TOWARDS SUBLEADING POWER FACTORISATION

Robin van Bijleveld
r.vanbijleveld@nikhef.nl

In collaboration with
Eric Laenen, Leonardo Vernazza, Guoxing Wang and Coenraad Marinissen

HP2 2024 TORINO
10 SEPTEMBER 2024

BASED ON: 2308.00230 AND ONGOING WORK

Nikhef

OUTLINE

- Threshold resummation at next-to-leading power (for leading logarithms)
  - Threshold limit
  - Rapidity distributions
- Towards factorisation at next-to-leading power
  - Subleading jet functions
  - Method of regions

THRESHOLD LIMIT

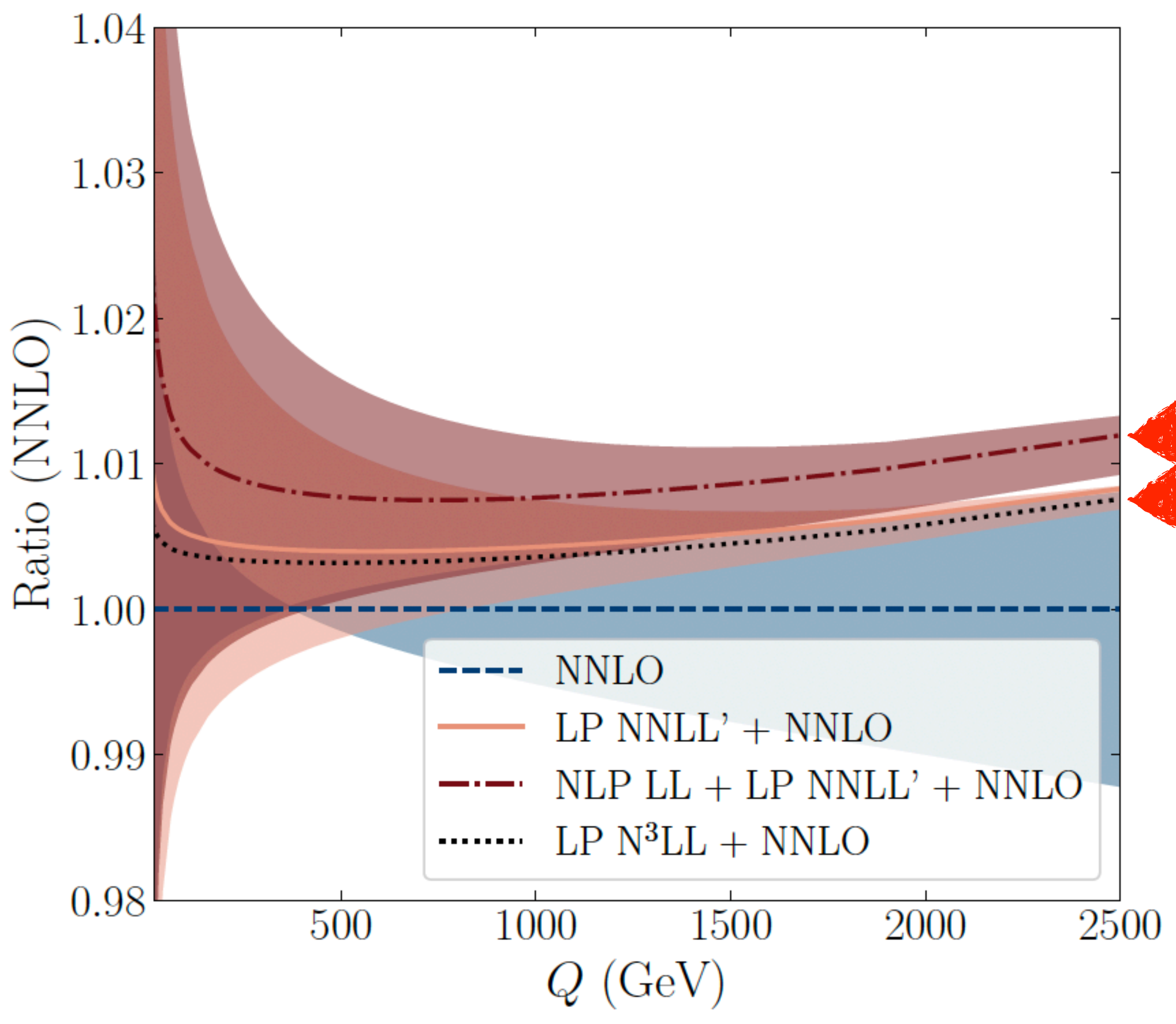
- Large logarithms appear at all orders in the perturbation theory
- Consider all orders in coupling constant to improve predictive power
- Threshold limit $z \equiv Q^2/\hat{s} \to 1, \ \lambda \sim \sqrt{1-z}$

$$\frac{d\hat{\sigma}}{dQ^2} = \sum_{n=0}^{\infty} \alpha_s^n \left(d_n \delta(1-z) + \sum_{m=0}^{2n-1} c_{nm}^{\mathrm{LP}} \left[\frac{\log^m(1-z)}{(1-z)} \right]_+ + c_{nm}^{\mathrm{NLP}} \log^m(1-z) + c_{nm}^{\mathrm{NNLP}} (1-z) \log^m(1-z) + \ldots \right)$$

NLP LL EFFECT

- DY cross section

$$\frac{d\sigma_{pp\to\ell^+\ell^-+X}}{dQ^2}$$



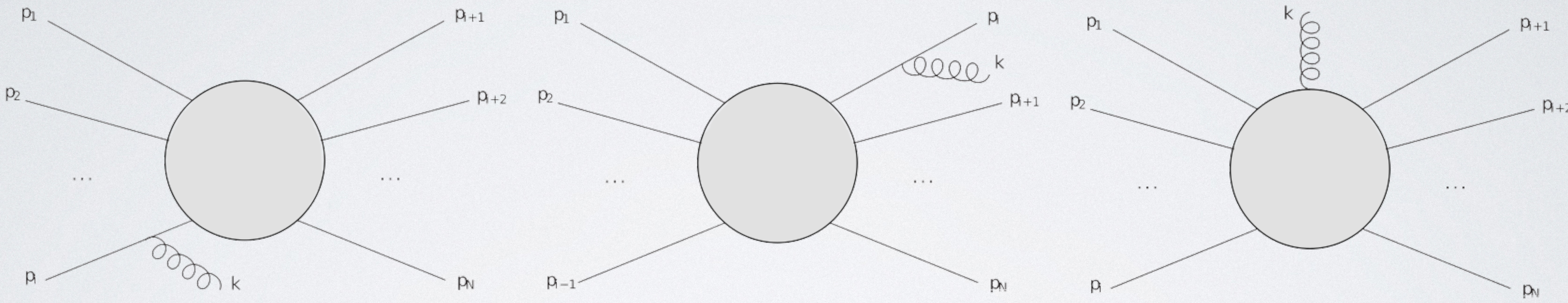
(b) *Ratio with respect to NNLO*

LP N2LL' + NLP LL
LP N3LL

Van Beekveld, Laenen, Sinninghe Damsté, Vernazza (2021)

THRESHOLD LIMIT

- Soft theorem



$$\mathcal{M}_{N+1}^{\text{NLP}} = \sum_{i=1}^{N} \frac{g_s \mathbf{T_i}}{2p_i \cdot k} \Big(\underbrace{2p_i^\mu + k^\mu}_{\text{scalar}} - \underbrace{2ik_\nu S_{(i)}^{\mu\nu}}_{\text{spin}} + \underbrace{2ik_\nu L_{(i)}^{\mu\nu}}_{\text{orbital}} \Big) \otimes \mathcal{M}_N(p_1, \ldots, p_N)\bar{\epsilon}_\mu(k)$$

Del Duca, Laenen, Magnea, Vernazza, White (2017)

Van Beekveld, Beenakker, Laenen, White (2020)

THRESHOLD LIMIT

- $q\bar{q}$ or gg initial state
- General colour-singlet final state
- Use soft theorem

$$\left|\mathcal{M}_{\text{NLO,NLP}}\right|^2 \sim \alpha_s \frac{p_1 \cdot p_2}{(p_1 \cdot k)(p_2 \cdot k)} \left|\mathcal{M}_{\text{LO}}(p_1 + \delta p_1, p_2 + \delta p_2)\right|^2$$

$$\frac{d\hat{\sigma}^{(q\bar{q}/gg)}_{\text{NLO, NLP}}}{dz} = C_{F/A} K_{\text{NLP}}(z, \alpha_s, \epsilon) \hat{\sigma}^{(q\bar{q}/gg)}_{\text{LO}}(z\hat{s})$$

Del Duca, Laenen, Magnea, Vernazza, White (2017)

THRESHOLD RESUMMATION

- Generalise K-factor
- Mellin transform $f(N) = \int_0^1 dz\, z^{N-1} f(z)$
- NLP phase space correction contributes beyond LL

$$\sim \epsilon(1-z)$$

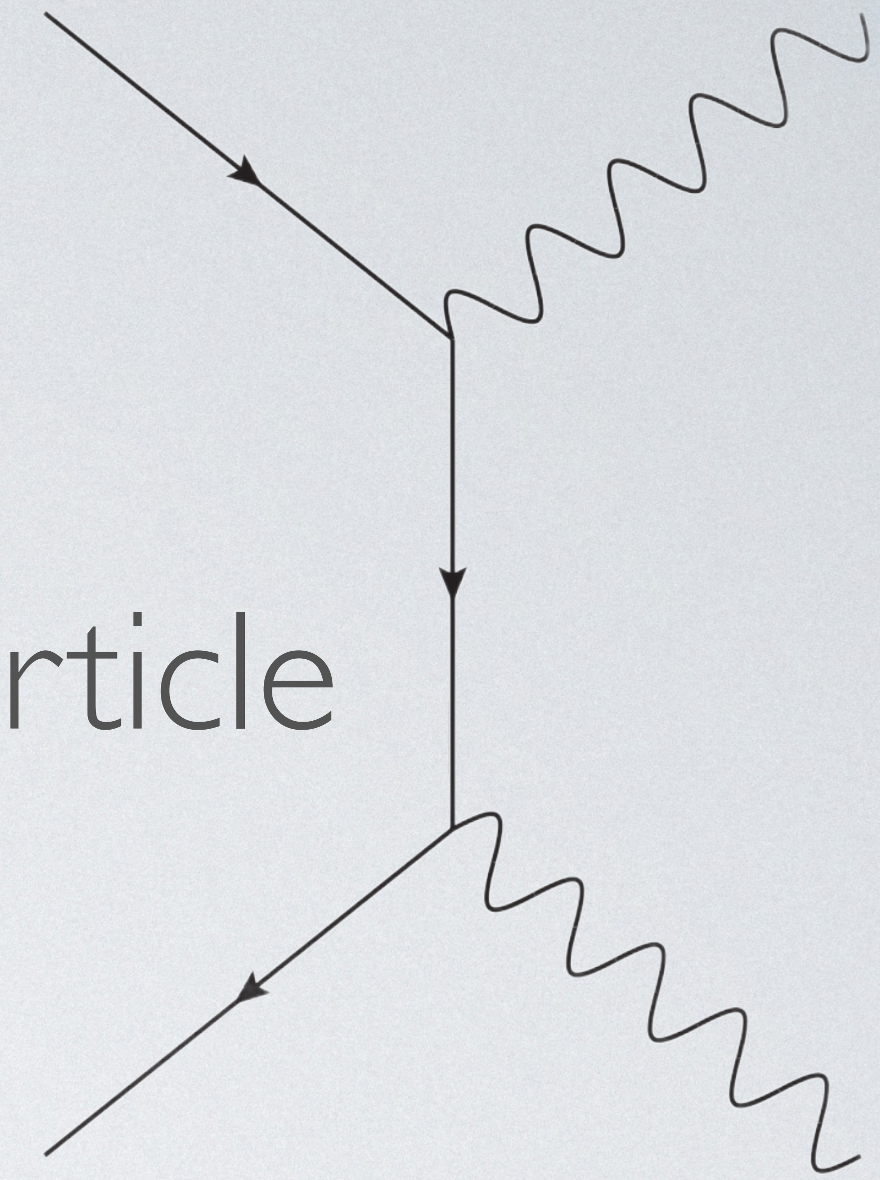
$$\hat{\sigma} \sim \int d\Phi_{\rm LP} \left[|\mathcal{M}|^2_{\rm LP} + |\mathcal{M}|^2_{\rm NLP} \right] + \int d\Phi_{\rm NLP} |\mathcal{M}|^2_{\rm LP} + \ldots$$

- Resummation for leading logarithms at LP and NLP

$$\frac{d\hat{\sigma}^{(q\bar{q}/gg)}_{\rm NLP}}{dz} = \hat{\sigma}^{(q\bar{q}/gg)}_{\rm LO}(Q^2)\, \exp\left[\frac{2\alpha_s C_{F/A}}{\pi} \log^2 N\right] \left(1 + \frac{2\alpha_s C_{F/A}}{\pi} \frac{\log N}{N}\right)$$

Bahjat-Abbas, Bonocore, Sinninghe Damsté, Laenen, Magnea, Vernazza, White (2019)

RAPIDITY DISTRIBUTIONS



- Extend these methods to include rapidity of final state particle
- QCD-induced diphoton production, Drell-Yan process
- Fixed order and resummed
- Challenging to push beyond

LL at NLP

$$\mathscr{L}_n(x) \equiv \log^n(x)$$

$$\mathscr{D}_n(x) \equiv \left.\frac{\log(x)}{x}\right|_+$$

$$\bar{x} = 1 - x$$

$$\begin{aligned}\Delta^{(2)}(z,y)\big|^{\rm NLP} = \left(\frac{\alpha_s}{4\pi}\right)^2 \Big\{ C_F^2 \Big[\big[\delta(y) + \delta(1-y)\big] \Big[-\frac{32\mathcal{L}_1(\bar{z}) + 16}{\epsilon^2} \\ + \frac{96\mathcal{L}_2(\bar{z}) + 40\mathcal{L}_1(\bar{z}) - 46 + 16\zeta_2}{\epsilon} - \frac{448}{3}\mathcal{L}_3(\bar{z}) - 52\mathcal{L}_2(\bar{z}) + \Big(94 + 64\zeta_2\Big)\mathcal{L}_1(\bar{z}) \\ - 64 + 40\zeta_2 - 32\zeta_3 \Big] + \mathcal{D}_0(y)\left(\frac{16}{\epsilon} + 20 - 16\mathcal{L}_1(\bar{y}) - 48\mathcal{L}_1(\bar{z})\right) + \mathcal{D}_0(\bar{y})\left(\frac{16}{\epsilon} + 20\right. \\ \left. -16\mathcal{L}_1(y) - 48\mathcal{L}_1(\bar{z})\right) - 32\Big(\mathcal{D}_1(y) + \mathcal{D}_1(\bar{y})\Big) + \mathcal{D}_0(y)\mathcal{D}_0(\bar{y})\Big[-\frac{16}{\epsilon^2} + \frac{96\mathcal{L}_1(\bar{z}) + 8}{\epsilon} \\ - 224\mathcal{L}_2(\bar{z}) - 16\mathcal{L}_1(\bar{z}) + 32 + 32\zeta_2 \Big] + \Big[\mathcal{D}_0(y)\mathcal{D}_1(\bar{y}) + \mathcal{D}_1(y)\mathcal{D}_0(\bar{y})\Big]\Big[\frac{48}{\epsilon} - 224\mathcal{L}_1(\bar{z}) - 8\Big] \\ - 112\mathcal{D}_1(y)\mathcal{D}_1(\bar{y}) - 56\Big[\mathcal{D}_0(y)\mathcal{D}_2(\bar{y}) + \mathcal{D}_2(y)\mathcal{D}_0(\bar{y})\Big]\Big]\end{aligned}$$

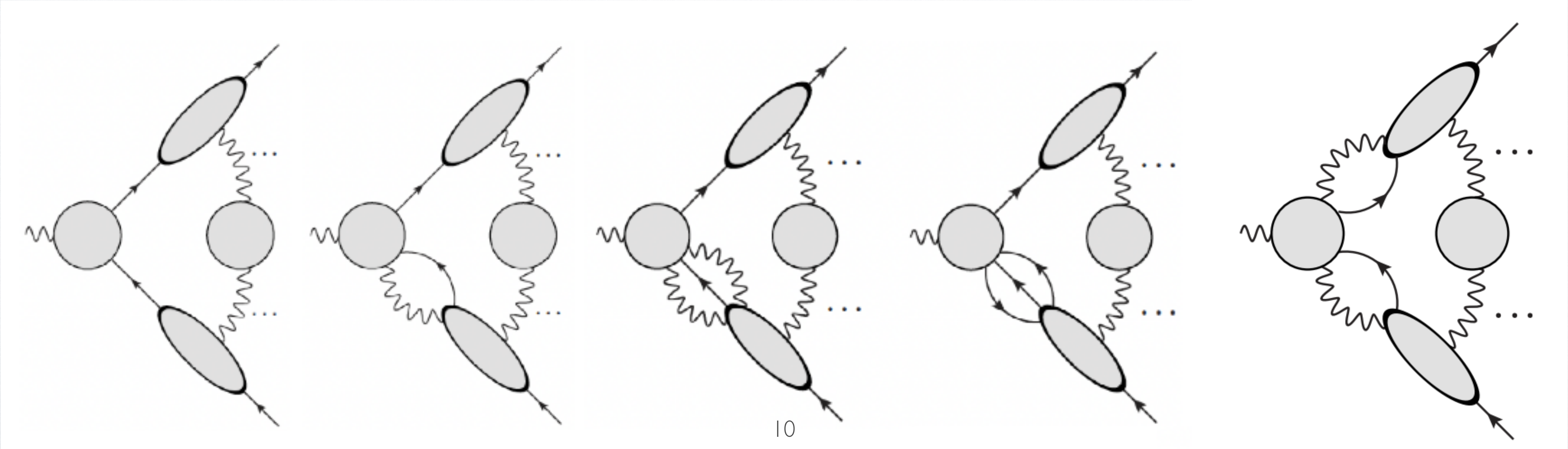
SO WHAT DO WE DO NOW?

FACTORISATION BEYOND LEADING POWER

- Reduced diagrams up to $\mathcal{O}(\lambda^2)$ in power counting,

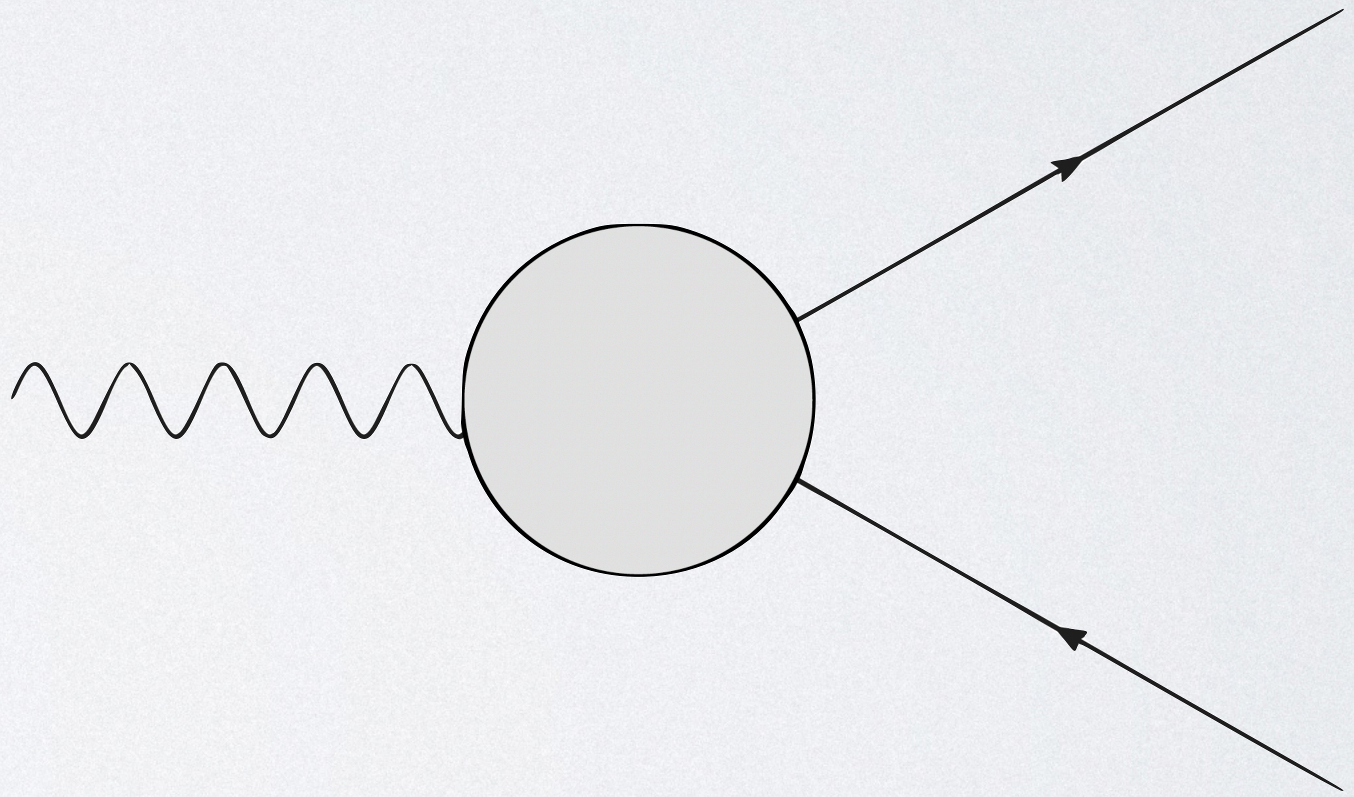
$$\mathcal{M} \sim \left(\prod_{i=1}^{n} J^i_{(f)}\right) \otimes H_{(f)} S + \sum_{i=1}^{n} \left(\prod_{j\neq i} J^{(j)}_{(f)}\right) J^i_{(f\gamma)} \otimes H^i_{(f\gamma)} S + (f\gamma\gamma) + (fff) + (f\gamma)(f\gamma) + \mathcal{O}(\lambda^3)$$

Gervais (2017)
Laenen, Sinninghe Damsté, Vernazza, Waalewijn, Zoppi (2020)



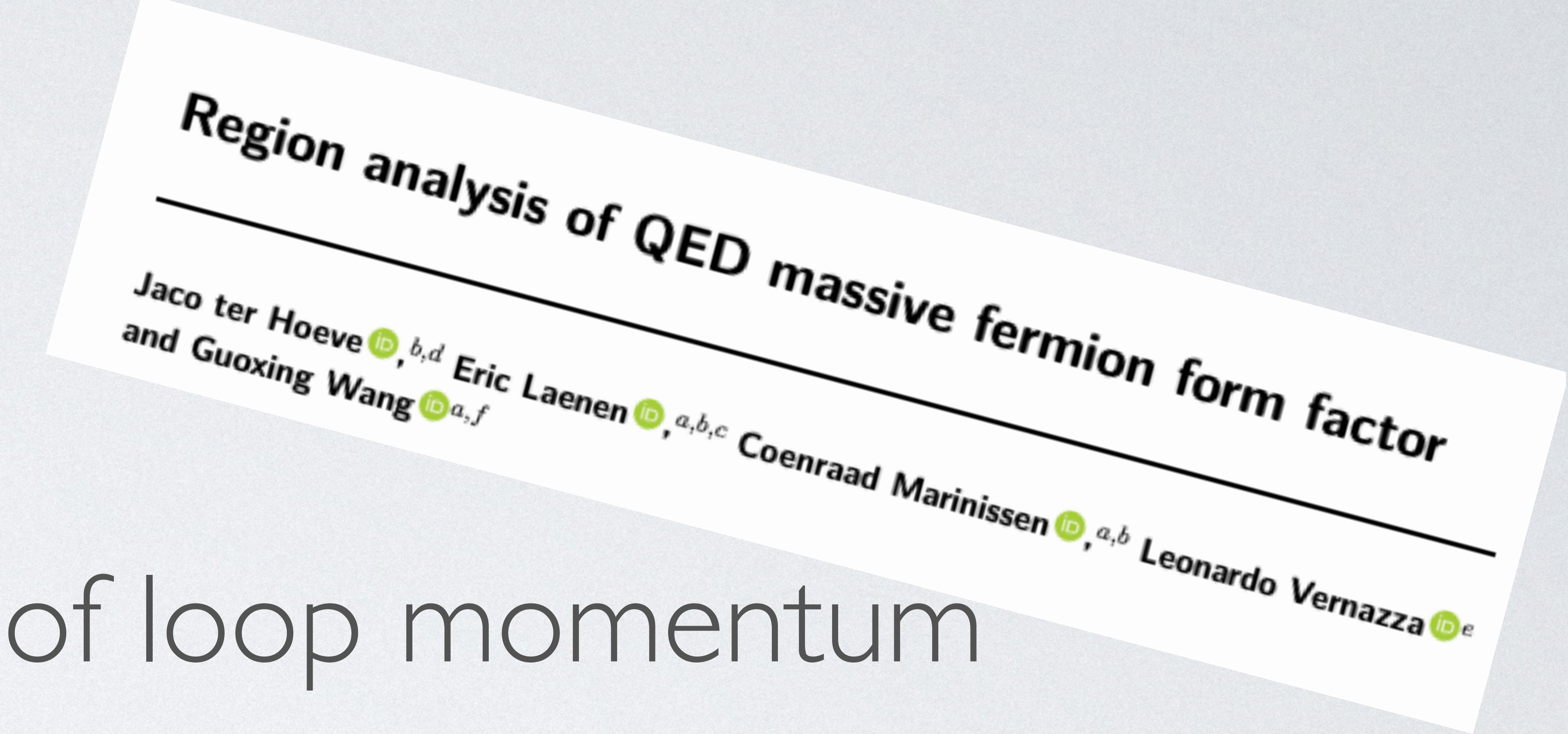
SETUP

- Nonzero, but small quark mass: $m \sim \lambda Q$
- Consider bare massive QED form factor up to two virtual loops
- Define and calculate jet functions
- Check with region analysis



Bonciani, Mastrolia, Remiddi (2004)
Bernreuther, Bonciani, Gehrmann, Heinesch, Leineweber, Mastrolia, Remiddi (2004)
Ter Hoeve, Laenen, Marinissen, Vernazza, Wang (2024)

METHOD OF REGIONS



Region analysis of QED massive fermion form factor

Jaco ter Hoeve[b,d], Eric Laenen[a,b,c], Coenraad Marinissen[a,b], Leonardo Vernazza[e] and Guoxing Wang[a,f]

- Decompose integral into regions
- Regions are defined by momentum scalings of loop momentum
- $p_c \sim \sqrt{\hat{s}}(1,\lambda^2,\lambda), \quad p_{\bar{c}} \sim \sqrt{\hat{s}}(\lambda^2,1,\lambda), \quad p_h \sim \sqrt{\hat{s}}(1,1,1)$
- $\int \frac{d^dk}{(2\pi)^d}\frac{1}{k^2}\frac{1}{(p_1-k)^2-m^2}\frac{1}{(p_2+k)^2-m^2} \rightarrow \int \frac{d^dk}{(2\pi)^d}\frac{1}{k^2}\frac{1}{(p_1-k)^2-m^2}\frac{1}{2p_2^- \cdot k}$
- Add all regions for full result

Beneke, Smirnov (1997)
Jantzen (2011)

LEADING POWER EXAMPLE

- Leading power is well-known

$$\mathscr{M}_{\text{LP}} = \left(\prod_{i=1}^{n} J^i_{(f)} \right) \otimes H_{(f)} S$$

$$J^i_{(f)}(p_i) = \langle p_i | \bar{\psi}(0) W_{n_{i+}}(0,\infty) | 0 \rangle$$

- E.g. one loop result:

$$J^{(1)}_{(f)}(p_1) H^{(0)\mu}_{(f)} \bar{J}^{(0)}_{(f)}(p_2) + J^{(0)}_{(f)}(p_1) H^{(1)\mu}_{(f)} \bar{J}^{(0)}_{(f)}(p_2) + J^{(0)}_{(f)}(p_1) H^{(0)\mu}_{(f)} \bar{J}^{(1)}_{(f)}(p_2)$$

- Check with region method

- NB: more regions than jet function contributions (e.g. ultracollinear)

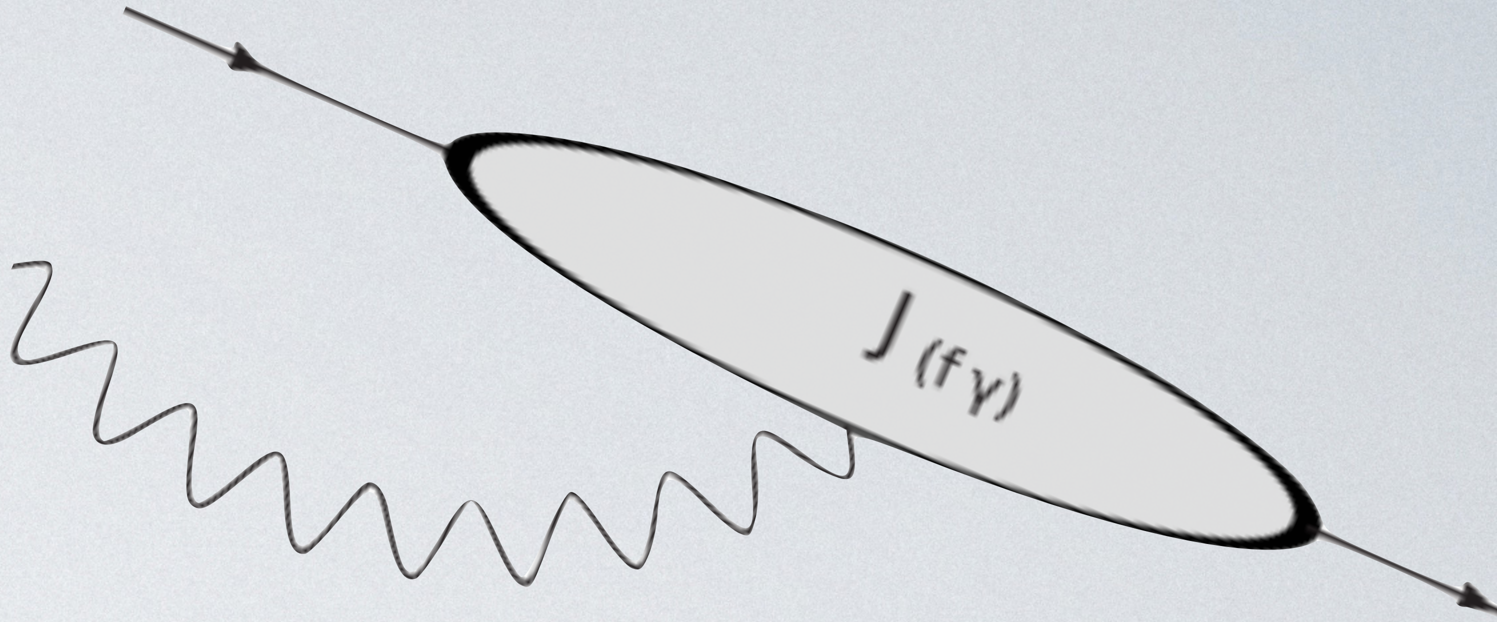
SUBLEADING PARTS

- Leading power jet contains essential subleading parts

- Two loop expressions

$$
\begin{aligned}
J^{(2)}_{(f)}(p_1) = {} & \left(\frac{\alpha_{\mathrm{EM}}}{4\pi}\right)^2 \left(\frac{\bar{\mu}^2}{m^2}\right)^{2\epsilon} \left(\frac{\tilde{\mu}^2}{\hat{s}}\right)^{\nu} \bar{u}(p_1) \\
& \times \left[\frac{1}{2\epsilon^4} + \frac{2}{3\epsilon^3} + \frac{1}{\epsilon^2}\left(\frac{4}{3\nu} + \frac{32}{3} + \frac{\zeta_2}{2}\right) + \frac{1}{\epsilon}\left(-\frac{20}{9\nu} + \frac{52}{9} - \frac{11\zeta_2}{3} + \frac{17\zeta_3}{3}\right) \right. \\
& \quad + \frac{1}{\nu}\left(\frac{112}{27} + \frac{4\zeta_2}{3}\right) + \frac{3040}{81} + \frac{218\zeta_2}{3} - 72\log(2)\zeta_2 - \frac{161\zeta_2^2}{20} + \frac{89\zeta_3}{9} \\
& \quad + n_f\left(-\frac{1}{3\epsilon^3} - \frac{17}{9\epsilon^2} - \frac{1}{\epsilon}\left(\frac{196}{27} + \frac{5\zeta_2}{3}\right) - \frac{2012}{81} - \frac{85\zeta_2}{9} - \frac{22\zeta_3}{9}\right) + \\
& \quad + \not{p}_2 \frac{m}{\hat{s}} \left\{ \frac{1}{\epsilon^3} - \frac{14}{3\epsilon^2} + \frac{1}{\epsilon}\left(\frac{20}{9} - 5\zeta_2\right) + \frac{796}{27} - 54\zeta_2 - \frac{92\zeta_3}{3} + 48\zeta_2\log(2) \right. \\
& \quad \left. \left. + n_f\left(\frac{4}{3\epsilon^2} + \frac{56}{9\epsilon} + \frac{616}{27} + \frac{20\zeta_2}{3}\right) \right\} + \mathcal{O}(\epsilon) \right],
\end{aligned}
$$

SUBLEADING JETS



- Use SCET building blocks for subleading operators and adapt

$$J^{\mu}_{(f\gamma)} \sim \int \frac{d\xi}{2\pi} e^{-i\ell(\xi n_+)} \langle p \,|\, \underbrace{\bar{\psi}(0) W(0,\infty)}_{\text{quark}} \left[W(\infty, \xi n_+) \underbrace{D^{\mu} W(\xi n_+, \infty)}_{\text{gauge boson}} \right] |\, 0 \rangle$$

- Momentum fraction $x = \dfrac{n_+ \ell}{n_+ p}$, hence convolution with hard function

$$J^{(1)\mu}_{(f\gamma)}(p_1, x) = \frac{e^2}{16\pi^2} \left(\frac{m^2}{4\pi\mu^2} \right)^{-\epsilon} \Gamma(\epsilon) \bar{u}(p_1) \Bigg[m x^{1-2\epsilon} \left(-\gamma^{\mu} + \frac{2\not{p}_2^- p_1^{\mu}}{\hat{s}} \right) + \frac{m^2}{\hat{s}} \left[\frac{1}{1-\epsilon} \left(\delta(1-x) - (1-2\epsilon) x^{1-2\epsilon} \right) \gamma^{\mu} \not{p}_2^- - 4 x^{-2\epsilon} (1-x) p_2^{-\mu} \right] \Bigg]$$

Becher, Broggio, Ferroglia (2014)
Beneke, Garny, Szafron, Wang (2017)
Laenen, Sinninghe Damsté, Vernazza, Waalewijn, Zoppi (2020)

CHECKS WITH REGIONS

- One loop can be reproduced up to $\mathcal{O}(\lambda^2)$
- Two loop result is already reproduced for all but one region up to $\mathcal{O}(\lambda^2)$
- Calculate form factor directly, hence not yet on jet function level
- Momentum rerouting in region approach is mimicked by different jet functions

CONCLUSIONS

- We resummed threshold logarithms including rapidity at NLP for LL
- Beyond NLP LL, more rigorous approach is needed
- Factorisation of matrix element beyond leading power
- We defined new jet functions and compare to region approach

WHAT IS NEXT?

- Finalise last region
- Calculate jet functions with momentum fraction dependence
- Use these results towards threshold resummation at next-to-leading power

WHAT IS NEXT?

- Finalise last region
- Calculate jet functions with momentum fraction dependence
- Use these results towards threshold resummation at next-to-leading power

THANK YOU!

BACK UP SLIDES

APPROXIMATE NLO CROSS SECTION

$$\delta p_1^\alpha = -\frac{1}{2}\left(\frac{p_2 \cdot k}{p_1 \cdot p_2} p_1^\alpha - \frac{p_1 \cdot k}{p_1 \cdot p_2} p_2^\alpha + k^\alpha\right), \quad \delta p_2^\alpha = -\frac{1}{2}\left(\frac{p_1 \cdot k}{p_1 \cdot p_2} p_2^\alpha - \frac{p_2 \cdot k}{p_1 \cdot p_2} p_1^\alpha + k^\alpha\right)$$

$$\left|\mathcal{M}_{\mathrm{NLP}}^{(q\bar{q})}\right|^2 = \frac{2 g_s^2 C_F p_1 \cdot p_2}{(p_1 \cdot k)(p_2 \cdot k)} \left|\mathcal{M}_{\mathrm{LO}}(p_1 + \delta p_1, p_2 + \delta p_2)\right|^2$$

$$K_{\mathrm{NLP}}(z, \alpha_s, \mu^2, \hat{s}, \epsilon) = \frac{\alpha_s}{\pi}\left(\frac{4\pi\mu^2}{\hat{s}}\right)^\epsilon z(1-z)^{-1-2\epsilon} \frac{\Gamma^2(-\epsilon)}{\Gamma(-2\epsilon)\Gamma(1-\epsilon)}$$

Del Duca, Laenen, Magnea, Vernazza, White (2017)

DIPHOTON

$$\overline{\left|\mathcal{M}\right|^2_{\text{LO}}} = \frac{2(ee_q)^4}{N_c}\left[\frac{1+\cos^2\theta}{1-\cos^2\theta}(1-\epsilon)^2 - \epsilon(1-\epsilon)\right]$$

$$\frac{d\hat{\sigma}^{(0)}_{q\bar{q}}}{dzd\eta}(\hat{s}, z, \eta) = \frac{\pi\alpha^2_{\text{EM}}e_q^4}{N_c\,\hat{s}}\,(1+\tanh^2\eta)\,\delta(1-z)$$

$$\frac{d\sigma}{dQ^2dY} = \frac{1}{s}\int_\tau^1 \frac{dz}{z}\int_{\log\left(\sqrt{\frac{\tau}{z}}e^Y\right)}^{\log\left(\sqrt{\frac{z}{\tau}}e^Y\right)} d\eta\,\mathcal{L}(z,\eta)\frac{d\hat{\sigma}}{dzd\eta}(\hat{s}, z, \eta)$$

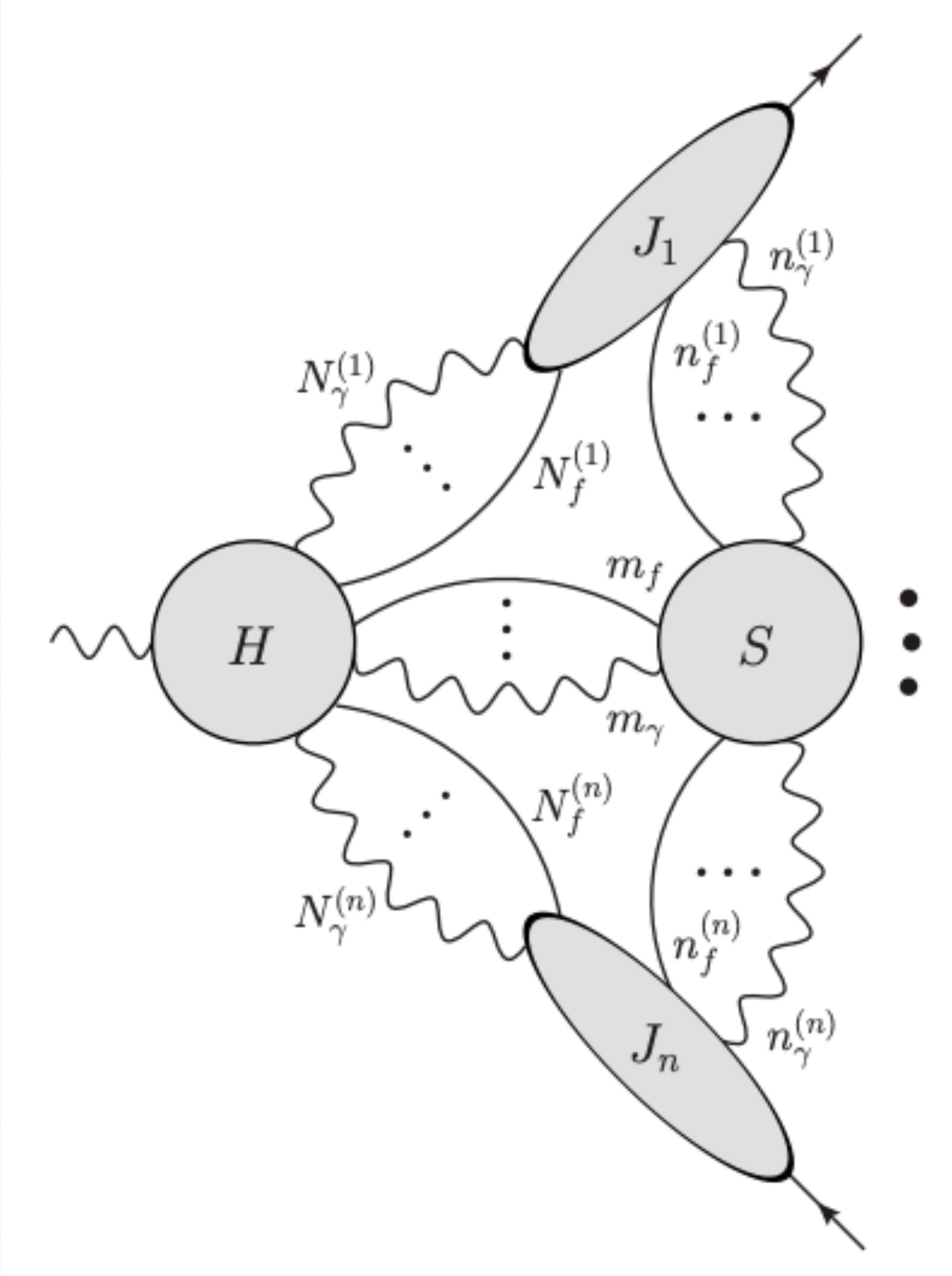
NLP PHASE SPACE

$$\hat{\sigma} = \frac{1}{2\hat{s}}\left[\int d\Phi_{\mathrm{LP}}\,|\mathcal{M}|^2_{\mathrm{LP}} + \int d\Phi_{\mathrm{LP}}\,|\mathcal{M}|^2_{\mathrm{NLP}} + \int d\Phi_{\mathrm{NLP}}\,|\mathcal{M}|^2_{\mathrm{LP}} + \mathcal{O}(\mathrm{NNLP})\right]$$

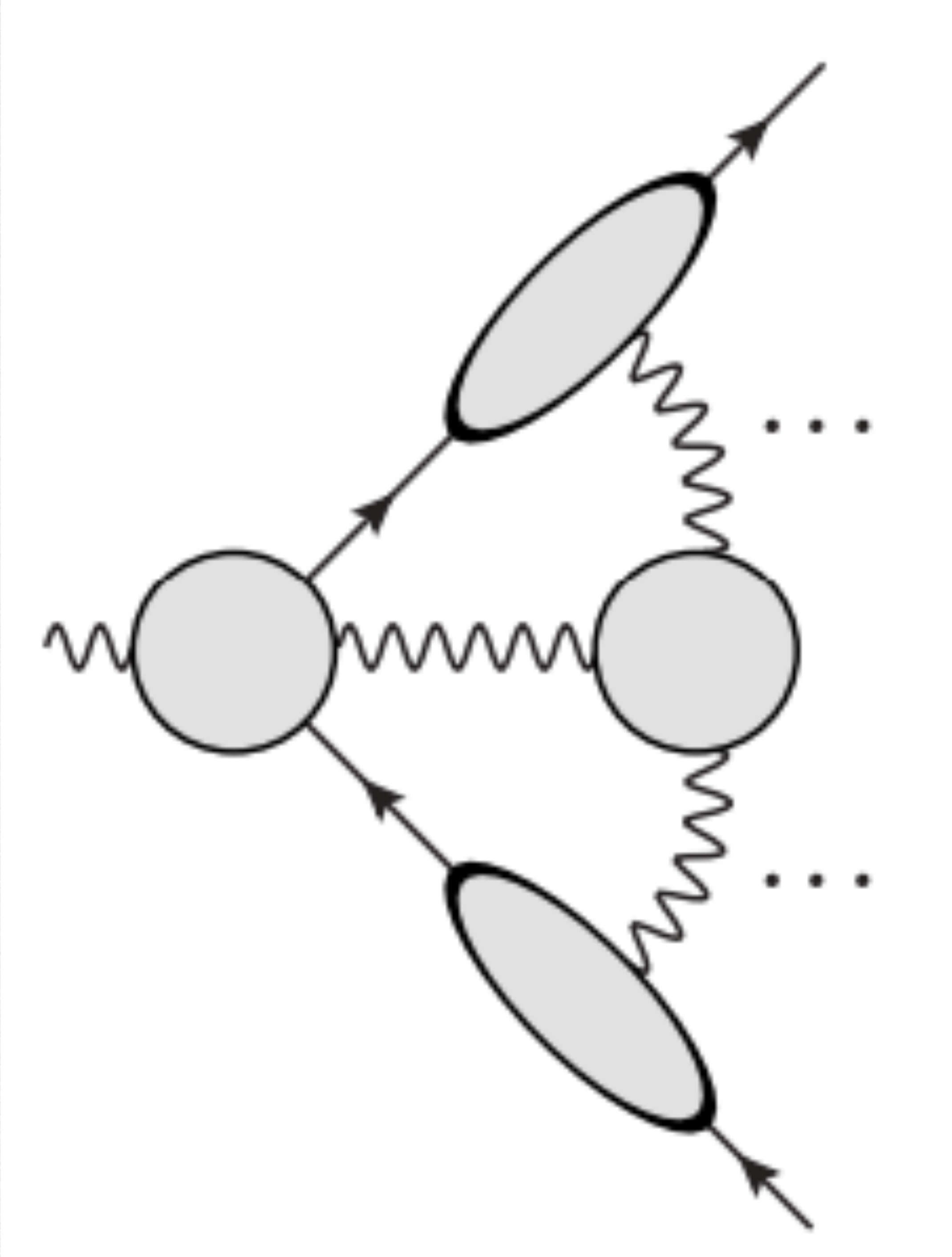
$$|\mathcal{M}|^2_{\mathrm{LP},n} = f(\alpha_s, \epsilon, \mu^2, \eta)\prod_{i=1}^{n}\frac{p_1\cdot p_2}{p_1\cdot k_i p_2\cdot k_i}$$

$$\delta(Q^2-(p_3+p_4)^2) = \frac{1}{\hat{s}}\delta\left(1-z-\frac{2\sum_i k_i\cdot(p_1+p_2)}{\hat{s}}+\frac{2\sum_{i<j}k_i\cdot k_j}{\hat{s}}\right)$$

REDUCED DIAGRAMS



General structure

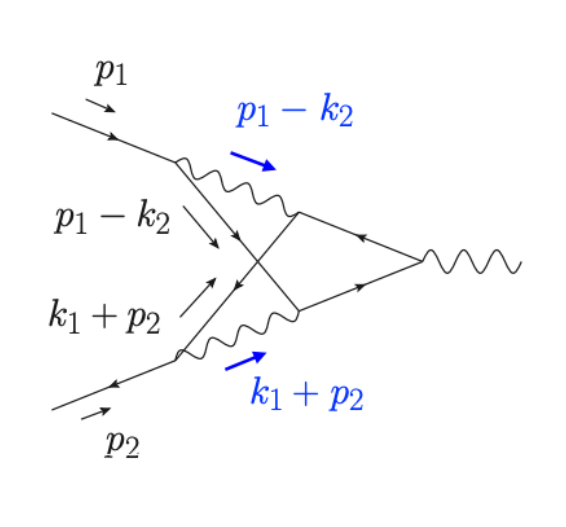


Another reduced diagram, which could contribute, but does not

Laenen, Sinninghe Damsté, Vernazza, Waalewijn, Zoppi (2020)

MOMENTUM REROUTING

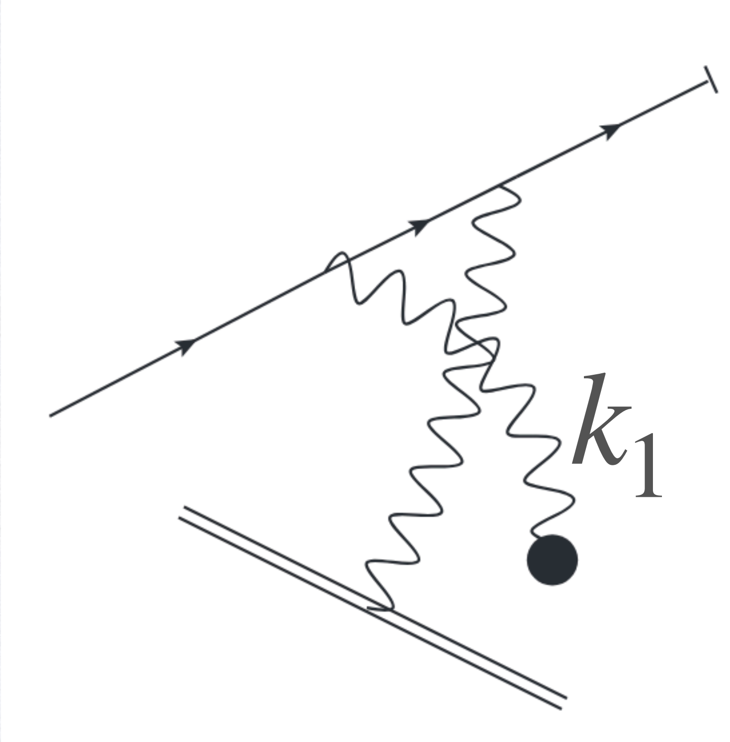
- Region analysis paper:

topology X		hh	cc	$\bar{c}\bar{c}$	$\bar{c}c$	hc	$\bar{c}h$	$\bar{c}uc$	$\overline{uc}c$	cc'	$\bar{c}\bar{c}'$
(h)		✓	ν_2	ν_1	ν_1, ν_2	✓	✓	✓	✓	ν_1, ν_2	ν_1, ν_2

Ter Hoeve, Laenen, Marinissen, Vernazza, Wang (2024)

- Jet function approach:

$(f\gamma)$ jet



(fff) jet