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# THE NNLO SOFT FUNCTION FOR N-JETTINESS IN HADRONIC COLLISIONS

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Based on [2312.11626] with Guido Bell, Bahman Dehnadi, and Tobias Mohrmann

High Precision for Hard Processes, Torino, 12th September 2024

# THE CRIME SCENES

u^b

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DESY.

# OUTLINE

- Background
- Calculation:
  SoftSERVE with N jets
- Results
- Endpoints

# N-JETTINESS

[Stewart, Tackmann, Waalewijn, '10]

- General definition:

$$\mathcal{T}_N = \sum_m \min_{\{q_j\}} \frac{2q_j \cdot k_m}{Q_j}$$

$k_i^\mu$ : Momenta of particles in the event

$q_j^\mu$ : Momenta of signal jets/beams

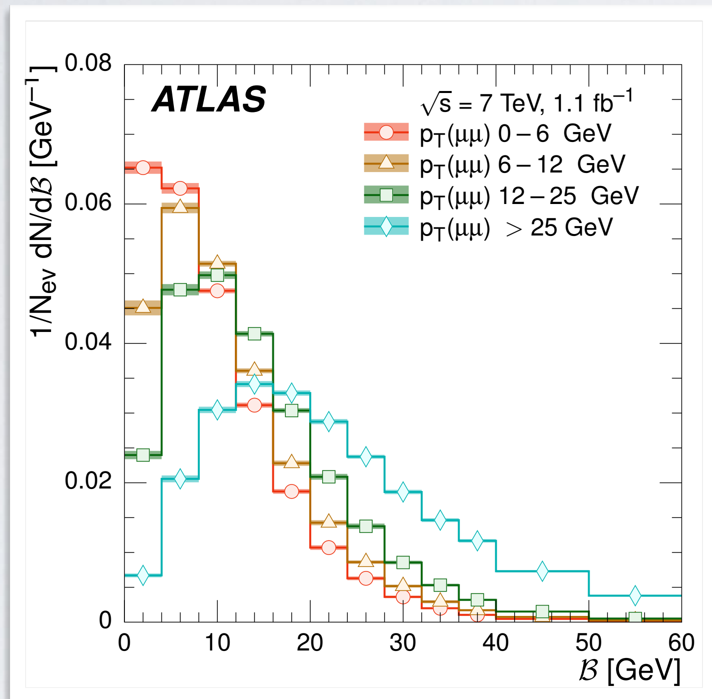
$Q_j$ : Weight factors

- We choose $Q_j = 2E_j$, then $n_j = (1, \vec{n}_j)$, and

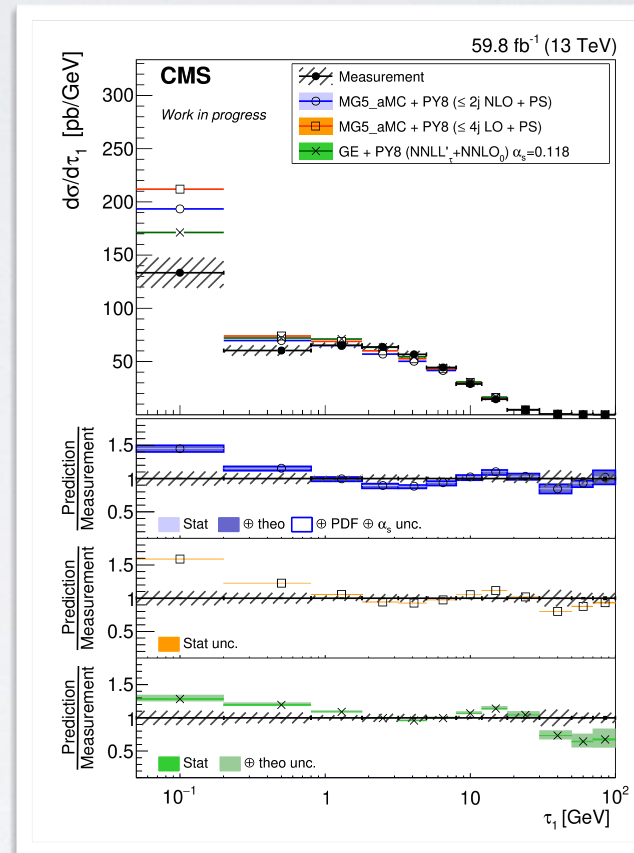
$$\mathcal{T}_N = \sum_m \min_j n_j \cdot k_m$$

# UN-OBSERVED OBSERVABLE

- Few measurements




[Aad et al, '16]




[Mijušković, '22]

- Reason: hadronisation, pileup

- Modifications: Leptons (Thrust), DIS, tracks,…

# PHASE SPACE SLICING

[Boughezal, Focke, Liu, Petriello, '15]
[Gaunt, Stahlhofen, Tackmann, Walsh, '15]

- How to get finite NNLO cross section predictions?

$$\sigma_{VV} + \sigma_{RV} + \sigma_{RR} = \text{finite}$$

- *Slice* phase space to add unresolved reals to virtuals

- N-jettiness to resolve additional emissions

$$\sigma_{\text{NNLO}} = \sigma_{\text{NNLO}}(\mathcal{T}_N < \mathcal{T}_N^{\text{cut}}) + \sigma_{\text{NLO+1}}(\mathcal{T}_N > \mathcal{T}_N^{\text{cut}})$$

approximate using small $\mathcal{T}_N^{\text{cut}}$

# STATE OF THE ART

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{T}_N} = \sum_{i,j,\{k_n\}} B_i \otimes B_j \otimes \prod_{n=1}^{N} J_{k_n} \otimes \mathrm{tr}[H_{ij\to\{k_n\}} * S_{ij\to\{k_n\}}] + \mathcal{O}(\mathcal{T}_N)$$

- Hard, beam, and jet functions known to NNLO+
  [Broggio et al., '14]
  [Stewart et al., '10; Berger et al., '11; Gaunt et al., '14; Ebert et al., '20; Baranowski et al. '22]
  [Bauer et al, '04; Bosch et al., '04; Becher et al., '06, '10, '11; Brüser et al., '18; Banerjee et al., '18]

- NLO soft function, NNLO for 0-, 1-, partial 2-jettiness known
  [Jouttenous et al., '11]
  [Kelley et al., '11; Monni et al., '11; Hornig et al., '17]
  [Campbell et al., '18; Boughezal et al., '15]
  [Bell et al., '18; Jin et al., '19]

$\Rightarrow$ Calculate NNLO Soft function for at least N=2

- Recently: recipe for NNLO N-jettiness (confirming/-ed)
  [Agarwal, Melnikov, Pedron, '24]

Soft function
Simulation and
Evaluation of
Real and
Virtual
Emissions

[Bell, RR, Talbert, '18, '20]

- Soft functions can be calculated from

$$S(\tau,\epsilon) = \int \mathrm{d}\Pi_i \, |\mathcal{A}(\{k_i\},\epsilon)|^2 \, \delta(\tau - \tau(\{k_i\}))$$

$$S(\tau,\mu) = \frac{1}{N_c} \sum_X \mathcal{M}(\tau,k_i) \ \mathrm{Tr}\, \langle 0|S_{\bar{n}}^\dagger(0) S_n(0)|X\rangle\langle X|S_n^\dagger(0) S_{\bar{n}}(0)|0\rangle$$

- Amplitude is divergent, measurement harmless

$\Rightarrow$ Isolate divergences analytically,
do all the rest numerically

# QUICK EXAMPLE: NLO

- Matrix element $$|\mathcal{A}(k)|^2 \sim \frac{1}{k\cdot n \;\; k\cdot \bar{n}}$$
- Expose divergences $$k^\mu = k_T \sqrt{y}\, n^\mu + \frac{k_T}{\sqrt{y}} \bar{n}^\mu + \ldots$$
- Divergences exposed: $$\int \mathrm{d}^d k\, |\mathcal{A}|^2 \sim \int \mathrm{d}k_T\, \mathrm{d}y\, k_T^{-1-\epsilon} y^{-1}$$ (Angles and observable omitted)
- Classify observable behaviour:

Leading behaviour

$$\tau(k) = k_T\, y^{\frac{n}{2}}\, f(y, \{\vartheta\})$$

Mass dimension

# QUICK EXAMPLE: NLO

- Master formula $$S_0(\tau) = 1 + \left(\frac{Z_\alpha \alpha_s}{4\pi}\right)(\mu^2 \bar{\tau}^2)\, S_R(\epsilon) + \mathcal{O}(\alpha_s^2)$$

$$S_R(\epsilon) = \frac{16 C_F e^{-\gamma_E \epsilon}}{\sqrt{\pi}} \frac{\Gamma(-2\epsilon)}{\Gamma(1/2-\epsilon)} \int_0^1 \mathrm{d}t \int_0^1 \mathrm{d}y\, y^{-1+n\epsilon}\, f(y,t)^{2\epsilon} \big(4t(1-t)\big)^{-1/2-\epsilon}$$

- Leads to a (bare) soft function involving integrals of *f*

$$S_R(\epsilon) = \frac{-4C_F}{n\epsilon^2} - \frac{8C_F}{n\epsilon}\left[\int_0^1 \mathrm{d}t \frac{\ln f(0,t)}{\pi\sqrt{t(1-t)}} + \ldots\right] + \ldots$$

- SoftSERVE wraps this in C++, and integrates using Cuba

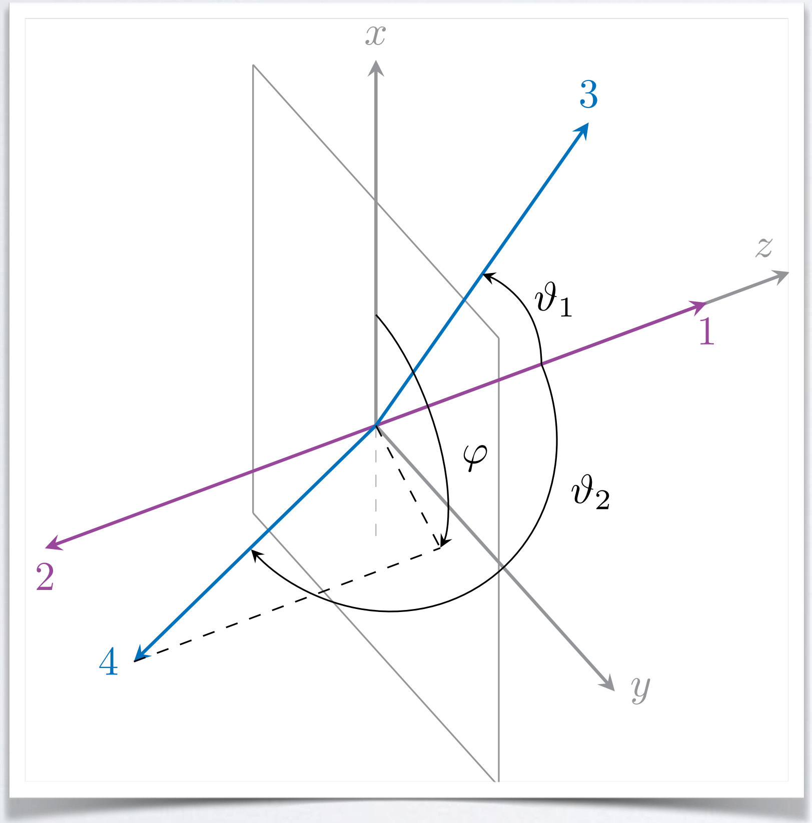
[Hahn, '05]

- Extension to N-jet cases:
  - More emitters
  - Nontrivial colour
  - Complicated geometry
  - Two more integration dimensions

$$C_1\, S_{12} \Rightarrow \sum_{i \neq j = 0}^{N+2} \mathbf{T}_i \cdot \mathbf{T}_j\, S_{ij}$$

(And tri-/quadrupoles)

- Complicated geometry, see e.g. 2-jettiness




1-jettiness: 1 angle
2-jettiness: 3 angles
3-jettiness: 5 angles
N-jettiness: 2N-1 angles

- Grids grow very large

# THE CALCULATIONS

- Sample in steps of $\frac{\pi}{25}$ , yielding 34476 samples (29904 2-jettiness, rest 0- and 1-jettiness)
- Fourfold symmetry reduces this to 8619
- 24 tripole and 18 dipole programs per point (NLO, $C_A$, and $n_f$ for 6 dipoles)
- 900GB binaries and log files, 30min per point (NNLO programs, particularly $C_A$, the slowest)

# THE COMPETITION

- [Agarwal et al, '24] use simplifications: $f_{\text{N-jn}}(0,t) = 1$

$$S_R(\epsilon) = \frac{-4C_F}{n\epsilon^2} - \frac{8C_F}{n\epsilon}\left[\int_0^1 \mathrm{d}t \frac{\ln f(0,t)}{\pi\sqrt{t(1-t)}} + \ldots\right] + \ldots$$

- Do you need a recipe or a grid? Do you need N>2?

# THE TARGET

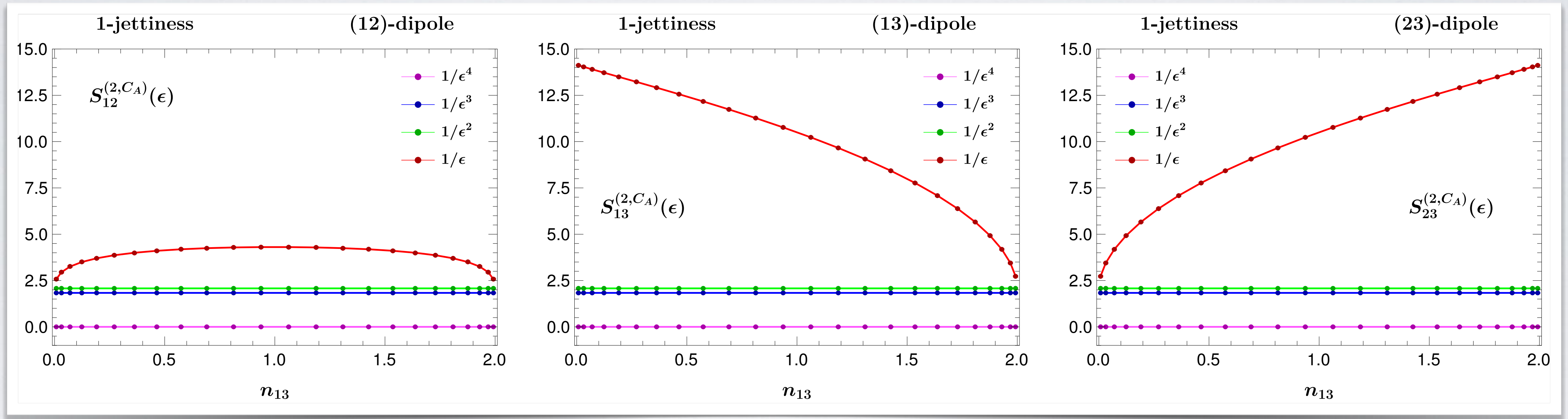
- What I'm about to plot:

$$S(\tau,\mu) = 1 + \left(\frac{\alpha_s}{4\pi}\right)\sum_{i\neq j}\mathbf{T}_i\cdot\mathbf{T}_j\left(\frac{\Gamma_0}{n}\,L_{ij}^2 + \frac{2\gamma_0^S}{n}\,L_{ij} + c_{ij}^{(1)}\right) + \left(\frac{\alpha_s}{4\pi}\right)^2$$
$$\times\Bigg\{\sum_{i\neq j}\mathbf{T}_i\cdot\mathbf{T}_j\left(\frac{2\beta_0\Gamma_0}{3n}\,L_{ij}^3 + \left(\frac{\Gamma_1}{n} + \frac{2\beta_0\gamma_0^S}{n}\right)L_{ij}^2 + 2\left(\frac{\gamma_1^S}{n} + \beta_0 c_{ij}^{(1)}\right)L_{ij} + c_{ij}^{(2)}\right)$$
$$+ 2\pi\sum_{i\neq j\neq k} f_{ABC}\,\mathbf{T}_i^A\,\mathbf{T}_j^B\,\mathbf{T}_k^C\left(\Gamma_0\lambda_{ij}\left(\frac{\Gamma_0}{3n}L_\mu^3 + \left(\frac{\Gamma_0}{n}\tilde{L}_{jk} + \frac{\gamma_0^S}{n}\right)L_\mu^2 + \left(\frac{\Gamma_0}{n}\tilde{L}_{jk}^2 + \frac{2\gamma_0^S}{n}\tilde{L}_{jk} + c_{jk}^{(1)}\right)L_\mu\right) + \tilde{c}_{ijk}^{(2)}\right)$$
$$+ \frac{1}{4}\sum_{i\neq j}\sum_{k\neq l}\{\mathbf{T}_i\cdot\mathbf{T}_j\,,\,\mathbf{T}_k\cdot\mathbf{T}_l\}\left(\frac{\Gamma_0}{n}\,L_{ij}^2 + \frac{2\gamma_0^S}{n}\,L_{ij} + c_{ij}^{(1)}\right)\left(\frac{\Gamma_0}{n}\,L_{kl}^2 + \frac{2\gamma_0^S}{n}\,L_{kl} + c_{kl}^{(1)}\right)\Bigg\}$$

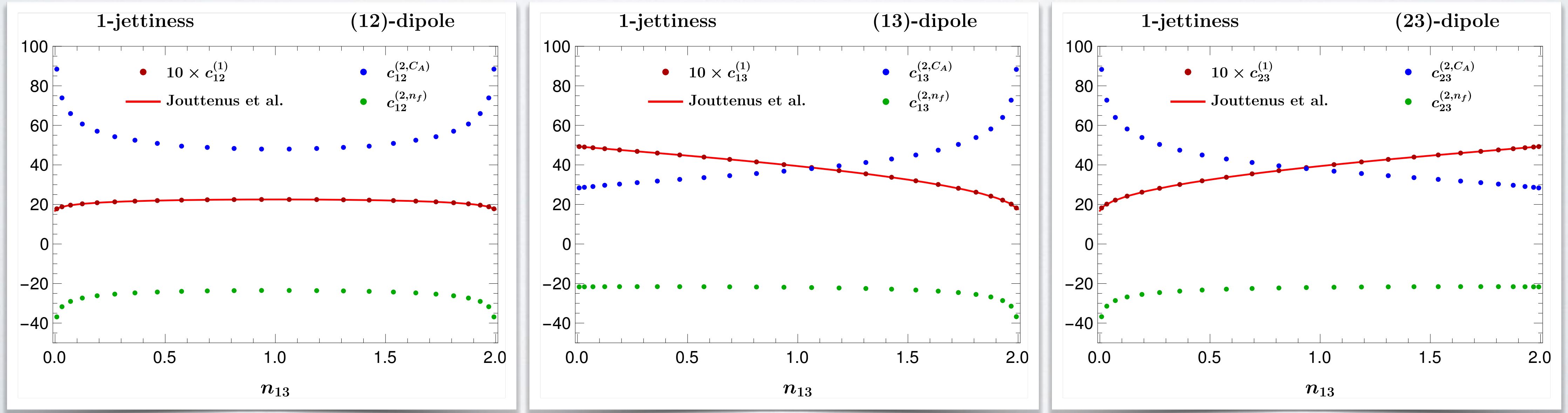
- Full results as ancillary file on the arXiv

# 1-JETTINESS

- Pole cancellations



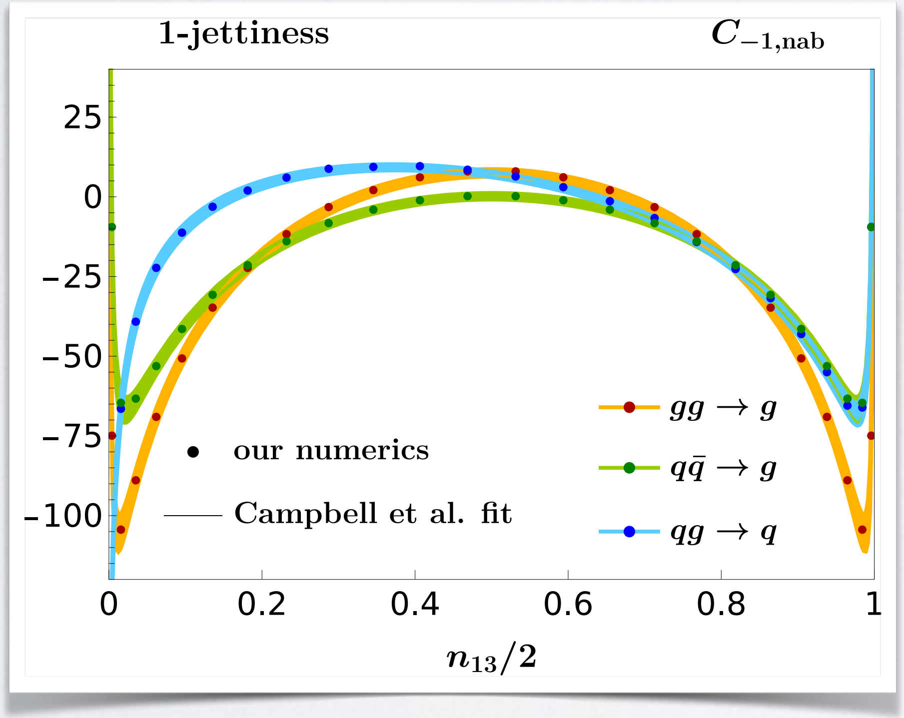

- Finite results

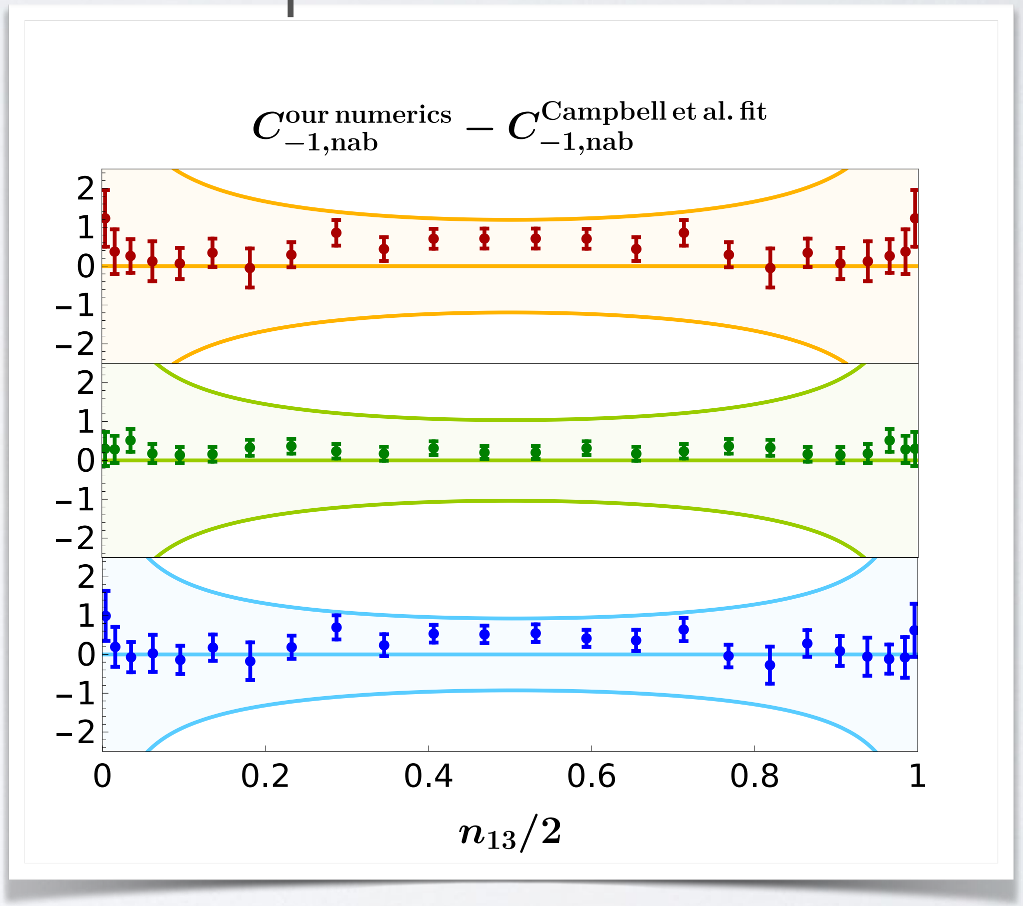


# 1-JETTINESS COMPARISON

[Campbell, Ellis, Mondini, Williams, '18]

- Compare with result from Campbell et al.

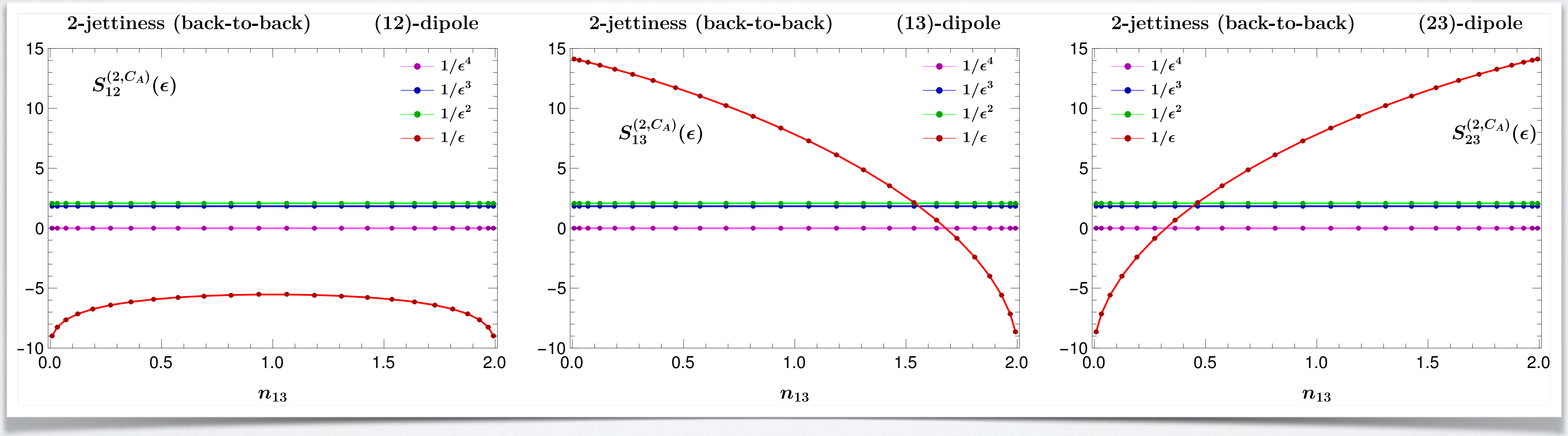



- Only fit functions for physical channels provided

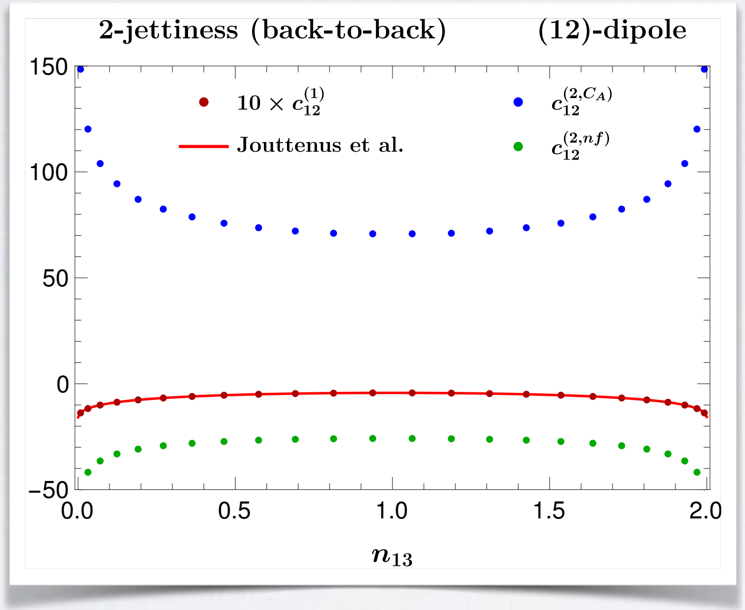
# 2-JETTINESS - B2B

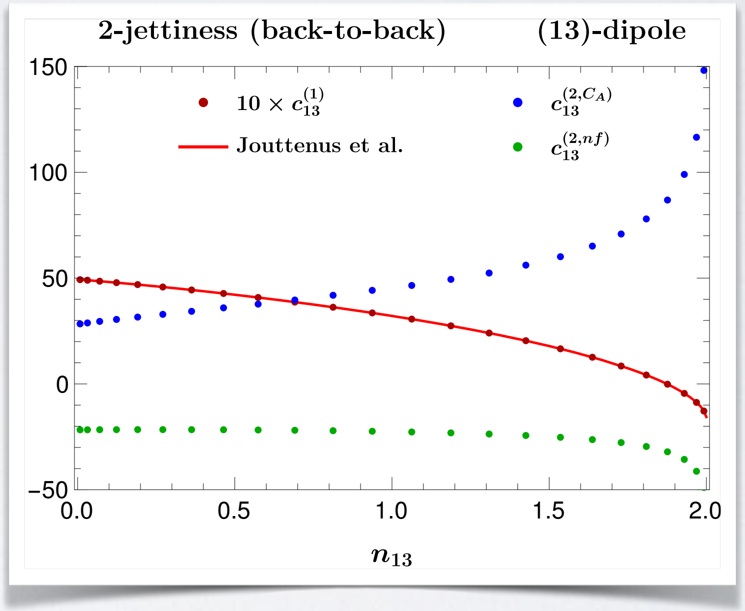
- Pole cancellations

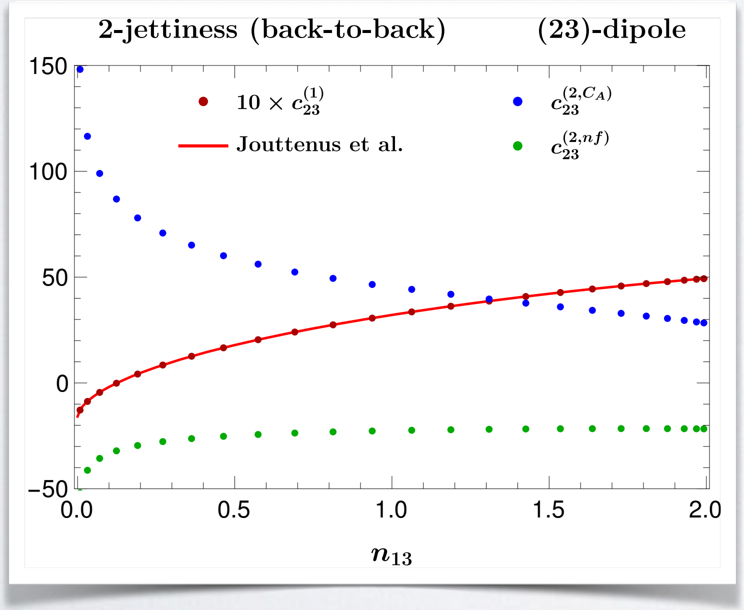


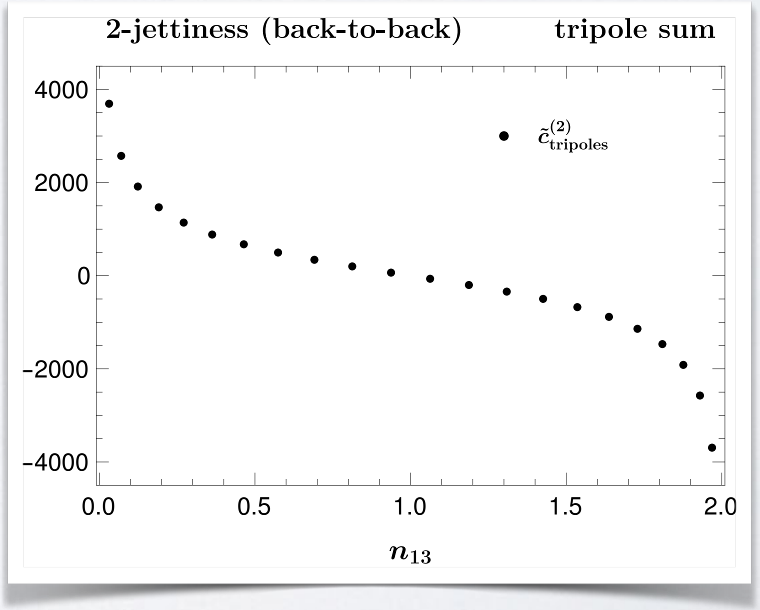
# 2-JETTINESS - B2B

- Finite results

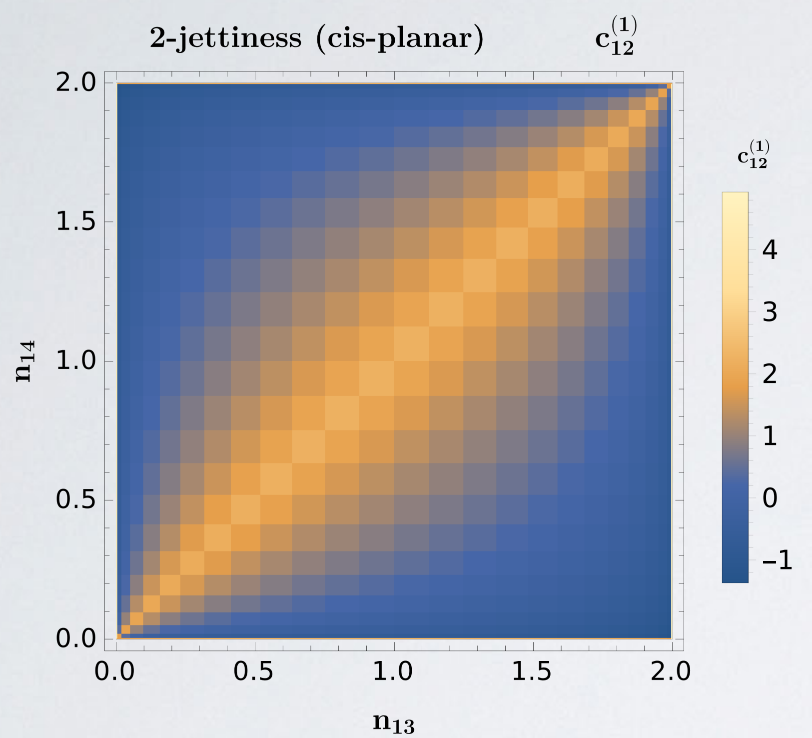


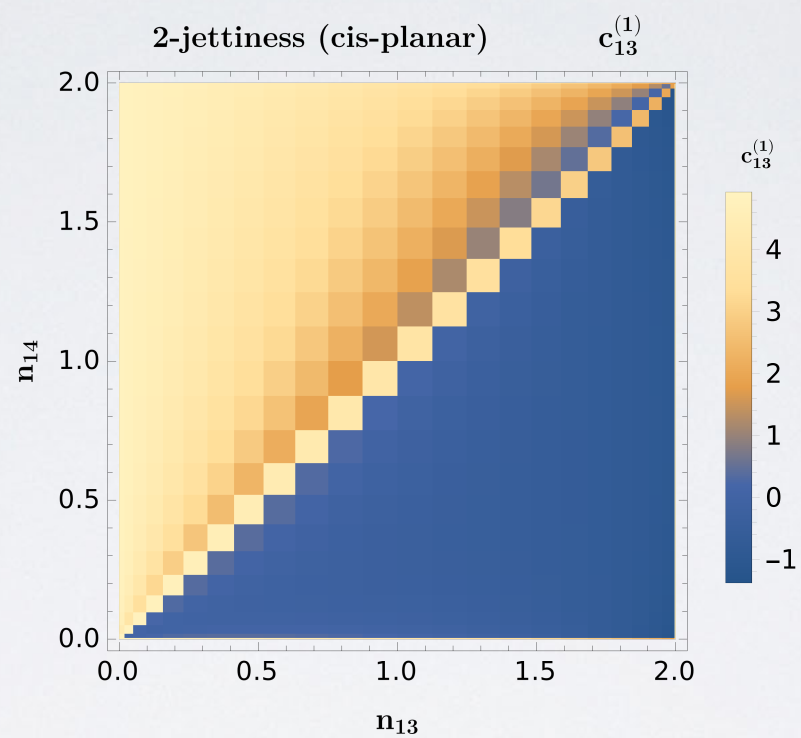



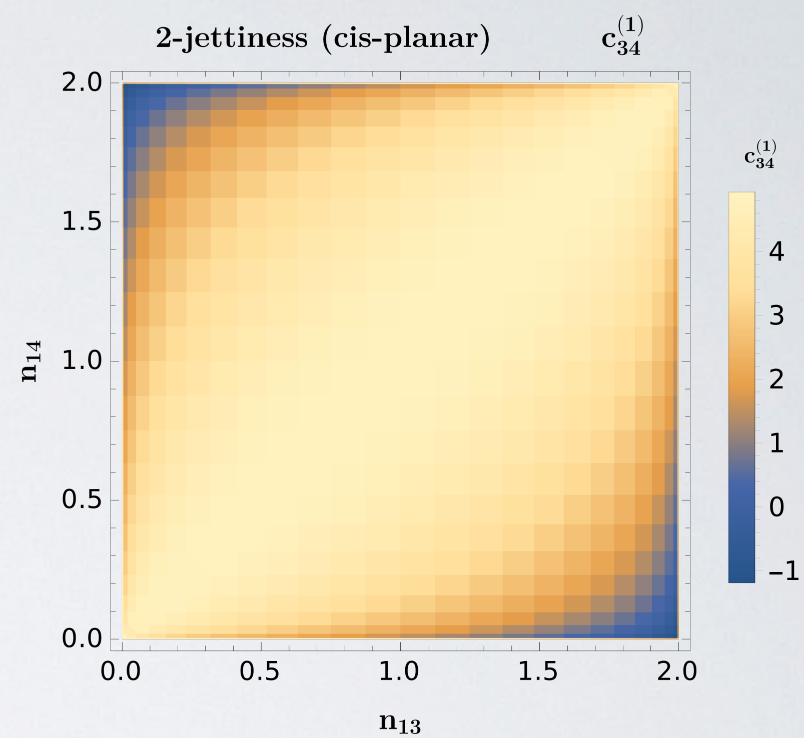



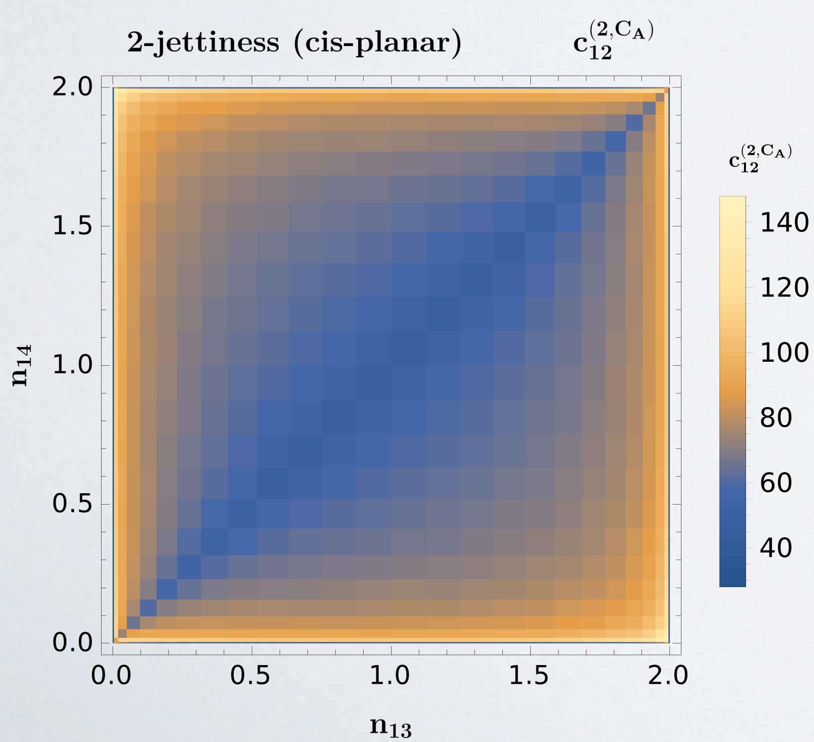



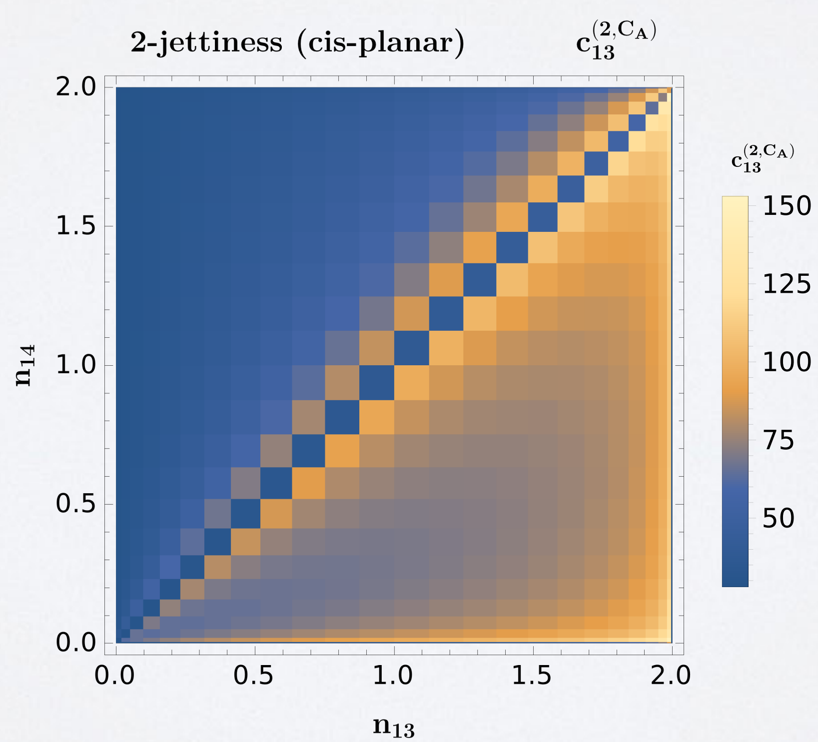
# 2-JETTINESS - PLANAR

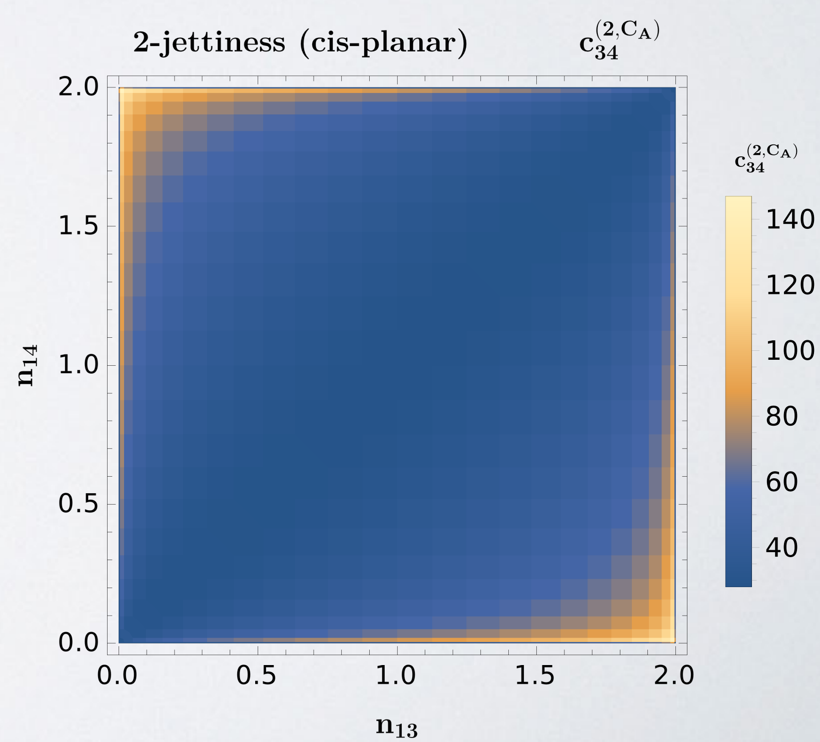


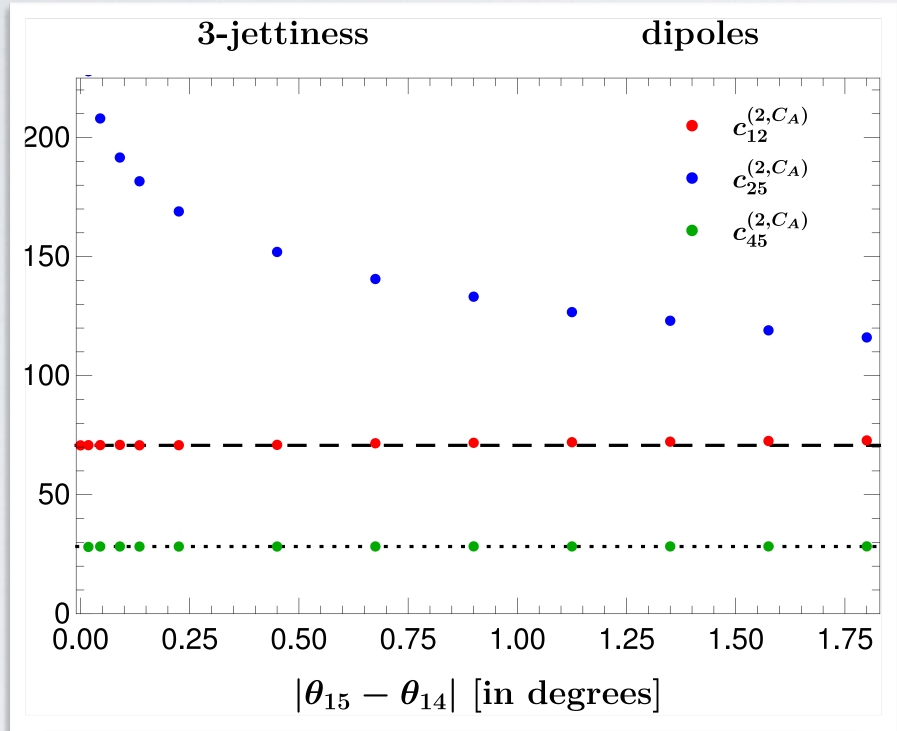



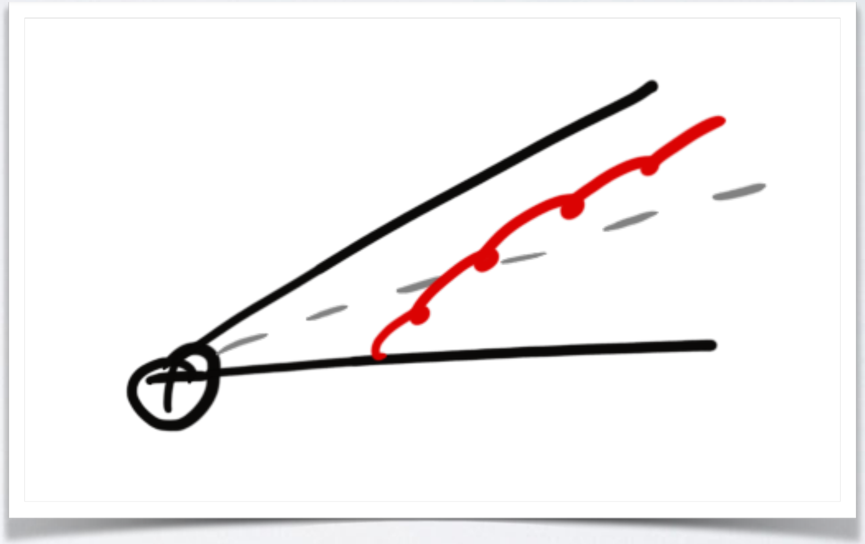



# EDGE CASES

- When jets become collinear, some results diverge



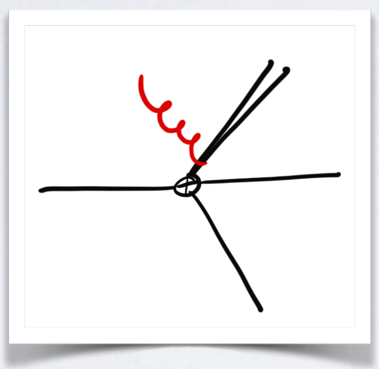
- Non-dipole jet collinear to dipole jet: divergence

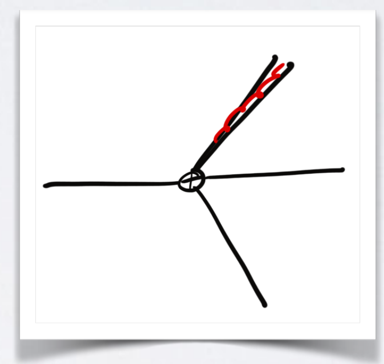
- Origin: contribution from non-dipole jet enhanced by collinearity to dipole jet

# METHOD OF REGIONS!

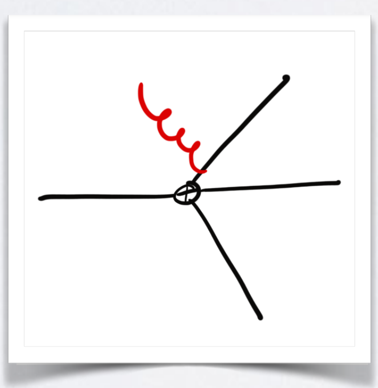
- Two regions:

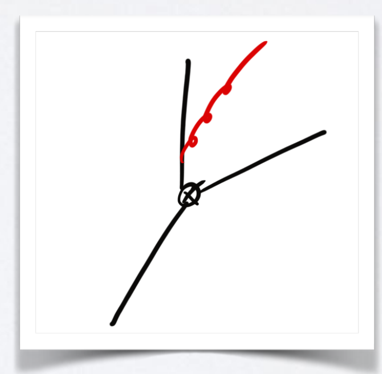
bulk and collinear





$\Downarrow$





$\Downarrow$

(N-1)-jet base + Universal correction

# IMPLICATIONS

- A kind of refactorisation

  (global soft base and collinear soft correction)

- Pattern should be general

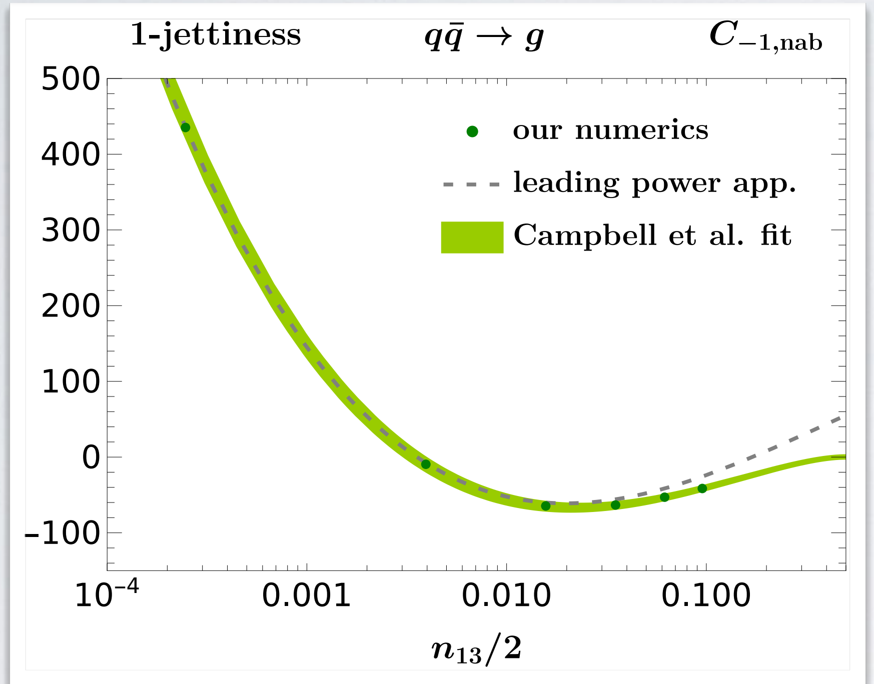
- Correction is observable and dipole dependent

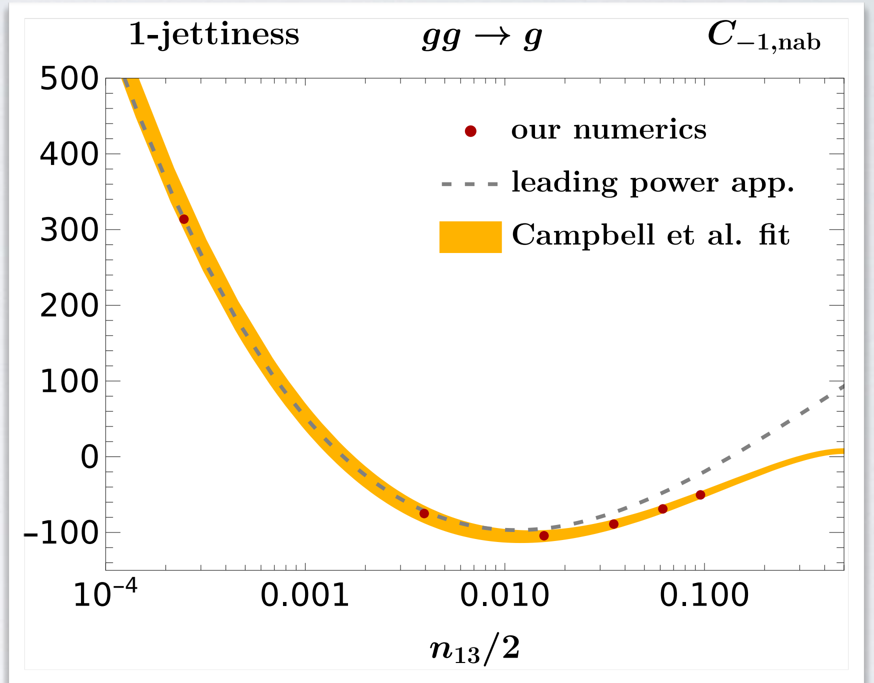
- N-Jettiness:

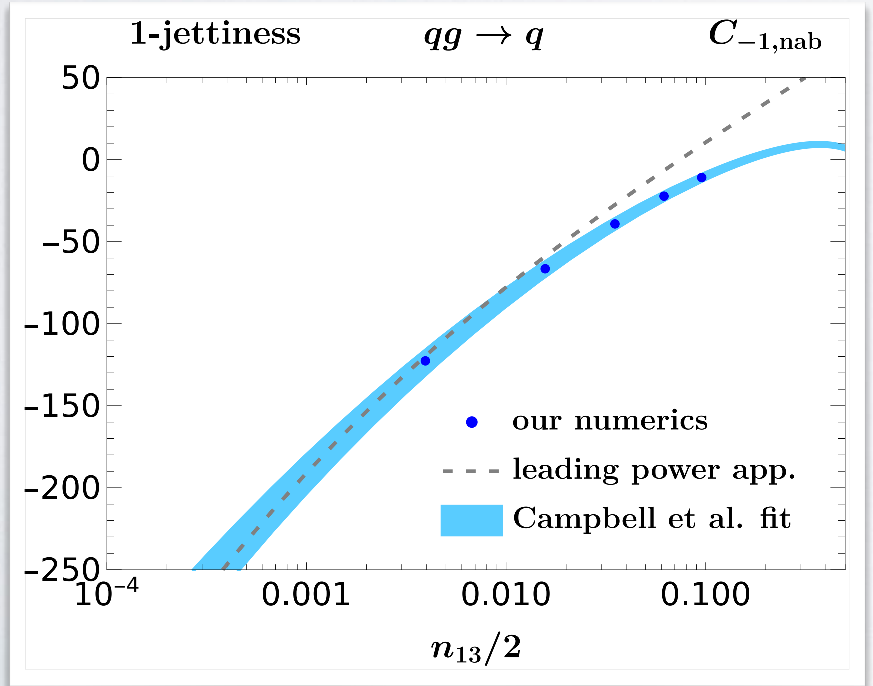
$$c_{ij}^{(1,\mathrm{corr})} = -\frac{\pi^2}{3}$$

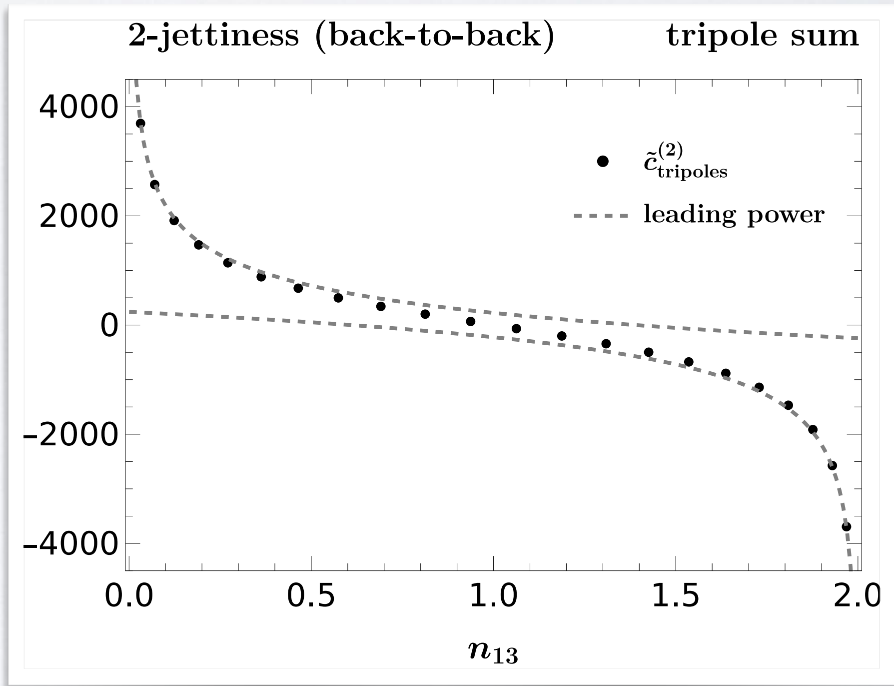
$$c_{ij}^{(2,\mathrm{corr})} = T_F n_f \left( \frac{4\pi^2}{9} \ln\left(\frac{2\delta}{n_{ij}}\right) - 8.023(2) \right) - C_A \left( \frac{11\pi^2}{9} \ln\left(\frac{2\delta}{n_{ij}}\right) - 10.0335(2) \right)$$

# CONVERGENCE



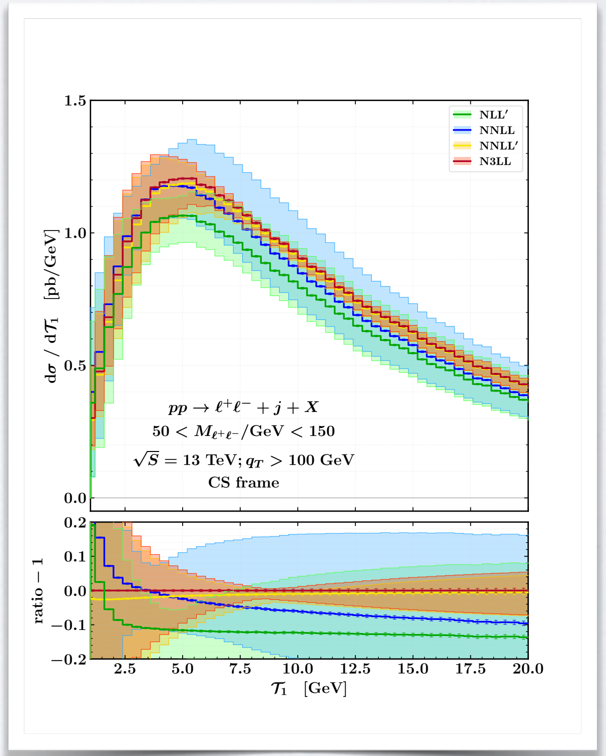



# IS THIS USEFUL?

- 1-jettiness at $N^3LL$ for GENEVA
  [Alioli, Bauer, Berggren, Tackmann, Walsh, '15]

- Z+jet production in lab frame vs CS-frame
  [Alioli, Bell, Billis, Broggio, Dehnadi, Lim, Marinelli, Nagar, Napoletano, RR '24]

- For some $Q_j$, small values for $n_{13}$ have to be probed



# CONCLUSIONS AND OUTLOOK

- We extended the SoftSERVE code to N-jet cases and applied it to the N-jettiness at NNLO
- Derived a grid for the 2-jettiness soft function
- Investigated logarithmic divergences at the edges of phase space using the Method of Regions
- Used our results for an $N^3LL$ resummation
- Next: non-global logarithms, N-gularities, …

# THANK YOU!

- Integration dimensions, dijet:
  - Three physical dimensions
    (two beams, one transverse reference)
  - One emission can probe 4th dimension,
    Two emissions can probe 4th and 5th dimension:

$$\frac{1}{\Gamma(-\epsilon)} \int_0^1 \mathrm{d}t_5 \, t_5^{-1-\epsilon} \sim \mathcal{O}(\epsilon^0)$$

- Integration dimensions, N-jet:
  - Four physical dimensions
    (all the beams and jets)
  - One emission can probe 5th dimension,
    Two emissions can probe 5th and 6th dimension:

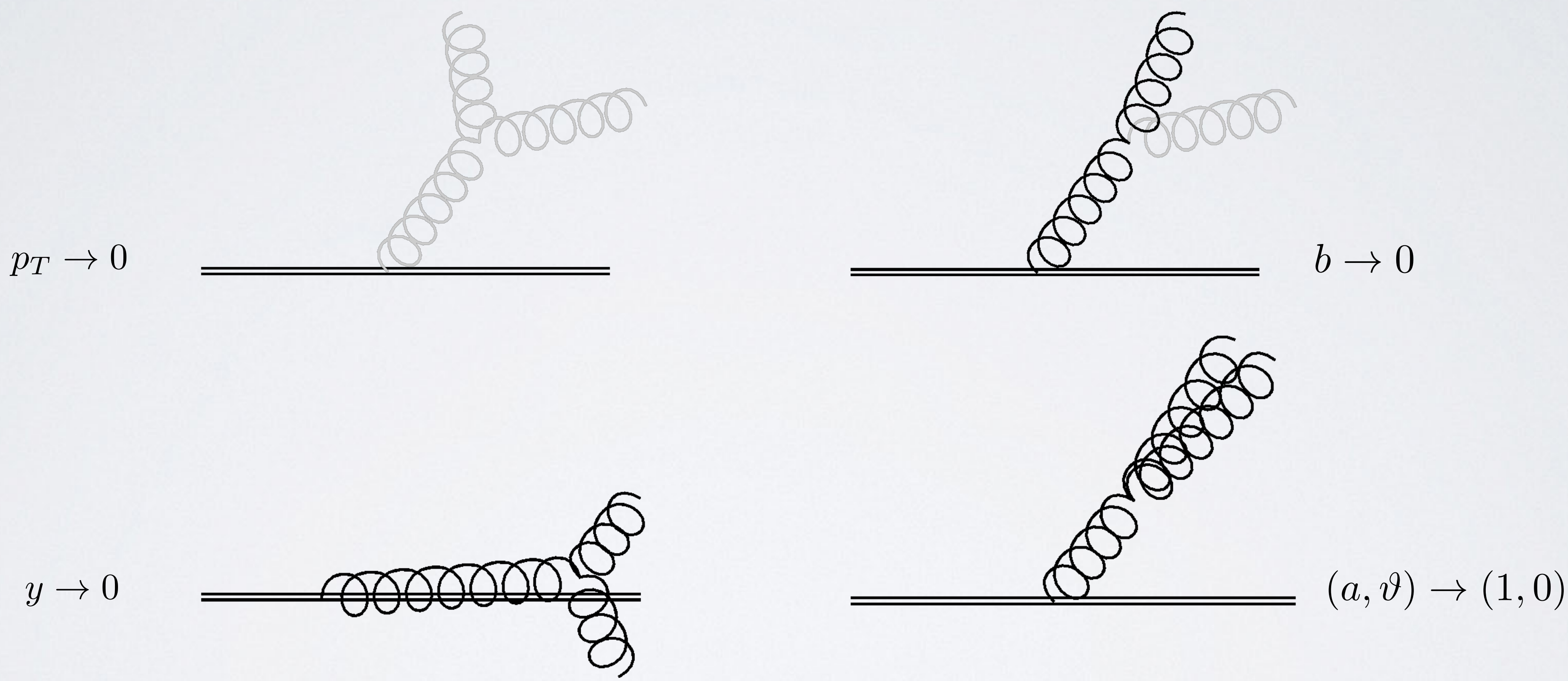
$$\frac{1}{\Gamma(-1/2-\epsilon)}\int_0^1 \mathrm{d}t_7\, t_7^{-3/2-\epsilon} \sim \mathcal{O}(\epsilon^0)$$

The "awful angle"

  - For N-jettiness: analytic integration

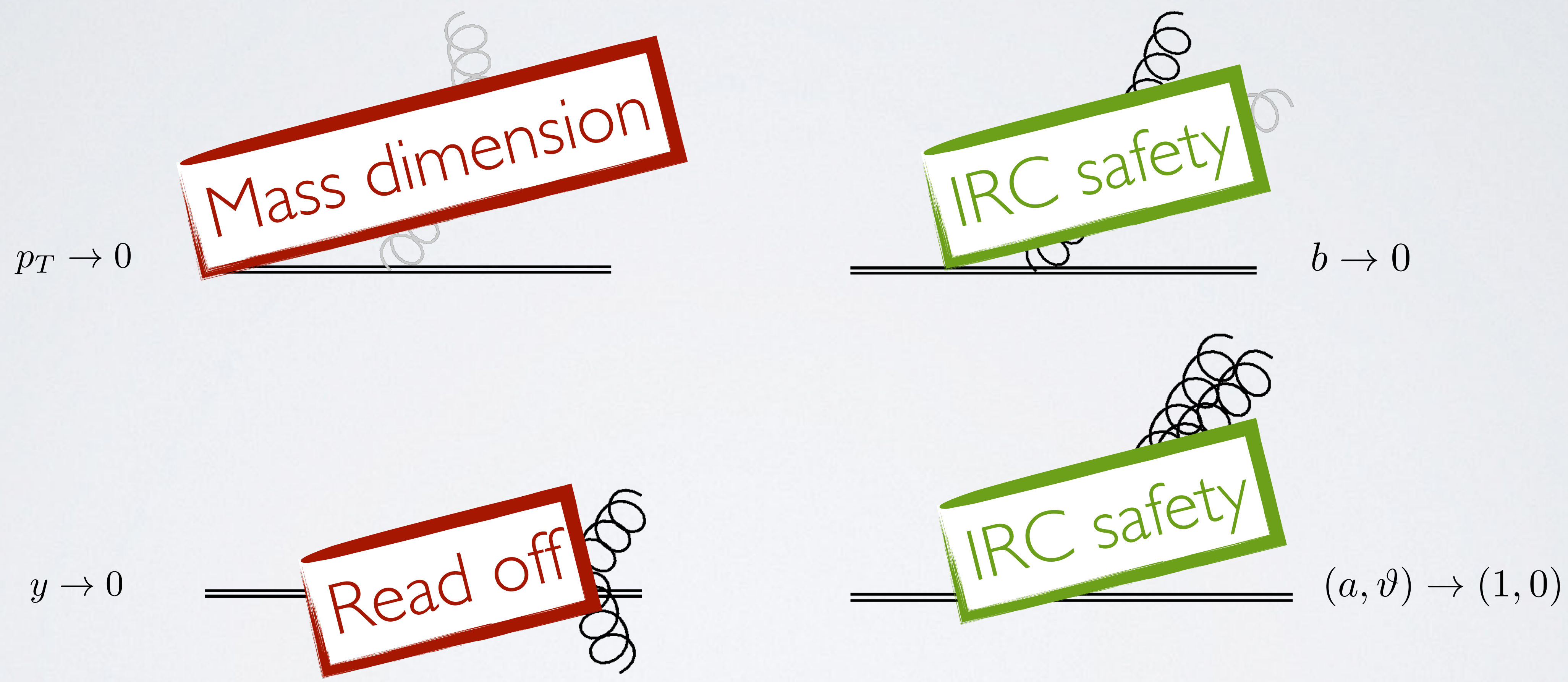
# TWO EMISSIONS

- Four divergence cases:



# TWO EMISSIONS

- Four divergence cases:




- Observable: $\tau(k,l) = p_T \, y^{\frac{n}{2}} \, F(a, b, y, \{\vartheta\})$