Energy-Energy Correlation in the back-to-back region at $N^3LL+NNLO$ in QCD

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In collaboration with:

U.G. Aglietti

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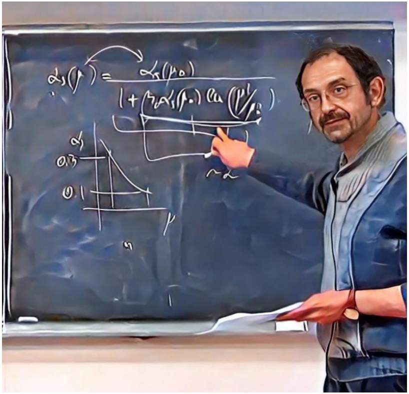
Stefano Catani (1958-2024)



Wonderful person, outstanding physicist

The idea: α_S from semi-inclusive processes

QCD COHERENT BRANCHING AND SEMI-INCLUSIVE PROCESSES AT LARGE x*

S. CATANI** and B.R. WEBBER

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Received 22 June 1990

$$\alpha_s^{(\mathrm{MC})} = \alpha_s^{(\overline{\mathrm{MS}})}\left(1 + K\frac{\alpha_s^{(\overline{\mathrm{MS}})}}{2\pi}\right),$$

$$\Lambda_{\mathrm{MC}} = \Lambda_{\overline{\mathrm{MS}}}\exp(K/4\pi\beta_0)$$
$$\simeq 1.569\,\Lambda_{\overline{\mathrm{MS}}} \quad \text{for } N_f = 5.$$

In this paper we have studied [...] the next-to-leading logarithmic terms in **semi-inclusive hard processes such as the DIS and DY processes** *at large x. Since the Monte Carlo algorithm with these improvements is accurate to next-to-leading order in the large-x region,* **it can be used to determine the fundamental QCD scale $\Lambda_{\overline{MS}}$**

The idea: α_S from semi-inclusive processes

Advantages:

- **higher sensitivity to α_S** w.r.t. *inclusive* observables;
- calculable at **higher theoretical accuracy** w.r.t. *exclusive* observables.

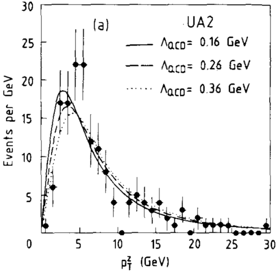
Challenges:

- sensitivity to **infrared (Sudakov) logs**;
- sensitivity **non perturbative QCD** effects.

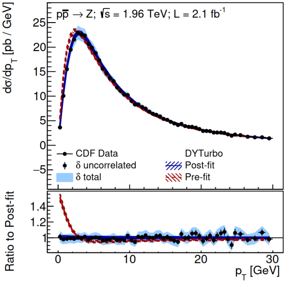
Classical semi-inclusive obs. at colliders:
high invariant-mass **Drell–Yan** lepton pair hadroproduction
at small transverse-momentum (q_T) and
energy-energy-correlation in e^+e^- annihilation **in the back-to-back limit.**

α_S from Z-boson q_T distribution

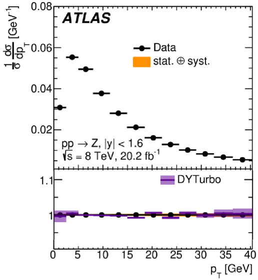
$Sp\bar{p}S$ ($\sqrt{s} = 0.63$ TeV) | **Tevatron ($\sqrt{s} = 1.96$ TeV)** | **LHC ($\sqrt{s} = 7 - 8$ TeV)**



[UA2 Coll.('92)]
compared with
[Altarelli et al.('84)]

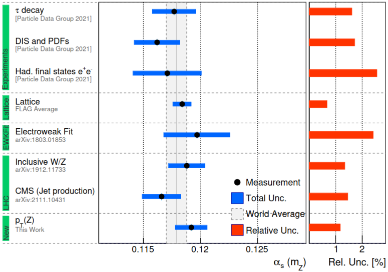


[D0 Coll.('08,'10)]
compared with DYTurbo:
[Catani et al.('10)],
[Camarda et al.('20)]



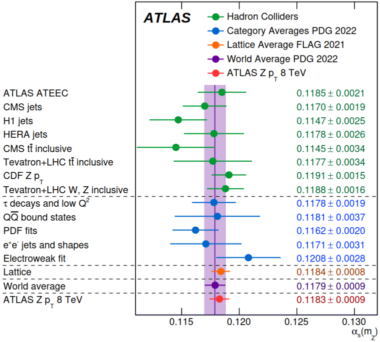
[ATLAS Coll.('14)]
compared with DYTurbo:
[Catani et al.('15)],
[Camarda et al.('20)]

α_S from Z-boson q_T distribution



$\alpha_S(m_Z)$ determination from Z-boson p_T at Tevatron [Camarda,G.F.,Schött('22)]

$\alpha_S(m_Z) = 0.1184^{+0.0013}_{-0.0015}$



$\alpha_S(m_Z)$ determination from Z-boson p_T at LHC [ATLAS Coll.('23)]

$\alpha_S(m_Z) = 0.1183 \pm 0.0009$

Energy-Energy Correlation (EEC) function

$$e^+ + e^- \rightarrow h_i + h_j + X$$

$$\frac{d\Sigma}{d\cos\chi} = \sum_{i,j=1}^{n} \int \frac{E_i}{Q}\frac{E_j}{Q}\,\delta(\cos\chi - \cos\theta_{ij})\, d\sigma_{e^+e^- \to h_i h_j + X}$$

where $Q = \sqrt{s}$ and θ_{ij} is the angle between momenta $\vec{p}_i$ and $\vec{p}_j$ [Basham,Brown, Ellis,Love('78)].

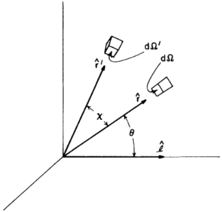


FIG. 2. Geometry for the experiment.

- EEC is IRC finite. While $d\sigma$ depends on parton fragmentation functions $D_{h,q}$, EEC does not: $\sum_h \int_0^1 dx\, x\, D_{h,q}(x,\mu_F^2) = 1$. EEC calculable in pure pQCD.
- Normalization gives

$$\int_{-1}^{+1} \frac{d\Sigma}{d\cos\chi}\, d\cos\chi = \int \left(\sum_{i=1}^{n} \frac{E_i}{Q}\right)^2 d\sigma = \sigma_{tot}.$$

- In the CoM frame at $\mathcal{O}(\alpha_S^0)$ we have a back-to-back $q\bar{q}$ pair:

$$\frac{1}{\sigma_{tot}}\frac{d\Sigma}{d\cos\chi} = \frac{1}{2}\delta(1-\cos\chi) + \frac{1}{2}\delta(1+\cos\chi) + \mathcal{O}(\alpha_S).$$

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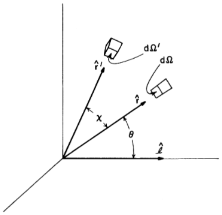

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EEC in fixed-order pQCD

At higher orders in QCD we have (we use $z = (1 - \cos\chi)/2 = \sin^2(\chi/2)$):

$$\frac{1}{\sigma_{tot}}\frac{d\Sigma}{dz} = \frac{1}{2}\left(\delta(1-z) + \delta(z)\right) + \frac{\alpha_S}{\pi}\mathcal{A}(z) + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{B}(z) + \left(\frac{\alpha_S}{\pi}\right)^3 \mathcal{C}(z) + \mathcal{O}(\alpha_S^4)\,,$$

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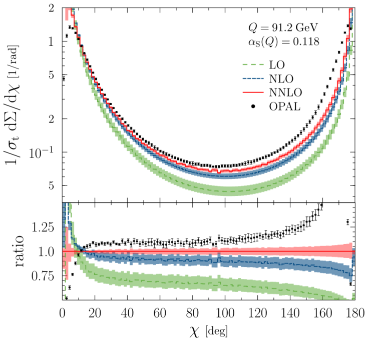
- The $\mathcal{O}(\alpha_S)$ function $\mathcal{A}(z)$ is known analytically from [Basham et al.('78)].
- At $\mathcal{O}(\alpha_S^2)$ function $\mathcal{B}(z)$ known analytically by [Dixon et al.('18)] (numerically by [Richards et al.('82,83)]).
- The $\mathcal{O}(\alpha_S^3)$ function $\mathcal{C}(z)$ known numerically by [Del Duca et al.('16)],[Tulipant,Kardos, Somogyi('17)] from (fully differential) NNLO calculation of 3-jets cross-section in e^+e^- ann. using ColoRFulNNLO subtraction method [Del Duca et al.('16)].



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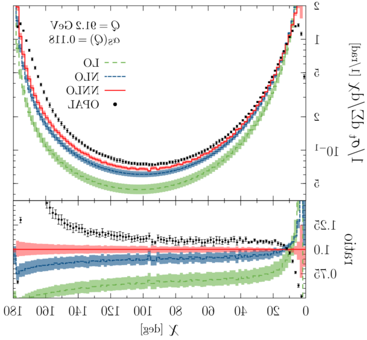

EEC in the back-to-back limit

In the back-to-back limit $z \to 1$ ($\chi \to \pi$) we have

$$\mathcal{A}(z) = C_F \left\{ -\frac{1}{2}\left[\frac{\ln(1-z)}{1-z}\right]_+ - \frac{3}{4}\left[\frac{1}{1-z}\right]_+ - \left(\frac{\pi^2}{12} - \frac{11}{8}\right)\delta(1-z) + \cdots . \right\}$$

- In general at any order α_S^n large infrared (Sudakov) logarithms appears

$$\alpha_S^n \left[\frac{\ln^k(1-z)}{1-z}\right]_+ , \quad 0 \le k \le 2n-1$$

- Large logs spoils the convergence of fixed-order perturbative expansion. Reliable QCD predictions requires all order Sudakov resummation.
- In the back-to-back region the q_T between 2 hadrons is

$$q_T^2 \simeq Q^2 \cos^2(\chi/2) = Q^2(1-z) \to 0$$

  and EEC is closely related to Drell–Yan process at small-q_T.
- EEC function also contains large (single) logarithmic corrections of hard-collinear nature in the forward region $z \to 0$ (or $\chi \to 0$), $\ln^{n-1}(z)/z$, where hadrons have small angular separations [Dixon et al.('19)].

EEC in the back-to-back limit

ECC can be written in terms of the unitegrated parton fragmentation functions $D_{h,q}(x, p_T, Q)$ [Kodaira,Trentadue('81)].

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CAN SOFT GLUON EFFECTS BE MEASURED IN ELECTRON-POSITRON ANNIHILATION?*

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Stanford University, Stanford, California 94305

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Stanford University, Stanford, California 94305

ABSTRACT

The energy-energy correlation at large angles in e^+e^- annihilation is calculated by resumming soft gluon contributions through two-loop level. The result is compared with experimental data. No agreement is obtained using a purely perturbative analysis. The relevance of nonperturbative effects at present energies is emphasized.

Using the unintegrated parton densities[4,8] $D(Q^2, p_T, x)$ the energy-energy correlation is written as[9] (see Fig. 1)

$$\frac{1}{\sigma_{TOT}} \frac{d\Sigma}{d^2 Q_T} = \frac{1}{\sigma_{TOT}} \frac{1}{2} \sum_{A,B} \int x_A dx_A \int x_B dx_B \sum_{q,\bar{q}} \int d^2p_T^A \, d^2p_T^B \, d^2p_T^S \times \delta^2\left(Q_T - \frac{p_T^A}{x_A} - \frac{p_T^B}{x_B} - p_T^S\right) D_q^A\left(Q^2, p_T^A, x_A\right) D_{\bar{q}}^B\left(Q^2, p_T^B, x_B\right) S\left(Q^2, p_T^S\right) \quad (1)$$

$$Q^2 \frac{\partial}{\partial Q^2} D_q\left(Q^2, b_T, x\right) = \int \frac{dz}{z} \int dq_T^2 \left[\frac{\alpha_s(q_T^2)}{2\pi} + K\left(\frac{\alpha_s(q_T^2)}{2\pi}\right)^2\right] \times C_F\left(\frac{1+z^2}{1-z}\right)_+ \delta\left[z(1-z)Q^2 - q_T^2\right] J_0\left(\frac{bq}{z}\right) D_q\left(Q^2, \frac{b_T}{z}, \frac{x}{z}\right) \, , \quad (2)$$

Sudakov resummation for EEC

[Kodaira,Trentadue('81)],
[Collins,Soper('83),de Florian,Grazzini('04),Tulipant et al.('17),Kardos et al.('18)]

$$\frac{1}{\sigma_{tot}}\frac{d\Sigma}{dz} = \frac{1}{\sigma_{tot}}\frac{d\Sigma_{res.}}{dz} + \frac{1}{\sigma_{tot}}\frac{d\Sigma_{fin.}}{dz}\ ;$$

In the impact parameter (b) space [Parisi,Petronzio('79)]:
$1-z \ll 1 \Leftrightarrow Qb \gg 1,\quad \ln(1-z) \gg 1 \Leftrightarrow \ln(Qb) \gg 1$

$$\frac{1}{\sigma_{tot}}\frac{d\Sigma_{res.}}{dz} = \frac{1}{4}\,H(\alpha_S)\int_0^\infty db\,Q^2\,b\,J_0(\sqrt{1-z}\,Qb)\,S(Q,b),$$

$$S(Q,b) \;=\; \exp\left\{-\int_{b_0^2/b^2}^{Q^2}\frac{dq^2}{q^2}\left[A(\alpha_S(q^2))\ln\frac{Q^2}{q^2} + B(\alpha_S(q^2))\right]\right\}$$

$$H(\alpha_S) = \sum_{n=1}^\infty H_n\,\alpha_S^n,\quad A(\alpha_S) = \sum_{n=1}^\infty A_n\,\alpha_S^n,\quad B(\alpha_S) = \sum_{n=1}^\infty B_n\,\alpha_S^n\ .$$

- Analogous results have been obtained, more recently, in the framework of SCET framework [Moult,Zhu('18)],[Ebert et al.('20)],[Duhr et al.('22)]

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Distinctive features of the formalism [Catani, de Florian, Grazzini('01)], [Bozzi, Catani, de Florian, Grazzini('03)], [de Florian, Grazzini('04)]:

$$S(Q,b) \;=\; \exp\left\{ L\, g^{(1)}(\alpha_S L) + g^{(2)}(\alpha_S L) + \frac{\alpha_S}{\pi} g^{(3)}(\alpha_S L) + \left(\frac{\alpha_S}{\pi}\right)^2 g^{(4)}(\alpha_S L) + \cdots \right\}$$

with $L = \ln(Q^2 b^2 / b_0^2), \quad \alpha_S L \sim 1, \quad b_0 = 2e^{-\gamma_E} \simeq 1.123$

LL ($\sim \alpha_S^n L^{n+1}$): $g^{(1)}$; NLL ($\sim \alpha_S^n L^n$): $g^{(2)}$, H_1; $\cdots$ N^{k}LL ($\sim \alpha_S^n L^{n+k-1}$): $g^{(k+1)}$, H_k;

- Introduction of resummation scale $\mu_Q \sim Q$ [Bozzi at al. ('03)]: variations give an estimate of the uncertainty from uncalculated logarithmic corrections.

$$\ln(Q^2 b^2) \rightarrow \ln(\mu_Q^2 b^2) + \ln(Q^2/\mu_Q^2)$$

- Perturbative unitarity constraint: recover *exactly* the fixed-order total cross-section (upon integration on z)

$$\ln(Q^2 b^2/b_0^2) \rightarrow \widetilde{L} \equiv \ln(Q^2 b^2/b_0^2 + 1) \;\Rightarrow\; \exp\{\alpha_S^n \widetilde{L}^k\}\big|_{b=0} = 1 \;\Rightarrow\; \int_0^1 dz \left(\frac{d\sigma}{dz}\right) = \sigma_{tot};$$

- No explicit NP models: Landau singularity of α_S regularized using a *Minimal Prescription* without power-suppressed corrections [Catani et al.('96)], [Laenen et al.('00)].

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- No explicit NP models: Landau singularity of α_S regularized using a *Minimal Prescription* without power-suppressed corrections [Catani et al.('96)],[Laenen et al.('00)].

Sudakov resummation in pQCD

Distinctive features of the formalism [Catani, de Florian, Grazzini ('01)], [Bozzi, Catani, de Florian, Grazzini ('03)], [de Florian, Grazzini ('04)]:

$$S(Q,b) = \exp\left\{ L\, g^{(1)}(\alpha_S L) + g^{(2)}(\alpha_S L) + \frac{\alpha_S}{\pi} g^{(3)}(\alpha_S L) + \left(\frac{\alpha_S}{\pi}\right)^2 g^{(4)}(\alpha_S L) + \cdots \right\}$$

with $L = \ln(Q^2 b^2/b_0^2)$, $\alpha_S L \sim 1$, $b_0 = 2e^{-\gamma_E} \simeq 1.123$

LL ($\sim \alpha_S^n L^{n+1}$): $g^{(1)}$; NLL ($\sim \alpha_S^n L^n$): $g^{(2)}$, H_1; $\cdots$ N^{k}LL ($\sim \alpha_S^n L^{n+k-1}$): $g^{(k+1)}$, H_k;

- Introduction of resummation scale $\mu_Q \sim Q$ [Bozzi at al. ('03)]: variations give an estimate of the uncertainty from uncalculated logarithmic corrections.

$$\ln(Q^2 b^2) \to \ln(\mu_Q^2 b^2) + \ln(Q^2/\mu_Q^2)$$

- Perturbative unitarity constraint: recover *exactly* the fixed-order total cross-section (upon integration on z)

$$\ln(Q^2 b^2/b_0^2) \to \widetilde{L} \equiv \ln(Q^2 b^2/b_0^2 + 1) \;\Rightarrow\; \exp\{\alpha_S^n \widetilde{L}^k\}\big|_{b=0} = 1 \;\Rightarrow\; \int_0^1 dz \left(\frac{d\sigma}{dz}\right) = \sigma_{tot};$$

- No explicit NP models: Landau singularity of α_S regularized using a *Minimal Prescription* without power-suppressed corrections [Catani et al.('96)], [Laenen et al.('00)].

EEC resummation: perturbative accuracy

- Coefficients A_1, A_2, A_3, A_4, B_1, B_2 and H_1 already known [Basham et al.('78)],Kodaira,Trentadue('82),de Florian,Grazzini('04),Becher,Neubert('11). We have determined the new coefficients B_3 and H_2 and H_3 in full QCD from results in SCET [Ebert,Mistlberger,Vita('20)].
- We thus performed all-order resummation up to N^{3}LL logarithmic accuracy all orders (i.e. up to $\exp(\sim \alpha_S^n L^{n-2})$) including hard-virtual contribution up to factor N^{3}LO.
- Matching with NNLO corrections (i.e. up to $\mathcal{O}(\alpha_S^3)$) from results in [Del Duca et al.('16)],[Tulipant,Kardos,Somogyi('17)];
- Results up to N^{3}LO (i.e. up to $\mathcal{O}(\alpha_S^3)$) recovered for the total cross section (from unitarity).
- Full three-loop ($\mathcal{O}(\alpha_S^3)$) result also includes three-loop solution of the QCD coupling (β_0–β_3).
- pQCD prediction fully determined from the knowledge of $\alpha_S(m_Z^2)$.

Asymptotic expansion

$$\frac{1}{\sigma_{\rm tot}}\frac{d\Sigma_{\rm (res.)}}{dz}\Big|_{\rm f.o.} = \frac{1}{2}\delta(1-z)+\mathcal{A}_{\rm (res.)}(z)\frac{\alpha_S}{\pi}+\mathcal{B}_{\rm (res.)}(z)\left(\frac{\alpha_S}{\pi}\right)^2+\mathcal{C}_{\rm (res.)}(z)\left(\frac{\alpha_S}{\pi}\right)^3+\cdots$$

$$\mathcal{A}_{\rm (res.)}(z) = -\frac{A_1}{4}I_2(z)-\frac{B_1}{2}I_1(z)+\frac{H_1}{2}\delta(1-z)$$

$$\mathcal{B}_{\rm (res.)}(z) = \frac{A_1^2}{16}I_4(z)+\frac{A_1}{2}\left(\frac{B_1}{2}-\frac{\beta_0}{3}\right)I_3(z)-\frac{1}{4}\left[A_2-B_1^2+B_1\beta_0+A_1H_1\right]I_2(z)-\frac{1}{2}\left[B_2+B_1H_1\right]I_1(z)+\frac{H_2}{2}\delta(1-z);$$

$$\begin{aligned}\mathcal{C}_{\rm (res.)}(z) = & -\frac{A_1^3}{96}I_6(z)-A_1^2\left(\frac{B_1}{16}-\frac{\beta_0}{12}\right)I_5(z)+\frac{A_1}{8}\left[A_2-B_1^2+\frac{7B_1\beta_0}{3}-\beta_0^2+A_1\frac{H_1}{2}\right]I_4(z)\\ & +\left[\frac{A_2B_1}{2}+\frac{A_1B_2}{2}-\frac{B_1^3}{6}-\frac{A_1\beta_1}{3}-\frac{2}{3}A_2\beta_0+\left(\frac{B_1}{2}-\frac{\beta_0}{3}\right)\left(A_1H_1+B_1\beta_0\right)\right]\frac{I_3(z)}{2}\\ & +\left[-\frac{A_3}{2}+(B_1-\beta_0)\left(\frac{B_1H_1}{2}+B_2\right)-\frac{B_1\beta_1}{2}-\frac{A_2H_1}{2}-\frac{A_1H_2}{2}\right]\frac{I_2(z)}{2}-(B_3+B_2H_1+B_1H_2)\frac{I_1(z)}{2}+\frac{H_3}{2}\delta(1-z)\end{aligned}$$

where $\quad I_n(z) \equiv \int_0^\infty d(Qb)\,\frac{Qb}{2}\,J_0\left(\sqrt{1-z}\,Qb\right)\,\ln^n\left(\frac{Q^2\,b^2}{b_0^2}\right);$

$$I_1(z)=-\left[\frac{1}{1-z}\right]_+,\; I_2(z)=2\left[\frac{\ln(1-z)}{1-z}\right]_+,\; I_3(z)=-4z_3\delta(1-z)-3\left[\frac{\ln^2(1-z)}{1-z}\right]_+,\;\cdots,\; I_n(z)=\cdots$$

with the unitarity constraint we have:

$$I_n(z)\to\widetilde{I}_n(z) \equiv \int_0^\infty d(Qb)\,\frac{Qb}{2}\,J_0\left(\sqrt{1-z}\,Qb\right)\,\ln^n\left(\frac{Q^2\,b^2}{b_0^2}+1\right).$$

Asymptotic expansion

$$\left.\frac{1}{\sigma_{\rm tot}}\frac{d\Sigma_{\rm (res.)}}{dz}\right|_{\rm f.o.} = \frac{1}{2}\delta(1-z) + \mathcal{A}_{\rm (res.)}(z)\frac{\alpha_S}{\pi} + \mathcal{B}_{\rm (res.)}(z)\left(\frac{\alpha_S}{\pi}\right)^2 + \mathcal{C}_{\rm (res.)}(z)\left(\frac{\alpha_S}{\pi}\right)^3 + \cdots$$

$$\mathcal{A}_{\rm (res.)}(z) = -\frac{A_1}{4}I_2(z) - \frac{B_1}{2}I_1(z) + \frac{H_1}{2}\delta(1-z)$$

$$\mathcal{B}_{\rm (res.)}(z) = \frac{A_1^2}{16}I_4(z) + \frac{A_1}{2}\left(\frac{B_1}{2} - \frac{\beta_0}{3}\right)I_3(z) - \frac{1}{4}\left[A_2 - B_1^2 + B_1\beta_0 + A_1H_1\right]I_2(z) - \frac{1}{2}\left[B_2 + B_1H_1\right]I_1(z) + \frac{H_2}{2}\delta(1-z);$$

$$\begin{aligned}\mathcal{C}_{\rm (res.)}(z) = & -\frac{A_1^3}{96}I_6(z) - A_1^2\left(\frac{B_1}{16} - \frac{\beta_0}{12}\right)I_5(z) + \frac{A_1}{8}\left[A_2 - B_1^2 + \frac{7B_1\beta_0}{3} - \beta_0^2 + A_1\frac{H_1}{2}\right]I_4(z) \\ & + \left[\frac{A_2B_1}{2} + \frac{A_1B_2}{2} - \frac{B_1^3}{6} - \frac{A_1\beta_1}{3} - \frac{2}{3}A_2\beta_0 + \left(\frac{B_1}{2} - \frac{\beta_0}{3}\right)\left(A_1H_1 + B_1\beta_0\right)\right]\frac{I_3(z)}{2} \\ & + \left[-\frac{A_3}{2} + (B_1-\beta_0)\left(\frac{B_1H_1}{2} + B_2\right) - \frac{B_1\beta_1}{2} - \frac{A_2H_1}{2} - \frac{A_1H_2}{2}\right]\frac{I_2(z)}{2} - (B_3 + B_2H_1 + B_1H_2)\frac{I_1(z)}{2} + \frac{H_3}{2}\delta(1-z)\end{aligned}$$

where $\quad I_n(z) \equiv \int_0^\infty d(Qb)\frac{Qb}{2}J_0\left(\sqrt{1-z}\,Qb\right)\ln^n\left(\frac{Q^2b^2}{b_0^2}\right);$

$$I_1(z) = -\left[\frac{1}{1-z}\right]_+,\ I_2(z) = 2\left[\frac{\ln(1-z)}{1-z}\right]_+,\ I_3(z) = -4z_3\delta(1-z) - 3\left[\frac{\ln^2(1-z)}{1-z}\right]_+,\ \cdots,\ I_n(z) = \cdots$$

with the unitarity constraint we have:

$$I_n(z) \to \widetilde{I}_n(z) \equiv \int_0^\infty d(Qb)\frac{Qb}{2}J_0\left(\sqrt{1-z}\,Qb\right)\ln^n\left(\frac{Q^2b^2}{b_0^2}+1\right).$$

Asymptotic expansion

$$\frac{1}{\sigma_{\rm tot}}\frac{d\Sigma_{\rm (res.)}}{dz}\Bigg|_{\rm f.o.} = \frac{1}{2}\delta(1-z) + \mathcal{A}_{\rm (res.)}(z)\frac{\alpha_S}{\pi} + \mathcal{B}_{\rm (res.)}(z)\left(\frac{\alpha_S}{\pi}\right)^2 + \mathcal{C}_{\rm (res.)}(z)\left(\frac{\alpha_S}{\pi}\right)^3 + \cdots$$

$$\mathcal{A}_{\rm (res.)}(z) = -\frac{A_1}{4} I_2(z) - \frac{B_1}{2} I_1(z) + \frac{H_1}{2}\delta(1-z)$$

$$\mathcal{B}_{\rm (res.)}(z) = \frac{A_1^2}{16} I_4(z) + \frac{A_1}{2}\left(\frac{B_1}{2} - \frac{\beta_0}{3}\right) I_3(z) - \frac{1}{4}\left[A_2 - B_1^2 + B_1\beta_0 + A_1 H_1\right] I_2(z) - \frac{1}{2}\left[B_2 + B_1 H_1\right] I_1(z) + \frac{H_2}{2}\delta(1-z);$$

$$\mathcal{C}_{\rm (res.)}(z) = -\frac{A_1^3}{96} I_6(z) - A_1^2\left(\frac{B_1}{16} - \frac{\beta_0}{12}\right) I_5(z) + \frac{A_1}{8}\left[A_2 - B_1^2 + \frac{7B_1\beta_0}{3} - \beta_0^2 + A_1\frac{H_1}{2}\right] I_4(z)$$
$$+ \left[\frac{A_2 B_1}{2} + \frac{A_1 B_2}{2} - \frac{B_1^3}{6} - \frac{A_1\beta_1}{3} - \frac{2}{3}A_2\beta_0 + \left(\frac{B_1}{2} - \frac{\beta_0}{3}\right)\left(A_1 H_1 + B_1\beta_0\right)\right]\frac{I_3(z)}{2}$$
$$+ \left[-\frac{A_3}{2} + (B_1-\beta_0)\left(\frac{B_1 H_1}{2} + B_2\right) - \frac{B_1\beta_1}{2} - \frac{A_2 H_1}{2} - \frac{A_1 H_2}{2}\right]\frac{I_2(z)}{2} - (B_3 + B_2 H_1 + B_1 H_2)\frac{I_1(z)}{2} + \frac{H_3}{2}\delta(1-z)$$

where $$I_n(z) \equiv \int_0^\infty d(Qb)\frac{Qb}{2} J_0\left(\sqrt{1-z}\,Qb\right)\ln^n\left(\frac{Q^2 b^2}{b_0^2}\right);$$

$$I_1(z) = -\left[\frac{1}{1-z}\right]_+, \; I_2(z) = 2\left[\frac{\ln(1-z)}{1-z}\right]_+, \; I_3(z) = -4z_3\delta(1-z) - 3\left[\frac{\ln^2(1-z)}{1-z}\right]_+, \; \cdots, \; I_n(z) = \cdots$$

with the unitarity constraint we have:

$$I_n(z) \to \widetilde{I}_n(z) \equiv \int_0^\infty d(Qb)\frac{Qb}{2} J_0\left(\sqrt{1-z}\,Qb\right)\ln^n\left(\frac{Q^2 b^2}{b_0^2} + 1\right).$$

Asymptotic expansion

$$\frac{1}{\sigma_{\rm tot}}\frac{d\Sigma_{\rm (res.)}}{dz}\bigg|_{\rm f.o.} = \frac{1}{2}\delta(1-z) + \mathcal{A}_{\rm (res.)}(z)\frac{\alpha_S}{\pi} + \mathcal{B}_{\rm (res.)}(z)\left(\frac{\alpha_S}{\pi}\right)^2 + \mathcal{C}_{\rm (res.)}(z)\left(\frac{\alpha_S}{\pi}\right)^3 + \cdots$$

$$\mathcal{A}_{\rm (res.)}(z) = -\frac{A_1}{4}I_2(z) - \frac{B_1}{2}I_1(z) + \frac{H_1}{2}\delta(1-z)$$

$$\mathcal{B}_{\rm (res.)}(z) = \frac{A_1^2}{16}I_4(z) + \frac{A_1}{2}\left(\frac{B_1}{2}-\frac{\beta_0}{3}\right)I_3(z) - \frac{1}{4}\left[A_2 - B_1^2 + B_1\beta_0 + A_1H_1\right]I_2(z) - \frac{1}{2}\left[B_2 + B_1H_1\right]I_1(z) + \frac{H_2}{2}\delta(1-z);$$

$$\begin{aligned}\mathcal{C}_{\rm (res.)}(z) = & -\frac{A_1^3}{96}I_6(z) - A_1^2\left(\frac{B_1}{16}-\frac{\beta_0}{12}\right)I_5(z) + \frac{A_1}{8}\left[A_2 - B_1^2 + \frac{7B_1\beta_0}{3} - \beta_0^2 + A_1\frac{H_1}{2}\right]I_4(z)\\ & + \left[\frac{A_2B_1}{2} + \frac{A_1B_2}{2} - \frac{B_1^3}{6} - \frac{A_1\beta_1}{3} - \frac{2}{3}A_2\beta_0 + \left(\frac{B_1}{2}-\frac{\beta_0}{3}\right)\left(A_1H_1 + B_1\beta_0\right)\right]\frac{I_3(z)}{2}\\ & + \left[-\frac{A_3}{2} + (B_1-\beta_0)\left(\frac{B_1H_1}{2}+B_2\right) - \frac{B_1\beta_1}{2} - \frac{A_2H_1}{2} - \frac{A_1H_2}{2}\right]\frac{I_2(z)}{2} - (B_3 + B_2H_1 + B_1H_2)\frac{I_1(z)}{2} + \frac{H_3}{2}\delta(1-z)\end{aligned}$$

where $$I_n(z) \equiv \int_0^\infty d(Qb)\,\frac{Qb}{2}\,J_0\left(\sqrt{1-z}\,Qb\right)\,\ln^n\left(\frac{Q^2\,b^2}{b_0^2}\right);$$

$$I_1(z) = -\left[\frac{1}{1-z}\right]_+,\quad I_2(z) = 2\left[\frac{\ln(1-z)}{1-z}\right]_+,\quad I_3(z) = -4z_3\delta(1-z) - 3\left[\frac{\ln^2(1-z)}{1-z}\right]_+,\ \cdots,\ I_n(z) = \cdots$$

with the unitarity constraint we have:

$$I_n(z) \to \widetilde{I}_n(z) \equiv \int_0^\infty d(Qb)\,\frac{Qb}{2}\,J_0\left(\sqrt{1-z}\,Qb\right)\,\ln^n\left(\frac{Q^2\,b^2}{b_0^2}+1\right).$$

Finite (remainder) function

Remainder function obtained subtracting the asymptotic expansion from the f.o. result:

$$\frac{1}{\sigma_{\rm tot}}\frac{d\Sigma_{(\rm fin.)}}{dz} = \frac{1}{\sigma_{\rm tot}}\frac{d\Sigma}{dz} - \frac{1}{\sigma_{\rm tot}}\frac{d\Sigma_{(\rm res.)}}{dz}\bigg|_{\rm f.o.} = \mathcal{A}_{(\rm fin.)}(z)\,\frac{\alpha_S}{\pi} + \mathcal{B}_{(\rm fin.)}(z)\left(\frac{\alpha_S}{\pi}\right)^2 + \mathcal{C}_{(\rm fin.)}(z)\left(\frac{\alpha_S}{\pi}\right)^3 + \cdots$$

$$\mathcal{A}_{(\rm fin.)}(z) = -\frac{2}{3\,z^5}\left(z^4+z^3-3z^2+15z-9\right)\ln(1-z) - \frac{z^3+z^2+7z-6}{z^4}\,,$$

$$\begin{aligned}\mathcal{B}_{(\rm fin.)}(z) &= \frac{1080z^6-3240z^5+4164z^4-2924z^3+1134z^2-229z+1}{9z(1-z)}\,z_3 + \cdots\\ &+ \quad \cdots\end{aligned}$$

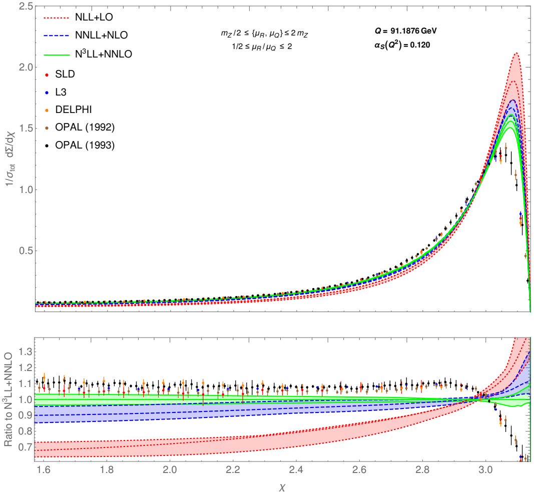
Third-order remainder function fitted with the following function:

$$\begin{aligned}\mathcal{C}_{(\rm fin.)}(z) &\approx 15\ln^5(1-z)+130\ln^4(1-z)+408\ln^3(1-z)+544\ln^2(1-z)+308\ln(1-z)+226\\ &+ 0.70545\,\frac{\ln^2(z)}{z} - 15.494\,\frac{\ln(z)}{z} + 39.568\,\frac{1}{z}\,,\end{aligned}$$

where the terms enhanced for $z \to 0$ known from analysis in [Dixon et al.('19)].

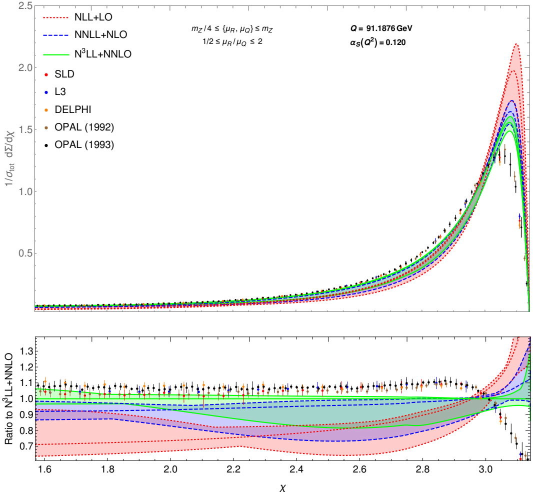
Similar results obtained with the unitarity constraint.

Numerical results: perturbative results



The resummed EEC spectrum at $\sqrt{s} = 91.1876$ GeV at various perturbative orders in QCD.

Numerical results: perturbative results



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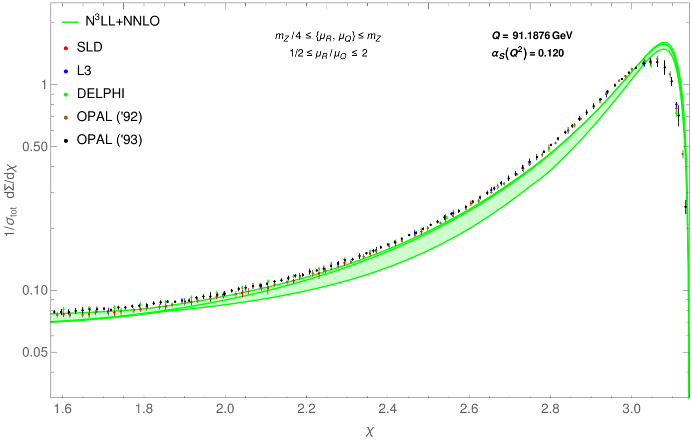
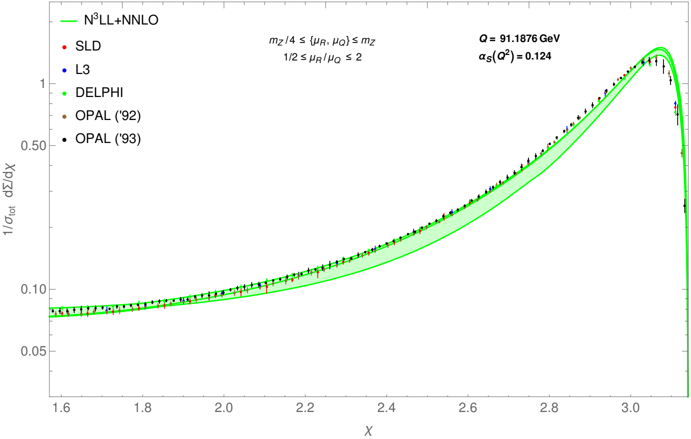
Numerical results: perturbative results



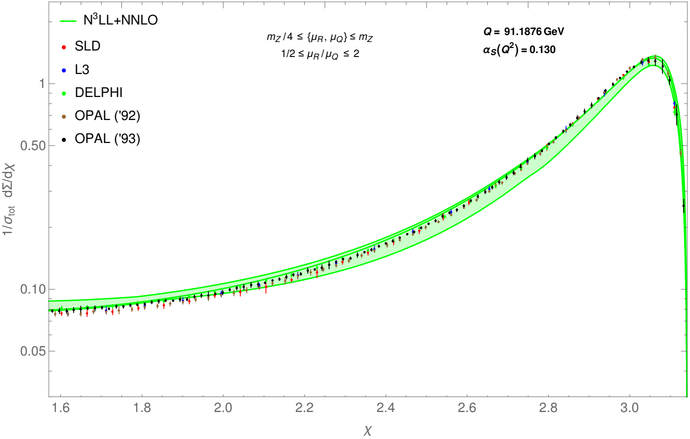
The resummed EEC spectrum at $\sqrt{s} = 91.1876$ GeV at N^{3}LL+NNLO in QCD with $\alpha_S(m_Z^2) = 0.120$, compared with LEP data from [OPAL Coll.('92)]

Numerical results: perturbative results



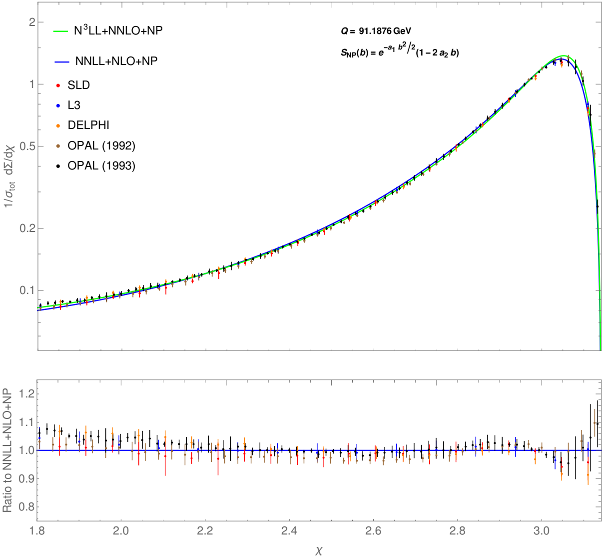
The resummed EEC spectrum at $\sqrt{s} = 91.1876$ GeV at N^{3}LL+NNLO in QCD with $\alpha_S(m_Z^2) = 0.124$, compared with LEP data from [OPAL Coll.('92)]

Numerical results: perturbative results



The resummed EEC spectrum at $\sqrt{s} = 91.1876$ GeV at N^{3}LL+NNLO in QCD with $\alpha_S(m_Z^2) = 0.130$, compared with LEP data from [OPAL Coll.('92)]

Numerical results: non perturbative effects



Preliminary comparison at N^3LL+NNLO and NNLL+NLO with NP effects parameterized by a form factor

$$S(Q,b) \to S(Q,b)S_{NP}(b)$$

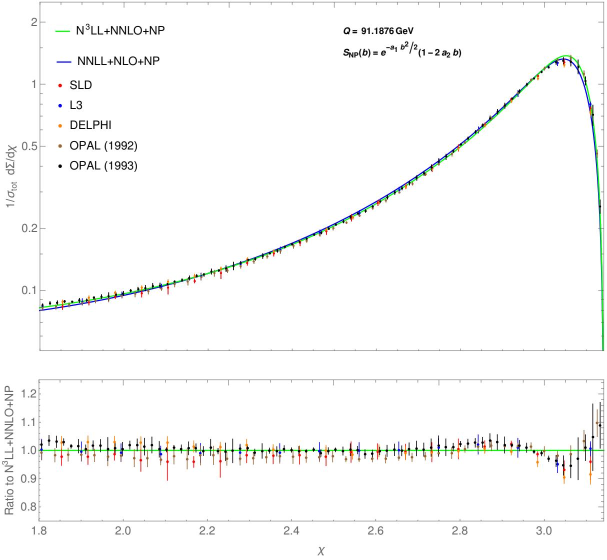
$$S_{NP} = \exp\{-a_1 b^2\}(1-a_2 b)$$

[Dokshitzer,Marchesini, Webber('99)]

Fit results

NNLL+NLO:
$\alpha_S(m_Z) = 0.121 \pm 0.002$,
$a_1 = 1.9 \pm 1.4 GeV^2$,
$a_2 = 0.4 \pm 0.1 GeV$

Numerical results: non perturbative effects



Preliminary comparison at $N^3LL+NNLO$ and $NNLL+NLO$ with NP effects parameterized by a form factor

$$S(Q,b) \to S(Q,b) S_{NP}(b)$$

$$S_{NP} = \exp\{-a_1 b^2\}(1 - a_2 b)$$

[Dokshitzer, Marchesini, Webber('99)]

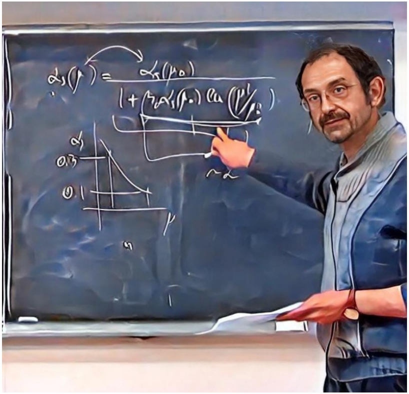
Fit results

NNLL+NLO:
$\alpha_S(m_Z) = 0.121 \pm 0.002$,
$a_1 = 1.9 \pm 1.4 GeV^2$,
$a_2 = 0.4 \pm 0.1 GeV$

$N^3LL+NNLO$:
$\alpha_S(m_Z) = 0.120 \pm 0.002$,
$a_1 = 1.8 \pm 1.4 GeV^2$,
$a_2 = 0.3 \pm 0.1 GeV$

Conclusions

- Semi-inclusive processes important to test pQCD predictions, extract information on NP QCD and determine the value of α_S.
- Presented resummed result for energy-energy-correlation in e^+e^- in the back-to-back region at full N^{3}LL accuracy (including N^{3}LO hard-virtual effects).
- Resummed results matched with the known NNLO results (important away the back-to-back region).
- Very precise pQCD: percent level perturbative uncertainty.
- Preliminary inclusion of NP QCD effects allows us to provide a very good description of precise experimental data from LEP and SLD at $\sqrt{s} = m_Z$.
- Extraction of $\alpha_S(m_Z)$ at N^{3}LO and NNLO accuracy consistent with the world average.



Thank you, Stefano!