Electroweak logarithms in OpenLoops

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Introduction

- In the energy range above the **EW** scale ($\sqrt{s} \gg M_W$), *Sudakov logs* represent the leading contribution of **EW** radiative corrections

- *Sudakov logarithms* from **N^{n}LO** **EW** corrections

$$\alpha^n \log^k \frac{|r_{kl}|}{m_i^2}, \qquad 1 \leq k \leq 2n \qquad\qquad |r_{kl}| = |(p_k + p_l)^2| \sim s$$

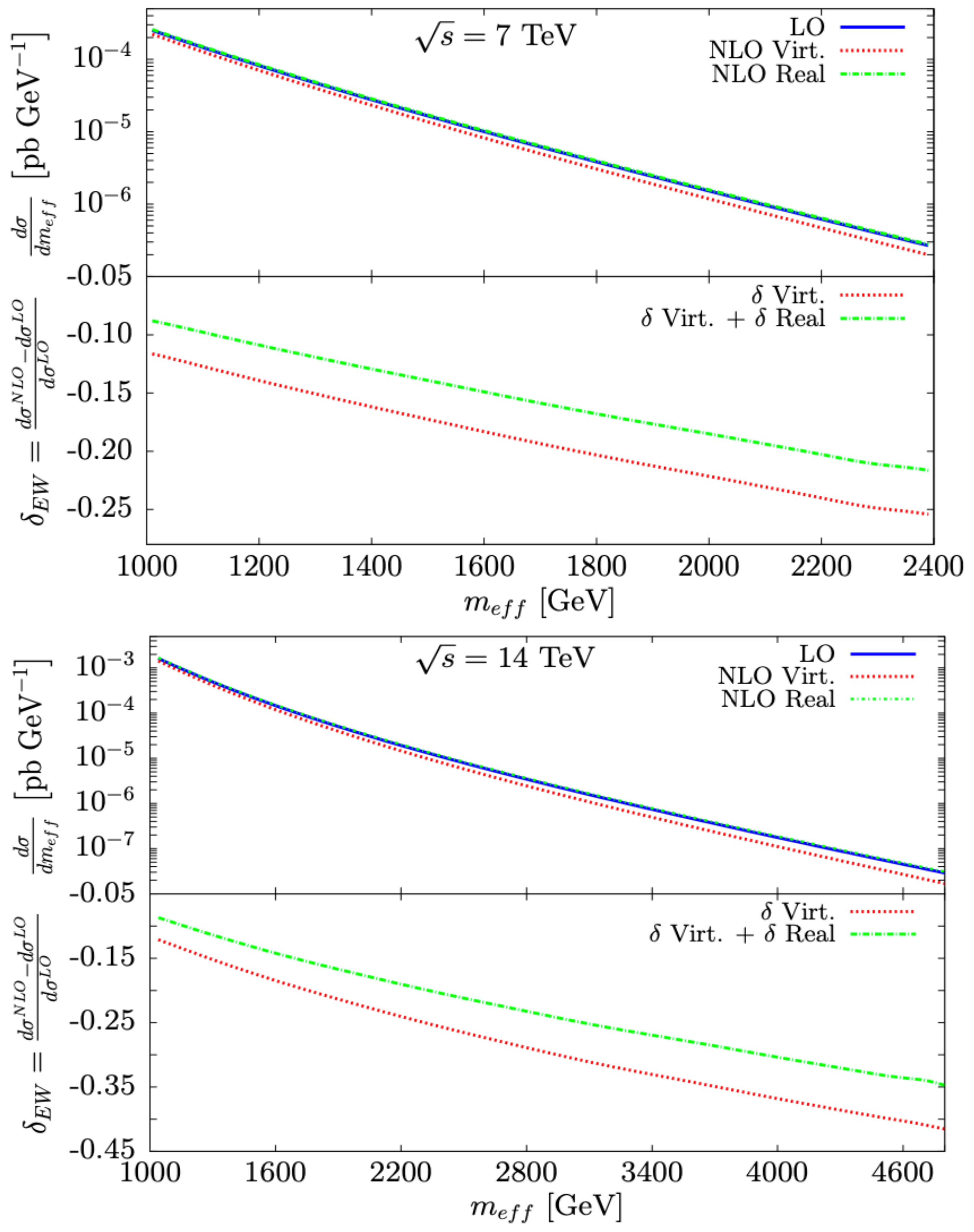
- At **NLO**

Double logs (DL): $$L(|r_{kl}|, m_i^2) = \frac{\alpha}{4\pi} \log^2 \frac{|r_{kl}|}{m_i^2}$$

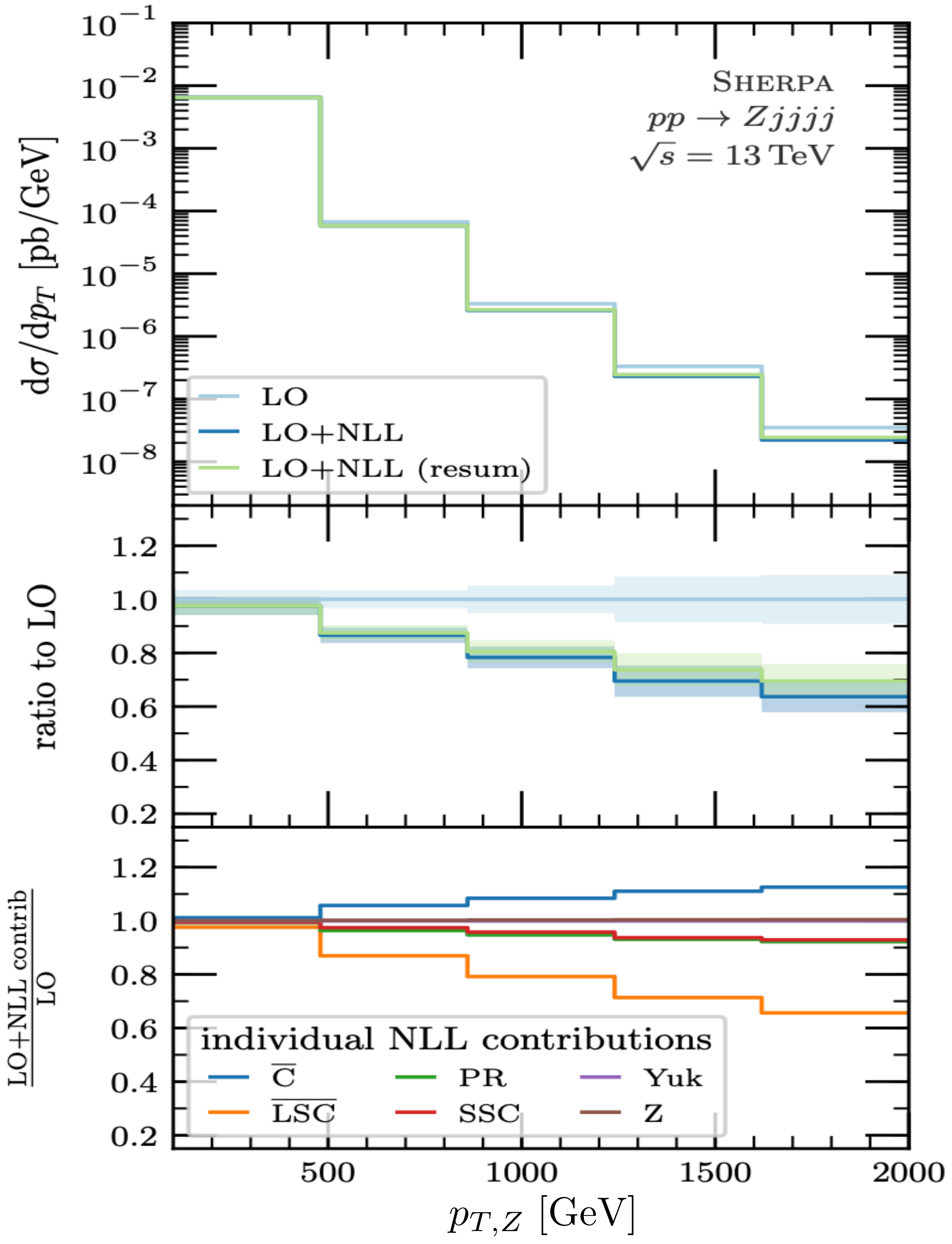
Single logs (SL): $$l(|r_{kl}|, m_i^2) = \frac{\alpha}{4\pi} \log \frac{|r_{kl}|}{m_i^2}$$

Introduction

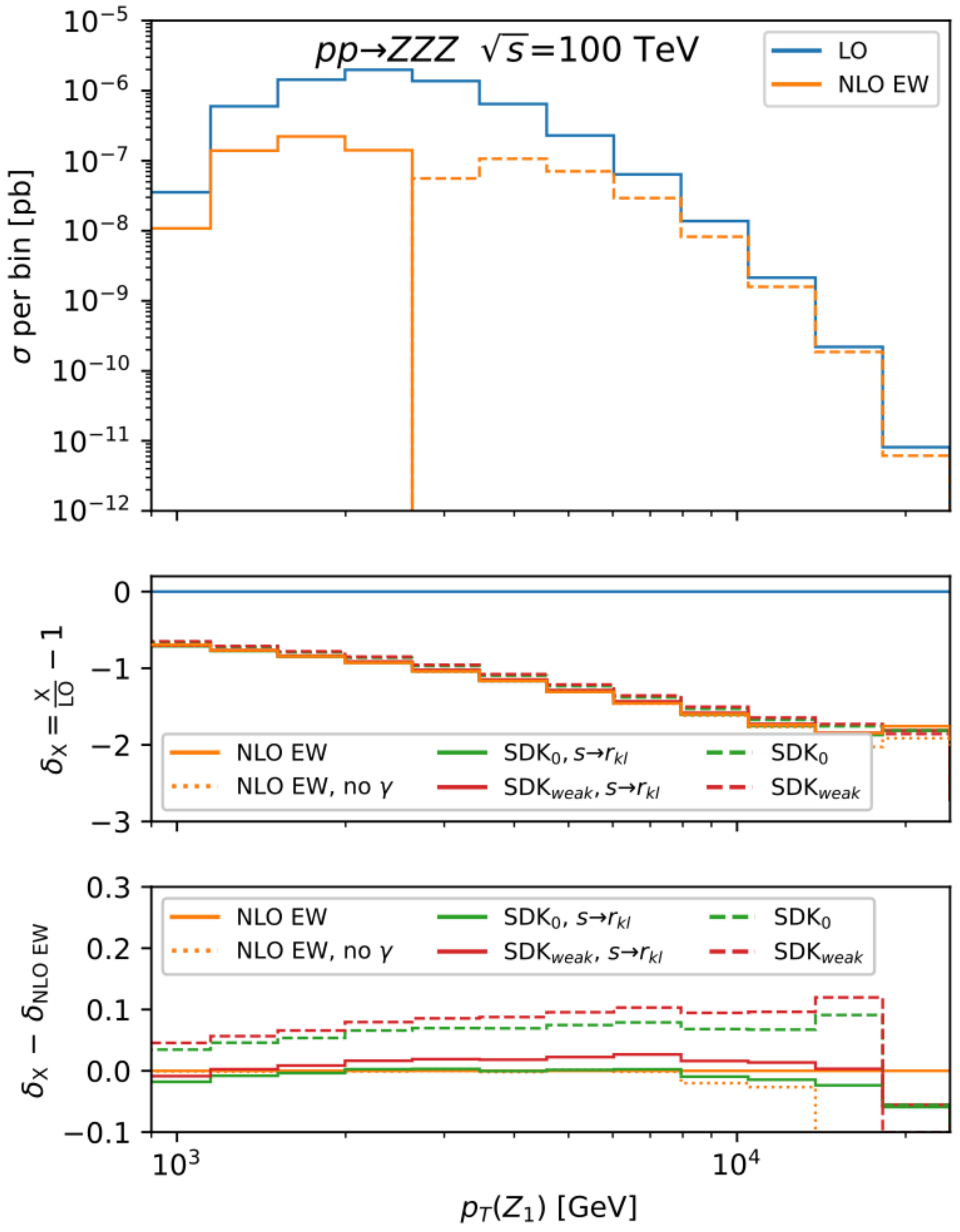
- Without clear signs of NP as resonances, small deviations in tails of kinematic distributions are under scrutiny
- **NLO EW** corrections and their *Sudakov approximation* are crucial as they can provide several ten % effects in tails



Alpgen: Chiesa *et al.* 1305.6837; *2013*



Sherpa: Bothmann, Napoletano 2006.14635; *2020*



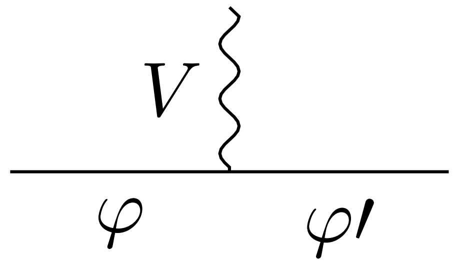
MadGraph: Pagani, Zaro 2110.03714; *2021*

- Still relevant at **2-loop**: $\alpha^2 \log^4\left(s/m_i^2\right) \simeq 3\%$ at $s = 1$ TeV

Framework: notation & conventions

- Convention:

  ⟶ All incoming particles, i.e. $n \to 0$ process $\varphi_{i_1}(p_1)\dots\varphi_{i_n}(p_n) \to 0$

  ⟶ **EW** Feynman rules 

$= ieI^V_{\varphi\varphi'}T$ as in [Denner 0709.1075; *2007*]

- DP algorithm based on logarithmic approximation (LA):

  ⟶ On-shell external momenta $p_k^2 = m_{\varphi_k}^2$ and all r_{kl} much larger than W/Z masses

$$|r_{kl}| = |(p_k + p_l)^2| \approx 2|p_k p_l| \sim s \gg m_W^2\,, \qquad k \neq l$$

  ⟶ Not mass-suppressed Born matrix element, i.e. $\mathcal{M}_0^{\varphi_{i_1}\dots\varphi_{i_n}} \sim E^d$

  ⟶ At one-loop keep only leading and universal DL and SL corrections

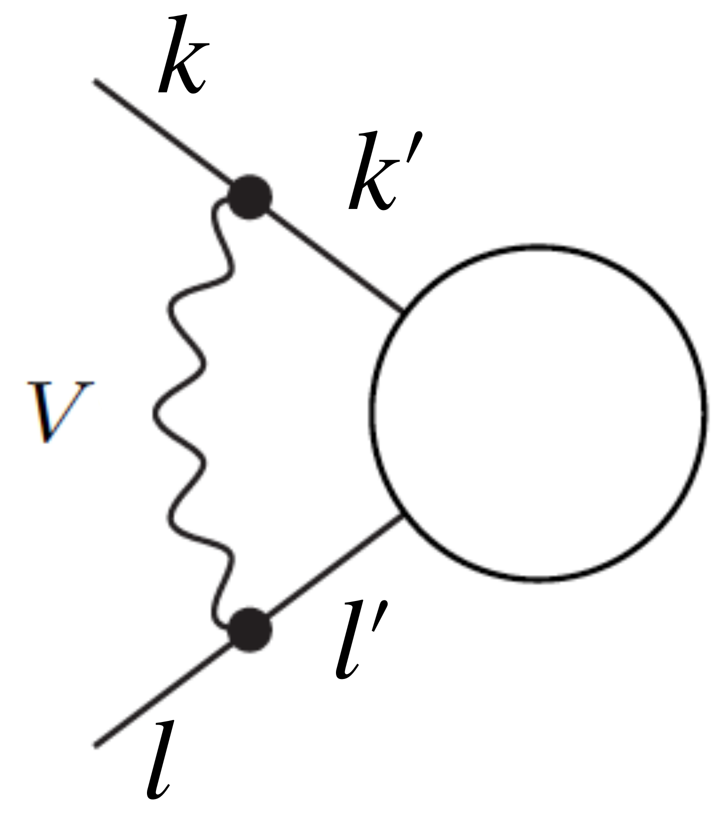
$$\delta^{\mathrm{DL}}\mathcal{M}^{\varphi_{i_1}\dots\varphi_{i_n}} \sim L\,\mathcal{M}_0^{\varphi_{i_1}\dots\varphi_{i_n}} \qquad \delta^{\mathrm{SL}}\mathcal{M}^{\varphi_{i_1}\dots\varphi_{i_n}} \sim l\,\mathcal{M}_0^{\varphi_{i_1}\dots\varphi_{i_n}}$$

neglecting constant $(\sim \alpha E^d)$ and mass suppressed $(\sim m^n E^{d-n} L)$ contributions

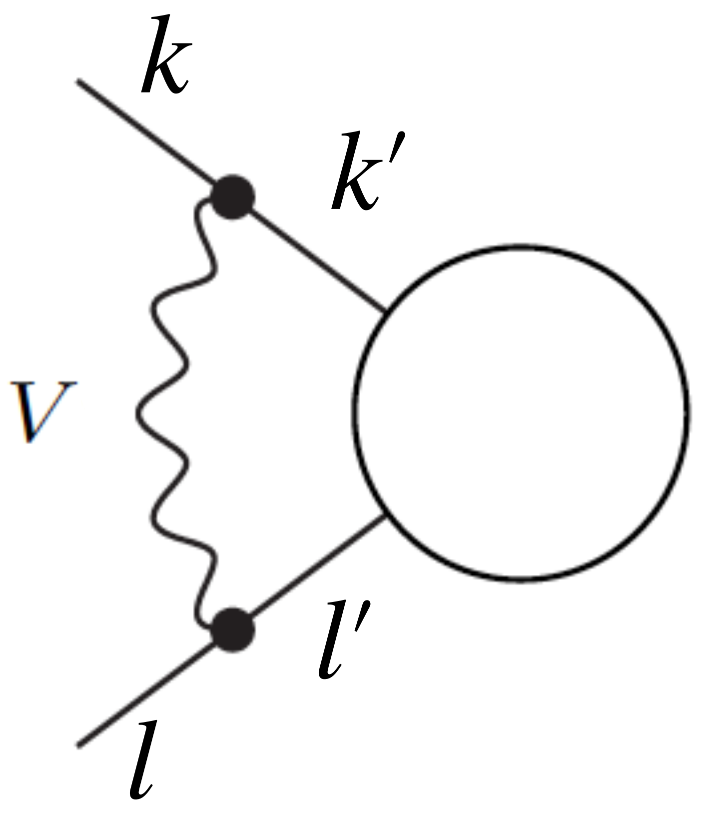
[Denner and Pozzorini 0010201; *2001*]

Double Logs

- DL originate from triangle diagrams where two external legs exchange a ***soft and collinear* (SC)** gauge boson V



Double Logs



- DL originate from triangle diagrams where two external legs exchange a ***soft and collinear* (SC)** gauge boson V

- In the *Eikonal approximation*[1], the loop integral reduces to the scalar three-point function C_0, which ***factorises***

$$\delta^{\mathrm{DL}}\mathcal{M}^{\varphi_{i_1}\cdots\varphi_{i_n}} \overset{\mathrm{LA}}{=} \sum_{k} \sum_{l<k} \sum_{V} \sum_{k',l'} \frac{\alpha}{4\pi} I^{V}_{kk'} I^{\bar{V}}_{ll'} \underbrace{\left[\log^2\frac{|r_{kl}|}{m_V^2} - 2i\pi\Theta(r_{kl})\log\frac{|r_{kl}|}{m_V^2}\right]}_{\propto\, C_0|_{\mathrm{LA}}} \mathcal{M}_0^{\varphi_{i_1}\cdots\varphi_{i'_k}\cdots\varphi_{i'_l}\cdots\varphi_{i_n}}$$

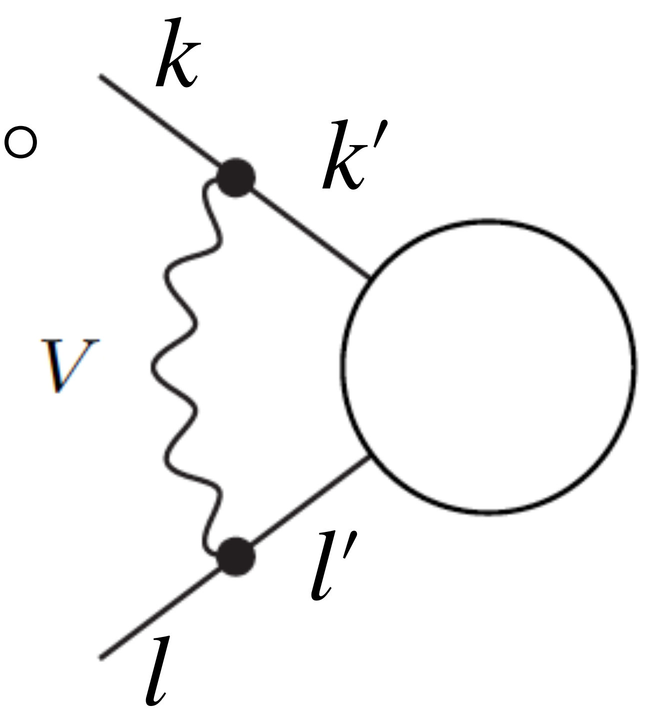
with $r_{kl} = (p_k + p_l)^2$

- Consequence of C_0 ***factorisation***: DL are ***universal***, i.e. process independent

[1] NB: external longitudinal gauge bosons require GBET

Double Logs: LSC, SSC, S-SSC

DL originate when two external legs exchange a ***soft and collinear* (SC)** gauge boson V



- DL can be split into

→ **Leading *Soft-Collinear* (LSC)**: angular independent, single sum over external legs

$$\delta^{\mathrm{LSC}}\mathcal{M}^{\varphi_{i_1}\cdots\varphi_{i_n}} = \sum_k \sum_{k'} \sum_V \delta^{\mathrm{LSC,V}}_{kk'} \mathcal{M}_0^{\varphi_{i_1}\cdots\varphi_{i'_k}\cdots\varphi_{i_n}}, \qquad \delta^{\mathrm{LSC,\ V}}_{kk'} \sim \frac{\alpha}{4\pi} I^V I^{\bar{V}} \boxed{\log^2\left(\frac{s}{m_V^2}\right)}$$

→ **Subleading *Soft-Collinear* (SSC)** and **Sub-SSC (SSSC)**: angular dependent, double sum over external legs

$$\delta^{\mathrm{(S-)SSC}}\mathcal{M}^{\varphi_{i_1}\cdots\varphi_{i_n}} = \sum_k \sum_{l<k} \sum_{k',l'} \sum_V \delta^{\mathrm{(S-)SSC,V}}_{kk'll'} \mathcal{M}_0^{\varphi_{i_1}\cdots\varphi_{i'_k}\cdots\varphi_{i'_l}\cdots\varphi_{i_n}}$$

$$\delta^{\mathrm{SSC,\ V}}_{kk'll'} \sim \frac{\alpha}{4\pi} I^V I^{\bar{V}} \boxed{\log\left(\frac{s}{m_V^2}\right) \log\left(\frac{|r_{kl}|}{s}\right)} \qquad \delta^{\mathrm{S\text{-}SSC,\ V}}_{kk'll'} \sim \frac{\alpha}{4\pi} I^V I^{\bar{V}} \boxed{\log^2\left(\frac{|r_{kl}|}{s}\right)}$$

→ Formally not part of LA and omitted in original DP, but needed for reliable estimates as firstly pointed out in [Pagani, Zaro 2110.03714; *2021*]

$$\boxed{\text{LA:}\quad s \sim r_{kl} \equiv (p_k + p_l)^2 \gg m^2_{\mathrm{Z,W}} \qquad \forall k, l}$$

[Denner and Pozzorini 0010201; *2001*]

Single Logs

- SL have a triple origin

Single Logs: PR

- SL have a triple origin

→ **PR**: UV renormalisation of **EW** dimensionless parameters

See also
C. Del Pio's talk

$$\boxed{g_{i,0} = g_i + \delta g_i = \left(1 + \delta Z_{g_i}\right) g_i}$$

$$\mu_{i,0}^2 = \mu_i^2 + \delta\mu_i^2$$

$$\varphi_{i,0} = \left(1 + \frac{1}{2}\delta Z_{\varphi_i\varphi_j}\right)\varphi_j$$

$$\boxed{g_{i,0} = g_i + \delta g_i = \left(1 + \delta Z_{g_i}\right) g_i}$$

Renormalisation of masses and couplings with mass dimensions brings only mass-suppressed corrections

Single Logs: PR & WFR

- SL have a triple origin

→ **PR**: UV renormalisation of **EW** dimensionless parameters

$$\boxed{g_{i,0} = g_i + \delta g_i = (1 + \delta Z_{g_i})\, g_i}$$

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→ **WF**: wave-function renormalisation of external fields

$$\boxed{\varphi_{i,0} = \left(1 + \frac{1}{2}\delta Z_{\varphi_i \varphi_j}\right) \varphi_j}$$

yields to the ***factorised*** correction

$$\delta^{\mathrm{WF}} \mathcal{M}^{\varphi_{i_1} \dots \varphi_{i_n}} \overset{\mathrm{LA}}{=} \sum_k \sum_{k'} \delta^{\mathrm{WF}}_{kk'} \mathcal{M}_0^{\varphi_{i_1} \dots \varphi_{i'_k} \dots \varphi_{i_n}}, \qquad \delta^{\mathrm{WF}}_{kk'} = \frac{1}{2}\delta Z_{kk'}$$

$$\mu^2_{i,0} = \mu^2_i + \delta\mu^2_i$$

$$\boxed{\varphi_{i,0} = \left(1 + \frac{1}{2}\delta Z_{\varphi_i \varphi_j}\right) \varphi_j}$$

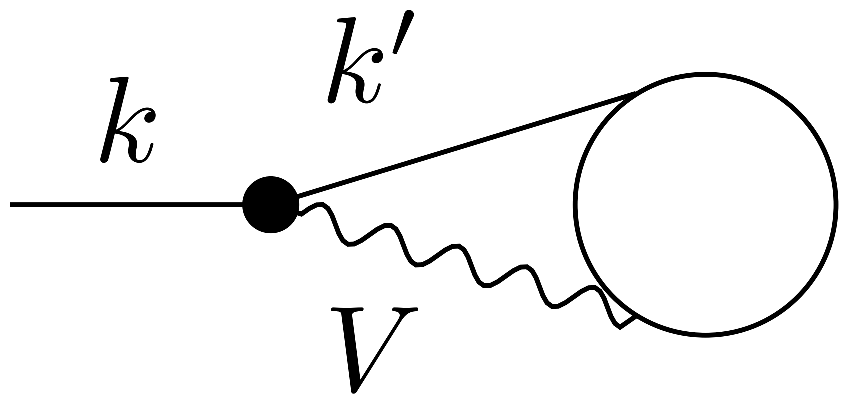
$$\boxed{g_{i,0} = g_i + \delta g_i = (1 + \delta Z_{g_i})\, g_i}$$

Renormalisation of masses and couplings with mass dimensions brings only mass-suppressed corrections

Single Logs: Coll

- SL have a triple origin

→ **Coll**: external leg emission of a collinear gauge boson



Its evaluation in *Collinear approximation* leads to the ***factorised*** contribution

$$\delta^{\mathrm{coll}}\mathcal{M}^{\varphi_{i_1}\cdots\varphi_{i_n}} \overset{\mathrm{LA}}{=} \sum_{k}\sum_{k'}\delta^{\mathrm{coll}}_{kk'}\mathcal{M}_0^{\varphi_{i_1}\cdots\varphi_{i'_k}\cdots\varphi_{i_n}}\,, \qquad \delta^{\mathrm{coll}}_{kk'} \sim \frac{\alpha}{4\pi}\sum_{V} I^{V} I^{\bar{V}} \log\left(\frac{s}{m_V^2}\right)$$

Single Logs: Coll & C

- SL have a triple origin

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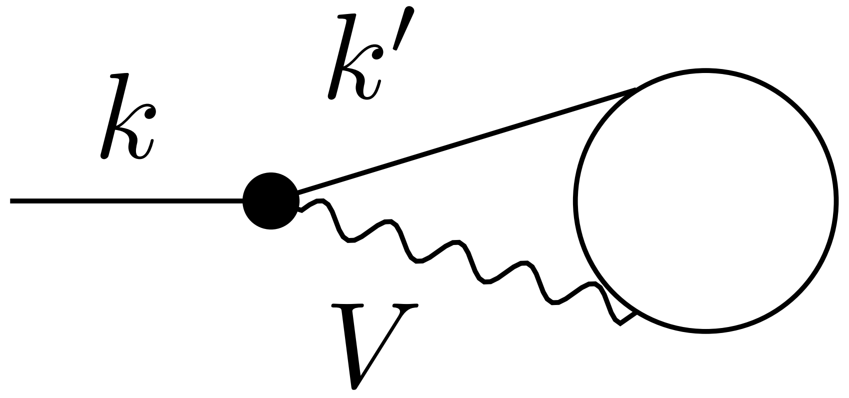

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$$\delta^{\mathrm{coll}}\mathcal{M}^{\varphi_{i_1}\dots\varphi_{i_n}} \overset{\mathrm{LA}}{=} \sum_{k}\sum_{k'} \delta^{\mathrm{coll}}_{kk'}\mathcal{M}_0^{\varphi_{i_1}\dots\varphi_{i'_k}\dots\varphi_{i_n}}\,, \qquad \delta^{\mathrm{coll}}_{kk'} \sim \frac{\alpha}{4\pi}\sum_V I^V I^{\bar{V}} \log\left(\frac{s}{m_V^2}\right)$$

→ **C**: Full gauge-invariant SL correction associated to external fields:

$$\delta^{\mathrm{C}}\mathcal{M}^{\varphi_{i_1}\dots\varphi_{i_n}} \overset{\mathrm{LA}}{=} \sum_{k}\sum_{k'} \delta^{\mathrm{C}}_{kk'}\mathcal{M}_0^{\varphi_{i_1}\dots\varphi_{i'_k}\dots\varphi_{i_n}}\,, \qquad \delta^{\mathrm{C}}_{kk'} = \left(\delta^{\mathrm{coll}}_{kk'} + \delta^{\mathrm{WF}}_{kk'}\right)\Big|_{\mu^2=s}$$

Implementation in OpenLoops

- OpenLoops (OL): automated tool for the calculation of tree and one-loop amplitudes [Buccioni *et al*, 1907.13071*; 2019*]

- Goal: exploit ***factorisation*** of *Sudakov logs* to evaluate **one-loop** **EW** corrections at **NLL** via tree amplitudes → up to two orders of magnitude faster w.r.t. full loop computation

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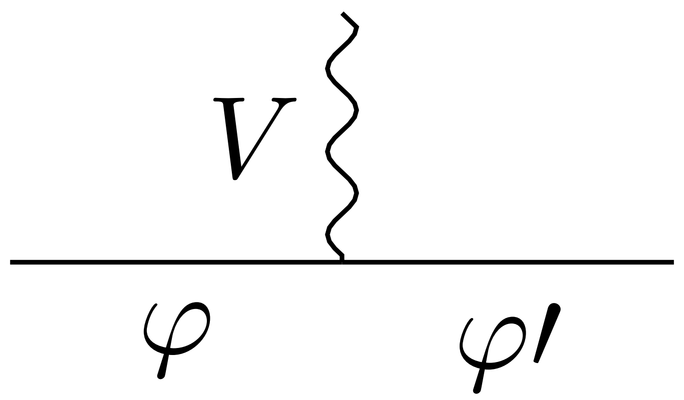
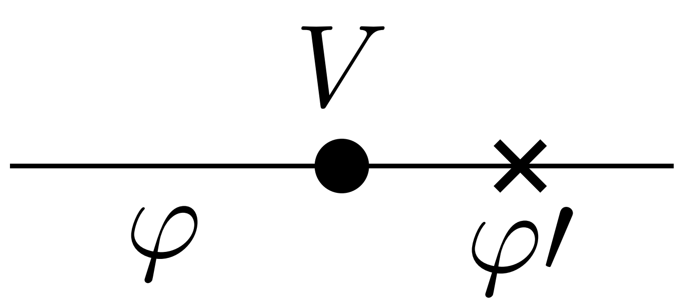
- How? Representation of DP algorithm via *helicity-dependent* two-point (effective) vertex rules



$$= ieI^{V}_{\varphi\varphi\prime}T \quad \longrightarrow$$

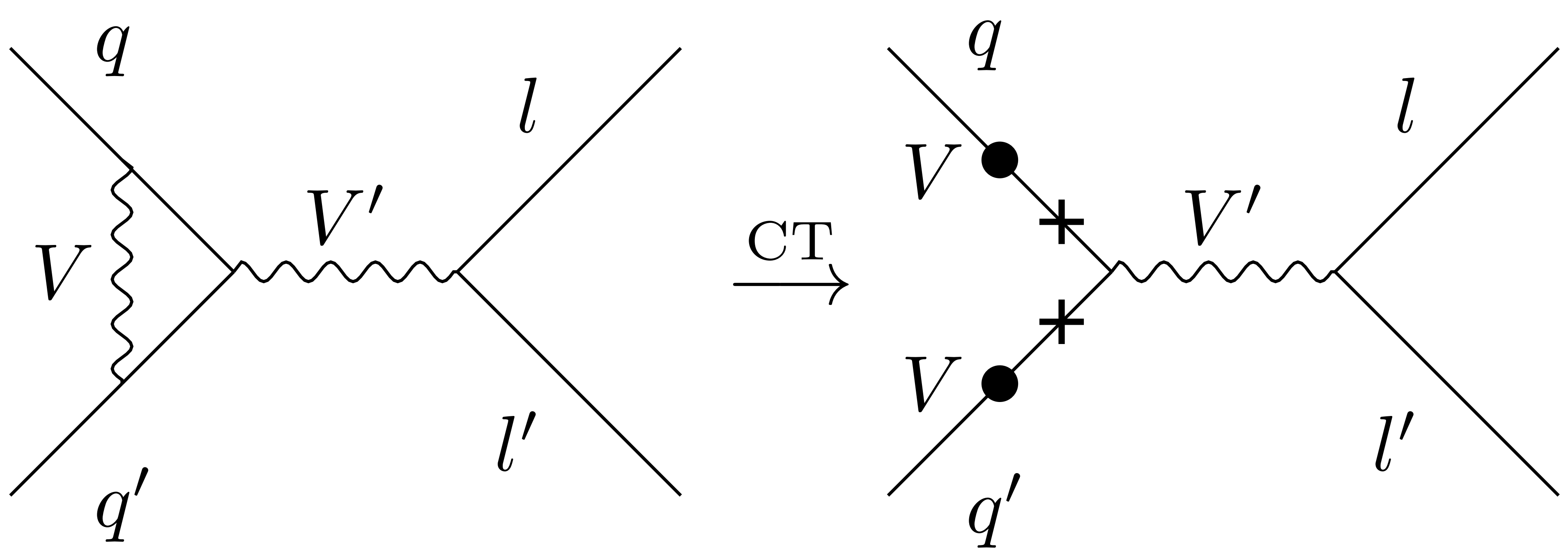


$$= ieI^{V}_{\varphi\varphi\prime}$$

- The virtual soft boson is cut and the internal propagator is removed, while the external particle remains on-shell

Implementation in OpenLoops

- Translation of soft-collinear **NLO** topology into corresponding Born one via double pseudo-CT insertions. E.g.: Drell-Yan



$$\Rightarrow \mathcal{M} \sim e^2 \underbrace{\sum_{V=A,Z,W^\pm} I_q^V \, I_{q'}^V \, K_{\text{ew}}^V \mathcal{M}_0}_{\sim\, \delta^{\text{DL, SL}}}$$

contains the desired logarithmic structure (DL, SL or any combination of them)

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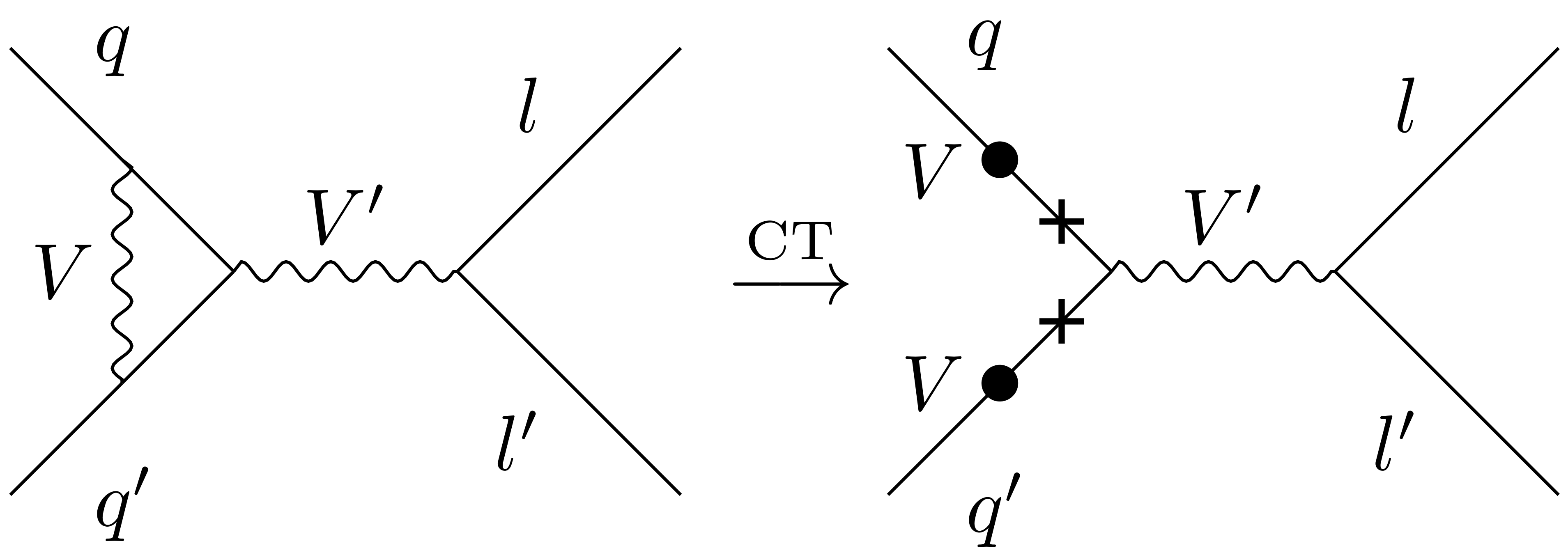

$$\Rightarrow \mathcal{M} \sim e^2 \underbrace{\sum_{V=A,Z,W^\pm} I_q^V\, I_{q'}^V\, K_{\mathrm{ew}}^V\, \mathcal{M}_0}_{\displaystyle \Downarrow \atop \displaystyle \sim \delta^{\,\mathrm{DL,\ SL}}}$$

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- Key question:

Is the gain in speed worth the loss in accuracy?

Results: $pp \to ZZ$

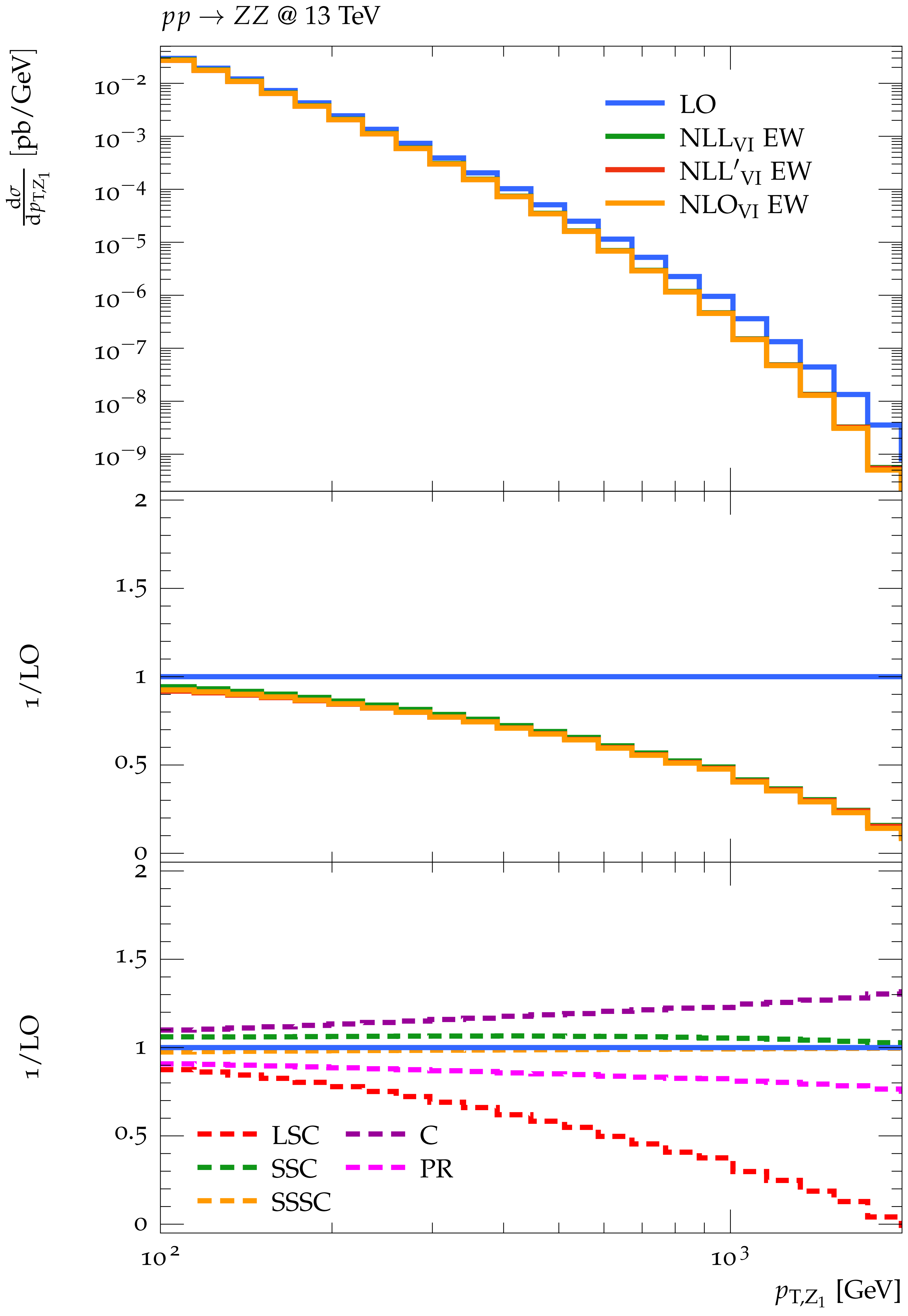
NLL EW: [Accomando *et al,* 0409247; 2004]

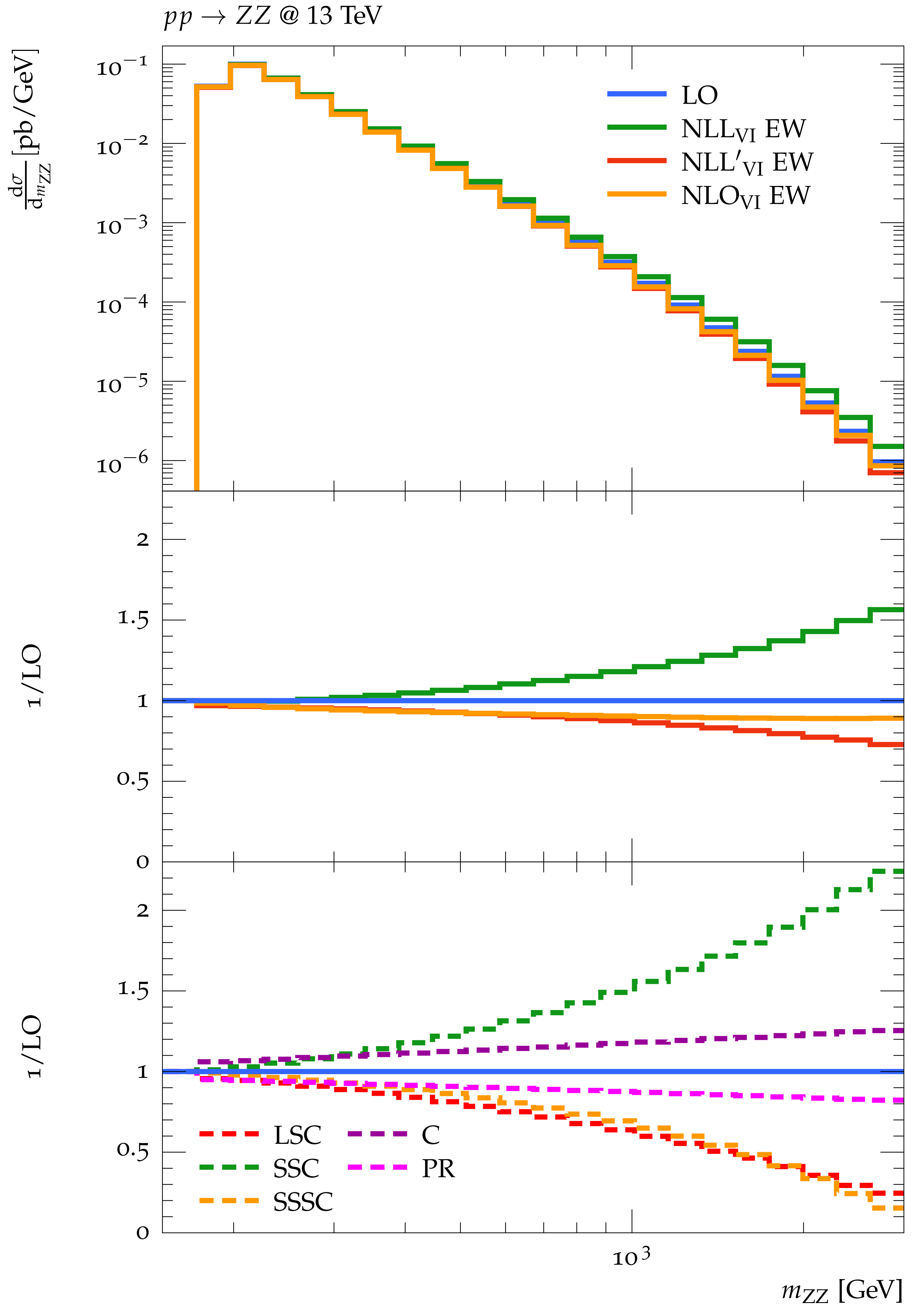
Full **NLO EW**: [Bierweiler *et al,* 1305.5402; 2013]

Full **NLO**: [Baglio *et al,* 1307.4331; 2016]

NNLO QCD+NLO EW: [Grazzini *et al,* 1912.00068; 2020]

NLO EW vs **NLL EW**: [Bothmann *et al,* 2111.13453; 2021]

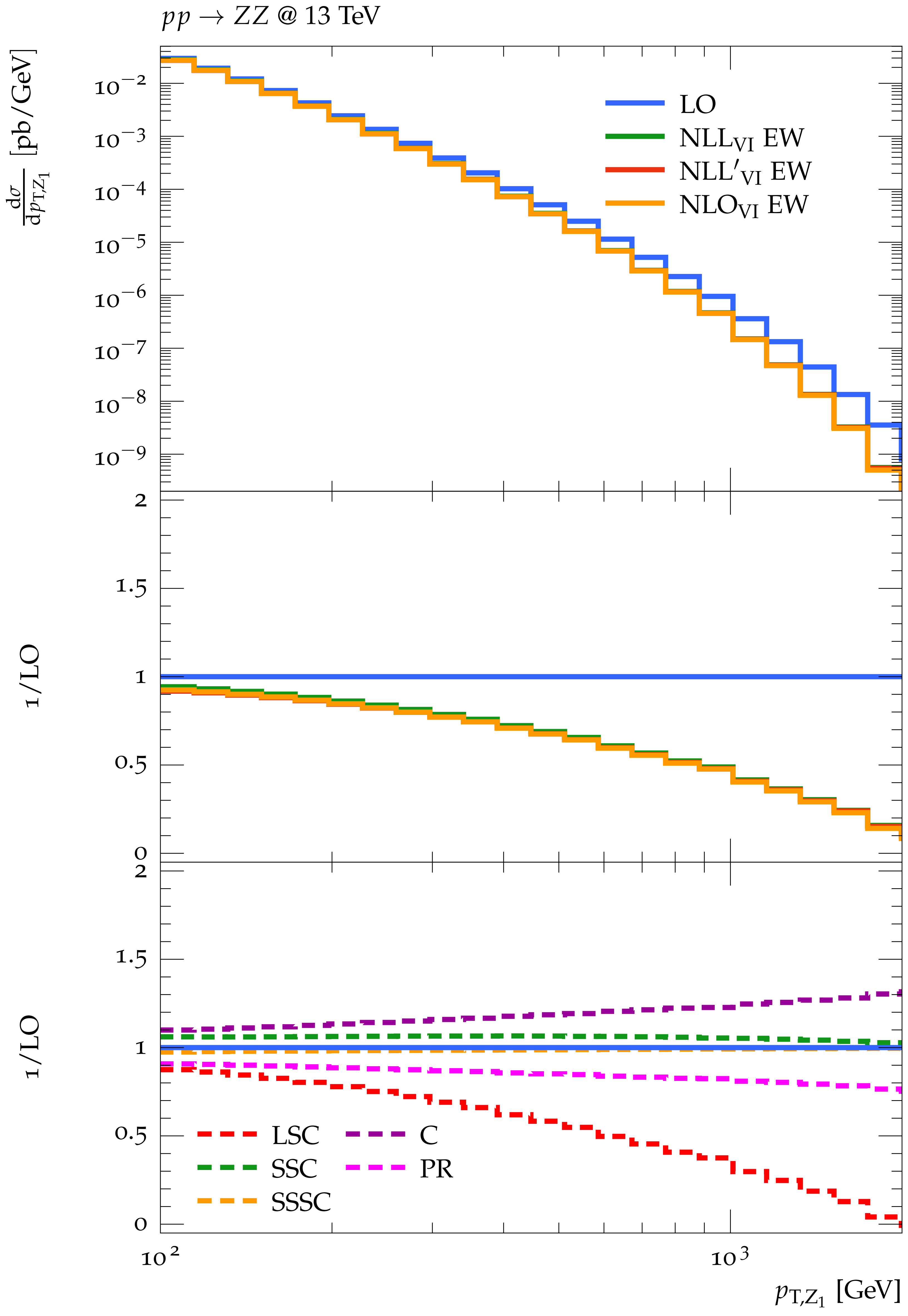


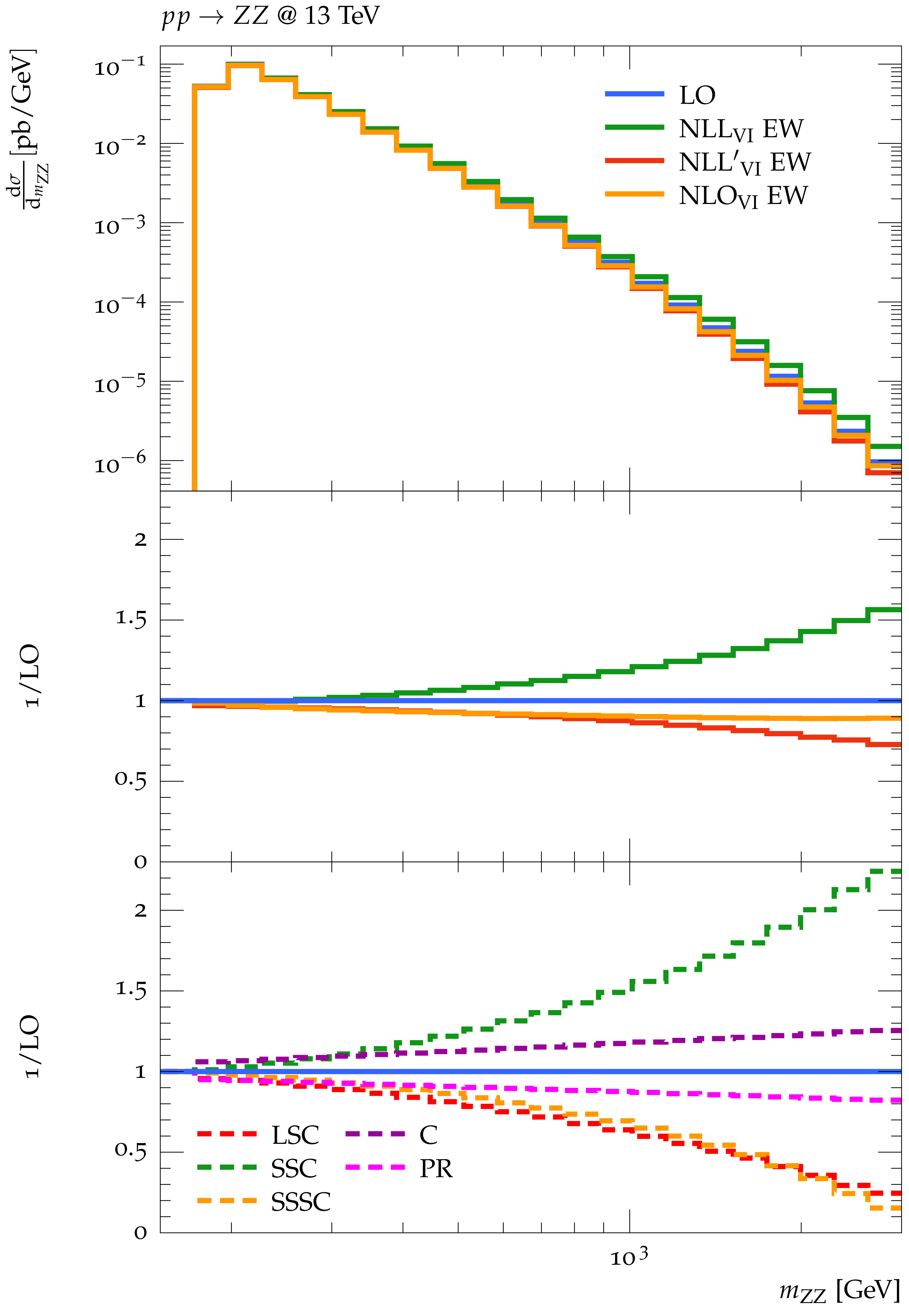


$$\boxed{\mathrm{NLL}_{\mathrm{VI}}\ \mathrm{EW} = (1 + \mathrm{LSC} + \mathrm{SSC} + \mathrm{C} + \mathrm{PR} + \mathbf{I})\mathrm{LO}}$$

$$\boxed{\mathrm{NLL'}_{\mathrm{VI}}\ \mathrm{EW} = (1 + \mathrm{LSC} + \mathrm{SSC} + \mathrm{SSSC} + \mathrm{C} + \mathrm{PR} + \mathbf{I})\mathrm{LO}}$$

Results: $pp \to ZZ$





SSC and **SSSC** become very sizeable for PS regions where LA condition

$$s \sim r_{kl} \equiv (p_k + p_l)^2 \gg m^2_{\mathrm{Z,W}} \quad \forall k, l$$

is violated, with hierarchy among invariants

$$s \sim r_{kl} \equiv (p_k + p_l)^2 \gg r_{k'l'} \equiv (p_{k'} + p_{l'})^2 \gg m^2_{\mathrm{Z,W}}$$

$$\delta^{\mathrm{SSC,\,V}}_{kk'll'} \sim \frac{\alpha}{4\pi} I^V I^{\bar{V}} \log\left(\frac{s}{m_V^2}\right) \log\left(\frac{|r_{kl}|}{s}\right)$$

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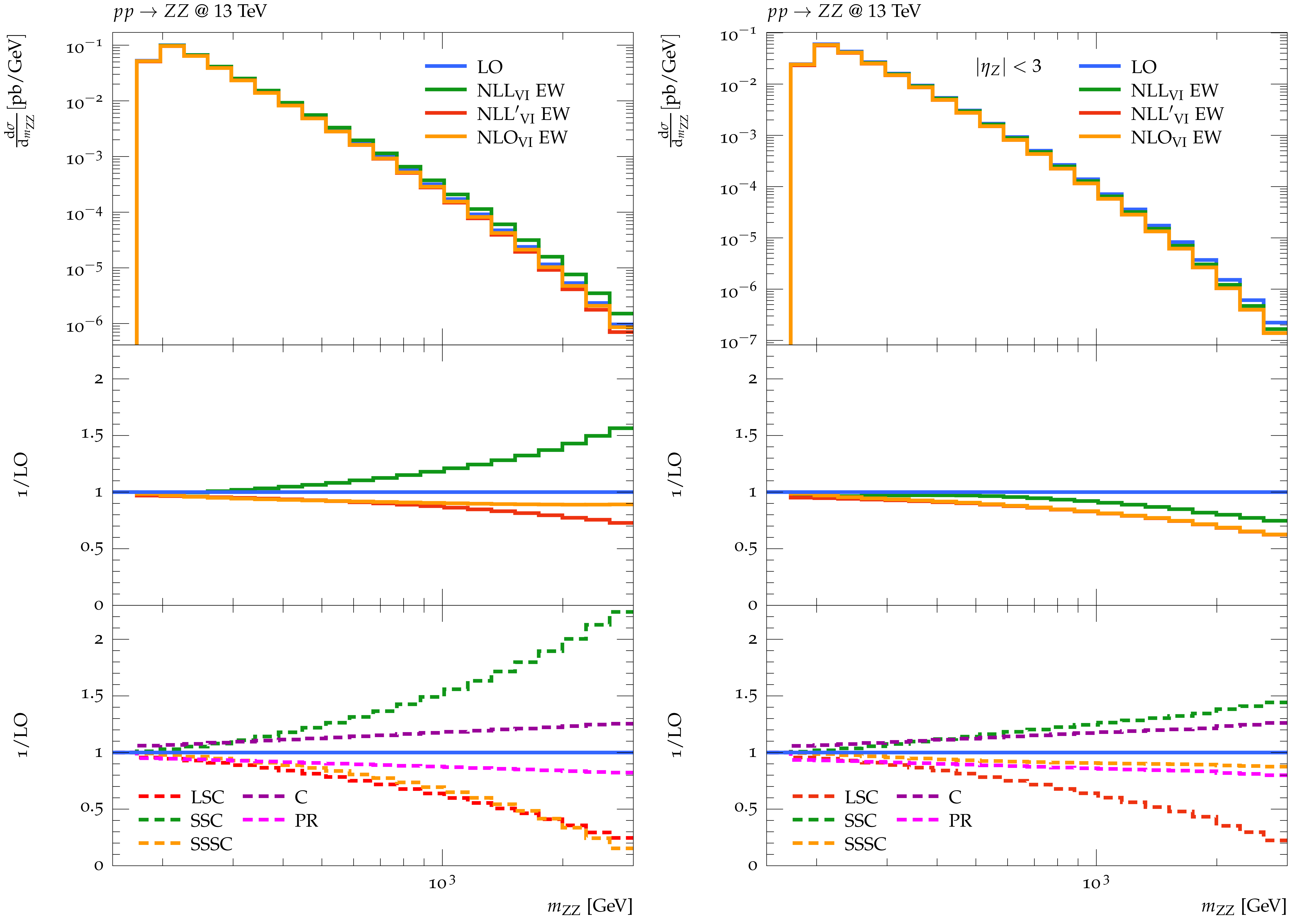
Results: $pp \to ZZ$



Pseudo-rapidity cut $|\eta_Z| < 3$ avoids pathological very forward configurations which violate LA; such cuts are anyway applied in any realistic analysis

Again, the inclusion of **SSSC** provides more accurate predictions. However, no full control on it as there are *non-universal* **SSSC**-like terms arising from high-energy expansion of 4-point functions. These angular contributions *cannot* be reliably controlled in LA

Looking at differences between NLL' and NLL opens two scenarios:

- $\mathrm{NLL}' - \mathrm{NLL} > \mathrm{NLL}' - \mathrm{NLO} \Rightarrow$ **SSSC** is a reliable estimate of sub-sub-leading angular terms beyond LA
- If NLO is unknown: **SSSC** might be interpreted as a conservative estimate of uncertainties of LA

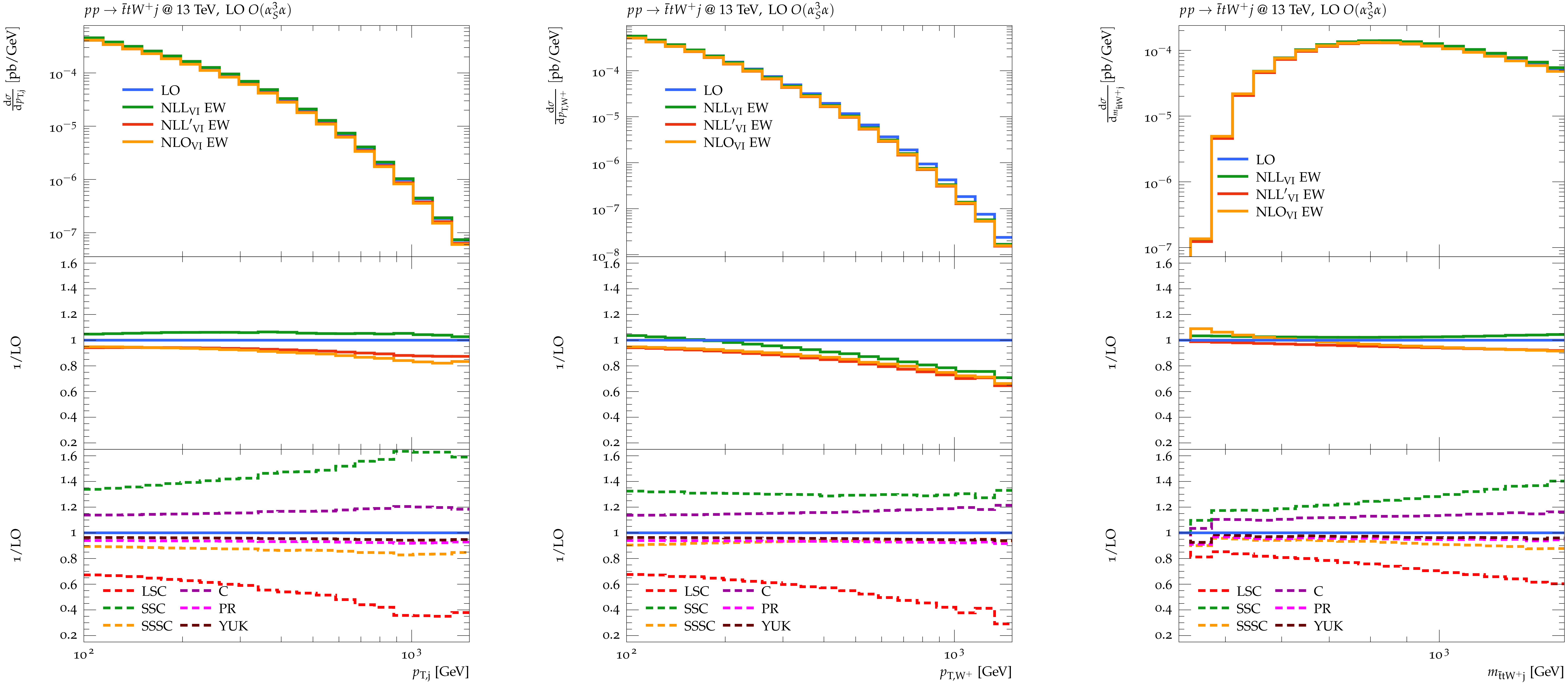
Results: $pp \to t\bar{t}W^+j$

Multijet merging @ **NLO**: [Frederix & Tsinikos, 2108.07826; 2021]

NLO QCD to $t\bar{t}W$: [Maltoni *et al*, 1406.3262; 2014]

NNLO QCD to $t\bar{t}W$: [Buonocore *et al*, 2306.16311; 2023]

NLO EW vs **NLL EW**: [Lindert & L.M., 2312.07927; 2023]



- $pp \to t\bar{t} + X$ are important backgrounds in Higgs analyses and/or BSM searches, but also for tests of EWSB
- Algorithm easily applicable to high multiplicity processes

Implementation in OpenLoops: resonances

- DP algorithm in a nutshell:
  - ▲ At $\sqrt{s} \gg m_W$, **NLO EW** radiative corrections are DL and SL
  - ▲ These corrections are ***universal***, i.e. are associated to *external* states only

- Not suitable for processes involving the two-body decay of an unstable particle $X \to ij$ as in the resonant region $s \gg r_{ij} \approx m_X^2$ → LA is violated

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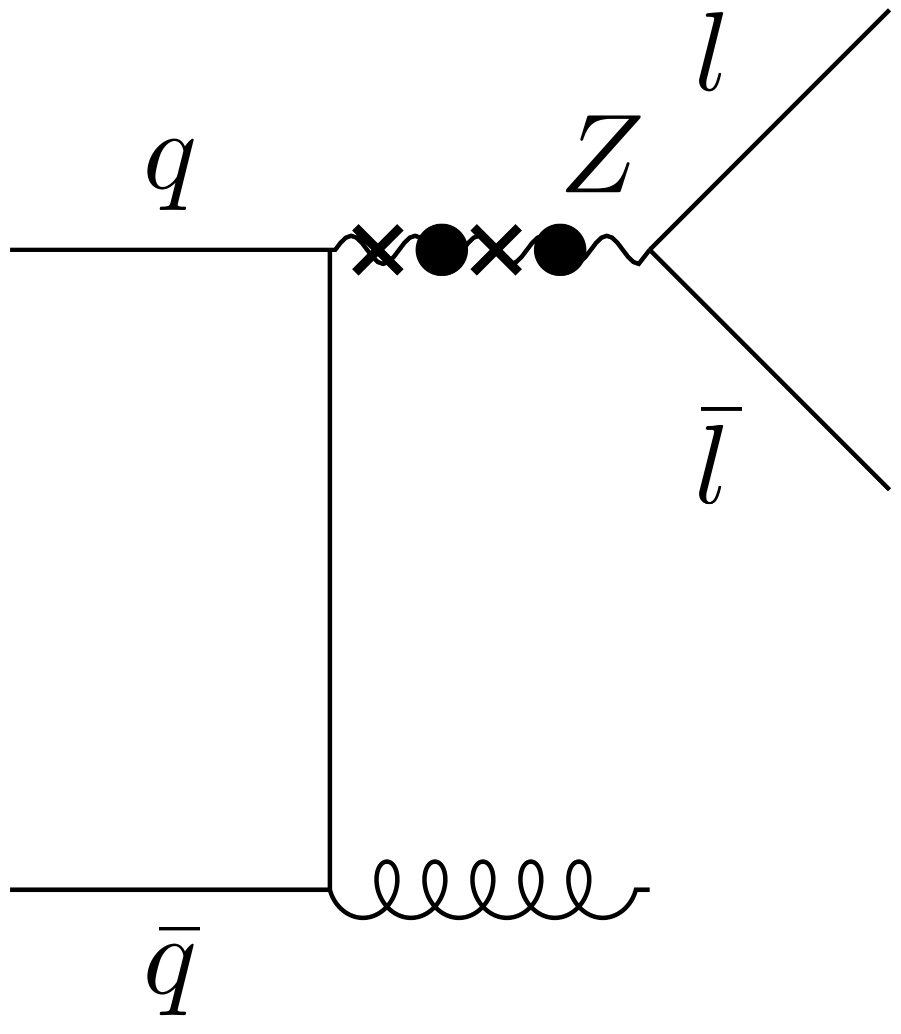
- A possible solution is the strategy adopted within Madspin [1212.3460] and via the HDH handler in Sherpa [1905.09127]:
  - ➞ Employ NWA to generate the hard scattering process including the associated $\mathcal{O}(\alpha)$ EW corrections, then adding the decay
  - ➞ LO off-shell effects and spin correlations can be retained via subsequent Breit-Wigner smearing

Implementation in OpenLoops: resonances

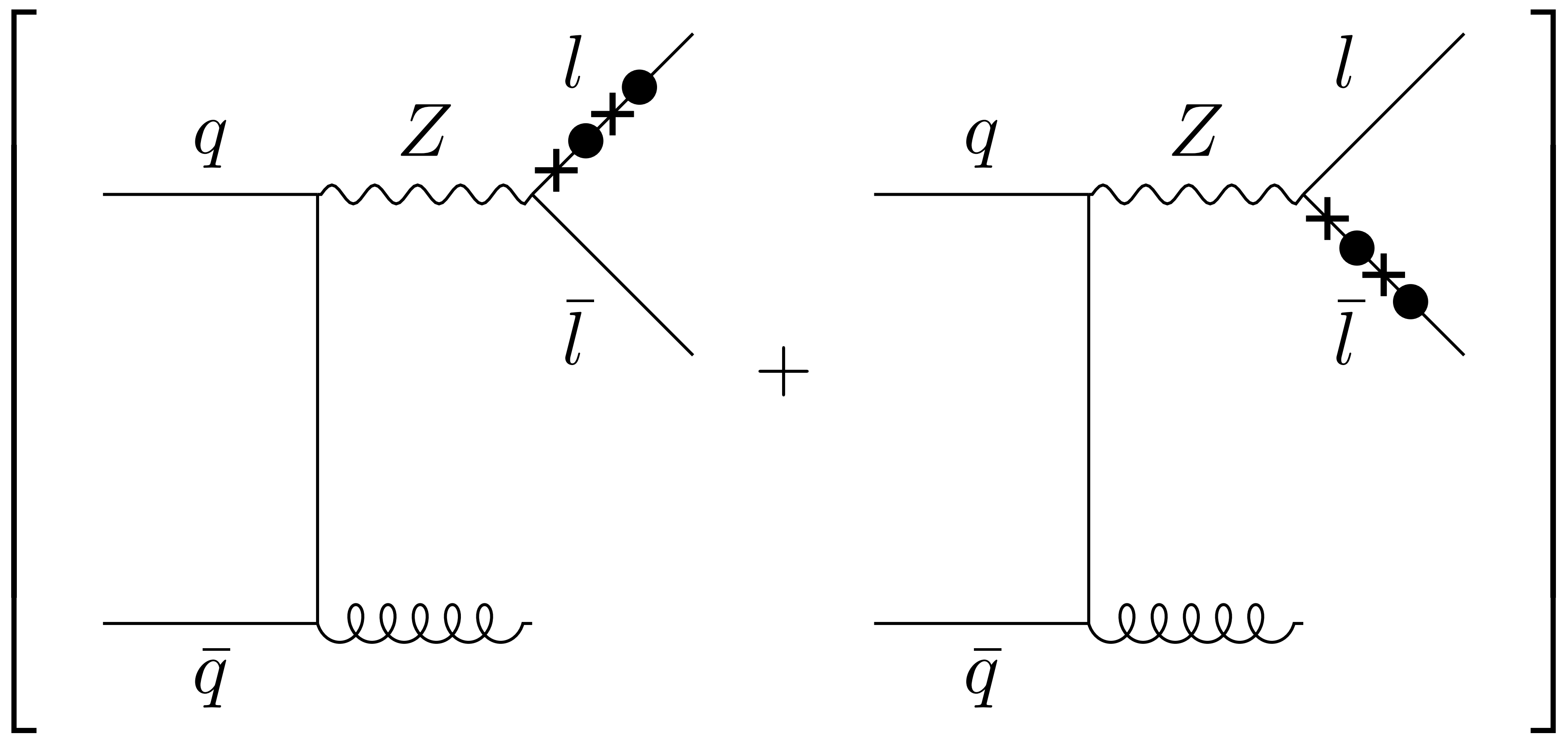
- Our approach: evaluation of *Sudakov* corrections associated to both X and $\{i,j\}$ with different weights $P_i(k_i)$



$\times\, P_Z(k)$

▼ Also the internal effective two-point counterterm vertices are *helicity- dependent* and project on the helicity of the combined $\{i,j\}$ current of the external states

$$P_{X_i}(k_i) = \left| \frac{\mu^2_{X_i} - m^2_{X_i}\Gamma^2_{X_i}}{(k_i^2 - \mu^2_{X_i})^2 + \mu^2_{X_i}} \right| = \begin{cases} 1 \text{ if } & k_i^2 \to m^2_{X_i} \\ 0 \text{ if } & k_i^2 \to \infty \end{cases}$$

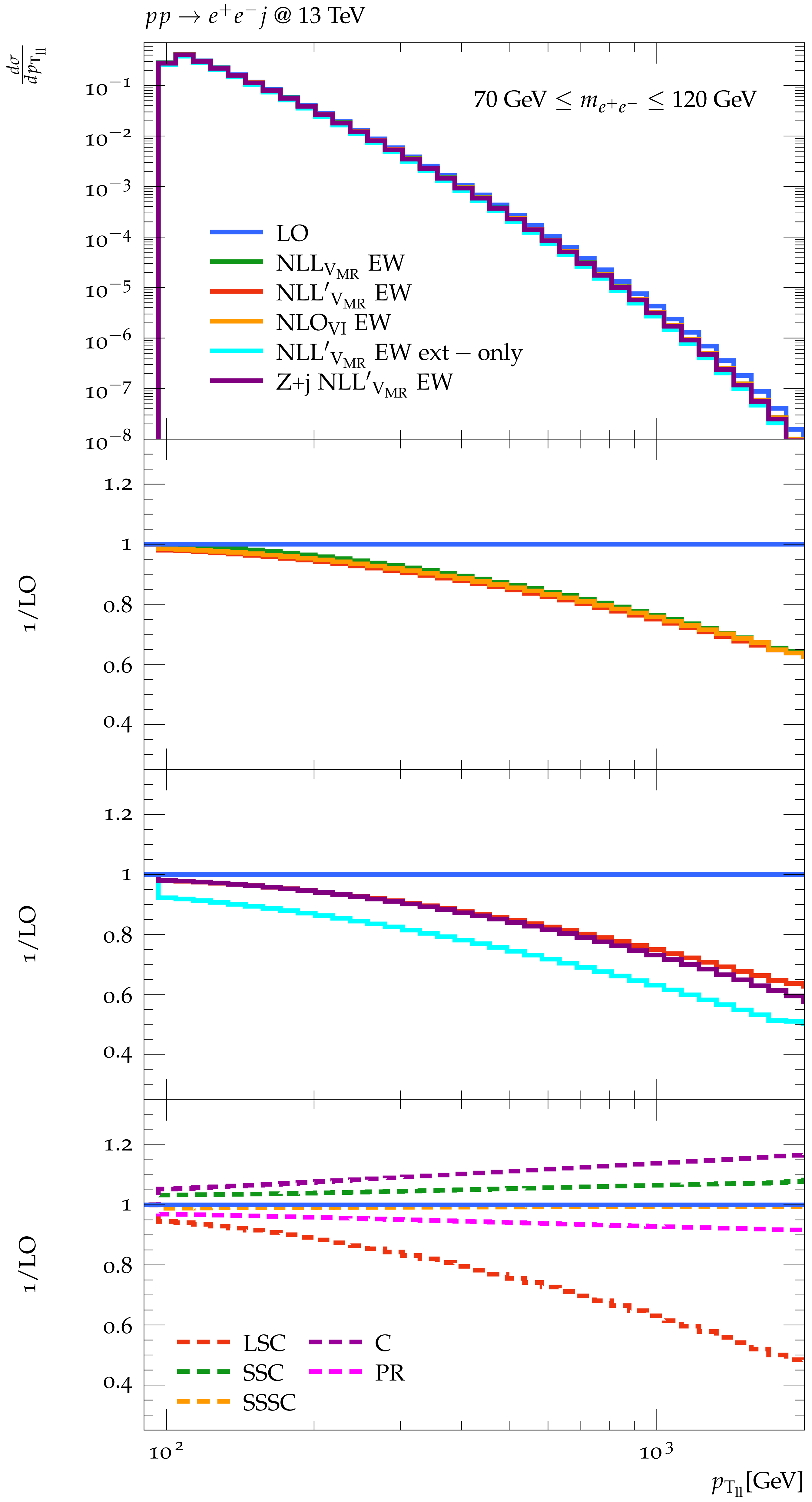


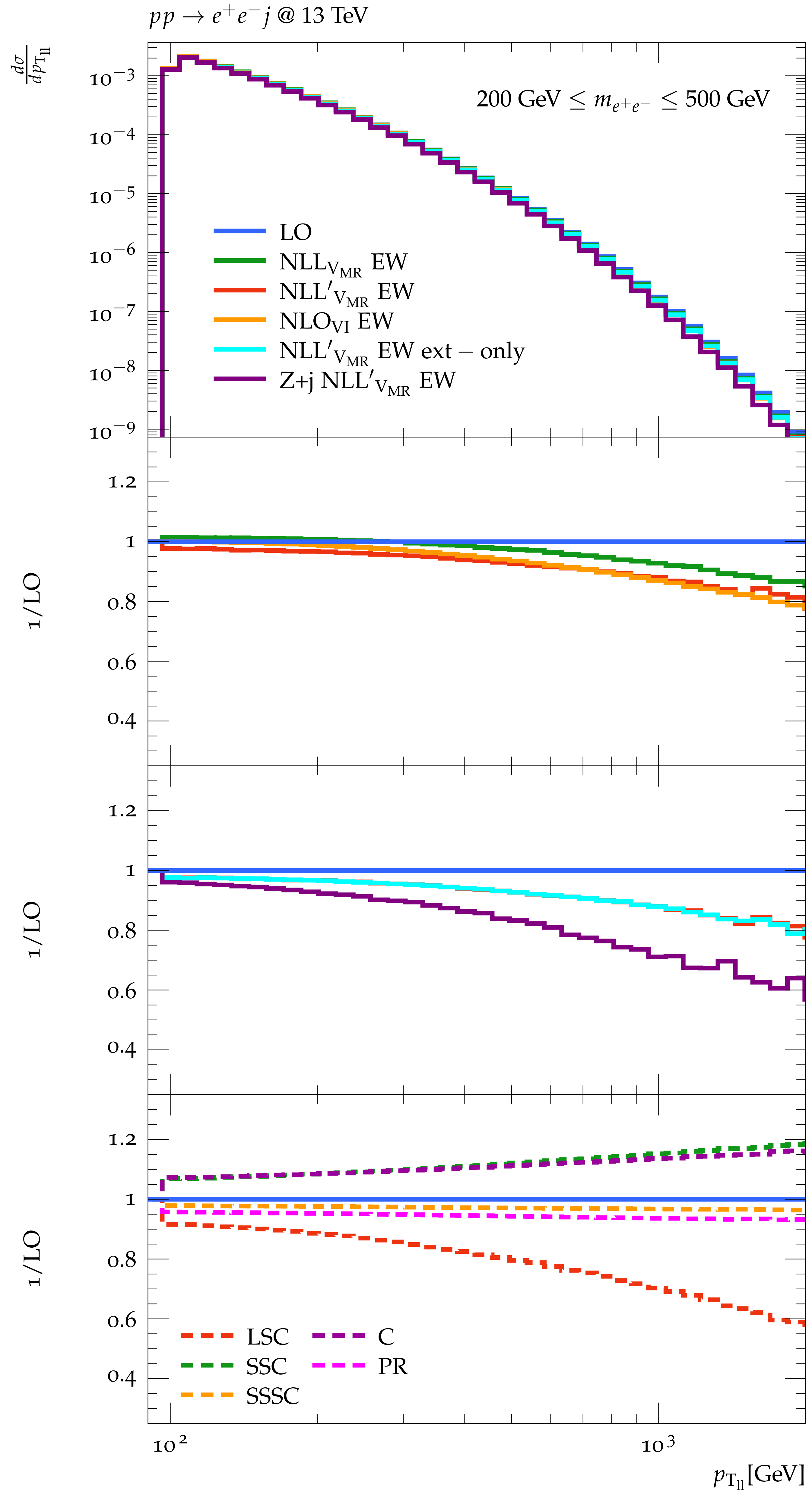
$\times\, P_{\{l,\bar{l}\}}(k) = 1 - P_Z(k)$

Results: $pp \to e^+e^-j$

NLO EW: [Denner *et al*, 1011.6674; 2010]

NLO QCD+EW: [Kallweit *et al*, 1511.08692; 2016]





The standard implementation based on external insertions fails in reproducing the full NLO prediction for the $m_{e^+e^-}$ range capturing the resonance

Issue naturally solved with internal insertions controlled by projectors

Automatic recover of standard algorithm when far from the resonance

NB: all NLL' and NLL predictions are evaluated in MR with $\lambda = m_W$ for QED contributions.
However, consistency of $\boxed{\mathrm{NLL}'_{\mathrm{V_{MR}}}}$ with:

- $\boxed{Z + \mathrm{j}\ \mathrm{NLL}'_{\mathrm{V_{MR}}}}$ in their respective on-shell phase spaces
- $\boxed{\mathrm{NLL}'_{\mathrm{V_{MR}}}\ \text{ext-only}}$ in the off-shell phase space

Conclusions and outlook

- In the **EW** sector, radiative corrections at high energies are dominated by *Sudakov logarithms* which significantly enhance tails of kinematic distributions (> 10%)
- Exploiting the ***universality*** of *Sudakov logs* we developed an effective CT vertex approach for the DP algorithm and implemented it in OpenLoops

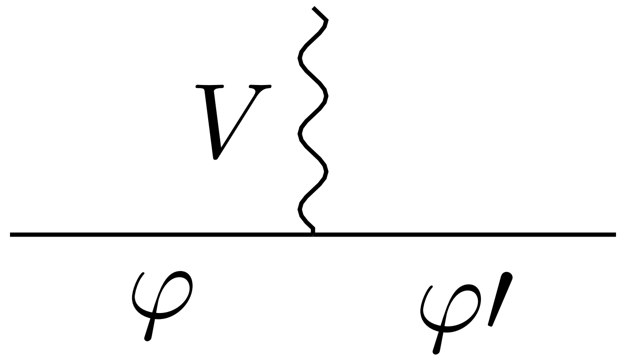
Reduction of one-loop **EW** corrections to a tree-level problem with percent level of agreement

- Additional aspects of the implementation:
    - ▶ Largely model independent (applicable to both **SM** and **BSM** scenarios)
    - ▶ Direct employment in PS Event Generators with OL interface
    - ▶ Can be used together with differential QED radiation at **NLO** (both MR and DR are available)
    - ▶ Support **EW** corrections for resonant processes
- Outlook:
    - ▶ Resummation for preservation of PT
    - ▶ Dressing **NLL** **EW** *Sudakov logs* with **QCD** loops, i.e. **mixed** **QCD-EW** corrections
    - ▶ Suitable for **NNLO/two-loop** extension (ongoing work)

Backup

DL sketch

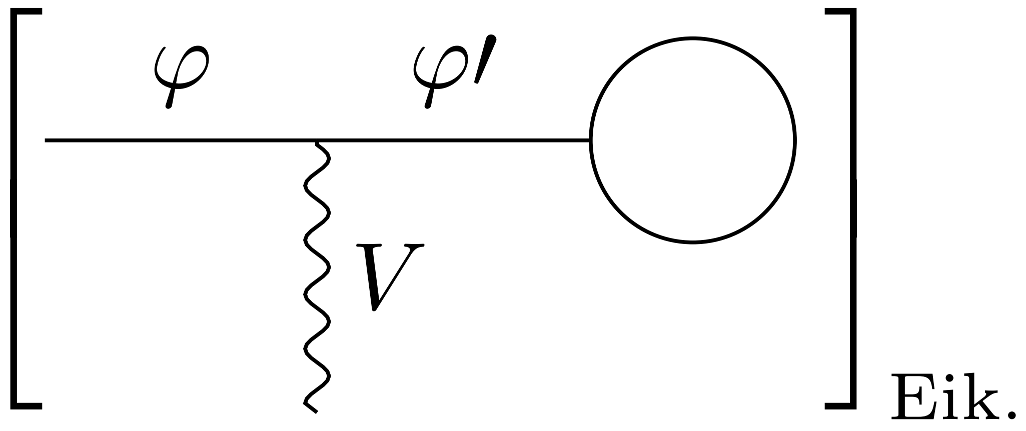
- General form of **EW** Feynman rules as in [Denner 0709.1075; *2007*]



$$= ieI^V_{\varphi\varphi\prime}T$$

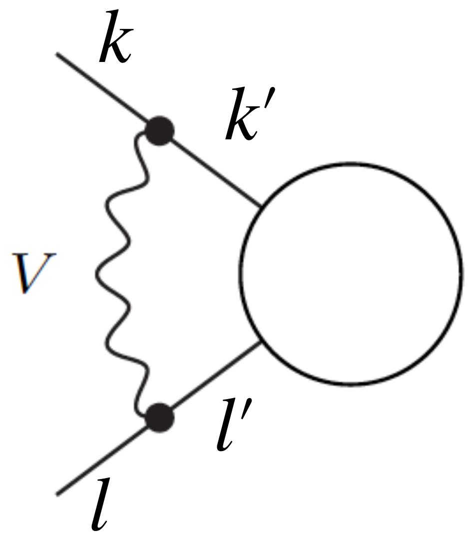
where T contains the Lorentz structure of the vertex (e.g. in $\gamma\bar{f}f\,:\; I^V_{\varphi\varphi'} = -Q_f, \quad T = \gamma^\mu$)

- When contracted with external w.f.s. in computing one-loop amplitudes, it follows



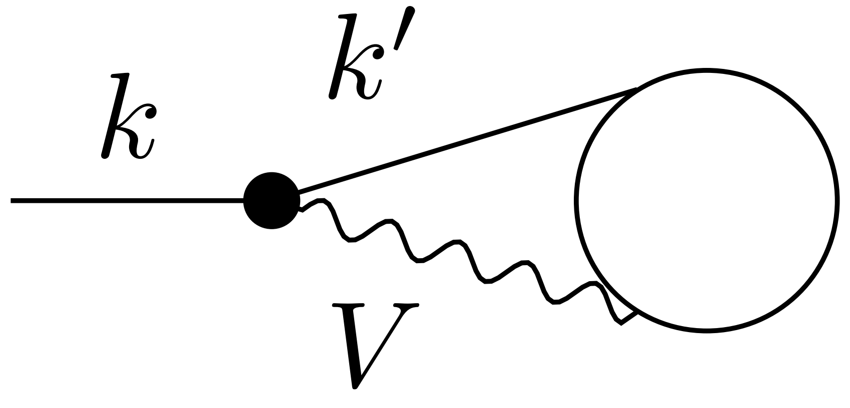
$$\propto 2ieI^V_{\varphi\varphi\prime}p^\mu_\varphi\mathcal{M}_0^{\dots\varphi'\dots}$$

so that



$$\sim \sum_V \sum_{k',l'} \frac{\alpha}{4\pi} I^V_{kk'} I^{\bar{V}}_{ll'}\, r_{\varphi\varphi'} C_0|_{\text{Eik.}}\mathcal{M}_0^{\varphi_{i_1}\cdots\varphi_{i'_k}\cdots\varphi_{i'_l}\cdots\varphi_{i_n}}, \qquad C_0|_{\text{Eik.}} \propto \frac{1}{r_{\varphi\varphi'}}\left[\log^2\frac{r_{\varphi\varphi'}}{m_V^2}\right]$$

Coll sketch



- Original integral

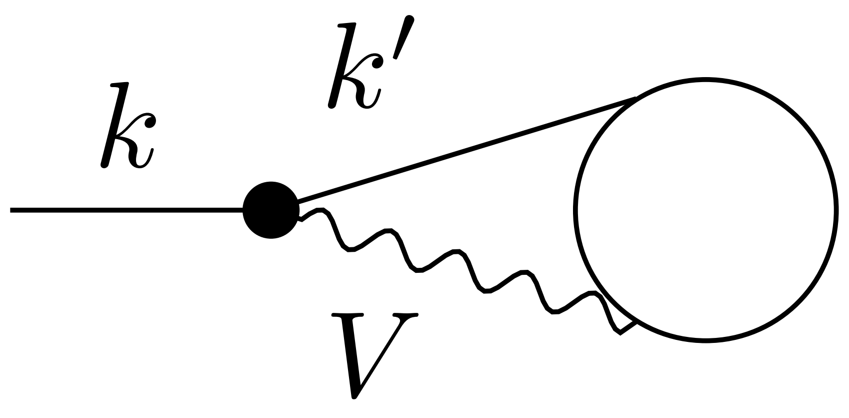
$$\sim \mu^{4-D}\int \frac{d^D q}{(2\pi)^D}\frac{N(q)}{(q^2-m_V^2+i0)[(p_k-q)^2-m_k^2+i0]}$$

- In Sudakov parametrisation $q^\mu = xp_k^\mu + yl^\mu + q_\mathrm{T}^\mu$, after y integration the integral reduces to

$$\sim \mu^{4-D}\int_0^1 dx \int \frac{d^{D-2}q_\mathrm{T}}{(2\pi)^{D-2}}\frac{N(x,y_i,q_\mathrm{T})}{|\vec{q}_\mathrm{T}|^2+\Delta(x)}$$

with $\Delta(x) = (1-x)m_V^2 + xm_{k'}^2 - x(1-x)p_k^2$ regulating the logarithmic singularity

Coll sketch



- In Sudakov parametrisation

$$\sim \mu^{4-D} \int_0^1 dx \int \frac{d^{D-2}q_\mathrm{T}}{(2\pi)^{D-2}} \frac{N(x, y_i, q_\mathrm{T})}{|\vec{q}_\mathrm{T}|^2 + \Delta(x)}$$

with $\Delta(x) = (1-x)m_V^2 + xm_{k'}^2 - x(1-x)p_k^2$ regulating the logarithmic singularity

- Since we restrict to logarithmic mass-singular contributions, all terms of order

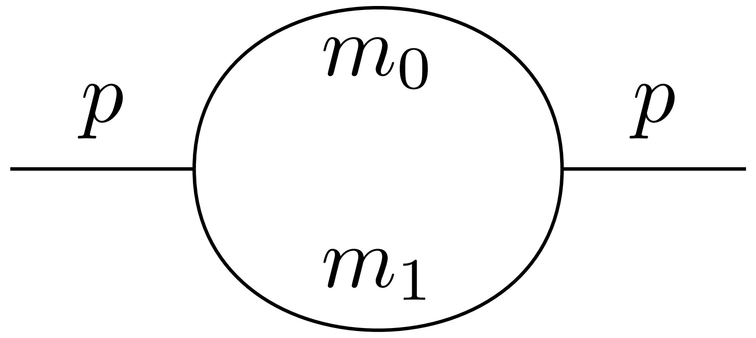
$$|\vec{q}_\mathrm{T}|^2,\ p_k^2,\ m_V,\ m_k,\ y_i\,(\propto |\vec{q}_\mathrm{T}|^2/p_k l)$$

can be neglected in $N(q)$

- *Collinear approximation*
    - ▲ Substitute $N(x, y, q_\mathrm{T}) \to N(x, 0, 0)$, i.e. $q^\mu \to xp_k^\mu$
    - ▲ Neglect all mass terms in $N(x, 0, 0)$

Single Logs: PR

- Generic two-point function



$$\frac{\mathrm{i}}{(4\pi)^2} B_{\{0,\mu,\mu\nu\}}\left(p, m_0, m_1\right) := \mu^{4-D} \int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{\{1, q_\mu, q_\mu q_\nu\}}{\left(q^2 - m_0^2 + \mathrm{i}\varepsilon\right)\left[(q+p)^2 - m_1^2 + \mathrm{i}\varepsilon\right]}$$

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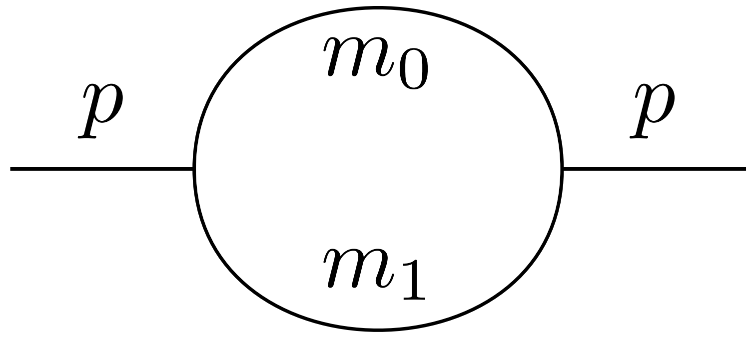

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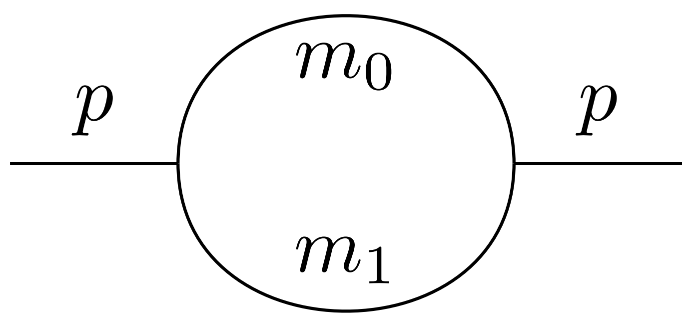
- In LA $\mu^2 = s \gg p^2, m_0^2, m_1^2 \Rightarrow$ four possible hierarchy of masses

$$\begin{aligned}
&(a) \quad m_i^2 \ll p^2 \text{ and } p^2 - m_{1-i}^2 \ll p^2 \quad \text{for } i = 0 \text{ or } \quad i = 1, \\
&(b) \quad \text{not} \quad (a) \text{ and } m_i^2 \not\gg p^2 \text{ for } i = 0, 1, \\
&(c) \quad m_0^2 = m_1^2 \gg p^2 \\
&(d) \quad m_i^2 \gg p^2 \not\gg m_{1-i}^2 \text{ for } i = 0 \text{ or } i = 1
\end{aligned}$$

Single Logs: PR



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- Results for two point functions and their derivatives

$$B_0\left(p, m_0, m_1\right) \overset{\text{LA}}{=} \log\frac{\mu^2}{M^2},$$
$$B_1\left(p^2, m_0, m_1\right) \overset{\text{LA}}{=} -\frac{1}{2}\log\frac{\mu^2}{M^2},$$
$$\frac{1}{p^2} B_{00}\left(p^2, m_0, m_1\right) \overset{\text{LA}}{=} \frac{3m_0^2 + 3m_1^2 - p^2}{12p^2}\log\frac{\mu^2}{M^2},$$
$$\frac{1}{p^2} g^{\mu\nu} B_{\mu\nu}\left(p^2, m_0, m_1\right) \overset{\text{LA}}{=} \frac{m_0^2 + m_1^2}{p^2}\log\frac{\mu^2}{M^2}$$

$$p^2 B_0'\left(p, m_0, m_1\right) \overset{\text{LA}}{=} \frac{1}{2}\log\frac{m_{1-i}^2}{m_i^2} = \frac{1}{2}\log\frac{p^2}{\lambda^2},$$
$$p^2 B_1'\left(p^2, m_0, m_1\right) + \frac{1}{2} p^2 B_0'\left(p, m_0, m_1\right) \overset{\text{LA}}{=} -\frac{1}{4}\log\frac{m_0^2}{m_1^2}$$

Implementation in OpenLoops: PR

- Two-point effective vertices are suitable for the evaluation of ***soft-collinear*** and ***collinear*** *Sudakov* corrections

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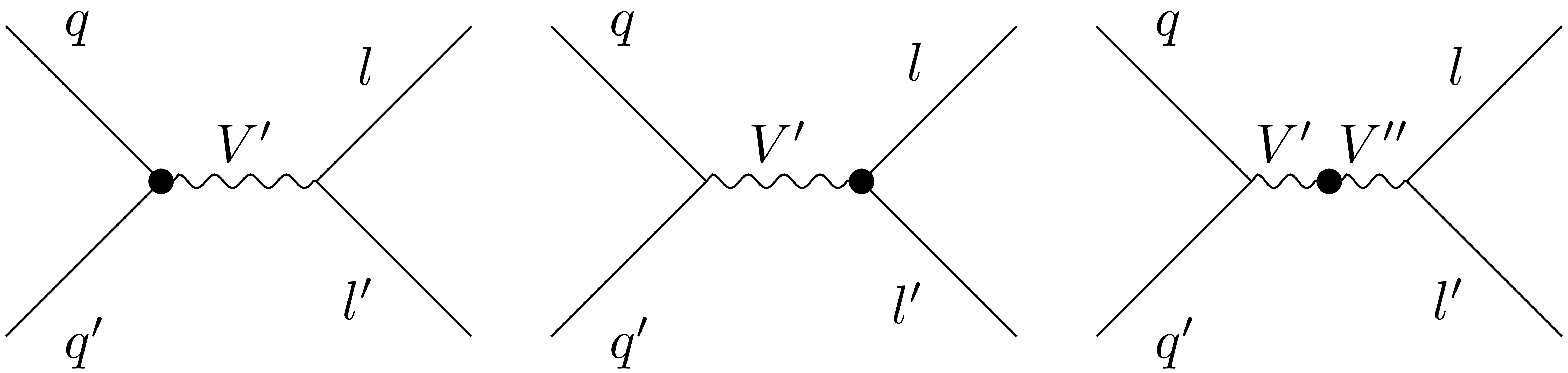

setting all the **WFRCs** to zero

Implementation in OpenLoops: PR

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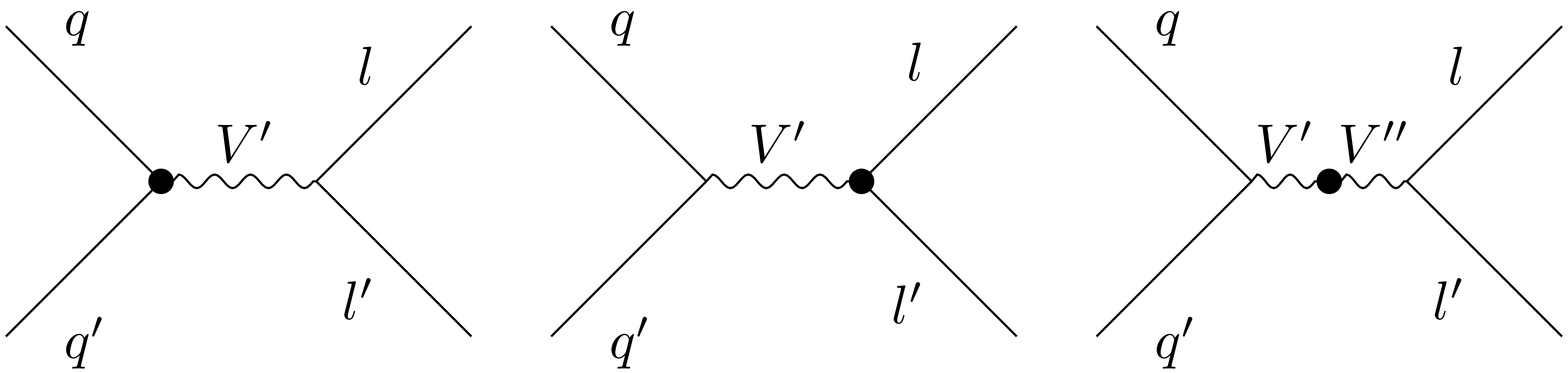

setting all the **WFRCs** to zero

- Alternative way: evaluate **WF** + **PR** via standard UV counterterms and set $\delta^{\mathrm{WF}}_{kk'}$ to zero to avoid double counting

Implementation in OpenLoops: projectors

- Explicit expression of the projectors for unstable particles X

$$P_{X_i}(k_i) = \left| \frac{\mu_{X_i}^2 - m_{X_i}^2 w_{\text{rescale}}^2 \Gamma_{X_i}^2}{(k_i^2 - m_{X_i}^2 + i m_{X_i} w_{\text{rescale}} \Gamma_{X_i})^2 + \mu_{X_i}^2} \right| = \begin{cases} 1 & \text{if} \quad k_i^2 \to m_{X_i}^2 \\ 0 & \text{if} \quad k_i^2 \to \infty \end{cases}$$

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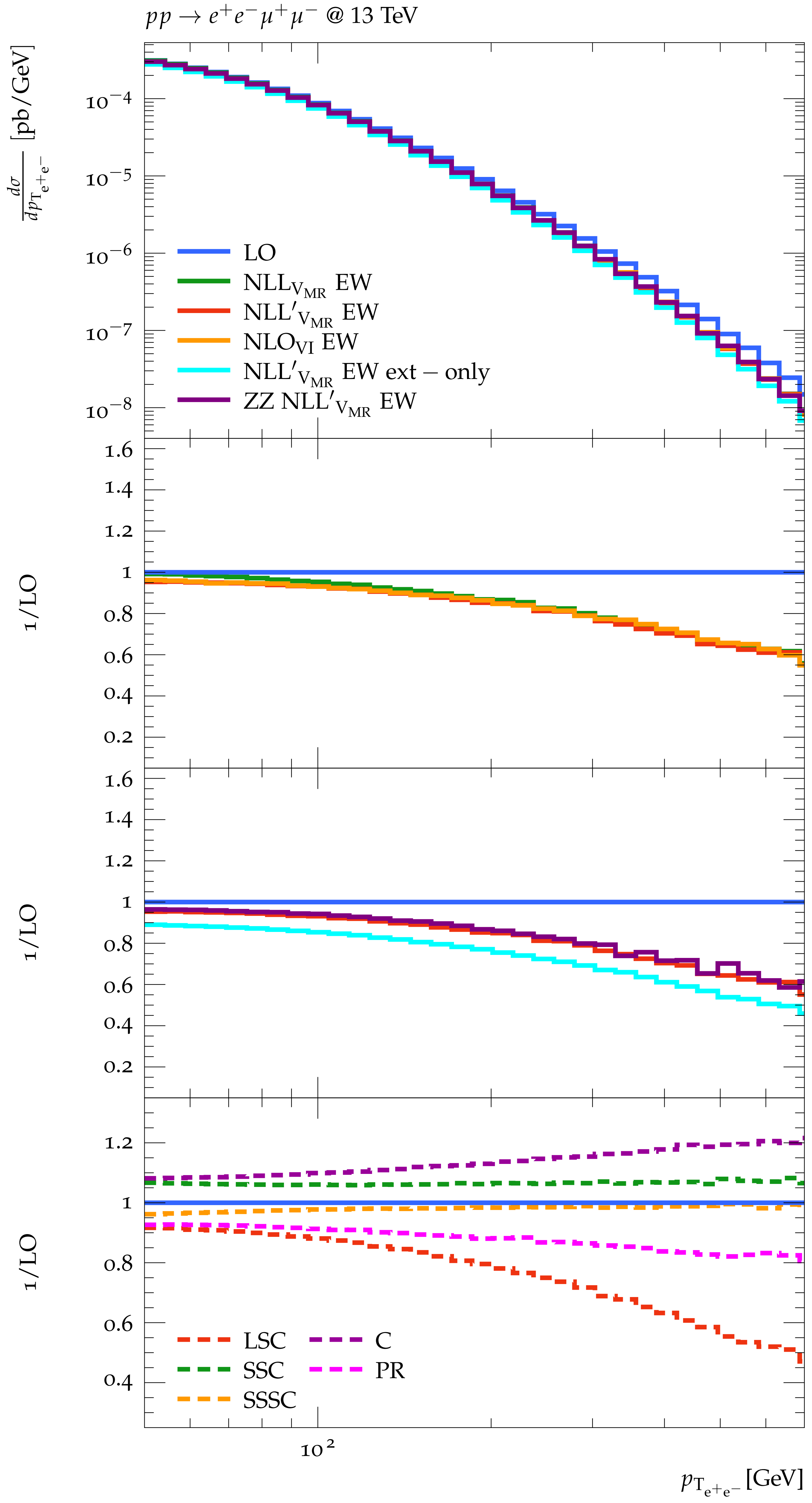
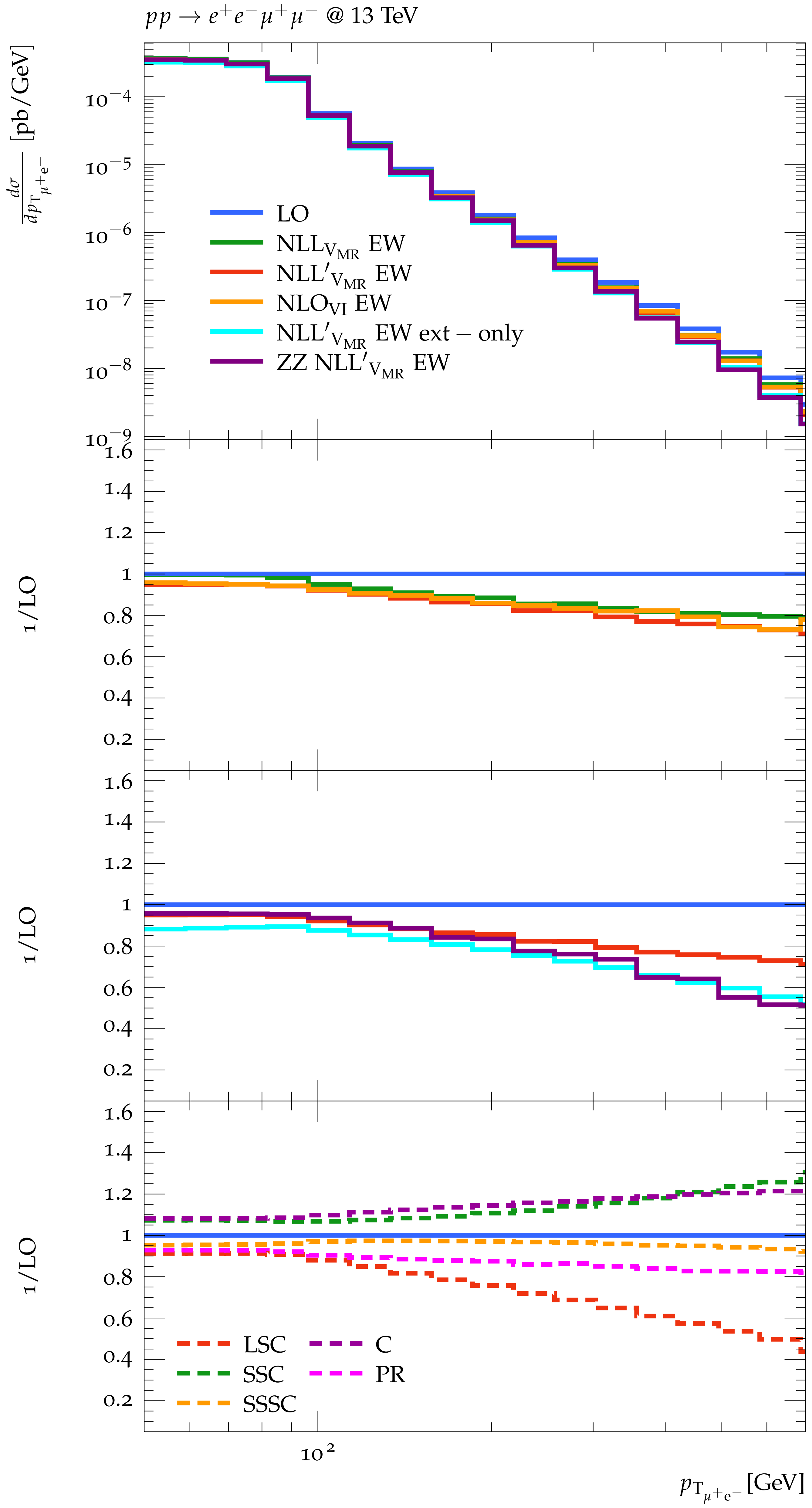
- The direct employment of projectors would violate unitarity but this can be prevented as follows:
  - Evaluation of $P_{X_i}(k_i)$ for a given psp
  - Generation of random number $0 \leq a \leq 1$
  - Choice $P_{X_i} = \begin{cases} 1 & \text{if } P_{X_i} \geq a \\ 0 & \text{if } P_{X_i} \leq a \end{cases}$

Results: $pp \to e^+e^-\mu^+\mu^-$



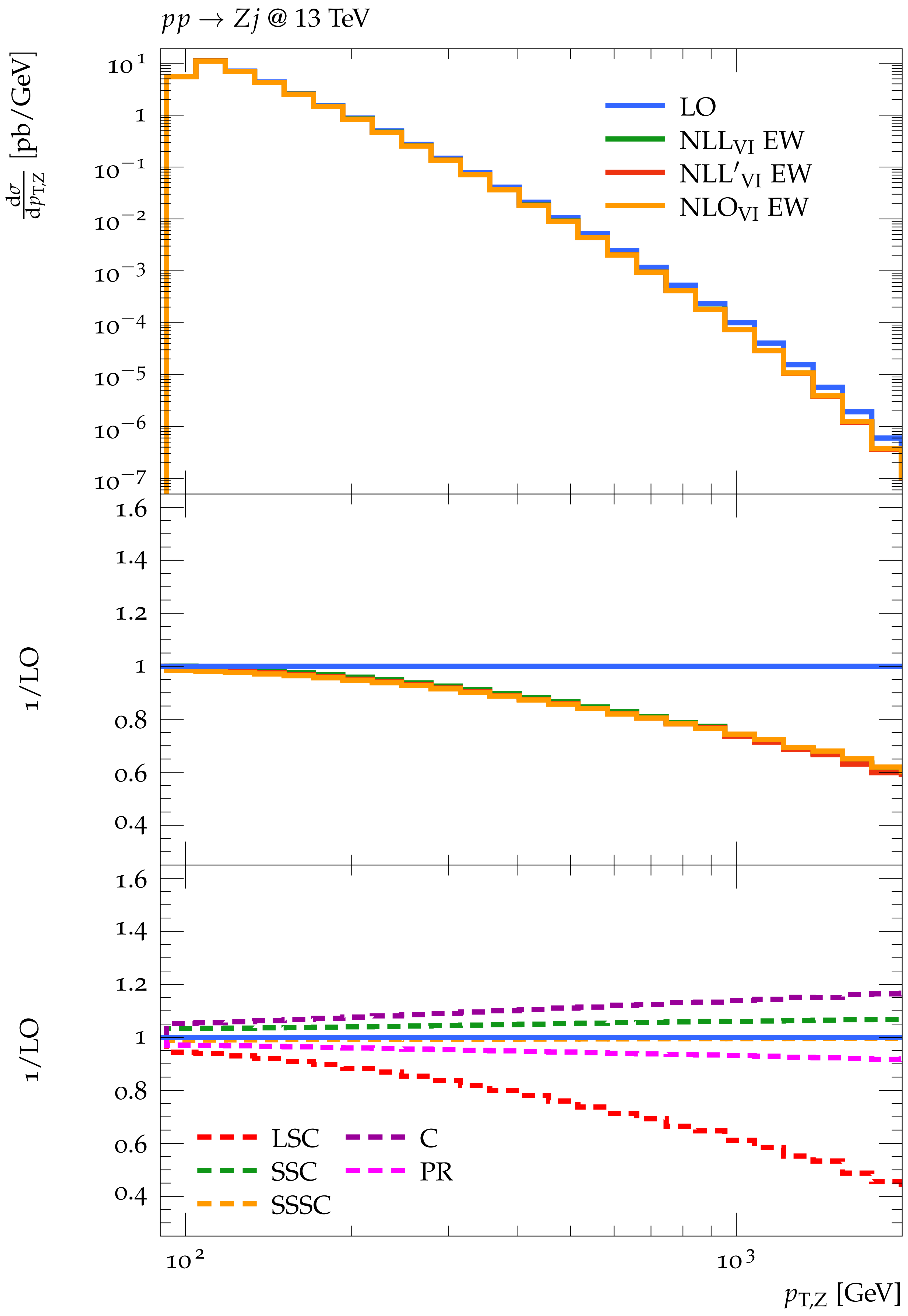


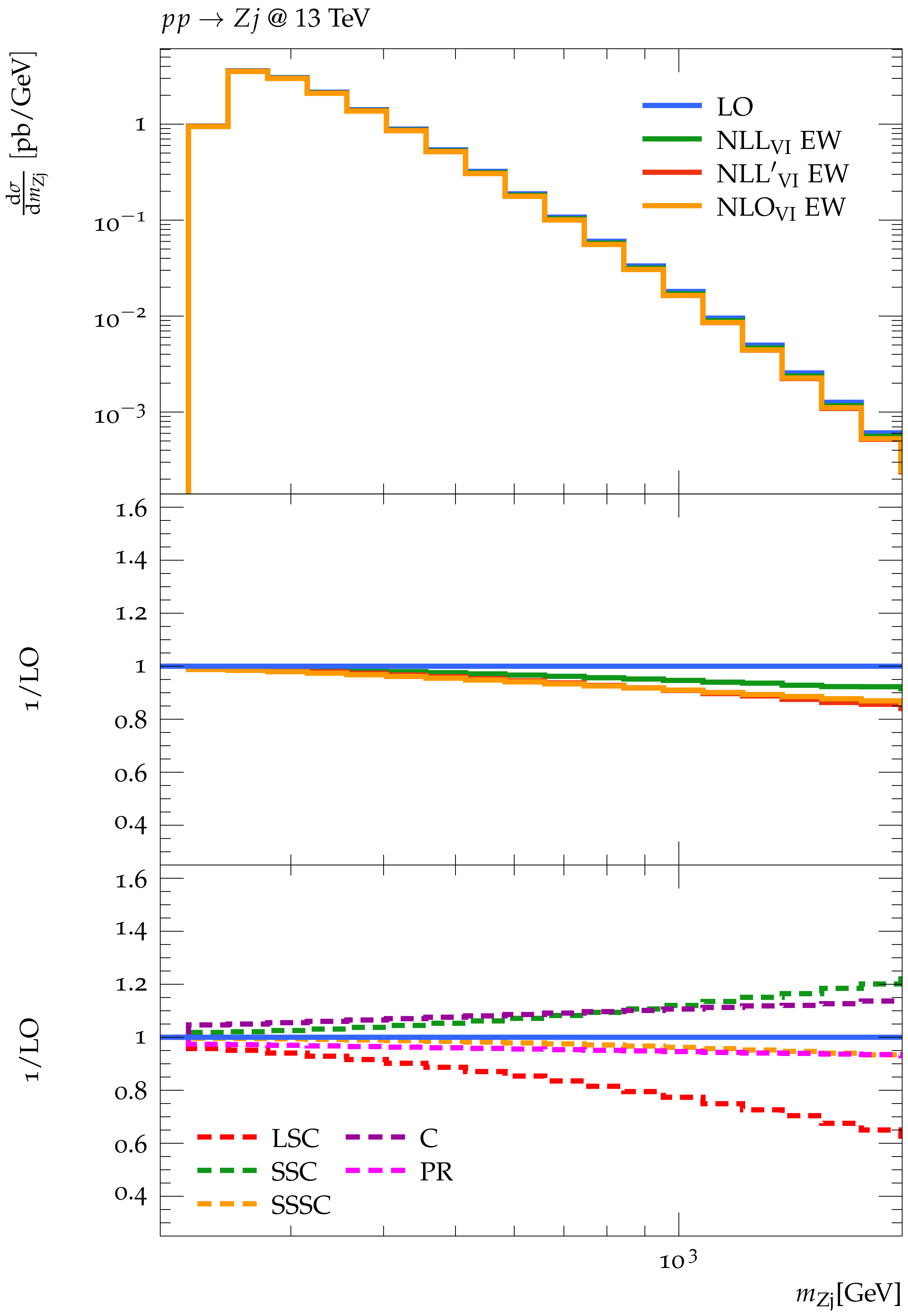
Internal insertions work accurately also for processes with more unstable particles:

- Resonant configuration correctly captured in $p_{\mathrm{T}_{e^+e^-}}$
- Overlapping on-shell and off-shell effects properly interpolated in $p_{\mathrm{T}_{\mu^+e^-}}$

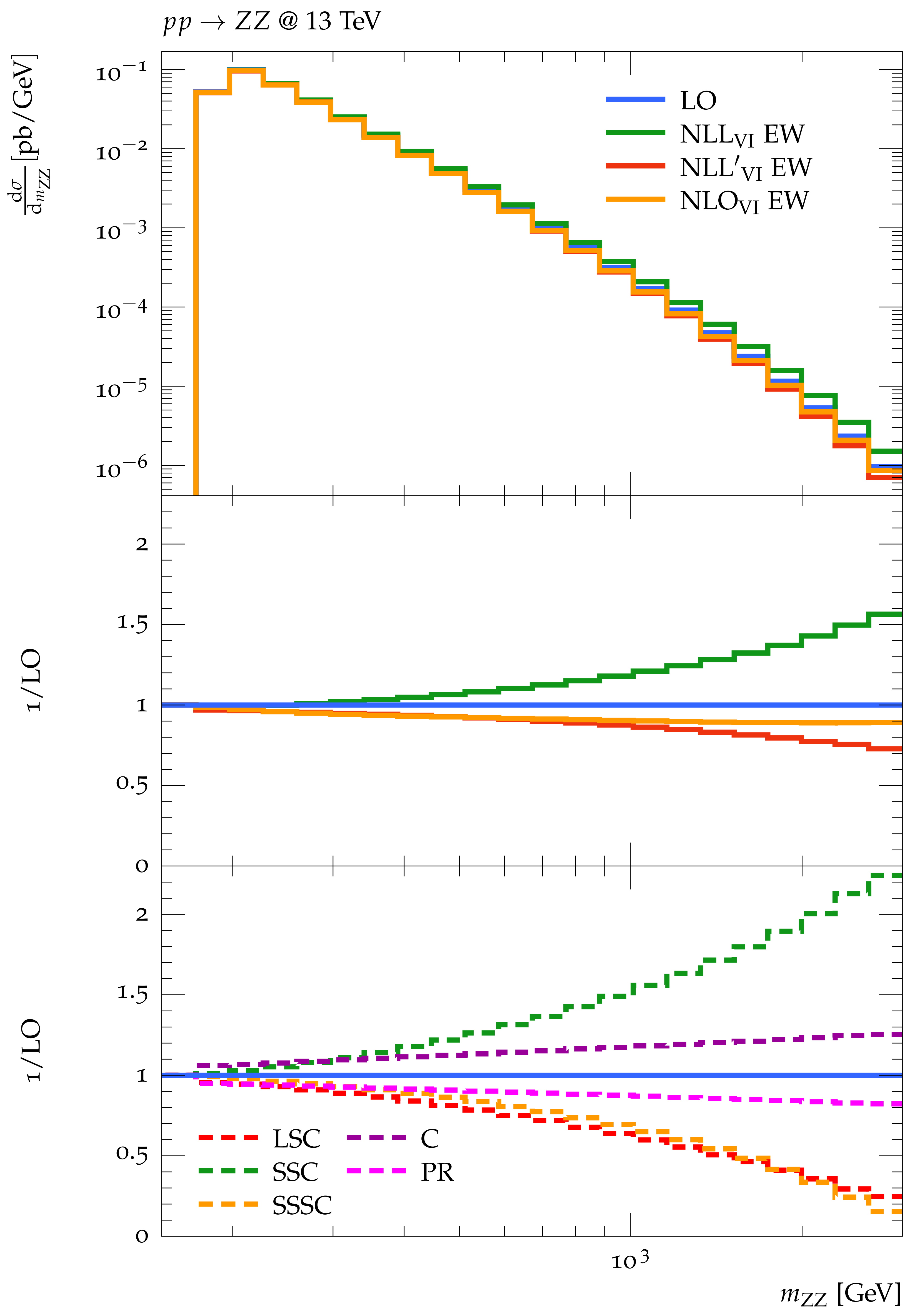
Additional results

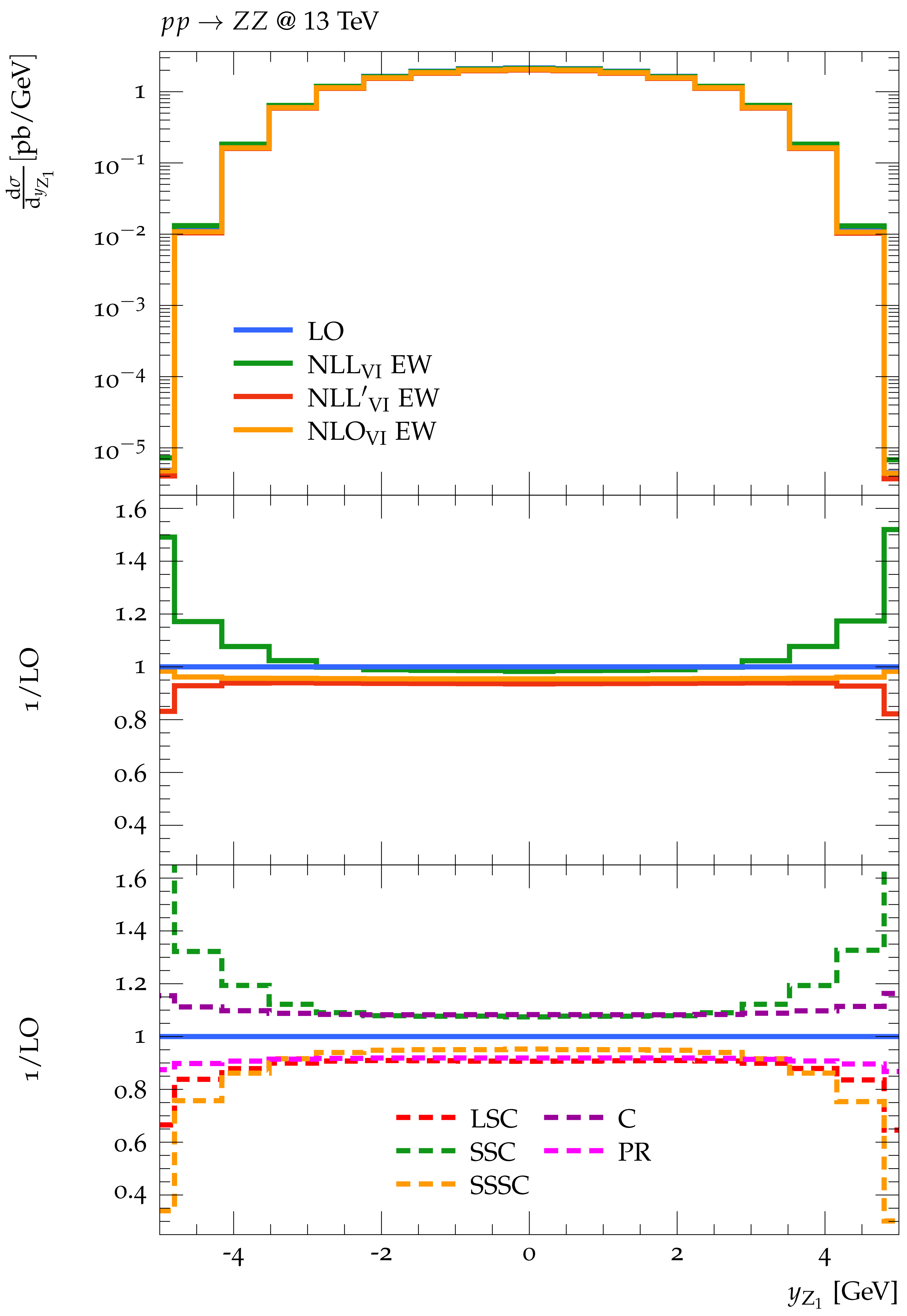
Results: $pp \to Zj$





Results: $pp \to ZZ$





Two considerations from the rapidity distribution:

- The inclusion of **SSSC** allows for a better *Sudakov* approximation, in particular for $|y_Z| < 3$
- For very forward configurations, i.e. outside the central region $|y_Z| < 3$, **SSC** and **SSSC** rapidly grow

Results: $pp \to W^+W^-$

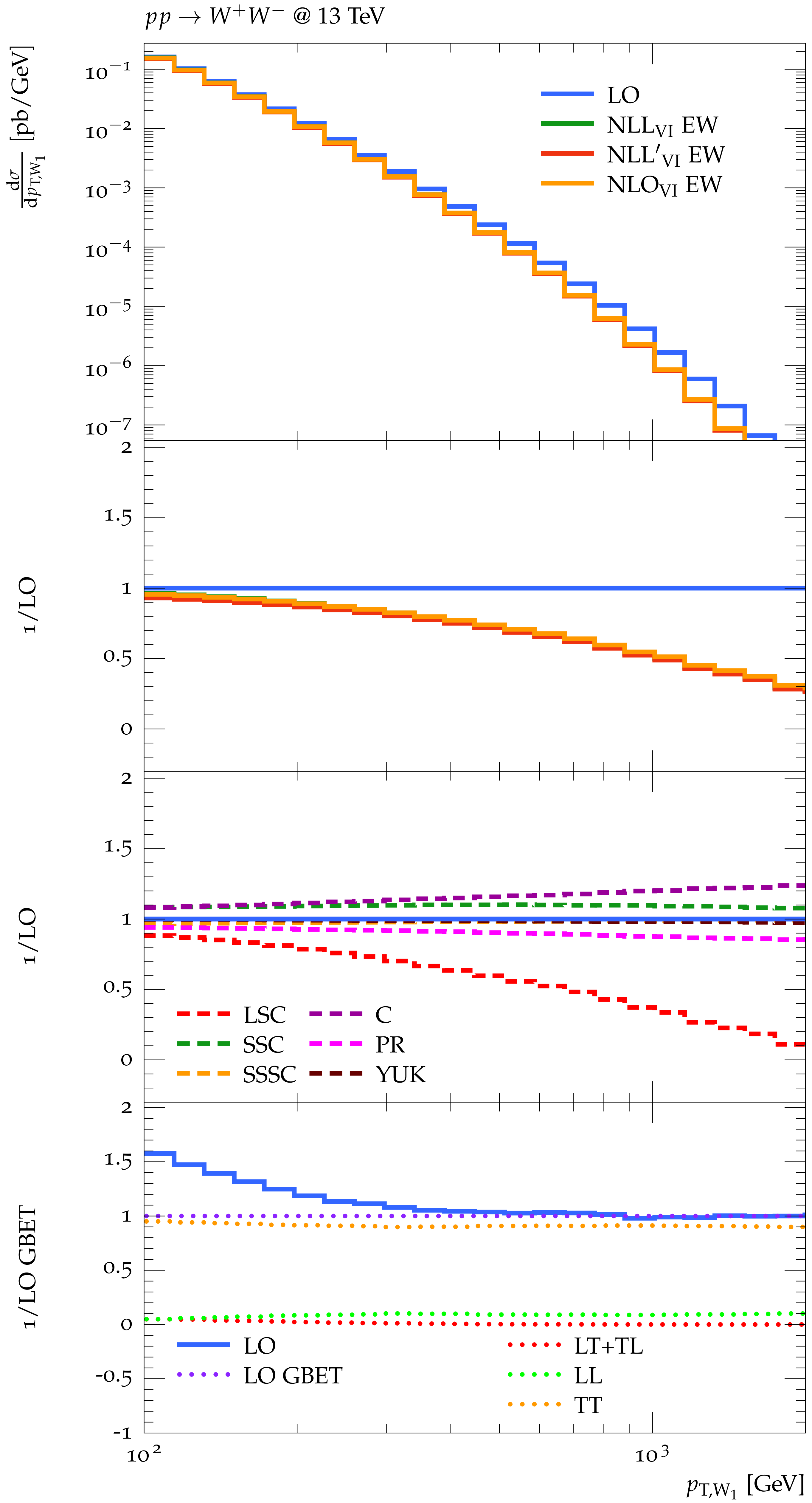
NLL EW: [Accomando et al, 0409247; 2004]

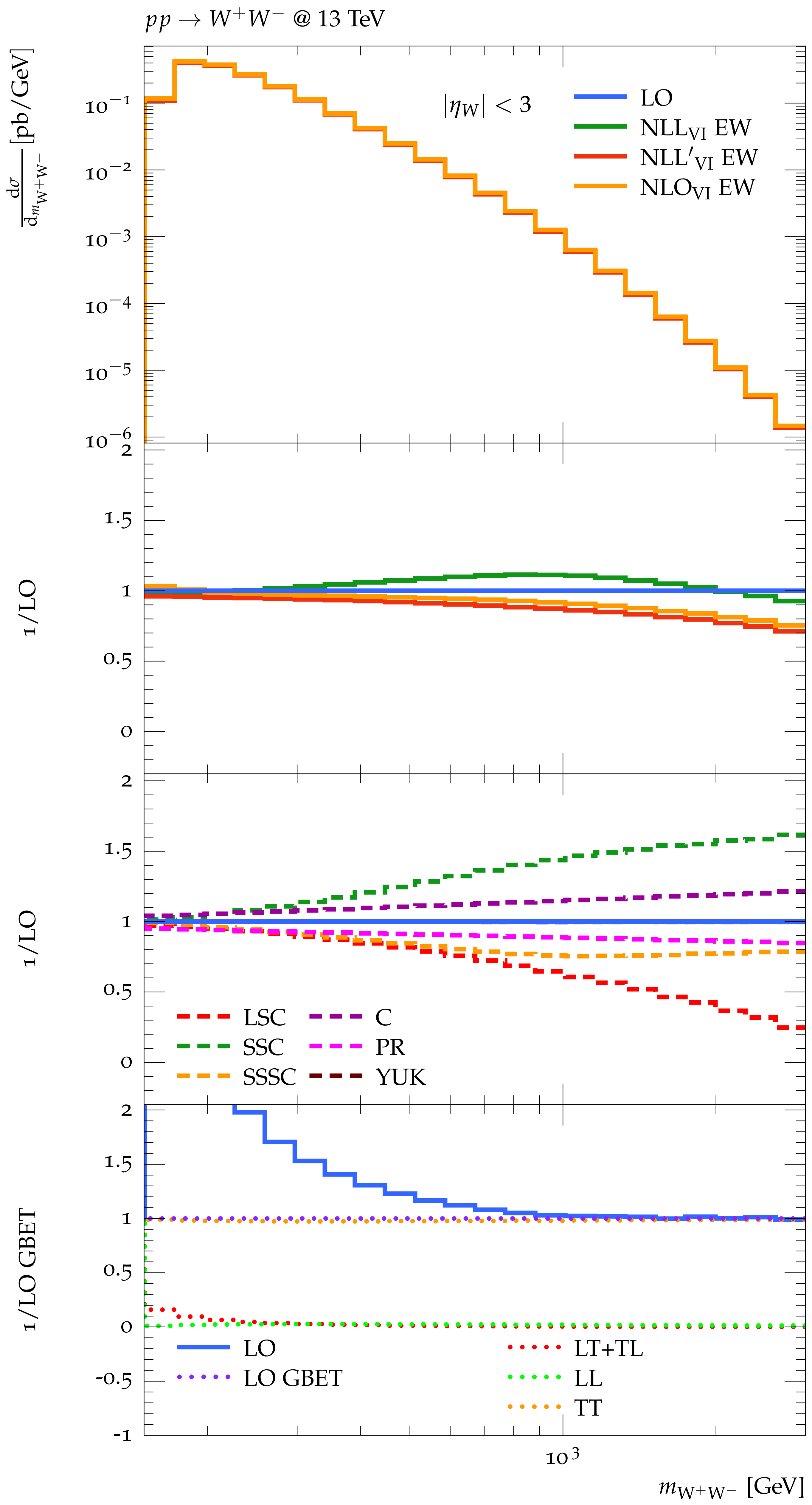
Full **NLO EW**: [Bierweile *et al*, 1208.3147; 2012]

Full **NLO**: [Baglio *et al*, 1307.4331; 2016]

Mixed NLO QCD - EW: [Bräuer *et al*, 2005.12128; 2020]

NNLO QCD+NLO EW: [Grazzini *et al*, 1912.00068; 2020]





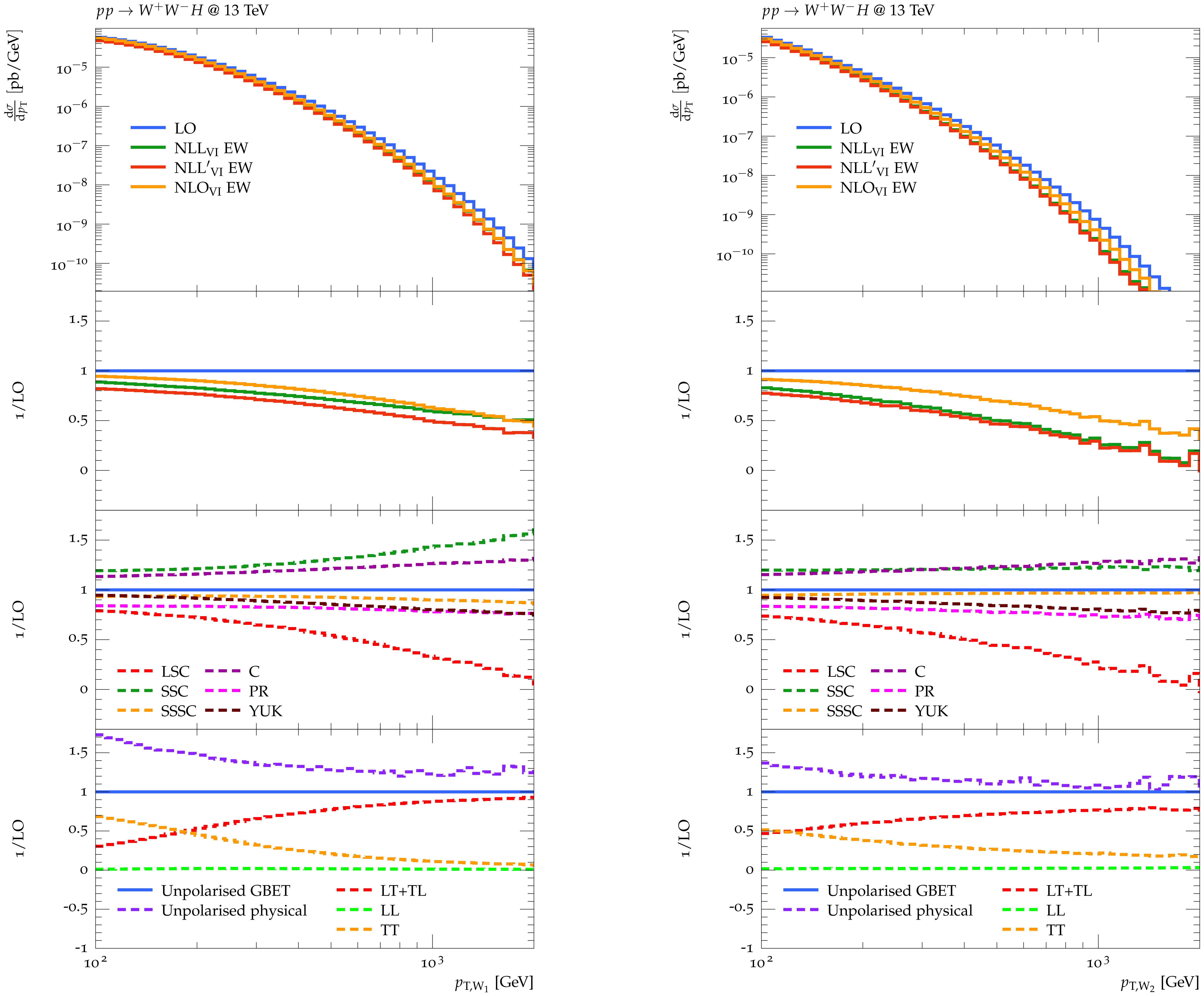
Here **LT** and **TL** polarisation configurations are mass-suppressed while mixed **TT** and **LL** are not.

However, **LT** and **TL** are several orders of magnitude smaller than both **TT** and **LL**.

Within this setup, *Sudakov* approximation can be directly employed for these observables

NLO QCD: [Mao et al, 0903.2885; 2009]
Full **NLO**: [Alwall *et al,* 1405.0301; 2014]
NLO QCD PS: [Baglio, 1609.05907; 2016]

Results: $pp \to W^+W^-H$



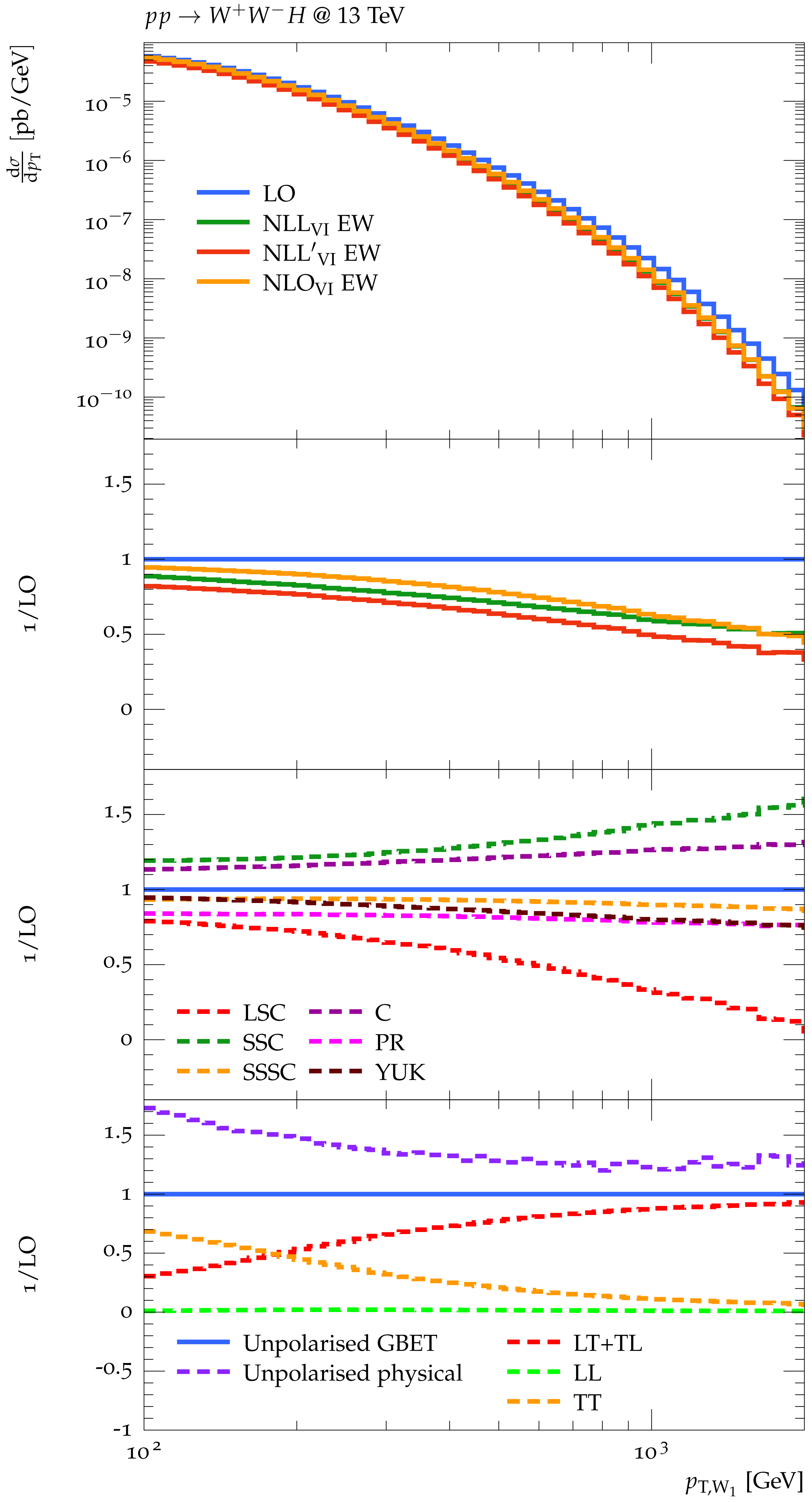
Here **TT** and **LL** polarisation configurations are mass-suppressed while mixed **LT** and **TL** are not

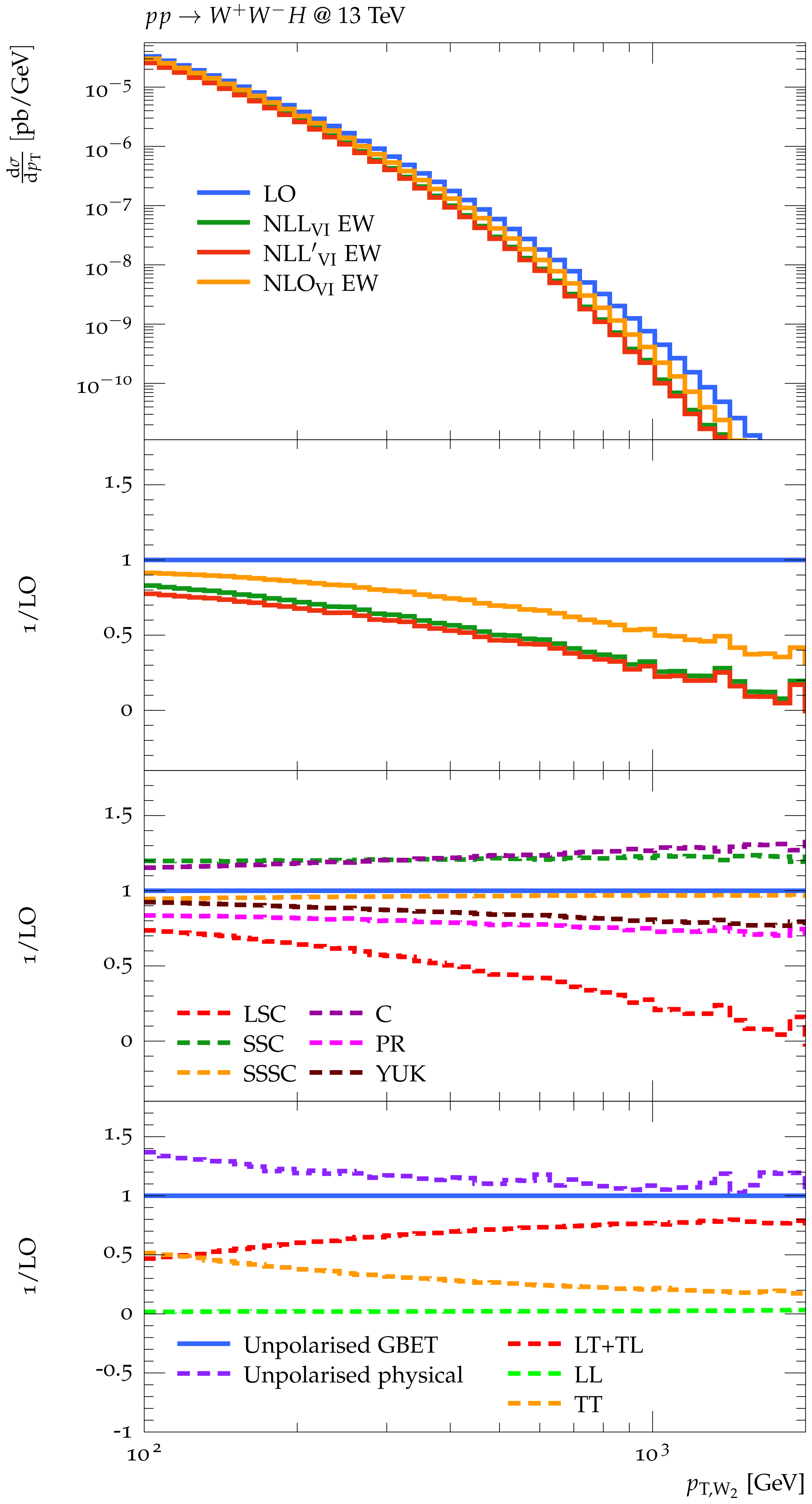
Small but sizeable contribution to the LO coming from **TT**. In the tail

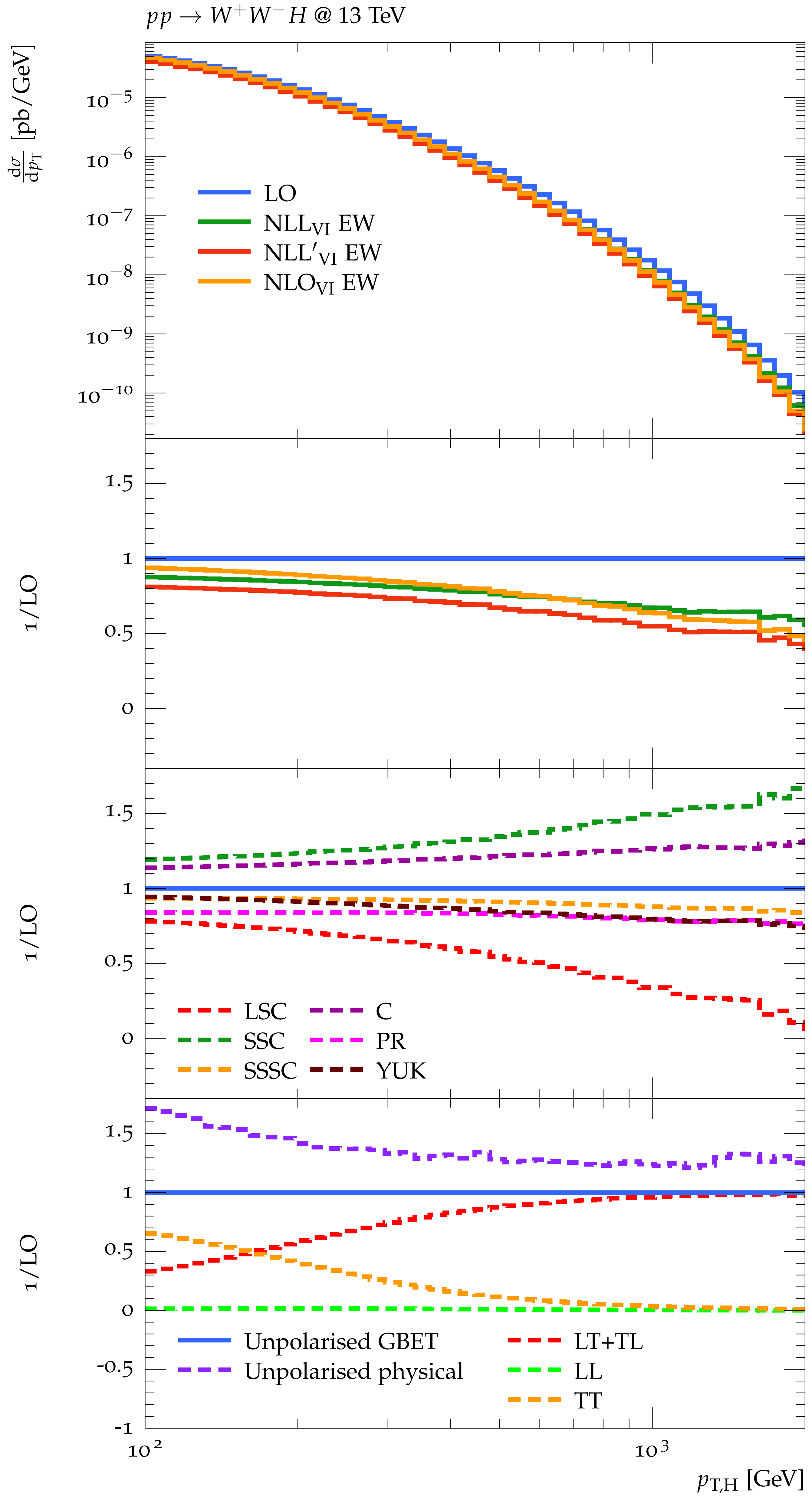
- $p_{\mathrm{T,W_1}}$: $\sim 5\,\%$
- $p_{\mathrm{T,W_2}}$: $\sim 15\,\%$

Within this setup, *Sudakov* approximation cannot be directly employed for these observables

Results: $pp \to W^+W^-H$







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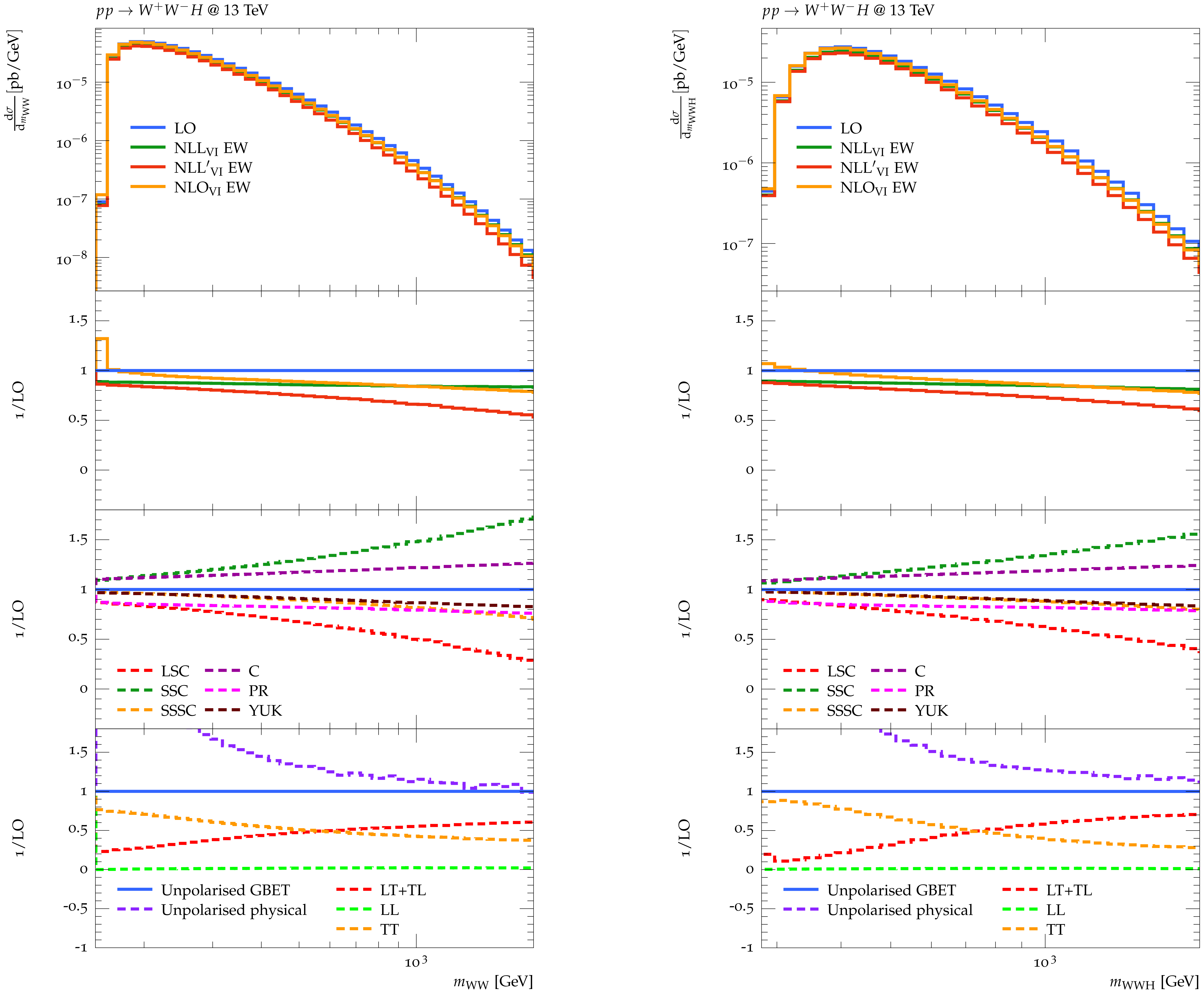
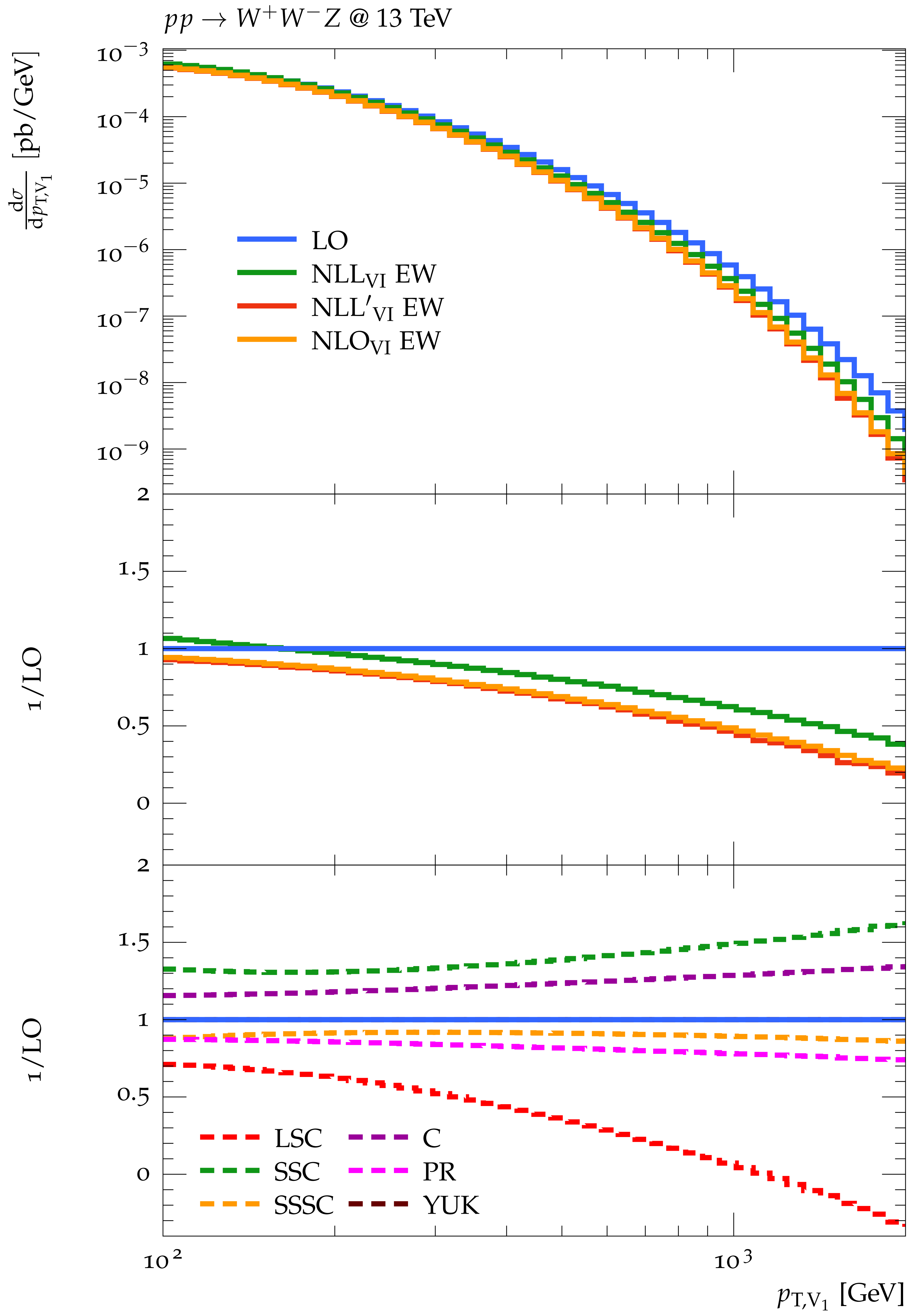


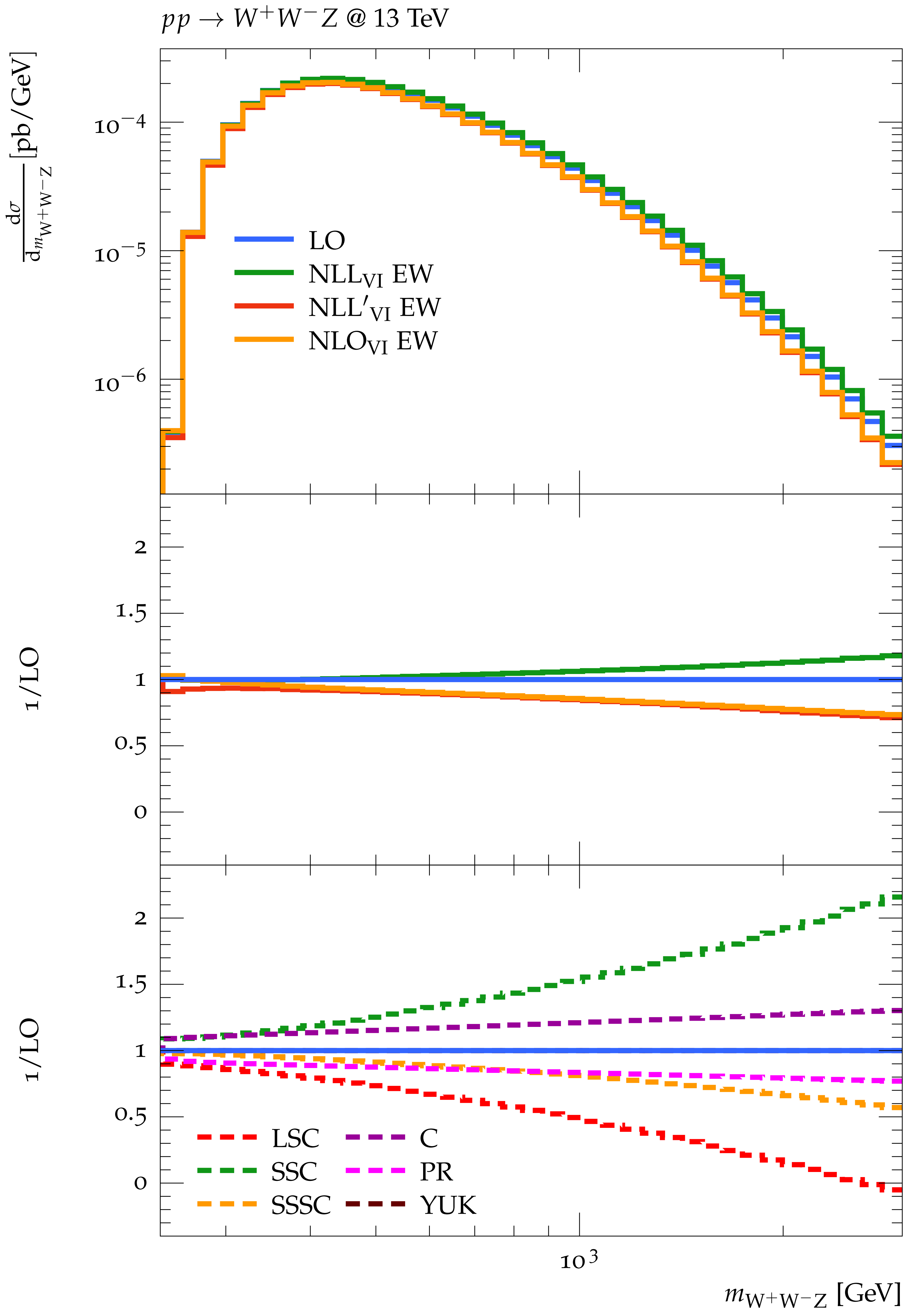
Too big contribution to the LO coming from the mass-suppressed **TT** fraction, around $30 - 40\,\%$

Significantly higher energies are required to further suppress **TT** and apply the *Sudakov* approximation

Less appealing solution: systematically derive and implement all mass-suppressed corrections

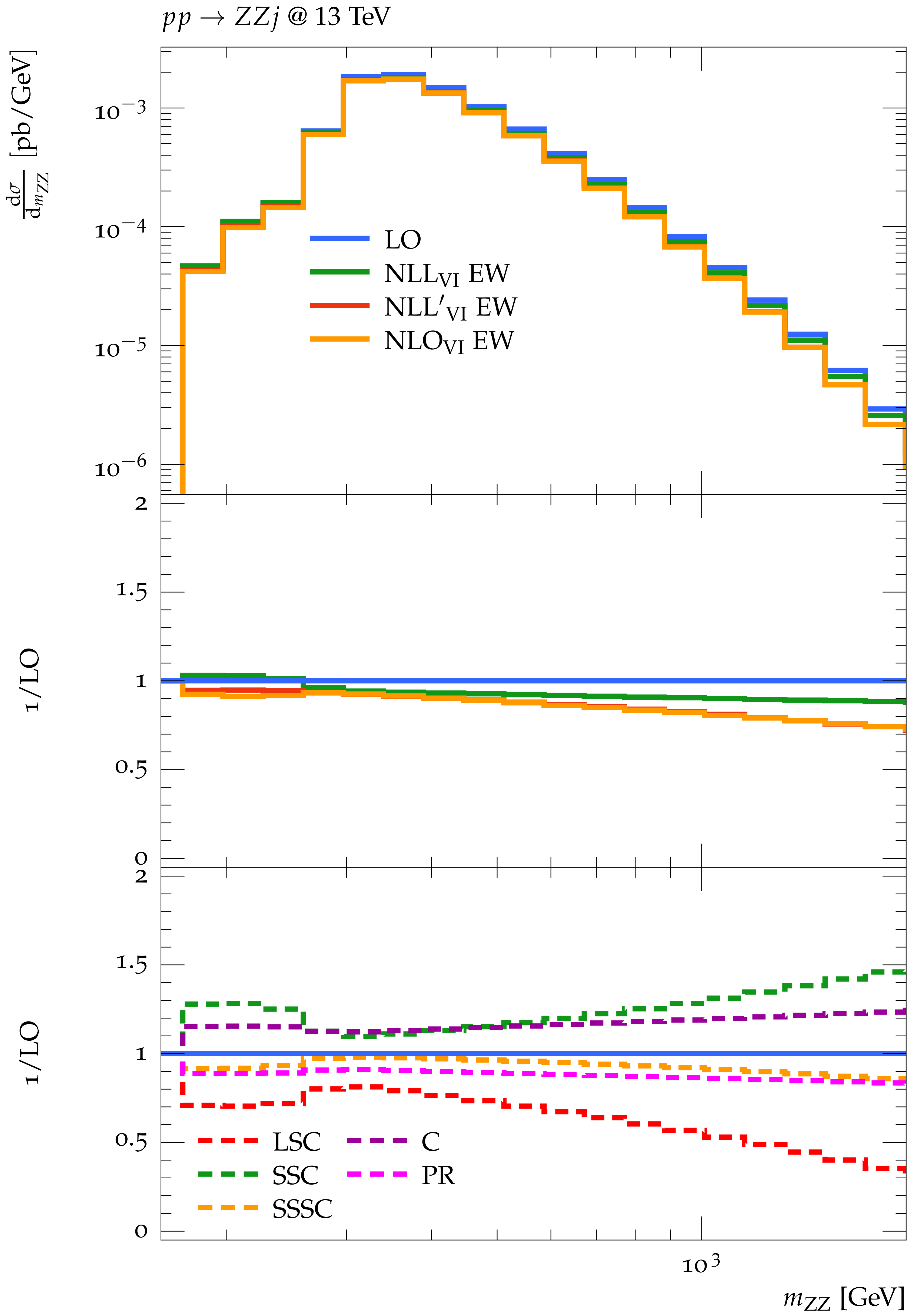
Results: $pp \to W^+W^-Z$

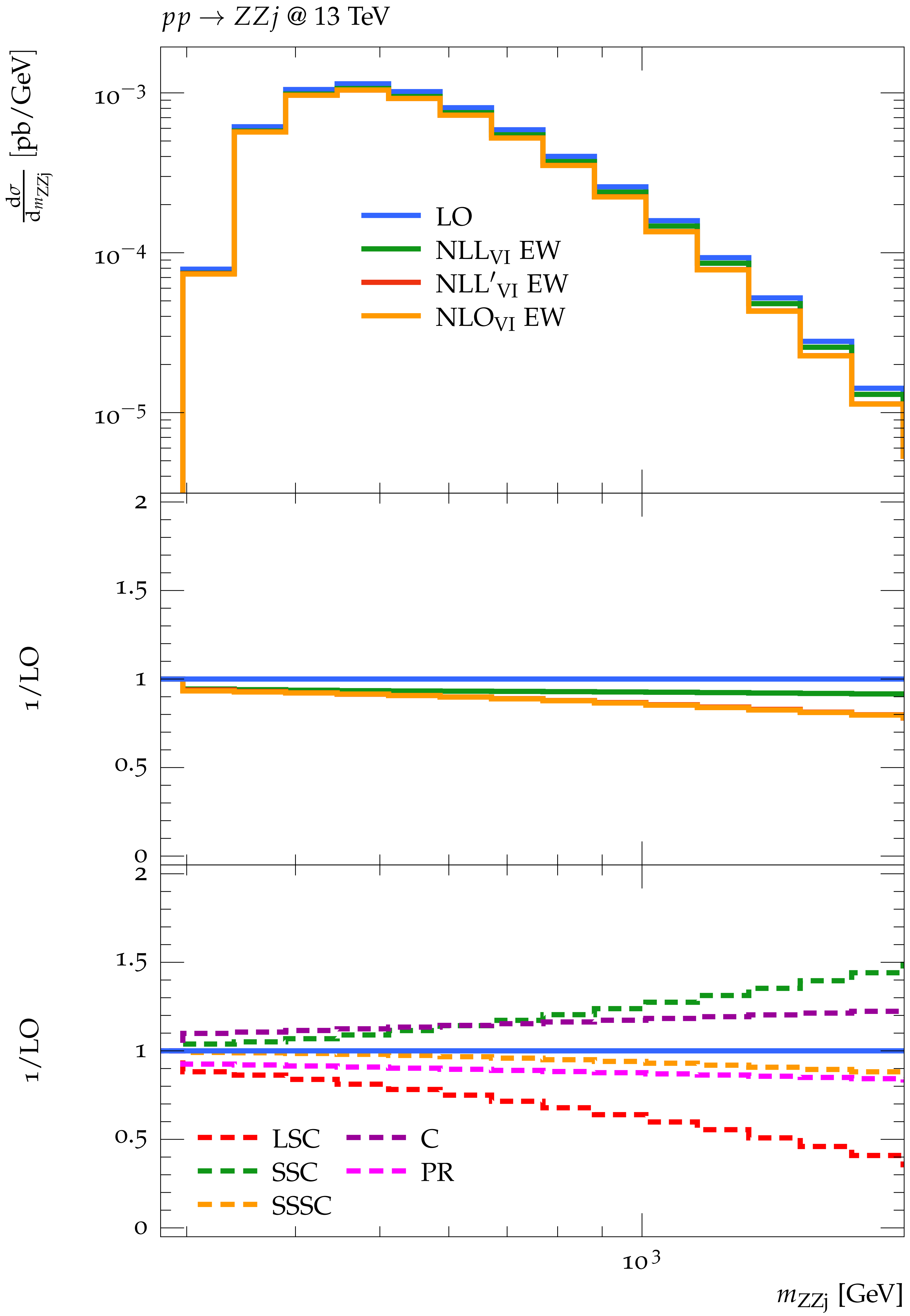




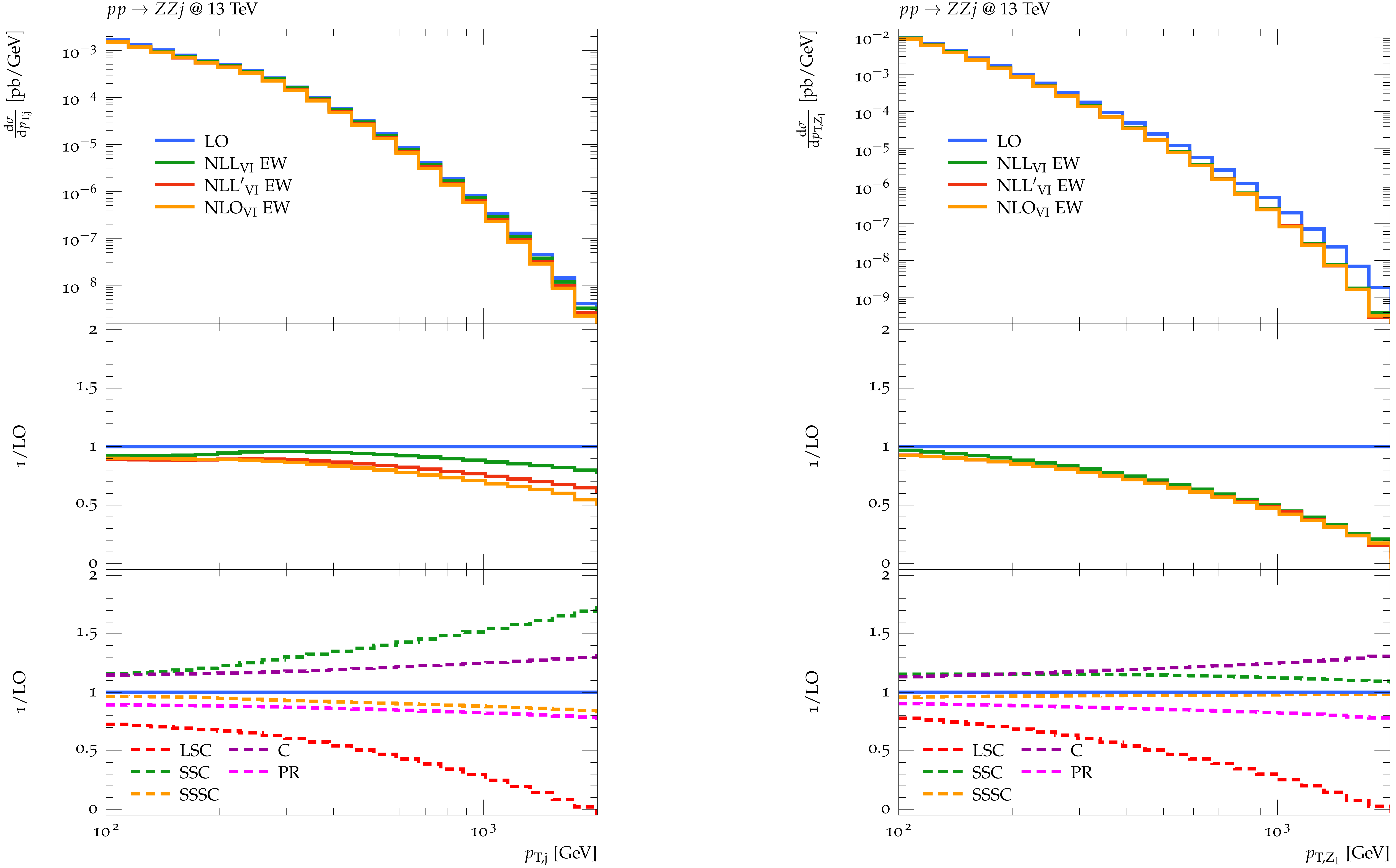
The inclusion of **SSSC** provides better predictions, but there is no full control on it! (Non-universal) **SSSC**-like terms arise also from LA of 4-point functions

Results: $pp \to ZZj$

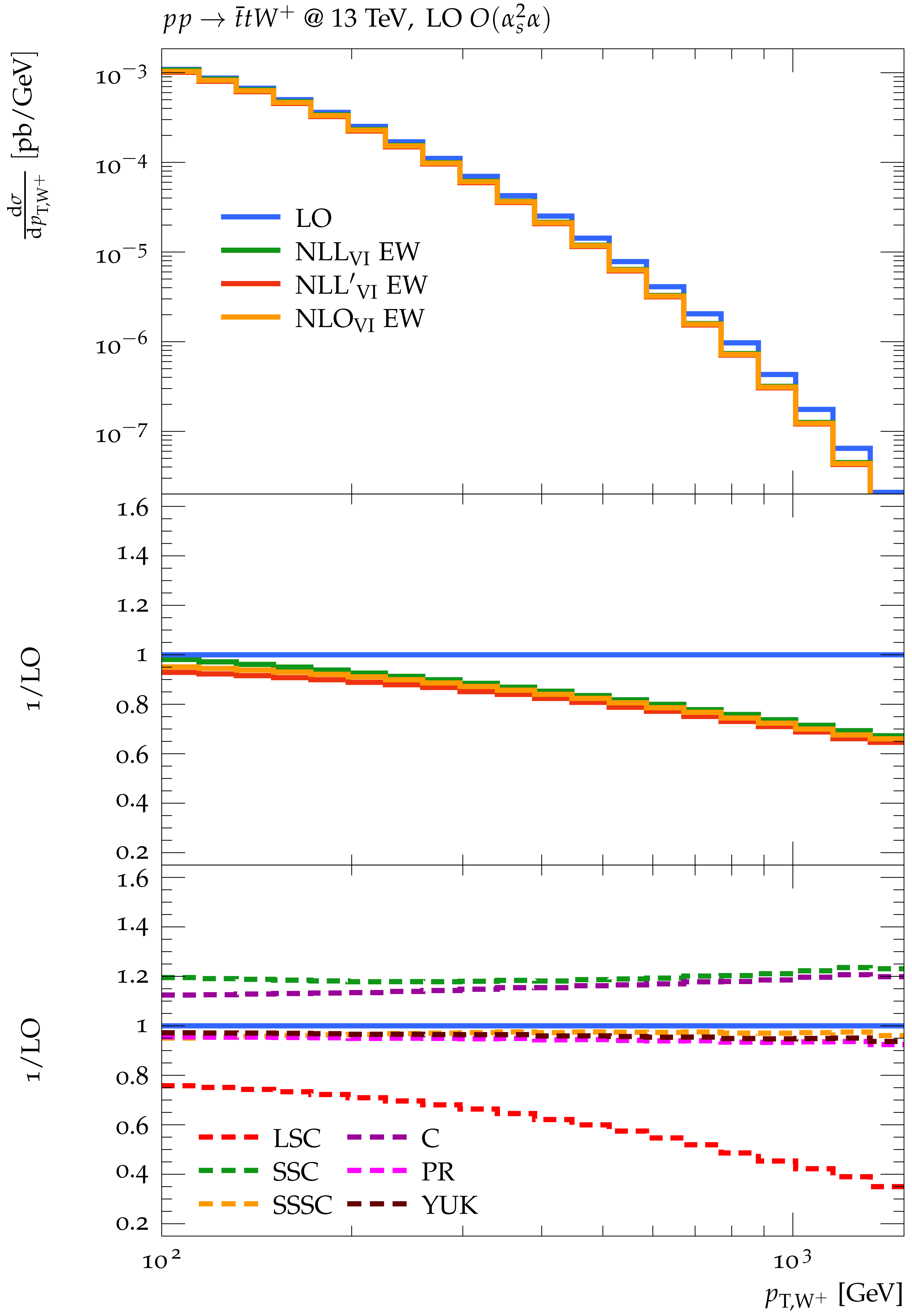


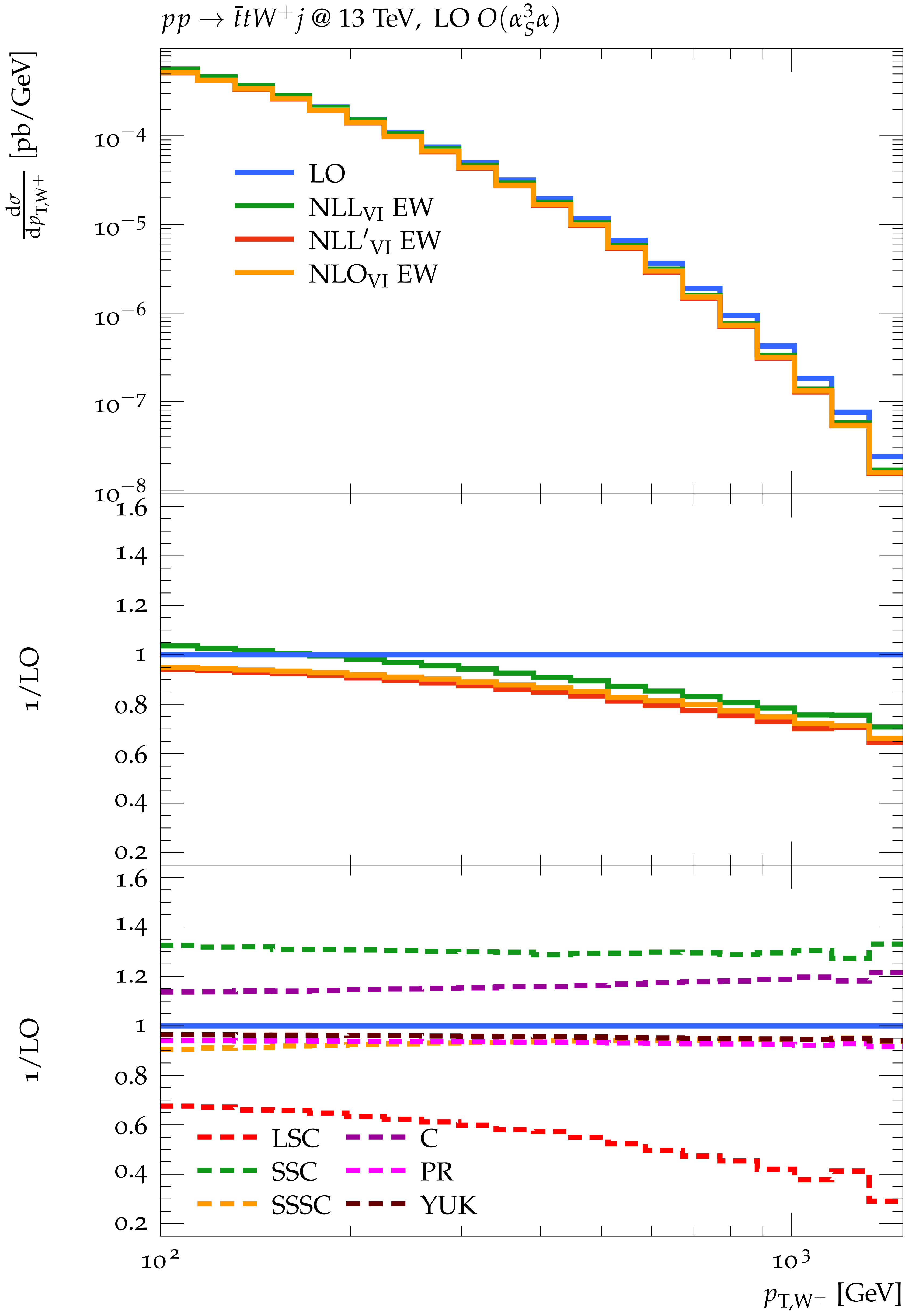


Results: $pp \to ZZj$



Results: $pp \to ttW^+$ & $pp \to ttW^+j$



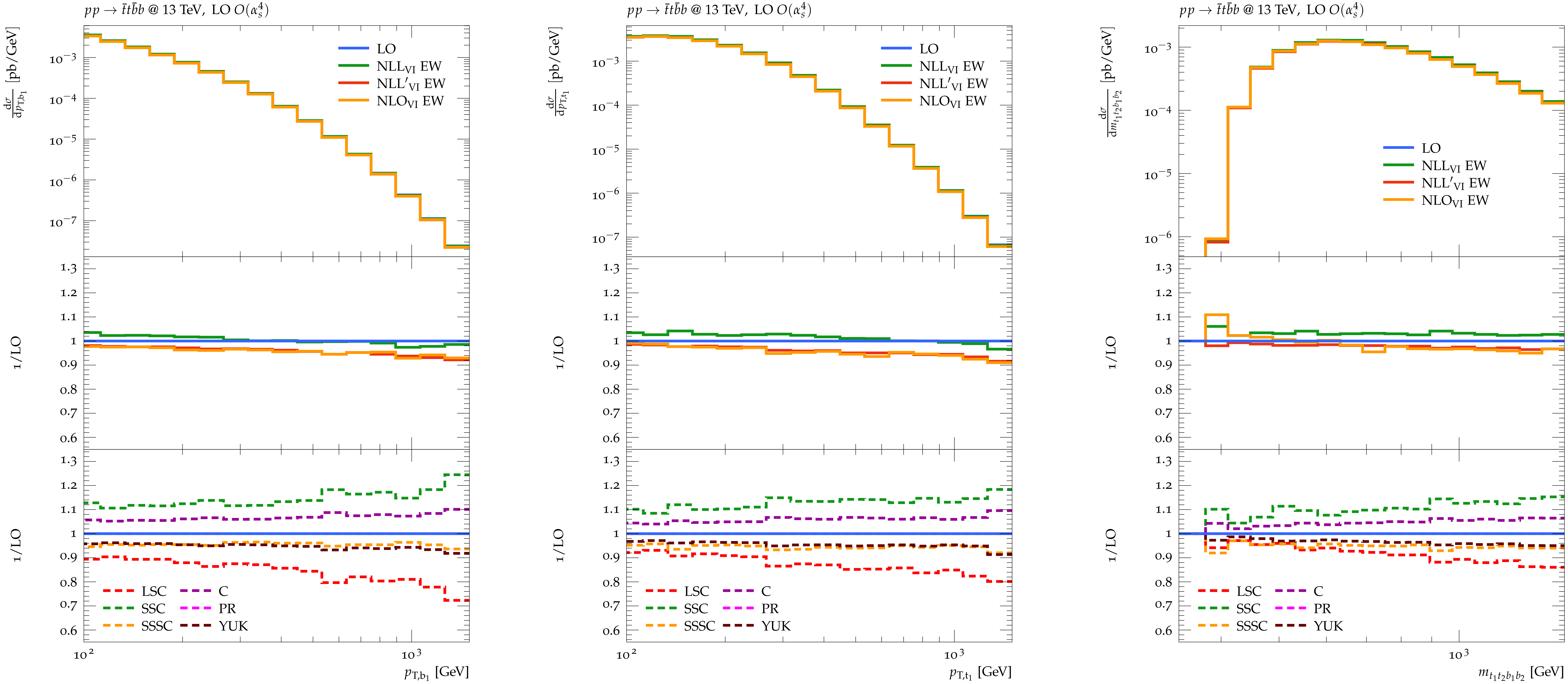


Results: $pp \to t\bar{t}b\bar{b}$

NLO QCD: [Bredenstein *et al*, 0807.1248; 2008]

NLO QCD to $t\bar{t}b\bar{b}j$: [Buccioni *et al*, 1907.13624; 2019]

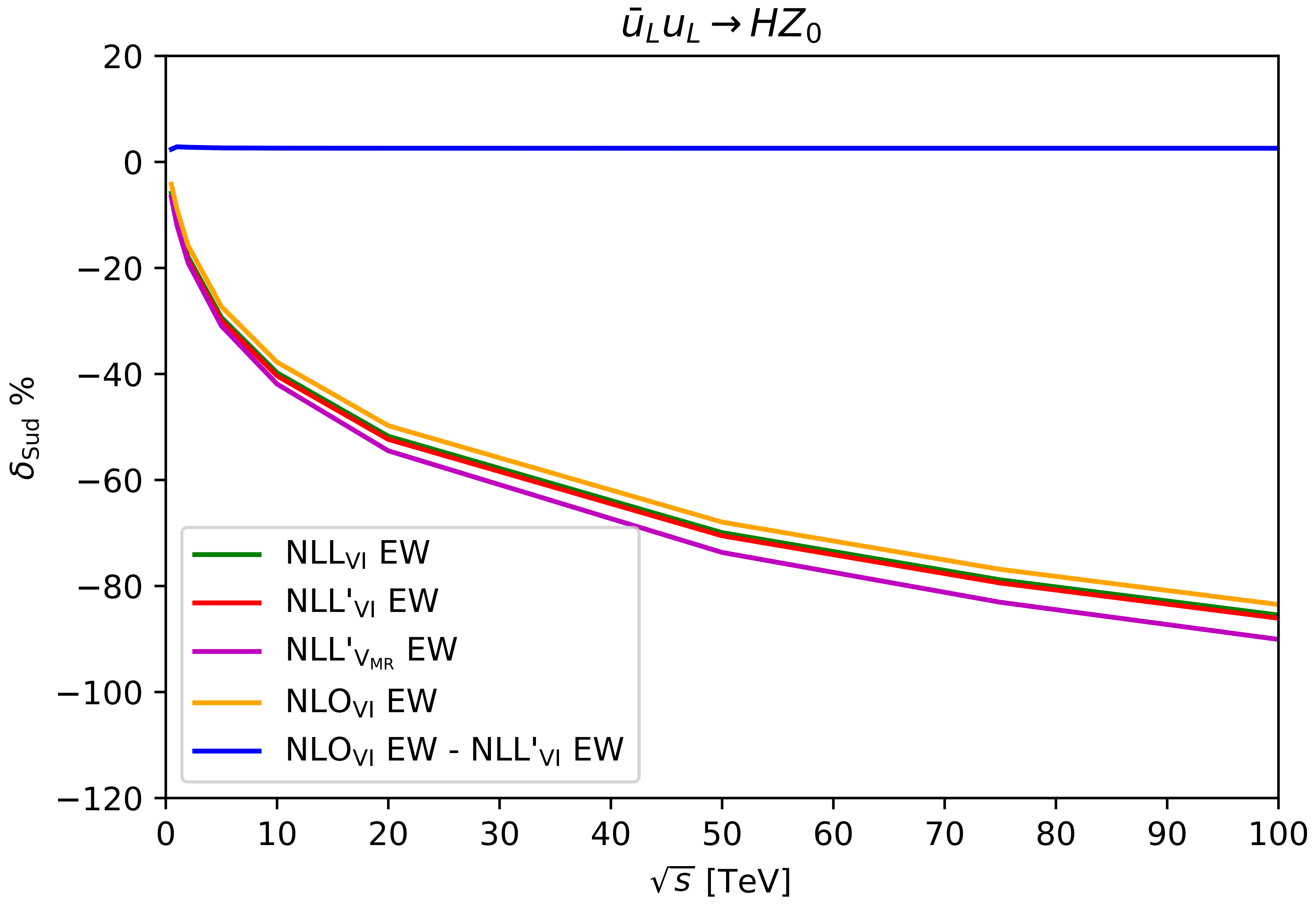
Status: [CMS collaboration, 2309.144422; 2023]

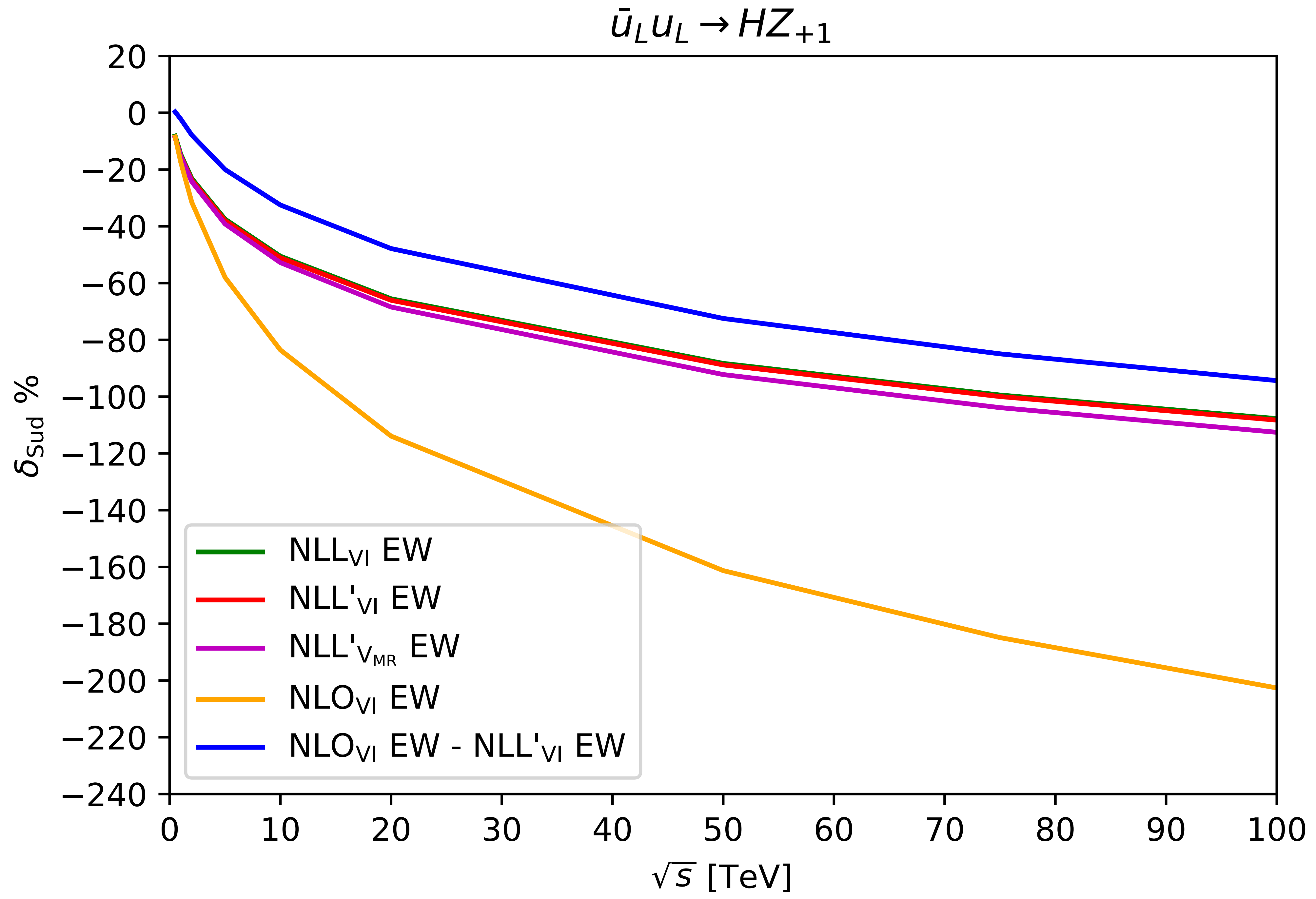


NLO EW never computed before and expected to be small. We explicitly checked and verified it, observing $\sim 6-7\,\%$ @ $p_T \approx 1$ TeV

Still a preliminary analysis! A more detailed study of **NLO EW** corrections will follow

Amplitude-level validation: $\sqrt{s}$ scan





- In *Sudakov* approximation: keep only double and singular logarithmic corrections

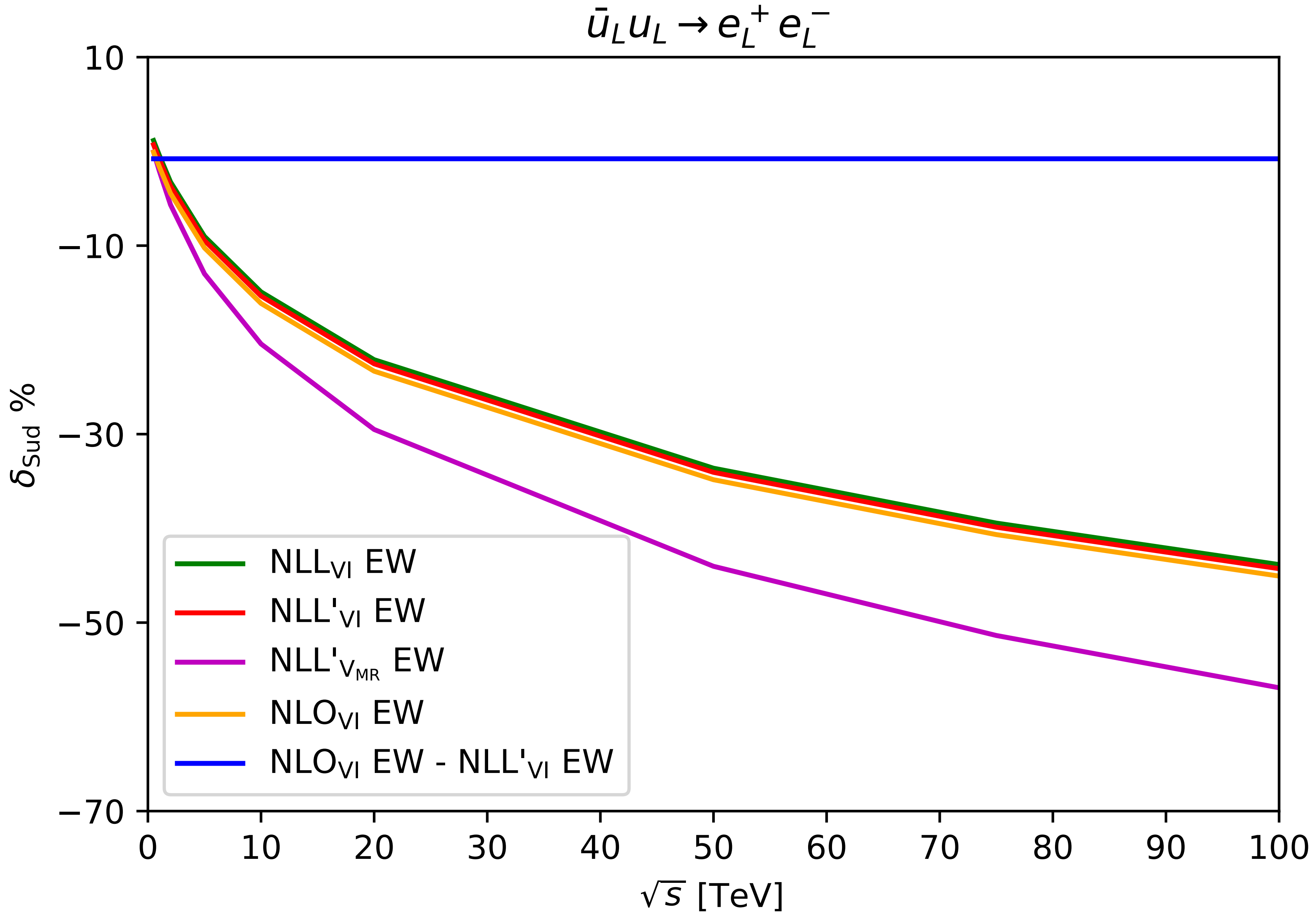
$$\delta^{\mathrm{DL}}\mathcal{M}^{\varphi_{i_1}\ldots\varphi_{i_n}} \sim E^d L \qquad \delta^{\mathrm{SL}}\mathcal{M}^{\varphi_{i_1}\ldots\varphi_{i_n}} \sim E^d l$$

neglecting constant $(\sim \alpha E^d)$ and mass suppressed $(\sim M^n E^{d-n} L)$ terms

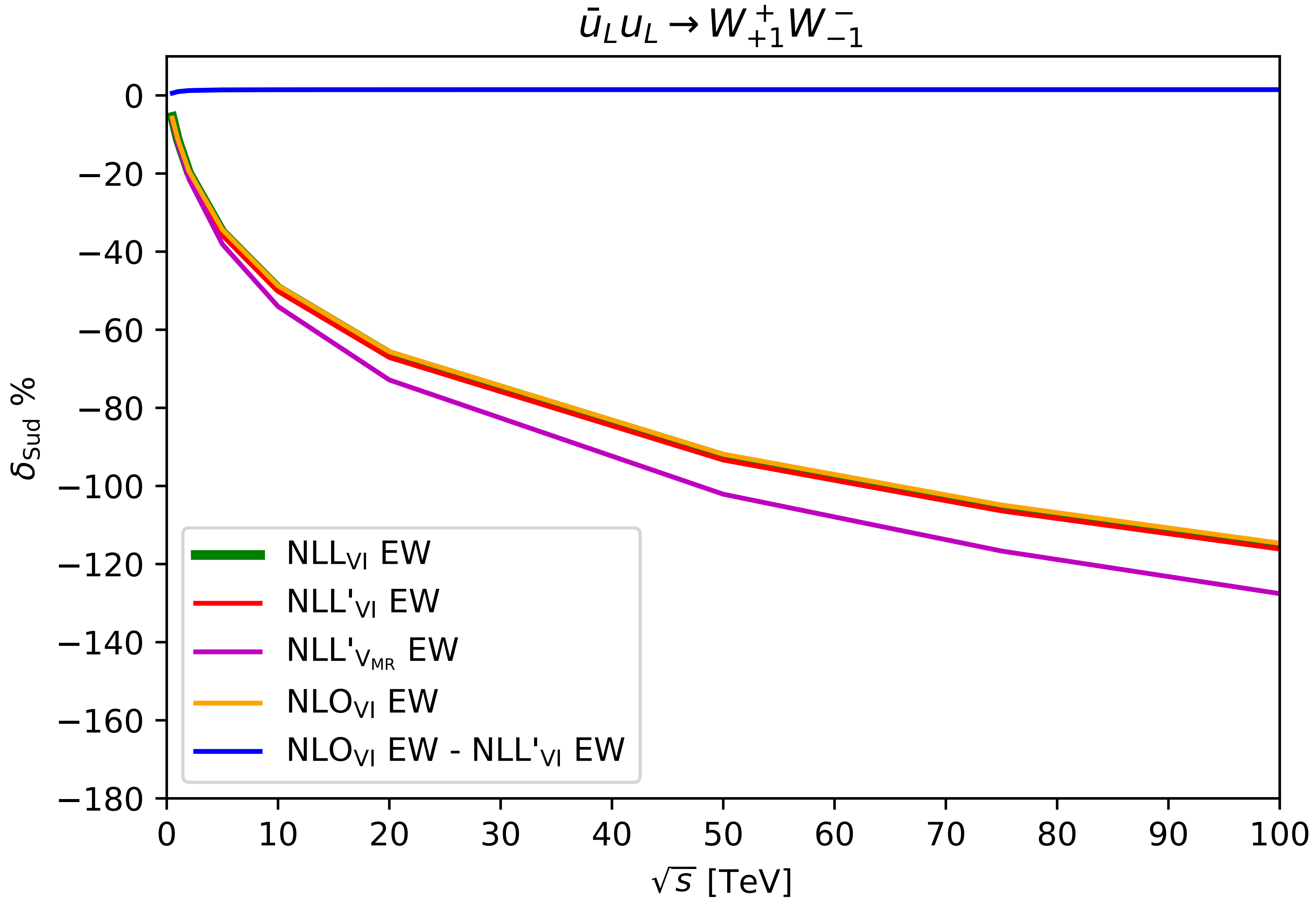
- In the high energy limit and for non mass-suppressed [2] matrix elements we expect $\mathrm{NLO}_{\mathrm{VI}}\ \mathrm{EW} - \mathrm{NLL'}_{\mathrm{VI}}\ \mathrm{EW} \propto\ const$

[2] NB: non mass-suppressed configurations scale like $\sim \sqrt{s}^{4-n}$

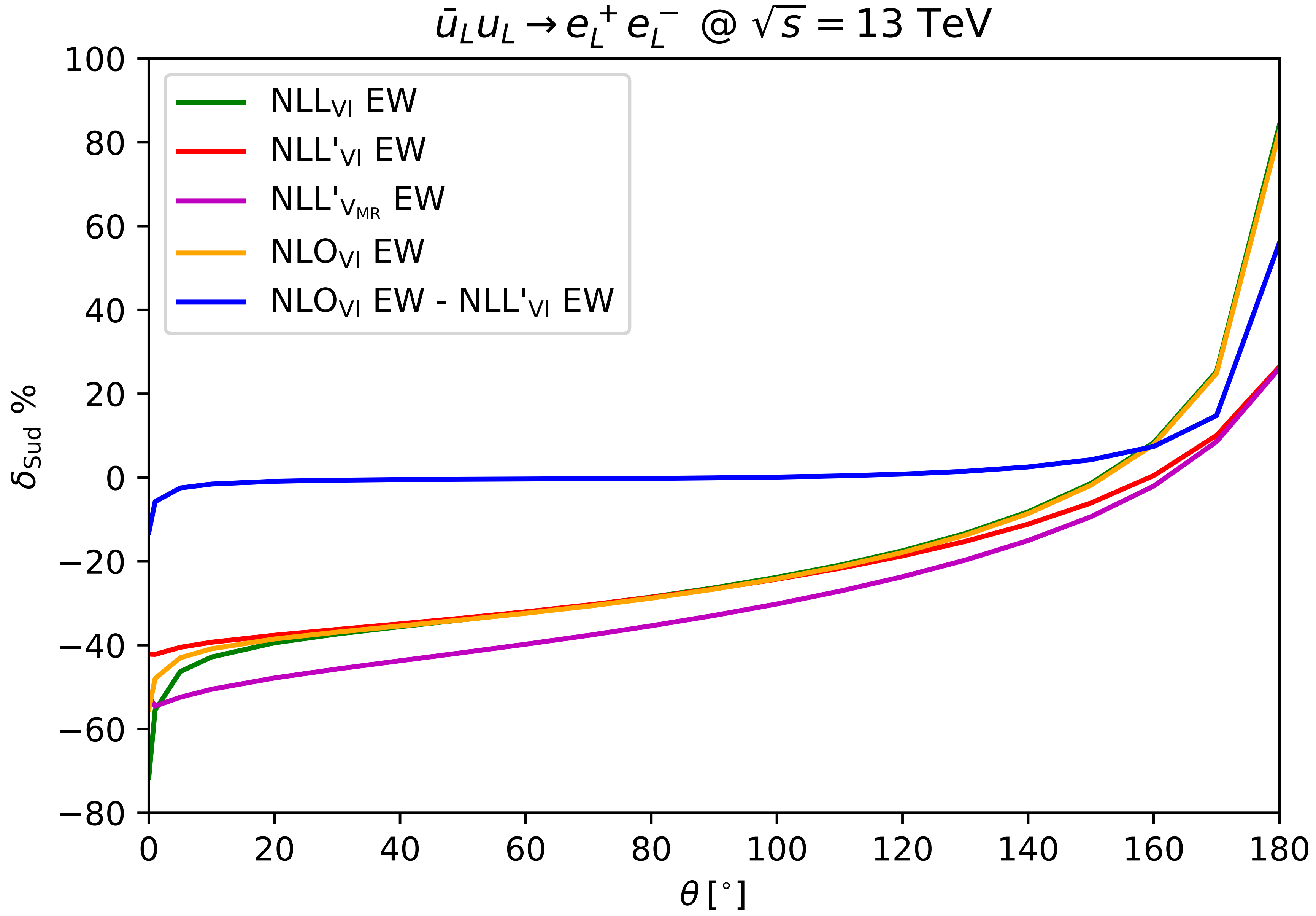
Amplitude-level validation: $\sqrt{s}$ and θ scans



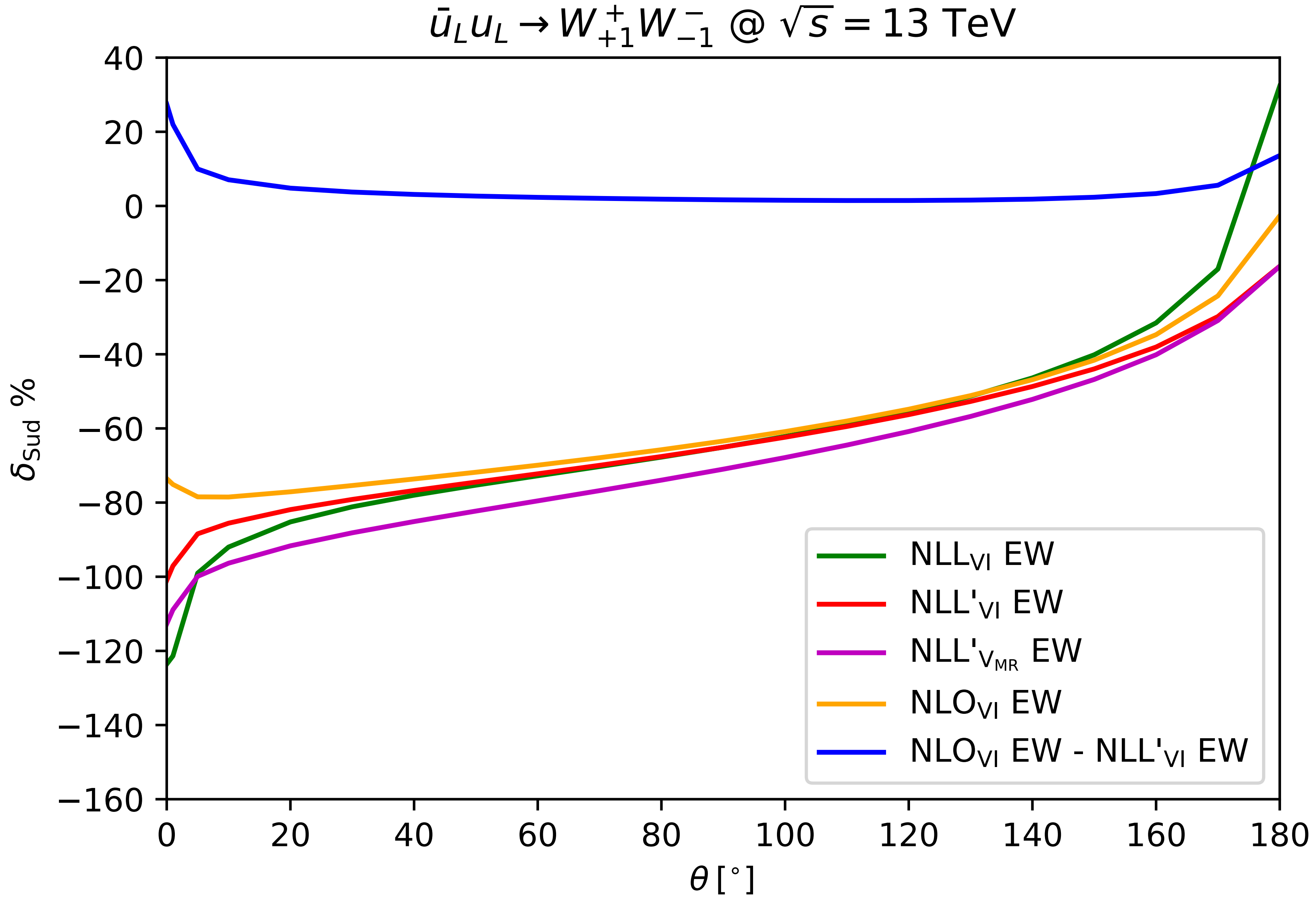
$\bar{u}_L u_L \to e_L^+ e_L^-$



$\bar{u}_L u_L \to W_{+1}^+ W_{-1}^-$



$\bar{u}_L u_L \to e_L^+ e_L^-$ @ $\sqrt{s} = 13$ TeV



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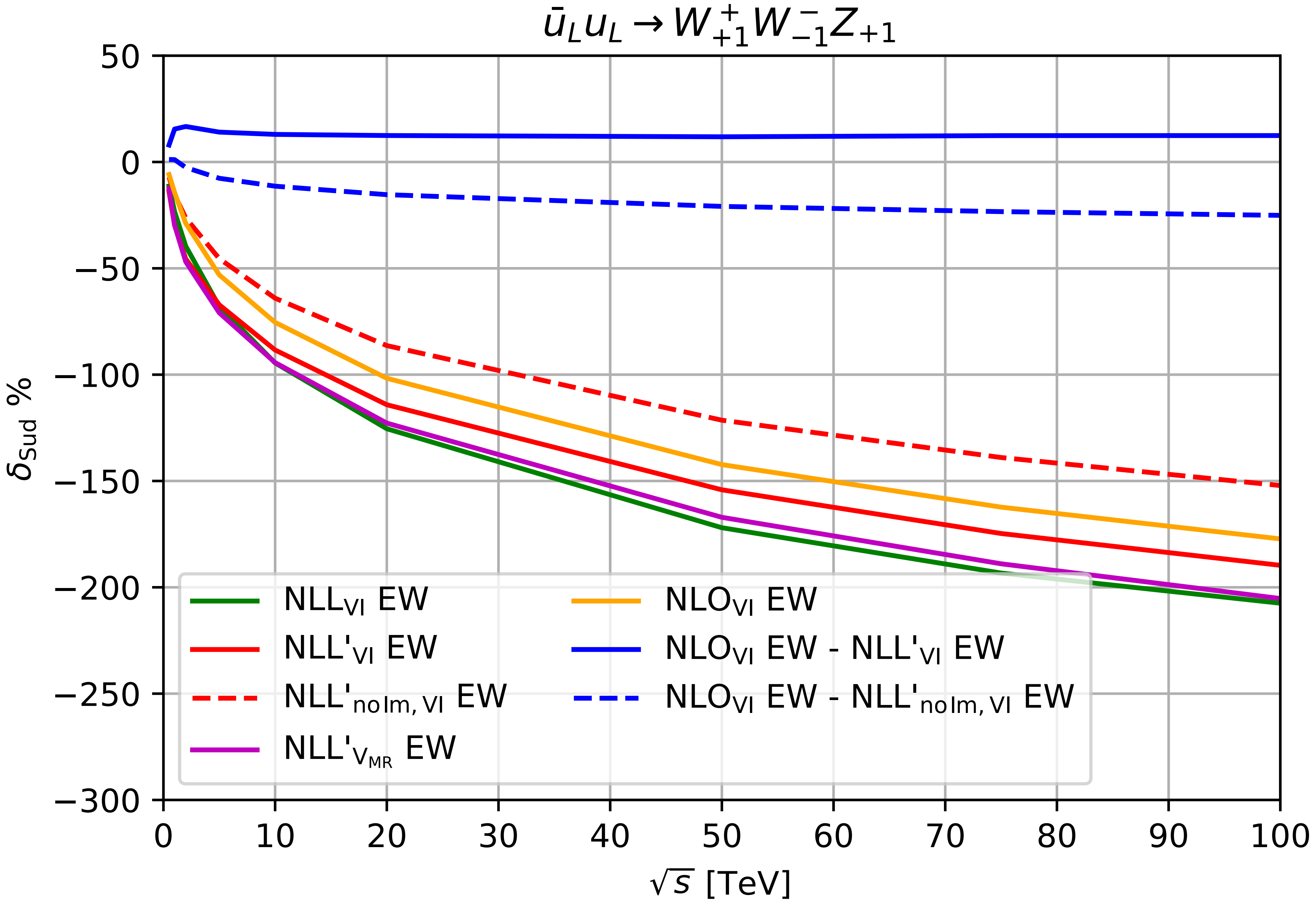
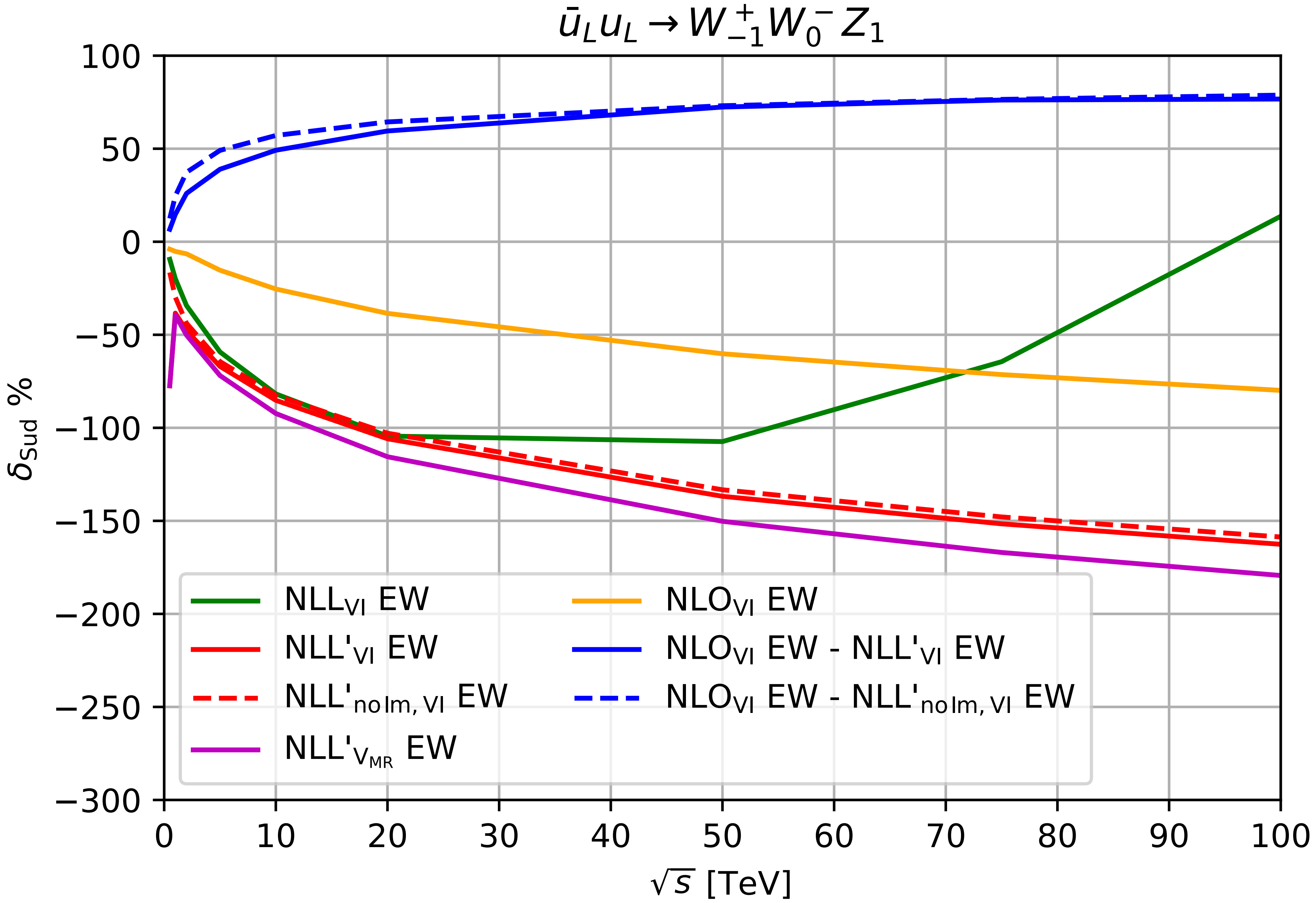
Amplitude-level validation: $\sqrt{s}$ scan





- In the high energy limit and for non mass-suppressed matrix elements we expect $\mathrm{NLO_{VI}\ \ EW - NLL'_{VI}\ \ EW} \propto \ const$

- Inclusion of the phase in DL from the LA of C_0, i.e.

$$C_0|_{\mathrm{LA}} \propto \left[\log^2 \frac{|r_{kl}|}{M_V^2} - 2i\pi\Theta(r_{kl}) \log \frac{|r_{kl}|}{M_V^2}\right]$$

is crucial in $2 \to n$ processes with $n \geq 3$: without phase $\mathrm{NLO_{VI}\ \ EW - NLL'_{VI}\ \ EW}$ shows a logarithmic dependence. This has been firstly noticed in [Pagani, Zaro 2110.03714; 2021]