Collaborative Research Center TRR 257

Particle Physics Phenomenology after the Higgs Discovery

NLO Yukawa and self-coupling corrections to $gg \to HH$

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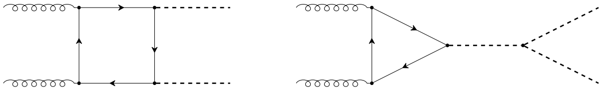
ITP - KIT, IPPP

Why calculate higher orders to $gg \to HH$

- Sensitivity to Higgs selfcoupling λ

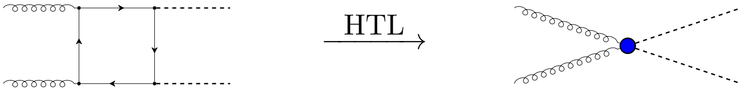


- Match expected experimental uncertainty at HL-LHC, corrections impact the extracted constraints
- Sizeable effects on differential cross sections expected
- Les Houches Wishlist $>$ 2015

Wishlist	known dσ	desired dσ
2016	$\text{N}^2\text{LO}_{\text{HTL}}$, NLO_{QCD}	$\text{N}^2\text{LO}_{\text{HTL}}+\text{NLO}_{\text{QCD}}+\text{NLO}_{\text{EW}}$
2021	$\text{N}^3\text{LO}_{\text{HTL}} \otimes \text{NLO}_{\text{QCD}}$	NLO_{EW}

A rudimentary history

- LO is already at loop level $\Rightarrow$ Challenging calculation for NLO
- LO was already calculated 1988 (Glover and van der Bij 1988)
- First full m_t dependent NLO QCD result from 2016 (Borowka et al. 2016), (Baglio et al. 2019)
- First full NLO EW result from 2023 (Bi et al. 2023)
- Simplification via expansions or heavy top limit is possible in certain kinematic ranges



- On the way to higher orders numerous combinations of these techniques are used, for QCD, e.g. (Bagnaschi et al. 2023; Grazzini et al. 2018), for EW, e.g. (Mühlleitner et al. 2022), or both, e.g. (J. Davies et al. 2023)

Our higher order calculation toolchain

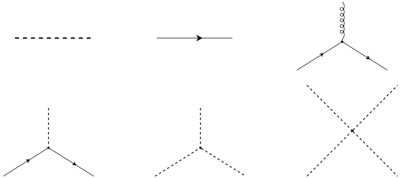
1. Produce contributing diagrams (`QGRAF`)
2. Project onto form factors (`Mathematica`)
3. Reduce the number of integrals (`kira`, `Reduze`, `Ratracer`)
4. Integrate the remaining master integrals (`pySecDec`)
5. Perform the Renormalization (blood, sweat and tears)
6. Crosschecks (`DiffExp`)
7. Put everything back together

The bare Lagrangian

- Gaugeless limit $\Rightarrow$ Weak bosons decouple
- Unitary gauge $\Rightarrow$ Goldstone bosons decouple

$$\mathcal{L} = -\frac{1}{4}\mathcal{G}_{0,\mu\nu}\mathcal{G}_0^{\mu\nu} + \frac{1}{2}(\partial_\mu H_0)^\dagger(\partial^\mu H_0) - \frac{m_{H,0}^2}{2}H_0^2 - \frac{m_{H,0}^2}{2v_0}H_0^3 - \frac{m_{H,0}^2}{8v_0^2}H_0^4$$
$$+ i\bar{t}_0 \not{D} t_0 - m_{t,0}\bar{t}_0 t_0 - \frac{m_{t,0}}{v_0}H_0\bar{t}_0 t_0 + \text{constant}$$

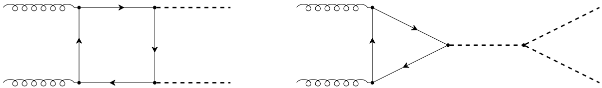
Yields Feynman rules for:



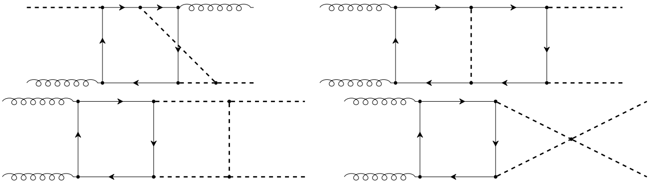
Reparametrized in terms of $m_{H,0}$, $m_{t,0}$ and v_0.

Contributing Diagrams

LO



NLO (examples)



Automated by the tool `QGRAF`. (Nogueira 1993)

Formfactors

Separate the matrix element into tensor structures and Form Factors

$$\mathcal{M}^{\mu\nu} = F_1 T_1^{\mu\nu} + F_2 T_2^{\mu\nu}$$

Form factors can be obtained by using projectors

$$\mathcal{P}_i^{\mu\nu} T_{j,\mu\nu} = \delta_{ij}$$

$$T_1^{\mu\nu} = g^{\mu\nu} - \frac{p_1^\nu p_2^\mu}{p_1 \cdot p_2}$$

$$T_2^{\mu\nu} = g^{\mu\nu} + \frac{m_H^2 p_1^\nu p_2^\mu}{p_T^2 p_1 \cdot p_2} - \frac{2 p_1 \cdot p_3 p_2^\mu p_3^\nu}{p_T^2 p_1 \cdot p_2} - \frac{2 p_2 \cdot p_3 p_1^\nu p_3^\mu}{p_T^2 p_1 \cdot p_2} + \frac{2 p_3^\mu p_3^\nu}{p_T^2}$$

with

$$p_T = \sqrt{\frac{ut - m_H^4}{s}}$$

Coupling Structures

Each diagram is sorted into different classes, according to the occurring couplings

$$g_{t,0} \equiv \frac{m_{t,0}}{v_0} \qquad g_{3,0} \equiv \frac{3m_{H,0}^2}{v_0} \qquad g_{4,0} \equiv \frac{3m_{H,0}^2}{v_0^2}$$

and whether it is a 1PI or 1PR contribution.

At NLO:

$$F_i = g_{s,0}^2 \Big(g_{3,0}\, g_{4,0}\, g_{t,0}\, F_{i,g_3 g_4 g_t} + g_{3,0}^3\, g_{t,0}\, F_{i,g_3^3 g_t} + g_{4,0}\, g_{t,0}^2\, F_{i,g_4 g_t^2} + g_{3,0}^2\, g_{t,0}^2\, F_{i,g_3^2 g_t^2} + g_{3,0}\, g_{t,0}^3\, F_{i,g_3 g_t^3} + g_{t,0}^4\, F_{i,g_t^4} \Big)$$

Type	$g_3 g_4 g_t$	$g_3^3 g_t$	$g_4 g_t^2$	$g_3^2 g_t^2$	$g_3 g_t^3$	g_t^4
1PI	0	0	3	6	24	60
1PR	12	6	1	6	24	26
Total	12	6	4	12	48	86

IBP Reduction

- Choose a suitable basis of master integrals M.I.:
  - prefer dots over numerators
  - search for finite coefficients for top-level M.I. from non-planar sectors
  - avoid poles on diagonal elements of differential equation system
- Have obtained a fully symbolic reduction to M.I.s retaining dependence on s, t, m_t and m_H using `kira` with `ratracer` (Klappert et al. 2021; Magerya 2022)
- Faster reduction: fix as many open parameters as possible, e.g.

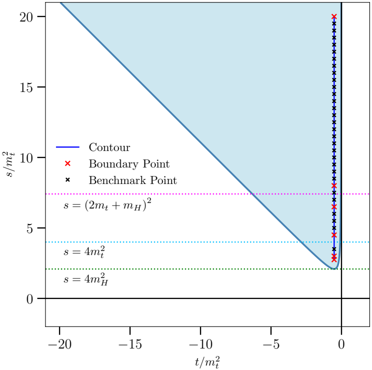
$$\frac{m_H^2}{m_t^2} = \frac{12}{23} .$$

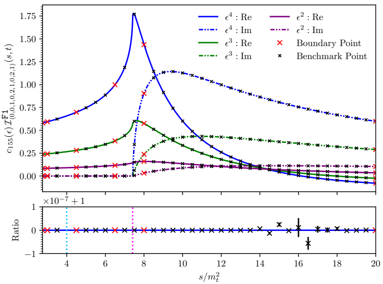
This is calculated with `reduze`. (Manteuffel and Studerus 2012)

The Master Integrals

- The total number of remaining master integrals is 492
- *d*-factorizing integrals, i.e. parts depending on dimensionality *d* are separated from parts containing the kinematic dependence
- Up to three dots, dimension shifts between $2 - 2\epsilon$ and $8 - 2\epsilon$
- Still, too many mass scales to solve analytically
- Numerical evaluation using `pySecDec` is feasible (Heinrich et al. 2024)
- Bottleneck: computation of rational coefficients in fully symbolic amplitude $\Rightarrow$ use preinserted $\frac{m_H^2}{m_t^2} \equiv \frac{12}{23}$
- Spurious poles at $\mathcal{O}(\epsilon^{-4}, \epsilon^{-3}, \epsilon^{-2})$

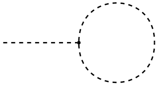
Crosscheck with `DiffExp`

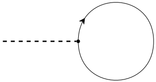




- Run contours in `DiffExp` between boundary points
- Check `pySecDec` vs `DiffExp` for benchmark points

Tadpole Renormalization





- At higher orders the vev gets shifted.
- Fleischer-Jegerlehner tadpole scheme: (Fleischer and Jegerlehner 1981)

$$H + v \to H + v + \Delta v$$

- Require the tadpole diagrams T_H to vanish also at NLO through the tadpole counterterm

$$\delta T = -T_H$$

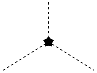
- Identify $\delta T = -\Delta v m_H^2$
- This corresponds to a redistribution of tadpole contributions.

Counterterms

$$\mathcal{M}_{\rm ren} = \mathcal{M}^{(0)}(m_t, m_H^2, v) + \mathcal{M}^{(1)}_{\delta \rm X}(m_t, m_H^2, v) + \mathcal{M}^{(1)}(m_t, m_H^2, v) + \mathcal{O}(\delta X^2)$$

Introduce CTs:

$$\begin{aligned}
H_0 &= \sqrt{Z_H} H = \sqrt{1+\delta_H} H \\
t_0 &= \sqrt{Z_t} t = \sqrt{1+\delta_t} t \\
m_{H,0}^2 &= m_H^2(1+\delta m_H^2) \\
m_{t,0} &= m_t(1+\delta m_t) \\
v_0 + \Delta v &= v(1+\delta_v) + \Delta v
\end{aligned}$$



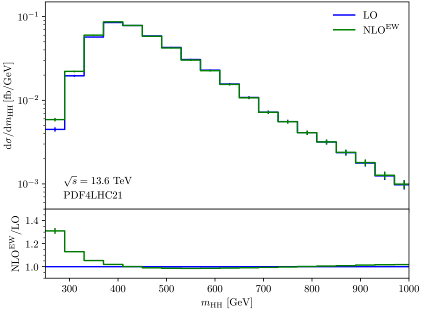
$$= -i3\frac{m_H^2}{v}\left(\delta m_H^2 + \frac{3}{2}\delta_H - \delta_v - \frac{\delta T}{v m_H^2}\right)$$

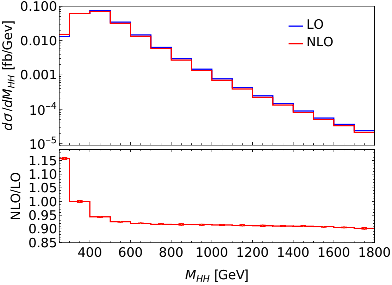
etc.

- $\delta_H, \delta_t, \delta m_H^2, \delta m_t$ fixed through on-shell renormalization conditions
- δ_v fixed in G_μ scheme according to (Biekötter et al. 2023)

The Cross Section

Corrections	Yukawa			Full EW (Bi et al. 2023)
$\sqrt{s}$	13 TeV	13.6 TeV	14 TeV	14 TeV
LO [fb]	16.45	18.26	19.52	19.96
$\mathrm{NLO}^{\mathrm{EW}}$ [fb]	16.69	18.52	19.79	19.12
$\mathrm{NLO}^{\mathrm{EW}}/\mathrm{LO}$	1.01	1.01	1.01	0.958





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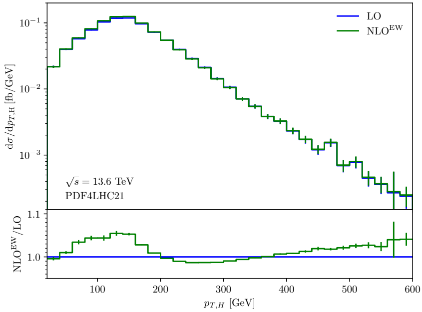
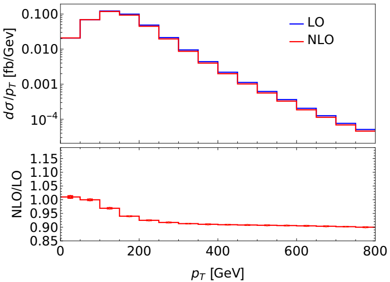




Conclusion

Where we are:

- Achieved fully symbolic reduction
- Crosschecked with (Joshua Davies et al. 2024), and `DiffExp` setup
- Found $K = 1.01$
- Acquired invariant Higgs pair mass and transverse momentum distributions of the cross section
  - Quite large enhancement in low m_{HH} region
  - No Sudakov logs $\Rightarrow$ tail of distributions only slightly changed
  - Dominant contributions from vector bosons expected

Where to go:

- Include the full EW corrections and cross-check the result of (Bi et al. 2023)
- Investigate the effects of the bottom quark
- Implement an EFT framework

Deriving the Tensor structure

General structure:

$$\begin{aligned}\mathcal{M}^{\mu\nu} = \quad & a_{00} g^{\mu\nu} + a_{21} p_2^\mu p_1^\nu + a_{31} p_3^\mu p_1^\nu + a_{23} p_2^\mu p_3^\nu + a_{33} p_3^\mu p_3^\nu \\ & + a_{11} p_1^\mu p_1^\nu + a_{22} p_2^\mu p_2^\nu + a_{12} p_1^\mu p_2^\nu + a_{13} p_1^\mu p_3^\nu + a_{32} p_3^\mu p_2^\nu\end{aligned}$$

Further constraints from Ward identities:

$$\epsilon_{1,\mu} p_1^\mu = 0 \qquad \epsilon_{2,\nu} p_2^\nu = 0$$

Basic example of Sector Decomposition

$$\mathfrak{I} = \int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}y x^{-1-a\epsilon} y^{-b\epsilon} \left(x + (1-x)y\right)^{-1}$$

Diverging for $x \to 0$ and $y \to 0$

$$\mathfrak{I} = \int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}y x^{-1-a\epsilon} y^{-b\epsilon} \left(x + (1-x)y\right)^{-1} \left[\Theta(x-y) + \Theta(y-x)\right]$$

Variable transformation $y = xt$ and $x = yt$

$$\begin{aligned}\mathfrak{I} = &\int_0^1 \frac{\mathrm{d}x}{x^{1+(a+b)\epsilon}} \int_0^1 \frac{\mathrm{d}t}{t^{b\epsilon}\left(1 + (1-x)t\right)} \\ &+ \int_0^1 \frac{\mathrm{d}x}{y^{1+(a+b)\epsilon}} \int_0^1 \frac{\mathrm{d}t}{t^{1+a\epsilon}\left(1 + (1-y)t\right)}\end{aligned}$$

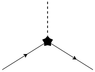
Both limits $x \to 0$ and $y \to 0$ are independent

On-Shell Renormalization

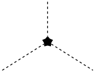
$$0 = \Big[\Sigma_i(\hat{p})\Big]_{\not{p}=m_i} \qquad\qquad 0 = \left[\frac{\mathrm{d}}{\mathrm{d}\not{p}}\Sigma_i(\not{p})\right]_{\not{p}=m_i}$$

$$= -i\left[(m_t - \not{p})\delta_t + m_t\delta m_t - \frac{m_t}{v m_H^2}\delta T\right]$$

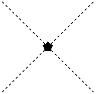
$$= -i\left[(m_H^2 - p^2)\delta_H + m_H^2\delta m_H^2 - 3\frac{\delta T}{v}\right]$$



$$= -i\frac{m_t}{v}\left(\delta m_t + \frac{\delta_H}{2} + \delta_t - \delta_v\right)$$



$$= -i3\frac{m_H^2}{v}\left(\delta m_H^2 + \frac{3}{2}\delta_H - \delta_v - \frac{\delta T}{v m_H^2}\right)$$



$$= -i3\frac{m_H^2}{v^2}(\delta m_H^2 + 2\delta_H - 2\delta_v)$$