

*High Precision for Hard Processes 2024 - 10.09.24*

# **MATCHING NLO CALCULATIONS WITH NLL PANSCALES SHOWERS WITH INITIAL-STATE PARTONS**

*Silvia Zanolì - University of Oxford*



*with M. van Beekveld, S. Ferrario Ravasio, J. Helliwell, A. Karlberg, G.P. Salam, L. Scyboz, A. Soto-Ontoso, G. Soyez*

# A LHC EVENT

ENERGY  
SCALE [GeV]

1000

**HARD SCATTERING**

100

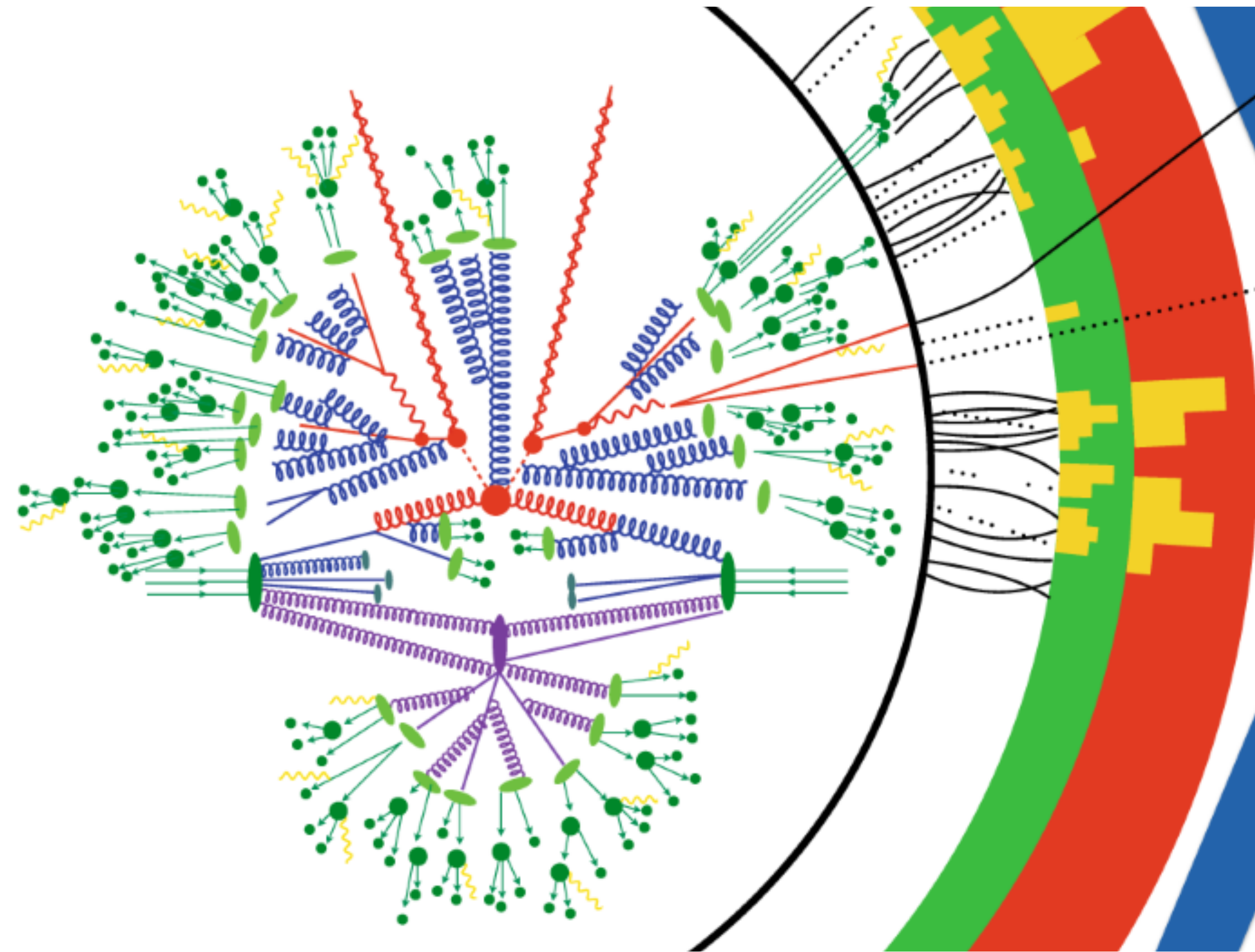
**PARTON SHOWER**

10

**HADRONIZATION**

1

**UNDERLYING EVENT**





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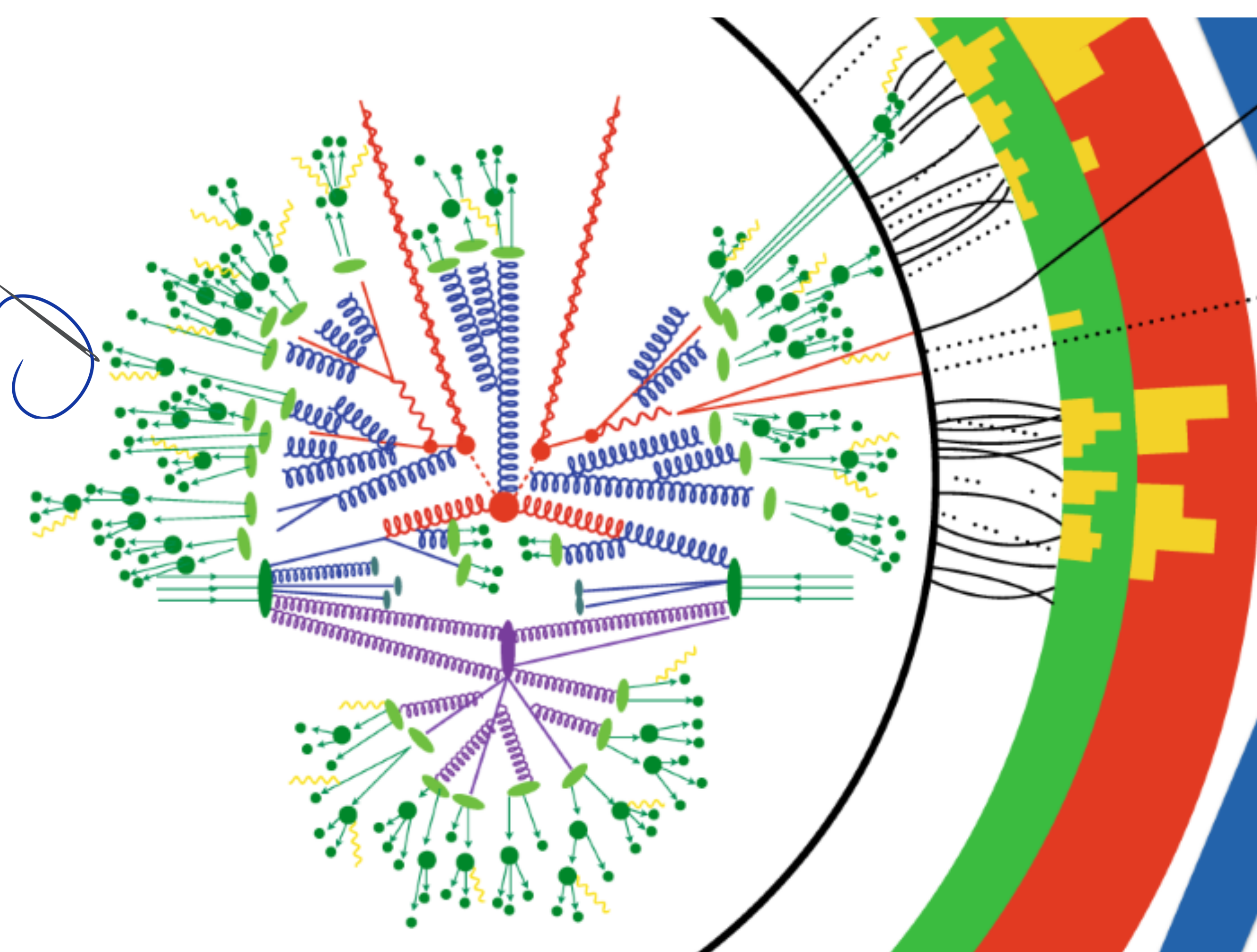
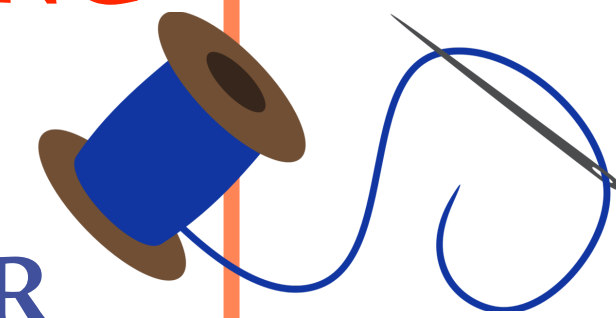
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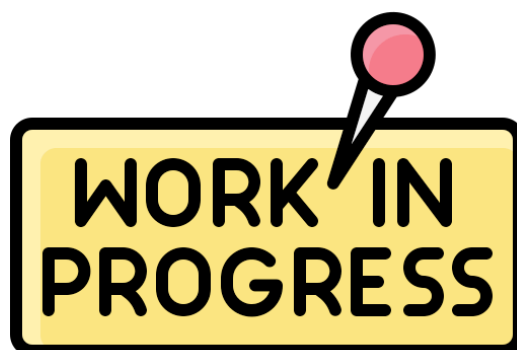
# MATCHING

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**NLO+LL<sub>PS</sub>**

- A **solved problem** for long time.
- Completely understood and **fully automatized**.
- Two main approaches available: POWHEG [Nason '04; Frixione, Nason, Oleari '07; Alioli, Nason, Oleari, Re '10] and MC@NLO [Frixione, Webber '02].



**NNLO+LL<sub>PS</sub>**

- **State-of-the-art** for precision LHC phenomenology.
- Lots of ongoing effort, **many processes already implemented**.
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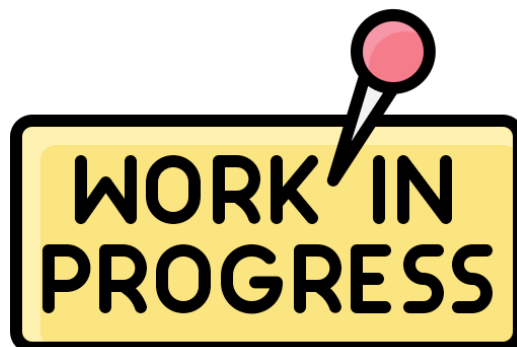
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*... what's the next step?*



# NLL SHOWERS

---

**NLL showers are now available**

Parton showers beyond leading logarithmic accuracy #1

Mrinal Dasgupta (U. Manchester (main)), Frédéric A. Dreyer (Oxford U., Theor. Phys.), Keith Hamilton (U. Coll. London), Pier Francesco Monni (CERN), Gavin P. Salam (Oxford U., Theor. Phys.) et al. (Feb 25, 2020)

Published in: *Phys.Rev.Lett.* 125 (2020) 5, 052002 • e-Print: [2002.11114](#) [hep-ph]

**PanScales**

A new approach to color-coherent parton evolution #1

Florian Herren (Fermilab), Stefan Höche (Fermilab), Frank Krauss (Durham U., IPPP), Daniel Reichelt (Durham U., IPPP), Marek Schoenherr (Durham U., IPPP) (Aug 11, 2022)

Published in: *JHEP* 10 (2023) 091 • e-Print: [2208.06057](#) [hep-ph]

**ALARIC**

A partitioned dipole-antenna shower with improved transverse recoil #1

Christian T. Preuss (Wuppertal U.) (Mar 28, 2024)

Published in: *JHEP* 07 (2024) 161 • e-Print: [2403.19452](#) [hep-ph]

**APOLLO**

Building a consistent parton shower #1

Jeffrey R. Forshaw (Manchester U. and Schrodinger Inst., Vienna), Jack Holguin (Manchester U. and Schrodinger Inst., Vienna), Simon Plätzer (Schrodinger Inst., Vienna and Vienna U.) (Mar 13, 2020)

Published in: *JHEP* 09 (2020) 014 • e-Print: [2003.06400](#) [hep-ph]

**Forshaw-Holguin-Plätzer**

Summations of large logarithms by parton showers #1

Zoltán Nagy (DESY), Davison E. Soper (Oregon U.) (Nov 9, 2020)

Published in: *Phys.Rev.D* 104 (2021) 5, 054049 • e-Print: [2011.04773](#) [hep-ph]

**DEDUCTOR**

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For  $e^+e^-$  matching, see [2301.09645]

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**THIS TALK:**

**NLO+NLL<sub>PS</sub> matching with PanScales showers with initial-state partons**

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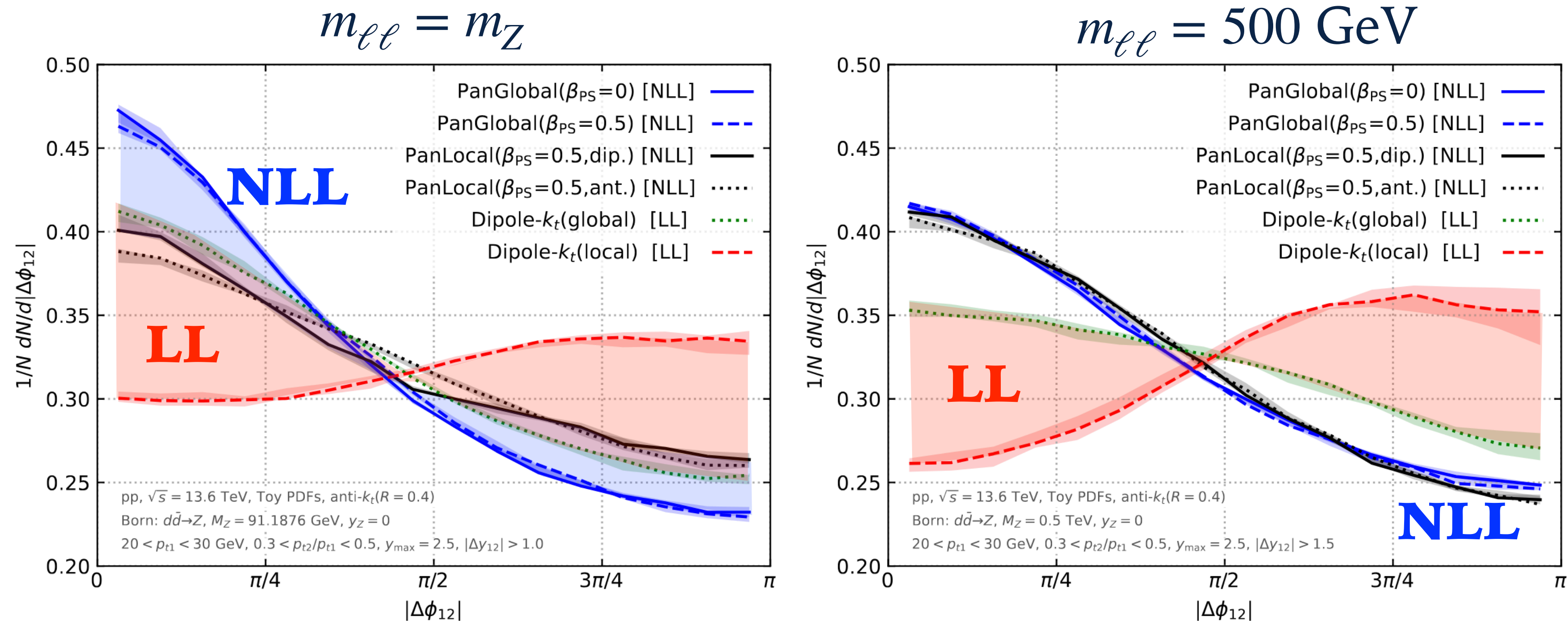
**DEDUCTOR**



# MOTIVATION

1. NLL showers are likely to become a fundamental ingredient of the precision physics program at the LHC.

DY - azimuthal angle between leading jets



Different shape between LL and NLL showers,  
especially at high energies



# MOTIVATION

1. NLL showers are likely to become a fundamental ingredient of the precision physics program at the LHC.
2. NLO matching can augment the shower accuracy.

NLO  
MATCHING

$$\Sigma(\alpha_s, L) = \exp\left(\underbrace{\frac{1}{\alpha_s} g_1(\alpha_s L)}_{\mathcal{O}(1/\alpha_s)} + \underbrace{g_2(\alpha_s L)}_{\mathcal{O}(1)} + \underbrace{\alpha_s g_3(\alpha_s L)}_{\mathcal{O}(\alpha_s)}\right)$$

$$\Sigma(\alpha_s, L) = (1 + \underbrace{\alpha_s C_1}_{\mathcal{O}(\alpha_s)}) \exp\left(\frac{1}{\alpha_s} g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L)\right)$$

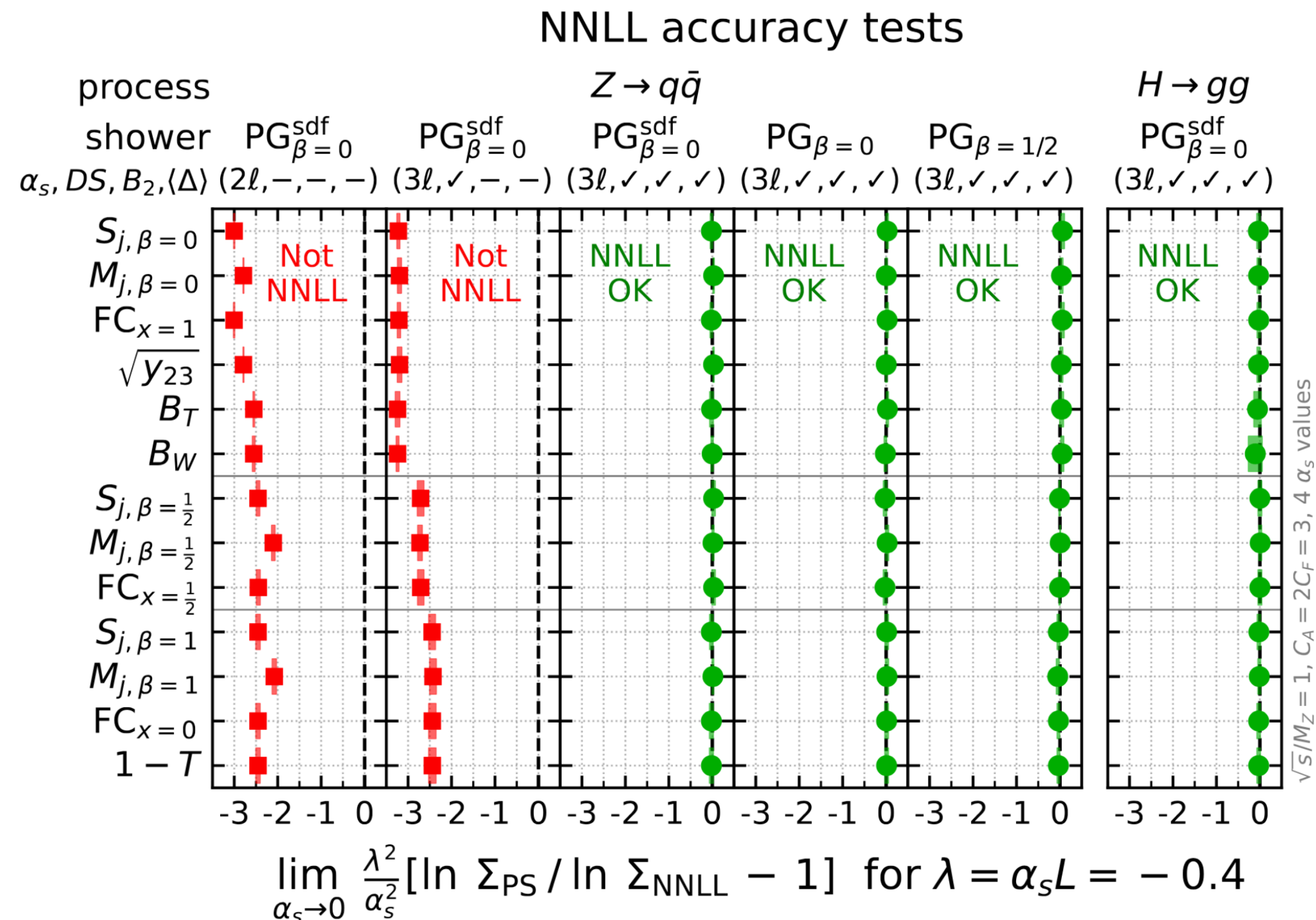
Using a double log expansion:

$$\Sigma(\alpha_s, L) = \underbrace{h_1(\alpha_s L^2)}_{\text{DL}} + \underbrace{\sqrt{\alpha_s} h_2(\alpha_s L^2)}_{\text{NDL}} + \underbrace{\alpha_s h_3(\alpha_s L^2)}_{\text{NNDL}}$$

For event-shapes:  
C<sub>1</sub> + NLL → NNDL

# MOTIVATION

1. NLL showers are likely to become a fundamental ingredient of the precision physics program at the LHC.
2. NLO matching can augment the shower accuracy.
3. NLO matching is needed for obtaining NNLL accurate showers.



[van Beekveld, et al. '24]

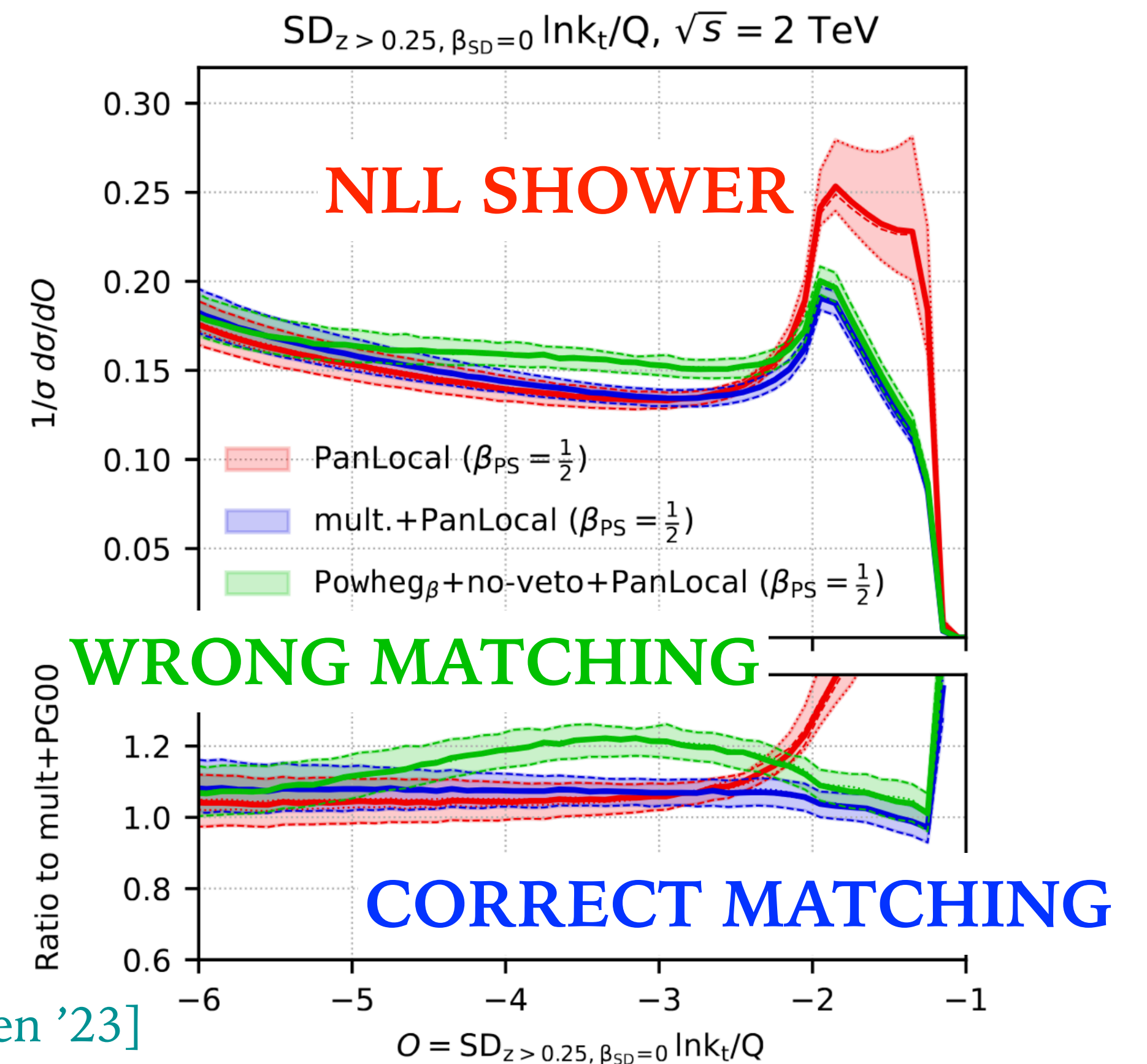
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IF MATCHING IS DONE  
WRONGLY, THE STRUCTURE OF  
THE SHOWER CAN BREAK.

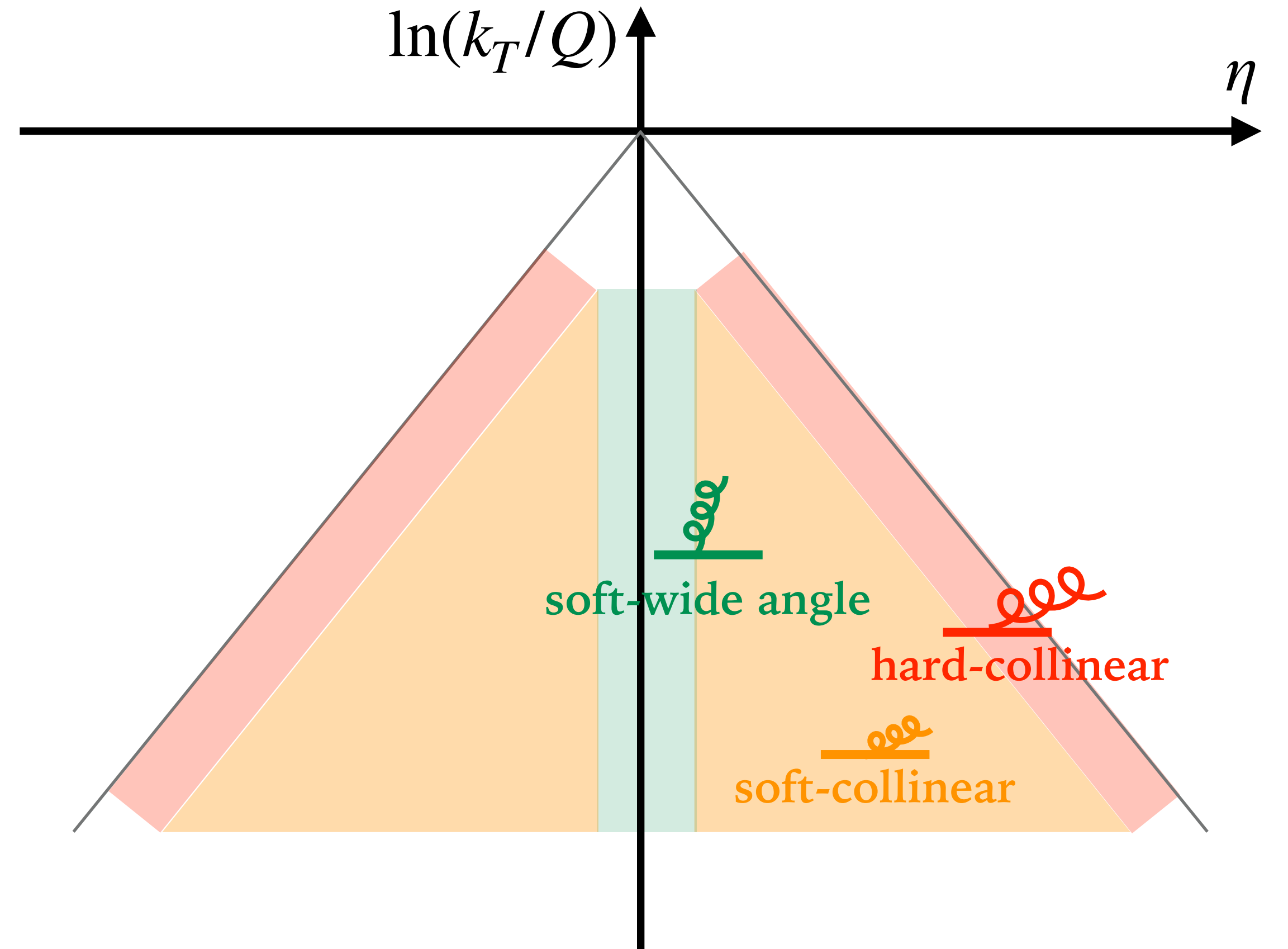
[Hamilton, Karlberg, Salam, Scyboz, Verheyen '23]





# NLL PanScales SHOWERS IN A NUTSHELL

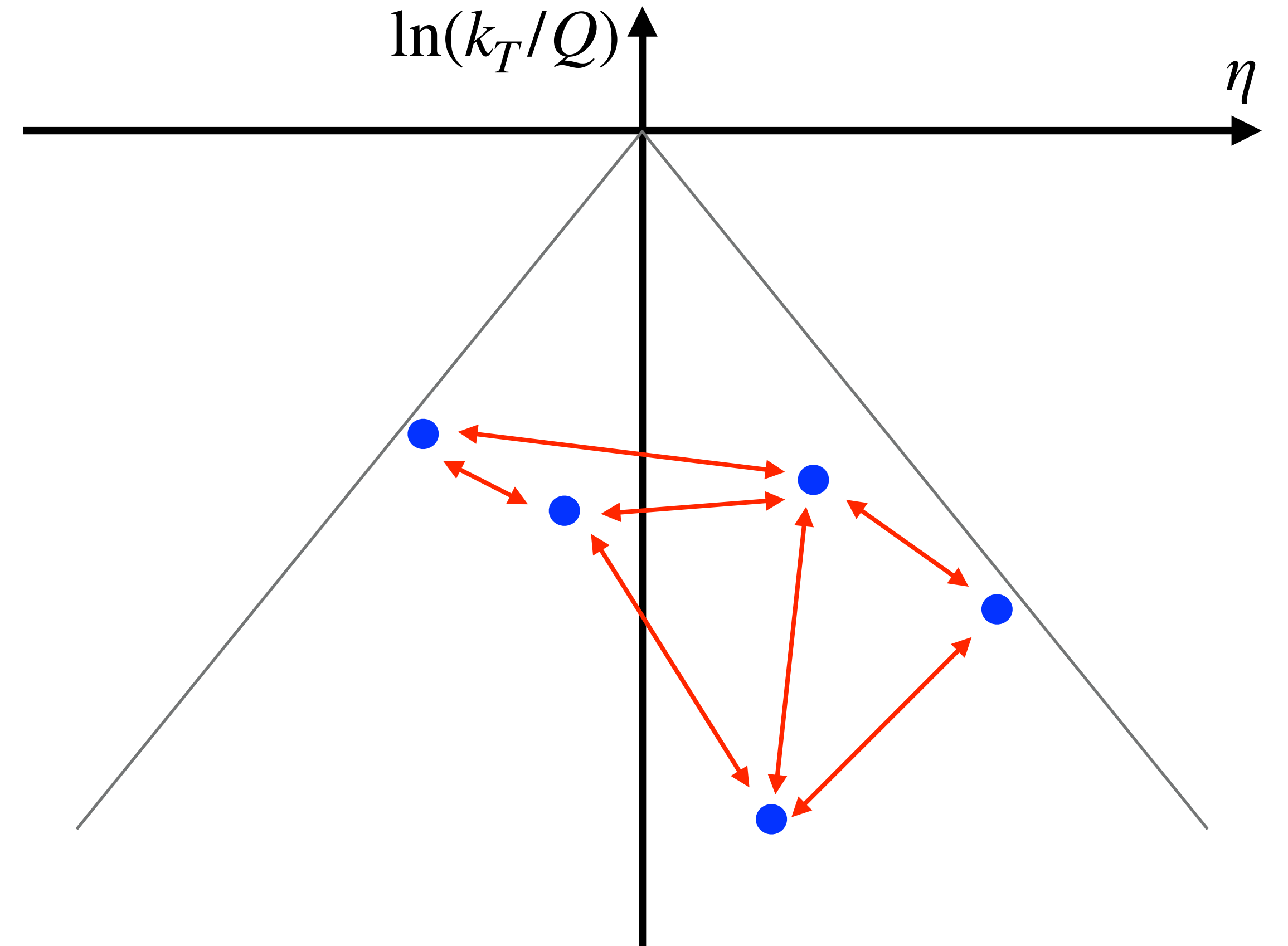
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# NLL PanScales SHOWERS IN A NUTSHELL

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1. The shower must reproduce the exact matrix element when all emissions are well separated in at least  $k_T$  or  $\eta$ .
2. The shower must reproduce NLL analytic results for a broad class of observables (e.g. event shapes, subjet multiplicities).

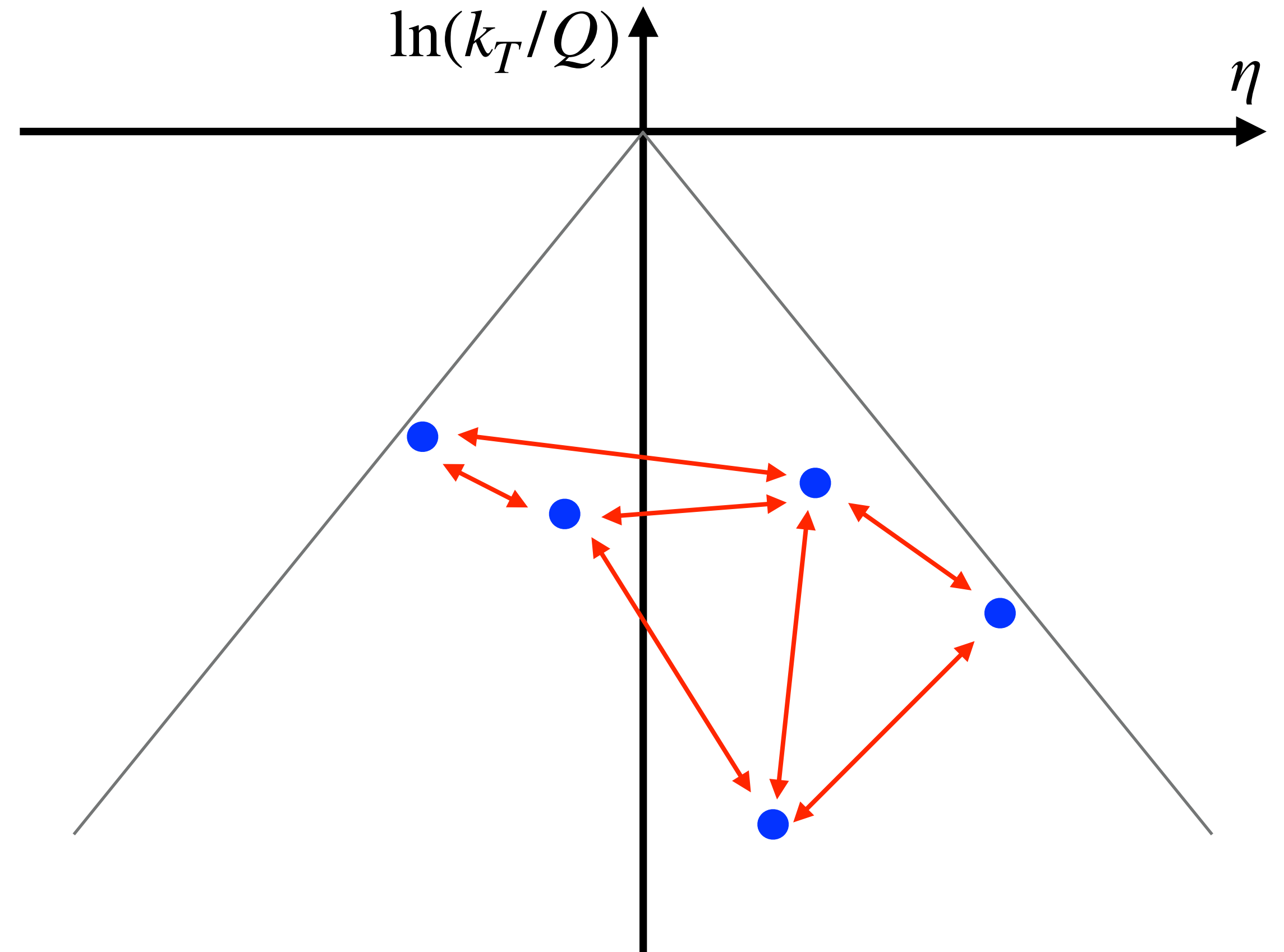


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## The PanScales Recipe

- Ordering variable:  $v = k_t e^{-\beta|\eta|}$
- Recoil scheme:  
PanLocal:  $\perp, +, -$  local  
PanGlobal:  $\perp$  global,  $\{+, -\}$  local
- Partitioning of the dipole in event com frame





# THE METHOD

---

## MULTIPLICATIVE MATCHING

$$d\sigma(\Phi_B) = \bar{B}(\Phi_B) \left[ S(\Phi_B) \times \frac{R(\Phi_B, \Phi_{\text{rad}})}{B(\Phi_B)} d\Phi \right] I_{\text{PS}}$$

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- a) Reach NLO accuracy in the normalization  $\bar{B}$ . Analytically when possible, otherwise numerically.

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- a) Reach NLO accuracy in the normalization  $\bar{B}$ . Analytically when possible, otherwise numerically.
- b) Generate the first emission with the exact real matrix element.
- c) Generate all the subsequent emissions through a NLL shower.

# a) Normalization

---

The Born kinematics is generated according to the NLO cross section as:

$$d\sigma = \bar{B}(\Phi_B) d\Phi_B \quad \text{with} \quad \bar{B}(\Phi_B) = B + V + \int R d\Phi_{\text{rad}}$$

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**Use analytic  $\bar{B}$  when available:**

$$\text{DIS (photon only): } \frac{d^2\sigma}{dx dy} = \frac{4\pi\alpha^2}{xyQ^2} \left( (1-y) F_2 + y^2 x F_1 \right)$$

with:

- x and y usual DIS variables
- $F_1$  and  $F_2$  NLO structure functions

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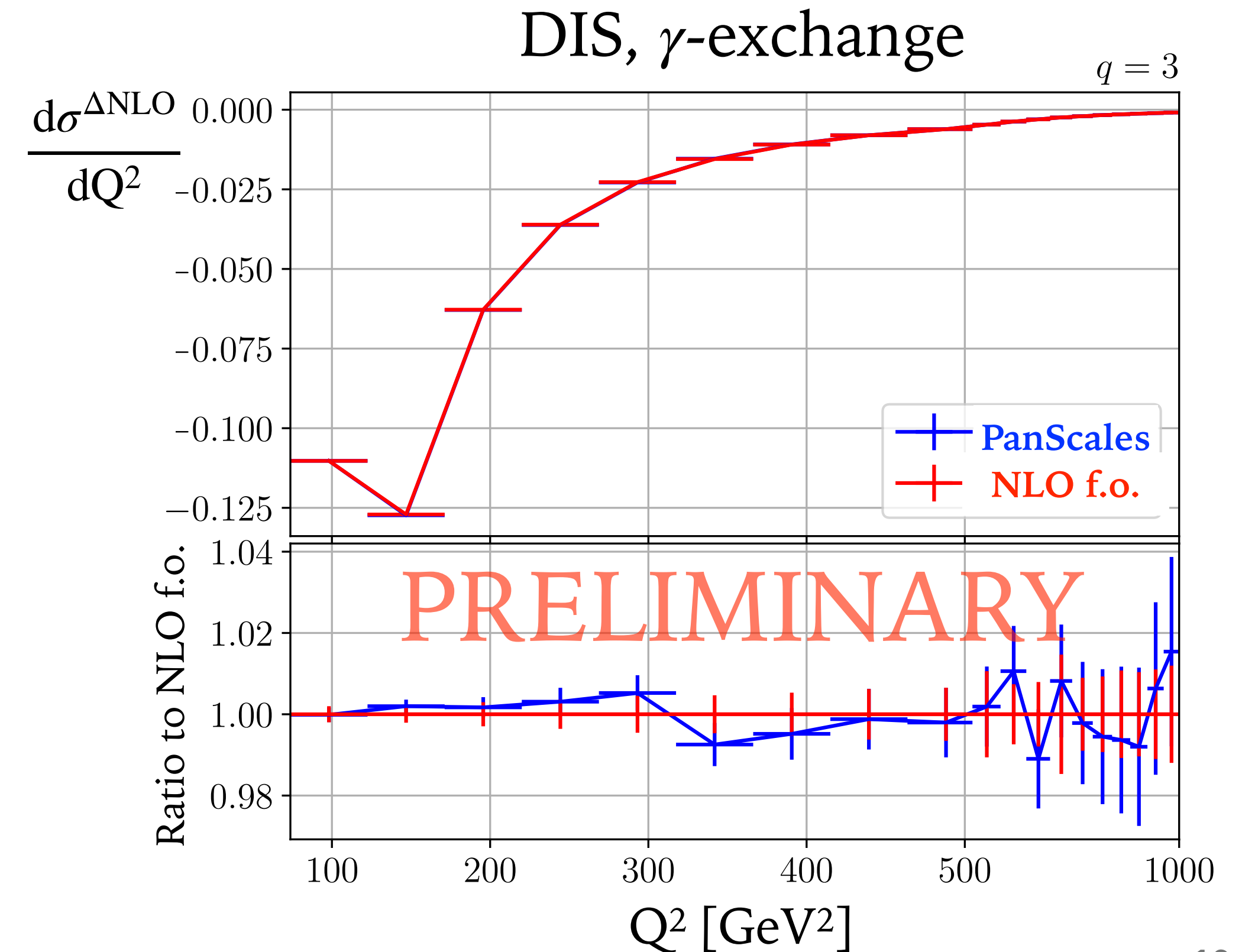
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At NLO this requires a careful flavour decomposition  
(more on this later)





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Obtain  $\bar{B}$  numerically:

POWHEG: 
$$\bar{B}(\Phi_B) = \int dX_1 dX_2 dX_3 \tilde{B}(\Phi_B, X_1, X_2, X_3)$$

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$$\tilde{B}(\Phi_B, X_1, X_2, X_3) \rightarrow \tilde{B}(\Phi_B, X_1, X_2, X_3) + \Delta\tilde{B}(\Phi_B, X_1, X_2, X_3)$$

$$\Delta\tilde{B} = \text{Jac} \cdot \text{R}(\Phi_{\text{rad}}^{\text{PS}}) - \text{R}(\Phi_{\text{rad}}^{\text{FKS}})$$



Correct FKS/PanScales mappings

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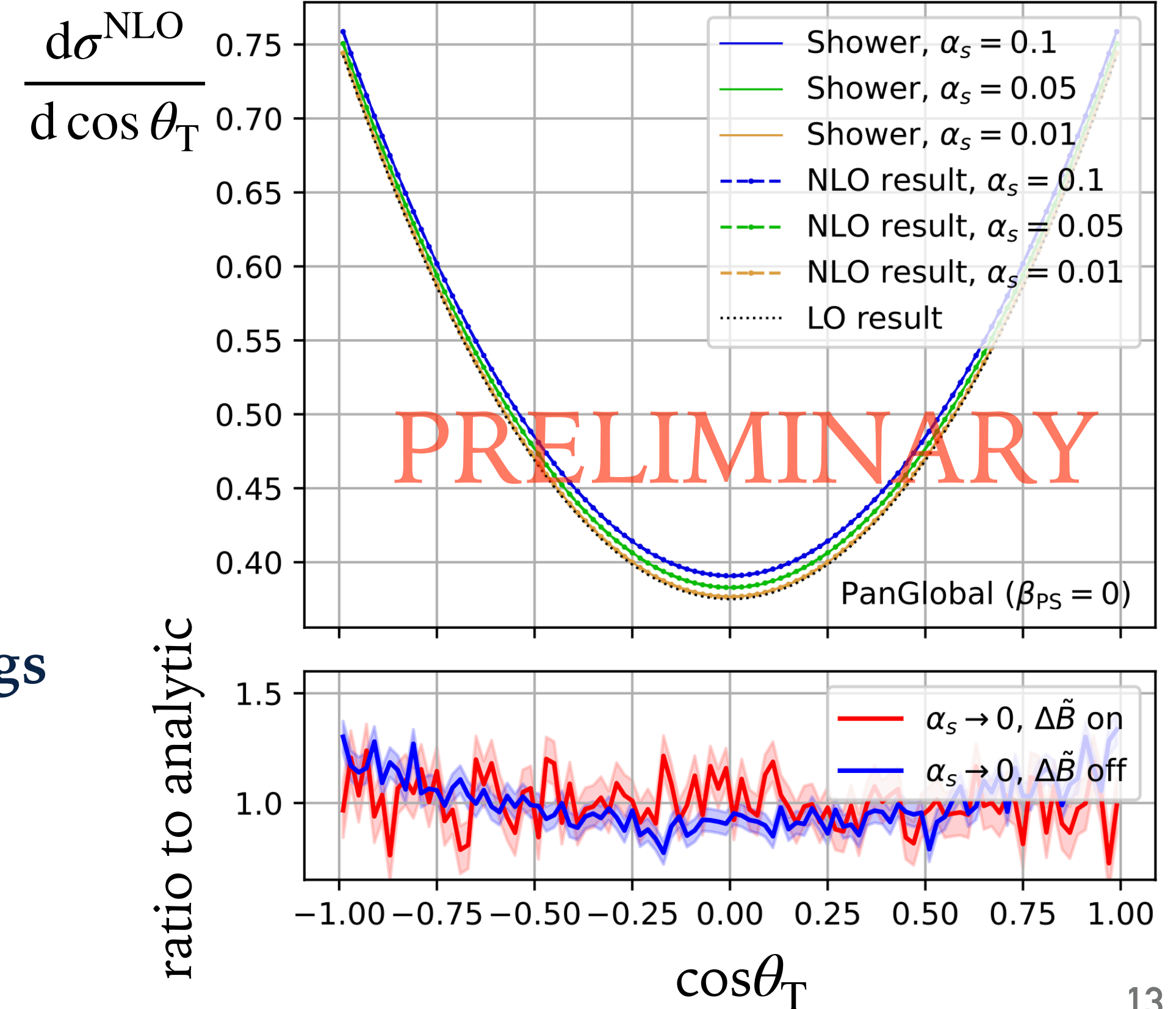
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Correct FKS/PanScales mappings

Applied to  $e^+e^- \rightarrow Z$  and  $pp \rightarrow Z$  but **easily extendable to more complicated processes taking  $\tilde{B}$  directly from POWHEG.**



## b) Hardest emission

---

The hardest emission is generated with the correct real matrix element.

$$\tilde{ij} \rightarrow ijk : P \sim \frac{R_{i,j,k}(\Phi_{n+1})}{B_{\tilde{i},\tilde{j}}(\Phi_n)}$$

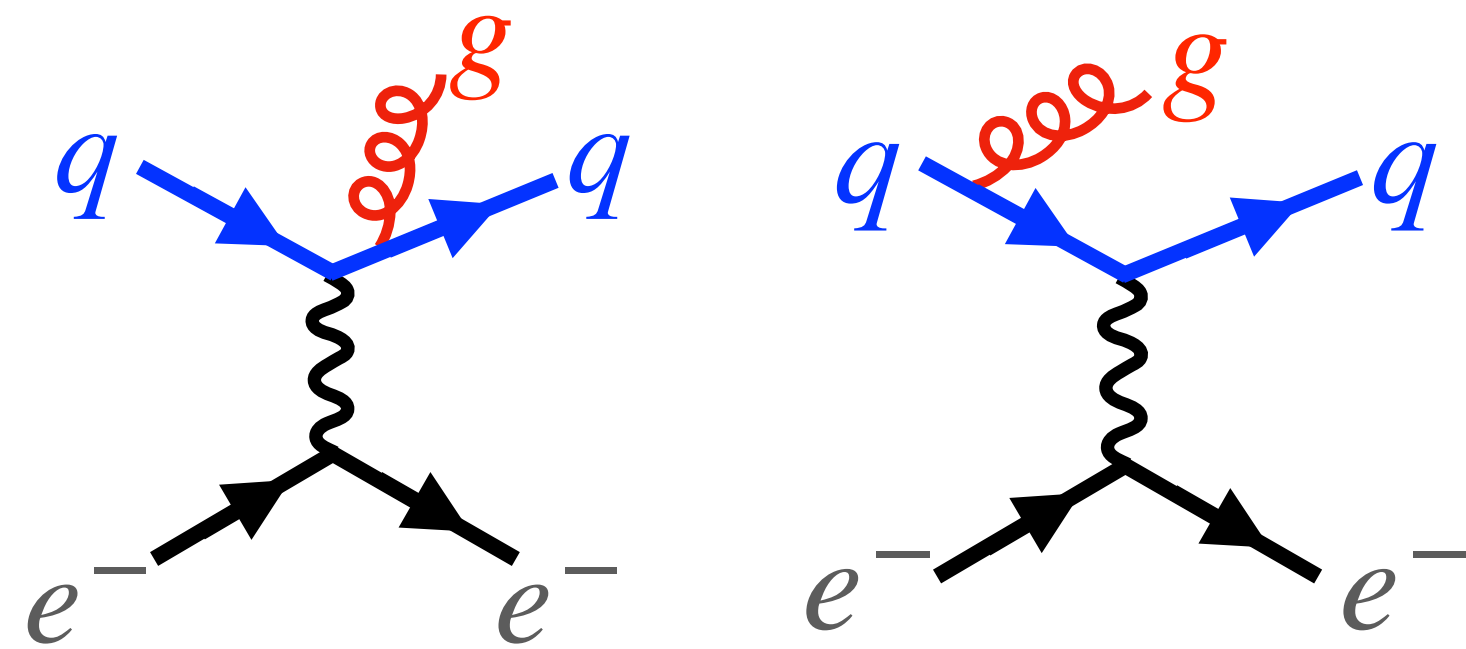


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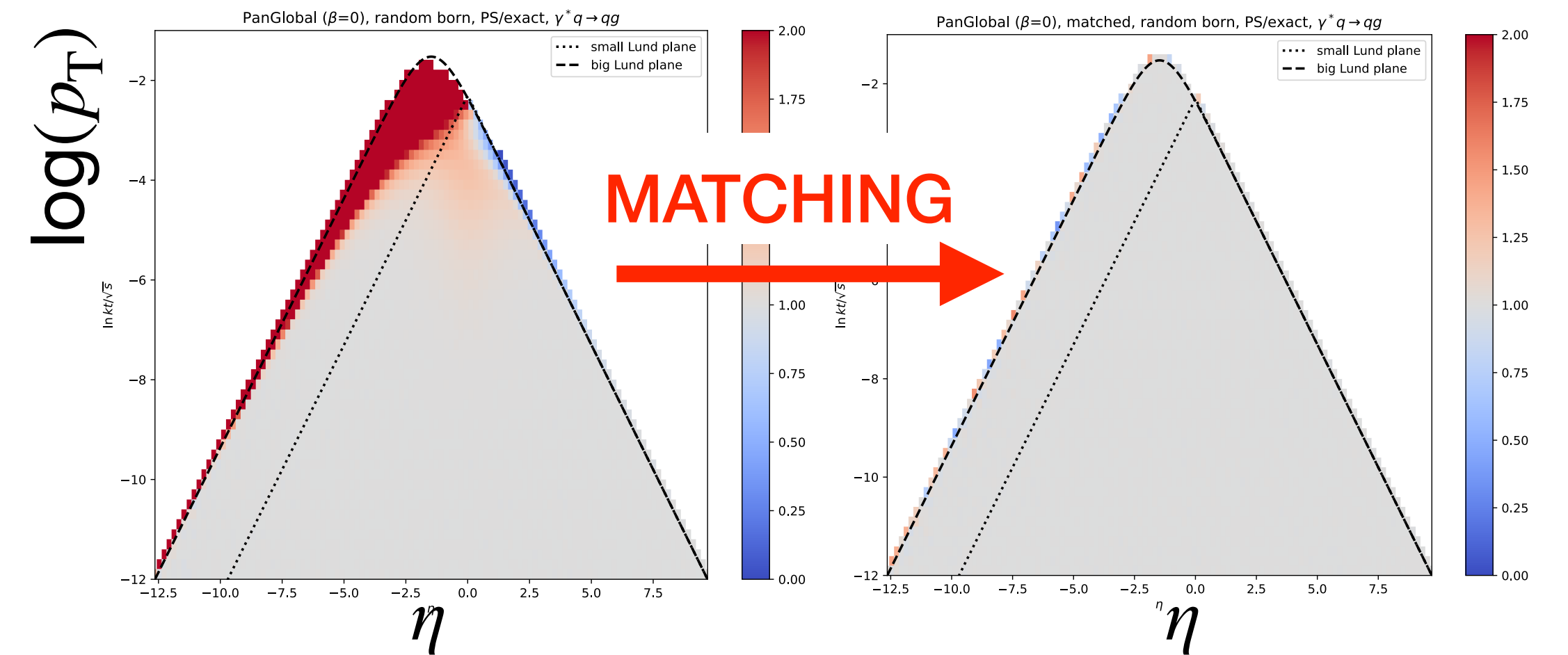
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Simple example: DIS, only  $\gamma^*q \rightarrow qg$  channel



(+  $\bar{q}$ -contributions)

## RATIO PS/exact ME

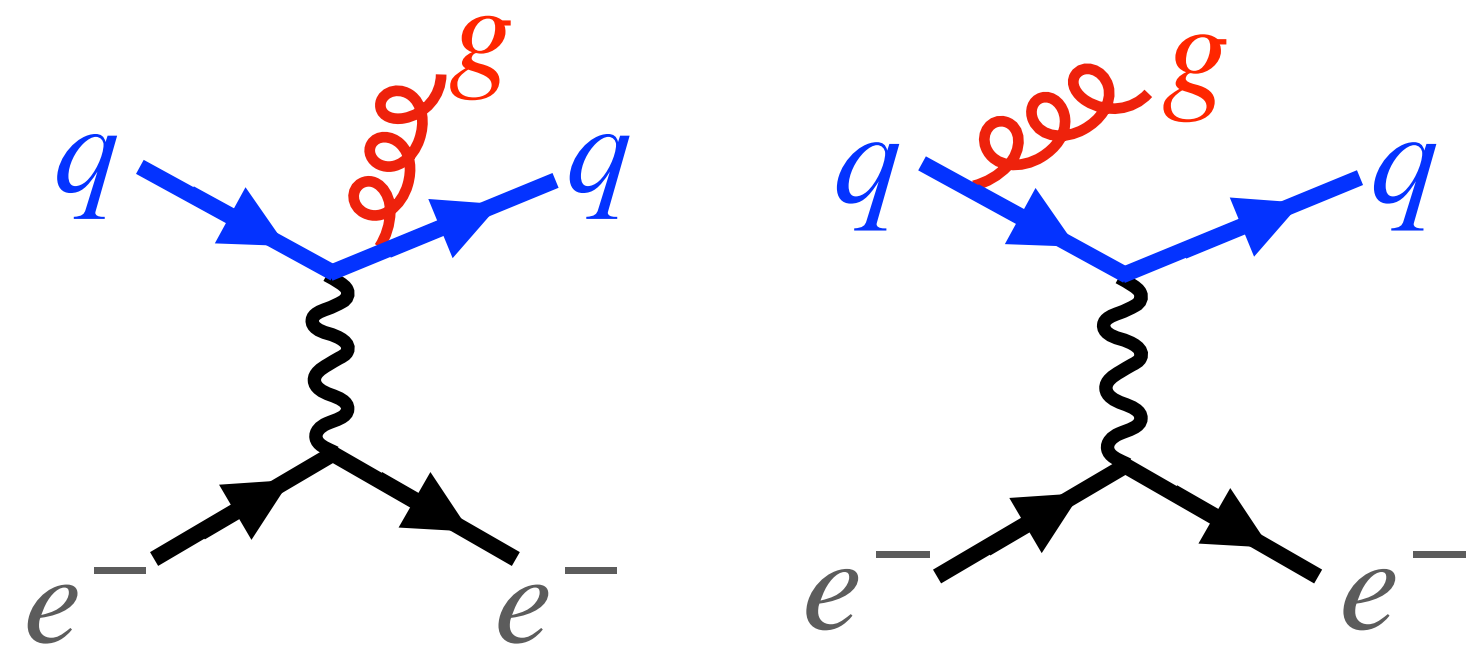


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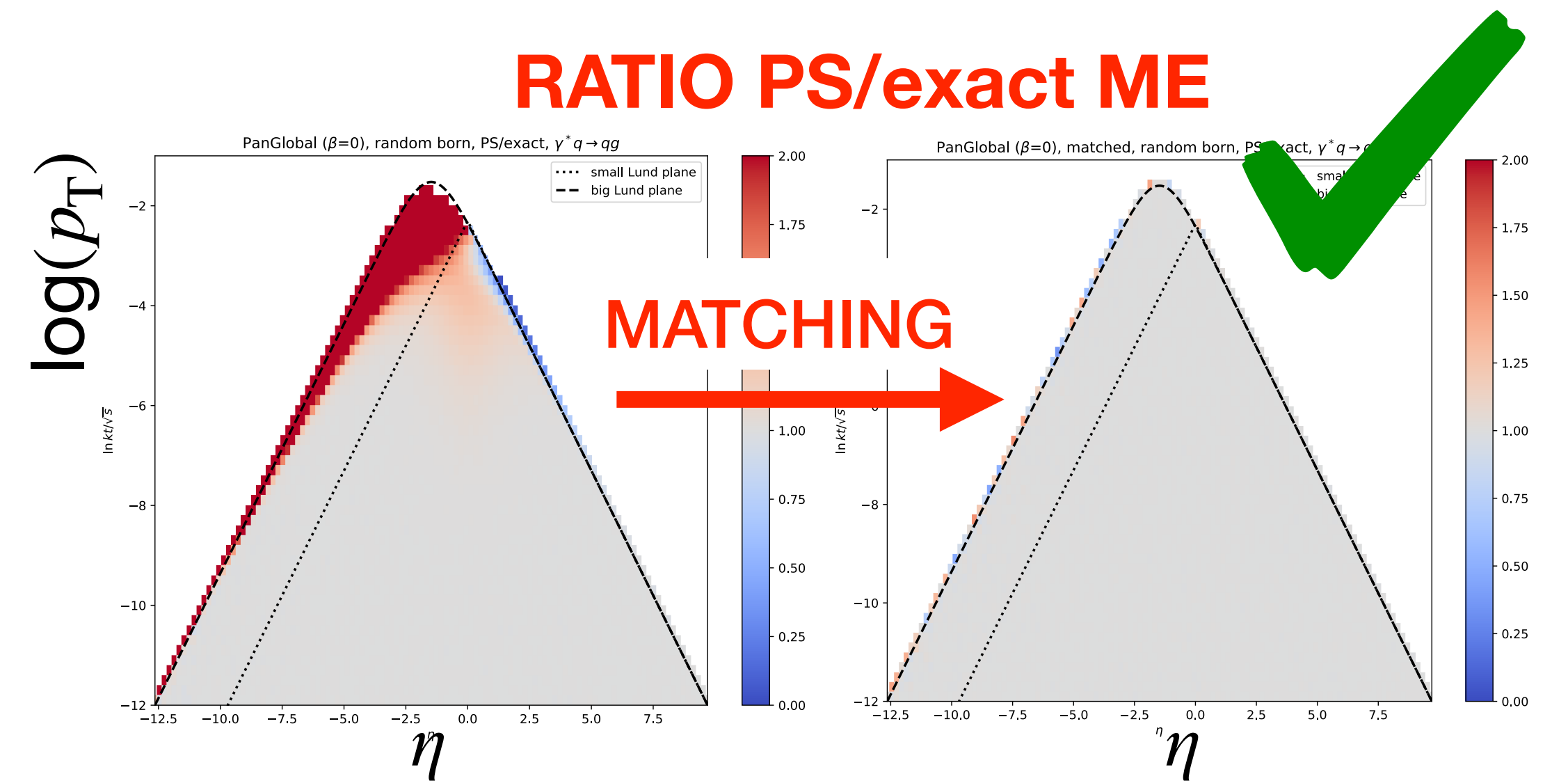
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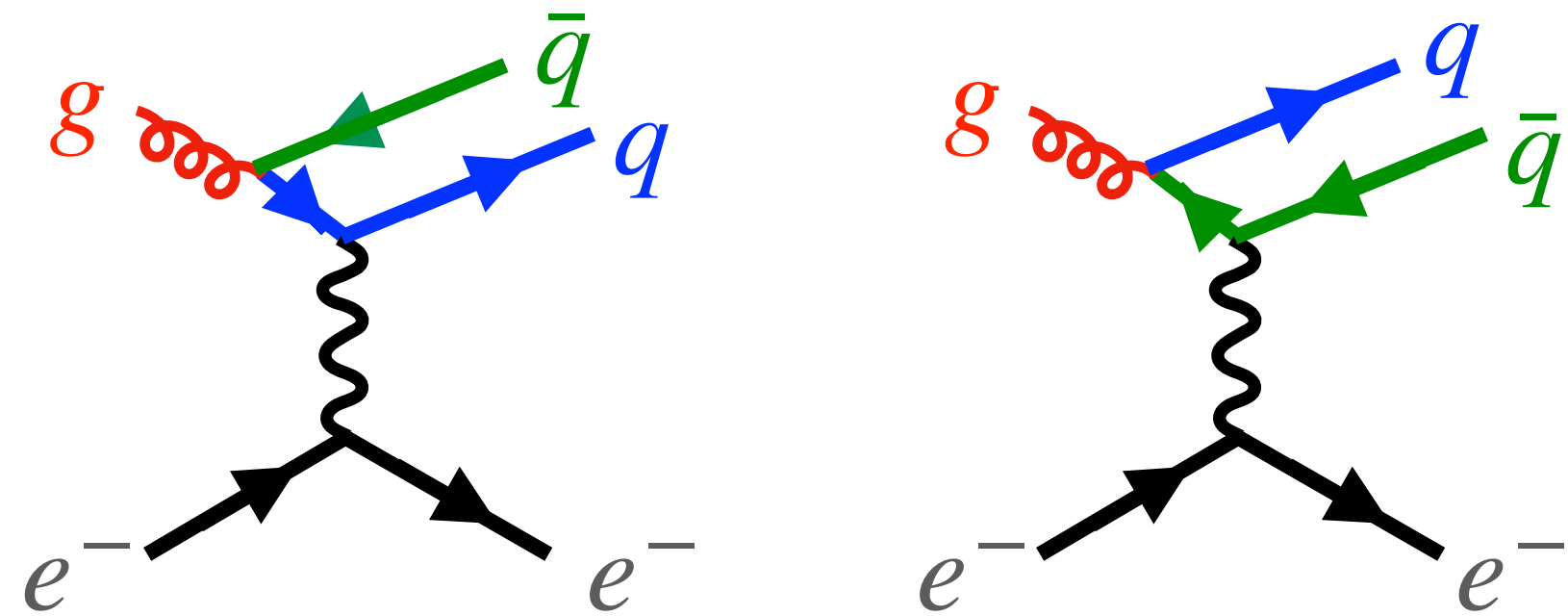


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Non-trivial example: DIS,  $\gamma^*g \rightarrow q\bar{q}$  channel



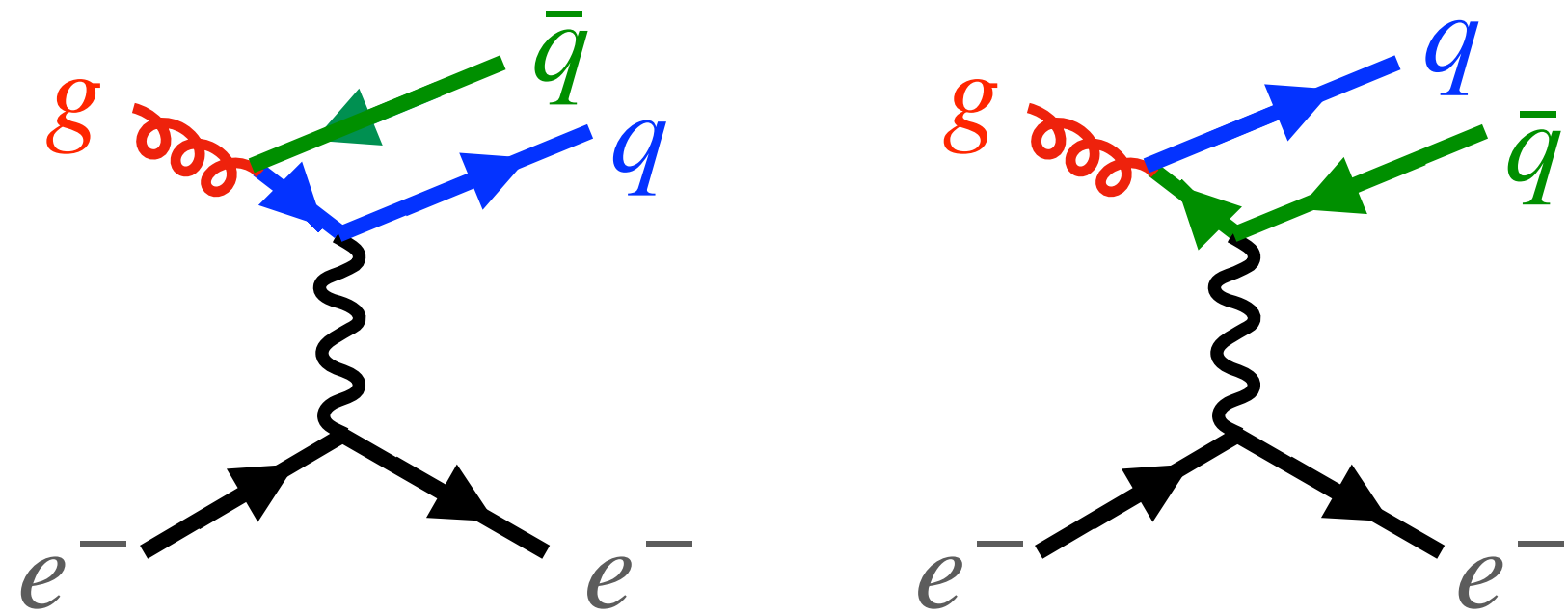
Two possible underlying Borns for the same real configuration!

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Two possible underlying Borns for the same real configuration!

Introduce a partitioning factor to split the full real matrix element in the two possible Borns.

The fraction of real associated to a given Born is taken as:

$$\left[ \begin{array}{l} \gamma^* q \rightarrow q \\ \gamma^* \bar{q} \rightarrow \bar{q} \end{array} \right. \begin{array}{l} \text{diagram 1} \\ \text{diagram 2} \end{array} \longrightarrow \begin{array}{l} \theta(p_q \cdot p_g < p_{\bar{q}} \cdot p_g) \\ \theta(p_{\bar{q}} \cdot p_g < p_q \cdot p_g) \end{array}$$

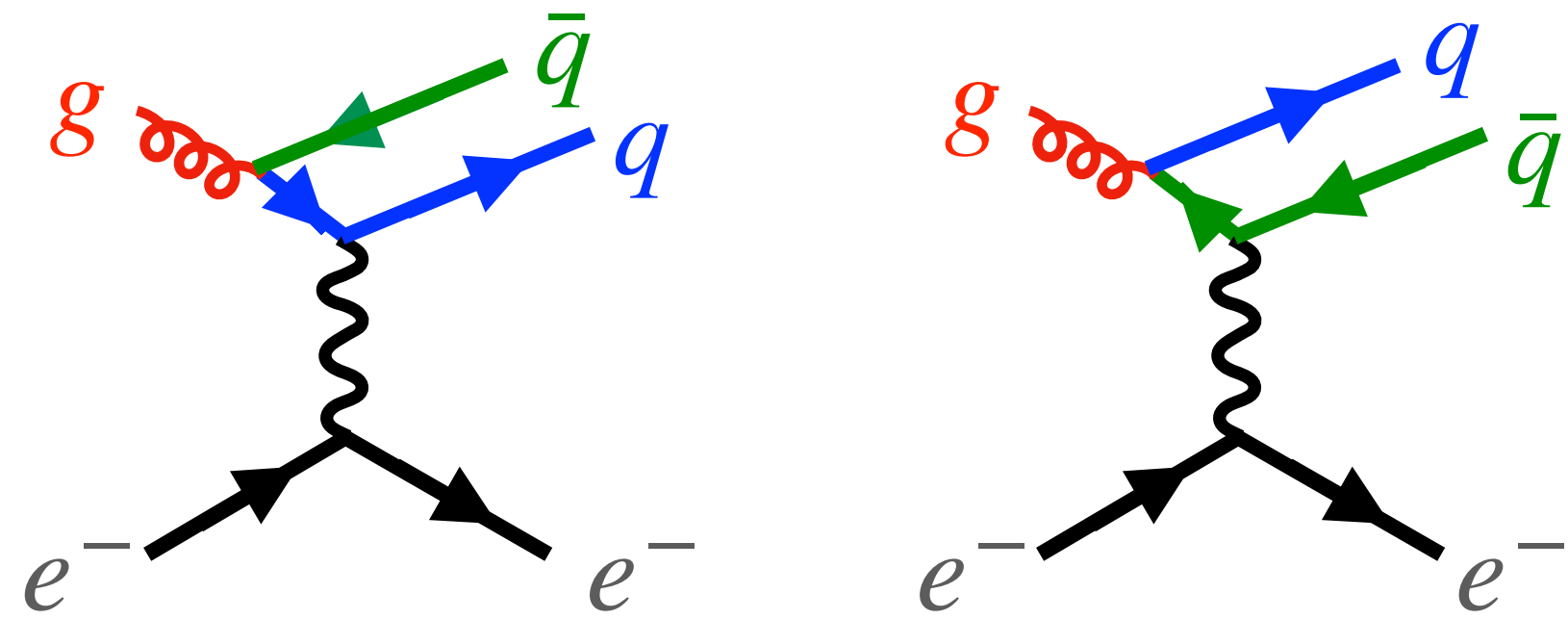


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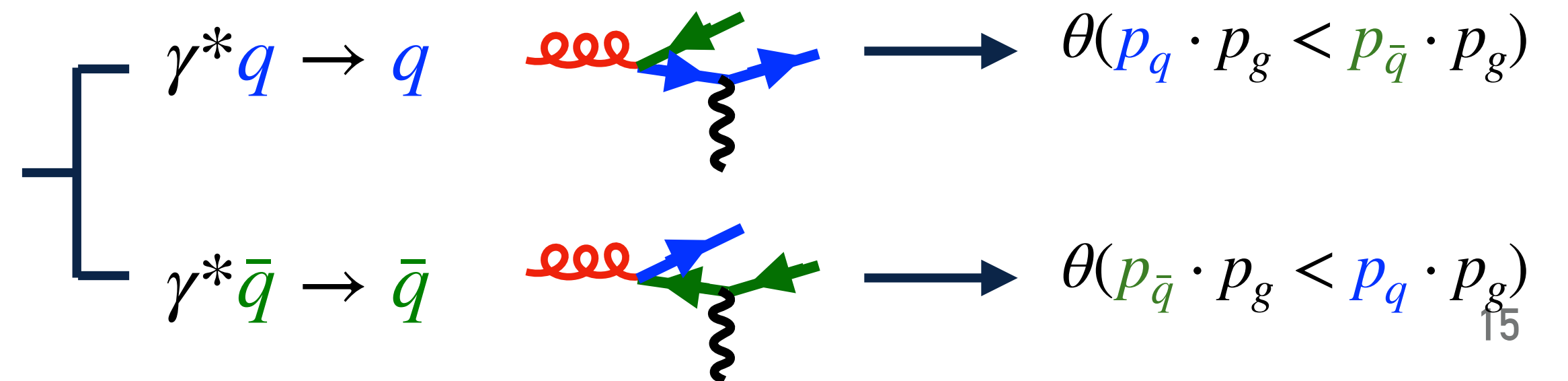
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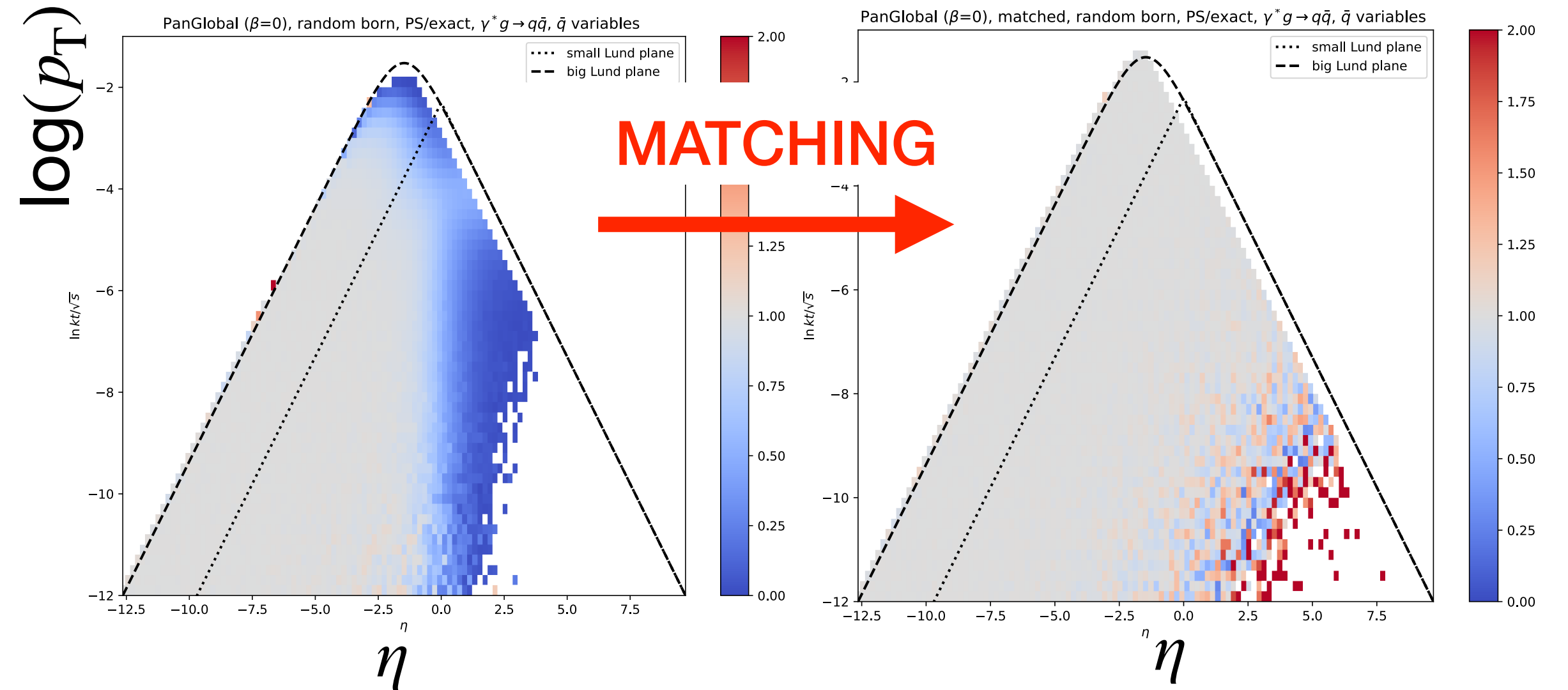
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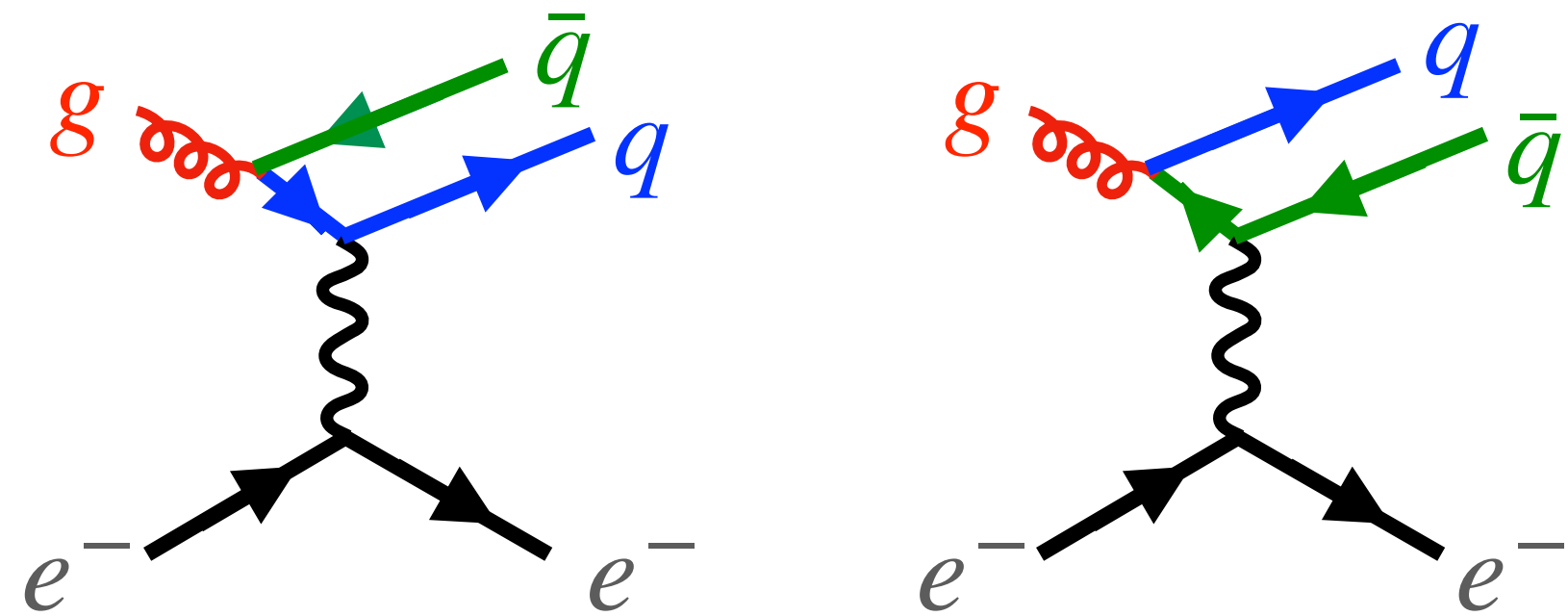
Separation of singular regions between the two underlying Borns.

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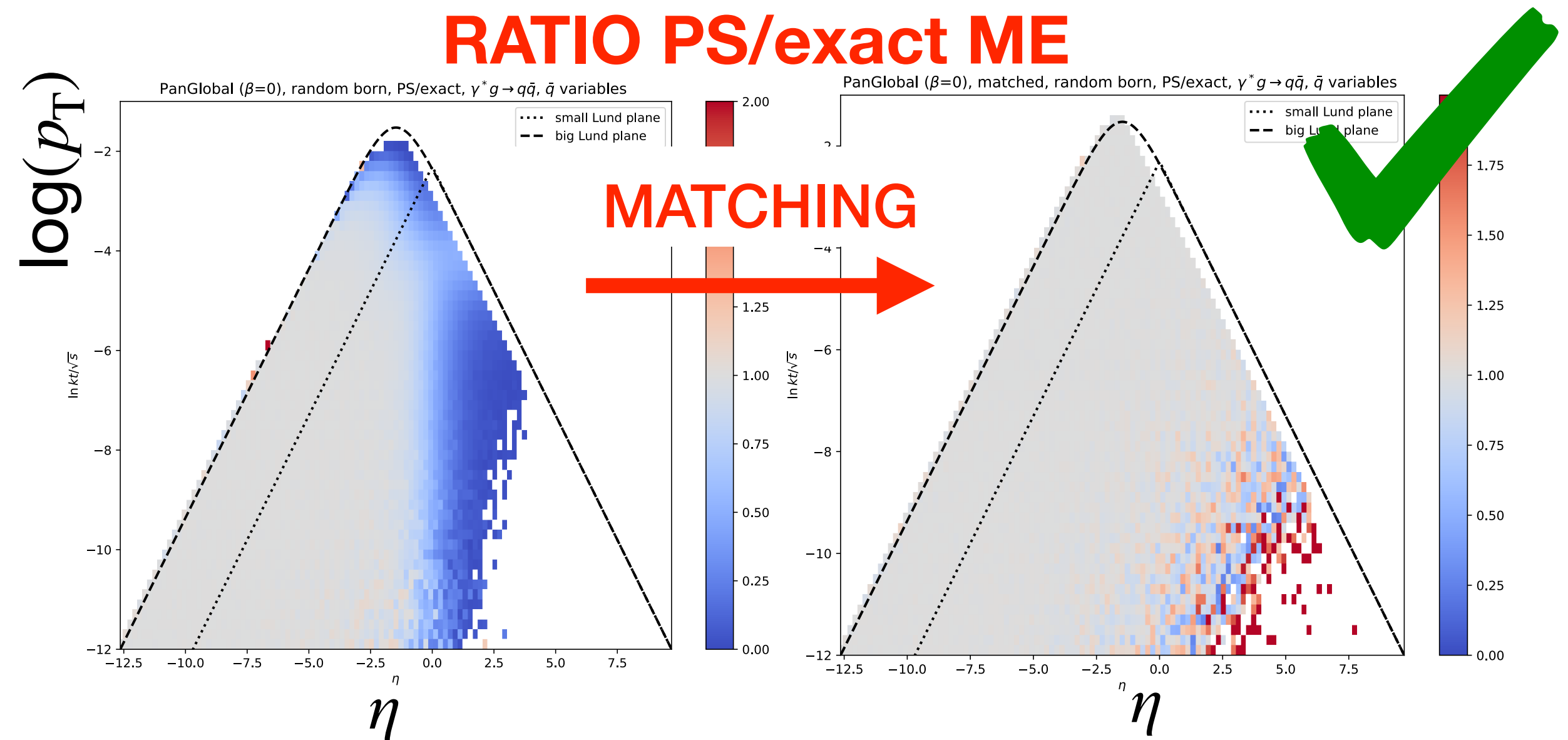
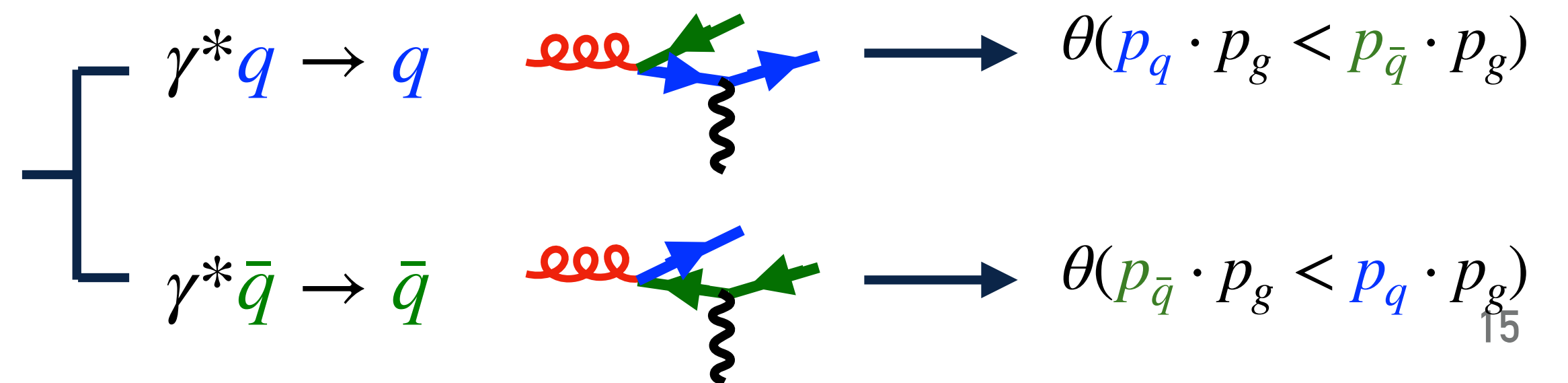
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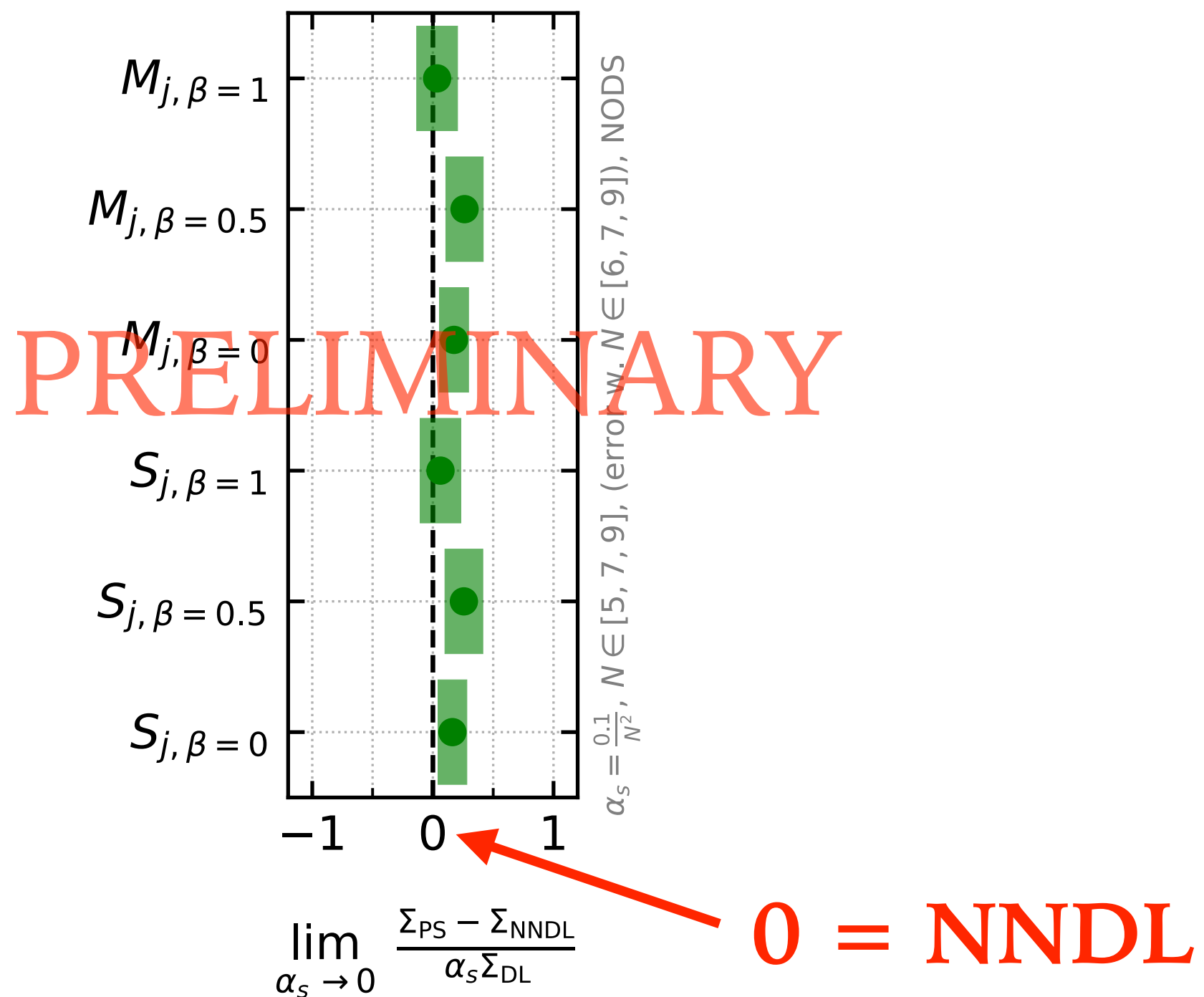


Separation of singular regions between the two underlying Borns.

# NNDL TEST

$$pp \rightarrow Z, \sqrt{s}/m_Z = 100, \alpha_s L^2 = 1.296$$

PanGlobal  
( $\beta_{PS} = 0$ )



- $\text{NNDL} = \mathcal{O}(\alpha_s^n L^{2n-2})$
- Compare the parton shower result with analytic calculations.
- Eliminate spurious contributions generated by the shower with:

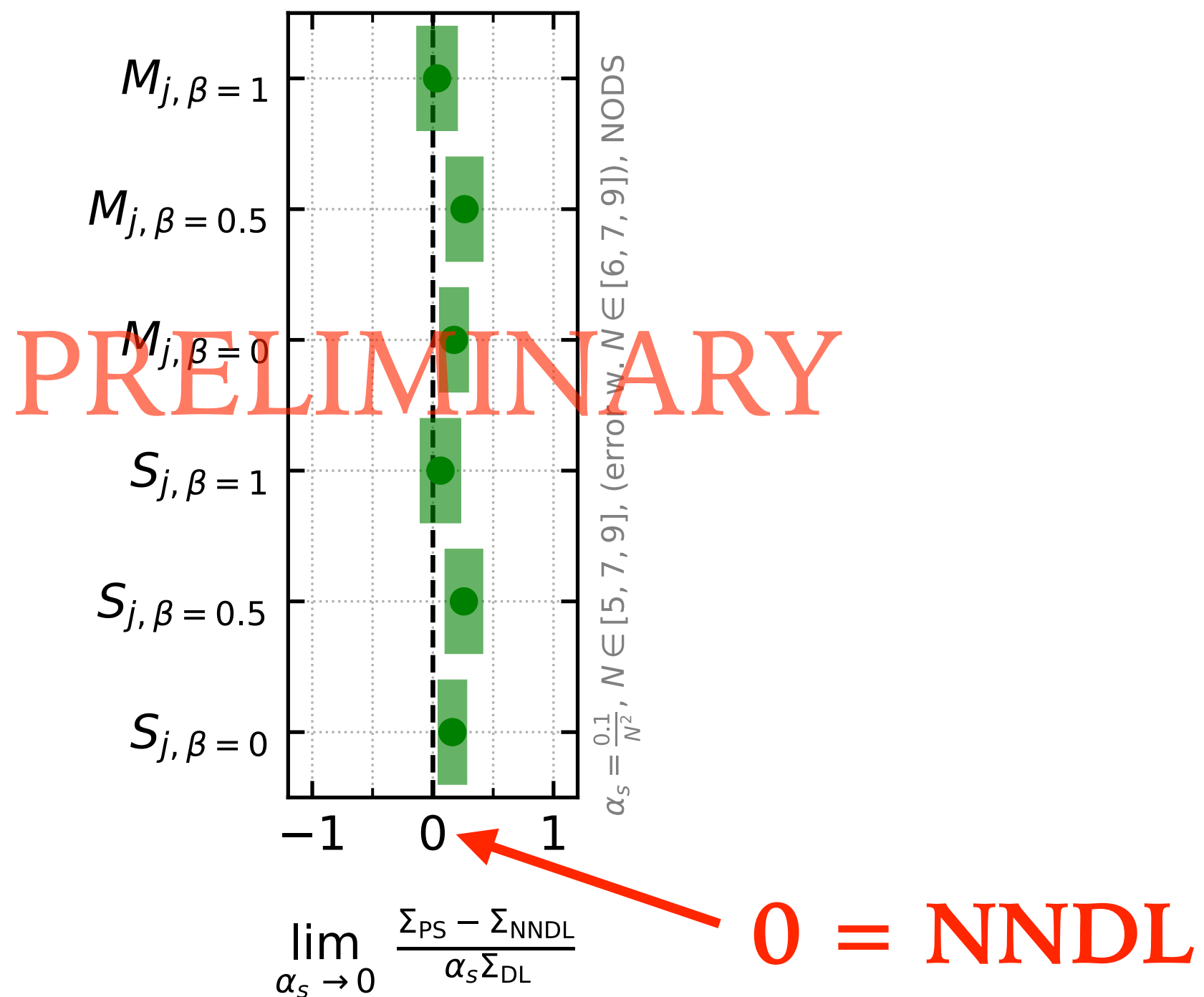
$$\lim_{\alpha_s \rightarrow 0} \frac{\Sigma_{\text{PS}} - \Sigma_{\text{NNDL}}}{\alpha_s \Sigma_{\text{DL}}} \Bigg|_{\alpha_s L^2 \text{ fixed}}$$

$$M_{j,\beta} = \max_{i \in \text{jets}} \frac{k_{Ti}}{Q} e^{-\beta |\eta_i - \eta_Z|} \quad S_{j,\beta} = \sum_{i \in \text{jets}} \frac{k_{Ti}}{Q} e^{-\beta |\eta_i - \eta_Z|}$$

# NNDL TEST

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- Compare the parton shower result with analytic calculations.
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$$\lim_{\alpha_s \rightarrow 0} \frac{\Sigma_{\text{PS}} - \Sigma_{\text{NNDL}}}{\alpha_s \Sigma_{\text{DL}}} \Bigg|_{\alpha_s L^2 \text{ fixed}}$$

First time a shower demonstrably generates NNDL accurate results across many observables for pp collisions.



# CONCLUSIONS and OUTLOOK

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- NLO matching with NLL showers is needed for precision physics.
- **Results for DIS, DY and oriented  $e^+e^-$  have been obtained at NLO+NLL accuracy through a multiplicative matching with PanScales showers.**
- **Extension to more complicated processes is possible through an on-the-fly conversion of  $\tilde{B}^{\text{FKS}}$  to  $\tilde{B}^{\text{PS}}$ .**
- **NNDL accurate results have been obtained for many observables in pp collisions.**

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The PanScales collaboration is currently working on a new NLO matching scheme with the nice feature of **generating only positive weights.**

IDEA: use a simple (=analytic)  $\bar{B}$  function and correct it à la MacNLOPS.

*...more on this soon!*