Two-loop QCD amplitudes for $t\bar{t}H$ production from high energy limit

G. Wang, Tianya Xia, Li Lin Yang and Xiaoping Ye, JHEP 05 (2024) 082, JHEP 07 (2024) 121

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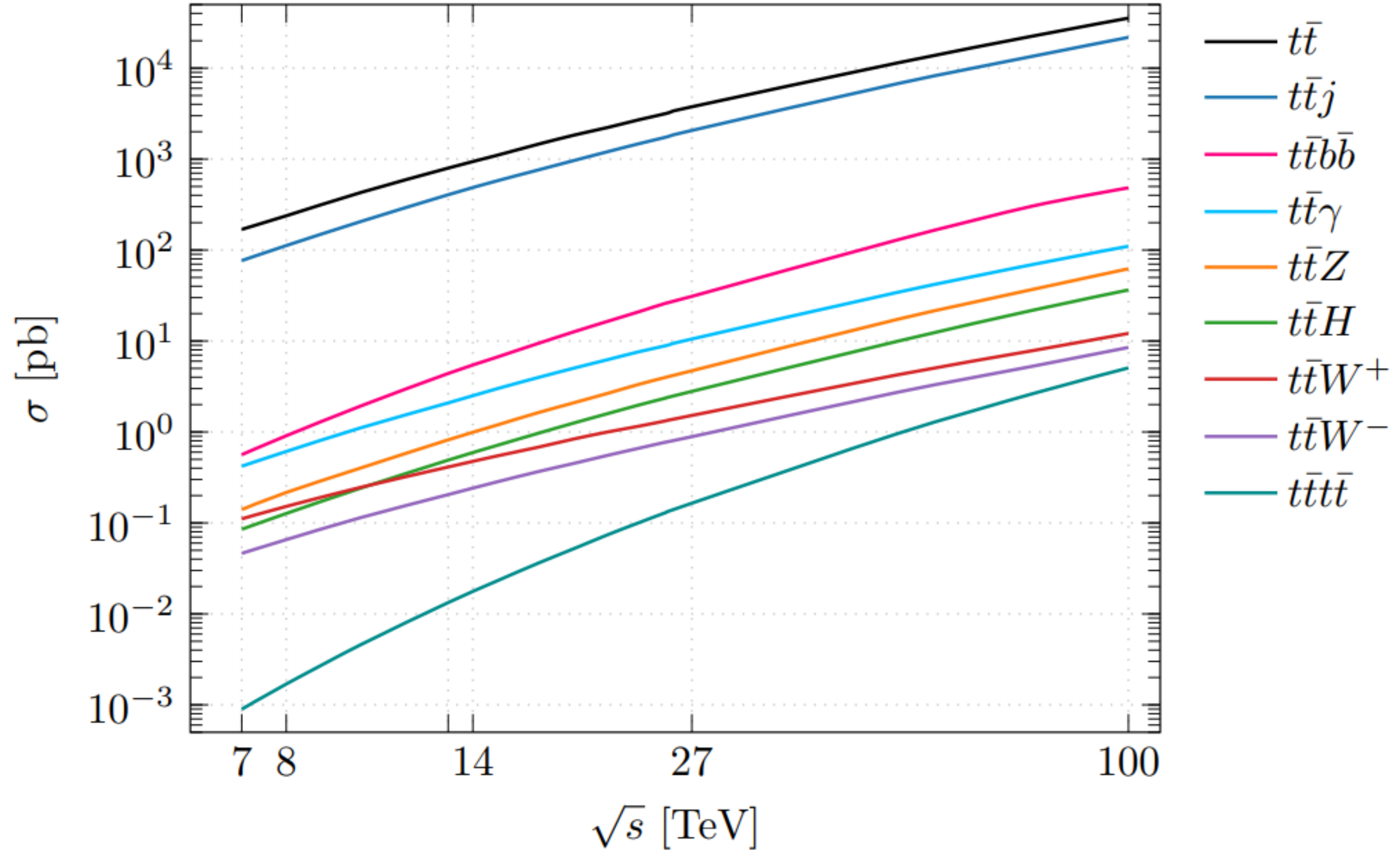
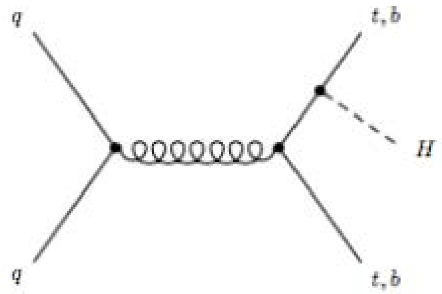
Catalog

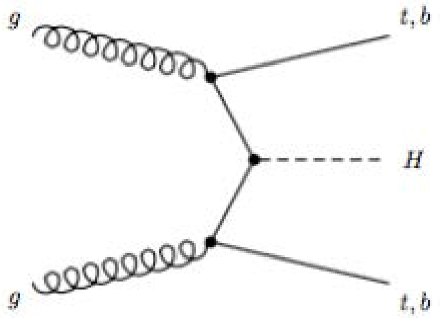
- Introduction to $t\bar{t}H$ production
- Factorization in the high energy limit at leading power
- $t\bar{t}H$ production in the high energy limit at leading power
- Toward the high energy limit at next-to-leading power
- Summary and outlook.

Introduction to $t\bar{t}H$ production

- First observation at the LHC: CMS, 1804.02610; ATLAS, 1806.00425
- Current Exp. acc. ~20% e.g. CMS, 2003.10866; ATLAS, 2004.04545







R. Schwienhorst, *et al.* 2209.11267

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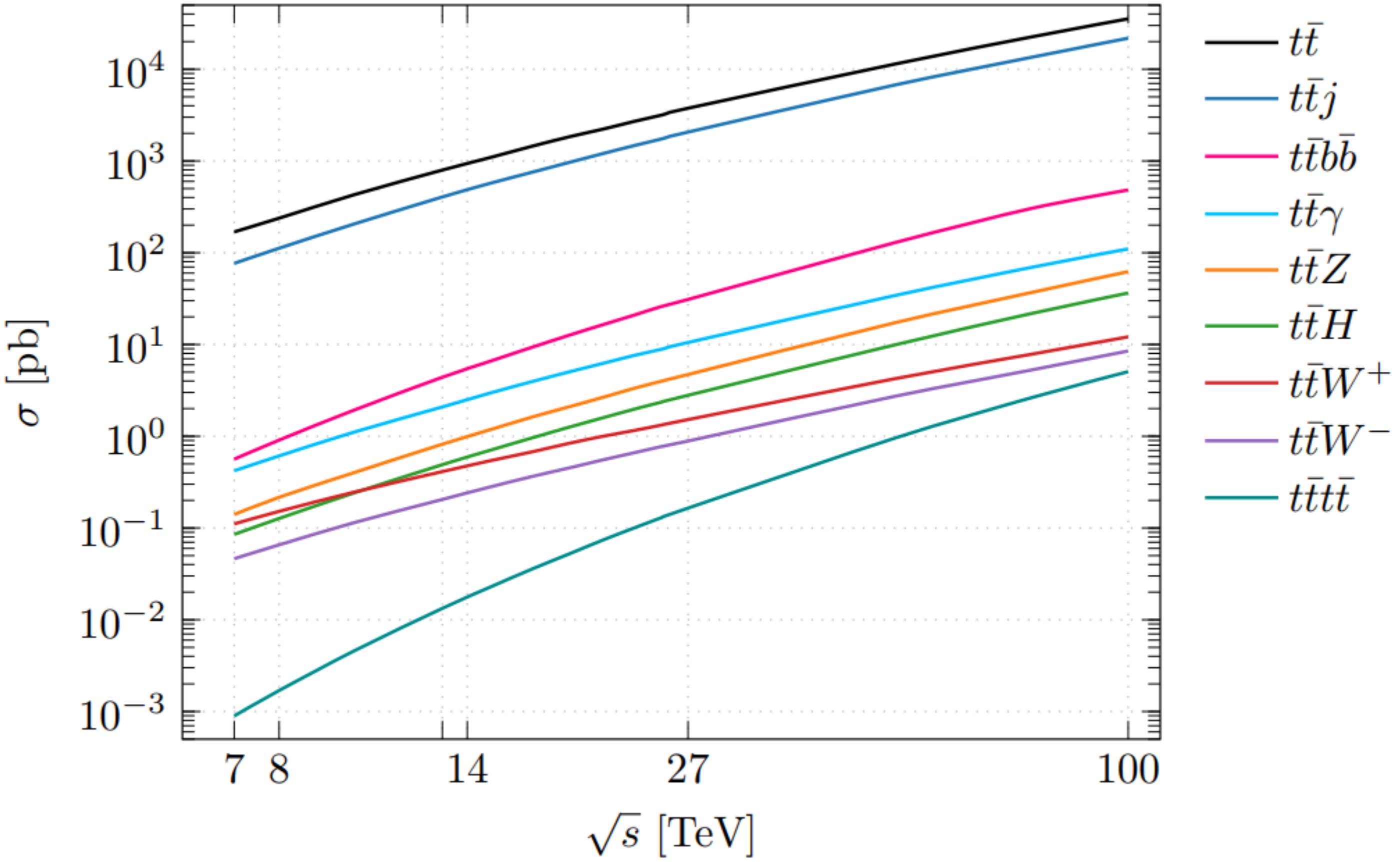

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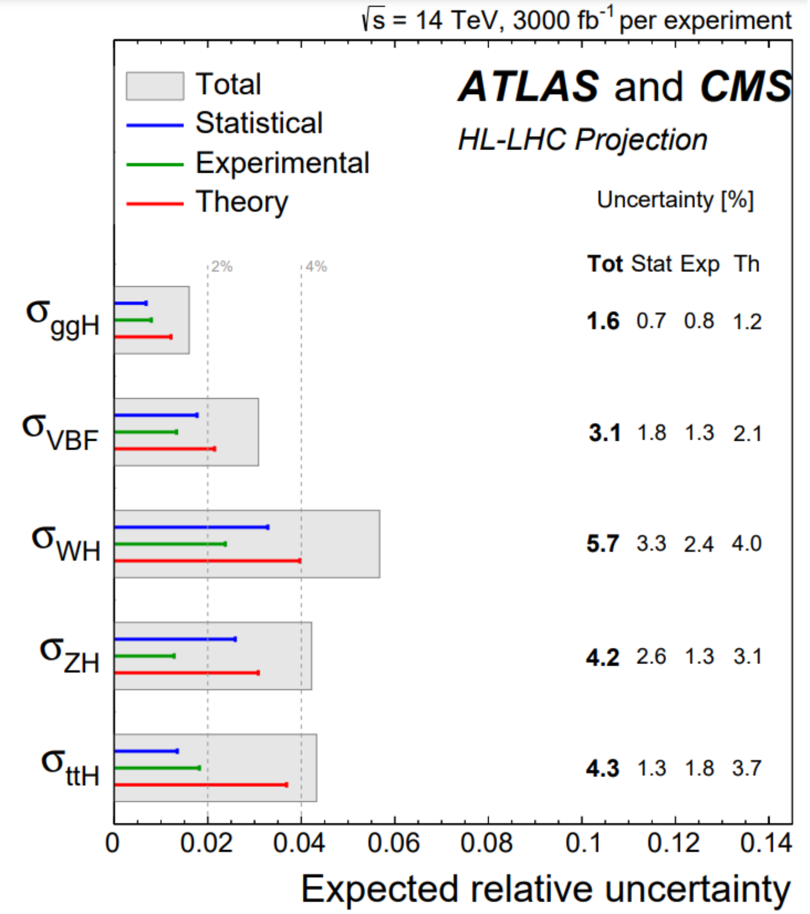

M. Cepeda, *et al.* 1902.00134

Introduction to $t\bar{t}H$ production

- Theory
  - NLO QCD
  - NLO EW
  - NNLL resummation
  - Toward NNLO
    - Off diagonal channels at NLO: less than 1%
      e.g. $qg \to t\bar{t}H + q$
    - Diagonal channels at NNLO: the quark-initiated N_f-part
      5 minutes per phase space point in the bulk region, slower in the high energy region.

e.g. Beenakker, *et al, Phys. Rev. Lett.* 87 (2001)201805
Reina, *et al, Phys. Rev. Lett.* 87 (2001) 201804
Frixione, *et al, JHEP* 06 (2015) 184
Kulesza , *et al, JHEP* 03 (2016) 065
Ju and Yang, *JHEP* 06 (2019) 050

Broggio, *et al. JHEP* 03 (2016) 124 LO: $378.7^{+31.8\%}_{-22.5\%}$

QCD correction:

NLO: $474.8^{+9.4\%}_{-10.9\%}$

NLO + NNLL: $486.4^{+6.1\%}_{-5.0\%}$

$(\mathrm{NLO}+\mathrm{NNLL})_{\mathrm{NNLOexp.}}$: $482.7^{+2.2\%}_{-4.4\%}$

Frixione, *et al, JHEP* 09 (2014) 065 NLO EW: $\sim 1.8\%$

13 TeV

Catani, *et al, Eur. Phys. J. C* 81 (2021) 491

Agarwal, *et al, JHEP* 05 (2024) 013

See talk by A. Olsson

Approximation method!

Introduction to $t\bar{t}H$ production

- Approximation at NNLO
  - Soft Higgs approximation — Catani, *et al. Phys. Rev. Lett.* 130 (2023) 111902

$$\mathcal{M}(\{p_i\},k) \simeq F(m_t) \sum_{i=t,\bar{t}} \frac{m_t}{p_i \cdot k} \mathcal{M}(\{p_i\})$$

$2 \to 3 \quad \Rightarrow \quad 2 \to 2$

σ [pb]	$\sqrt{s} = 13$ TeV	$\sqrt{s} = 100$ TeV
σ_{LO}	$0.3910\,^{+31.3\%}_{-22.2\%}$	$25.38\,^{+21.1\%}_{-16.0\%}$
σ_{NLO}	$0.4875\,^{+5.6\%}_{-9.1\%}$	$36.43\,^{+9.4\%}_{-8.7\%}$
σ_{NNLO}	$0.5070\,(31)^{+0.9\%}_{-3.0\%}$	$37.20(25)\,^{+0.1\%}_{-2.2\%}$

See talk by C. Savoini.

  - High energy limit — GW, Xia, Yang and Ye: *JHEP* 05 (2024) 082

$$\left|\mathcal{M}^{\mathrm{massive}}(\{p\},\{m\})\right\rangle = \prod_i \left(\mathcal{Z}^{(m|0)}_{[i]}(\{m\})\right)^{1/2} \boldsymbol{\mathcal{S}}(\{p\},\{m\}) \left|\mathcal{M}^{\mathrm{massless}}(\{p\})\right\rangle$$

Massive amplitude $\Rightarrow$ Massless amplitude

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Factorization in the high energy limit at LP

$$h_1(p_1,m_1)+h_2(p_2,m_2)\to h_3(p_3,m_3)+h_4(p_4,m_4)+\cdots+h_{n+2}(p_{n+2},m_{n+2})+X(\{p_X\},\{m_X\})$$

- High energy limit: $|s_{ij}|\gg m_k^2,\ i\neq j$

Mitov and Moch: *JHEP* 05 (2007) 001
GW , Xia, Yang and Ye: *JHEP* 05 (2024) 082

$$\left|\mathcal{M}^{\rm massive}(\{p\},\{m\})\right\rangle=\prod_i\left(\mathcal{Z}_{[i]}^{(m|0)}(\{m\})\right)^{1/2}\boldsymbol{\mathcal{S}}(\{p\},\{m\})\left|\mathcal{M}^{\rm massless}(\{p\})\right\rangle \quad i=q,\,Q,\,g$$

- Up to NNLO

$$\mathcal{S}(\{p\},\{m\})=1+\left(\frac{\alpha_s}{4\pi}\right)^2\sum_{j,l,\,j\neq l}(-\boldsymbol{T}_j\cdot\boldsymbol{T}_l)\sum_h\mathcal{S}^{(2)}(s_{jl},m_h^2)+\mathcal{O}(\alpha_s^3)$$

$$\mathcal{Z}_{[i]}^{(m|0)}(\{m\})=1+\sum_{n=1}^{\infty}\left(\frac{\alpha_s}{4\pi}\right)^n\mathcal{Z}_{[i]}^{(n)}$$

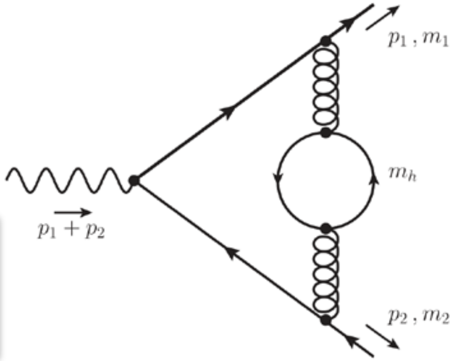
$$\mathcal{Z}_{[Q]}^{(1)}$$

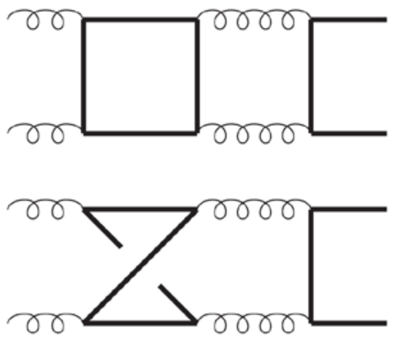
$$\mathcal{Z}_{[g]}^{(1)}=\sum_h\mathcal{Z}_{[g]}^{(1),h}$$

$$\mathcal{Z}_{[q]}^{(2)}=\sum_h\mathcal{Z}_{[q]}^{(2),h}$$

$$\mathcal{Z}_{[g]}^{(2)}=\left(\mathcal{Z}_{[g]}^{(1)}\right)^2+\frac{4}{3\epsilon}(n_l+n_h)T_F\,\mathcal{Z}_{[g]}^{(1)}+\sum_h\mathcal{Z}_{[g]}^{(2),h}$$

$$\mathcal{Z}_{[Q]}^{(2)}=\mathcal{Z}_{[Q]}^{(2),l}+\mathcal{Z}_{[Q]}^{(2),Q}+\sum_{h\neq Q}\mathcal{Z}_{[Q]}^{(2),h}$$



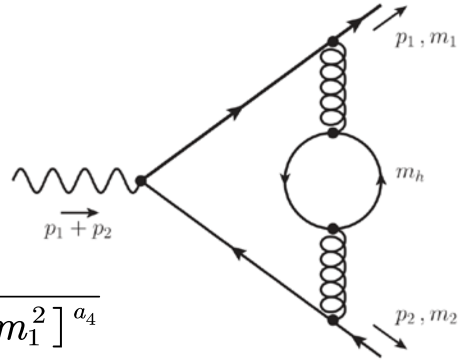


Czakon, Mitov and Moch:
Nucl.Phys.B 798 (2008) 210-250

Factorization in the high energy limit at LP

- Determine soft function and $\mathcal{Z}$-factor:
  - $q\bar{q}$-vector vertex

$F_2 \propto m_Q^2$



$$\Gamma^\mu(p_1,p_2) = F_1(s,m_Q^2,m_h^2)\,\gamma^\mu + \frac{1}{2m_Q}F_2(s,m_Q^2,m_h^2)\,i\,\sigma^{\mu\nu}(p_1+p_2)_\nu$$

$$I_{\{a_i\}} \equiv \mu^{4\epsilon}\int \frac{dk_1}{(2\pi)^d}\frac{dk_2}{(2\pi)^d}\frac{1}{[k_1^2-m_h^2]^{a_1}}\frac{1}{[k_2^2-m_h^2]^{a_2}}\frac{1}{[(k_1+k_2)^2]^{a_3}}\frac{1}{[(k_1+k_2-p_1)^2-m_1^2]^{a_4}}$$
$$\times\frac{(-\tilde{\mu}^2)^\nu}{[(k_1+k_2+p_2)^2-m_2^2]^{a_5+\nu}}\frac{1}{[(k_1-p_1)^2]^{a_6}}\frac{1}{[(k_1+p_2)^2]^{a_7}},$$

  - Light-cone coordinate $\quad n_i^2 = \bar{n}_i^2 = 0\,,\quad n_i\cdot\bar{n}_i = 2$

$$n^\mu = n_1^\mu = (1,0,0,1)$$
$$\bar{n}^\mu = n_2^\mu = (1,0,0,-1)$$

  - Region expansion

$$\text{hard}: k^\mu \sim \sqrt{|s|}\,,$$
$$n_i\ \text{collinear}: (n_i\cdot k,\ \bar{n}_i\cdot k,\ k_\perp) \sim \sqrt{|s|}\,(\lambda^2,\ 1,\ \lambda)\,,$$
$$\text{soft}: k^\mu \sim \sqrt{|s|}\,\lambda\,.$$
$$\vdots$$

$$q^\mu = q^-\frac{\bar{n}^\mu}{2} + q^+\frac{n^\mu}{2} + q_\perp^\mu$$

$$\lambda = \frac{m}{\sqrt{|\hat{s}|}}$$

$$cc:\ (k_1+k_2+p_2)^2 - m_2^2 \to \bar{n}\cdot(k_1+k_2)\,n\cdot p_2\,,\quad (k_1+p_2)^2 \to \bar{n}\cdot k_1\,n\cdot p_2$$

Factorization in the high energy limit at LP

- Determine soft function and $\mathcal{Z}$-factor:
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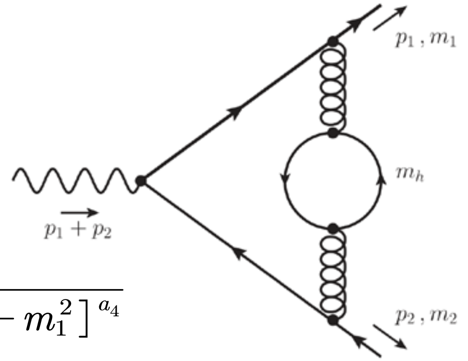
$F_2 \propto m_Q^2$

$$\Gamma^\mu(p_1,p_2) = F_1(s,m_Q^2,m_h^2)\,\gamma^\mu + \frac{1}{2m_Q} F_2(s,m_Q^2,m_h^2)\, i\sigma^{\mu\nu}(p_1+p_2)_\nu$$



$$I_{\{a_i\}} \equiv \mu^{4\epsilon} \int \frac{dk_1}{(2\pi)^d} \frac{dk_2}{(2\pi)^d} \frac{1}{[k_1^2 - m_h^2]^{a_1}} \frac{1}{[k_2^2 - m_h^2]^{a_2}} \frac{1}{[(k_1+k_2)^2]^{a_3}} \frac{1}{[(k_1+k_2-p_1)^2 - m_1^2]^{a_4}}$$
$$\times \frac{(-\tilde{\mu}^2)^\nu}{[(k_1+k_2+p_2)^2 - m_2^2]^{a_5+\nu}} \frac{1}{[(k_1-p_1)^2]^{a_6}} \frac{1}{[(k_1+p_2)^2]^{a_7}},$$

$$x = \frac{m_h}{m_Q}$$

$$F_{1,cc}^{(2),\text{bare}}(s,m_Q^2,m_h^2) = C_F T_F \left(\frac{\mu^2}{m_h^2}\right)^{2\epsilon} \left(\frac{1}{\nu} + \ln\frac{\tilde{\mu}^2}{-s}\right) \left(\frac{4}{3\epsilon^2} - \frac{20}{9\epsilon} + \frac{112}{27} + \frac{4\zeta_2}{3}\right)$$
$$+ C_F T_F \left(\frac{\mu^2}{m_Q^2}\right)^{2\epsilon} \left\{ -\frac{4}{3\epsilon^3} - \frac{1}{\epsilon^2}\left[\frac{4}{3} - \frac{8}{3}H(0,x)\right] - \frac{1}{\epsilon}\left[\frac{8\zeta_2}{3} + \frac{56}{9} + \frac{16}{3}H(0,0,x)\right]\right.$$
$$- \frac{4\zeta(3)}{9} - \left(\frac{20x^3}{3} - 36x + \frac{80}{9}\right)\zeta_2 + \frac{40x^2}{9} - \frac{2504}{81} + \left(\frac{40x^2}{9} + \frac{8\zeta_2}{3} - \frac{224}{27}\right)H(0,x)$$
$$- \left(\frac{20x^3}{9} - 12x - \frac{88}{9}\right)H(-1,0,x) - \left(\frac{20x^3}{9} - 12x + \frac{88}{9}\right)H(1,0,x) - \frac{80}{9}H(0,0,x)$$
$$\left. + \frac{16}{3}H(0,-1,0,x) - \frac{16}{3}H(0,1,0,x)\right\},$$

$$\boxed{x \to 0:\ \text{divergent}, \qquad x \to 1:\ \text{finite}}$$

$$F_{1,\bar{c}\bar{c}}^{(2),\text{bare}}(s,m_Q^2,m_h^2) = C_F T_F \left(\frac{\mu^2}{m_h^2}\right)^{2\epsilon} \left(\frac{1}{\nu} + \ln\frac{\tilde{\mu}^2}{m_h^2}\right) \left(-\frac{4}{3\epsilon^2} + \frac{20}{9\epsilon} - \frac{112}{27} - \frac{4\zeta_2}{3}\right)$$
$$+ C_F T_F \left(\frac{\mu^2}{m_Q^2}\right)^{2\epsilon} \left\{ \frac{2}{3\epsilon^3} - \frac{1}{\epsilon^2}\left[\frac{28}{9} + \frac{16}{3}H(0,x)\right] - \frac{1}{\epsilon}\left[\frac{2\zeta_2}{3} + \frac{212}{27} - \frac{64}{9}H(0,x)\right.\right.$$
$$\left. - \frac{80}{3}H(0,0,x) + \right] - \frac{40\zeta_3}{9} - \left(\frac{20x^3}{3} - 36x + \frac{32}{3}\right)\zeta_2 + \frac{40x^2}{9} - \frac{1652}{81}$$
$$+ \left(\frac{40x^2}{9} - \frac{16\zeta_2}{3} - \frac{16}{9}\right)H(0,x) - \frac{112}{3}H(0,0,x) - \left(\frac{20x^3}{9} - 12x - \frac{88}{9}\right)H(-1,0,x)$$
$$- \left(\frac{20x^3}{9} - 12x + \frac{88}{9}\right)H(1,0,x) + \frac{16}{3}H(0,-1,0,x) - 128H(0,0,0,x)$$
$$\left. - \frac{16}{3}H(0,1,0,x)\right\}.$$

Factorization in the high energy limit at LP

- Determine soft function and $\mathcal{Z}$-factor:

$$\mathcal{Z}^{(2),h}_{[Q]} = F^{(2),\text{bare}}_{1,cc}(s,m_Q^2,m_h^2) + F^{(2),\text{bare}}_{1,\bar{c}\bar{c}}(s,m_Q^2,m_h^2) + Z^{(1),h}_{\alpha_s}\left[F^{(1),\text{bare}}_{1,c}(s,m_Q^2) + F^{(1),\text{bare}}_{1,\bar{c}}(s,m_Q^2)\right] + Z^{(2)}_Q - C_F\,\mathcal{S}^{(2)}(s,m_h^2)\,,$$

$$\mathcal{Z}^{(2),Q}_{[Q]} = \mathcal{Z}^{(2),h}_{[Q]}\Big|_{x\to 1}$$

$$\mathcal{Z}^{(2),h}_{[q]} = F^{(2),\text{bare}}_{1,cc}(s,0,m_h^2) + F^{(2),\text{bare}}_{1,\bar{c}\bar{c}}(s,0,m_h^2) + Z^{(2)}_q - C_F\,\mathcal{S}^{(2)}(s,m_h^2)\,,$$

$$\mathcal{Z}^{(2),h}_{[g]} = F^{(2),\text{bare}}_{gg,cc}(s,m_h^2) + F^{(2),\text{bare}}_{gg,\bar{c}\bar{c}}(s,m_h^2) + Z^{(2)}_g - C_A\,\mathcal{S}^{(2)}(s,m_h^2)\,,$$

  - Soft function: $\mathcal{S}^{(2)}(s,m_h^2) = T_F\left(\frac{\mu^2}{m_h^2}\right)^{2\epsilon}\left(-\frac{4}{3\epsilon^2} + \frac{20}{9\epsilon} - \frac{112}{27} - \frac{4\zeta_2}{3}\right)\ln\frac{-s}{m_h^2}$
  - $\mathcal{Z}$-factor:

$$\mathcal{Z}^{(2),Q}_{[Q]} = C_F T_F\left[\frac{2}{\epsilon^3} + \frac{1}{\epsilon^2}\left(\frac{4}{3}\ln\frac{\mu^2}{m_Q^2} + \frac{8}{9}\right) + \frac{1}{\epsilon}\left(\frac{4}{9}\ln\frac{\mu^2}{m_Q^2} - \frac{65}{27} - 2\zeta_2\right) - \frac{4}{9}\ln^3\frac{\mu^2}{m_Q^2} - \frac{2}{9}\ln^2\frac{\mu^2}{m_Q^2} - \left(\frac{274}{27} + \frac{16\zeta_2}{3}\right)\ln\frac{\mu^2}{m_Q^2} + \frac{5107}{162} - \frac{70\zeta_2}{9} - \frac{4\zeta_3}{9}\right]$$

- Validation: full form factor, top quark pair production and IR structures.

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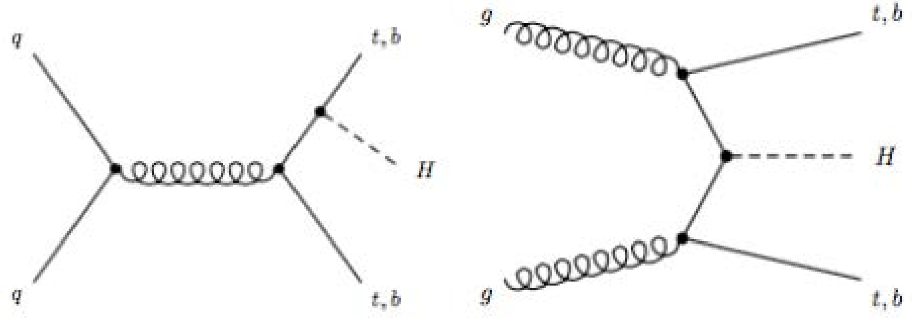
$t\bar{t}H$ production in the high energy limit

- The partonic processes:

$$q_\beta(p_1) + \bar{q}_\alpha(p_2) \to t_k(p_3) + \bar{t}_l(p_4) + H(p_5)$$
$$g_a(p_1) + g_b(p_2) \to t_k(p_3) + \bar{t}_l(p_4) + H(p_5)$$



- UV and IR singularities:

$$s_{ij} \equiv (\sigma_i p_i + \sigma_j p_j)^2, \quad \tilde{s}_{ij} = 2\sigma_i\sigma_j \tilde{p}_i \cdot \tilde{p}_j$$
$$p_1^2 = p_2^2 = 0, \quad p_3^2 = p_4^2 = m_t^2, \quad p_5^2 = m_H^2$$

$$\left|\mathcal{M}^R_{q,g}(\alpha_s, g_Y, m_t, \mu, \epsilon)\right\rangle = \left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^{-3\epsilon/2} Z_{q,g} Z_Q \left|\mathcal{M}^{\text{bare}}_{q,g}(\alpha_s^0, g_Y^0, m_t^0, \epsilon)\right\rangle$$

$$\boldsymbol{Z}^{-1}_{q,g}(\epsilon, m_t, \mu)\left|\mathcal{M}^R_{q,g}(\alpha_s, g_Y, m_t, \mu, \epsilon)\right\rangle = \text{finite}$$

$$\left|\mathcal{M}^{(1),\,\text{sing}}_{q,g}\right\rangle = \boldsymbol{Z}^{(1)}_{q,g}\left|\mathcal{M}^{(0)}_{q,g}\right\rangle,$$
$$\left|\mathcal{M}^{(2),\,\text{sing}}_{q,g}\right\rangle = \left[\boldsymbol{Z}^{(2)}_{q,g} - \left(\boldsymbol{Z}^{(1)}_{q,g}\right)^2\right]\left|\mathcal{M}^{(0)}_{q,g}\right\rangle + \left(\boldsymbol{Z}^{(1)}_{q,g}\left|\mathcal{M}^{(1)}_{q,g}\right\rangle\right)_{\text{poles}}$$

Chen, Ma, **GW** , Yang and Ye, JHEP 04 (2022) 025

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- The partonic processes:

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$$g_a(p_1) + g_b(p_2) \to t_k(p_3) + \bar{t}_l(p_4) + H(p_5)$$

- Massive cases:

$$\left|\mathcal{M}^{(l)}_{q,g}\right\rangle = \sum_{I,i} c^{(l)q,g}_{Ii} \left|c^{q,g}_I\right\rangle \otimes \left|d^{q,g}_i\right\rangle$$

$$c^{R;q,g}_{Ii} = \sum_j \frac{\left(D^{-1}_{q,g}\right)_{ij}}{\left\langle c^{q,g}_I \middle| c^{q,g}_I\right\rangle} \left[\left\langle d^{q,g}_j\right| \otimes \left\langle c^{q,g}_I \middle| \mathcal{M}^R_{q,g}\right\rangle\right]$$

$\left|d^q_i\right\rangle$: 28

$\left|d^g_i\right\rangle$: 40

$$D^{q,g}_{ij} = \left\langle d^{q,g}_i \middle| d^{q,g}_j\right\rangle$$

- Massless cases:

$$\left|\tilde{\mathcal{M}}^{R}_{q,g}\right\rangle = \sum_{I,i} \tilde{c}^{R;q,g}_{Ii} \left|c^{q,g}_I\right\rangle \otimes \left|\tilde{d}^{q,g}_i\right\rangle$$

$$\tilde{c}^{R;q,g}_{Ii} = \sum_j \frac{\left(\tilde{D}^{-1}_{q,g}\right)_{ij}}{\left\langle c^{q,g}_I \middle| c^{q,g}_I\right\rangle} \left[\left\langle \tilde{d}^{q,g}_j\right| \otimes \left\langle c^{q,g}_I \middle| \tilde{\mathcal{M}}^R_{q,g}\right\rangle\right]$$

$\left|\tilde{d}^q_i\right\rangle$: 14

$\left|\tilde{d}^g_i\right\rangle$: 20

$$\tilde{D}^{q,g}_{ij} = \left\langle \tilde{d}^{q,g}_i \middle| \tilde{d}^{q,g}_j\right\rangle$$

$t\bar{t}H$ production in the high energy limit

- Amplitudes in high energy limit:

Massless scheme

$$\left|\bar{\mathcal{M}}^{R}_{q,g}\left(\epsilon,\{p\},m_t,m_H,\mu\right)\right\rangle = \mathcal{Z}^{(m|0)}_{[q,g]}(\epsilon,m_t,\mu)\,\mathcal{Z}^{(m|0)}_{[t]}(\epsilon,m_t,\mu) \times \boldsymbol{\mathcal{S}}(\epsilon,\{\tilde{p}\},m_t,\mu)\sum_{I,i}\tilde{c}^{R;q,g}_{Ii}\,|c^{q,g}_I\rangle\otimes|\tilde{d}^{q,g}_i\rangle$$

Massive scheme

$$\left|\hat{\mathcal{M}}^{R}_{q,g}\left(\epsilon,\{p\},m_t,m_H,\mu\right)\right\rangle = \mathcal{Z}^{(m|0)}_{[q,g]}(\epsilon,m_t,\mu)\,\mathcal{Z}^{(m|0)}_{[t]}(\epsilon,m_t,\mu) \times \boldsymbol{\mathcal{S}}(\epsilon,\{\tilde{p}\},m_t,\mu)\sum_{I,i}\tilde{c}^{R;q,g}_{Ii}\,|c^{q,g}_I\rangle\otimes|\hat{d}^{q,g}_i\rangle$$

- Squared Amplitudes:

$$\left\langle\mathcal{M}^{R}_{q,g}\middle|\mathcal{M}^{R}_{q,g}\right\rangle \qquad \left\langle\bar{\mathcal{M}}^{R}_{q,g}\middle|\bar{\mathcal{M}}^{R}_{q,g}\right\rangle \qquad \left\langle\hat{\mathcal{M}}^{R}_{q,g}\middle|\hat{\mathcal{M}}^{R}_{q,g}\right\rangle$$

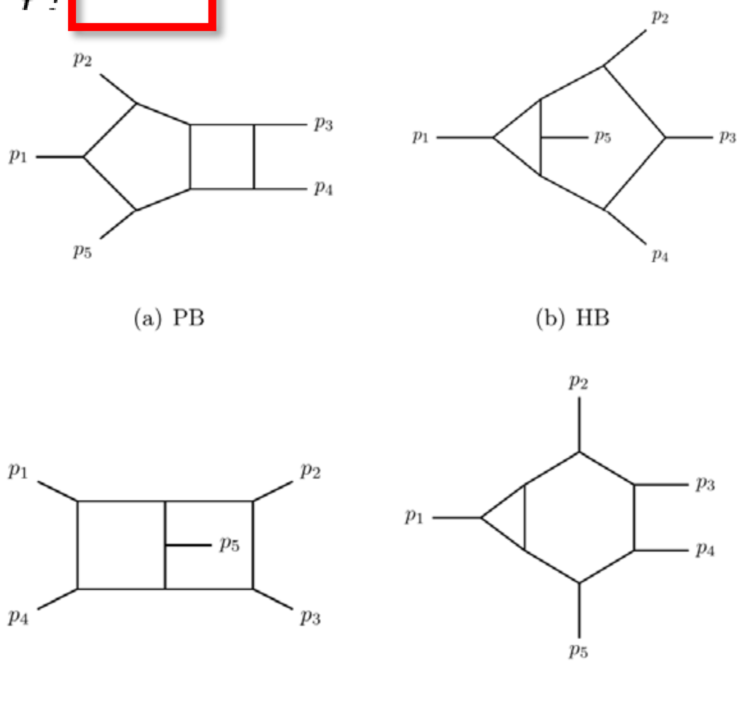
$$\left\langle\mathcal{M}^{(0)R}_{q,g}\middle|\mathcal{M}^{(2)R}_{q,g}\right\rangle \qquad \left\langle\bar{\mathcal{M}}^{(0)R}_{q,g}\middle|\bar{\mathcal{M}}^{(2)R}_{q,g}\right\rangle \qquad \left\langle\mathcal{M}^{(0)R}_{q,g}\middle|\hat{\mathcal{M}}^{(2)R}_{q,g}\right\rangle$$

$t\bar{t}H$ production in the high energy limit

- Calculation of massless form factors:

$$\left|\hat{\mathcal{M}}^{R}_{q,g}\left(\epsilon,\{p\},m_t,m_H,\mu\right)\right\rangle = \mathcal{Z}^{(m|0)}_{[q,g]}(\epsilon,m_t,\mu)\,\mathcal{Z}^{(m|0)}_{[t]}(\epsilon,m_t,\mu)$$
$$\times\,\boldsymbol{\mathcal{S}}(\epsilon,\{\tilde{p}\},m_t,\mu)\sum_{I,i}\tilde{c}^{R;q,g}_{Ii}\,|c^{q,g}_{I}\rangle\otimes|\hat{d}^{q,g}_{i}\rangle$$

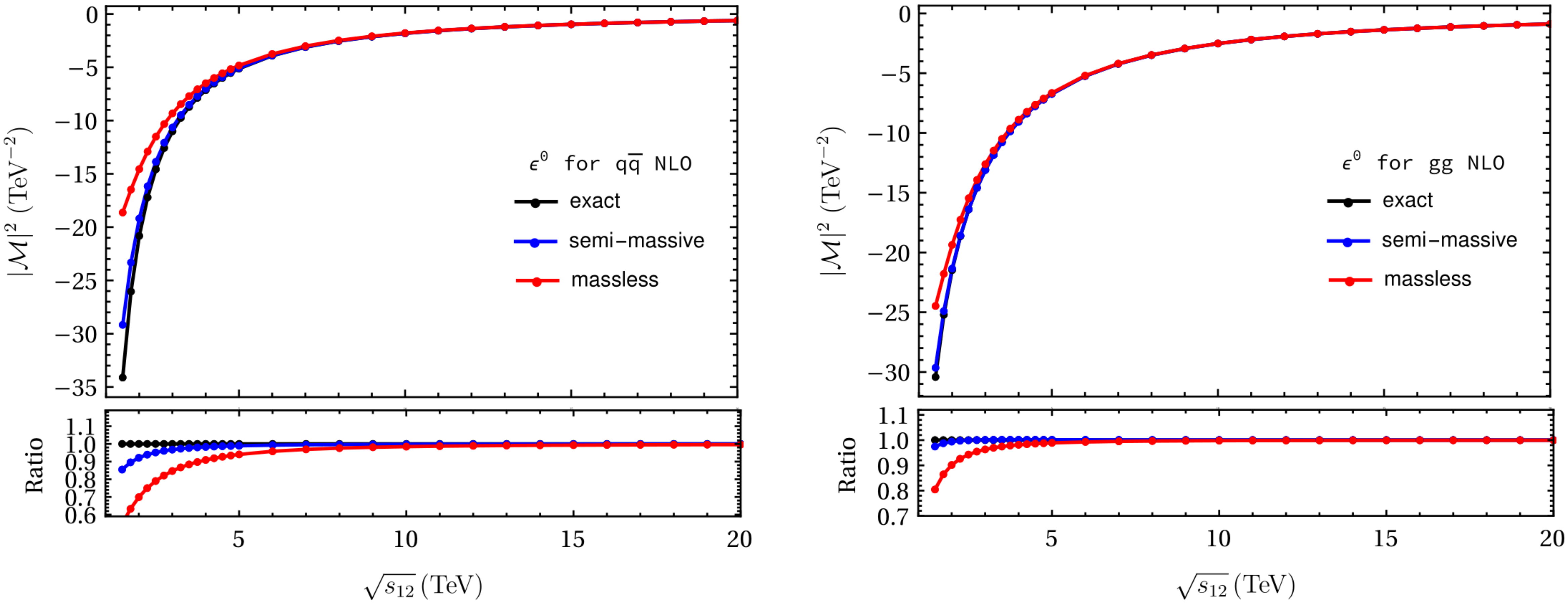
	$\vec{\mathbf{D}}_{a,\sigma_0}$	$\vec{\mathbf{D}}_{b,\sigma_0}$	$\vec{\mathbf{D}}_{c,\sigma_0}$	$\vec{\mathbf{D}}_{d,\sigma_0}$
1	$(l_1)^2$	$(l_1)^2$	$(l_1)^2$	$(l_1)^2$
2	$(l_1+p_1)^2$	$(l_1-p_1)^2$	$(l_1-p_1)^2$	$(l_1-p_1)^2$
3	$(l_1+p_1+p_2)^2$	$(l_1-p_1-p_2)^2$	$(l_1-p_1-p_2)^2$	$(l_1-p_1-p_2)^2$
4	$(l_1-p_4-p_5)^2$	$(l_1+p_4+p_5)^2$	$(l_2)^2$	$(l_2)^2$
5	$(l_2)^2$	$(l_2)^2$	$(l_2+p_4+p_5)^2$	$(l_2+p_4+p_5)^2$
6	$(l_2-p_4-p_5)^2$	$(l_2+p_5)^2$	$(l_2+p_5)^2$	$(l_2-p_1-p_2)^2$
7	$(l_2-p_5)^2$	$(l_1-l_2)^2$	$(l_1-l_2)^2$	$(l_2+p_5)^2$
8	$(l_1-l_2)^2$	$(l_1-l_2+p_4)^2$	$(l_1-l_2+p_3)^2$	$(l_1-l_2)^2$
9	$(l_1-p_5)^2$	$(l_2-p_1)^2$	$(l_1+p_5)^2$	$(l_1-l_2+p_3)^2$
10	$(l_2+p_1)^2$	$(l_2-p_1-p_2)^2$	$(l_2-p_1)^2$	$(l_1+p_5)^2$
11	$(l_2+p_1+p_2)^2$	$(l_2+p_4+p_5)^2$	$(l_2-p_1-p_2)^2$	$(l_2-p_1)^2$



(a) PB (b) HB (c) DP (d) HT

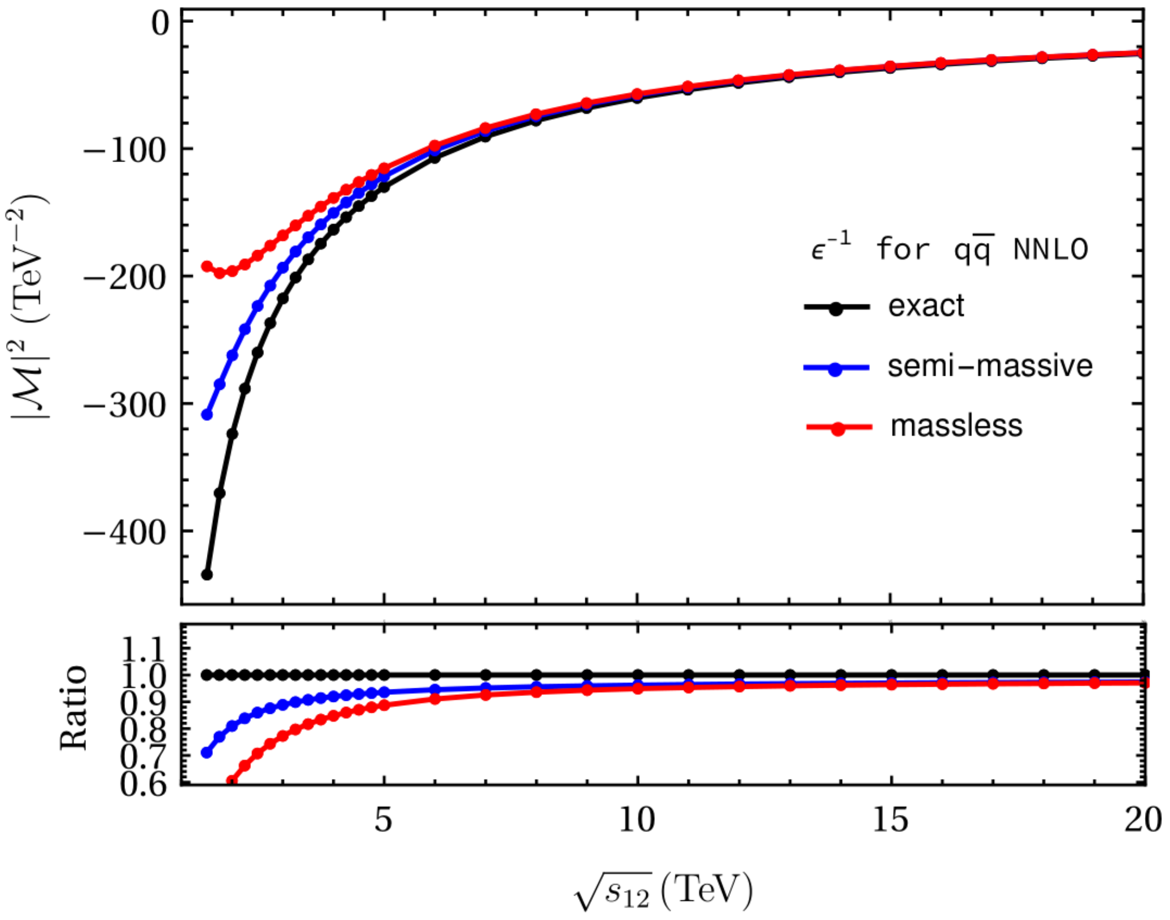
D. Chicherin and V. Sotnikov, JHEP 12 (2020) 167

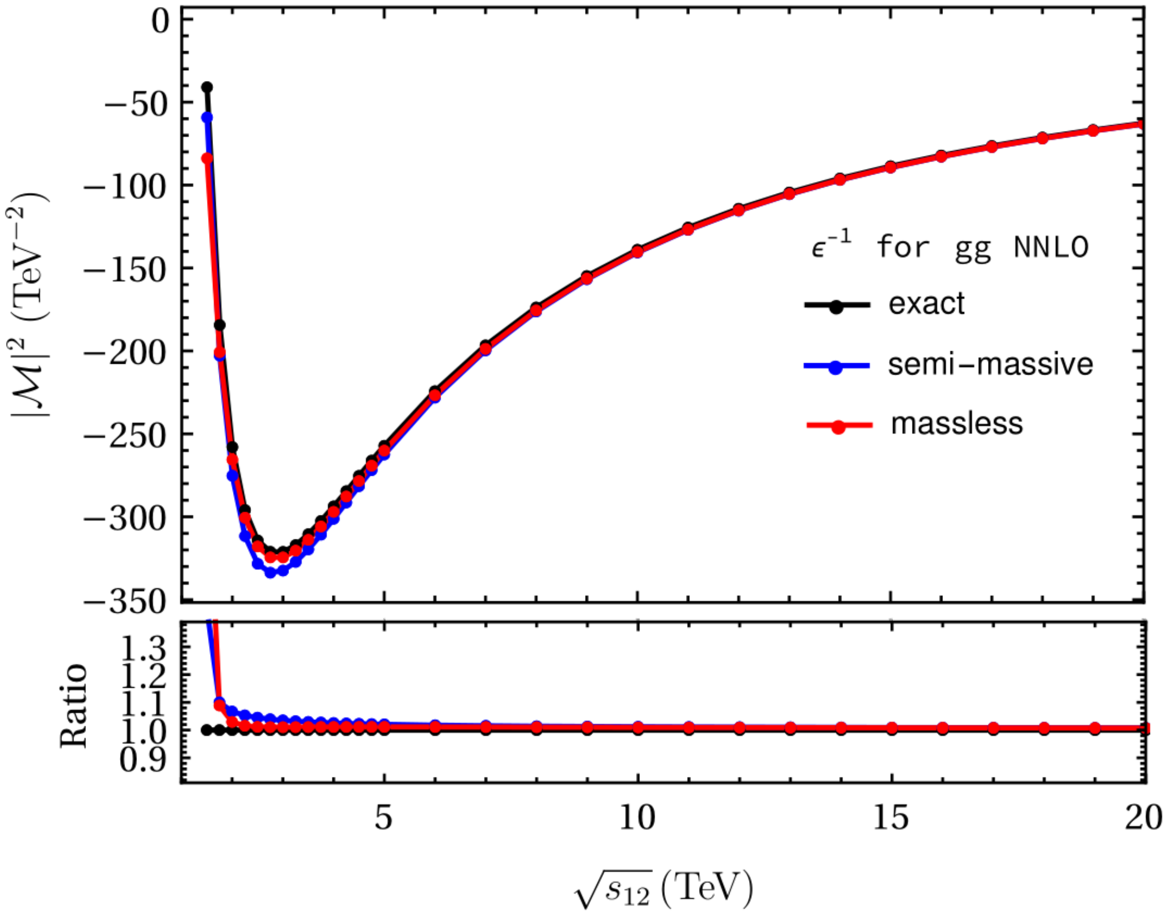
$t\bar{t}H$ production in the high energy limit



$$\theta_3 = 14\pi/29\,,\phi_3 = 34\pi/29\,,\theta_5 = 15\pi/29 \text{ and } q_5 = 20\,q_{5,\max}/29$$

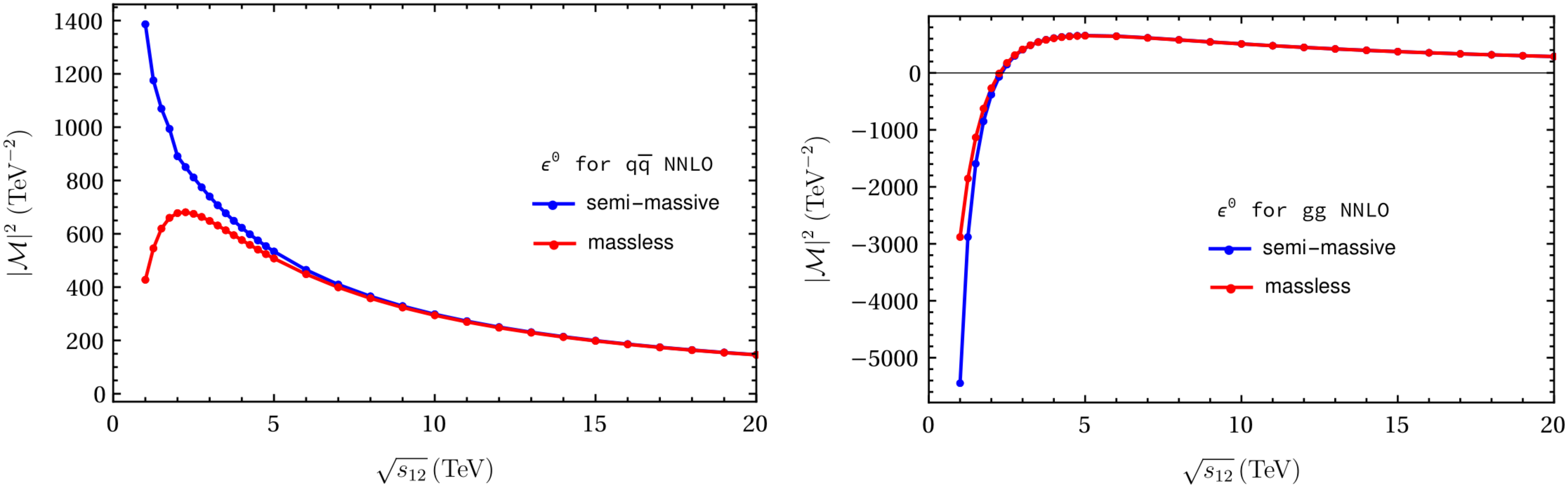
$t\bar{t}H$ production in the high energy limit





$$\theta_3 = 14\pi/29\,, \phi_3 = 34\pi/29\,, \theta_5 = 15\pi/29 \text{ and } q_5 = 20\, q_{5,\max}/29$$

$t\bar{t}H$ production in the high energy limit



$$\theta_3 = 14\pi/29\,,\phi_3 = 34\pi/29\,,\theta_5 = 15\pi/29 \text{ and } q_5 = 20\,q_{5,\max}/29$$

Catalog

- Introduction to $t\bar{t}H$ production
- Factorization in the high energy limit at leading power
- $t\bar{t}H$ production in the high energy limit at leading power
- Toward the high energy limit at next-to-leading power
- Summary and outlook.

Toward the high energy limit at NLP

- Region expansion in the high energy limit beyond LP
  - Mssive quark form factor in QED $Q\bar{Q} \to \gamma^*$
  - High energy expansion

Hoeve, Laenen, Marinissen, Vernazza and **GW**
JHEP 02 (2024) 024

See talks by, e.g., H. Zhang, K. Schönwald, R. Groeber.

- Factorization at NLP in the high energy
  - Massive quark form factor in QED $Q\bar{Q} \to \gamma^*$

Laenen, *et al.*, *Phys. Rev. D.* 103 (2021) 034022

$$\mathcal{M}^{\rm NLP}_{\rm coll} = \left(\prod_{i=1}^{n} J^{i}_{(f)}\right) H_{(f)}\, S + \sum_{i=1}^{n}\left(\prod_{j\neq i} J^{j}_{(f)}\right)\left[J^{i}_{(f\gamma)} \otimes H^{i}_{(f\gamma)} + J^{i}_{(f\partial\gamma)} \otimes H^{i}_{(f\partial\gamma)}\right] S$$
$$+ \sum_{i=1}^{n}\left(\prod_{j\neq i} J^{j}_{(f)}\right) J^{i}_{(f\gamma\gamma)} \otimes H^{i}_{(f\gamma\gamma)}\, S + \sum_{i=1}^{n}\left(\prod_{j\neq i} J^{j}_{(f)}\right) J^{i}_{(fff)} \otimes H^{i}_{(fff)}\, S$$
$$+ \sum_{1\leq i\leq j\leq n}\ \prod_{k\neq i,j} J^{k}_{(f)} J^{i}_{(f\gamma)} J^{j}_{(f\gamma)} \otimes H^{ij}_{(f\gamma)(f\gamma)}\, S\,.$$

LP

See talk by R. van Bijleveld

$$\left|\mathcal{M}^{\rm massive}(\{p\},\{m\})\right\rangle = \prod_{i}\left(\mathcal{Z}^{(m|0)}_{[i]}(\{m\})\right)^{1/2} \boldsymbol{\mathcal{S}}(\{p\},\{m\}) \left|\mathcal{M}^{\rm massless}(\{p\})\right\rangle$$

GW , Xia, Yang and Ye: *JHEP* 05 (2024) 082

Summary and outlook

- The factorization in the high energy limit at LP.
- Two-loop amplitudes for the production of a Higgs boson associated with a top-quark pair in the high energy limit at LP.
- The factorization or expansion in the high energy limit at NLP

- Resummation in the high energy limit at LP for $t\bar{t}H$.
- Combing the real correction with our two-loop results to present the NNLO contribution to the $t\bar{t}H$.
- NLP factorization effect in the high energy limit to a process at the collider.

Thanks!

Introduction to factorization

- Power counting and method of region

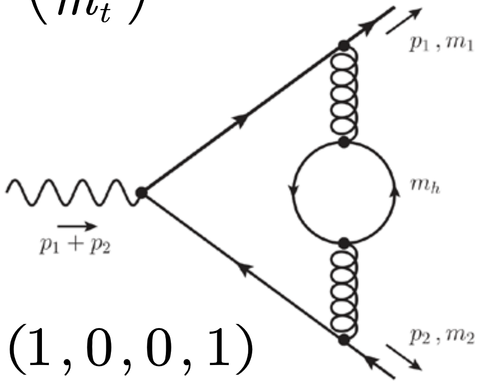
$$\frac{1}{l^2-m_t^2}+\mathcal{O}\left(\frac{1}{m_t^2}\right)^2 \quad \text{hard}: l\sim m_t$$

  - Higgs Effective Field Theory

$$\frac{1}{(l+p_1)^2-m_t^2}$$

$m_t \to \infty$

$$\frac{1}{-m_t^2}+\mathcal{O}\left(\frac{1}{m_t^2}\right)^2 \quad \text{soft}: l\sim m_H$$



  - Soft-Collinear Effective Field Theory

$$n_i^2=\bar{n}_i^2=0\,, \quad n_i\cdot\bar{n}_i=2$$

$$\lambda=\frac{m}{\sqrt{|\hat{s}|}}$$

$$\text{hard}: k^\mu\sim\sqrt{|s|}\,,$$

$$n_i \text{ collinear}: (n_i\cdot k,\, \bar{n}_i\cdot k,\, k_\perp)\sim\sqrt{|s|}\,(\lambda^2,\,1\,,\,\lambda)\,,$$

$$\text{soft}: k^\mu\sim\sqrt{|s|}\,\lambda\,.$$

$$\vdots$$

$$n^\mu=n_1^\mu=(1\,,0\,,0\,,1)$$

$$\bar{n}^\mu=n_2^\mu=(1\,,0\,,0\,,-1)$$

$$q^\mu=q^-\frac{\bar{n}^\mu}{2}+q^+\frac{n^\mu}{2}+q_\perp^\mu$$

$$\frac{1}{(l+p_1)^2-m_t^2} \Rightarrow \frac{1}{2l^-p_1^+}+\mathcal{O}(\lambda^2) \quad n_2 \text{ collinear}$$

Factorization in the high energy limit at LP

- High energy limit: $|s_{ij}| \gg m_k^2,\ i \neq j$

Mitov and Moch: *JHEP* 05 (2007) 001
GW , Xia, Yang and Ye: *JHEP* 05 (2024) 082

$$\left|\mathcal{M}^{\text{massive}}(\{p\},\{m\})\right\rangle = \prod_i \left(\mathcal{Z}_{[i]}^{(m|0)}(\{m\})\right)^{1/2} \boldsymbol{\mathcal{S}}(\{p\},\{m\}) \left|\mathcal{M}^{\text{massless}}(\{p\})\right\rangle \qquad i = q,\ Q,\ g$$

- In QED case (Bhabha scattering at NNLO): Becher and Melnikov: *JHEP* 06 (2007) 084

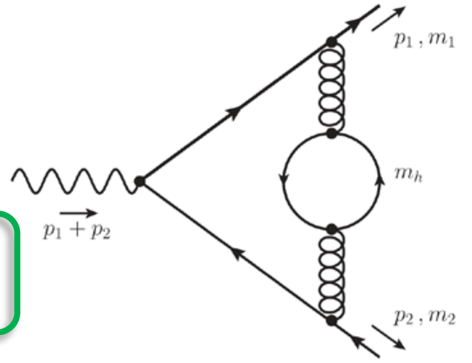
$$S(\{p\},\{m\}) = 1 + \sum_{i=e,\mu} \delta S(s, m_i^2)$$

$$\delta S(s, m_i^2, N_i) = -N_i (4\pi\alpha_0)^2 \int \frac{d^d k}{(2\pi)^d} \frac{p_1 \cdot p_2}{(p_1 \cdot k)\ (p_2 \cdot k)\ k^2}\ i\Pi(k^2, m_i^2)$$

$$= N_i a_0^2 m_i^{-4\epsilon} \ln\left(\frac{Q^2}{m_e^2}\right) \left(-\frac{1}{12\epsilon^2} + \frac{5}{36\epsilon} - \frac{7}{27} - \frac{\pi^2}{72} + \mathcal{O}(\epsilon)\right),$$

- Not a complete region expansion
- Dependence on the external mass
- Invalid for massless external legs

Rapidity divergence

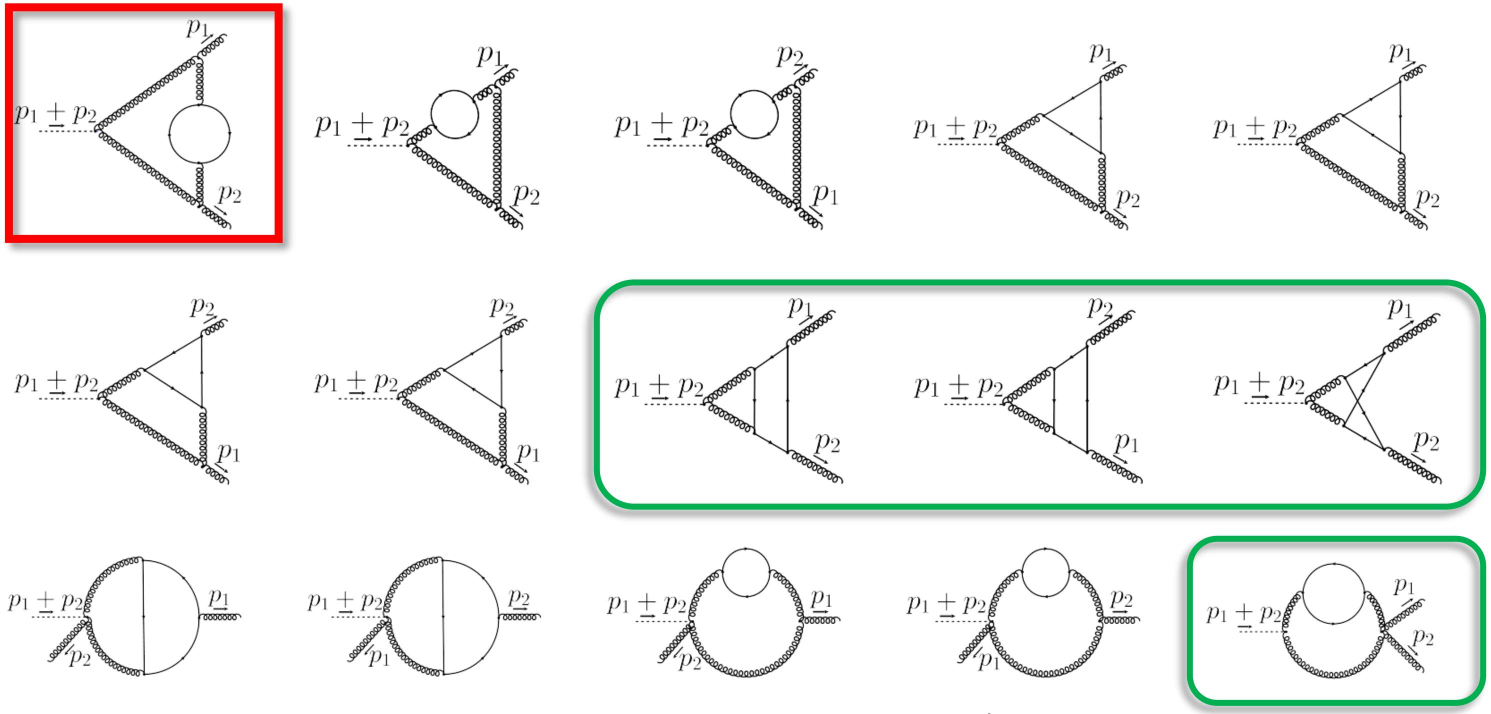


Factorization in the high energy limit at LP

- Determine soft function and $\mathcal{Z}$-factor:

GW , Xia, Yang and Ye: *JHEP* 05 (2024) 082

  - Gluon scalar form factor $\mathcal{L}_{\text{int}} = -\frac{\lambda}{4} H G_a^{\mu\nu} G_{a,\mu\nu} \quad \rightarrow \quad F_{gg} = \frac{p_1 \cdot p_2\, g_{\mu\nu} - p_{1,\mu} p_{2,\nu} - p_{1,\nu} p_{2,\mu}}{2(1-\epsilon)} \Gamma_{gg}^{\mu\nu}$



$$I^A_{\{a_i\}} \equiv \mu^{4\epsilon} \int \frac{dk_1}{(2\pi)^d} \frac{dk_2}{(2\pi)^d} \frac{1}{[k_1^2 - m_h^2]^{a_1}} \frac{1}{[k_2^2 - m_h^2]^{a_2}} \frac{1}{[(k_1+k_2)^2]^{a_3}} \frac{1}{[(k_1+k_2-p_1)^2]^{a_4}} \times \frac{(-\tilde{\mu}^2)^{\nu}}{[(k_1+k_2+p_2)^2]^{a_5+\nu}} \frac{1}{[(k_1-p_1)^2 - m_h^2]^{a_6}} \frac{1}{[(k_1+p_2)^2 - m_h^2]^{a_7}}$$

$$I^B_{\{a_i\}} \equiv \mu^{4\epsilon} \int \frac{dk_1}{(2\pi)^d} \frac{dk_2}{(2\pi)^d} \frac{1}{[k_1^2]^{a_1}} \frac{1}{[(k_1-p_1-p_2)^2]^{a_2}} \frac{1}{[k_2^2 - m_h^2]^{a_3}} \frac{1}{[(k_2+p_1)^2 - m_h^2]^{a_4}} \times \frac{1}{[(k_1+k_2)^2 - m_h^2]^{a_5}} \frac{1}{[(k_1+k_2-p_2)^2 - m_h^2]^{a_6}} \frac{1}{[(k_2+p_2)^2]^{a_7}}$$

Factorization in the high energy limit at LP

- High energy limit: $|s_{ij}| \gg m_k^2,\ i \neq j$

Mitov and Moch: *JHEP* 05 (2007) 001
GW , Xia, Yang and Ye: *JHEP* 05 (2024) 082

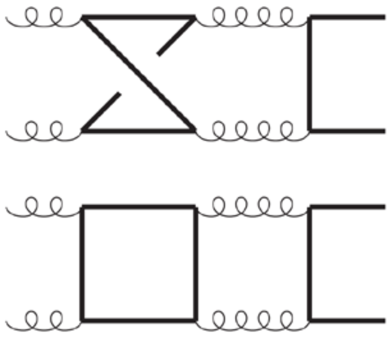
$$\left|\mathcal{M}^{\text{massive}}(\{p\},\{m\})\right\rangle = \prod_i \left(\mathcal{Z}_{[i]}^{(m|0)}(\{m\})\right)^{1/2} \boldsymbol{\mathcal{S}}(\{p\},\{m\}) \left|\mathcal{M}^{\text{massless}}(\{p\})\right\rangle \qquad i = q,\, Q,\, g$$

- Top-pair production at NNLO:

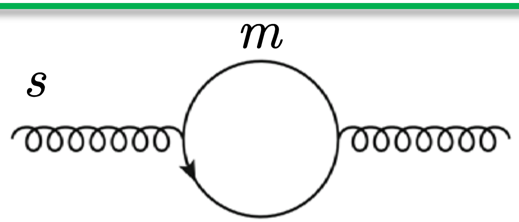
Czakon, Mitov and Moch: *Nucl.Phys.B* 798 (2008) 210

  - 8-fold MB representation

$$\begin{aligned}\bar{I}_{NP} = {} & (-1)^a(-s)^{-a-2\varepsilon+4}\int_{-i\infty}^{i\infty}\prod_{i=1}^{8}\frac{dz_i}{\Gamma(a_i)}\left(-\frac{s}{m^2}\right)^{z_1}\left(-\frac{t}{m^2}\right)^{z_2}\left(-\frac{u}{m^2}\right)^{z_3}\\ & \times\Gamma(-z_3)\Gamma(-z_4)\Gamma(-z_5)\Gamma(-z_6)\Gamma(-z_7)\Gamma(-z_8)\Gamma(-z_2+z_4+z_5)\Gamma(z_1+z_2+z_3+z_6)\\ & \times\Gamma(a+2\varepsilon-z_1+z_7+z_8-4)\Gamma(z_2+z_3-z_4-z_5+z_8+a_1)\Gamma(z_5+z_8+a_4)\Gamma(z_7+z_8+a_6)\\ & \times\Gamma(-2z_1-2z_2-2z_3+z_4+z_5-2z_6+a_7)\Gamma(z_2-z_5+a_8)\Gamma(-\varepsilon-z_6-a_{13}+2)\\ & \times\Gamma(-\varepsilon-z_6-a_{24}+2)\Gamma(-2z_1-z_2-z_3+z_4-2z_6+a_{78})\\ & \times\Gamma(-\varepsilon+z_1+z_2+z_3-z_4+z_6-z_7-a_{5678}+2)\Gamma(-2\varepsilon+z_1-z_8-a_{1234678}+4)\\ & \times\Gamma(-2\varepsilon+z_1+z_5-z_7-z_8-a_{1245678}+4)\\ & \times\Gamma(-2\varepsilon+z_1+z_2+z_3-z_4-z_5-z_7-z_8-a_{1345678}+4)\\ & \times\Big(\Gamma(-a-3\varepsilon-z_6+6)\Gamma(-2z_1-z_2-2z_3+z_4-2z_6+a_{78})\Gamma(-2\varepsilon-2z_6-a_{1234}+4)\\ & \times\Gamma(-2\varepsilon+z_1+z_2+z_3-z_4-z_7-a_{135678}+4)\\ & \times\Gamma(-2\varepsilon+z_1+z_2+z_3-z_4-z_7-a_{245678}+4)\Big)^{-1},\end{aligned}$$

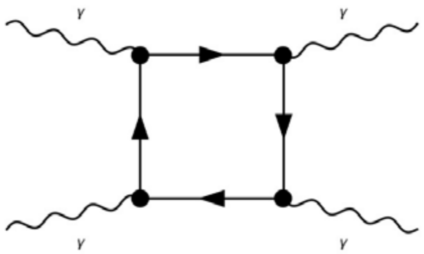


Not contribute at LP?



$$\mathcal{A}(s\neq 0, m=0) = \mathcal{A}(s\neq 0, m\neq 0)|_{\text{LP}}$$

$$\mathcal{Z}_{[g]}^{(1)} = \sum_h T_F\left(\frac{\mu^2}{m_h^2}\right)^{\epsilon}\left(-\frac{4}{3\epsilon} - \frac{2\zeta_2}{3}\epsilon + \frac{4\zeta_3}{9}\epsilon^2\right)$$
$$= Z_3$$



Four-photon scattering

$t\bar{t}H$ production in the high energy limit

- Phase space point:

$$p_1 = \frac{\sqrt{s_{12}}}{2}(1,0,0,1)^{\mathrm{T}} \,,$$
$$p_2 = \frac{\sqrt{s_{12}}}{2}(1,0,0,-1)^{\mathrm{T}} \,,$$
$$p_3 = \left(\sqrt{m_t^2+q_3^2}, q_3\sin\theta_3\sin\phi_3, q_3\sin\theta_3\cos\phi_3, q_3\cos\theta_3\right)^{\mathrm{T}} \,,$$
$$p_4 = \left(\sqrt{m_t^2+q_4^2}, q_4\sin\theta_4\sin\phi_4, q_4\sin\theta_4\cos\phi_4, q_4\cos\theta_4\right)^{\mathrm{T}} \,,$$
$$p_5 = \left(\sqrt{q_5^2+m_H^2}, q_5\sin\theta_5, 0, q_5\cos\theta_5\right)^{\mathrm{T}} \,,$$

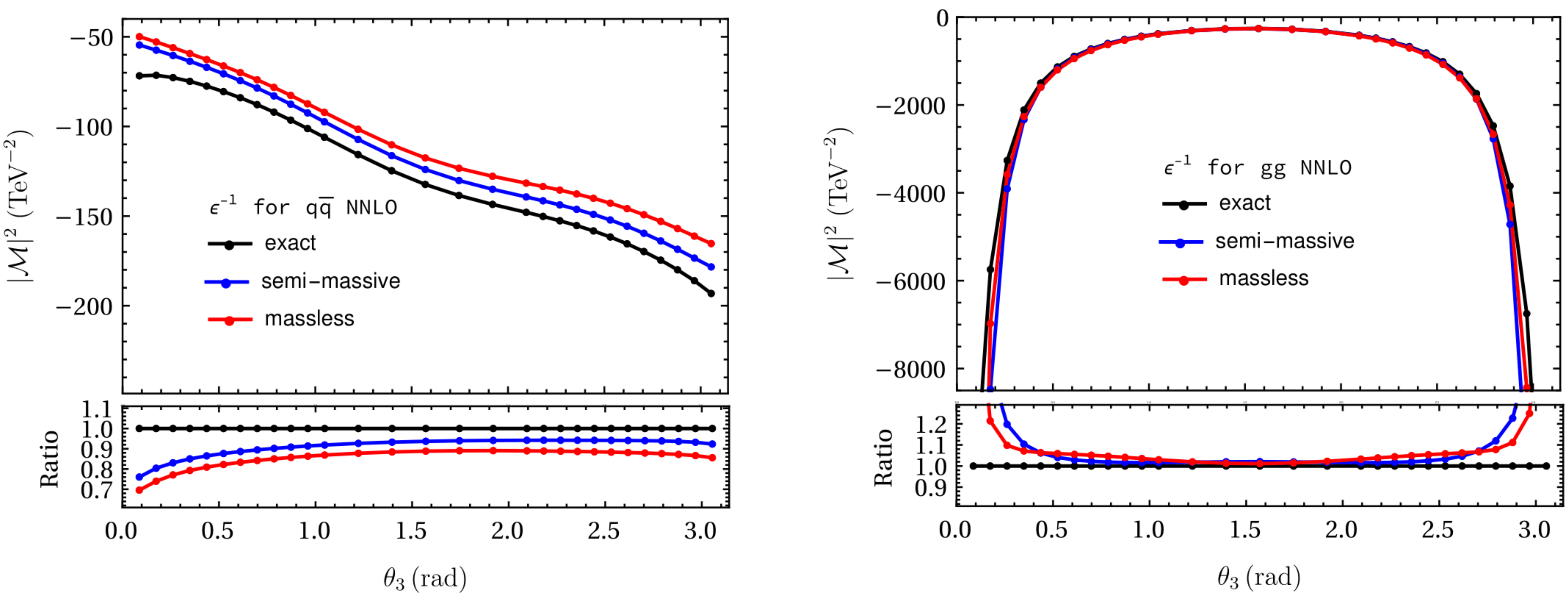
Massive

$$\tilde{p}_1 = \frac{\sqrt{\tilde{s}_{12}}}{2}(1,0,0,1)^{\mathrm{T}} \,,$$
$$\tilde{p}_2 = \frac{\sqrt{\tilde{s}_{12}}}{2}(1,0,0,-1)^{\mathrm{T}} \,,$$
$$\tilde{p}_3 = \left(\tilde{q}_3, \tilde{q}_3\sin\tilde{\theta}_3\sin\tilde{\phi}_3, \tilde{q}_3\sin\tilde{\theta}_3\cos\tilde{\phi}_3, \tilde{q}_3\cos\tilde{\theta}_3\right)^{\mathrm{T}} \,,$$
$$\tilde{p}_4 = \left(\tilde{q}_4, \tilde{q}_4\sin\tilde{\theta}_4\sin\tilde{\phi}_4, \tilde{q}_4\sin\tilde{\theta}_4\cos\tilde{\phi}_4, \tilde{q}_4\cos\tilde{\theta}_4\right)^{\mathrm{T}} \,,$$
$$\tilde{p}_5 = \left(\tilde{q}_5, \tilde{q}_5\sin\tilde{\theta}_5, 0, \tilde{q}_5\cos\tilde{\theta}_5\right)^{\mathrm{T}} \,,$$

Massless

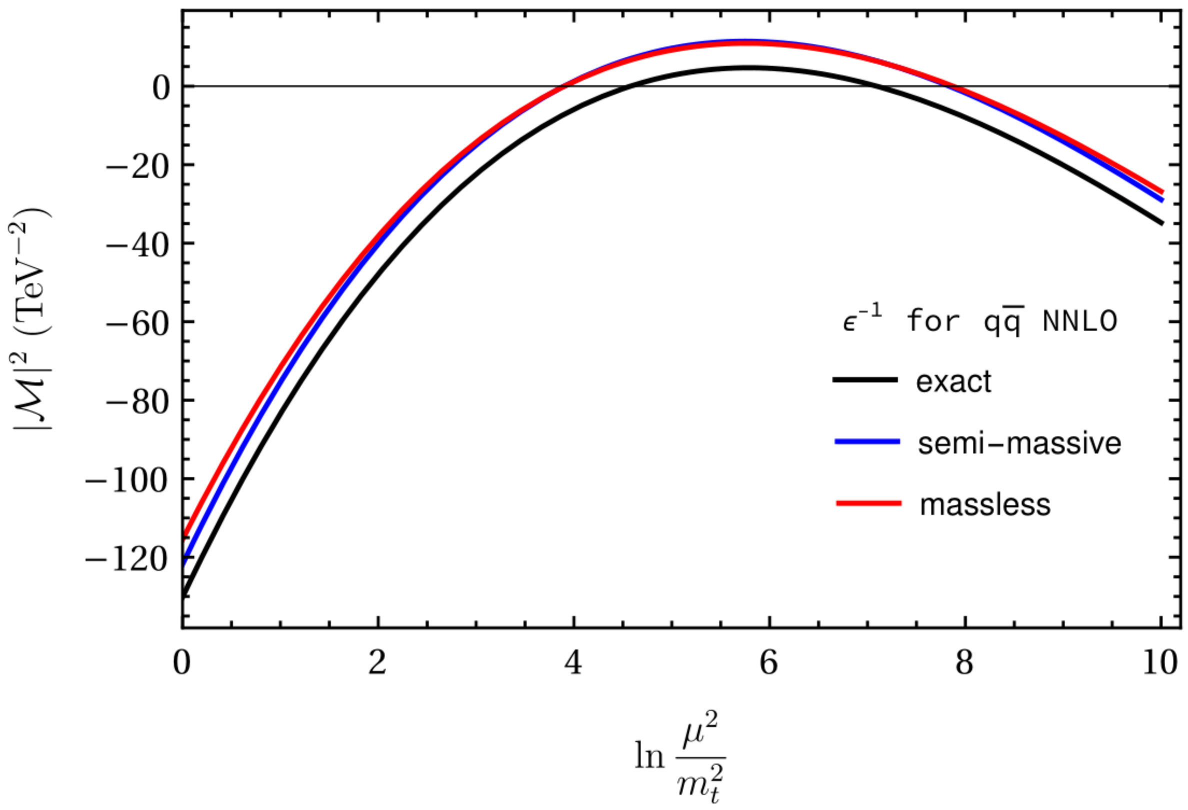
$$\tilde{s}_{12} = s_{12},\ \tilde{\theta}_i = \theta_i \text{ and } \tilde{\phi}_i = \phi_i$$

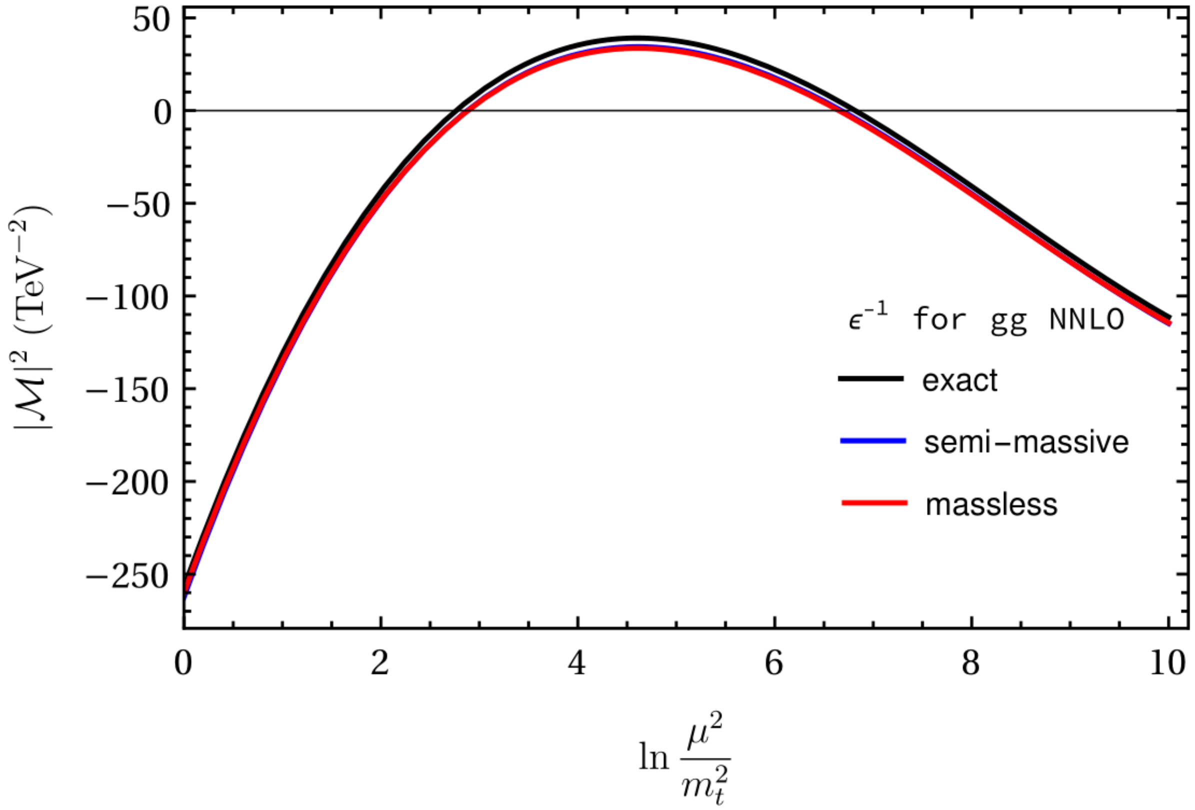
$t\bar{t}H$ production in the high energy limit



$$s_{12} = 5 \text{ TeV}, \phi_3 = 34\pi/29\,, \theta_5 = 15\pi/29 \text{ and } q_5 = 20\, q_{5,\max}/29$$

$t\bar{t}H$ production in the high energy limit





$$s_{12} = 5 \text{ TeV}, \theta_3 = 14\pi/29\,, \phi_3 = 34\pi/29\,, \theta_5 = 15\pi/29 \text{ and } q_5 = 20\, q_{5,\max}/29$$

Factorization in the high energy limit at LP

- Validation by using form factor

$$\left|\mathcal{M}^{\text{massive}}(\{p\},\{m\})\right\rangle = \prod_i \left(\mathcal{Z}_{[i]}^{(m|0)}(\{m\})\right)^{1/2} \boldsymbol{\mathcal{S}}(\{p\},\{m\}) \left|\mathcal{M}^{\text{massless}}(\{p\})\right\rangle$$

GW , Xia, Yang and Ye: *JHEP* 05 (2024) 082

$$F_{1,Q\bar{Q}}^{(2),h}(s,m_h^2,m_h^2,m_h^2) = F_{1,q\bar{q}}^{(2),l}(s) + C_F\,\mathcal{S}^{(2)}(s,m_h^2) + \mathcal{Z}_{[Q]}^{(2),h}(m_h^2,m_h^2)$$

$$F_{Q\bar{q}}^{(2),h}(s,m_h^2,m_h^2) = F_{q\bar{q}}^{(2),l}(s) + C_F\,\mathcal{S}^{(2)}(s,m_h^2) + \frac{1}{2}\mathcal{Z}_{[Q]}^{(2),h}(m_h^2,m_h^2) + \frac{1}{2}\mathcal{Z}_{[q]}^{(2),h}(m_h^2)$$

$$F_{q\bar{q}}^{(2),h}(s,m_h^2) = F_{q\bar{q}}^{(2),l}(s) + C_F\,\mathcal{S}^{(2)}(s,m_h^2) + \mathcal{Z}_{[q]}^{(2),h}(m_h^2)$$

$$F_{gg}^{(2),h}(s,m_h^2) = F_{gg}^{(2),l}(s) + \mathcal{Z}_{[g]}^{(1)}(m_h^2)\,F_{gg}^{(1),\text{no}-\text{quark}}(s) + \mathcal{Z}_{[g]}^{(2),h}(m_h^2) + N_c\,\mathcal{S}^{(2)}(s,m_h^2)$$

- Validation by using top-pair production

Moch, *et al.* *JHEP* 08 (2005) 049
Bernreuther, *et al.* Nucl. Phys. B 706 (2005) 245
Bonciani , *et al.* *JHEP* 11 (2008) 065
Czakon, *et al.* *Nucl.Phys.B* 798 (2008) 210
Anastasiou , *et al.* *Nucl.Phys.B 605* (2001) 486
……

$$\mathcal{M}_I^{\text{massive}} = \sum_J \prod_i \left(\mathcal{Z}_{[i]}^{(m|0)}\right)^{1/2} \boldsymbol{\mathcal{S}}_{IJ}\,\mathcal{M}_J^{\text{massless}}$$

$$\boldsymbol{\mathcal{S}}_{IJ} = \frac{\langle c_I|\boldsymbol{\mathcal{S}}|c_J\rangle}{\langle c_I|c_I\rangle}$$

$$\boldsymbol{T}_1\cdot\boldsymbol{T}_2|_{gg} = \begin{pmatrix} -N_c & 0 & 0 \\ 0 & -\frac{N_c}{2} & 0 \\ 0 & 0 & -\frac{N_c}{2} \end{pmatrix}$$

$$\boldsymbol{T}_1\cdot\boldsymbol{T}_2|_{q\bar{q}} = \begin{pmatrix} -C_F & 0 \\ 0 & \frac{1}{2N_c} \end{pmatrix}$$

Factorization in the high energy limit at LP

- Validation by IR pole
  - IR singularities of QCD

$$\left|\mathcal{M}^{\text{massive}}(\epsilon,\{p\},\{m\})\right\rangle = \prod_i \left(\mathcal{Z}^{(m|0)}_{[i]}(\epsilon,\{m\})\right)^{1/2} \boldsymbol{\mathcal{S}}(\epsilon,\{p\},\{m\}) \left|\mathcal{M}^{\text{massless}}(\epsilon,\{p\})\right\rangle$$

renormalized amplitudes

GW , Xia, Yang and Ye: *JHEP* 05 (2024) 082

$$\boldsymbol{Z}^{-1}_{\text{massive}}(\epsilon,\{p\},\{m\}) \left|\mathcal{M}^{\text{massive}}(\epsilon,\{p\},\{m\})\right\rangle = \text{finite}$$

$$\boldsymbol{Z}^{-1}_{\text{massless}}(\epsilon,\{p\}) \left|\mathcal{M}^{\text{massless}}(\epsilon,\{p\})\right\rangle = \text{finite}$$

$$\boldsymbol{Z}^{-1}\frac{d}{d\ln\mu}\boldsymbol{Z} = -\Gamma$$

$$\Gamma(\{\underline{p}\},\{\underline{m}\},\mu) = \sum_{(i,j)} \frac{\boldsymbol{T}_i\cdot\boldsymbol{T}_j}{2}\gamma_{\text{cusp}}(\alpha_s)\ln\frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i(\alpha_s) = \Gamma(\{p\},\mu)$$

$$- \sum_{(I,J)} \frac{\boldsymbol{T}_I\cdot\boldsymbol{T}_J}{2}\gamma_{\text{cusp}}(\beta_{IJ},\alpha_s) + \sum_I \gamma^I(\alpha_s) + \sum_{I,j} \boldsymbol{T}_I\cdot\boldsymbol{T}_j\,\gamma_{\text{cusp}}(\alpha_s)\ln\frac{m_I\mu}{-s_{Ij}}$$

$$+ \sum_{(I,J,K)} if^{abc}\,\boldsymbol{T}_I^a\boldsymbol{T}_J^b\boldsymbol{T}_K^c\,F_1(\beta_{IJ},\beta_{JK},\beta_{KI})$$

$$+ \sum_{(I,J)}\sum_k if^{abc}\,\boldsymbol{T}_I^a\boldsymbol{T}_J^b\boldsymbol{T}_k^c\,f_2\left(\beta_{IJ},\ln\frac{-\sigma_{Jk}\,v_J\cdot p_k}{-\sigma_{Ik}\,v_I\cdot p_k}\right) + \mathcal{O}(\alpha_s^3)\,.$$

Becher, *et al*, *Phys. Rev. D* 79 (2009) 125004
Ferroglia, *et al*, *Phys. Rev. Lett.* 103 (2009) 201601
Ferroglia, *et al*, *JHEP 11* (2009) 062

$$\boldsymbol{Z}^{-1}_{\text{massive}}(\epsilon,\{p\},\{m\}) \prod_i \left(\mathcal{Z}^{(m|0)}_{[i]}(\epsilon,\{m\})\right)^{1/2} \boldsymbol{\mathcal{S}}(\epsilon,\{p\},\{m\})\,\boldsymbol{Z}_{\text{massless}}(\epsilon,\{p\}) = \text{finite}$$

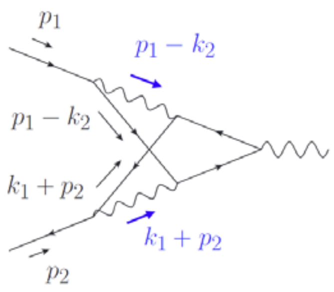
Collinear divergences:

$$\ln(m^2) \leftrightarrow \frac{1}{\epsilon}$$

Toward the high energy limit at NLP

- Massive quark form factor in QED $Q\bar{Q} \to \gamma^*$
  - Left hand side

Hoeve, Laenen, Marinissen, Vernazza and **GW** : *JHEP* 02 (2024) 024

topology X		hh	cc	$\bar{c}\bar{c}$	$\bar{c}c$	hc	$\bar{c}h$	$\bar{c}uc$	$\overline{uc}c$	cc'	$\bar{c}\bar{c}'$
(h)		✓	ν_2	ν_1	ν_1, ν_2	✓	✓	✓	✓	ν_1, ν_2	ν_1, ν_2

$$I^{\nu_1,\nu_2}_{X;\{n_i\}} = \int [dk_1][dk_2] \frac{1}{k_1^{2n_1}} \frac{1}{k_2^{2n_2}} \frac{\mu_1^{2\nu_1}}{[(k_2-p_1)^2 - m^2]^{n_3+\nu_1}} \frac{\mu_1^{2\nu_1}}{[(k_1+k_2-p_1)^2 - m^2]^{n_4+\nu_1}}$$
$$\times \frac{\mu_2^{2\nu_2}}{[(k_1+p_2)^2 - m^2]^{n_5+\nu_2}} \frac{\mu_2^{2\nu_2}}{[(k_1+k_2+p_2)^2 - m^2]^{n_6+\nu_2}} \frac{1}{[(k_1+k_2)^2]^{n_7}}$$

$$I'^{\,\nu_1,\nu_2}_{X;\{n_i\}} = \int [dk_1][dk_2] \frac{1}{(k_1+p_2)^{2n_1}} \frac{1}{(k_2-p_1)^{2n_2}} \frac{1}{[k_2^2 - m^2]^{n_3+\nu_1}}$$
$$\times \frac{\mu_1^{2\nu_1}}{[(k_1+k_2+p_2)^2 - m^2]^{n_4+\nu_1}} \frac{\mu_1^{2\nu_1}}{[k_1^2 - m^2]^{n_5+\nu_2}} \frac{\mu_2^{2\nu_2}}{[(k_1+k_2-p_1)^2 - m^2]^{n_6+\nu_2}} \frac{\mu_2^{2\nu_2}}{[(k_1+k_2)^2]^{n_7}}$$

Toward the high energy limit at NLP

- Massive quark form factor in QED $Q\bar{Q}\to\gamma^*$
  - Left hand side

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$$
\begin{aligned}
\mathcal{I}^X &= (4\pi)^4 \hat{s}^2\, I^{\nu_1,\nu_1,\nu_2,\nu_2}_{X;1,1,1,1,1,1,0} \\
&= (4\pi)^4 \hat{s}^2 \int [dk_1]\,[dk_2]\, \frac{1}{k_1^2}\frac{1}{k_2^2} \frac{\tilde{\mu}_1^{2\nu_1}}{[(k_2-p_1)^2-m^2]^{1+\nu_1}} \frac{\tilde{\mu}_1^{2\nu_1}}{[(k_1+k_2-p_1)^2-m^2]^{1+\nu_1}} \\
&\quad\times \frac{\tilde{\mu}_2^{2\nu_2}}{[(k_1+p_2)^2-m^2]^{1+\nu_2}} \frac{\tilde{\mu}_2^{2\nu_2}}{[(k_1+k_2+p_2)^2-m^2]^{1+\nu_2}}\,,
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{I}^X_{\rm full}\big|_{\rm NLP} = \left(\frac{\mu^2}{m^2}\right)^{2\epsilon}\bigg[&-\frac{1}{\epsilon}\left(\frac{1}{3}L^3+\zeta_2 L+\zeta_3\right)-\frac{1}{2}L^4+\zeta_2 L^2-\zeta_3 L-\frac{37\zeta_2^2}{10} \\
&-\frac{m^2}{\hat{s}}\left(4L^2-8L+4\zeta_2\right)+\mathcal{O}(\epsilon)\bigg],
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{I}^X\big|_{\bar{c}c} = \left(\frac{\mu^2}{m^2}\right)^{2\epsilon}\left(\frac{\tilde{\mu}_1^2}{-m^2}\right)^{\nu_1}\left(\frac{\tilde{\mu}_1^2}{\hat{s}}\right)^{\nu_1}\left(\frac{\tilde{\mu}_2^2}{-m^2}\right)^{\nu_2}\left(\frac{\tilde{\mu}_2^2}{\hat{s}}\right)^{\nu_2}\bigg[&\frac{1}{4\epsilon^4}+\frac{1}{\epsilon^3}\left(\frac{1}{2\nu_2}+\frac{1}{2\nu_1}\right) \\
&+\frac{1}{\epsilon^2}\left(\frac{5\zeta_2}{4}-\frac{1}{\nu_1\nu_2}\right)+\frac{1}{\epsilon}\left(\frac{3\zeta_2}{2\nu_1}+\frac{3\zeta_2}{2\nu_2}+\frac{17\zeta_3}{6}\right)-\frac{\zeta_2}{\nu_1\nu_2}+\frac{14\zeta_3}{3\nu_1}+\frac{14\zeta_3}{3\nu_2}+\frac{279\zeta_2^2}{40} \\
&+\frac{m^2}{\hat{s}}\left(\frac{1}{\epsilon}\left(\frac{2}{\nu_2}+\frac{2}{\nu_1}+4\right)-4\zeta_2+\frac{2}{\nu_1}+\frac{2}{\nu_2}-4\right)\bigg]
\end{aligned}
$$

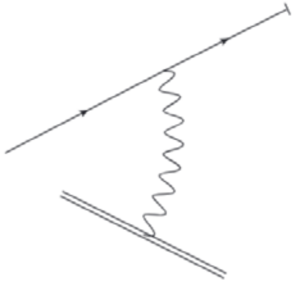
Toward the high energy limit at NLP

- Massive quark form factor in QED $Q\bar{Q} \to \gamma^*$
  - Right hand side

Bijleveld, Laenen, Marinissen, Vernazza and **GW** : on progress

$$J^{(1)}_{(f)}(p_1) = (-ie\mu^\epsilon)^2 \int_\infty^0 d\lambda \int dx \, \langle p_1| \bar{\psi}(0) n_+ A(\lambda n_+) \bar{\psi}(x) \not{A}(x) \psi(x) \, |0\rangle$$

$$= ie^2 \bar{u}(p_1) \int [dk] \frac{\not{n}_+ (\not{p}_1 + \not{k} + m)}{((p_1+k)^2 - m^2 + i\eta)(k^2 + i\eta)(-n_+ k + i\eta)}$$

$=$  .

$$J^{(1)\mu}_{(f\gamma)}(p_1, n_+\ell) = ie^2 \bar{u}(p_1) \int [dk] \frac{\gamma^\mu \left(\not{p}_1 - \not{k} + m\right)}{(k^2 + i\eta)((k-p_1)^2 - m^2 + i\eta)} \delta(n_+ k - n_+ \ell)$$

$$- ie^2 \bar{u}(p_1) \int [dk] \frac{\not{n}_+ \left(\not{p}_1 - \not{k} + m\right) k^\mu}{(k^2 + i\eta)((k-p_1)^2 - m^2 + i\eta)(n_+ k + i\eta)} \delta(n_+ k - n_+ \ell)$$

$=$  ,