# Analytic results for double Higgs production at the LHC

High Precision for Hard Processes, 2024

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University of Zürich

Based on: [JHEP 08 (2022) 259, JHEP 06 (2023) 063, JHEP 10 (2023) 033, JHEP 08 (2024) 096]

# Outline

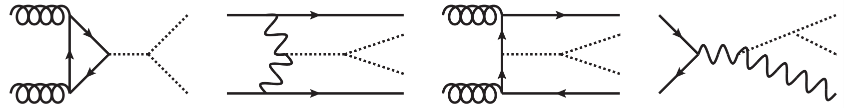
# Introduction

# Higgs Self Coupling

- Standard Model Higgs potential:

$$\boxed{V(H) = \frac{1}{2}m_H^2 H^2 + \lambda v H^3 + \frac{\lambda}{4}H^4}, \quad \text{where } \lambda = m_H^2/(2v^2) \approx 0.13.$$

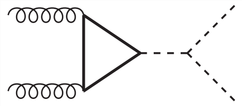
- Want to measure $\lambda$, to determine if $V(H)$ is consistent with nature.
    - Challenging! Cross-section $\approx 10^{-3} \times H$ prod.
    - $-1.24 < \lambda/\lambda_{SM} < 6.49$ [CMS '22] ; $-0.6 < \lambda/\lambda_{SM} < 6.6$ [Atlas '22]
- $\lambda$ appears in various production channels:

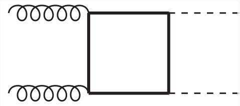


- Gluon fusion – dominant, 10x
- VBF
- $t\bar{t}$ associated production
- $H$-strahlung

# Gluon Fusion

- Leading order (1 loop) partonic amplitude:





$$\mathcal{M}^{\mu\nu} \sim \mathcal{A}_1^{\mu\nu}(\mathcal{F}_{tri} + \mathcal{F}_{box1}) + \mathcal{A}_2^{\mu\nu}(\mathcal{F}_{box2})$$

  - $\mathcal{F}_{tri}$ contains the dependence on $\lambda$ at LO

- Form factors:
  - LO: known exactly [Glover, van der Bij '88]
  - Beyond LO... no fully-exact (analytic) results to date
    - **QCD:** numerical evaluation, expansion in various kinematic limits
    - **EW:** heavy top expansion, high-energy expansion [Davies, Mishima, Schönwald, Steinhauser, Zhang '22]
    - see also Yuakwa corrections in (partial) HTL [Mühlleitner,Schlenk,Spira '22]
    - full (numerical) EW corrections [Bi, Huang, Huang, Ma, Yu '23]
    - numerical Yukawa- and Higgs self-coupling corrections [Heinrich, Jones, Kerner, Stone, Vestner '24]

# $gg \to HH$ Beyond LO

NLO QCD:

- large-$m_t$ [Dawson, Dittmaier, Spira '98] [Grigo, Hoff, Melnikov, Steinhauser '13]
- numeric [Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke '16] [Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher '19]
- large-$m_t$ + threshold exp. Padé [Gröber, Maier, Rauh '17]
- high-energy expansion [Davies, Mishima, Steinhauser, Wellmann '18,'19]
- small-$p_T$ expansion [Bonciani, Degrassi, Giardino, Gröber '18]
  + high-energy expansion [Bagnaschi, Degrassi, Gröber '23]

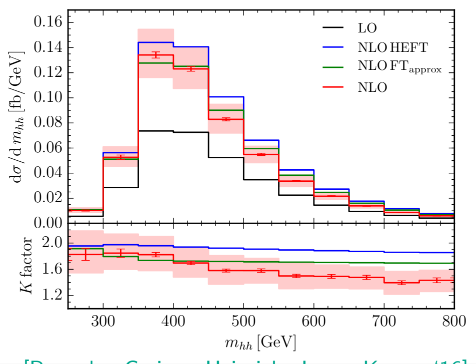
NNLO QCD:

- large-$m_t$ virtuals [de Florian, Mazzitelli '13] [Grigo, Hoff, Steinhauser '15, Davies; Steinhauser '19]
- HTL+numeric real ("FTapprox") [Grazzini, Heinrich, Jones, Kallweit, Kerner, Lindert, Mazzitelli '18]
- large-$m_t$ reals [Davies, Herren, Mishima, Steinhauser '19 '21]
- light fermion corrections at $p_T = 0$ [Davies, Schönwald, Steinhauser '23]

N3LO QCD:

- Wilson coefficient $C_{HH}$ [Spira '16; Gerlach, Herren, Steinhauser '18]
- HTL [Chen, Li, Shao, Wang '19]

# $gg \to HH$ Beyond LO



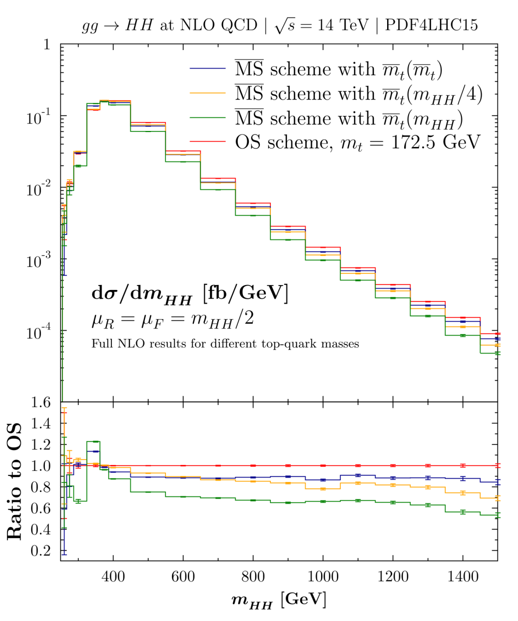

[Borowka, Greiner, Heinrich, Jones, Kerner '16]

Total cross section (14TeV):

| | $\sigma_{LO}$ | $\sigma_{NLO}$ | $\sigma_{NNLO}$ |
|---|---|---|---|
| B-i HTL | – | $38.32^{+18.1\%}_{-14.9\%}$ | $39.58^{+1.4\%}_{-4.7\%}$ |
| FTapprox | – | $34.25^{+14.7\%}_{-13.2\%}$ | $36.69^{+2.1\%}_{-4.9\%}$ |
| Full | $19.85^{+27.6\%}_{-20.5\%}$ | $32.88^{+13.5\%}_{-12.5\%}$ | – |

# $gg \to HH$ Beyond LO

- Large uncertainties due to the $m_t$ renormalization scheme.
- Can only be reliably reduced with an NNLO calculation.

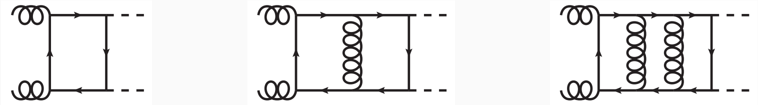



[Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira, Streicher '20]

# QCD corrections to $gg \to HH$

# QCD Corrections

Example diagrams at LO, NLO, NNLO:



Diagrams depend on $\epsilon$, $s$, $t$, $m_t$, $m_H$:

- analytic result is very involved
- simplify by expanding in certain kinemtic limits

We will consider:

- high-energy expansion: description for larger $p_T$ vales $\qquad s, |t| > m_t^2 > m_H^2$
- small-t expansion: description for smaller $p_T$ values $\qquad s, m_t^2 > |t|, m_H^2$

$\rightarrow$ The two expansions will cover the whole physically interesting phase space.

# High-energy expansion

Seek an expansion where $s, |t| > m_t^2 > m_H^2$ [Davies, Mishima, Steinhauser, Wellmann '18-'19]

1. Form factors in terms of scalar Feynman integrals: $I(m_H^2, m_t^2, s, t, \epsilon)$
2. Taylor expand for $m_H^2 \to 0$ (with `LiteRed`): [Lee '14]

$$I(m_H^2, m_t^2, s, t, \epsilon) = I(0, m_t^2, s, t, \epsilon) + m_H^2 I'(0, m_t^2, s, t, \epsilon) + \ldots$$

3. IBP reduce to master integrals: $J(0, m_t^2 s, t, \epsilon)$ (`FIRE`, `Kira`) [Smirnov '15]
[Klappert, Lange, Maierhöfer, Usovitsch '21]
4. Determine MIs as an expansion around $m_t \to 0$:

$$J(0, m_t^2, s, t, \epsilon) = \sum_{i,j,k} C_{ijk}(s,t)\epsilon^i (m_t^2)^j \log(m_t^2)^k$$

   - Insert ansatz into differential equation $\to$ linear equations for $c_{ijk}$.
   - Compute boundary conditions with expansion-by-regions.

Result: power series in $m_t^2$ and $\log(m_t^2)$.

- coefficients: functions of $s, t$ written in terms of harmonic polylogarithms

# High-energy expansion: Padé approximants

The expansion diverges for $\sqrt{s} \lesssim 750\,\text{GeV}$.

The convergence can be improved by making use of Padé approximants:

- Approximate a function using a rational polynomial:

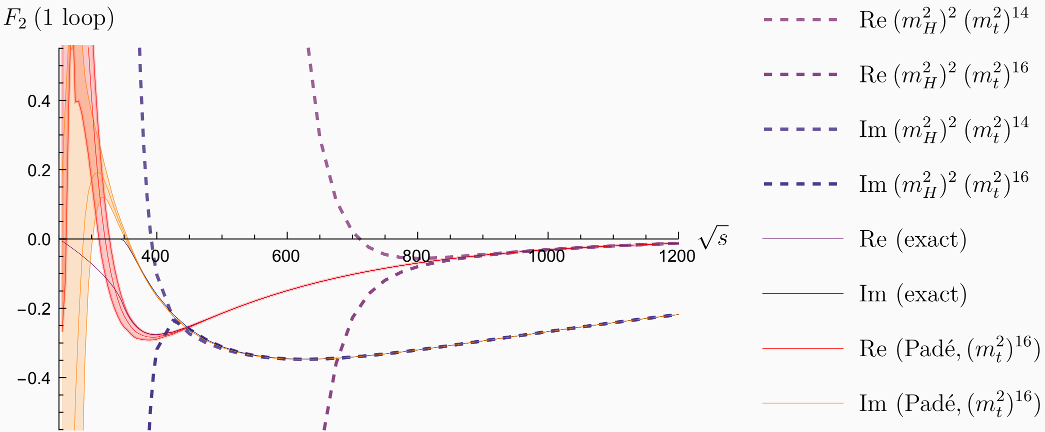
$$f(x) \approx [n/m](x) = \frac{a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n}{1 + b_1 x + b_2 x^2 + \cdots + b_m x^m}\,,$$

  where the coefficients $a_i$, $b_j$ are fixed by the series expansion of $f(x)$.

Compute a set of approximants (various choices of $n$, $m$):

- combine to give a central value and error estimate
- deeper expansions $\Rightarrow$ larger $n + m$ $\Rightarrow$ smaller error
- expansions to $m_t^{120}$ allows for very high-order approximants

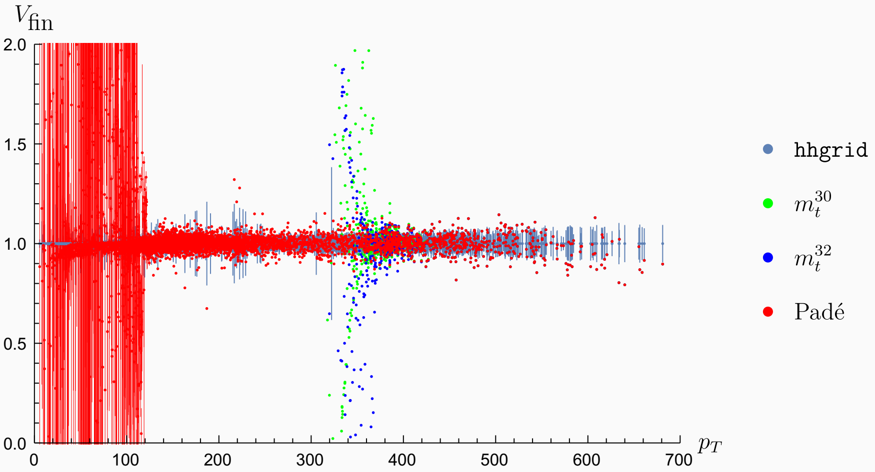
# High-energy expansion: Padé approximants



# High-energy expansion: $V_{fin}$

Comparison with `hhgrid`: [https://github.com/mppmu/hhgrid]

- interpolation grid of 6320 points evaluated numerically by `pySecDec`
- grid points normalized to `hhgrid` as function of $p_T$:

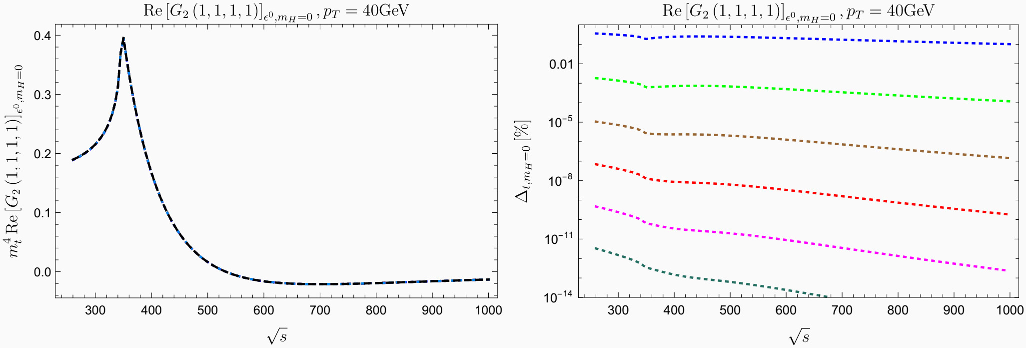


# Small-$t$ expansion

As for high-energy expansion, first expand around $m_H \to 0$.

Then two possible (and finally equivalent) approaches:

1. Take the IBP-reduced amplitude of the high-energy expansion:
   - expand the master integrals around $t \to 0$ instead of $m_t \to 0$
2. Expand the unreduced amplitude around $q_3 \to -q_1$ $(t \to 0)$:
   - IBP reduce to new master integrals which only depend on $\epsilon$, $s$, $m_t$
   - this approach can be applied at NNLO, but only to restricted expansion depth



# Small-$t$ expansion: evaluation of the MIs

"Semi-analytic" determination of the $t \to 0$ MIs: [Fael, Lange, Schönwald, Steinhauser '21]

1. Establish system of DEs for the MIs, w.r.t. $\hat{s} = s/m_t^2$.
2. Expand around $\hat{s} = 0$:
   - insert ansatz into DE: $J(\epsilon, \hat{s} = 0) = \sum_{i,j} c_{ijk}\, \epsilon^i\, \hat{s}^j \ln^k(\hat{s})$
   - determine minimal set of $c_{ijk}$ (`Kira`+`FireFly`)
   - evaluate minimal boundary constants analytically (in the large-mass expansion)
3. Expand around a new point $\hat{s} = \hat{s}_0$ (repeat the above, modify ansatz).
4. Match the expansions (numerically) at a point where they both converge.

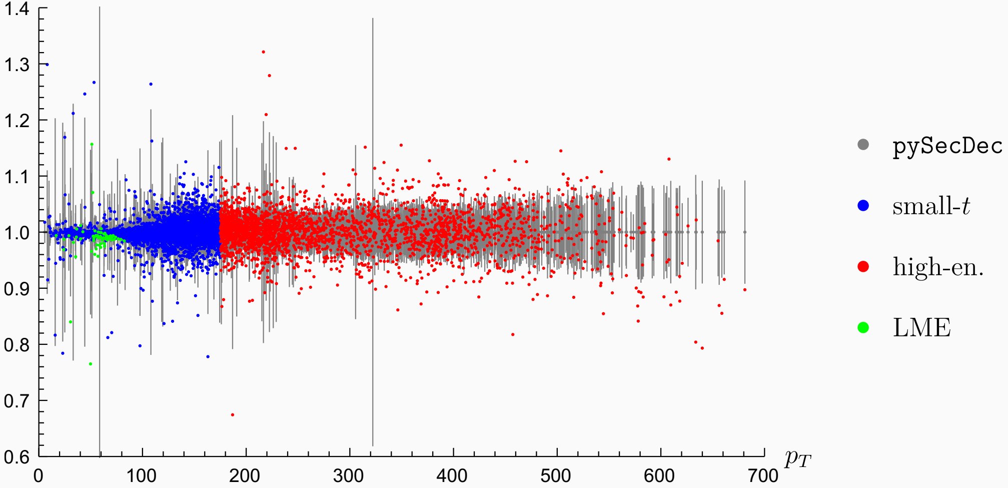
Here we have such "semi-analytic" expansions for the MIs at:

$$\hat{s} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 16, 20, 25, 30, 40, 50, \infty\}$$

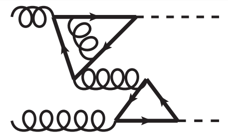
# HE and $t \to 0$ combination: "$V_{fin}$"

Comparison with `hhgrid`: [[https://github.com/mppmu/hhgrid]]

- merge both results, switch at $p_T = 175$ GeV.



Split the amplitude into parts:

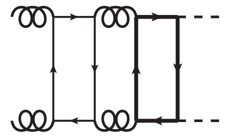


1PR

expand $m_H$,
rest exact

"$(gg \to H)^2$" w/
off-shell gluon

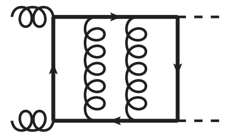
[Davies, Schönwald, Steinhauser, Vitti '24]



$n_l n_h \{C_A, C_F\}$

expand $m_H$,
small-$t$ exp.

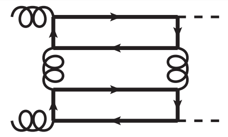
[Davies, Schönwald, Steinhauser '23]



$n_h \{C_A^2, C_A C_F, C_F^2\}$

expand $m_H$,
small-$t$ exp.

In progress

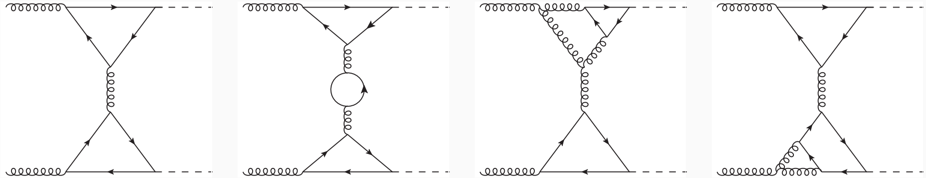


$n_h^2 \{C_A, C_F\}$

expand $m_H$,
small-$t$ exp. (!)

in progress
massless $t$-channel cut

# $gg \to HH$ at NNLO QCD: reducible contributions

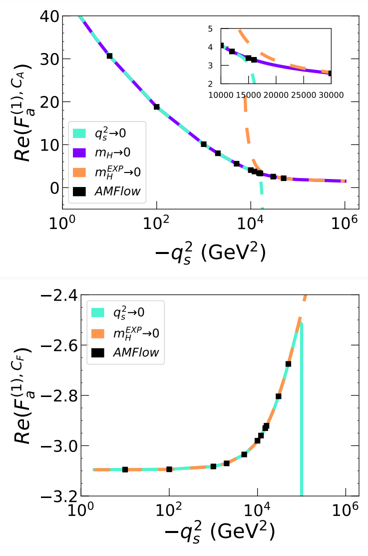


- Need to compute the of-shell $g(q_g)g^*(q_s)H(q_H)$ vertex up to 2 loops.
- Perform asymptotic expansions in:
  1. $m_H^2 \ll q_s^2, m_t^2$: hard region reduces to the same master integrals as the $t \to 0$ expansion
  2. $q_s^2 \ll m_H^2, m_t^2$: new analytic solutions for 2-loop master integrals in terms of HPLs

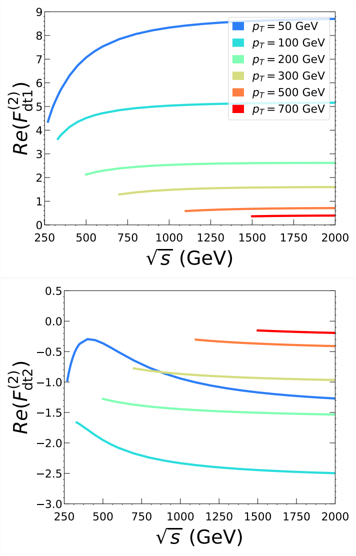
$\Rightarrow$ cover the whole phase space for $\{s, t; m_t, m_H\}$

# $gg \to HH$ at NNLO QCD: reducible contributions

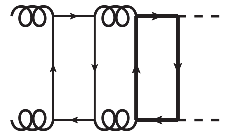
## $gg^*H$ form factor




## $gg \to HH$ form factor



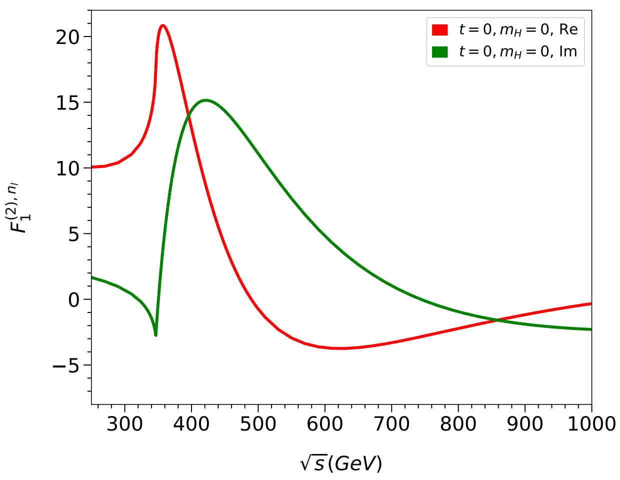
# $gg \to HH$ at NNLO QCD: $n_l$ part



$n_l n_h \{C_A, C_F\}$, leading expansion term $(m_H^0 t^0)$: [Davies, Schönwald, Steinhauser '23]

1. Expand $m_H \to 0$, $q_3 \to -q_1$ (FORM) [Ruijl, Ueda, Vermaseren '17]
2. Partial fraction decomposition (tapir, LIMIT) [Gerlach,Herren,Lang '23; Herren '20]
3. 60 integral topologies. 28 after common (sub-)sector identification
   - LiteRed, Feynson [Lee '14; Magerya '22]]
4. IBP (Kira) 85K→**176 MIs** (symm by Kira) [Klappert, Lange, Maierhöfer, Usovitsch '21]
   - (to compute $m_H^1 t^0 + m_H^0 t^1$: 4.5M integrals...)
5. Compute MIs with "expand and match".

# $gg \to HH$ at NNLO QCD: $n_l$ part




Sharp variation around $\sqrt{s} = 2m_t$ threshold:

- Leading behaviour $v \log^2 v$, cf. $v \log v$ at NLO ($v = \sqrt{1 - 4m_t^2/s}$).

$F_2$ **vanishes at** $m_H^0 t^0$**.**

# $gg \to HH$ at NNLO QCD: $n_h$ part

$n_h\{C_A^2, C_A C_F, C_F^2\}$, leading expansion term $(m_H^0 t^0)$:

1. Expand $m_H \to 0$, $q_3 \to -q_1$ (FORM)
2. Partial fraction decomposition (LIMIT)
3. 522 integral topologies. 203 after common (sub-)sector identification
   - Feynson (LiteRed is much too slow) [Magerya '22]
4. IBP (Kira) 2.6M→**33K MIs** across all topologies
   - Total: 330 days (16 core jobs)
   - Hardest single topology: 41 days, >2TB mem. Took several attempts:
     - master integral basis improvement, using ImproveMasters.m
     - change of momentum routings for smaller IBP relations

Cannot reduce master integrals between topologies with Kira:

- Symmetry finding and equation generation for each topology too slow.

# $gg \to HH$ at NNLO QCD: $n_h$ part, MI basis reduction

First step:

- Apply `FIRE`'s `FindRules` to MI list: 33K→**4313** [Smirnov, Chuharev '20]

Next:

- Apply `FindRules` to the 2.6M input integrals: 1.3M pairs
- Apply IBP tables to the pairs: 820K equations involving 4029 MIs
- Solve with `Kira`'s `user_defined_system`: 4313→**1647**

The basis is still not minimal.

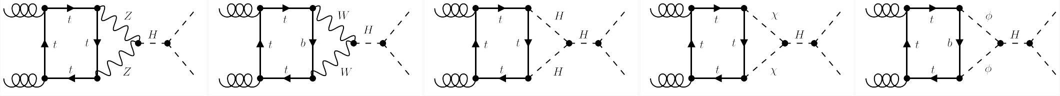
`FIRE` test reduction for all topologies (to a different basis):

- Repeat the above steps: 35K→1817→**1561**
  - Now the differential equations look better, and we can try to solve it.
  - (Probably, the basis is still not minimal)

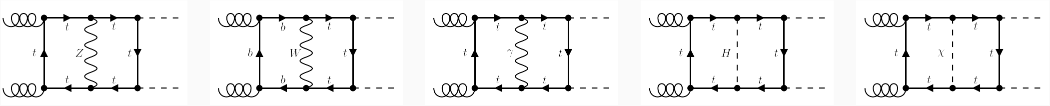
## Electroweak corrections to $gg \to HH$

# Full Electroweak Corrections in the Large-$m_t$ Expansion

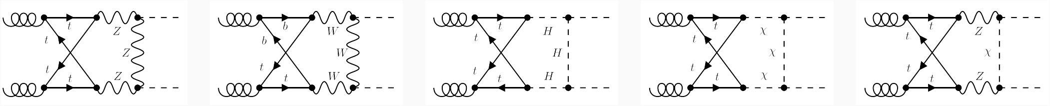
- Sample Feynman diagrams involving:
  - SM fields: $\{t,\ b,\ H,\ \gamma,\ Z,\ W^{\pm},\ \chi,\ \phi^{\pm}\}$
  - ghosts: $\{u^{\gamma},\ u^{Z},\ u^{\pm}\}$



(a-1) (a-2) (a-3) (a-4) (a-5)



(b-1) (b-2) (b-3) (b-4) (b-5)

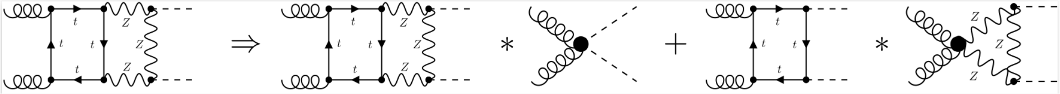


(c-1) (c-2) (c-3) (c-4) (c-5)

**Goal:** obtain analytic expressions in the large-$m_t$ expansion

# Large-$m_t$ Expansion and Renormalization

- Expand and calculate in general $R_\xi$ gauge with `qgraf` [Nogueira '93], `tapir` [Gerlach, Herren, Lang '23], `q2e&exp` [Harlander, Seidensticker, Steinhauser '97-'99], `form` [Ruijl, Ueda, Vermaseren '17], `LiteRed` [Lee '12] and `MATAD` [Steinhauser '01].
- Expansion hierarchy: $m_t^2 \gg \xi_W m_W^2, \xi_Z m_Z^2 \gg s, t, m_H^2, m_W^2, m_Z^2$

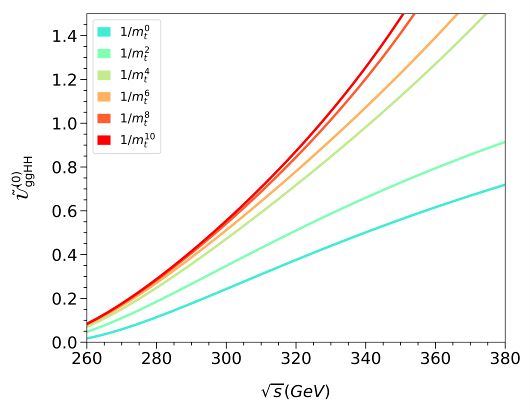



- We renormalize the input parameters $\{e,\ m_W,\ m_Z,\ m_t,\ m_H\}$ and the Higgs wave function on-shell and transform to the $G_\mu$ scheme.
  - $\xi_W, \xi_Z, \mu^2$ cancel analytically

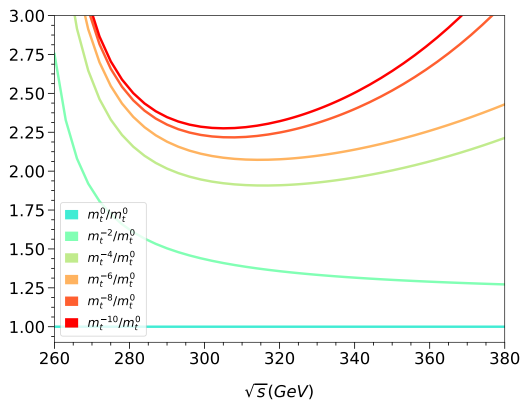
# LO Matrix Elements for $gg \to HH$

$$\mathcal{M} = \frac{1}{8^2 2^2} \sum_{\text{col}} \sum_{\text{pol}} |\mathcal{A}|^2 = \frac{1}{16} \left( X_0^{ggHH} \right)^2 \tilde{U}_{ggHH}$$

$$\tilde{U}_{ggHH} = \tilde{U}^{(0)}_{ggHH} + \frac{\alpha}{\pi} \tilde{U}^{(0,1)}_{ggHH}$$




$\tilde{U}_{ggHH}$ up to different expansion orders in $1/m_t$.



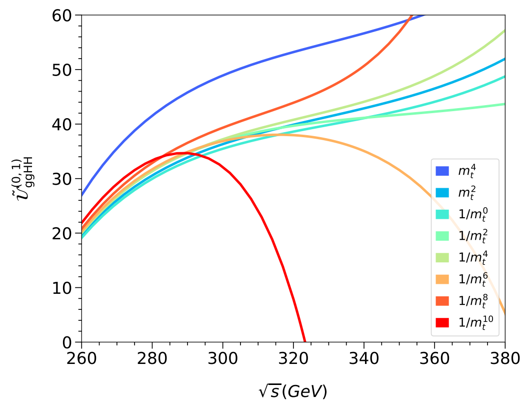

Different expansion orders normalized to $m_t^0$.

We see a nice convergence up to roughly $\sqrt{s} = 2m_t \approx 350$ ,GeV.

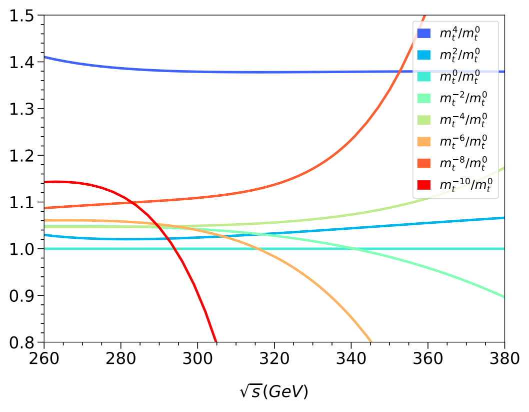
# NLO Electroweak Matrix Elements for $gg \to HH$

$$\mathcal{M} = \frac{1}{8^2 2^2} \sum_{\text{col}} \sum_{\text{pol}} |\mathcal{A}|^2 = \frac{1}{16} \left( X_0^{ggHH} \right)^2 \tilde{U}_{ggHH}$$

$$\tilde{U}_{ggHH} = \tilde{U}^{(0)}_{ggHH} + \frac{\alpha}{\pi} \tilde{U}^{(0,1)}_{ggHH}$$




$\tilde{U}_{ggHH}$ up to different expansion orders in $1/m_t$.

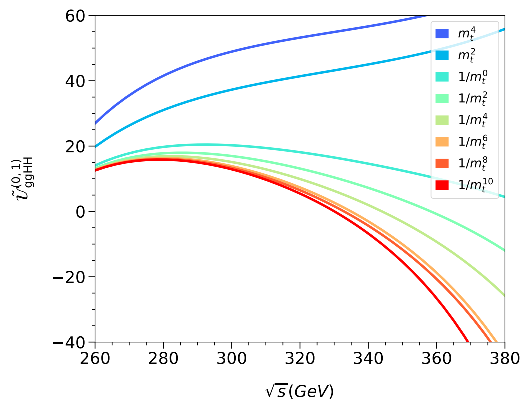


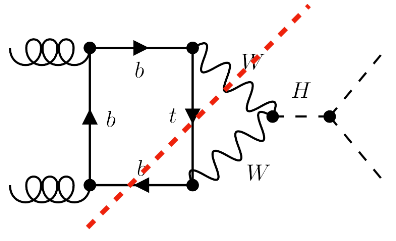

Different expansion orders normalized to $m_t^0$.

We do not see such a nice convergence at NLO.

# NLO Electroweak Matrix Elements for $gg \to HH$

$$\mathcal{M} = \frac{1}{8^2 2^2} \sum_{\text{col}} \sum_{\text{pol}} |\mathcal{A}|^2 = \frac{1}{16} \left(X_0^{ggHH}\right)^2 \tilde{U}_{ggHH}$$



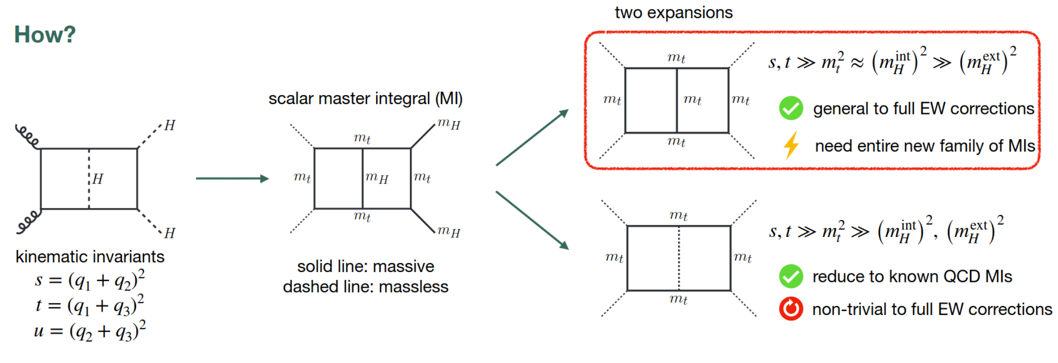

Cut through $W$-$t$-$b$ affects convergence of the large-$m_t$ expansion:

$m_t + m_b + m_W \approx 250\,\text{GeV}$

We can restore convergence by excluding diagrams with $W$-$t$-$b$ cuts.

# Beyond the Large-$m_t$ Expansion – High Energy Expansion

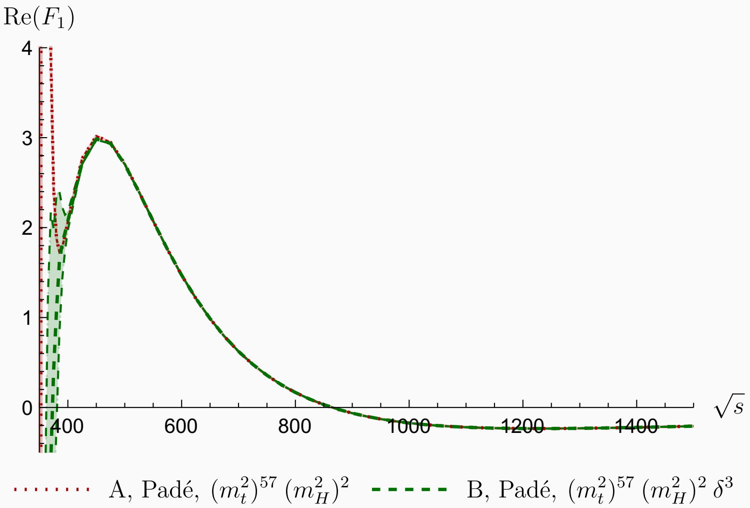
- Start with diagrams with internally propagating Higgs:
  - expansion parameter not small $\alpha_t = \alpha m_t^2/(2 s_W^2 m_W^2) \sim \alpha_s/2$
  - only planar integrals contribute in this subset



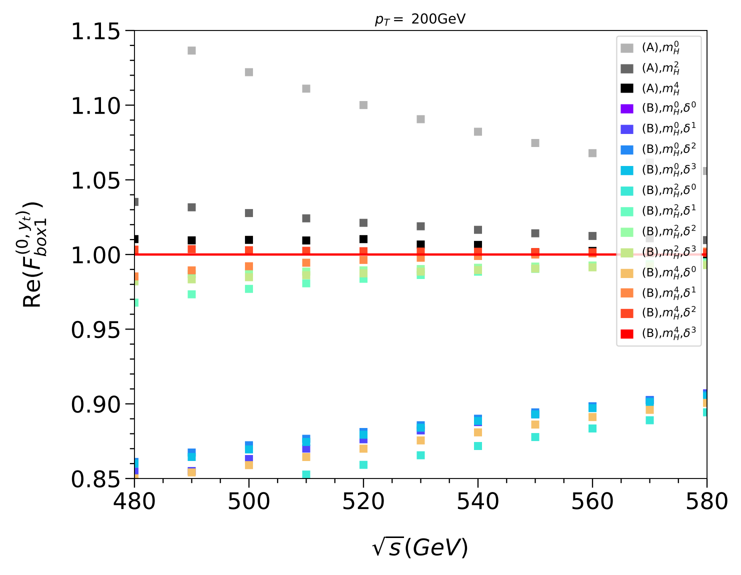
# High-energy Expansion: “A”, “B” comparison

$\text{Re}(F_{box1})$, fixed $\cos\theta = 0$, best “A” and “B” Padé

- “A”, “B” differ by at most 2% for $\sqrt{s} \geq 400\text{GeV}$,
- 0.1% for $\sqrt{s} \geq 500\text{GeV}$



# Beyond the Large-$m_t$ Expansion – High Energy Expansion




$$\mathcal{A}^{\mu\nu} = T_1^{\mu\nu} F_{box1} + T_2^{\mu\nu} F_{box2}$$

- We benchmark against the expansion to $O(m_H^4, \delta^3, m_t^{116})$, with $\delta = 1 - m_H/m_t$.
- Convergence of different expansion orders at fixed $p_T = 200$ GeV.
- Verified agreement with the pySecDec group.

# Conclusions and Outlook

# Conclusions and Outlook

**Conslusions:**

- Multi-scale, multi-loop integrals are hard to evaluate:
  $\rightarrow$ Reduce complexity by expanding in physically relevant regions.
- Expansions give a good description for $gg \rightarrow HH$ at NLO QCD.
- We made first steps toward NNLO by considering light-fermion corrections.
- We have calculated full NLO electroweak corrections to $gg \rightarrow HH$ and $gg \rightarrow gH$ in the large-$m_t$ expansion.
  - The convergence of these expansions is hindered by $W$-$t$-$b$ cuts.
- We have calculated parts of the leading-Yukawa corrections in the high-energy region and see a good convergence of our approach.

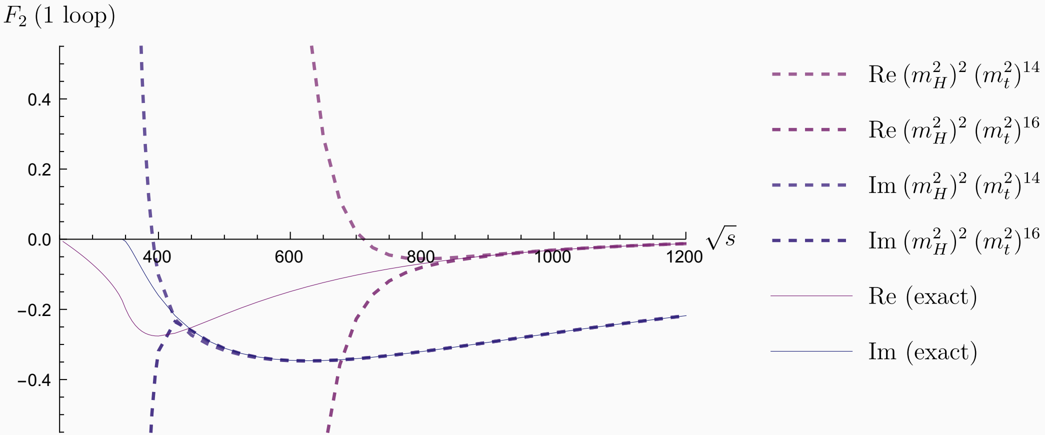
# Conclusions and Outlook

**Outlook:**

- Calculate full NNLO QCD corrections.
  - to come: remaining diagrams to leading expansion order
  - are deeper expansion orders possible? (very challenging IBP reduction)
- Calculate the full EW corrections in the
  1. high-energy expansion.
  2. small-$t$ expansion.
- Provide a numerical program, which can be incorporated into Monte-Carlo studies.

# Backup

# High-energy expansion: LO comparison



# Padé-Improved High-Energy Expansion

The master integrals for both methods are computed as an expansion in $m_t \ll s, |t|$.

The expansions diverge for $\sqrt{s} \sim 750\text{GeV}$ ("A"), $\sqrt{s} \sim 1000\text{GeV}$ ("B").

The situation can be improved using Padé Approximants:

- Approximate a function using a rational polynomial

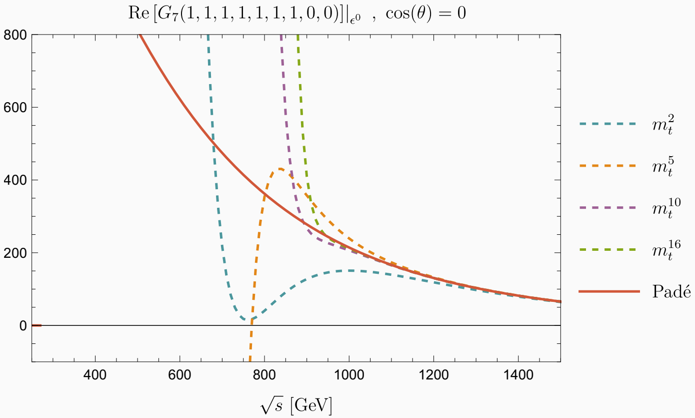
$$\boxed{f(x) \approx \frac{a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n}{1 + b_1 x + b_2 x^2 + \cdots + b_m x^m}},$$

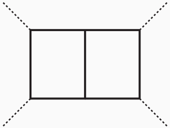
  where $a_i, b_j$ coefficients are fixed by the series coefficients of $f(x)$.

We compute a set of various Padé Approximants:

- combine to give a central value and error estimates
- a deeper input expansion $\Rightarrow$ larger $n + m \Rightarrow$ smaller error
- here, $m_t^{120}$ expansion allows for very high-order Padé Approximants

# Master Integrals Results

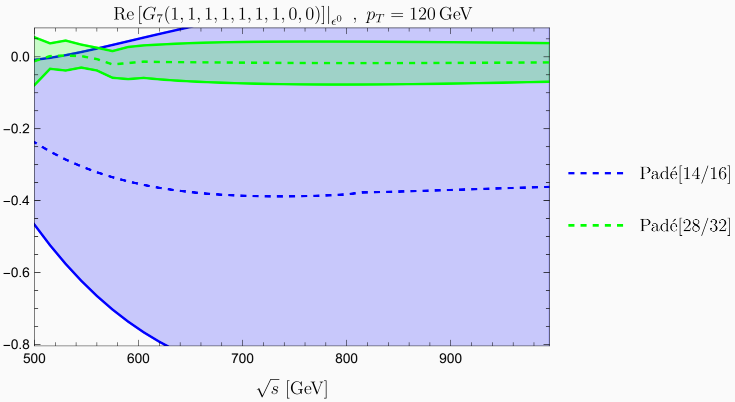


$$\cos(\theta) = \frac{s + 2t - 2m_h^2}{s\sqrt{1 - 4m_h^2/s}}$$

- Fixed order $m_t$ expansions diverge at $\sqrt{s} \sim 1000\,\mathrm{GeV}$.
- The Padé approximation extends the range of validity.

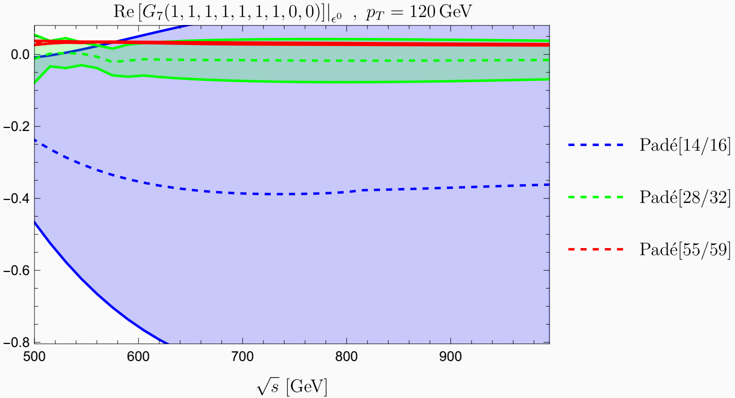
# Padé Improvement




$$p_T^2 = \frac{tu - m_h^4}{s}$$

- Lower order Padé approximantions cannot reach low values of $p_T$.
- For QCD corrections expansions up to $m_t^{32}$ were available: $p_T \gtrsim 150\,\mathrm{GeV}$
- With expansions up to $m_t^{120}$ we reach: $p_T \gtrsim 120\,\mathrm{GeV}$.
- Error estimate from Padé approximations is reliable.

# Padé Improvement

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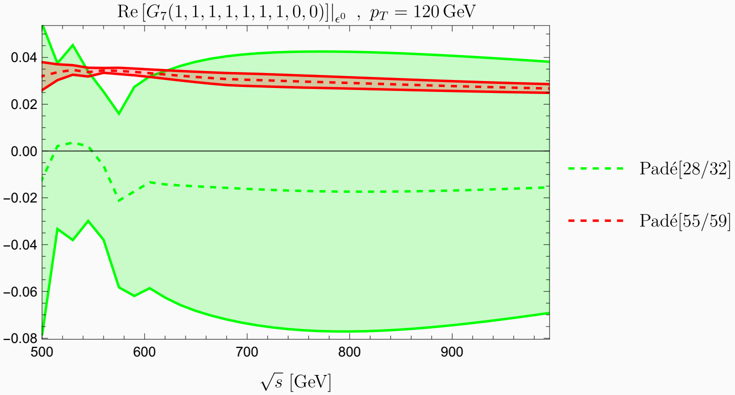


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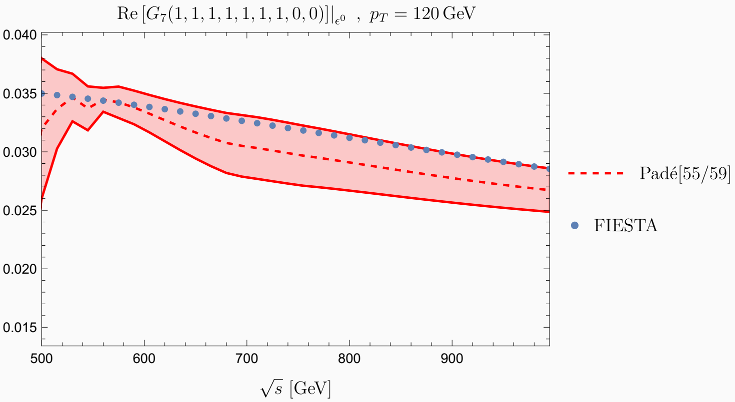


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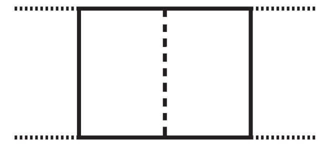
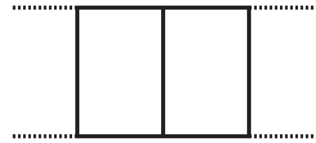
# Comparison to the $m_H \to 0$ Expansion



**Approach A**:

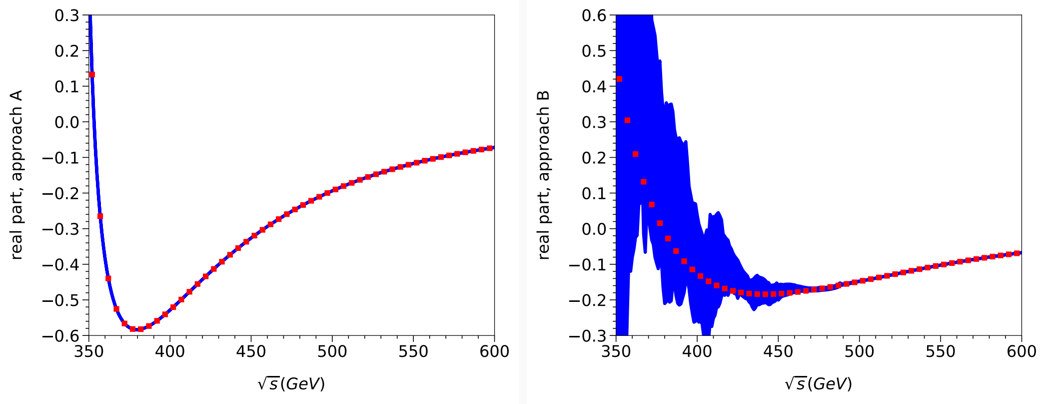
- middle line massless $m_H^{\text{int}} \approx 0$
- calculated in the context of QCD corrections [Davies, Mishima, Steinhauser, Wellmann '18, '19]



**Approach B**:

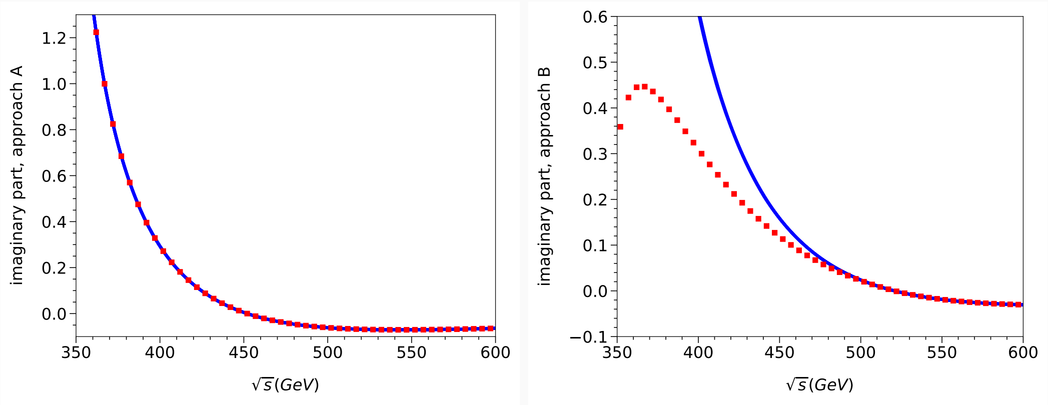
- middle line massive $m_H^{\text{int}} \approx m_t$

# Comparison with Approach A




Approach A: threshold at $\sqrt{s} = 2m_t = 346\,\text{GeV}$ Approach B: threshold at $\sqrt{s} = 3m_t = 519\,\text{GeV}$

# Comparison with Approach A




Approach A: threshold at $\sqrt{s} = 2m_t = 346\,\text{GeV}$ Approach B: threshold at $\sqrt{s} = 3m_t = 519\,\text{GeV}$

# Beyond the Large-$m_t$ Expansion – High Energy Expansion

Analytic high-energy expansion:

- Expansion hierarchy: $s, t \gg m_t^2 \approx (m_H^2)^{int} \gg (m_H^2)^{ext}$
- We get a system of differential equations for 140 master integrals

$$\boxed{\frac{\partial}{\partial m_t^2}\vec{I} = M(s,t,m_t^2,\epsilon)\cdot\vec{I}}, \quad \text{with } \vec{I} = (I_1,\ldots,I_{140})$$

- Plug in power-log ansatz for each master integral

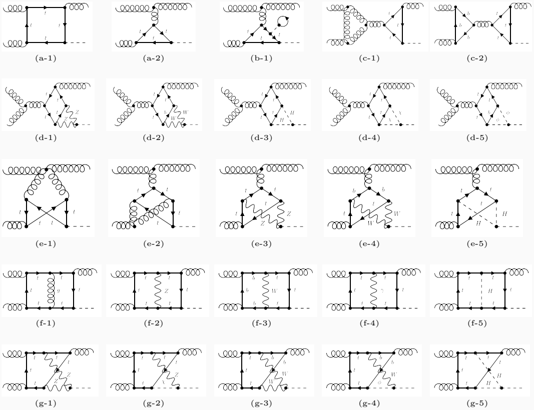
$$\boxed{I_n = \sum_{i=-2}^{0}\sum_{j=-1}^{60}\sum_{k=0}^{i+4} c_n^{ijk}(s,t)\epsilon^i (m_t^2)^j \ln^k(m_t^2)}$$

- Solve the system of linear equations for a small set of boundary constants with `Kira` and `FireFly` [Klappert, Lange, Maierhöfer, Usovitsch '21] .
- Solve boundary master integrals in the asymptotic limit $m_t \to 0$ with Mellin-Barnes methods and symbolic summation using `Asy` [Pak, Smirnov '11] , `MB.m` [Czakon '05] , `HarmonicSums` [Ablinger '10] and `Sigma` [Schneider '07] .

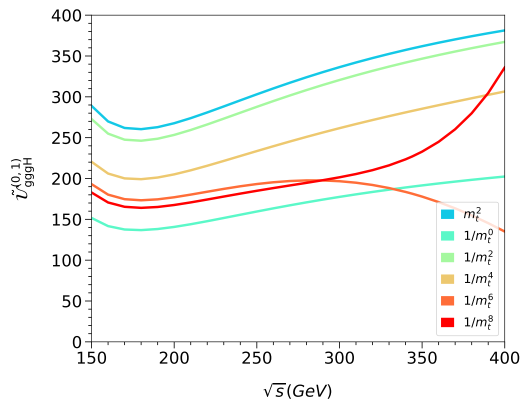
# NLO Electroweak Matrix Elements for $gg \to Hg$

$$\mathcal{M} = \frac{1}{8^2 2^2} \sum_{\text{col}} \sum_{\text{pol}} |\mathcal{A}|^2 = \frac{3}{32} \left( X_0^{gggH} \right)^2 s\, \tilde{U}_{gggH}$$

$$\tilde{U}_{gggH} = \tilde{U}^{(0)}_{gggH} + \frac{\alpha}{\pi} \tilde{U}^{(0,1)}_{gggH}$$




Graphs contributing to $gg \to Hg$.




Different expansion orders in $1/m_t$.

We observe a nice convergence at NLO.

# High energy expansion: Option A

Option A: asymptotic expansion around $m_H^{int} = 0$. Expnsion-by-subgraphs:

- two sub-graphs:


The two-loop subgraph is a Taylor expansion of the Higgs propagator:

- results in integrals with a massless internal line. Scales: $s, t, m_t$.
- IBP reduce with `FIRE` and `Kira` [Smirnov '15; Klappert, Lange, Maierhöfer, Usovitsch '21]
- these coincide with the QCD master integrals – reuse the old results

[Davies, Mishima, Steinhauser, Wellmann '18,'19]

The massive tadpoles are easily computed by `MATAD`. [Steinhauser '00]

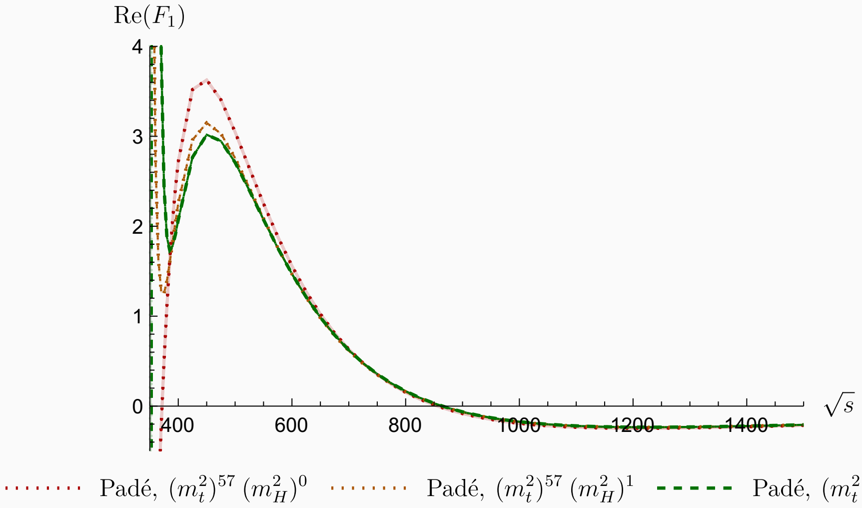
The asymptotic expansion procedure is done by `exp` and `FORM`. [Harlander, Seidelsticker, Steinhauser '97] [Ruijl, Ueda, Vermaseren '17]

We expand to quartic order: $(m_H^{int})^a \, (m_H^{ext})^b, \;\; 0 \leq (a+b) \leq 4$.

# High-Energy Expansion "A": convergence

Re($F_{box1}$), fixed $\cos\theta = 0$, expansion "A" Padé (to $(m_H^2)^{\{0,1,2\}}$):

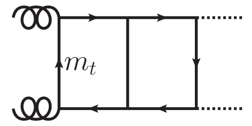
- $(m_H^2)^1$ and $(m_H^2)^2$ terms differ by at most 5% for $\sqrt{s} \geq 400$GeV



# High-energy Expansion "B"

Option B: expand around $m_H^{int} \approx m_t$,

- simple Taylor expansion, easy to implement




Write Higgs propagator as: $\frac{1}{p^2 - m_H^2} = \frac{1}{p^2 - m_t^2(1 - [2-\delta]\delta)}$

- expand around $\delta \to 0$ where $\delta = 1 - m_H/m_t \approx 0.28$.

This yields new integral families compared to the QCD computation:

- All lines have the mass $m_t$.
- IBP reduce and compute the master integrals (140) in the high-energy limit.

Expand to $(m_H^{ext})^4$ and $\delta^3$.

# High-energy Expansion “B”: convergence

Re($F_{box1}$), fixed $\cos\theta = 0$, expansion “B” Padé (to $(m_H^2)^2\delta^{\{0,1,2,3\}}$):

- $\delta^2$ and $\delta^3$ terms differ by at most 0.5% for $\sqrt{s} \geq 400$GeV