Bethe Center for Theoretical Physics

UNIVERSITÄT BONN

Massive variable flavour scheme for Drell–Yan process

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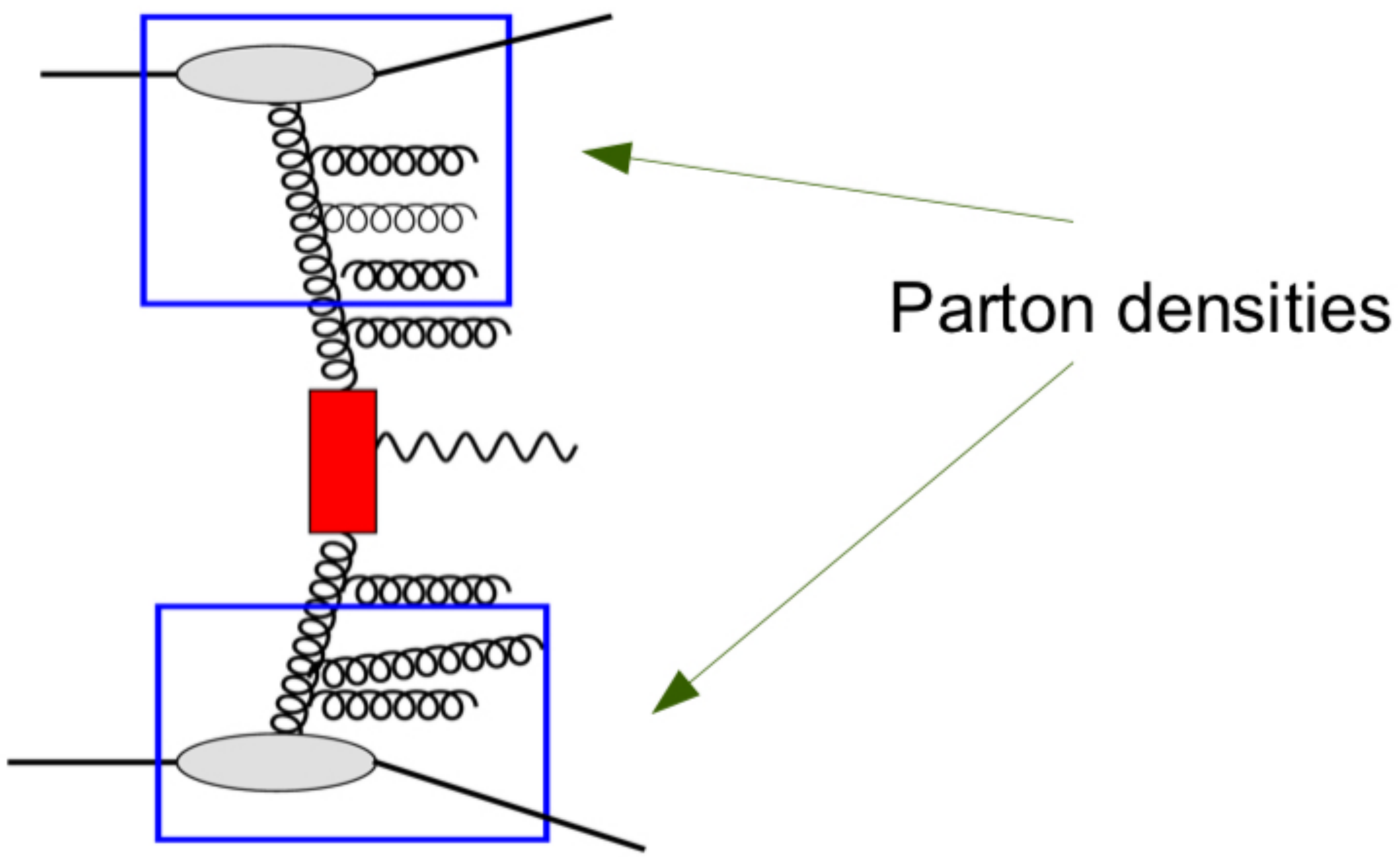
In collaboration with Prof. Dr. Claude Duhr, Dr. Ekta Chaubey and Dr. Rhorry Gauld

High Precision for Hard Processes (HP2 2024) | Sep 12, 2024

When to consider a heavy–flavour quark massive or massless?

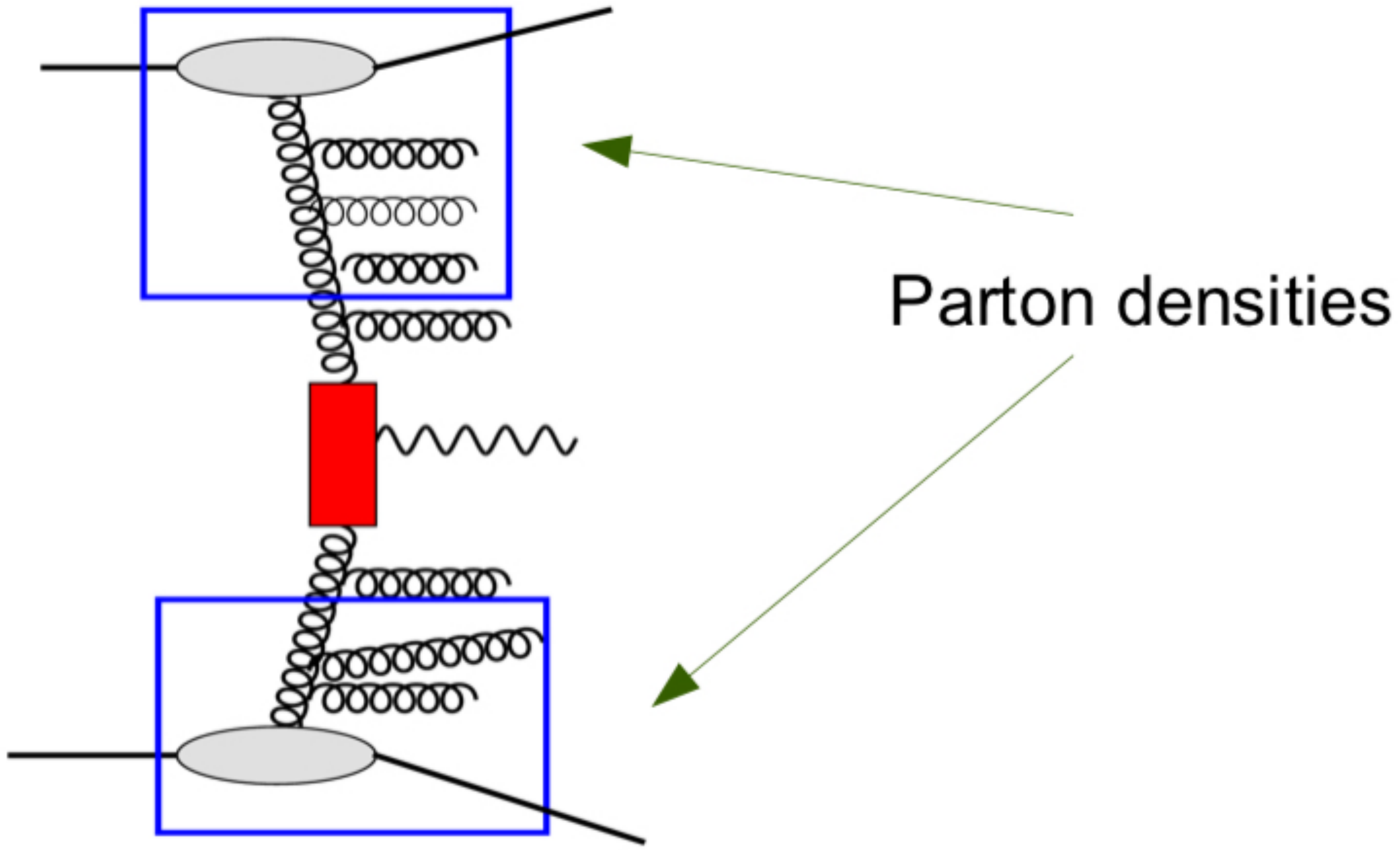


Observables ~ parton density ⊗ Matrix element ⊗ parton density

When to consider a heavy–flavour quark massive or massless?

Massless Parton :

Collinear logarithms
$(\alpha_s^i \ln[m/Q]^j$ **for** $i \geq j)$
are summed upto all orders



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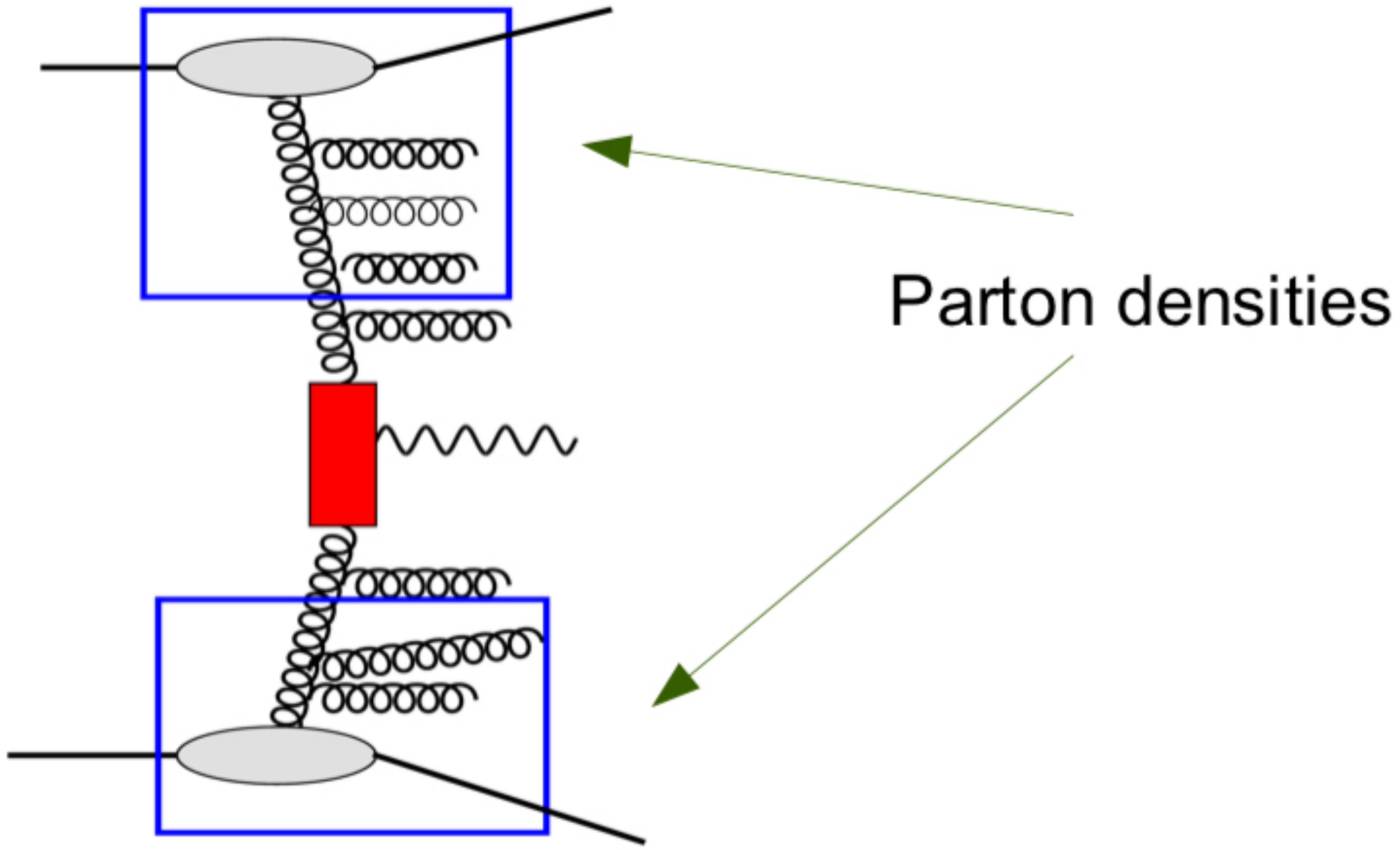


Massive Parton :

Collinear logarithms $(\alpha_s^i \ln[m/Q]^j$ **for** $i \geq j)$ **are considered exactly upto known perturbative order**

Observables ~ parton density $\otimes$ Matrix element $\otimes$ parton density

Constructing the massive variable flavour Scheme (M–VFNS)

(De)construct the massive computation of heavy–quark (Q) of mass (m):

$$d\sigma^{M} = d\sigma^{m=0,n_f} + d\sigma^{\ln[m]} + d\sigma^{pc}$$

massive contributions | massless contributions | logarithmic contributions | power corrections contributions

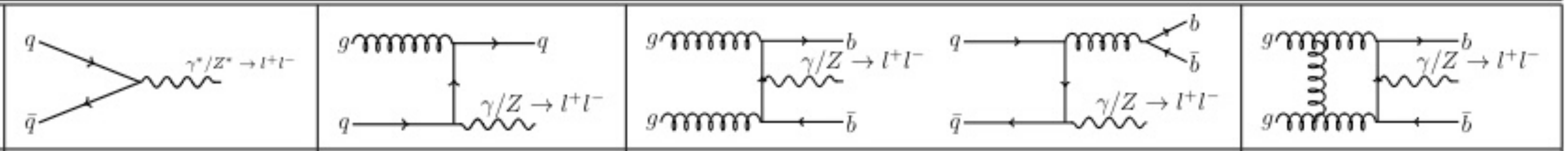
Construct the matching formula for predictions in massive variable flavour scheme :

$$d\sigma^{M-VFNS} = d\sigma^{m=0} + \sum_{i=c,b} d\sigma_i^{pc}$$

[Rhorry Gauld , 2021]

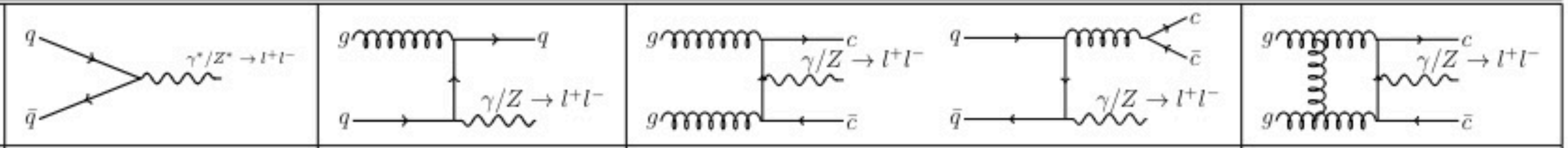
Variable flavour scheme in neutral Drell–Yan process

Representative diagrams for $pp \to l\bar{l} + X$ in four–flavour scheme (4FS)



4FS	–	–	LO	NLO
5FS	LO	NLO	NNLO	N^{3}LO

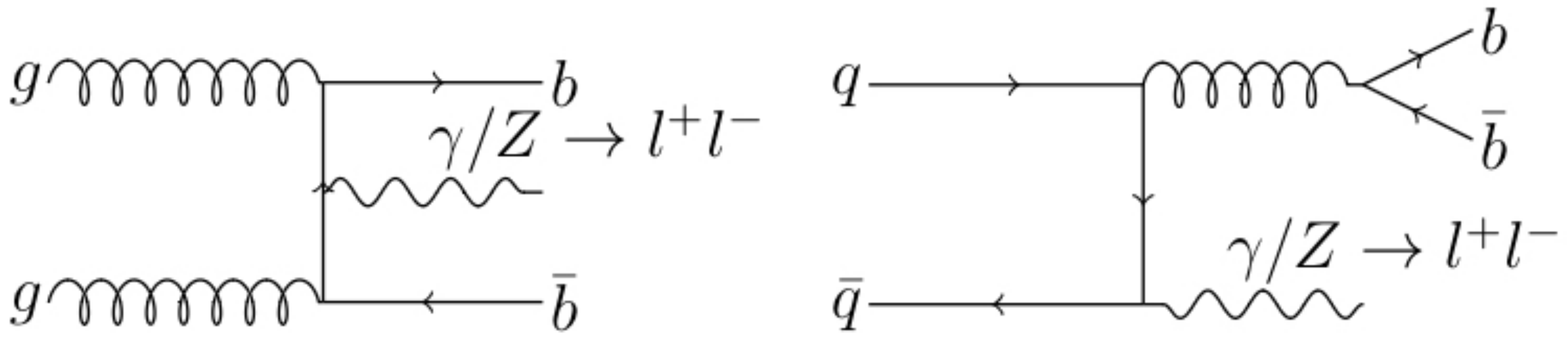
Representative diagrams for $pp \to l\bar{l} + X$ in three–flavour scheme (3FS)



3FS	–	–	LO	NLO
5FS	LO	NLO	NNLO	N^{3}LO

Massive and massless components in 4FS

Massive computation: Using in–house code and verified against aMC@NLO



4FS	LO
5FS	NNLO

Massless computation: using n3loxs

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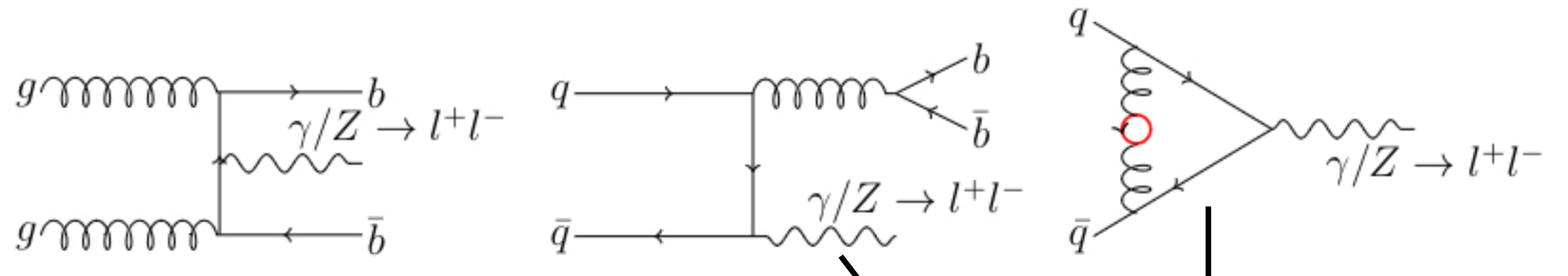
	
4FS 5FS	LO NNLO

Massless computation: using n3loxs

Significant cancellations

Sources of logarithmic contributions in 4FS

- The logarithmic dependence of the massive cross–section on heavy–quark mass is of collinear origin.

- This behaviour is universal and is governed by decoupling relations of strong coupling constants (α_s) and parton distribution functions (PDFs).

- For instance the decoupling relation for the PDFs in terms of massive operator matrix elements (OMEs, denoted by $\alpha_s^n A_{Qg}^{(n)}$) reads as :

$$f_b = f_{\bar{b}} = \alpha_s A_{bg}^{(1)} \otimes f_g + \alpha_s^2 \left[A_{bg}^{(2)} \otimes f_g + \sum_{i=-4, i\neq 0}^{4} A_{bi}^{(2)} \otimes f_i \right] + O(\alpha_s^3)$$

[M. Buza, et.al. 1996]

Sources of logarithmic contributions in 4FS

- Hence using the decoupling relations one obtains different logarithmic contributions as:

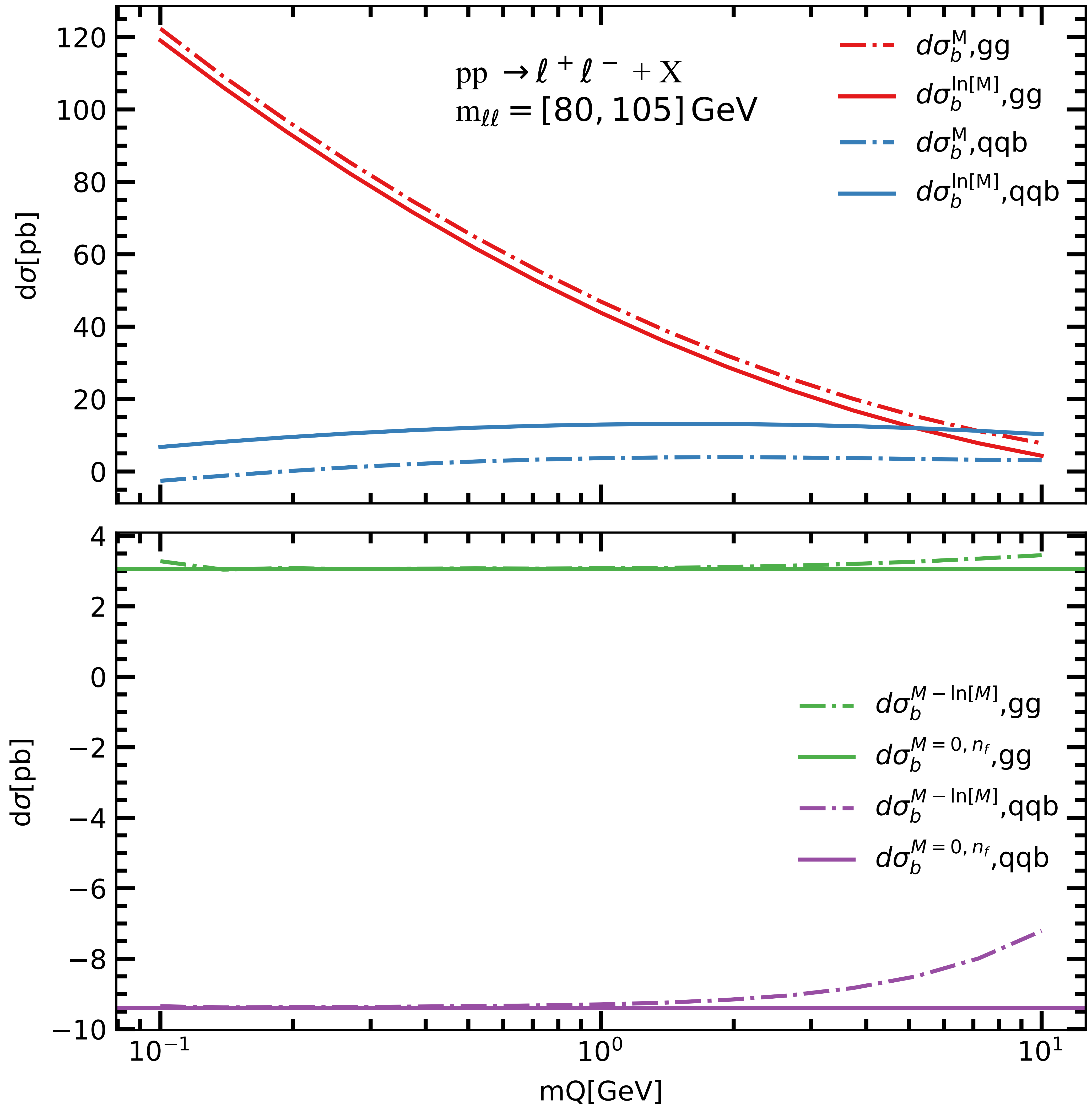
In gg–channel : $$d\sigma_{i,gg}^{\ln[\mathrm{M}],\mathrm{n_f}} \propto A_{gb}^{(1)} \otimes A_{gb}^{(1)} \otimes \sigma_{q\bar{q}}^{(0)} + A_{gb}^{(1)} \otimes \sigma_{bg}^{(1)}$$

In $q\bar{q}$–channel : $$d\sigma_{i,q\bar{q}}^{\ln[\mathrm{M}],\mathrm{n_f}} \propto A_{qq}^{(2)} \otimes \sigma_{q\bar{q}}^{(0)} + \Delta^{(1)}(\alpha_s)\sigma_{q\bar{q}}^{(1)}$$

In qg–channel : $$d\sigma_{i,qg}^{\ln[\mathrm{M}],\mathrm{n_f}} \propto \left(\Delta^{(1)}(\alpha_s) - A_{gg}^{(1)}\right) \otimes \sigma_{qg}^{(1)}$$

- All these convolutions are performed analytically using PolyLogTools.

Preliminary analysis to show the behaviour in 4FS



Prediction	Order	$m_{l\bar{l}} \in [80, 105]$ GeV
$d\sigma^{m=0}$	$\mathcal{O}(\alpha_s^2)$	$1824.63^{+0.7\%}_{-1.04\%}$
$d\sigma_b^{pc}(gg)$	$\mathcal{O}(\alpha_s^2)$	$+0.21$
$d\sigma_b^{pc}(q\bar{q})$	$\mathcal{O}(\alpha_s^2)$	$+0.89$

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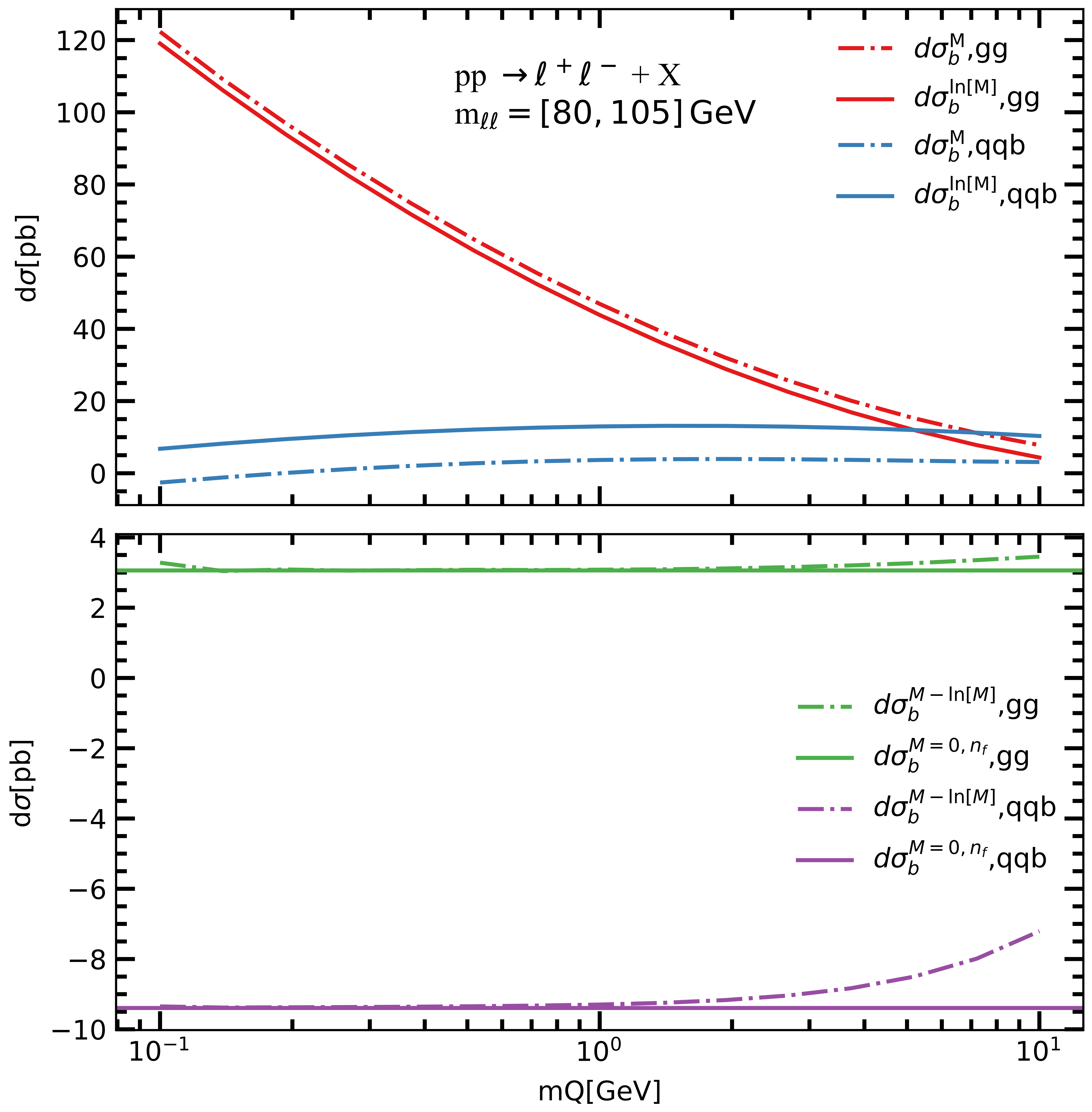

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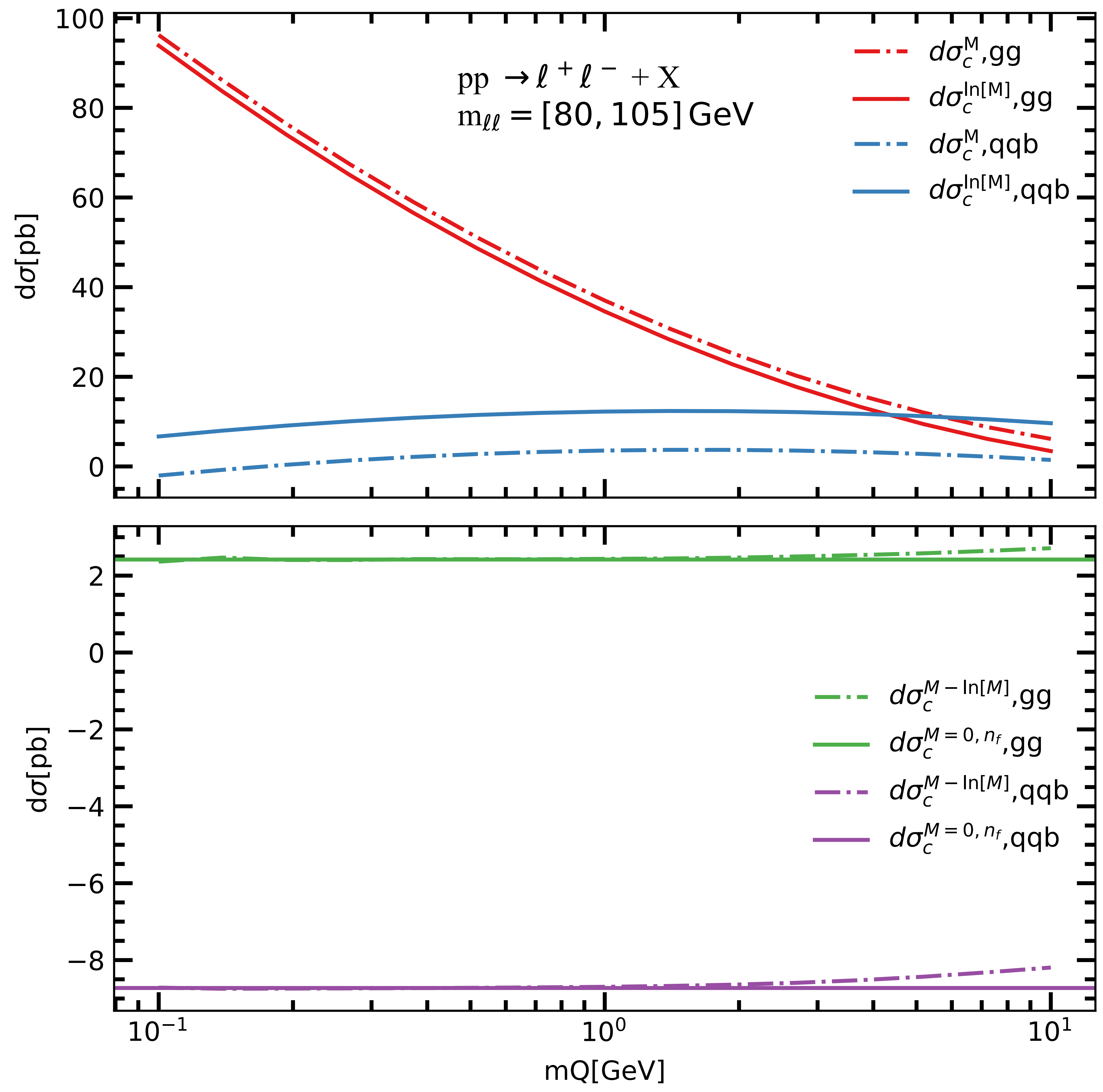

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$\mathrm{d}\sigma_c^{\mathrm{pc}}(gg)$	$\mathcal{O}(\alpha_s^2)$	$+0.03$
$\mathrm{d}\sigma_c^{\mathrm{pc}}(q\bar{q})$	$\mathcal{O}(\alpha_s^2)$	$+0.29$

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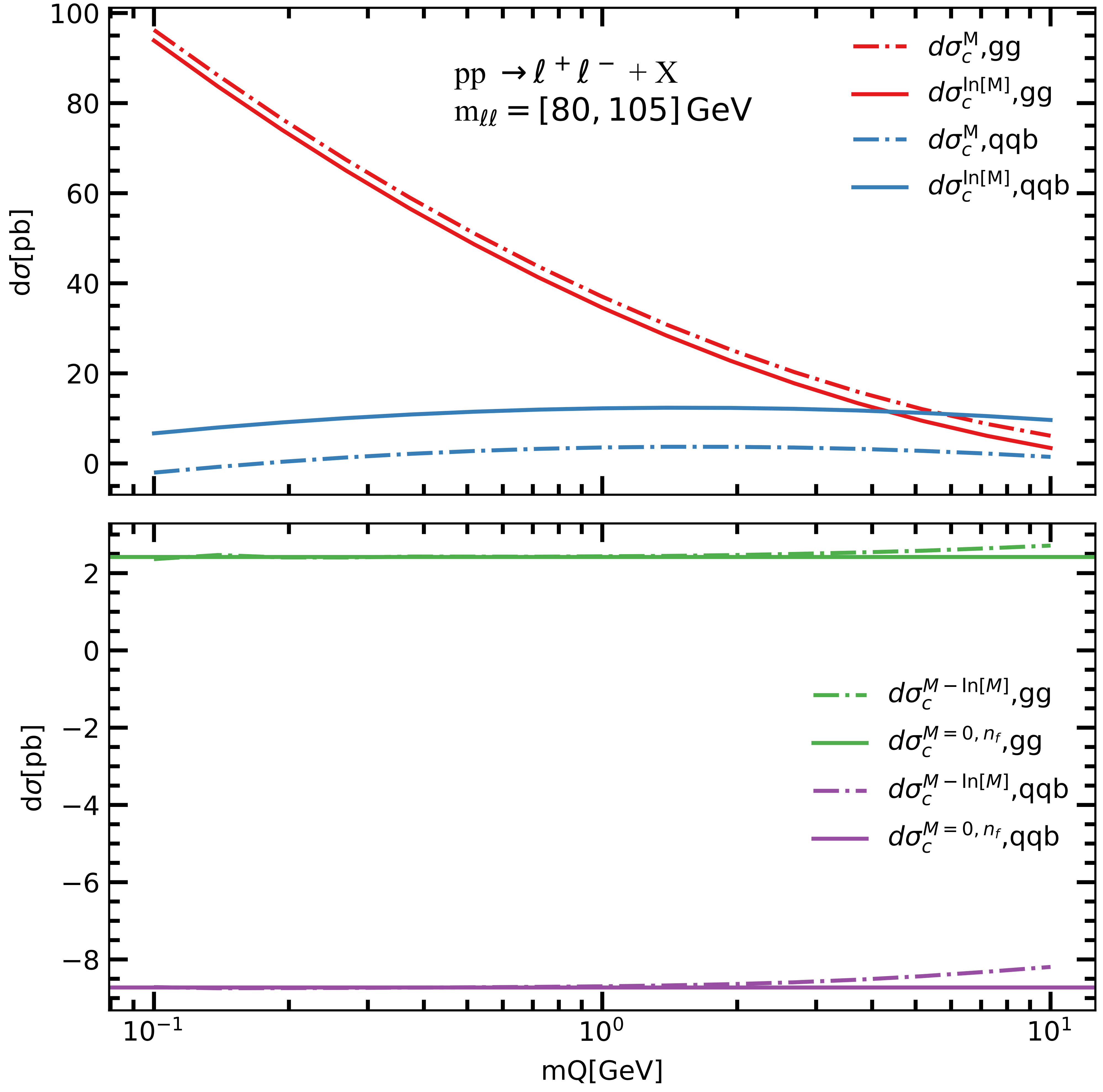

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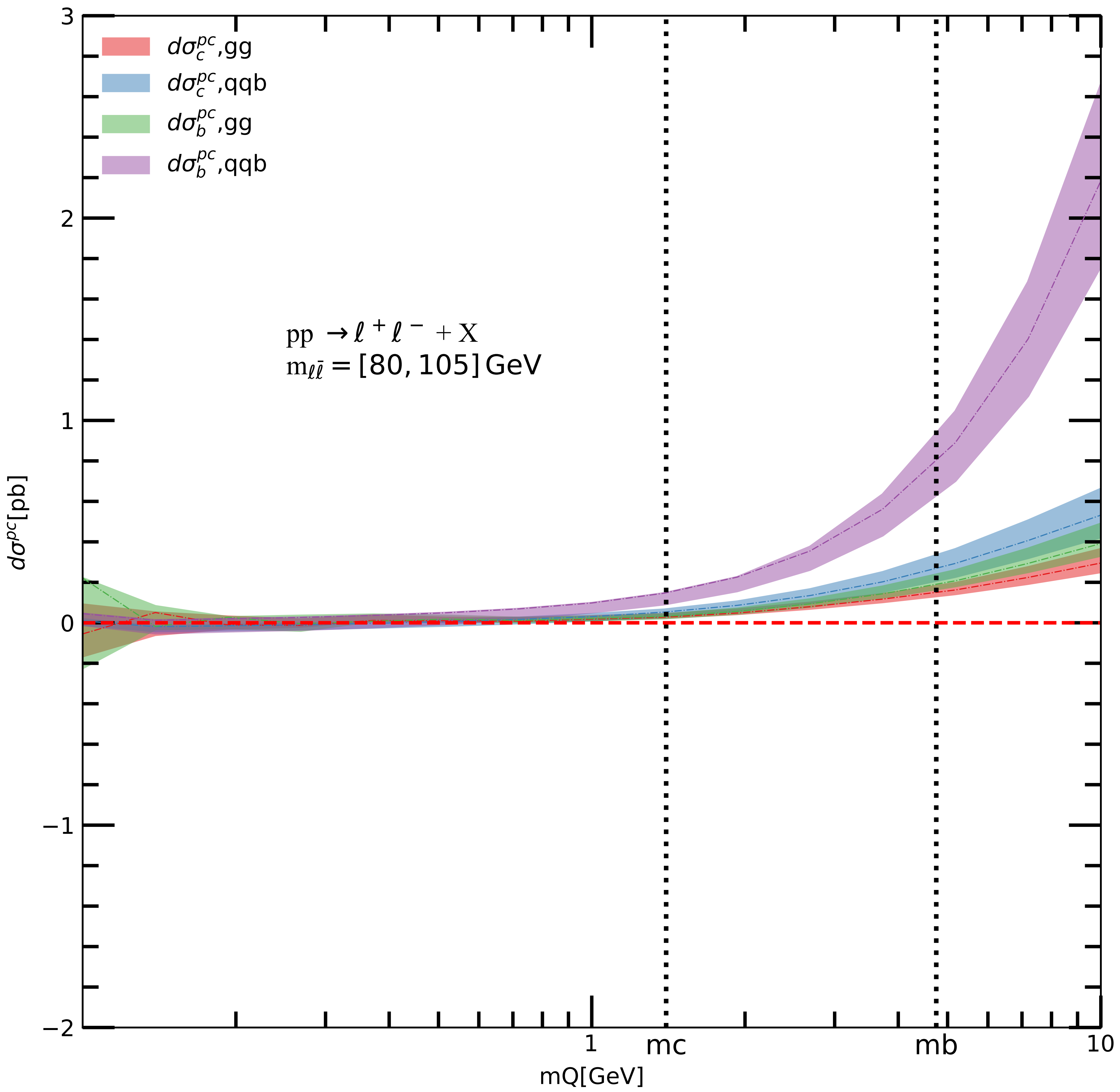
$d\sigma_c^{pc}(q\bar{q})$	$\mathcal{O}(\alpha_s^2)$	+0.29

Preliminary analysis to show the behaviour in $d\sigma^{pc}$

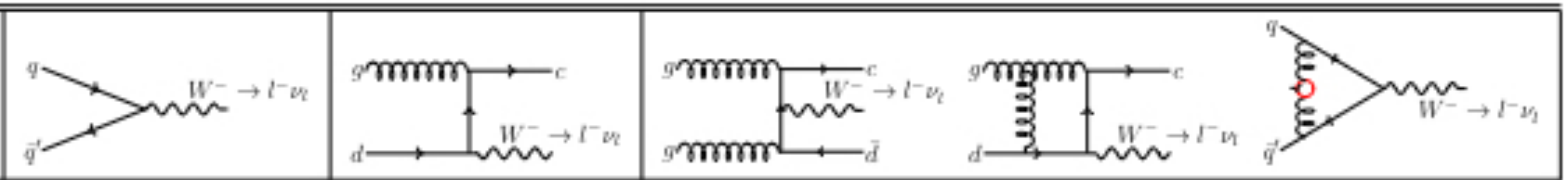


Re–arrange the cross section formula:

$$(d\sigma^{M} - d\sigma^{m=0,n_f}) - d\sigma^{\ln[m]} = d\sigma^{pc}$$

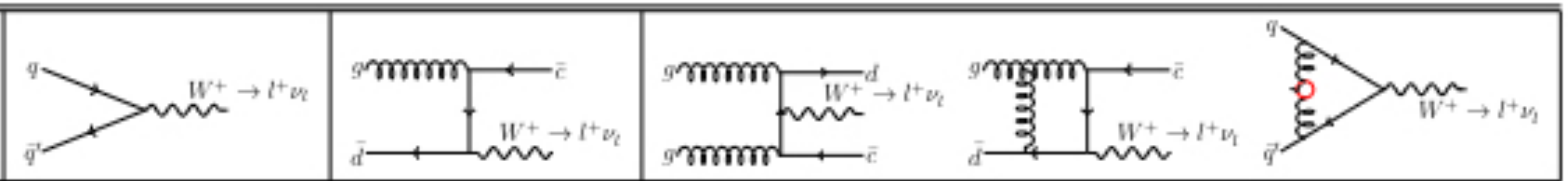
Variable flavour scheme in charged Drell–Yan process

Representative diagrams for $pp \to l^{\pm}\nu_l + X$ in 3FS for W^- production



3FS	–	LO	NLO
5FS	LO	NLO	NNLO

Representative diagrams for $pp \to l^{\pm}\nu_l + X$ in 3FS for W^+ production



3FS	–	LO	NLO
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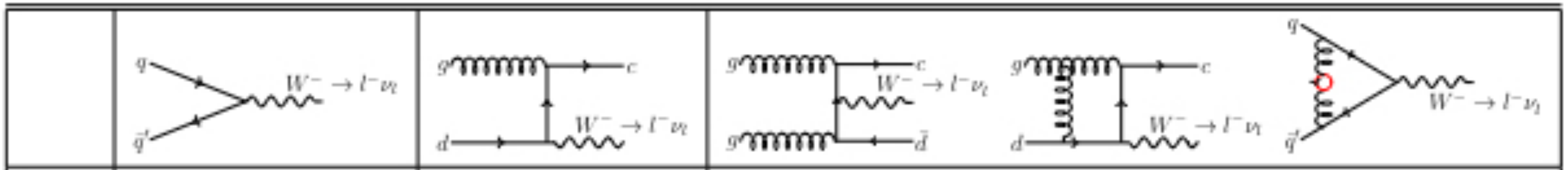

3FS 5FS	– LO	LO NLO	NLO NNLO

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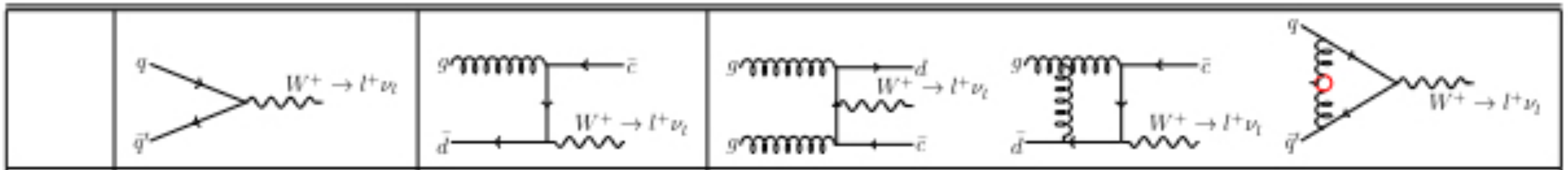
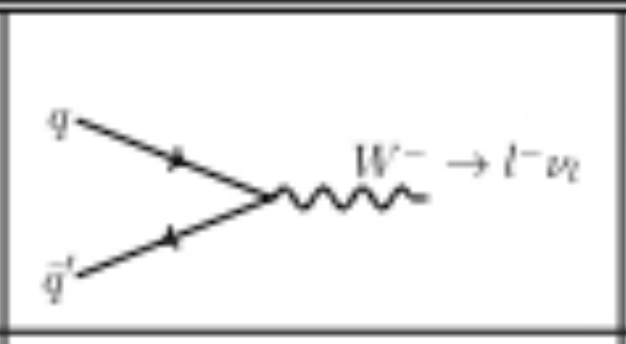
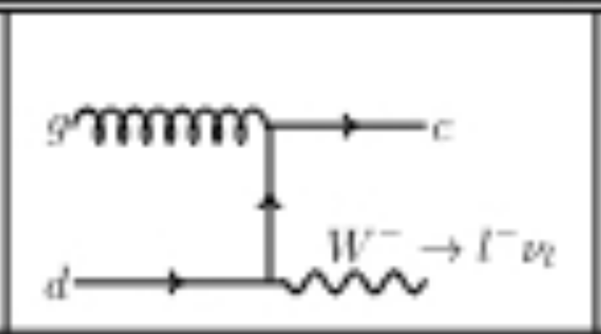
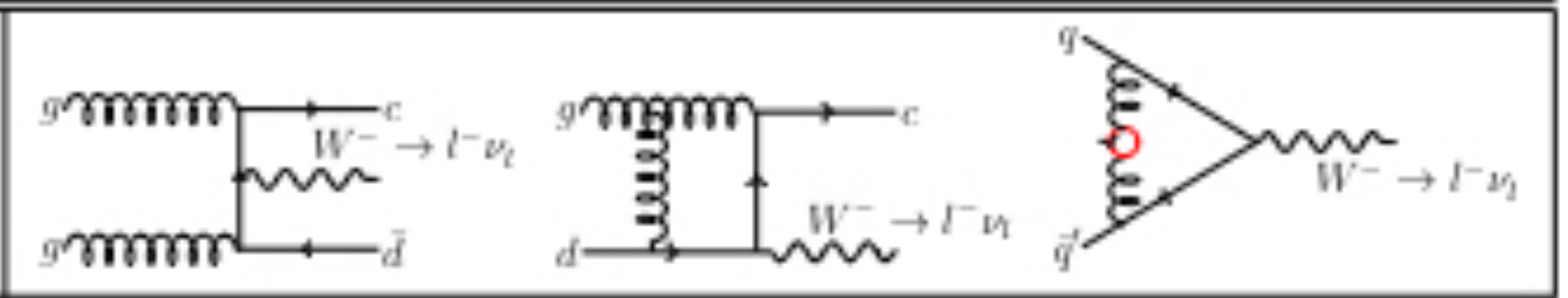
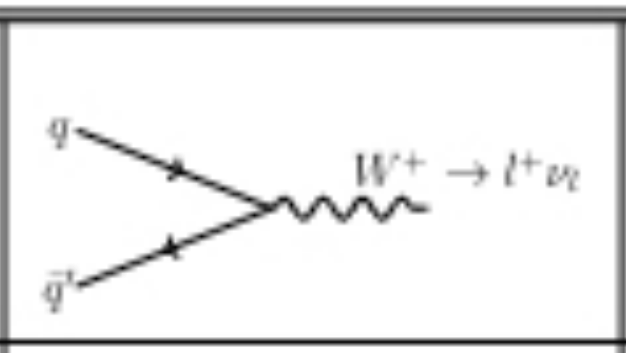
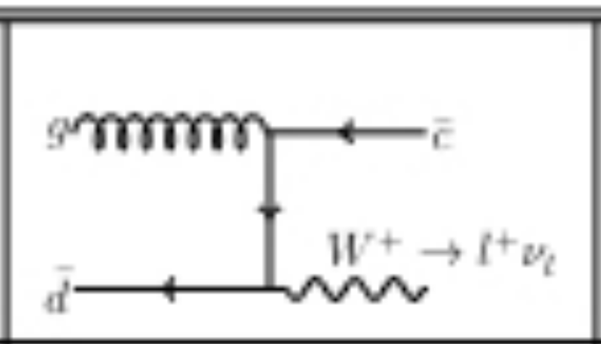
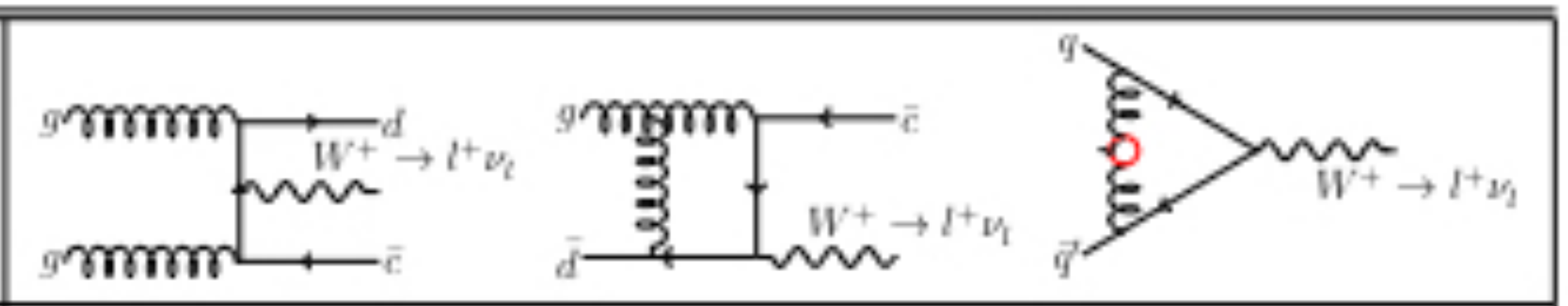


3FS 5FS	– LO	LO NLO	NLO NNLO

Massive and massless components in 3FS

Massive computation: Using in–house code and verified against aMC@NLO

			
3FS 5FS	– LO	LO NLO	NLO NNLO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	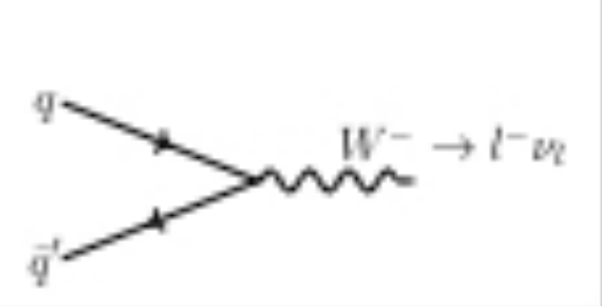	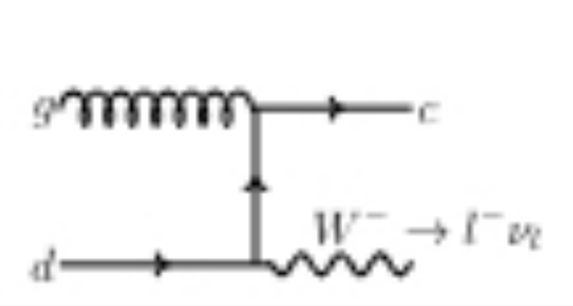	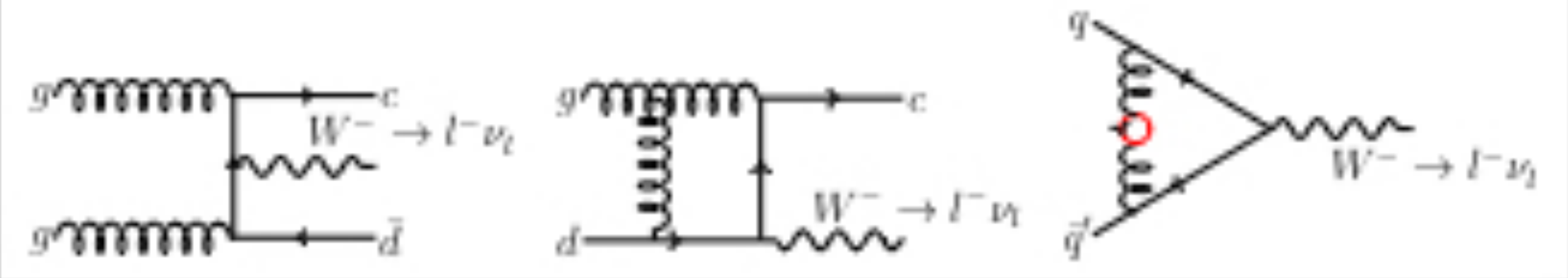
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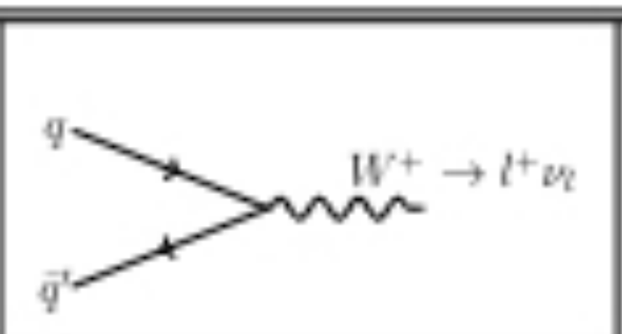
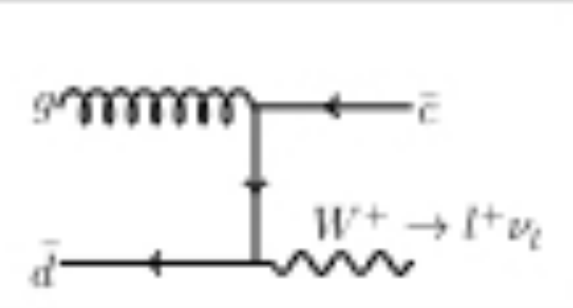
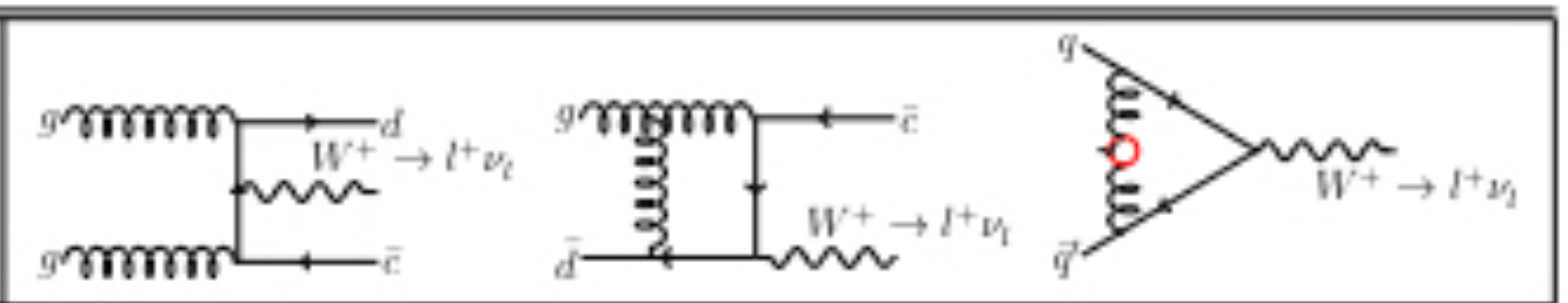
			
3FS 5FS	– LO	LO NLO	NLO NNLO

Massless computation: using n3loxs

Decoupling relations for two–mass variable flavour scheme

- Starting at 3–loop order, Feynman diagrams carrying internal fermion lines of different mass contribute to the OMEs.

- But up to 2–loop order there are no diagrams containing fermion lines of different mass contribute and so we use decoupling relations of charm and bottom quark one at a time.

$$f_b = f_{\bar{b}} = \alpha_s A_{bg}^{(1)} \otimes f_g + \alpha_s^2 \left[A_{bg}^{(2)} \otimes f_g + \sum_{i=-4, i\neq 0}^{4} A_{bi}^{(2)} \otimes f_i \right] + O(\alpha_s^3)$$

$$f_c = f_{\bar{c}} = \alpha_s A_{cg}^{(1)} \otimes f_g + \alpha_s^2 \left[A_{cg}^{(2)} \otimes f_g + \sum_{i=-3, i\neq 0}^{3} A_{ci}^{(2)} \otimes f_i \right] + O(\alpha_s^3)$$

[M. Buza, et.al. 1996]

Sources of logarithmic contributions in 3FS

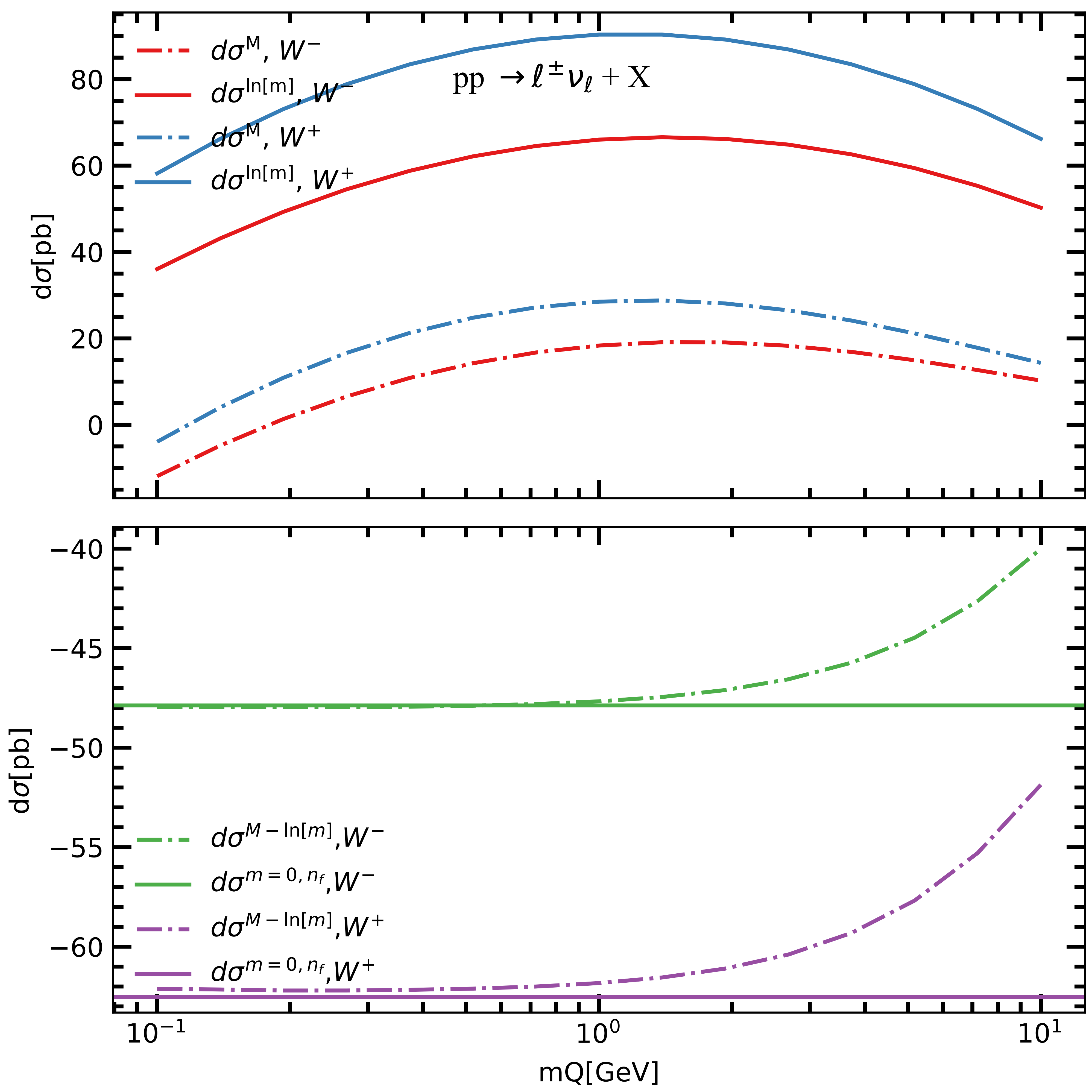
- The logarithmic corrections at 1–loop is simply given by the OMEs and they read as the following for W^- production:

$$d\sigma_{g\bar{u}}^{\ln[\mathrm{M}],\mathrm{n_f}} \propto A_{bg}^{(1)} \otimes \sigma_{b\bar{u}}^{(0)} \qquad d\sigma_{gd}^{\ln[\mathrm{M}],\mathrm{n_f}} \propto A_{cg}^{(1)} \otimes \sigma_{d\bar{c}}^{(0)} \qquad d\sigma_{gs}^{\ln[\mathrm{M}],\mathrm{n_f}} \propto A_{cg}^{(1)} \otimes \sigma_{s\bar{c}}^{(0)}$$

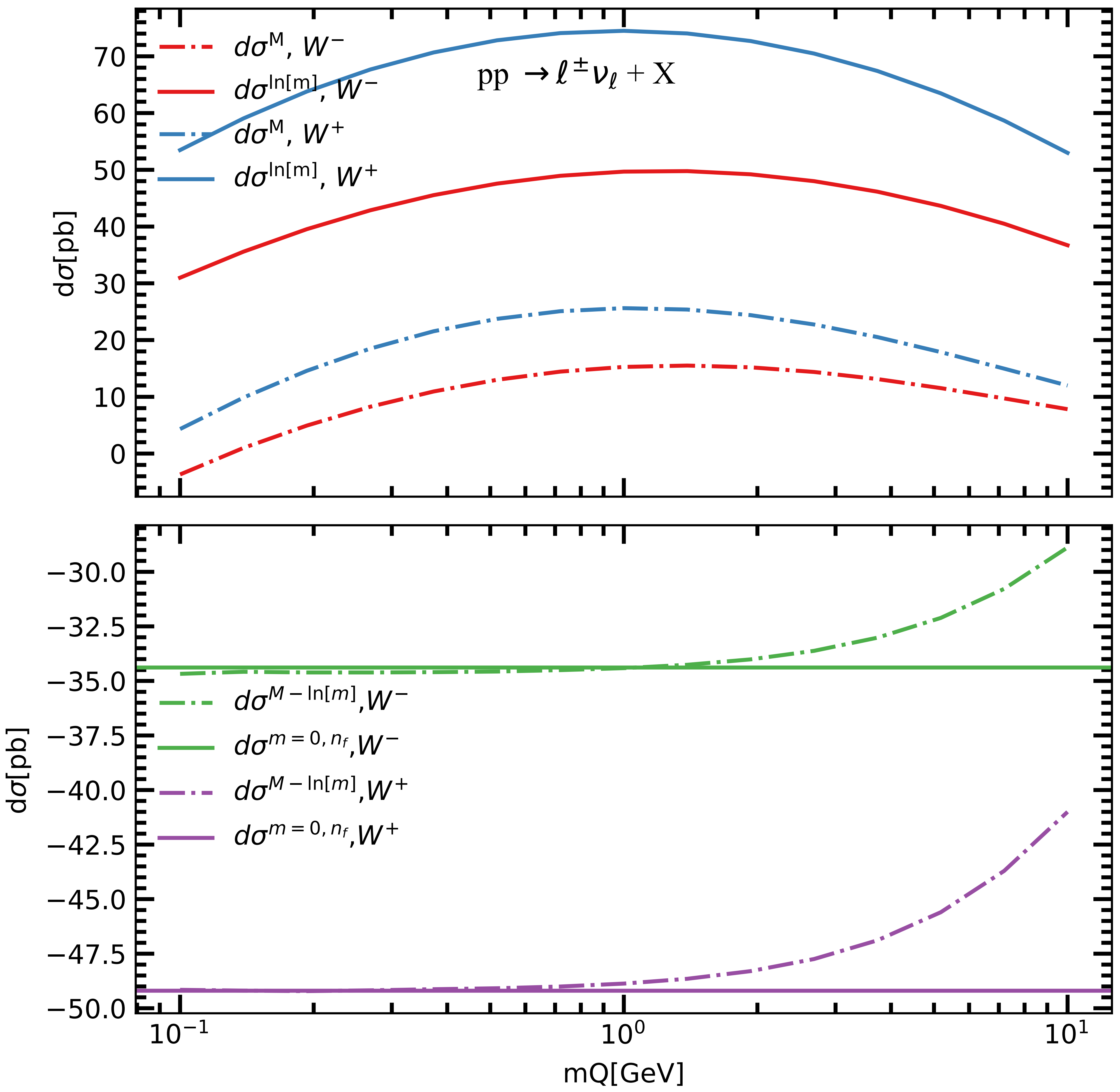
- 2–loop logarithmic corrections are obtained from $\{gq, g\bar{q}, gg\}$ channels containing two–loop single mass OMEs and one–loop partonic coefficient functions. For instance:

$$\delta\eta_{g\bar{u}}^{(2)} \propto A_{bq}^{(2)} \otimes \eta_{b\bar{u}}^{(0)} + A_{bg}^{(2)} \otimes \eta_{b\bar{u}}^{(0)} + A_{bg}^{(1)} \otimes \eta_{b\bar{u}}^{(1)} + A_{cq}^{(2)} \otimes \left(\eta_{\bar{c}d}^{(0)} + \eta_{\bar{c}s}^{(0)}\right)$$

Preliminary analysis to show the behaviour in CDY



With only bottom mass



With charm and bottom mass

Summary

- In this ongoing work we aim to provide a deeper theoretical understanding of the treatment and role of massive quarks in Drell–Yan process.

- We find that the massive power–corrections to the massless predictions are small in general.

- There are a number of cancellations between different partonic channels and different quark flavours (charm and bottom).

- Overall, sum of these corrections is negligible compared to the size of perturbative uncertainty of the massless calculation.