Anomalous Dimensions of Soft Functions at Subleading Power

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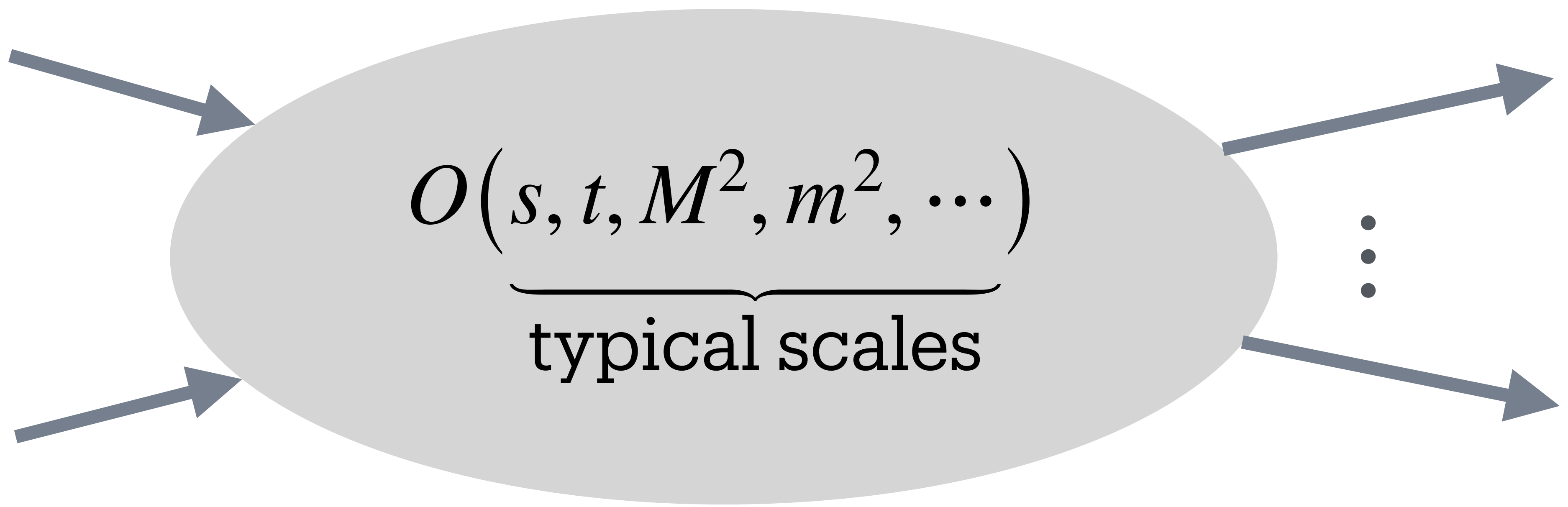
based on 2403.17738 and 2410.xxxxx with Martin Beneke, Erik Sünderhauf and Yao Ji

Factorization for High Precision



- Typical scales are usually widely separated
  - Leads to **factorization** when $\lambda = m/M \ll 1$
  - Large log's $\alpha_s^i \ln^j \lambda$ need to be **resummed** to improve precision predictions by **RGE**

$$O(m, M, \cdots) = \lambda^0 \overbrace{H_{\mathrm{LP}}(M,\mu) \otimes \cdots \otimes S_{\mathrm{LP}}(m,\mu)}^{\text{leading-power (LP)}} + \lambda \sum_i \overbrace{H_{\mathrm{NLP},\,i}(M,\mu) \otimes \cdots \otimes S_{\mathrm{NLP},\,i}(m,\mu)}^{\text{next-to-leading-power (NLP)}} + \mathcal{O}\left(\lambda^2\right)$$

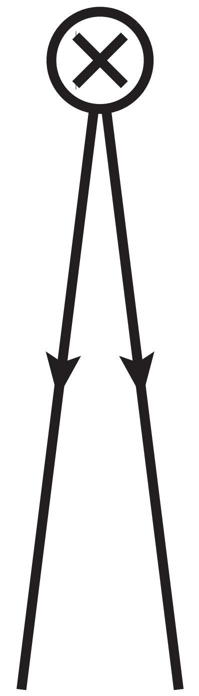
λ expansion

$$\begin{cases} H_{\mathrm{LP}}(M,\mu) = \sum_n \alpha_s^n H_{\mathrm{LP}}^{(n)}(M,\mu) \\ S_{\mathrm{LP}}(m,\mu) = \sum_n \alpha_s^n S_{\mathrm{LP}}^{(n)}(m,\mu) \end{cases}$$

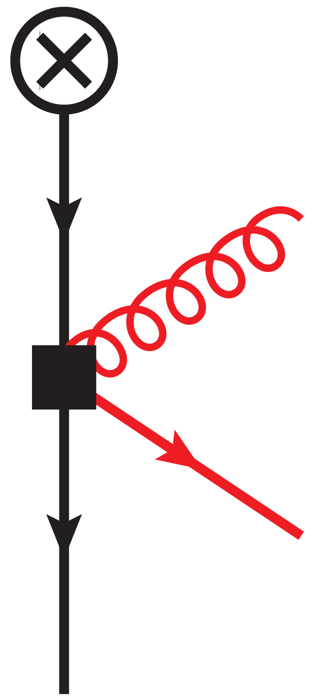
$$\begin{cases} H_{\mathrm{NLP},\,i}(M,\mu) = \sum_n \alpha_s^n H_{\mathrm{NLP},\,i}^{(n)}(M,\mu) \\ S_{\mathrm{NLP},\,i}(m,\mu) = \sum_n \alpha_s^n S_{\mathrm{NLP},\,i}^{(n)}(m,\mu) \end{cases}$$

α_s (loop) expansion (w./ $m \ll M$ limit)

A Glimpse of NLP Factorization



multi. fields in a sector



power-suppressed interaction, e.g., soft quark

- At LP, soft functions are built from **(semi-infinite) Wilson lines**, e.g., Drell-Yan threshold [Korchemsky, Marchesini, 1993]:

$$\frac{1}{N_c}\langle 0|\,\mathbf{Tr}\,\bar{\mathbf{T}}(Y^\dagger_{n_-}(x_0)Y_{n_+}(x_0))\,\mathbf{T}(Y^\dagger_{n_+}(0)Y_{n_-}(0))\,|0\rangle$$

- At NLP, soft operators contain soft fields on the light-cone, e.g., $q_s(x\_)$, from NLP SCET Lagranigian insertsions.
  ↪**soft-quark (soft) functions** appear as building blocks.

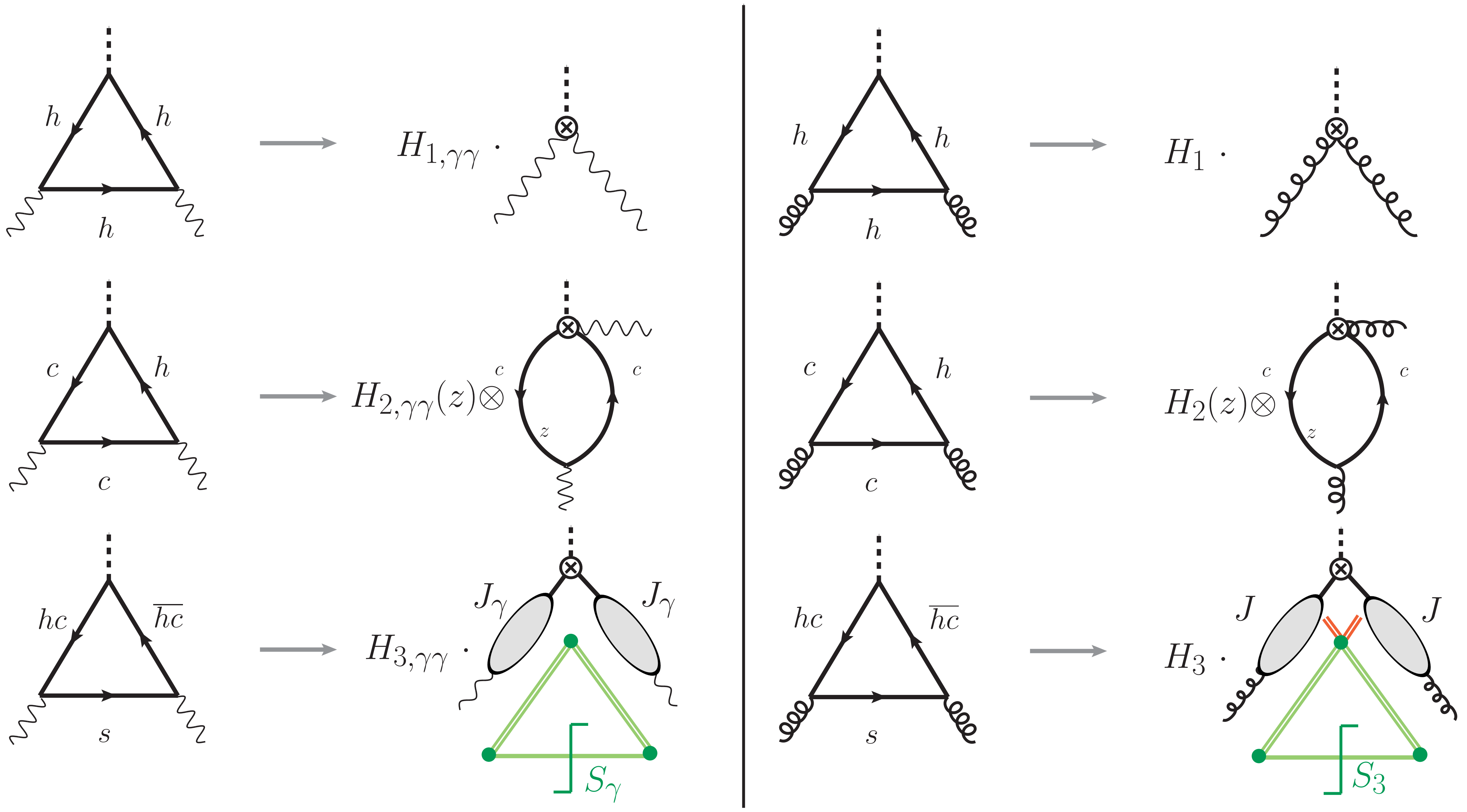
- Soft-quark functions are key ingredients in NLP factorizations and phenomenologically relevant.
- Large log's generated by soft-quark functions can be systematically obtained from RGE:

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}S\left(\{\omega\},\mu\right)=\int\{\mathrm{d}\omega'\}\,\boxed{\gamma_S\left(\{\omega\};\{\omega'\}\right)}\,S\left(\{\omega'\},\mu\right)$$

Outline

- $\gamma\gamma \to h$ form factor induced by light quarks
  - basic tools: position-space formalism + background-field method
- $gg \to h$ form factor induced by light quarks
  - supplement: extra regulator for IR (rapid) divergence in the appearance of semi-infinite Wilson lines
- $\gamma\gamma(gg) \to h$ anomalous dimensions beyond one-loop
  - conformal techniques come into play
- Drell-Yan $g\bar{q}$ channel @ NLP

Higgs Form Factors via Light Quarks



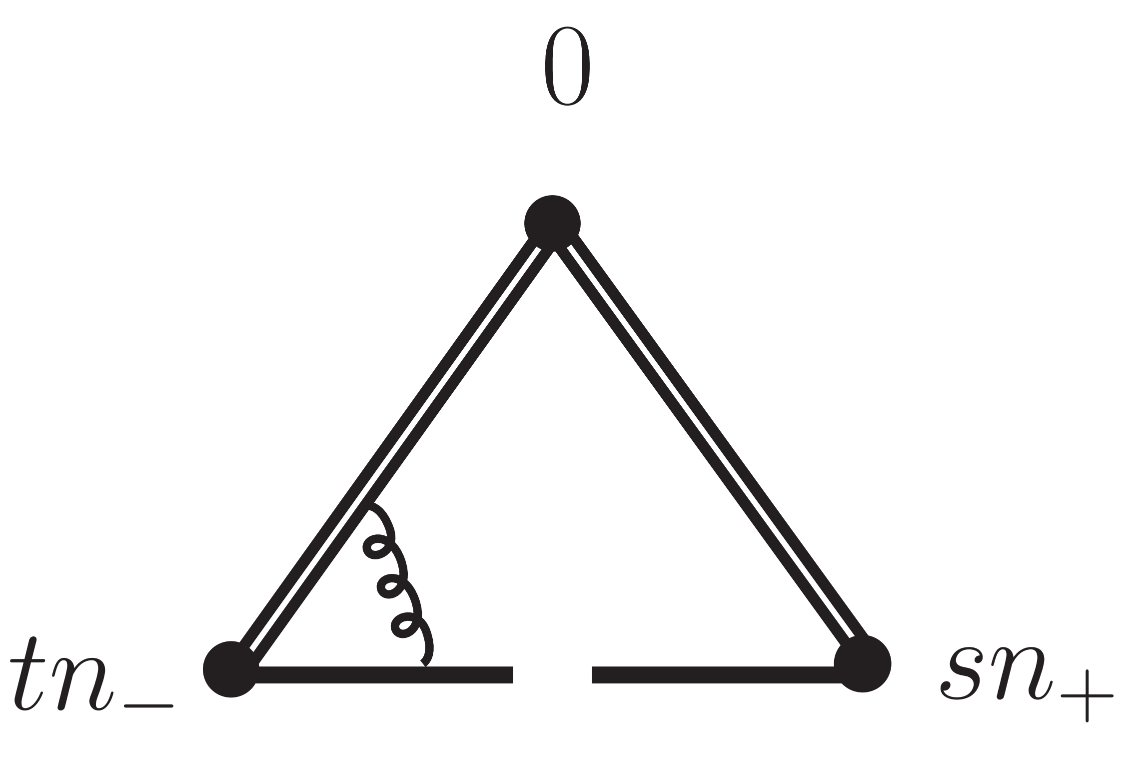
[Liu, Neubert, 1912.08818; Liu, Neubert, Schnubel, XW, 2212.10447]

$$O_\gamma(s,t) = \mathbf{T}\left\{\bar q(tn_-) Y_{n_-}(t) Y^\dagger_{n_-}(0) \frac{\not n_- \not n_+}{4} Y_{n_+}(0) Y^\dagger_{n_+}(s) q(sn_+)\right\}$$
$$= \mathbf{T}\left\{\bar q(tn_-)[tn_-,0]\frac{\not n_- \not n_+}{4}[0,sn_+]q(sn_+)\right\}$$

$$O_g^{\rm uns}(s,t) = \mathbf{T}\left\{\bar q(tn_-) Y_{n_-}(t) T^a Y^\dagger_{n_-}(0) \frac{\not n_- \not n_+}{4} Y_{n_+}(0) T^b Y^\dagger_{n_+}(s) q(sn_+)\right\}$$
$$= \mathbf{T}\left\{\bar q(tn_-)\Big(\mathcal{Y}_{n_-}(tn_-)\Big)^{ac} T^c [tn_-,0]\frac{\not n_- \not n_+}{4}[0,sn_+]\Big(\mathcal{Y}_{n_+}(sn_+)\Big)^{bd} T^d q(sn_+)\right\}$$

“abelian” $\gamma\gamma \to h$

- Anomalous dimension / RG kernel originally [Liu, Mecaj, Neubert, XW, Fleming, 2005.03013] inferred from RG consistency of the factorization formula;
- Direct computation by [Bodwin, Ee, Lee, X.P. Wang, 2101.04872] by a complicated excursion into transverse-momentum dependent soft functions.
- Our method:
  - **Background-field method** [Abbott, 1980; Balitsky, Braun 1988/89] $\Longrightarrow$ at the operator level
  - Calculate directly in the **position space** $\Longrightarrow$ compact and easier for conformal techniques



(a)

most complicated one in our case

$$= (ig_s)^2 \int \mathrm{d}^D z\, \bar{q}(z) \not{A}(z) q(z)\, \bar{q}(tn_-) \int_0^1 \mathrm{d}u\, tn_- \cdot \not{A}(utn_-) \frac{\not{n}_- \not{n}_+}{4} q(sn_+)$$

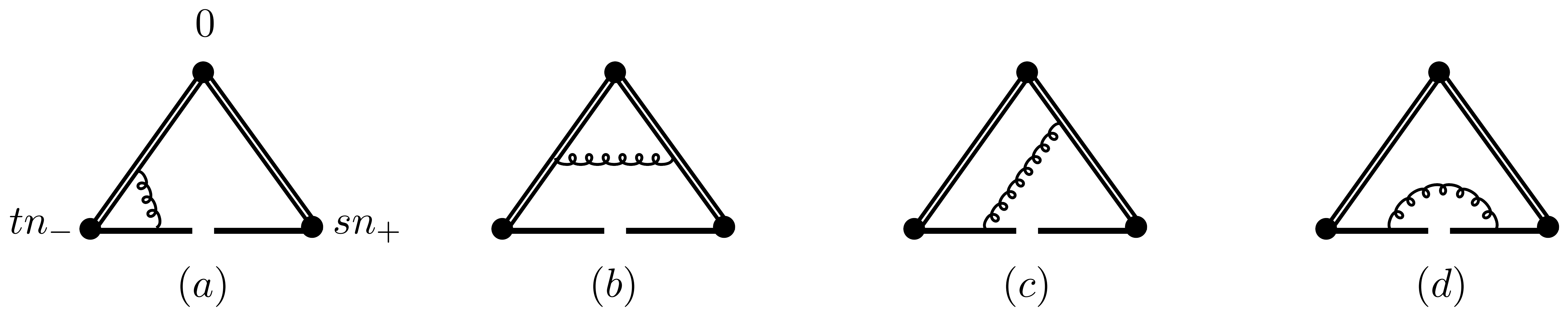
$$= -g_s^2 C_F \mu^{2\varepsilon} \frac{e^{\varepsilon\gamma_E}}{(4\pi)^\varepsilon} \int_0^1 \mathrm{d}u\, t \int \frac{\mathrm{d}^D p}{(2\pi)^D} \int \frac{\mathrm{d}^D l}{(2\pi)^D} e^{it(\bar{u}n_-\cdot l - un_-\cdot p)} \frac{\bar{q}(p)\, \not{n}_- \not{l}}{l^2 (p+l)^2} \frac{\not{n}_- \not{n}_+}{4} q(sn_+)$$

$$= \frac{\alpha_s(\mu)}{4\pi} \frac{2C_F}{\varepsilon} \int_0^1 \mathrm{d}u\, \frac{u}{1-u} \Big[\bar{q}(utn_-) - \bar{q}(tn_-)\Big] \frac{\not{n}_- \not{n}_+}{4} q(sn_+) + \mathcal{O}(\varepsilon^0)$$

$$= \frac{\alpha_s(\mu)}{4\pi} \frac{2C_F}{\varepsilon} \int_0^1 \mathrm{d}u \left[\frac{u}{1-u}\right]_+ O_\gamma(s, ut) + \mathcal{O}(\varepsilon^0)$$

$\varepsilon_{\mathrm{UV}}$

"abelian" $\gamma\gamma \to h$



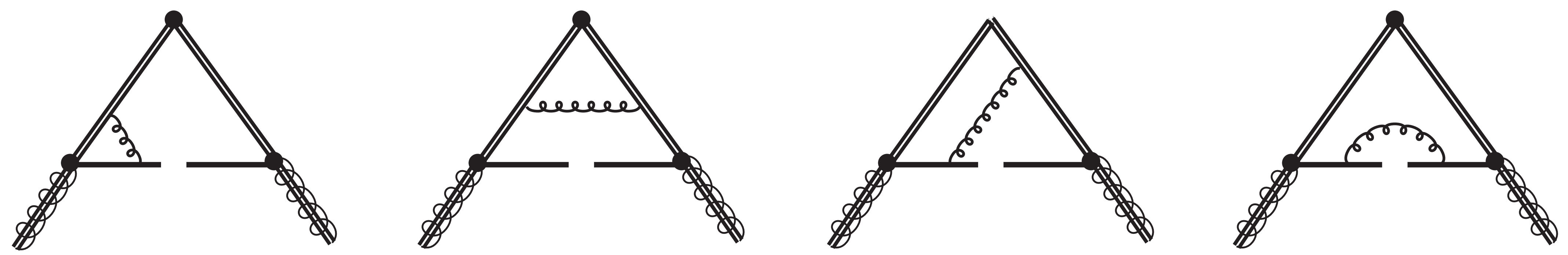
$$O_\gamma(s,t;\mu) = O_\gamma^{\text{bare}}(s,t) + \frac{\alpha_s(\mu)C_F}{4\pi}\int_0^1 \mathrm{d}u \Bigg[\Bigg(\underbrace{\frac{1}{\varepsilon^2} + \frac{2\ln\left(st\mu^2 e^{2\gamma_E}\right) - 1 + \xi}{2\varepsilon}}_{(b)} + \underbrace{\frac{1-\xi}{\varepsilon}}_{(c)} + \underbrace{\frac{\xi}{2\varepsilon}}_{Z_q}\Bigg)\delta(1-u)$$

$$\underbrace{-\frac{2}{\varepsilon}\left[\frac{u}{1-u}\right]_+}_{(a)}\Bigg]\left[O_\gamma^{\text{bare}}(us,t) + O_\gamma^{\text{bare}}(s,ut)\right] + \mathcal{O}(\alpha_s^2)$$

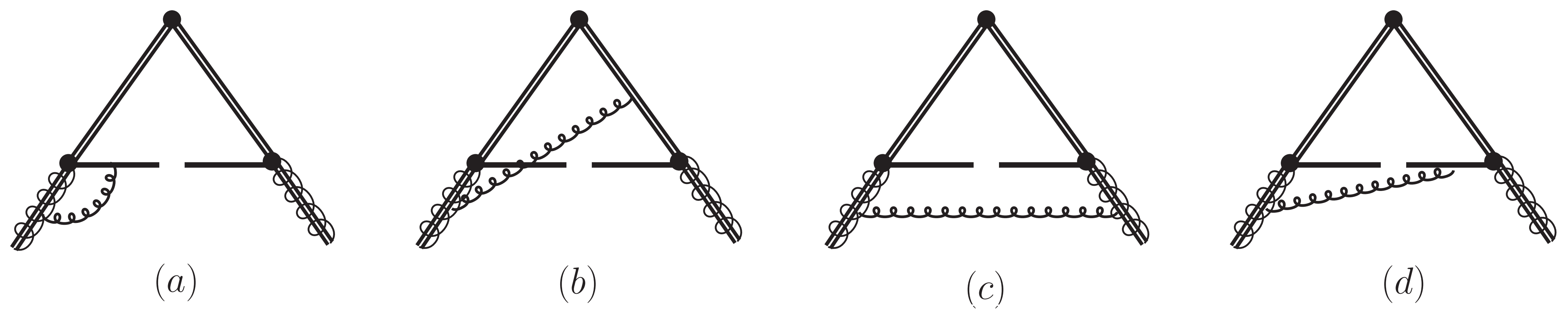
$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}O_\gamma(s,t,\mu) = -[\gamma_\gamma O_\gamma](s,t;\mu) \quad \Longleftarrow \quad [\gamma_\gamma O_\gamma](s,t;\mu) = -\frac{\alpha_s C_F}{\pi}\Bigg\{-\left(\ln\left(st\mu^2 e^{2\gamma_E}\right) + \frac{1}{2}\right)O_\gamma(s,t;\mu)$$

$$+\int_0^1 \mathrm{d}u \left[\frac{u}{1-u}\right]_+ \left(O_\gamma(us,t;\mu) + O_\gamma(s,ut;\mu)\right)\Bigg\} + \mathcal{O}(\alpha_s^2)$$

"non-abelian" $gg \to h$



similar as the abelian case, with replacement of color factors



correlations involving semi-infinite Wilson lines ⇝ rapidity div.

$$\left(\mathcal{Y}_{n_\pm}(x)\right)^{ab} = \hat{\mathrm{P}} \exp\left[-g_s f^{abc} \int_{-\infty}^{0} \mathrm{d}\lambda \, e^{\lambda(-i\delta_\pm + 0^+)} n_\pm \cdot A^c(x + \lambda n_\pm)\right]$$

δ regulators in WL's are related to off-shell regulators in the full theory!

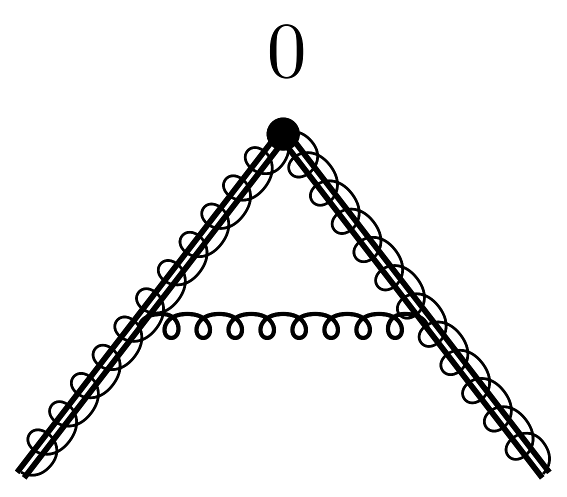
$$\frac{i n_+ \cdot p_c}{(p_c + \ell)^2 + i0^+} \longrightarrow \frac{i}{n_- \cdot \ell + \frac{p_c^2}{n_+ \cdot p_c} + i0^+} \equiv \frac{i}{n_- \cdot \ell + \delta_- + i0^+},$$

$$\frac{i n_- \cdot p_{\bar{c}}}{(p_{\bar{c}} + \ell)^2 + i0^+} \longrightarrow \frac{i}{n_+ \cdot \ell + \frac{p_{\bar{c}}^2}{n_- \cdot p_{\bar{c}}} + i0^+} \equiv \frac{i}{n_+ \cdot \ell + \delta_+ + i0^+}.$$

“non-abelian” $gg \to h$

$\varepsilon = \varepsilon_{\rm UV}$ ⟶

$$
\begin{aligned}
I_{O_g^{\rm uns}}^{(\delta_\pm,\xi)} = {}& O_g^{\rm uns}(s,t) + \frac{\alpha_s(\mu)}{4\pi}\frac{2}{\varepsilon}\left(C_F - \frac{C_A}{2}\right)\int_0^1 {\rm d}u \left[\frac{u}{1-u}\right]_+ \left(O_g^{\rm uns}(us,t) + O_g^{\rm uns}(s,ut)\right) \\
&+ \frac{\alpha_s(\mu)}{4\pi}\left[\frac{C_A}{\varepsilon}\left(2 + \ln\left(st\mu^2 e^{2\gamma_E}\right) + 2\ln\frac{-\delta_-\delta_+}{\mu^2} - \ln\frac{\partial_s\partial_t}{\mu^2}\right)\right. \\
&\left. - 2C_F\left(\frac{1}{\varepsilon^2} + \frac{\ln\left(st\mu^2 e^{2\gamma_E}\right)}{\varepsilon}\right) + \frac{C_A}{\varepsilon}(1-\xi) - \frac{C_F}{\varepsilon}\right] O_g^{\rm uns}(s,t) + \mathcal{O}(\alpha_s^2)\,.
\end{aligned}
$$



$$\mathbb{S}_g(0) = \left(\mathcal{Y}_{n_-}(0)\right)^{ac}\left(\mathcal{Y}_{n_+}(0)\right)^{cb} = \left[-\infty n_-, 0n_-\right]^{ac}\left[0n_+, -\infty n_+\right]^{cb}$$

$$\langle \mathbb{S}_g(0)\rangle = 1 + \frac{\alpha_s(\mu)}{4\pi}C_A\left[-\frac{2}{\varepsilon^2} + \frac{2\ln(-\delta_-\delta_+/\mu^2) + (1-\xi)}{\varepsilon}\right] + \mathcal{O}(\alpha_s^2)$$

$$O_g(s,t) \equiv \frac{O_g^{\rm uns}(s,t)}{\langle \mathbb{S}_g(0)\rangle}$$

- subtract (divide out) Wilson lines for the “charges from/to infinity”, and rearrange to other parts in the **factorization** formula: **IR rearrangement** [MB, Bobeth, Szafron, 1908.0711];
- reproduce the AD from consistency in [Liu, Neubert, Schnubel, XW, 2212.10447].

“non-abelian” $gg \to h$

equal S_g if onshell

equal 1 to all oders if onshell

$$\mathcal{M}_{gg\to h} \supseteq H_3 \cdot \left[J_g(M_h \ell_-) J_g(-M_h \ell_+)\right] \otimes S_g^{\text{uns}}(\ell_- \ell_+)\, \mathfrak{A}_g^{+,(\xi)}(k_1^2)\, \mathfrak{A}_g^{-,(\xi)}(k_2^2)$$

[Liu, Neubert, Schnubel, XW, 2212.10447]

$$= H_3 \cdot \left[J_g(M_h \ell_-) J_g(-M_h \ell_+)\right] \otimes \left[\frac{S_g^{\text{uns}}(\ell_- \ell_+)}{R_+^{(\xi)} R_-^{(\xi)}}\right] \left[R_+^{(\xi)} \mathfrak{A}_g^{+,(\xi)}(k_1^2)\right] \left[R_-^{(\xi)} \mathfrak{A}_g^{-,(\xi)}(k_2^2)\right]$$

$$S_g(\ell_- \ell_+)$$

$$\langle \mathbb{S}_g(0) \rangle = R_-^{(\xi)} R_+^{(\xi)}$$

- Each part in the bracket is well defined and gauge independent.

beyond one-loop

- At one loop, both AD's factorize in position space: $\gamma_i(s,t) = \gamma_i(s) + \gamma_i(t),\ \ i = \gamma, g;$
- Factorized pieces can be rewritten by collinear conformal generators

$$\hookrightarrow\ \widehat{S}_+ = s^2\partial_s + 2js = s\theta_s + 2js, \quad \widehat{S}_0 = s\partial_s + j = \theta_s + j, \quad \widehat{S}_- = -\partial_s$$

$$\widehat{T}_+ = t^2\partial_t + 2jt = t\theta_t + 2jt, \quad \widehat{T}_0 = t\partial_t + j = \theta_t + j, \quad \widehat{T}_- = -\partial_t$$

$$\gamma_\gamma(s,t) = \frac{\alpha_s(\mu)}{4\pi}\left[4C_F \ln\left(\mu^2 e^{4\gamma_E}\widehat{S}_+\widehat{T}_+\right) - 6C_F\right] + \mathcal{O}(\alpha_s^2)$$

$$\gamma_g(s,t) = \frac{\alpha_s(\mu)}{4\pi}\left[4\left(C_F - \frac{C_A}{2}\right)\ln\left(\mu^2 e^{4\gamma_E}\widehat{S}_+\widehat{T}_+\right) + 2C_A \ln\frac{\widehat{S}_-\widehat{T}_-}{\mu^2} - 6C_F\right] + \mathcal{O}(\alpha_s^2)$$

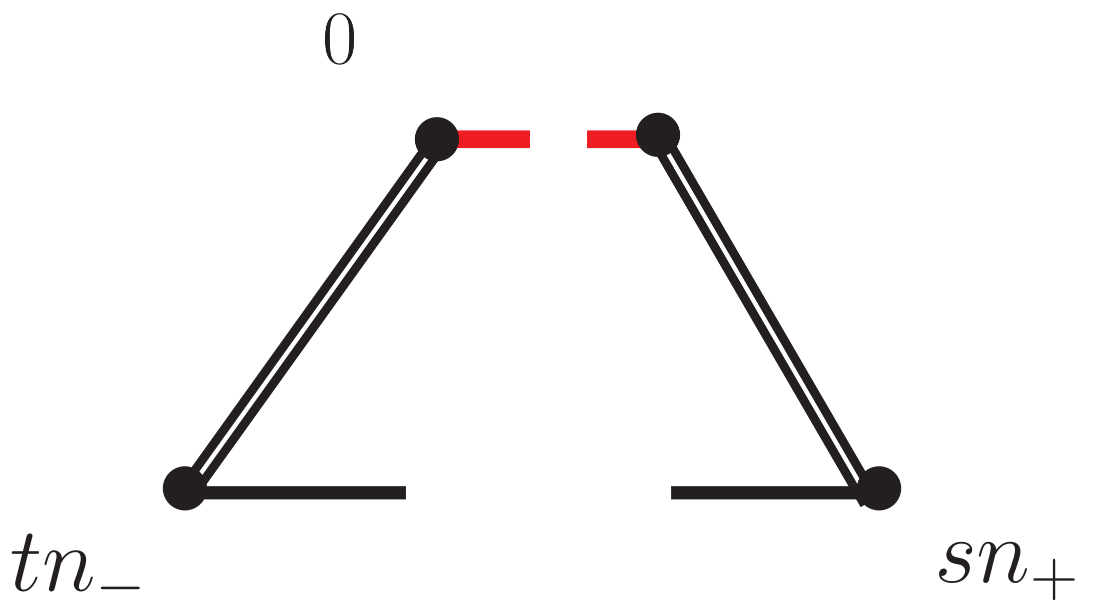
$$\hookrightarrow\quad \gamma_\gamma(s) = \frac{\alpha_s(\mu)}{4\pi}\left[4C_F \ln\left(\mu e^{2\gamma_E}\widehat{S}_+\right) - 3C_F\right] + \mathcal{O}(\alpha_s^2)$$

$$\gamma_g(s) = \frac{\alpha_s(\mu)}{4\pi}\left[4\left(C_F - \frac{C_A}{2}\right)\ln\left(\mu e^{2\gamma_E}\widehat{S}_+\right) + 2C_A \ln\frac{-\widehat{S}_-}{\mu} - 3C_F\right] + \mathcal{O}(\alpha_s^2)$$

beyond one-loop

twist-2 B-LCDA operator: $\mathbf{T}\left\{\bar{q}(tn_-)[tn_-,0]\frac{\not{n}_-}{2}h_v(0)\right\}$

$$O_\gamma(s,t) = \mathbf{T}\left\{\underbrace{\bar{q}(tn_-)[tn_-,0]\frac{\not{n}_-}{2}}_{n_-}\underbrace{\frac{\not{n}_+}{2}[0,sn_+]q(sn_+)}_{n_+}\right\}$$



"double-copy" of twist-2 B-LCDA

- The "abelian" case is essentially a "double-copy" of twist-2 B-LCDA AD!

$$\gamma_\gamma(s,t,\mu) = \mathscr{H}_B(s,\mu) + \mathscr{H}_B(t,\mu) - 2\gamma_Q$$

- AD of the twist-2 B-LCDA case is calculated to two loops [Braun, Ji, Manashov, 1905.04998].
  ↪ Two-loop AD for the "abelian" case is for free. Agree with [Liu, Mecaj, Neubert, XW, Fleming, 2005.03013].

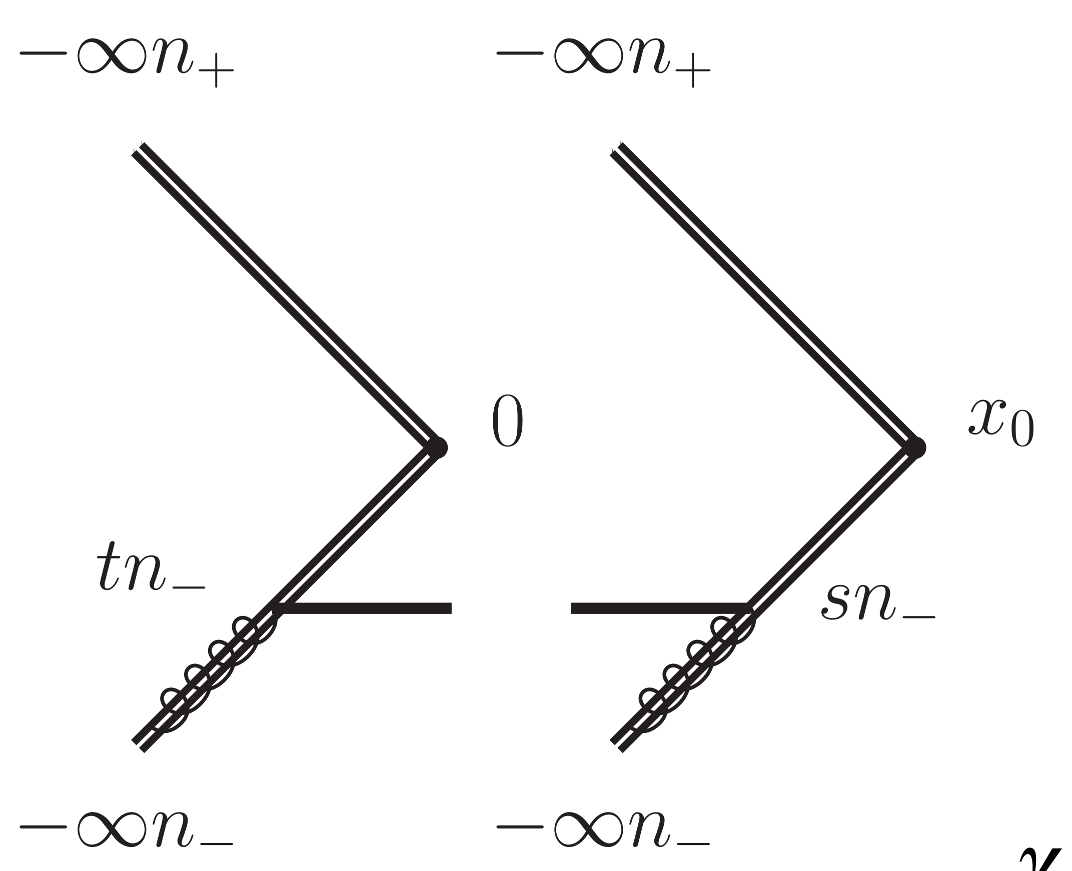
$$\mathscr{H}_B(s,\mu) = \Gamma_{\text{cusp}}\left(\alpha_s\right)\ln\left(\mathscr{K}\left(\alpha_s;s\right)\mu e^{2\gamma_E}\right) + \Gamma_+\left(\alpha_s\right), \quad \mathscr{K}\left(\alpha_s;s\right) = \hat{S}_+ + \mathcal{O}(\alpha_s)$$

- No "double-copy" for the "non-abelian" case due to $\hat{S}_-\hat{T}_-$. A two-loop **ansatz** in the position space with the **indirect constraint** from [Liu, Neubert, Schnubel, XW, 2112.00018] on the constant $\Gamma_g(\alpha_s)$:

$$\gamma_g(s,t) = \left(\Gamma^F_{\text{cusp}}(\alpha_s) - \frac{1}{2}\Gamma^A_{\text{cusp}}(\alpha_s)\right)\ln\left(\mathscr{K}\left(\alpha_s;s\right)\mu e^{2\gamma_E}\right) + \frac{1}{2}\Gamma^A_{\text{cusp}}(\alpha_s)\ln\frac{\hat{S}_-}{\mu} + \Gamma_g(\alpha_s) + (s\to t)$$

Drell-Yan $g\bar{q}$ channel @ NLP

[Beneke, Broggio, Jaskiewicz, Vernazza, 1912.01585]



$$O^{\rm NLP}_{g\bar{q},{\rm uns}}(x_0,\{s\}) = \frac{g_s^2}{N_c C_F}\,\mathbf{Tr}\,\bar{\mathbf{T}}\left[\bar{q}_s(x_0+s_1 n_-)\mathcal{Y}^{ca}_{n_-}(x_0+s_1n_-)T^c\left[x_0+s_1n_-,x_0\right]Y_{n_+}(x_0)\right]\frac{\not{n}_-}{4}$$
$$\times\,\mathbf{T}\left[Y^\dagger_{n_+}(0)\left[0,s_2n_-\right]T^d\,\mathcal{Y}^{ad}_{n_-}(s_2n_-)\,q_s(s_2n_-)\right]$$

$$\gamma_{g\bar{q}}(x_0;s,t) = \frac{\alpha_s(\mu)}{4\pi}\Bigg[\overbrace{4\left(C_F-\frac{C_A}{2}\right)\ln\left(\mu^2 e^{4\gamma_E}\hat{S}_+\hat{T}_+\right)+2C_A\ln\frac{\hat{S}_-\hat{T}_-}{\mu^2}}^{gg\to h}-6C_F$$
$$\underbrace{-4(C_F+C_A)\ln\left(i\mu e^{\gamma_E}x^0/2\right)}_{\text{DY LP-like}}+\beta_0\Bigg]+\mathcal{O}(\alpha_s^2)$$

- LP-like contribution factorizes from the NLP one;
- The soft-quark effect is universal!

Conclusion and Outlook

- NLP SCET is important in the precision era, and the soft-quark effect plays a key role;
- Position-space formalism, together with the background-field method, is powerful in deriving the anomalous dimensions directly at the operator level;
- Immediate steps for deriving AD's may involve IR (rapidity) divergences. δ regulators have a deep connection with factorization.

➡ Apply the formalism to more NLP observables and try to classify the involved building blocks of AD's;

➡ How much can conformal symmetry techniques help us on QCD and its EFTs?

Thank you!