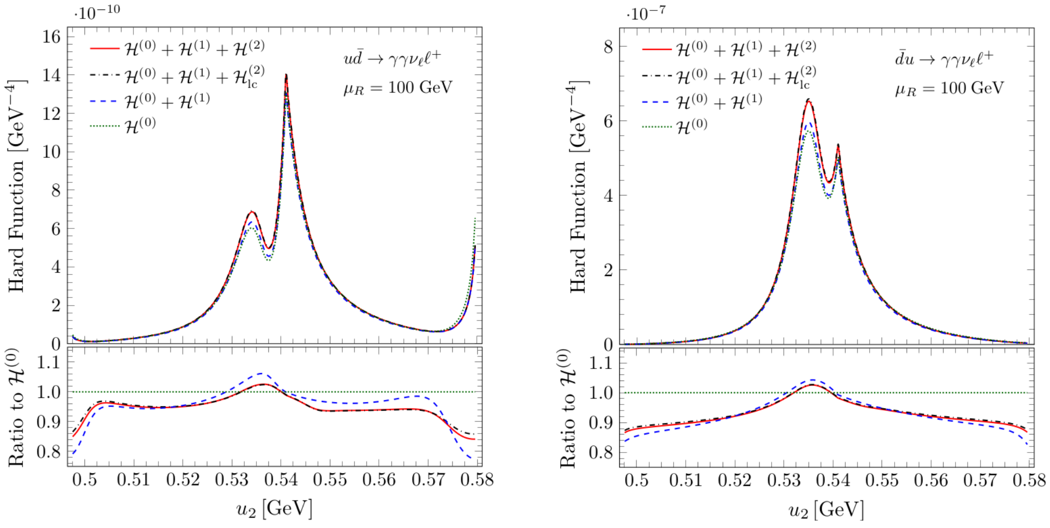
Employ 4-dim tensor decomposition and numerically evaluate contracted amplitudes over FF
[Peraro,Tancredi(2019,2020)]

$$\tilde{A}^{(L)}_{5,uu/ud,i} = \sum_{j=1}^{8} \chi^{(L)}_{uu/ud,i;j} T_j \,,$$

$$\chi^{(L)}_{uu/ud,k;i} = \sum_j \left(\Theta^{-1}\right)_{ij} T_j^\dagger \cdot \tilde{A}^{(L)}_{5,uu/ud,k} \,,$$

$$T_1 = \bar{u}(p_1)\not{p}_3 v(p_2)\, p_1\cdot\varepsilon(p_3,q_3)\, p_1\cdot\varepsilon(p_4,q_4)\,,$$
$$T_2 = \bar{u}(p_1)\not{p}_3 v(p_2)\, p_1\cdot\varepsilon(p_3,q_3)\, p_2\cdot\varepsilon(p_4,q_4)\,,$$
$$T_3 = \bar{u}(p_1)\not{p}_3 v(p_2)\, p_2\cdot\varepsilon(p_3,q_3)\, p_1\cdot\varepsilon(p_4,q_4)\,,$$
$$T_4 = \bar{u}(p_1)\not{p}_3 v(p_2)\, p_2\cdot\varepsilon(p_3,q_3)\, p_2\cdot\varepsilon(p_4,q_4)\,,$$
$$T_5 = \bar{u}(p_1)\not{p}_4 v(p_2)\, p_1\cdot\varepsilon(p_3,q_3)\, p_1\cdot\varepsilon(p_4,q_4)\,,$$
$$T_6 = \bar{u}(p_1)\not{p}_4 v(p_2)\, p_1\cdot\varepsilon(p_3,q_3)\, p_2\cdot\varepsilon(p_4,q_4)\,,$$
$$T_7 = \bar{u}(p_1)\not{p}_4 v(p_2)\, p_2\cdot\varepsilon(p_3,q_3)\, p_1\cdot\varepsilon(p_4,q_4)\,,$$
$$T_8 = \bar{u}(p_1)\not{p}_4 v(p_2)\, p_2\cdot\varepsilon(p_3,q_3)\, p_2\cdot\varepsilon(p_4,q_4)\,,$$

	original	stage 1	stage 2
$T_j^\dagger \cdot \tilde{A}^{(2),1/N_c^2}_{5,uu,1}$	100/97	100/97	100/0
$T_j^\dagger \cdot \tilde{A}^{(2),1/N_c^2}_{5,uu,2}$	99/96	99/96	99/0
$T_j^\dagger \cdot \tilde{A}^{(2),1/N_c^2}_{5,uu,3}$	101/97	101/97	101/0
$T_j^\dagger \cdot \tilde{A}^{(2),1/N_c^2}_{5,uu,4}$	101/97	101/97	101/0
$T_j^\dagger \cdot \tilde{A}^{(2),1/N_c^2}_{5,ud,1}$	97/93	96/92	96/0
$T_j^\dagger \cdot \tilde{A}^{(2),1/N_c^2}_{5,ud,2}$	97/93	97/93	97/0
$T_j^\dagger \cdot \tilde{A}^{(2),1/N_c^2}_{5,ud,3}$	97/93	96/92	96/0
$T_j^\dagger \cdot \tilde{A}^{(2),1/N_c^2}_{5,ud,4}$	97/93	96/92	96/0

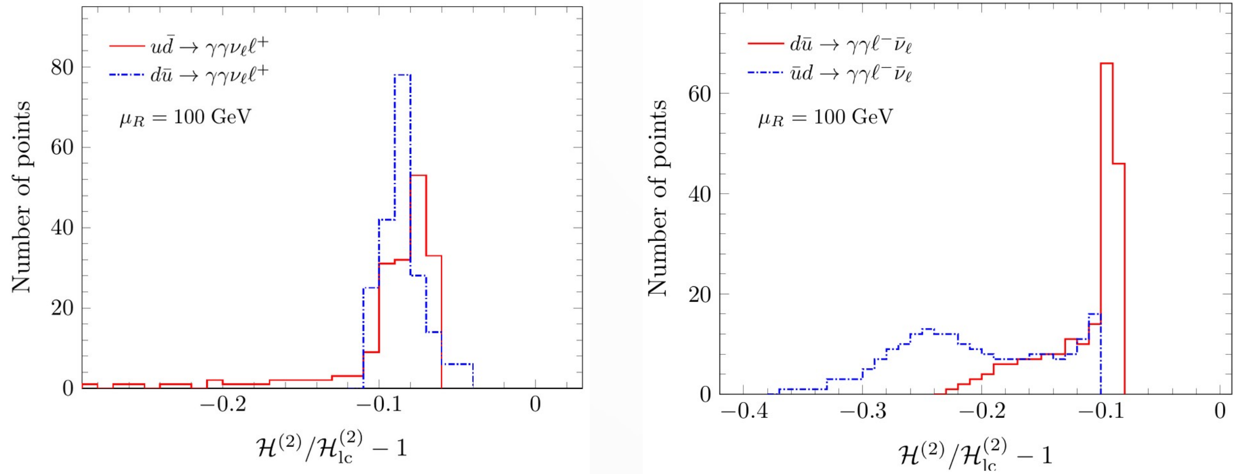


(a) $u\bar{d}\to\gamma\gamma\nu_\ell\ell^+$ (b) $\bar{d}u\to\gamma\gamma\nu_\ell\ell^+$

generate a univariate slice of
199 physical points:

O(30) prime fields needed
(depending on rationalisation)

much more expensive compared to
evaluation of analytic expressions



include SLC contributions to
the cross section via reweighting

SLC corrections peaked around 10%
→ more complete picture with actual
PS points from cross section run

Analytic structure of 5-pt SLC amplitudes

taken from V. Sotnikov's talk at QCD meets Gravity 2023

Planar scattering

Only linear or quadratic letters vanish in $\mathcal{P}_{\text{phys}}$, poles always canceled, i.e. $h^{(0)} = 0$

New feature of nonplanar scattering

Square roots of quartic polynomials $\sqrt{\Sigma_5}$ can vanish in $\mathcal{P}_{\text{phys}}$ $\implies$ new types of divergences

1. Integrable square root: $\mathrm{d}\log\frac{a+\sqrt{\Sigma_5}}{a-\sqrt{\Sigma_5}} \xrightarrow{\Sigma_5\to 0} \frac{\mathrm{d}\Sigma_5}{a\sqrt{\Sigma_5}} \xrightarrow{t\to t^\star} \frac{C}{\sqrt{t-t^\star}} + \ldots$
2. Uncompensated poles: $\mathrm{d}\log\sqrt{\Sigma_5} \xrightarrow{\Sigma_5\to 0} \frac{\mathrm{d}\Sigma_5}{2\Sigma_5} \xrightarrow{t\to t^\star} \frac{C}{t-t^\star} + \ldots \implies$ log divergence!

- Choose basis functions to localize non-analytic behavior
- Functions with type 2 divergence cancel out in physical results?
- Numerical evaluation more challenging

YES!!

(at the level of bare amplitude)

!!! need to check for other processes !!!

Summary

- ✔ Two-loop amplitudes for $W\gamma\gamma$ production at the LHC
- ✔ Analytic reconstruction from numerical evaluations over finite field
- ✔ Crucial ingredient: optimised IBP systems from NeatIBP
- ✔ Analytic results are obtained except for 2-loop 5-pt subleading colour amplitude
- ✔ Numerical approach for 2-loop 5-pt subleading colour amplitude

- ✗ Deployment for cross section: check VV contribution to NNLO cross section
  → expected size of 2-loop subleading colour contributions?
- ✗ Improved analytic reconstruction strategies
  [De Laurentis,Ita,Klinkert,Sotnikov(2023)][Chawdhry(2023)][De Laurentis,Maitre,(2020)]
- ✗ Construct $Z\gamma\gamma$ amplitudes from $W\gamma\gamma$
  → amplitude with Z coupled to closed quark loop is needed