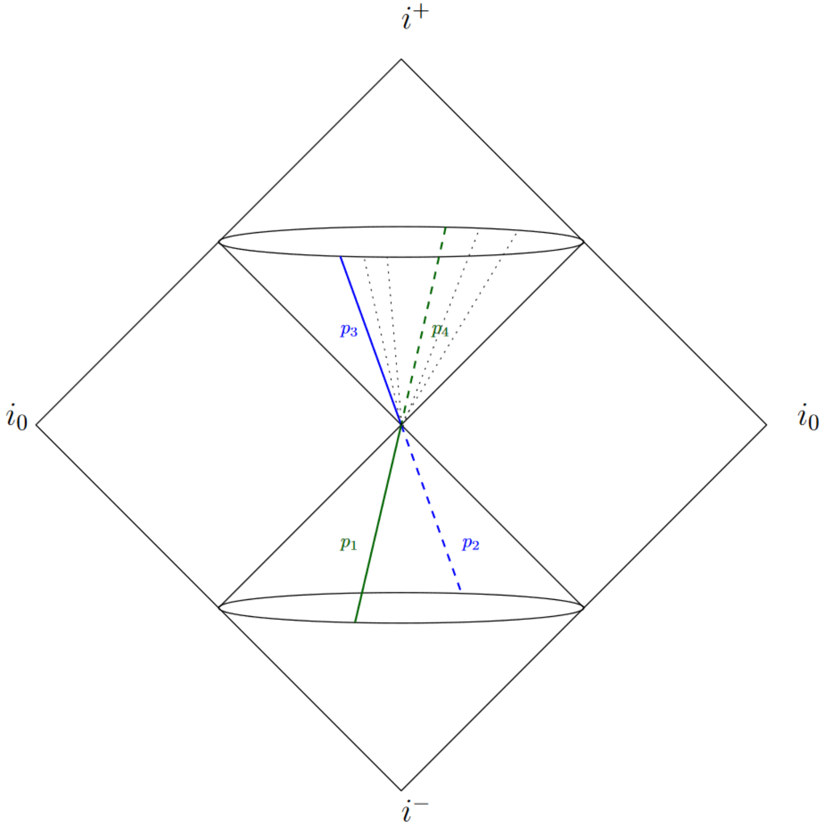
Minimal set of variables and high-energy building blocks at high multiplicity

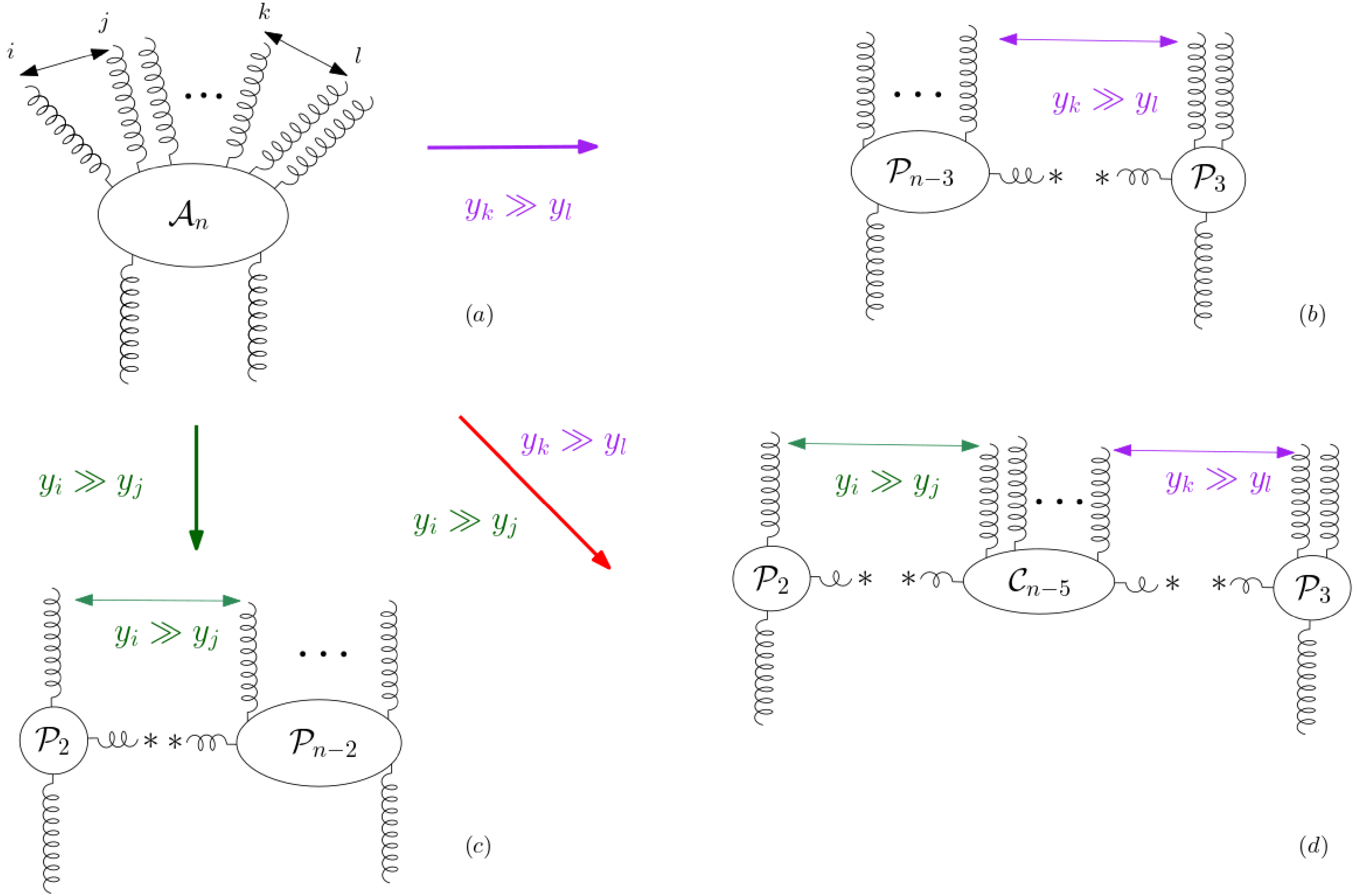


Joint work with

Emmet P. Byrne, Vittorio Del Duca, Einan Gardi, Jennifer M. Smillie

High-energy building blocks from Amplitudes

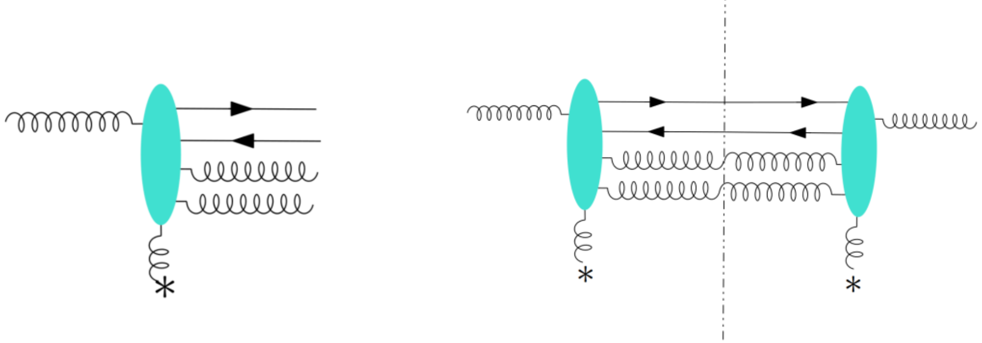
$$p_i = (p_i^+, p_i^-, p_{i\perp}) = |p_{i\perp}|\left(e^{y_i}, e^{-y_i}; \cos\phi_i, \sin\phi_i\right)$$



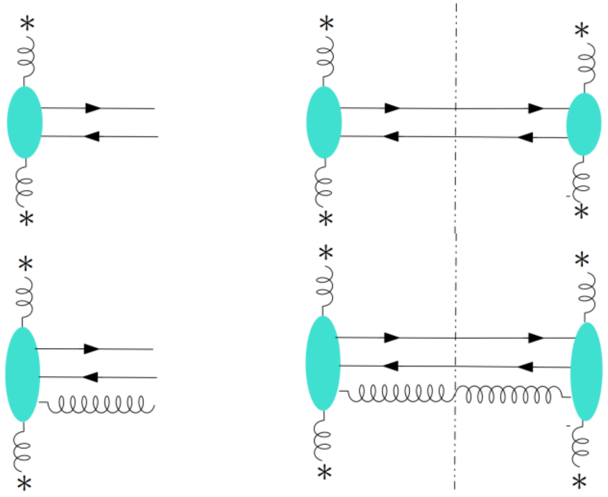
High-energy building blocks:
CEV: $\mathcal{C}$,
PEV: $\mathcal{P}$

Motivation

- Contribute to NNNLL Jet impact factor



- Prepare complete set of PEVs and CEVs for high logarithmic correction (radiative) for BFKL program (kernel)



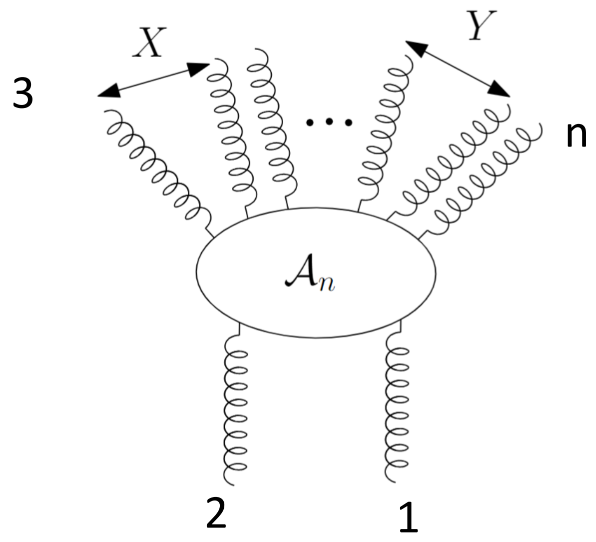
- The amplitude-like identity, photon decoupling, Kleiss Kuijf, SUSY Ward Identities

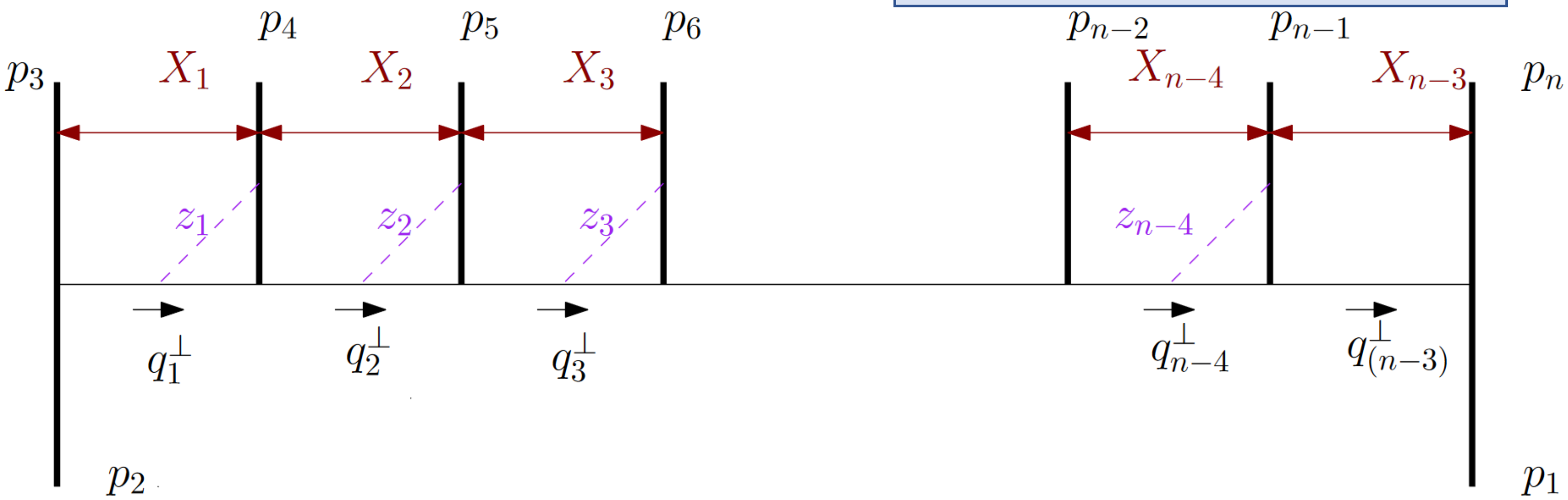
- Summarizing all known PEVs and CEVs and tabulating them to Mathematica files (using minimal set of variables)

Extraction of high energy building block:

Expressing amplitude in suitable minimal set of variables

In total **3(n-3)** variables to describe n point amp with spin





Suitable minimal set of variable

- Separate longitudinal and transverse
- Convenient for factorization
- Free of spurious pole

$$X_i \equiv \frac{p_{i+2}^+}{p_{i+3}^+}, \qquad z_i = -\frac{q_{(i+1)}^\perp}{p_{(i+3)}^\perp}, \qquad \mu \equiv q_1^\perp;$$

$$X_i \in \mathbb{R}^+ \text{ and } z_i, q_1^\perp \in \mathbb{C}$$

$$p_i = |p_{i\perp}| \left(e^{y_i}, e^{-y_i}; \cos\phi_i, \sin\phi_i\right)$$

In practice, we do

Spinor product

Mandelstam invariants

→

Momentum components

i.e. p^+ p^- $p^\perp$ $p^{\perp *}$

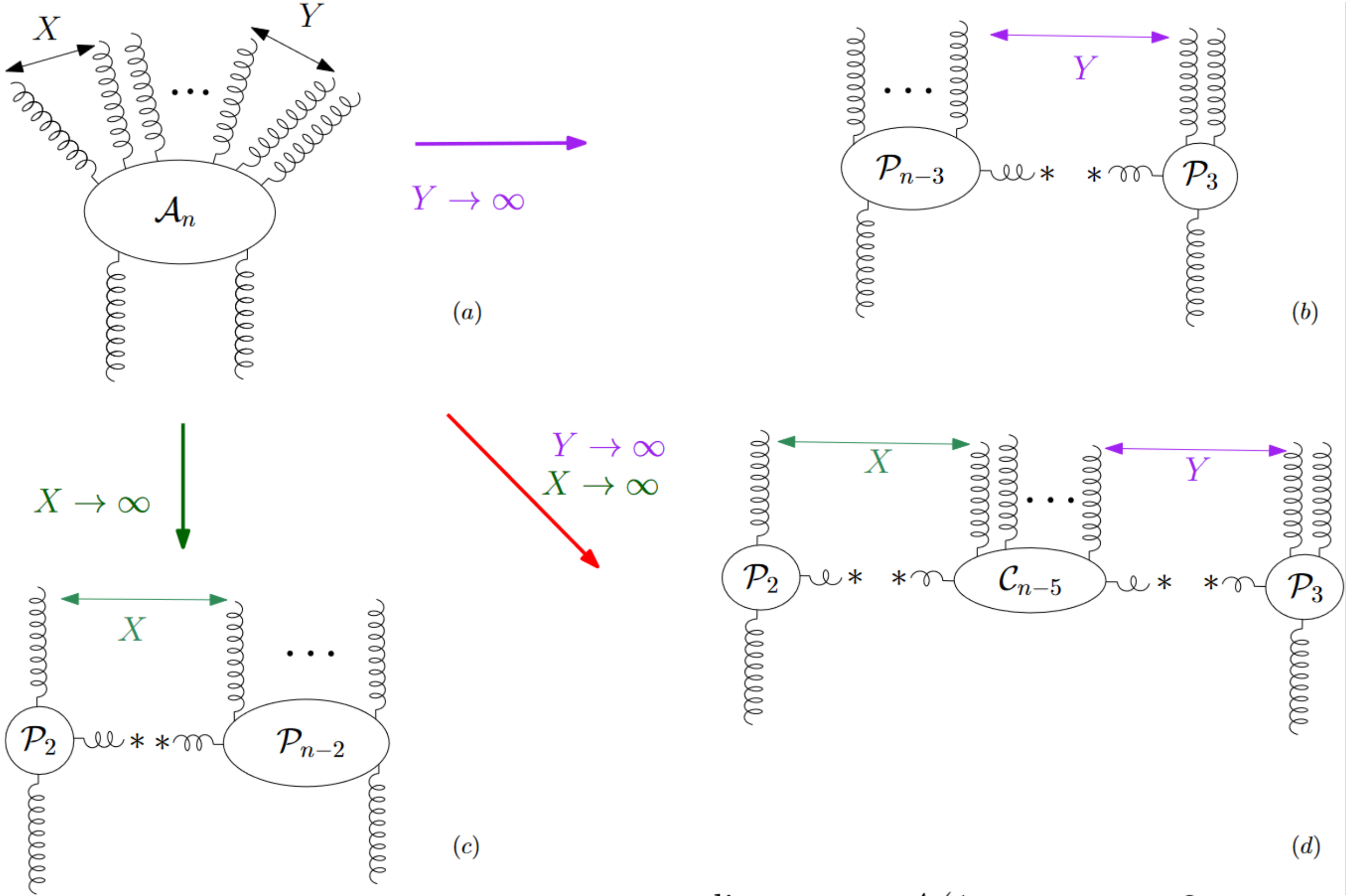
→

Minimal set of variables

X, z $q_1^\perp$ $q_1^{\perp,*}$

Convenient for high energy factorization

The large rapidity difference Δy corresponds to $X \to \infty$



$$p_i = (p_i^+, p_i^-, p_{i\perp}) = |p_{i\perp}| \left(e^{y_i}, e^{-y_i}; \cos\phi_i, \sin\phi_i\right)$$

$$X_i \equiv \frac{p_{i+2}^+}{p_{i+3}^+}, \qquad z_i = -\frac{q_{(i+1)}^\perp}{p_{(i+3)}^\perp}, \qquad \mu \equiv q_1^\perp;$$

The extraction formula CEV

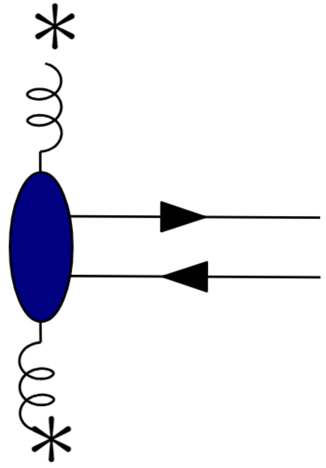
$$\lim_{\substack{X_1 \to \infty, \\ X_{n-3} \to \infty,}} A(1, \sigma_1, \ldots, \sigma_m, 2, \sigma_{m+1}, \ldots, \sigma_n) = s \frac{1}{t_1} \mathcal{P}(p_2^{\nu_2}, p_3^{\nu_3}) \frac{1}{t_{m+n+1}} \mathcal{P}(p_1^{\nu_1}, p_{m+n+4}^{\nu_{m+n+4}}) \times$$
$$\mathcal{C}_{b_m}\left(p_{\sigma_1}^{h_{\sigma_1}}, p_{\sigma_2}^{h_{\sigma_2}}, \ldots, p_{\sigma_m}^{h_{\sigma_m}}; (g_{q_1}^*), p_{\sigma_{m+1}}^{h_{\sigma_{m+1}}}, p_{\sigma_{m+2}}^{h_{\sigma_{m+2}}}, \ldots, p_{\sigma_n}^{h_{\sigma_n}}, (g_{q_{n+1}}^*)\right) \tag{1}$$

Convenient for high energy factorization

Some results: parton colour ordered CEV

$$\mathcal{C}_a\left(g^*,\bar{q}_4^{\ominus},q_5^{\oplus},g^*\right)$$

$$= -\frac{X_2^{5/2}z_1^3\left(\bar{z}_1-1\right)\bar{z}_1\bar{z}_2}{\left(X_2z_1+1\right)\left(X_2z_1\bar{z}_1+1\right)\left(X_2z_1\bar{z}_1-\bar{z}_2+1\right)} + \frac{X_2^{3/2}\left(z_1-1\right)z_1z_2\bar{z}_1\bar{z}_2}{\left(X_2+1\right)\left(\bar{z}_1+\bar{z}_2-1\right)\left(X_2z_1+1\right)\left(X_2z_1-z_2+1\right)} - \frac{X_2^{3/2}\left(z_1-1\right)z_1z_2\left(\bar{z}_1-1\right)\bar{z}_1\left(\bar{z}_2-1\right)^3}{\left(\bar{z}_1+\bar{z}_2-1\right)\left(X_2\bar{z}_1-\bar{z}_2+1\right)\left(X_2z_1\bar{z}_1-\bar{z}_2+1\right)\left(X_2z_1\bar{z}_1+z_2\bar{z}_2-z_2-\bar{z}_2+1\right)}.$$



Colour in blue: physical poles

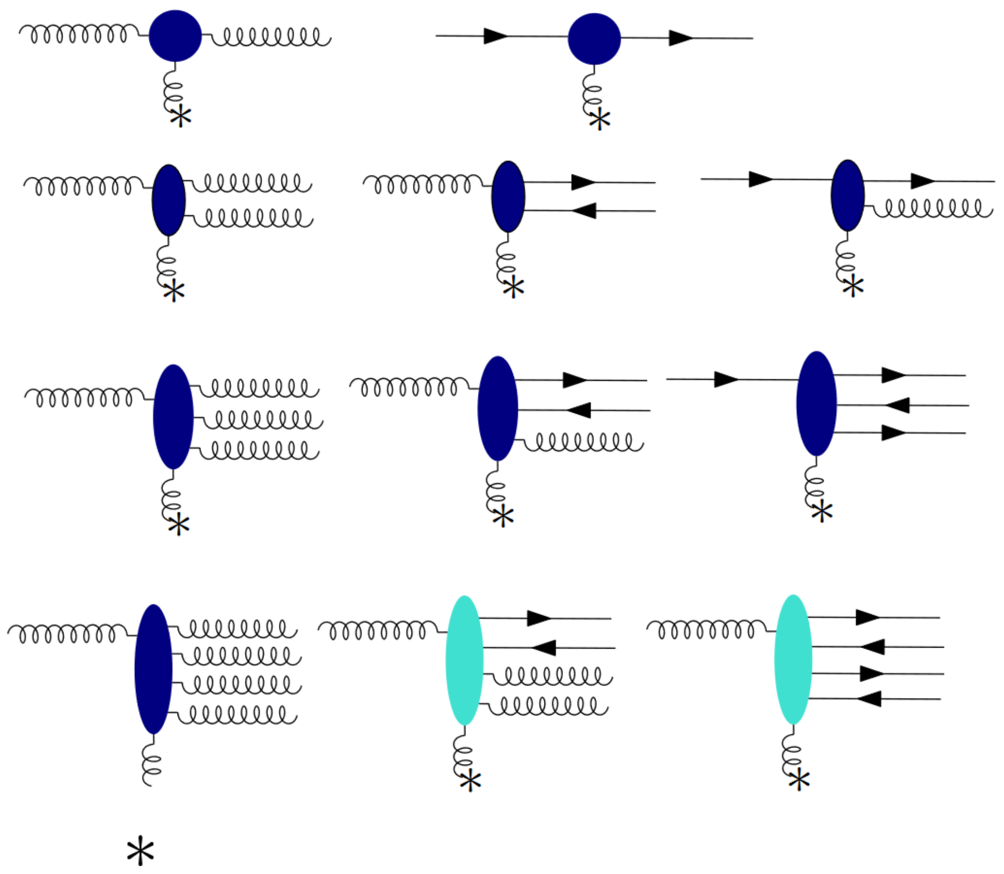
```
CEV[qP, qM][X2, q1⊥, q1⊥,*, z1, z̄1, z2, z̄2]//Together
```

$$\mathcal{C}\left(q_4^{\oplus},q_5^{\ominus}\right) = -\frac{1}{R_{\langle 45\rangle}R_{S_{234}}R_{S_{345}}R_{S_{456}}\bar{R}_{\langle 45\rangle}}X_2^{3/2}z_1\bar{z}_1\Big\{R_{S_{456}}X_2z_1^2\left(\bar{z}_1-1\right)\bar{z}_2R_{\langle 45\rangle} - z_2^2\left(\bar{z}_2-1\right)\left(R_{S_{456}}\left(z_1-1\right)+\bar{z}_2\left(z_1\left(X_2\left(\bar{z}_1-1\right)-1\right)+1\right)\right) + z_2\Big(X_2^2z_1\left(z_1\left(-\left(\left(\bar{z}_1-1\right)\left(\bar{z}_1-\bar{z}_2+2\right)\bar{z}_2\right)-1\right)+\left(\bar{z}_1-1\right)\bar{z}_2+1\right) + X_2\left(z_1+1\right)\left(\bar{z}_2-1\right)\left(z_1\left(\left(\bar{z}_1-1\right)\bar{z}_2+1\right)-1\right)-\left(z_1-1\right)\left(\bar{z}_2-1\right)^2\Big)\Big\}$$

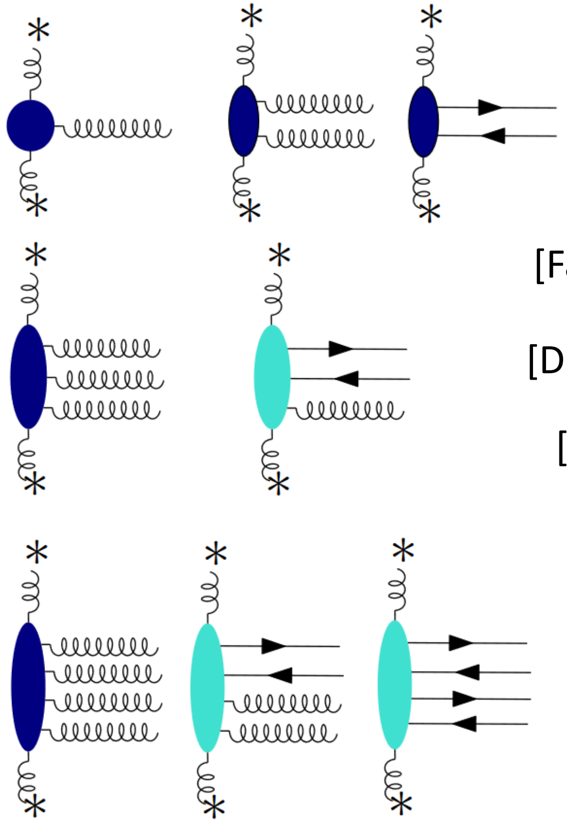
All tower of results extracted for PEV and CEV up to 4 emissions

Amp sourced from GGT package

PEV



CEV



[Fadin and Lipatov, 96]

[Del Duca, Fabio Maltoni , Frizzo 99]

[Duhr, 09]

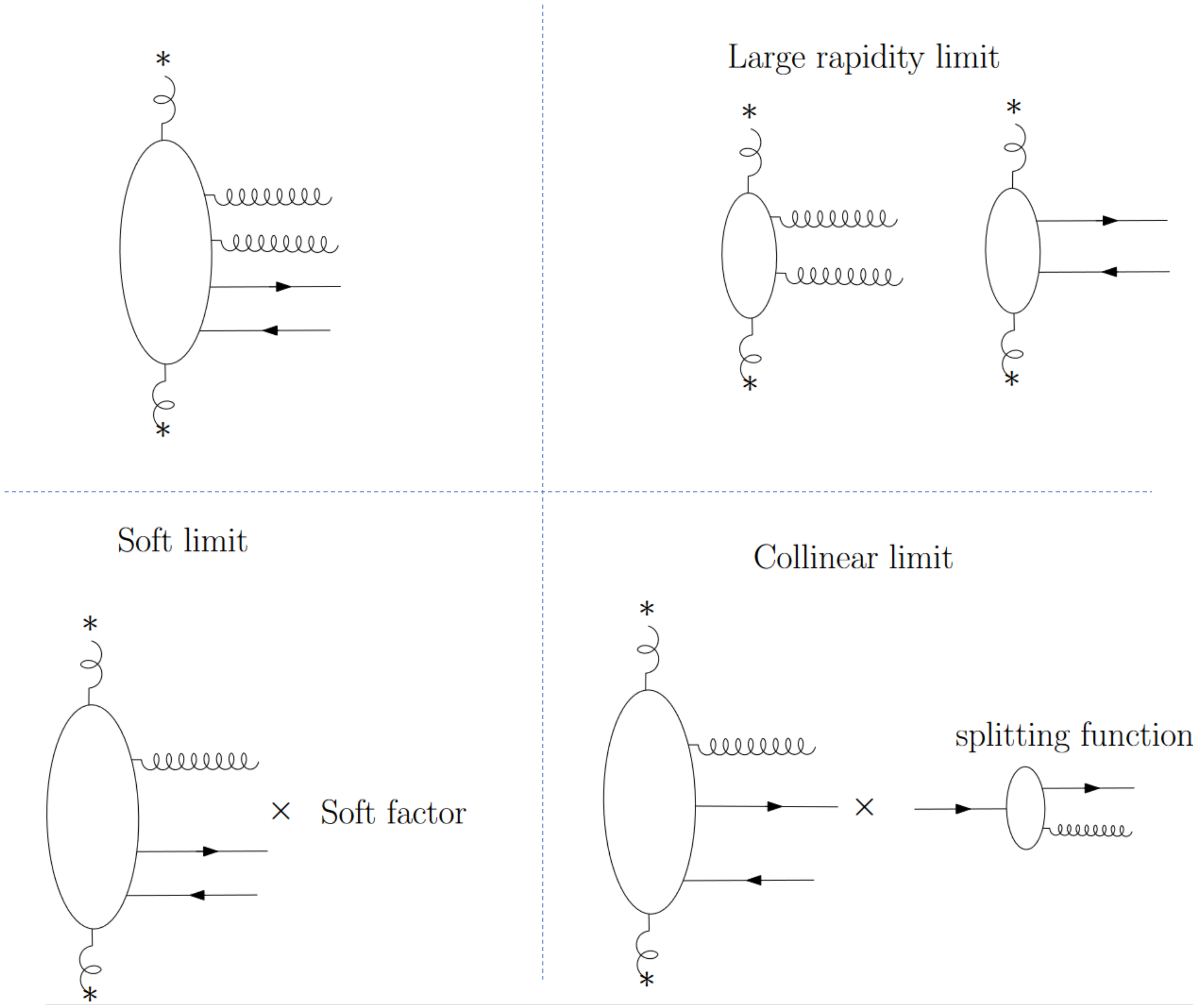
$$\mathcal{P}_{b_m}\left(p_{\sigma_1}^{h_{\sigma_1}}, p_{\sigma_2}^{h_{\sigma_2}}, \ldots, p_{\sigma_m}^{h_{\sigma_m}}; p_2^{\nu_2}, p_{\sigma_{m+1}}^{h_{\sigma_{m+1}}}, p_{\sigma_{m+2}}^{h_{\sigma_{m+2}}}, \ldots, p_{\sigma_n}^{h_{\sigma_n}}, (g^*_{q_{n+1}})\right)$$

$$\mathcal{C}_{b_m}\left(p_{\sigma_1}^{h_{\sigma_1}}, p_{\sigma_2}^{h_{\sigma_2}}, \ldots, p_{\sigma_m}^{h_{\sigma_m}}; (g^*_{q_1}), p_{\sigma_{m+1}}^{h_{\sigma_{m+1}}}, p_{\sigma_{m+2}}^{h_{\sigma_{m+2}}}, \ldots, p_{\sigma_n}^{h_{\sigma_n}}, (g^*_{q_{n+1}})\right)$$

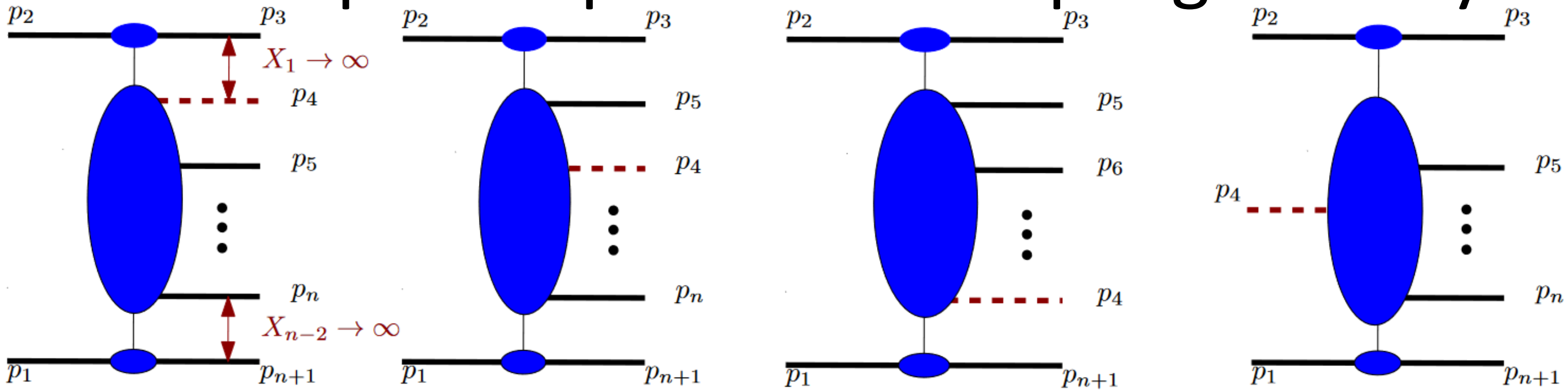
[Results summarized in Github codes]

We have tabulated them all in the Mathematica in minimal set of variable representation and checked

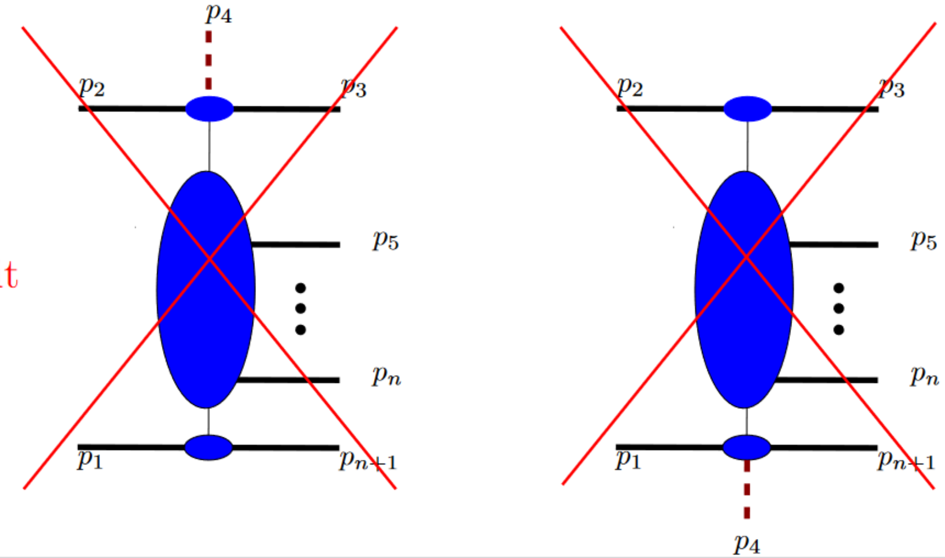
Checking the result: soft, collinear, or high energy factorization



Checking the result: parton photon decoupling identity



Power supressed
in high energy limit

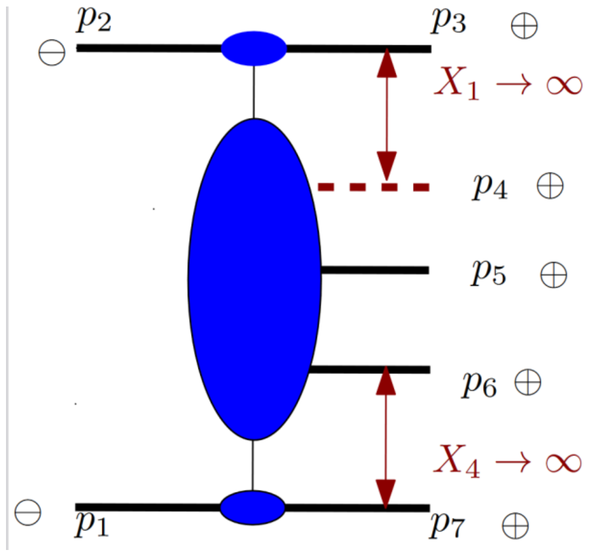


$$
\begin{aligned}
0 \;=\; & \mathcal{C}_{b_m}\left(p_{\sigma_1}^{h_{\sigma_1}}, p_{\sigma_2}^{h_{\sigma_2}}, \ldots p_{\sigma_m}^{h_{\sigma_m}}; (g_{q_1}^*), p_{\sigma_{m+1}}^{h_{\sigma_{m+1}}}, \ldots, p_{\sigma_n}^{h_{\sigma_n}}, (g_{q_{n+1}}^*)\right) + \ldots \\
& + \mathcal{C}_{b_m}\left(p_{\sigma_2}^{h_{\sigma_2}}, p_{\sigma_1}^{h_{\sigma_1}}, \ldots p_{\sigma_m}^{h_{\sigma_m}}; (g_{q_1}^*), p_{\sigma_{m+1}}^{h_{\sigma_{m+1}}}, \ldots, p_{\sigma_n}^{h_{\sigma_n}}, (g_{q_{n+1}}^*)\right) + \ldots \\
& + \mathcal{C}_{b_{m-1}}\left(p_{\sigma_2}^{h_{\sigma_2}}, \ldots p_{\sigma_m}^{h_{\sigma_m}}; (g_{q_1}^*), p_{\sigma_{m+1}}^{h_{\sigma_{m+1}}}, \ldots, p_{\sigma_n}^{h_{\sigma_n}}, p_{\sigma_1}^{h_{\sigma_1}}, (g_{q_{n+1}}^*)\right)
\end{aligned}
$$

Holds also for parton emissions

Checking the result: parton photon decoupling identity

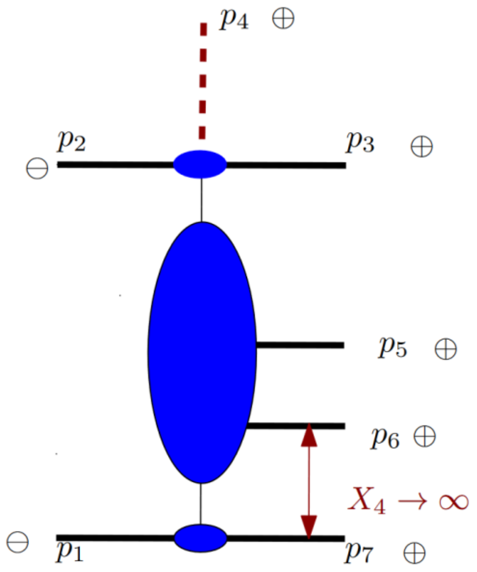
Power suppression



$$\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 56\rangle\langle 67\rangle\langle 71\rangle}$$

$$\langle 23\rangle = \frac{i\sqrt{X_1X_2X_3X_4+X_2X_3X_4+X_3X_4+X_4+1}q_1^\perp}{\sqrt{X_1}\sqrt{X_2}\sqrt{X_3}\sqrt{X_4}} \sim 1$$

$$\langle 45\rangle = \frac{(X_2z_1-z_2+1)\,q_1^\perp}{\sqrt{X_2}\,(z_1-1)\,(z_2-1)} \sim 1$$



$$\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 24\rangle\langle 43\rangle\langle 35\rangle\langle 56\rangle\langle 67\rangle\langle 71\rangle}$$

$$\langle 24\rangle = \frac{i\sqrt{X_1X_2X_3X_4+X_2X_3X_4+X_3X_4+X_4+1}q_1^\perp}{\sqrt{X_2}\sqrt{X_3}\sqrt{X_4}\,(z_1-1)} \sim \sqrt{X_1}$$

$$\langle 35\rangle = \frac{(X_1X_2z_1-z_2z_1+z_1+z_2-1)\,q_1^\perp}{\sqrt{X_1}\sqrt{X_2}\,(z_1-1)\,(z_2-1)} \sim \sqrt{X_1}$$

Checking the result: photon decoupling identity (off shell)

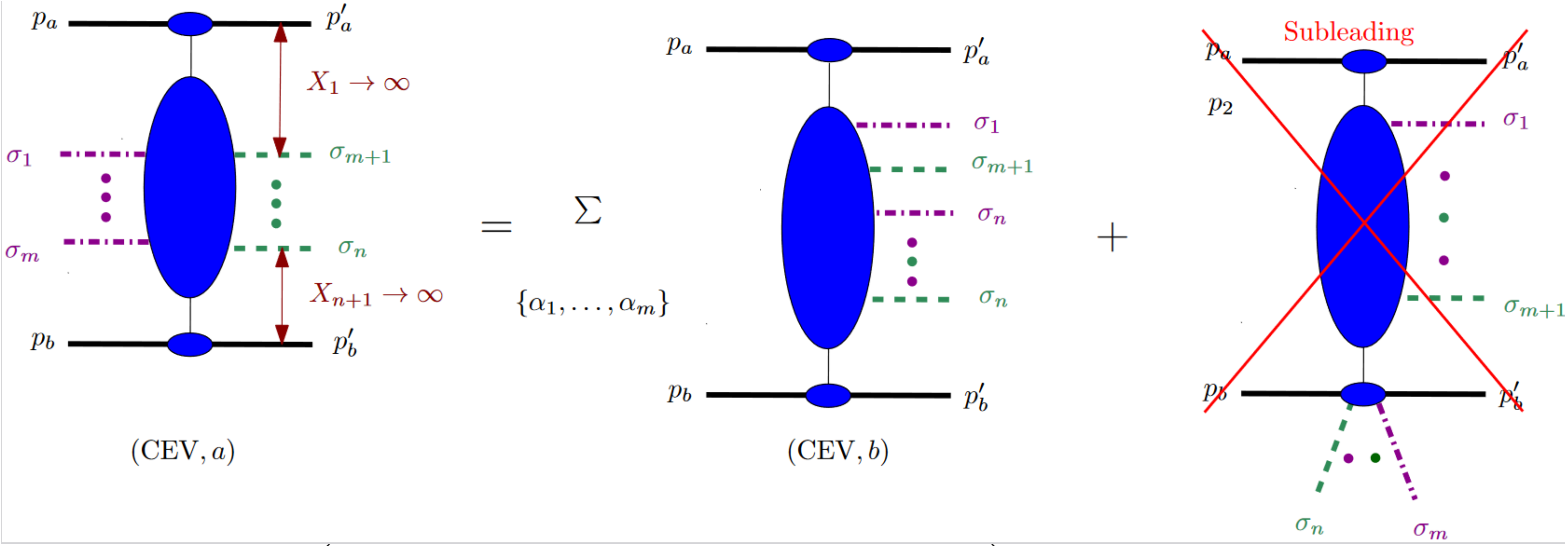
$$
\begin{aligned}
0 \quad = \quad & \mathcal{C}_{b_m}\left(\sigma_1, \sigma_2, ..., \sigma_m; (g^*_{q_1^\perp}), \sigma_{m+1}, ..., \sigma_n, (g^*_{q_{n+1}^\perp})\right) + \\
& \mathcal{C}_{b_{m+1}}\left(\sigma_1, \sigma_2, ..., \sigma_m, \sigma_{m+1}; (g^*_{q_1^\perp}), ..., \sigma_n, (g^*_{q_{n+1}^\perp})\right) + ... \\
& \mathcal{C}_{b_n}\left(\sigma_1, \sigma_2, ..., \sigma_m, \sigma_{m+1}, ..., \sigma_n; (g^*_{q_1^\perp}), (g^*_{q_{n+1}^\perp})\right) + \\
& \mathcal{C}_{a}\left((g^*_{q_1^\perp}), \sigma_1, \sigma_2, ..., \sigma_m, \sigma_{m+1}, ..., \sigma_n, (g^*_{q_{n+1}^\perp})\right) + \\
& \mathcal{C}_{b_1}\left(\sigma_1; (g^*_{q_1^\perp}), \sigma_2, ..., \sigma_m, \sigma_{m+1}, ..., \sigma_n, (g^*_{q_{n+1}^\perp})\right) + ... \\
& \mathcal{C}_{b_{m-1}}\left(\sigma_1, \sigma_2, ...; (g^*_{q_1^\perp}), \sigma_m, \sigma_{m+1}, ..., \sigma_n, (g^*_{q_{n+1}^\perp})\right)
\end{aligned}
$$

Example

$$
\mathcal{C}_a\left((g^*_{q_1^\perp}), q_4^{\oplus}, \bar{q}_5^{\ominus}, (g^*_{q_3^\perp})\right) + \mathcal{C}_{b_1}\left(q_4^{\oplus}; (g^*_{q_1^\perp}), \bar{q}_5^{\ominus}, (g^*_{q_3^\perp})\right) + \mathcal{C}_{b_1}\left(q_4^{\oplus}, \bar{q}_5^{\ominus}; (g^*_{q_1^\perp}), (g^*_{q_3^\perp})\right) = 0,
$$

The leg does not need to be Onshell

Checking the result: parton Kleiss-Kuijf relation



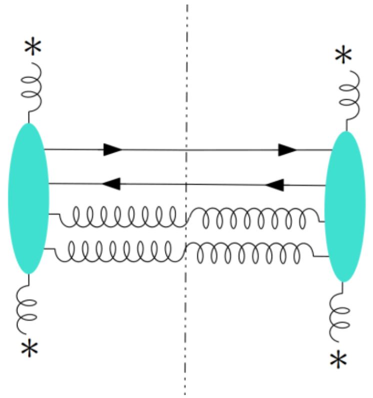
$$\mathcal{C}_{b_n}\left((\sigma_m,\ldots,\sigma_2,\sigma_1;(g^*_{q_1^\perp}),\sigma_{m+1},\ldots,\sigma_n,(g^*_{q_{n+1}^\perp})\right)$$
$$=\ (-1)^m\sum\mathcal{C}_a\left((g^*_{q_1^\perp}),\{\sigma_1,\ldots,\sigma_m\}\sqcup\{\sigma_{m+1},\ldots,\sigma_n\},(g^*_{q_{n+1}^\perp})\right),$$

Example $$\mathcal{C}_{b_2}\left(g_7^\ominus,g_4^\oplus;(g^*_{q_1^\perp}),Q_5^\ominus,\bar{Q}_6^\oplus,(g^*_{q_5^\perp})\right)\ =\ \sum\mathcal{C}_a\left((g^*_{q_1^\perp}),\{g_4^\oplus,g_7^\ominus\}\sqcup\{Q_5^\ominus,\bar{Q}_6^\oplus\},(g^*_{q_5^\perp})\right)$$

Conclusion

- We expressed amplitude in a suitable minimal set of variable
- We extract parton PEVs and CEVs up to 4 emissions with all colour orderrings and all helicity configurations.
- We check them by examining factorization properties or on amplitude-like identities, (onshell/offshell) photon decoupling, Kleiss-Kuijf, SUSY Ward Identities.

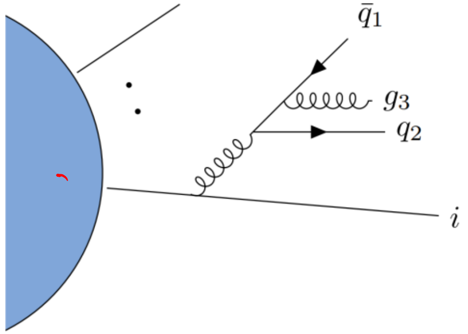
Future Directions



- “Squaring” PEVs and CEVs

and get radiative corrections to jet impact factors or BFKL kernel

- We can extract 4 parton soft current from 4 emission CEVs

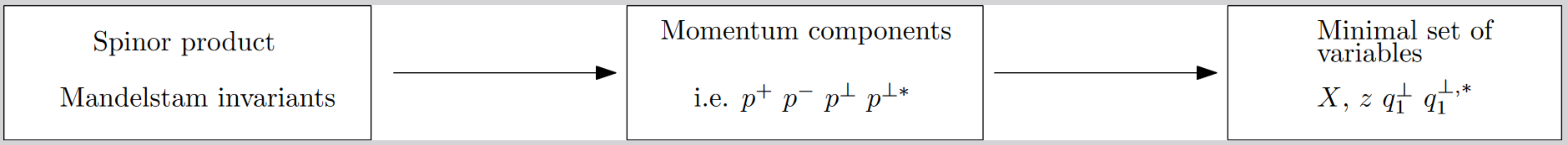


Plot by [Del Duca, Duhr, Haindl, Liu, 22]

The end

- Thanks !

Additional slides



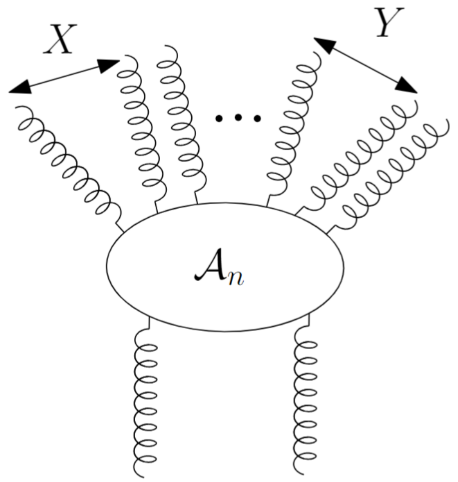
Spinor product
Mandelstam invariants
Momentum components
i.e. p^+ p^- $p^\perp$ $p^{\perp *}$
Minimal set of variables
X, z $q_1^\perp$ $q_1^{\perp,*}$

Aside: Minimal set of variable: easy to get rid of spurious pole

$$A_{\text{BCFW}}\left(q_1^{\oplus}\bar{q}_2^{\ominus}g_3^{\ominus}g_4^{\ominus}g_5^{\oplus}g_6^{\oplus}\right) = -\frac{\langle 4|5+6|1]^2\langle 4|5+6|2]}{s_{456}[12]\,[23]\,\langle 45\rangle\langle 56\rangle[3|4+5|6\rangle} + \frac{\langle 2|3+4|5]^2\langle 1|3+4|5]}{s_{345}\langle 12\rangle\langle 61\rangle[34]\,[45]\,[3|4+5|6\rangle}.$$
$$= r_1 + r_2 \tag{1.3}$$

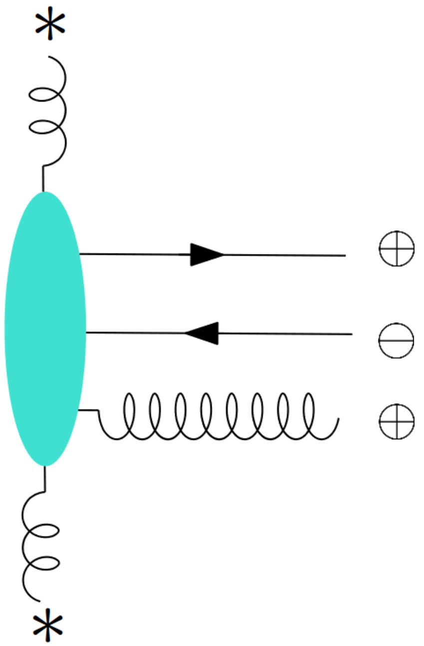
$$A_{\text{BG}}(q_1^{\oplus}\bar{q}_2^{\ominus}g_3^{\ominus}g_4^{\ominus}g_5^{\oplus}g_6^{\oplus}) = ig^4\left[\frac{P_1}{s_{123}s_{12}s_{23}s_{45}s_{56}} + \frac{P_2}{s_{234}s_{23}s_{34}s_{56}s_{61}} + \frac{P_3}{s_{345}s_{34}s_{45}s_{61}s_{12}} + \frac{P_s}{s_{12}s_{23}s_{34}s_{45}s_{56}s_{61}}\right],$$

- Spurious pole cancellation: finding common denominator
- Can show analytically BG=BCFW
- No need to tedious spinor algebras



From high energy limit of [GGT package]

Some results: parton colour ordered CEV



$$
\begin{aligned}
&\mathcal{C}\left(q_4^{\oplus}, \bar{q}_5^{\ominus}, g_6^{\oplus}\right) \\
&= \frac{X_2^{3/2} X_3^3 z_2 z_3 \bar{z}_1 \bar{z}_2 \bar{z}_3 \hat{z}_1^2 \hat{z}_2}{R_{<45>} R_{<56>} R_{S_{123}} \Delta_1 \Delta_4 q_1^{\perp}} - \frac{X_2^{5/2} X_3^5 z_1^2 z_2 z_3 \bar{z}_1 \bar{z}_2 \bar{z}_3 \hat{z}_1 \hat{z}_2 \hat{z}_1}{R_{<56>} R_{S_{234}} \Delta_4 \Delta_5 \Delta_6 q_1^{\perp}} - \\
&\frac{X_2^{3/2} X_3^3 z_2 z_3 \left(-\bar{z}_1 + \bar{z}_1 \bar{z}_2 + \bar{z}_1 \bar{z}_3\right)\left(1 - \bar{z}_2 - \bar{z}_3 + \bar{z}_2 \bar{z}_3\right)^3 \hat{z}_1^2 \hat{z}_2 \hat{z}_1}{R_{<56>} \Delta_1 \Delta_3 \Delta_6 \Delta_8 q_1^{\perp}} - \frac{X_2^{5/2} X_3 z_1^2 \bar{z}_1 \bar{z}_2 \bar{z}_3 \hat{z}_1 \hat{z}_2 \hat{z}_1 \hat{z}_2}{R_{<45>} R_{S_{671}} \Delta_2 \Delta_5 q_1^{\perp}} + \\
&\frac{X_2^{11/2} X_3^3 z_1^4 z_2 z_3 \bar{z}_1^4 \bar{z}_2^4 \hat{z}_1^2 \hat{z}_2^2 \hat{z}_1 \hat{z}_2}{R_{<45>} R_{S_{345}} R_{S_{712}} \Delta_2 \Delta_7 \Delta_8 q_1^{\perp}} - \frac{X_2^{3/2} X_3 z_1 z_2 z_3 \hat{z}_1 \hat{z}_2 \hat{z}_1 \hat{z}_2 \left(x_2 x_3 \bar{z}_1 \bar{z}_2 + \bar{z}_3 - \bar{z}_2 \bar{z}_3 + \hat{z}_2\right)^3}{R_{[45]} R_{S_{456}} \Delta_3 \Delta_7 q_1^{\perp}}
\end{aligned}
$$

R's physical pole (rational), Δ's spurious pole (rational)

Finding common denominator eliminate all spurious poles (Δ)'s

Checking the result: SUSY Ward Identity

$$0 = \left\langle \Omega \middle| \left[Q, a_1^{\text{out}} \ \ldots a_n^{\text{out}} \ a_1^{\dagger \ \text{in}} \ \ldots a_m^{\dagger \ \text{in}} \right] \middle| \Omega \right\rangle$$
$$= \sum_i \langle \Omega | a_1^{\text{out}} \ \ldots [Q, a_i^{\text{out}}] \ldots | \Omega \rangle + \sum_j \left\langle \Omega \middle| a_1^{\text{out}} \ \ldots \left[Q, a_j^{\dagger \ \text{in}} \right] \ldots \middle| \Omega \right\rangle .$$

Example Define an effective operator $\tilde{Q}_{\dot{\alpha}}$

$$0 = [k|^{\dot{\alpha}} \, \tilde{Q}_{\dot{\alpha}} A(g_2^{\ominus}, g_{\sigma_3}^{\oplus}, \bar{q}_{\sigma_4}^{\ominus}, g_5^{\oplus}, g_1^{\ominus})$$
$$= [k\sigma_3] \, A(g_2^{\ominus}, q_{\sigma_3}^{\oplus}, \bar{q}_{\sigma_4}^{\ominus}, g_5^{\oplus}, g_1^{\ominus}) + [k\sigma_4] \, A(g_2^{\ominus}, g_{\sigma_3}^{\oplus}, g_{\sigma_4}^{\ominus}, g_5^{\oplus}, g_1^{\ominus}) - [k\sigma_5] \, A(g_2^{\ominus}, g_{\sigma_3}^{\oplus}, \bar{q}_{\sigma_4}^{\ominus}, q_5^{\oplus}, g_1^{\ominus}).$$

Example: large Rapidity gap between {1,5} and {2,3,4}

$$-\mathcal{P}\left(g_2^{\ominus}, q_{\sigma_3}^{\oplus}, \bar{q}_{\sigma_4}^{\ominus}, (g_{q_2^{\perp}}^{*})\right) \left(\lim_{y_k \gg y_{\sigma_3} \sim y_{\sigma_4}} \frac{[k\sigma_3]}{[k\sigma_4]} \right) = \mathcal{P}\left(g_2^{\ominus}, g_{\sigma_3}^{\oplus}, g_{\sigma_4}^{\ominus}, (g_{q_2^{\perp}}^{*})\right) \qquad \hat{Q}\mathcal{P} = 0$$

Checking the result: SUSY Ward Identity

$$\hat{Q}\mathcal{P} = 0$$

MHV

$$0 = [k|^{\dot\alpha}\,\tilde{Q}_{\dot\alpha}\mathcal{P}(g_2^{\ominus}, g_{\sigma_3}^{\oplus}, \bar{q}_{\sigma_4}^{\ominus}, (g^*_{q_2^\perp}))$$
$$= [k\sigma_3]\,\mathcal{P}(g_2^{\ominus}, q_{\sigma_3}^{\oplus}, \bar{q}_{\sigma_4}^{\ominus}, (g^*_{q_2^\perp})) + [k\sigma_4]\,\mathcal{P}(g_2^{\ominus}, g_{\sigma_3}^{\oplus}, g_{\sigma_4}^{\ominus}, (g^*_{q_2^\perp}))$$

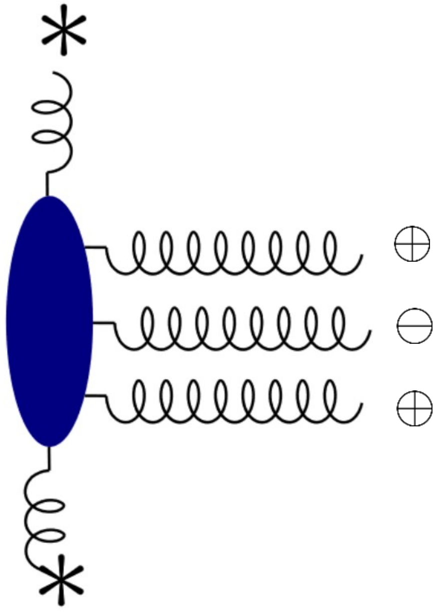
$$-\mathcal{P}\left(g_2^{\ominus}, q_{\sigma_3}^{\oplus}, \bar{q}_{\sigma_4}^{\ominus}, (g^*_{q_2^\perp})\right)\left(\lim_{y_k \gg y_{\sigma_3} \sim y_{\sigma_4}} \frac{[k\sigma_3]}{[k\sigma_4]}\right) = \mathcal{P}\left(g_2^{\ominus}, g_{\sigma_3}^{\oplus}, g_{\sigma_4}^{\ominus}, (g^*_{q_2^\perp})\right)$$

NMHV

$$\mathcal{P}\left(g_2^{\oplus}, g_{\sigma_3}^{\ominus}, g_{\sigma_4}^{\oplus}, g_{\sigma_5}^{\ominus}, (g^*_{q_2^\perp})\right)$$
$$= \left(\lim_{y_k \gg y_{2,3,4,5}} \frac{\langle k\sigma_5\rangle}{\langle k\sigma_4\rangle}\right)\mathcal{P}\left(g_2^{\oplus}, g_{\sigma_3}^{\ominus}, q_{\sigma_4}^{\oplus}, q_{\sigma_5}^{\ominus}, (g^*_{q_2^\perp})\right) - \left(\lim_{y_k \gg y_{2,3,4,5}} \frac{\langle k\sigma_3\rangle}{\langle k\sigma_4\rangle}\right)\mathcal{P}\left(g_2^{\oplus}, q_{\sigma_3}^{\ominus}, q_{\sigma_4}^{\oplus}, g_{\sigma_5}^{\ominus}, (g^*_{q_2^\perp})\right)$$

From high energy limit of [GGT package]

Compare with known gluon CEV



$$\mathcal{C}\left(g_4^{\oplus}, g_5^{\ominus}, g_6^{\oplus}\right)$$

$$=\frac{X_2^3 X_3^3 z_2 z_3 \bar{z}_1 \bar{z}_2 \bar{z}_3 \hat{z}_1^2 \hat{z}_2}{C_{<45>} C_{<56>} C_{S_{123}} \Delta_1 \Delta_4 q_1^{\perp}}+\frac{X_2 X_3^2 \left(1+X_3\right)^3 z_2 z_3 \bar{z}_1 \bar{z}_2 \bar{z}_3 \hat{z}_1 \hat{z}_2 \hat{z}_1}{C_{<56>} C_{S_{234}} z_1 \Delta_4 \Delta_5 \Delta_6 q_1^{\perp}}+\frac{C_{[56]}^3 X_2^3 X_3 z_1 z_2 z_3 \bar{z}_1^3 \hat{z}_1 \hat{z}_2 \hat{z}_1 \hat{z}_2}{C_{[45]} C_{S_{456}} \Delta_3 \Delta_7 q_1^{\perp}}$$

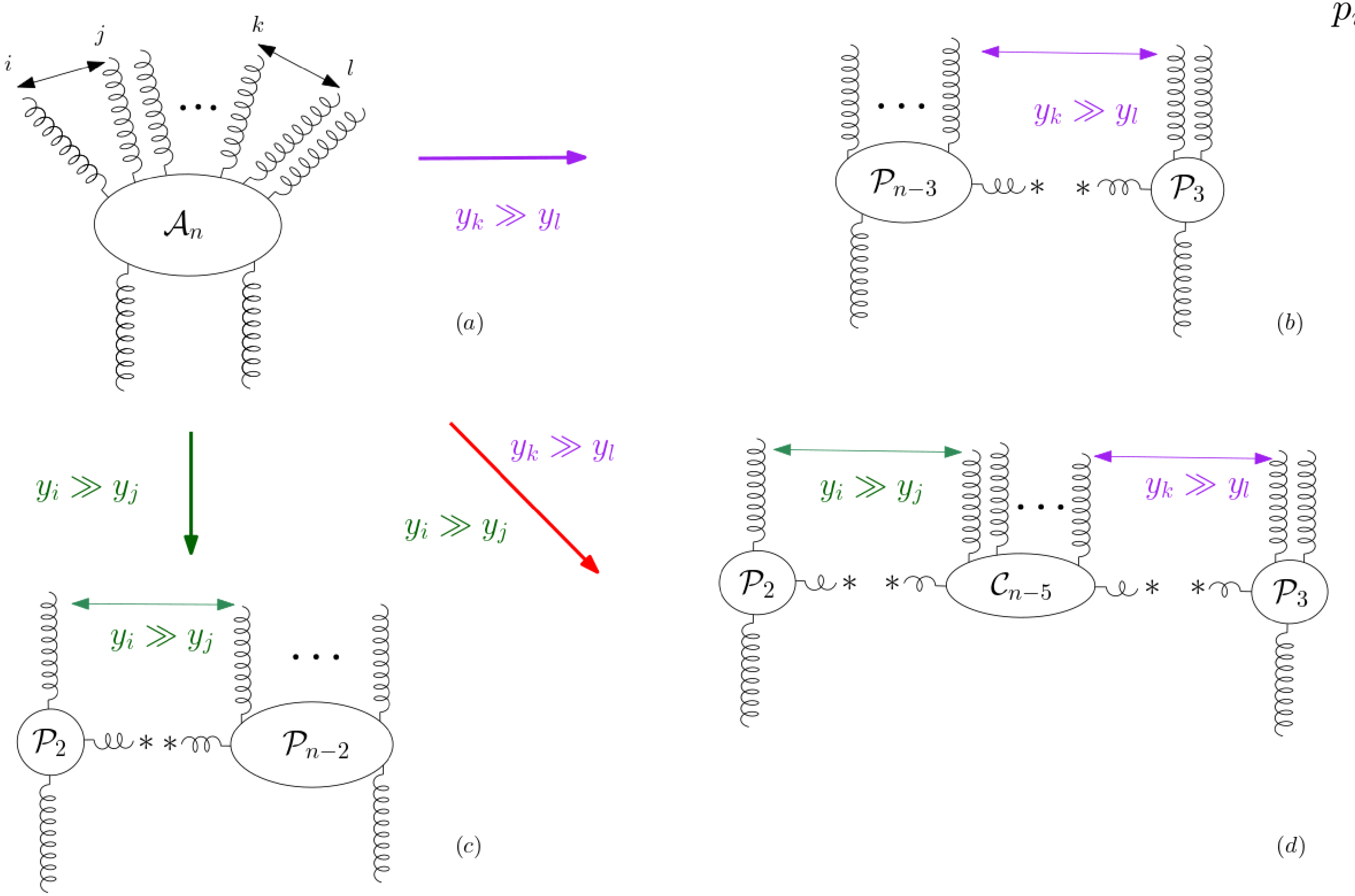
$$\frac{X_2 X_3 \bar{z}_1 \bar{z}_2 \bar{z}_3 \hat{z}_1 \hat{z}_2^4 \hat{z}_1 \hat{z}_2}{C_{<45>} C_{s_{671}} z_1 \Delta_2 \Delta_5 q_1^{\perp}}+\frac{X_2^4 X_3^3 z_1 z_2 z_3 \bar{z}_1^4 \bar{z}_2^4 \hat{z}_1^2 \hat{z}_2^5 \hat{z}_1 \hat{z}_2}{C_{<45>} C_{S_{345}} C_{S_{712}} \Delta_2 \Delta_7 \Delta_8 q_1^{\perp}}+\frac{X_2^3 X_3^3 z_2 z_3 \bar{z}_1^4 \hat{z}_1^2 \hat{z}_2 \hat{z}_1 \left(\bar{z}_3+\hat{z}_2\right)^4}{C_{<56>} \Delta_1 \Delta_3 \Delta_6 \Delta_8 q_1^{\perp}}$$

C's physical pole, Δ's spurious pole

Finding common denominator eliminate all spurious poles (Δ)'s

Extraction of high energy building block

$$p_i = (p_i^+, p_i^-, p_{i\perp}) = |p_{i\perp}| \left(e^{y_i}, e^{-y_i}; \cos\phi_i, \sin\phi_i\right)$$



High-energy building blocks:
CEV: $\mathcal{C}$,
PEV: $\mathcal{P}$

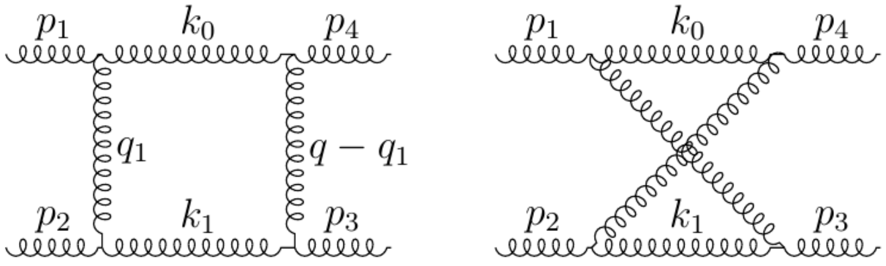
$$s=(p_1+p_2)^2,\quad t=(p_1+p_4)^2,\quad u=(p_1+p_3)^2,$$

leading logarithmic accuracy (LL): $\alpha_s^n L^n$

next to leading logarithmic accuracy (NLL): $\alpha_s^n L^{n-1}\ldots$

where $L=\ln\frac{s}{-t}-\frac{i\pi}{2}$ $\quad(s\gg -t)$.

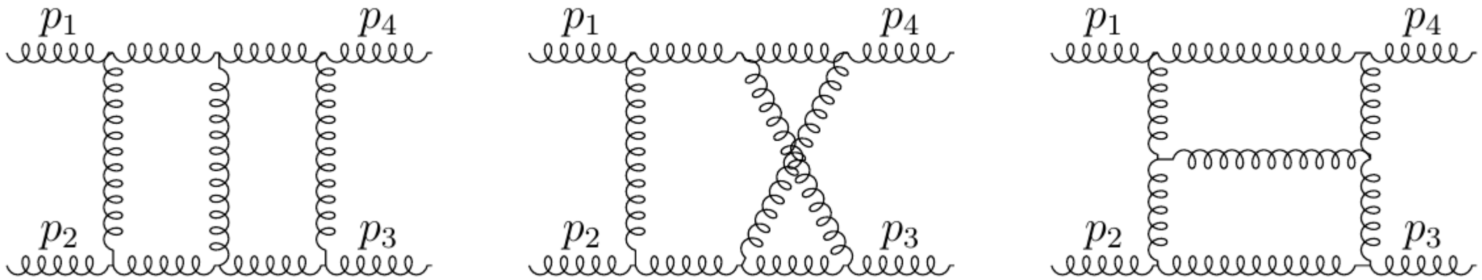
One loop

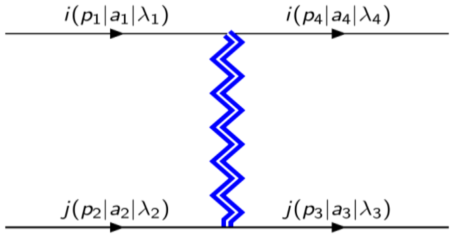


$$\mathcal{M}_{\mathrm{LL}}^{(-,1)}=N_c\alpha_g\left(t,\mu^2\right)L\mathcal{M}_{ij\to ij}^{(\mathrm{tree})}$$

$$\mathcal{M}_{\mathrm{LL}}^{(-,2)}=\frac{1}{2!}N_c^2\alpha_g\left(t,\mu^2\right)^2L^2\mathcal{M}_{ij\to ij}^{\mathrm{tree}}$$

Two loop





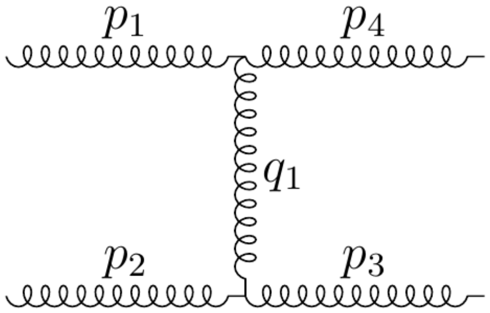
$$\mathcal{M}_{\mathrm{LL}}^{(8_a)}=e^{N_c\alpha_g\left(t,\mu^2\right)L}\mathcal{M}_{ij\to ij}^{(\mathrm{tree}\ )}$$

$$\alpha_g\left(t,\mu^2\right)=\frac{\alpha_S}{2\pi\epsilon}\left(\frac{\mu^2}{-t}\right)^{\epsilon}$$

$$s=(p_1+p_2)^2, \quad t=(p_1+p_4)^2, \quad u=(p_1+p_3)^2,$$ $(s \gg -t)$.

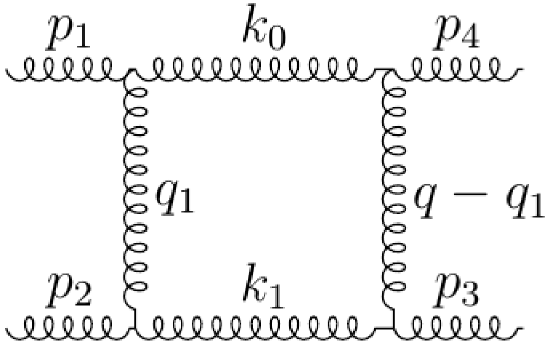
$$L=\ln\frac{s}{-t}-\frac{i\pi}{2}$$

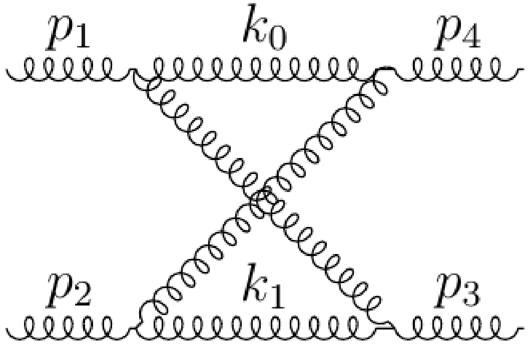
Tree level



$$\mathcal{M}_{ij\to ij}^{(\text{tree})}=4\pi\alpha_s\frac{2s}{t}\left(T_i^b\right)_{a_1a_4}\left(T_j^b\right)_{a_2a_3}\delta_{\lambda_1\lambda_4}\delta_{\lambda_2\lambda_3}$$

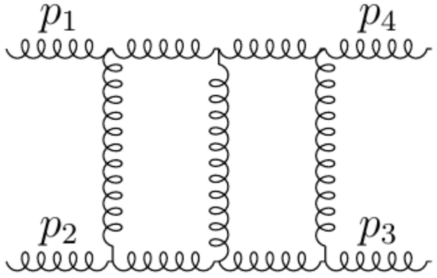
One loop

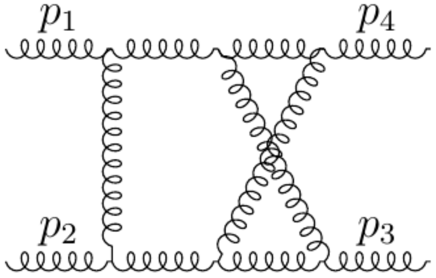


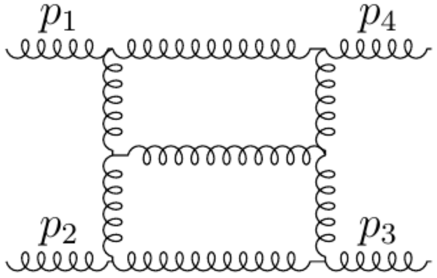


$$\mathcal{M}_{\text{LL}}^{(-,1)}=N_c\alpha_g\left(t,\mu^2\right)L\mathcal{M}_{ij\to ij}^{(\text{tree})}$$

Two loop







$$\mathcal{M}_{\text{LL}}^{(-,2)}=\frac{1}{2!}N_c^2\alpha_g\left(t,\mu^2\right)^2L^2\mathcal{M}_{ij\to ij}^{\text{tree}}$$

Unique representation of Amplitudes also PEV and CEV

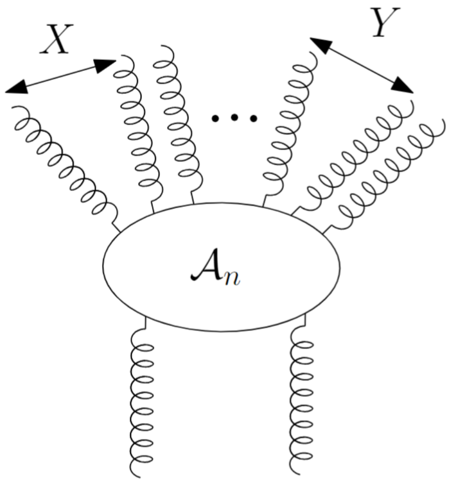
$$A_{\mathrm{BCFW}}\left(q_1^{\oplus}\bar{q}_2^{\ominus}g_3^{\ominus}g_4^{\ominus}g_5^{\oplus}g_6^{\oplus}\right) = -\frac{\langle 4|5+6|1]^2\langle 4|5+6|2]}{s_{456}[12]\,[23]\,\langle 45\rangle\langle 56\rangle[3|4+5|6\rangle} + \frac{\langle 2|3+4|5]^2\langle 1|3+4|5]}{s_{345}\langle 12\rangle\langle 61\rangle[34]\,[45]\,[3|4+5|6\rangle}.$$
$$= r_1 + r_2 \qquad (1.3)$$

$$A_{\mathrm{BG}}(q_1^{\oplus}\bar{q}_2^{\ominus}g_3^{\ominus}g_4^{\ominus}g_5^{\oplus}g_6^{\oplus}) = ig^4\left[\frac{P_1}{s_{123}s_{12}s_{23}s_{45}s_{56}} + \frac{P_2}{s_{234}s_{23}s_{34}s_{56}s_{61}} + \frac{P_3}{s_{345}s_{34}s_{45}s_{61}s_{12}} + \frac{P_s}{s_{12}s_{23}s_{34}s_{45}s_{56}s_{61}}\right],$$

- Spurious pole cancellation: finding common denominator
- BG=BCFW

$$r_2 = \frac{\sqrt{c_{61}}\sqrt{X_1X_2X_3}\sqrt{\hat{z}_1}\hat{z}_2^{5/2}\bar{z}_1\sqrt{\bar{\hat{z}}_1}\sqrt{\bar{\hat{z}}_2}\left(X_3\left(X_2z_1+1\right)+1\right)^2\left(X_2z_1\left(X_3z_2\bar{z}_2+1\right)+\hat{z}_2\bar{\hat{z}}_2\right)}{\sqrt{c_{23}}c_{56}s_{456}z_1c_{45}j^*_{3456}\sqrt{q_1^{\perp}q_1^{\perp*3/2}}}$$

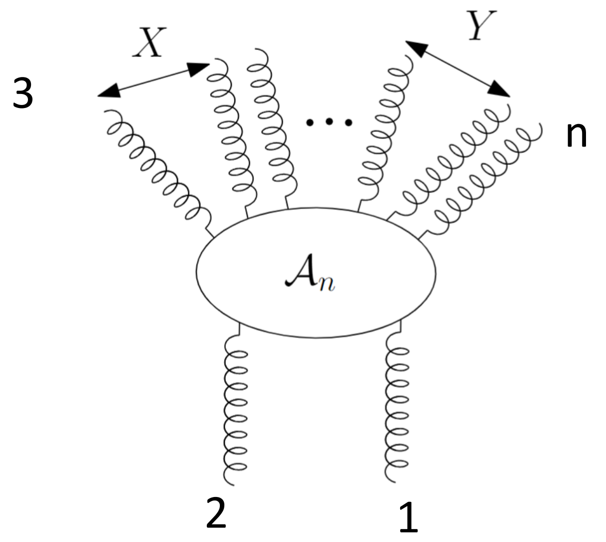
$$r_1 = \frac{\sqrt{c_{23}}\sqrt{X_1X_2X_3}\sqrt{\hat{z}_1}\hat{z}_2^{5/2}\bar{z}_1\bar{\hat{z}}_1^{3/2}\sqrt{\bar{\hat{z}}_2}\left(\bar{\hat{z}}_2-\left(X_1+1\right)X_2\right)\left(X_1\left(\bar{\hat{z}}_2-X_2z_1\bar{z}_1\right)+\hat{z}_1\bar{\hat{z}}_1\bar{\hat{z}}_2\right)^2}{\sqrt{c_{61}}s_{345}c^*_{34}c^*_{45}j^*_{3456}\sqrt{q_1^{\perp}q_1^{\perp*3/2}}}$$

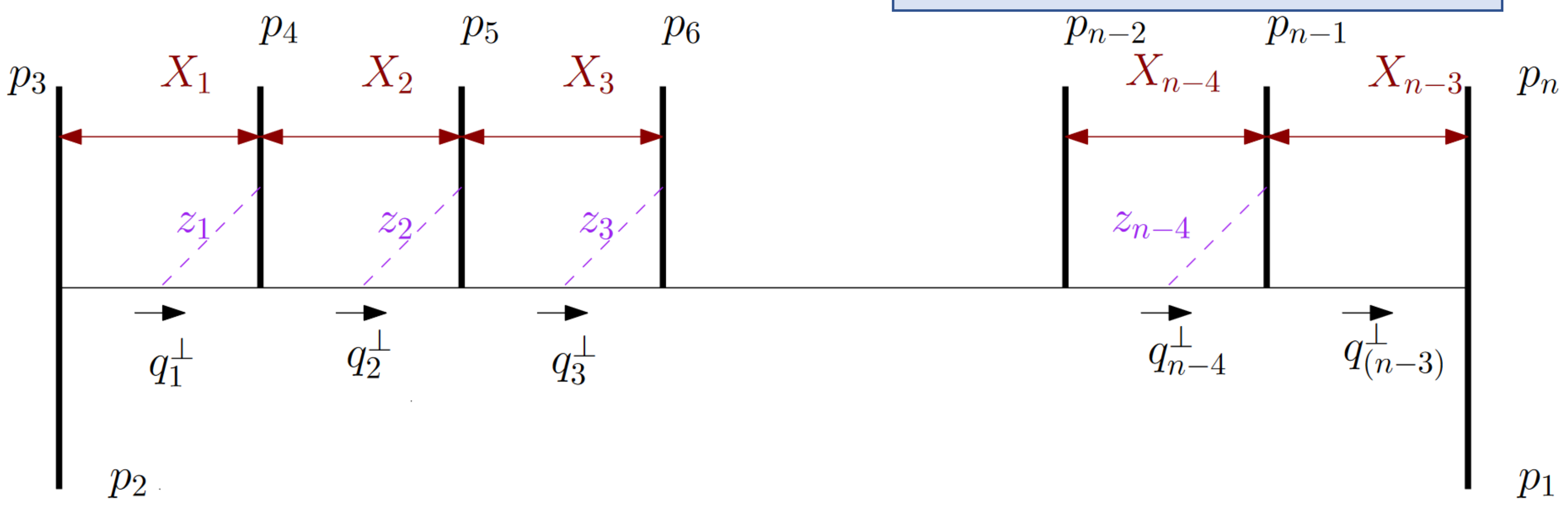


Extraction of high energy building block:

Expressing amplitude in suitable minimal set of variables

In total **3(n-3)** variables to describe n point amp with spin





$$X_i \equiv \frac{p_{i+2}^+}{p_{i+3}^+}, \qquad z_i = -\frac{q_{(i+1)}^\perp}{p_{(i+3)}^\perp}, \qquad \mu \equiv q_1^\perp;$$

Suitable minimal set of variable

- Separate longitudinal and transverse
- Convenient for high energy factorization
- Free of spurious pole

$$X_i \equiv \frac{s_{1(i+2)}}{s_{1(i+3)}}, \quad z_i \equiv \frac{\left\langle 2 \left| \sum_{k=3}^{i+3} p_k \right| 1 \right]}{\langle 2 \left| p_{i+3} \right| 1]}, \quad \mu \equiv \frac{\langle 2|3|1]}{[21]}.$$

$$\begin{aligned} p_i &= \left(p_i^+, p_i^-, p_{i\perp}\right) \\ &= |p_{i\perp}| \left(e^{y_i}, e^{-y_i}; \cos\phi_i, \sin\phi_i\right) \end{aligned}$$