Universität Zürich[UZH]

NNLO Predictions for Tribosons Processes at the LHC

High Precision for Hard Processes (HP2 2024)

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Importance of triboson processes and current bottleneck

- triboson production is a rare process which enables testing the non-Abelian structure of the SM (quartic gauge coupling)
- any deviation from the SM predictions can point to the presence of New Physics
- WWW,$WZ\gamma$ and $WW\gamma$ only recently observed

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While waiting for them, we can try to find a reasonable approximation
$\rightarrow$ Soft-Approximation

Ingredients used for the computation

- the q_T-subtraction formalism: cross section for the production of a colourless final state system at N^{k}LO

transverse momentum of the FS

$$d\sigma_{N^kLO} = \mathscr{H}_{N^kLO} \otimes d\sigma_{LO} + \left[d\sigma^{R}_{N^{k-1}LO} - d\sigma^{CT}_{N^kLO}\right]_{q_T > q_T^{cut}} + \mathcal{O}(\left(q_T^{cut}\right)^p)$$

[Catani, Grazzini (2007)]

- local subtraction method for NLO singularities [Catani, Seymour (1998)]
- the MATRIX framework and its multi-channel MC integrator MUNICH [Grazzini, Kallweit, Wiesemann (2017)]
- automated tools for the required 1-loop amplitudes:
  - OpenLoops2 (default tool) [Buccioni, Lang, Lindert, Maierhöfer, Pozzorini, Zhang, Zoller (2019)]
  - Recola2 (mainly checks against OL2 and 1-loop squared for $WW\gamma$) [Denner, Lang, Uccirati (2017)]
- exact 2-loop amplitudes for $W^-\gamma$ (validation process) and Drell-Yan [Matsuura, van der Marck, van Neerven (1989), Gehrmann, Tancredi (2012)]
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Soft-Photon Approximation

How do we approximate the 2-loop amplitude?

Only unknown ingredient: 2-loop contributing to the q_T hard-collinear coefficient

$|M^{(2)}_{fin}\rangle = |M^{(2)}\rangle - I^{(1)}|M^{(1)}\rangle - I^{(2)}|M^{(0)}\rangle$, IR counter-terms + IR-finite terms

$$\mathscr{H}_{NNLO} = H_2(M^2)\delta(1-z_1)\delta(1-z_2) + \delta\mathscr{H}_2(z_1, z_2), \quad H_2 = \frac{2\Re\left\langle M^{(0)} \middle| \boxed{M^{(2)}_{fin}} \right\rangle}{\left|M^{(0)}\right|^2}$$

scale at which the SA is performed

[Catani, Cieri, de Florian, Ferrera, Grazzini (2014)]

$\rightarrow$ we subtract IR divergences in the SCET scheme [Becher, Neubert (2013)]

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Dealing with a photon, we can exploit the *Soft-Photon Approximation*:

[Yennie, Frautschi, Suura (1961)]

- the photon is let to be soft $\rightarrow$ $\boxed{Soft - Factor}$ (see next slide)
- need to preserve momentum-conservation $\rightarrow$ projection from the full phase space to the reduced one (i.e., no γ)

symmetric recoil in the IS

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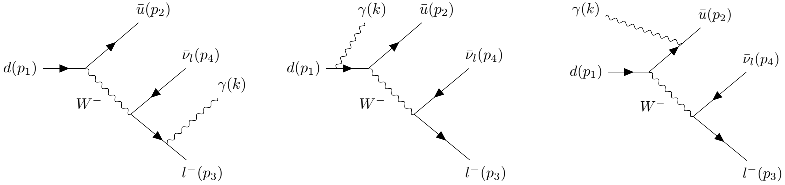
SA has been already applied to other processes such as
- $t\bar{t}H$ [Phys.Rev.Lett. 130 (2023)]
- $t\bar{t}W$ [Phys.Rev.Lett. 131 (2023)]

See Chiara's plenary on Friday!

Computation of the Soft-Factor (SF)

Example: $W^-\gamma$ (easily extended, e.g. to triboson production), dominant contributions from γ emitted from external legs:



When $k \to 0$, the amplitude for this process reads:

$$|M|^2 = 2\,|M_0|^2 \left[Q_e Q_d \frac{p_3 \cdot p_2}{(p_3 \cdot k)(p_2 \cdot k)} - Q_e Q_u \frac{p_3 \cdot p_1}{(p_3 \cdot k)(p_1 \cdot k)} + Q_d Q_u \frac{p_2 \cdot p_1}{(p_2 \cdot k)(p_1 \cdot k)} + \cancel{\text{terms} \propto p_i^2} \right]$$

SF

non-radiative amplitude

Being the photon massless, we expect the SA to perform better here than for $t\bar{t}H$ and $t\bar{t}W$!

The re-weighting (RW) procedure

- approximate 2-loop is given by the loop corrections to $|M_0|^2$
- works quite well at the inclusive level, but going differentially this agreement turns out to be mostly accidental
  - $\rightarrow$ compensation between over/under-estimations of the exact around the bulk
- we can try to extend the validity range of this approximation

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We approximate the $H_{1,2}$ contributions as:

$$H_1^{SA} = \frac{2\Re\{\langle M_{SA}^{(0)}|M_{SA,fin}^{(1)}\rangle\}}{|M^{(0)}|^2}, \qquad H_2^{SA} = \frac{2\Re\{\langle M_{SA}^{(0)}|M_{SA,fin}^{(2)}\rangle\}}{|M^{(0)}|^2}$$

$\Rightarrow$ 'non-re-weighted' amplitudes (no-RW).

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Different RWs to try to extend the validity range of our approximation:

- Born-squared-RW: $H_{1,2} \longrightarrow H_{1,2} \times |M^{(0)}|^2/|M_{SA}^{(0)}|^2$ $\quad |M^{(0)}|^2$-RW
- interference-RW: $H_2 \longrightarrow H_2 \times \frac{2\Re\{\langle M^{(0)}|M_{fin}^{(1)}\rangle\}}{2\Re\{\langle M_{SA}^{(0)}|M_{SA,fin}^{(1)}\rangle\}}$ $\quad M_1M_0$-RW
- 1-loop squared-RW: $H_2 \longrightarrow H_2 \times |M^{(1)}|^2/|M_{SA}^{(1)}|^2$ $\quad |M^{(1)}|^2$-RW

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Diboson production studies are nowadays well-estabilished:

- test of the non-Abelian trilinear gauge coupling in the SM
- 2-loop amplititudes known exactly

  [Gehrmann, Tancredi (2012), Gehrmann, von Manteuffel, Tancredi (2015)]
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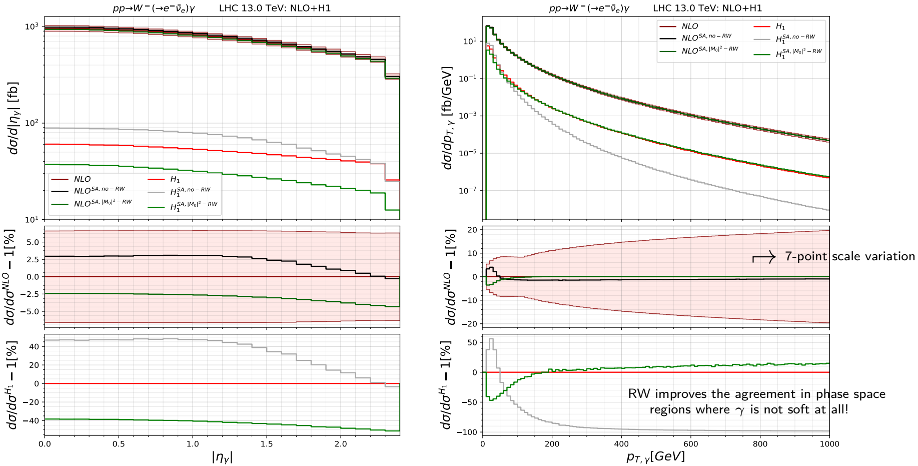
We exploited the exact NNLO result for $W^-\gamma$ to

- test our SA to pave the way to its application to triboson production $(pp \to W^-(e^-\bar{\nu}_e)\gamma \Rightarrow pp \to W^-(e^-\bar{\nu}_e)W^+(\mu^+\nu_\mu)\gamma)$
- build a solid error estimate of the procedure, able to cover the real error of our SA and which we can then apply to $WW\gamma$

$W^-\gamma$: SA performance at NLO

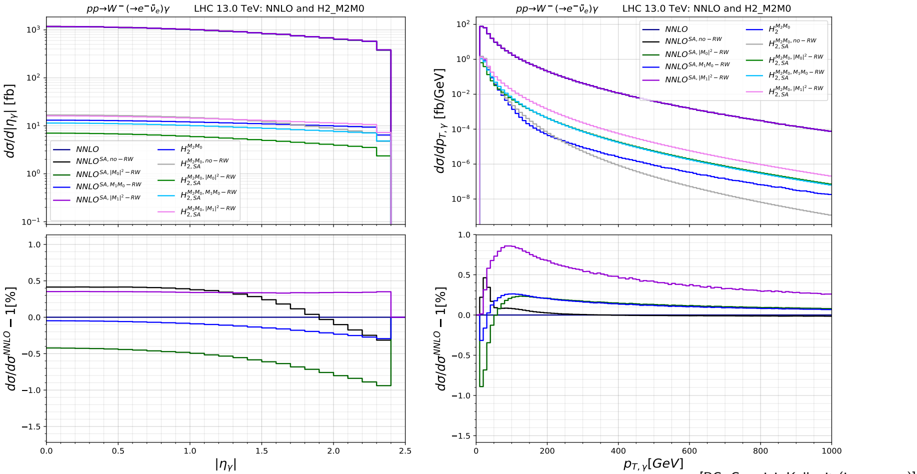
[PG, Grazzini, Kallweit (in progress)]



- very close differential results (compatible in different IR-schemes)
- RW: reduction of the phase space dependence in the difference between exact and SA
- NLO^{SA} in very good agreement with NLO, with differences well inside the 7-point band

$\Rightarrow$ extension to NNLO sounds promising

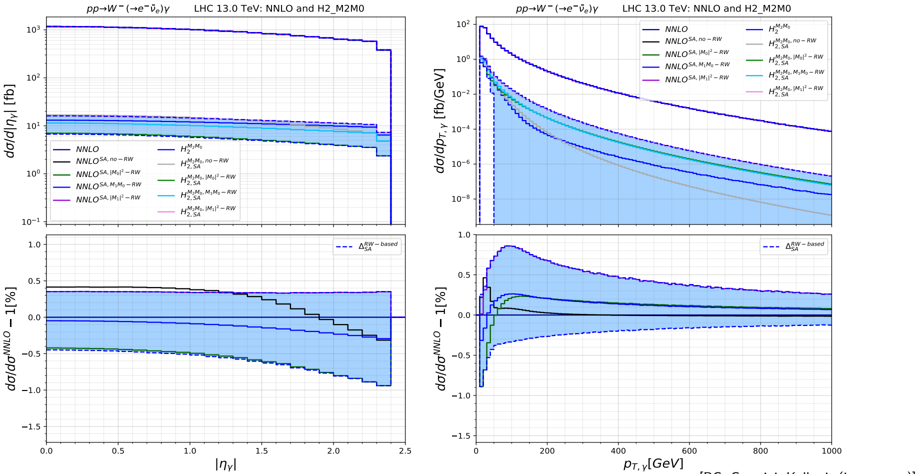
$W^-\gamma$ at NNLO: error estimate



[PG, Grazzini, Kallweit (in progress)]

Comparison of different RWs: they all produce very close results (few permille from the exact!) $\rightarrow M_1M_0$-RW chosen as our best prediction

$W^-\gamma$ at NNLO: error estimate



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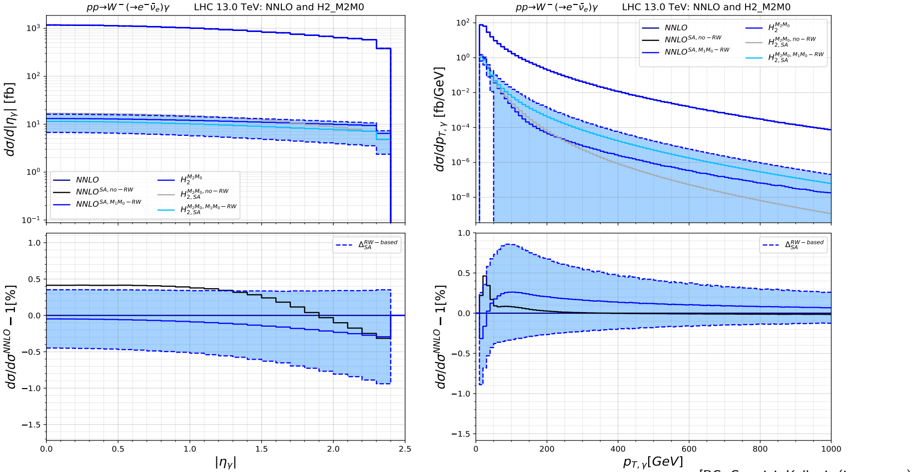

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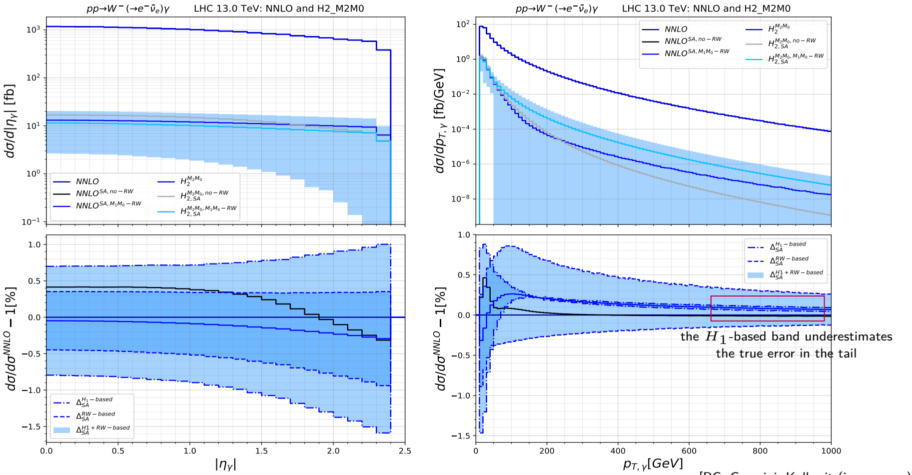

[PG, Grazzini, Kallweit (in progress)]

$$H_1\text{-}\textit{based band}:\ 2\times\left|\frac{H_{1,SA}^{|M_0|^2-RW}}{H_1}-1\right|,\quad |M_0|^2-RW:\ \text{best prediction for the } H_1$$

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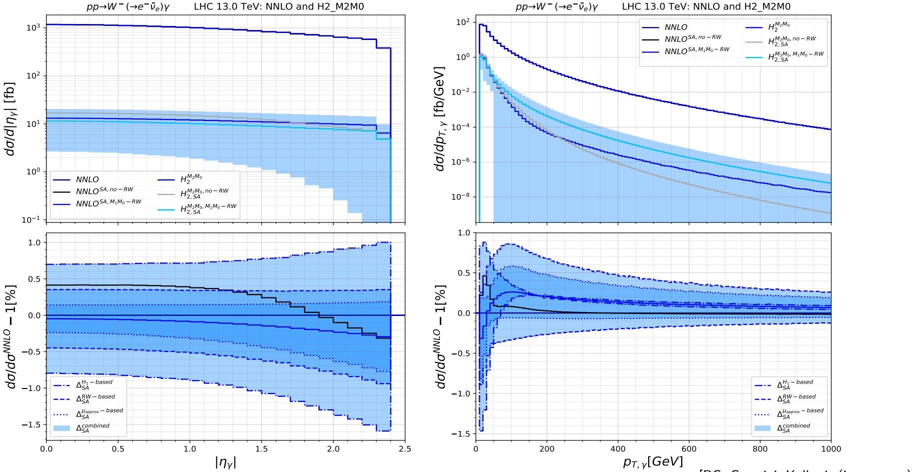

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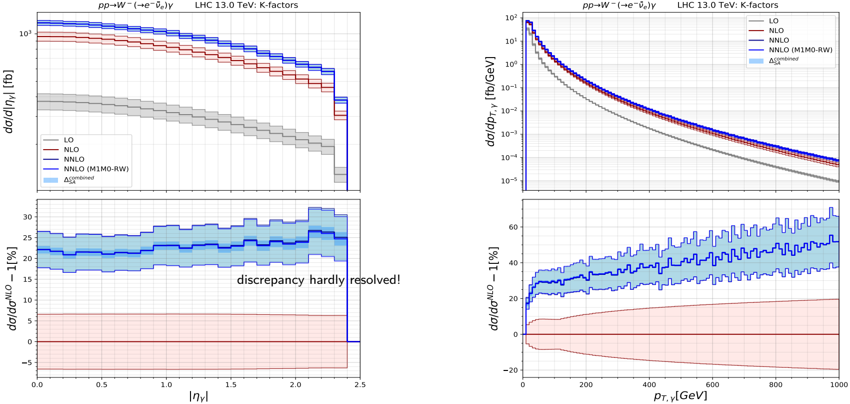

[PG, Grazzini, Kallweit (in progress)]

- the exact NNLO result is well covered by our SA plus its total error band
- $\Delta_{SA}^{combined}$ well inside the 7-point band
- H_2 very small $\to$ the overall prediction is reasonable even if at 2-loop the approximation is not extremely accurate (as for the H_1)
- in general, the 7-point band rather underestimates the true perturbative uncertainty

$W^-\gamma$: inclusive results

Total rates:

$\sigma_{LO}[fb]$	$\sigma_{NLO}[fb]$	$\sigma^{SA}_{NLO}[fb]$	$\sigma_{NNLO}[fb]$	$\sigma^{SA}_{NNLO}[fb]$
$701.594(8)^{10.6\%}_{-11.5\%}$	$1786.0(2)^{6.6\%}_{-5.3\%}$	$1734(1)^{6.4\%}_{-5.2\%}$	$2189(5)^{3.8\%}_{-3.6\%}$	$2186(5)^{3.7\%}_{-3.6\%} \pm (0.9\%)_{SA}$

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- NLO and NLO^{SA} are very close, $< 3\%$
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- different IR-subtraction schemes lead to compatible results
- $\Delta^{combined}_{SA}$ well inside the 7-point band $\rightarrow$ our predictions can be considered
  ↳ $\sim 1\%$ of the central result! NNLO-accurate

We can extend our method to triboson production $\rightarrow W^+W^-\gamma$

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$WW\gamma$ production in Soft-Approximation

- $WW\gamma$ observed by ATLAS (8 TeV) and more recently by CMS (13 TeV)

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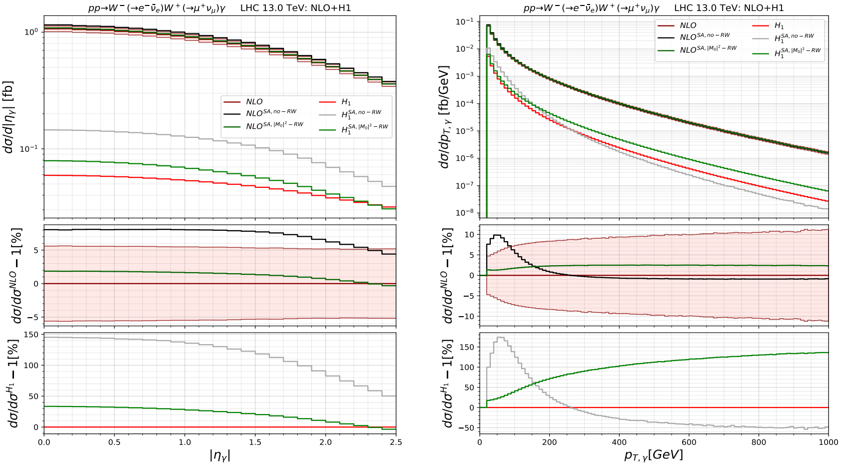
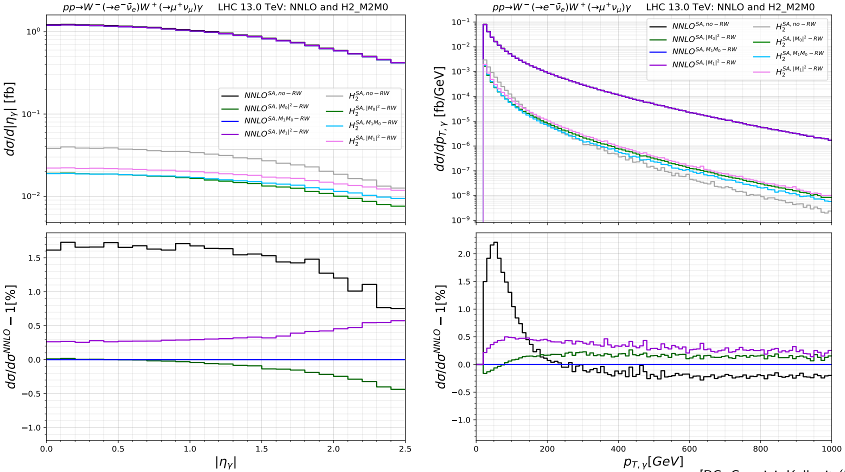
$WW\gamma$: SA performance at NLO

[PG, Grazzini, Kallweit (in progress)]



- similarly to $W^-\gamma$, the RW curve shows a reduced phase space dependence
- need for a RW procedure even more evident than for $W\gamma$

$WW\gamma$ at NNLO: error estimate



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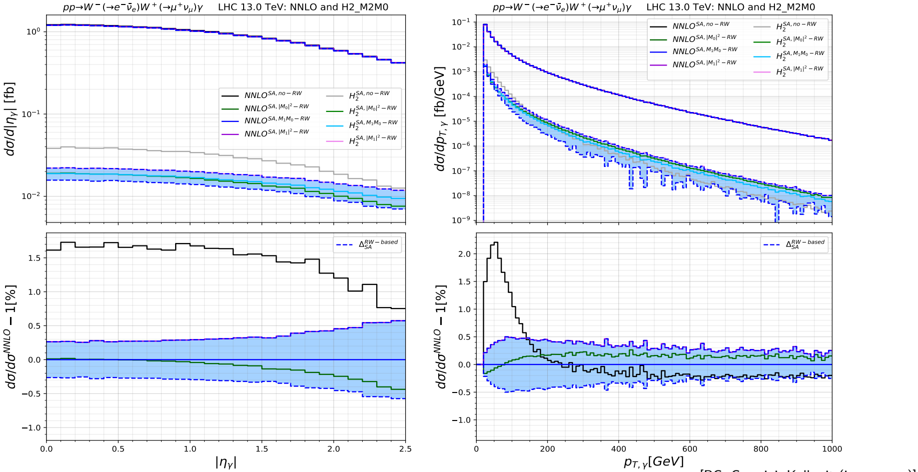

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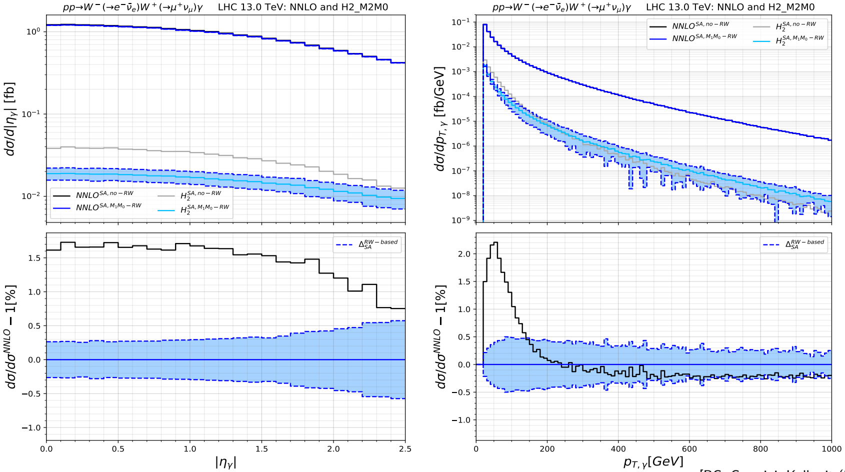

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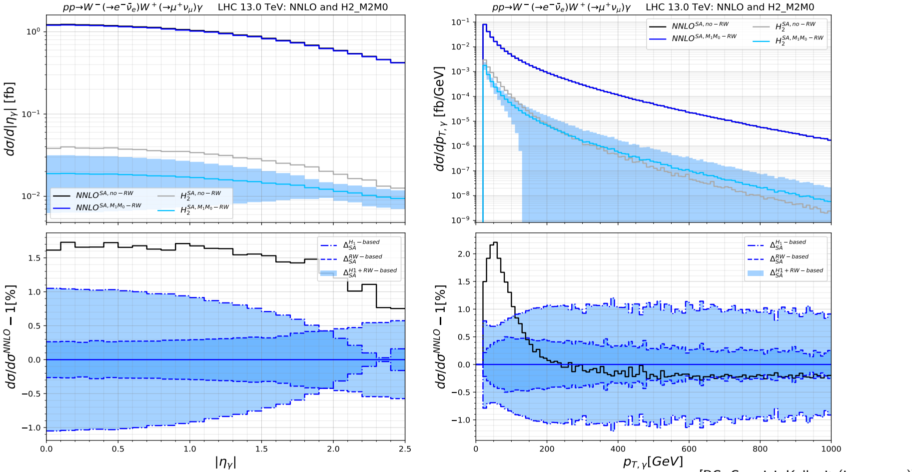

[PG, Grazzini, Kallweit (in progress)]

H_1*-based band*: $2 \times \left| \frac{H_{1,SA}^{|M_0|^2-RW}}{H_1} - 1 \right|$, $|M_0|^2 - RW$: best prediction for the H_1

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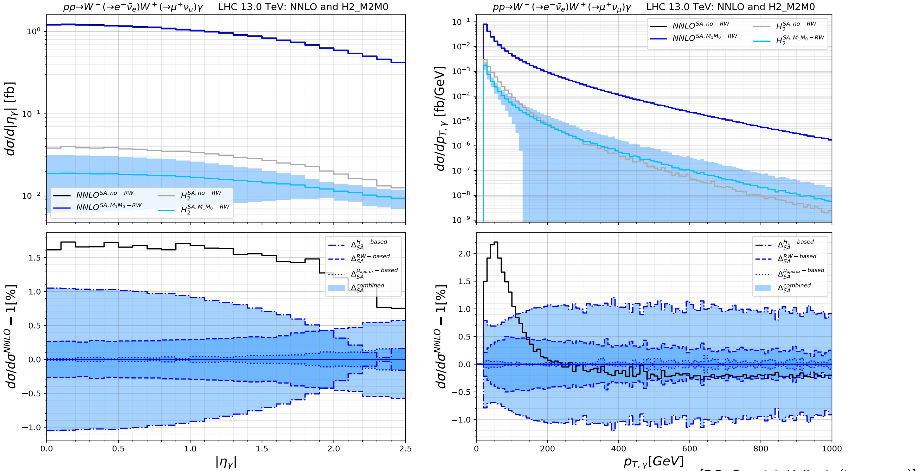

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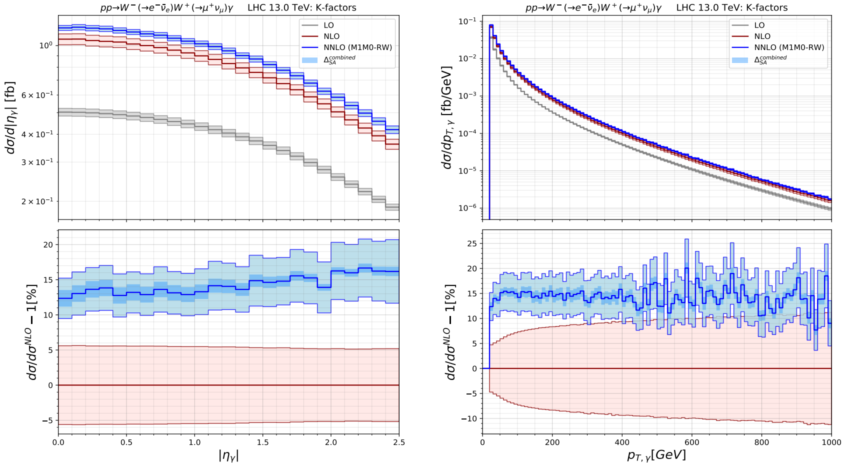

[PG, Grazzini, Kallweit (in progress)]

- NNLO corrections are quite sizeable, $\boxed{\sim 15\% \text{ of the NLO}}$ and not covered by its 7-point band
- $\Delta_{SA}^{combined}$ well inside the 7-point band as for $W^-\gamma$

$WW\gamma$: inclusive results

Total rates:

$\sigma_{LO}[fb]$	$\sigma_{NLO}[fb]$	$\sigma^{SA}_{NLO}[fb]$	$\sigma^{SA}_{NNLO}[fb]$
$0.95340(8)^{3.1\%}_{-3.8\%}$	$1.9622(3)^{5.4\%}_{-4.3\%}$	$1.991(1)^{5.5\%}_{-4.4\%}$	$2.232(3)^{2.9\%}_{-2.7\%} \pm (0.8\%)_{SA}$

[PG, Grazzini, Kallweit (in progress)]

- again very close NLO results, $\sim 1.5\%$
- NLO result increased by $\sim 14\%$ by NNLO corrections
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[Abreu, De Laurentis, Ita, Klinkert, Page, Sotnikov (2023)]

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Thank you!

Setup for $W^-\gamma$

The tests performed on $W^-\gamma$ have been run with a center of mass energy of $\sqrt{s} = 13$ TeV with $N_f = 5$ active flavors, using the central scale

$$\mu = \sqrt{(p_e + p_\nu)^2 + p_{T,\gamma}^2}$$

the following cuts

$\mathbf{p_T}$ cuts	η cuts	$\mathbf{dR}$ cuts
$p_{T,j} \geq 30GeV$	$\lvert\eta_j\rvert \leq 4.4$	$dR_{l\gamma} > 0.7$
$p_{T,l} \geq 25GeV$	$\lvert\eta_l\rvert \leq 2.47$	$dR_{lj} > 0.3$
$p_{T,\gamma} \geq 15GeV$	$\lvert\eta_\gamma\rvert \leq 2.37$	$dR_{\gamma j} > 0.3$
$p_{T,miss} > 35GeV$		

and the PDF sets NNPDF30_(n)(n)lo_as_0118.

Setup for $WW\gamma$

The results for $WW\gamma$ have been obtained with $N_f = 4$ (to exclude the strong $t\bar{t}$ background) and using the central scale

$$\mu = \sqrt{\frac{1}{4}(p_e + p_{\nu_e} + p_\mu + p_{\nu_\mu})^2 + p_{T,\gamma}^2}$$

and cuts [Phys. Rev. Lett. 132, 121901]

$\mathbf{p_T}$ cuts	η cuts	$\mathbf{dR}$ cuts	$\mathbf{m}$ cuts
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$p_{T,\gamma} \geq 20$ GeV	$\lvert\eta_\gamma\rvert \leq 2.5$		
$p_{T,\text{miss}} > 20$ GeV			
$p_{T,ll} > 15$ GeV			

with

$$m_T^{WW} = \sqrt{2p_{T,ll}p_{T,miss}[1 - \cos\Delta\Phi(\vec{p}_{T,ll}, \vec{p}_{T,miss})]}$$

and the PDFs sets NNPDF31_(n)(n)lo_as_0118_nf_4.

q_T and SCET schemes

The difference between the two schemes is only in finite terms and they both start from the same UV-renormalized all-order amplitude and apply different IR-subtraction operators:

$$
\begin{aligned}
Z^{-1}(\epsilon,\mu)|M(\epsilon,\mu)\rangle &= |M_{fin,N}(\mu)\rangle \\
I(\epsilon,\mu)|M(\epsilon,\mu)\rangle &= |M_{fin,q_T}(\mu)\rangle
\end{aligned}
$$

such that we can move from q_T to SCET with

$$
[Z^{-1}][I^{-1}]|M_{fin,q_T}\rangle = |M_{fin,N}\rangle
$$