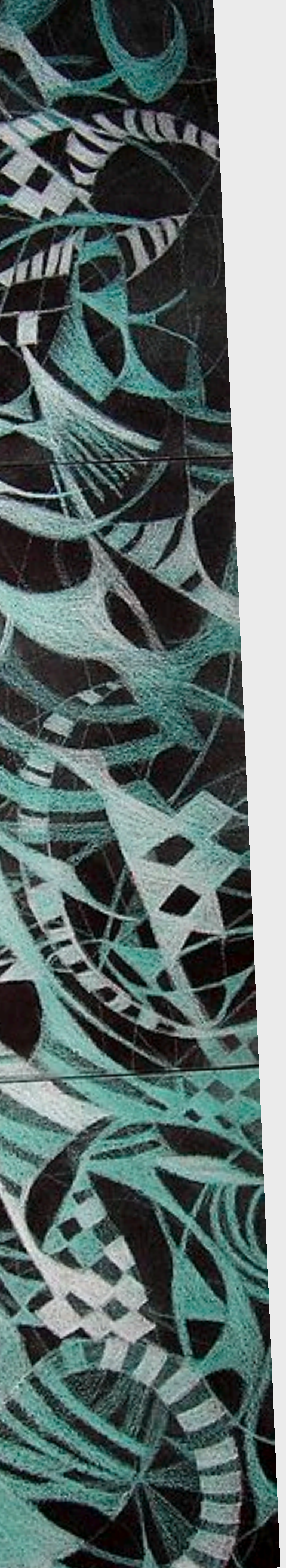
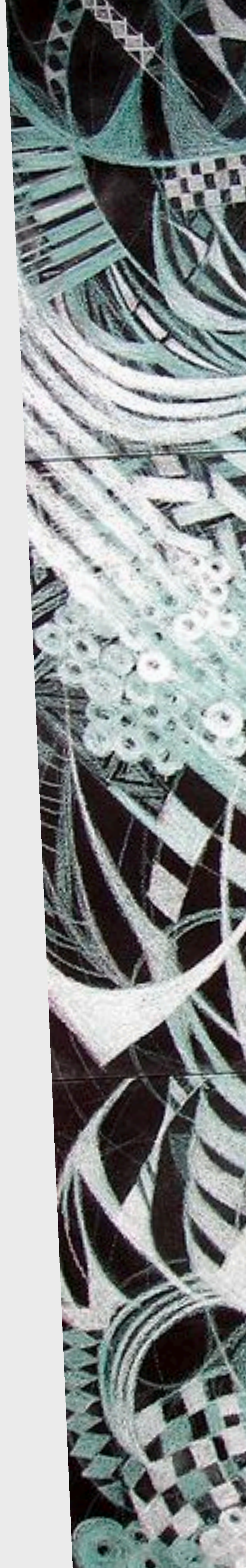


High precision Drell-Yan and electroweak input schemes

Clara Lavinia Del Pio - INFN Pavia

High Precision for Hard Processes 2024

Based on M. Chiesa, C. L. Del Pio, F. Piccinini, Eur.Phys.J. C 84 (2024) 5, 539

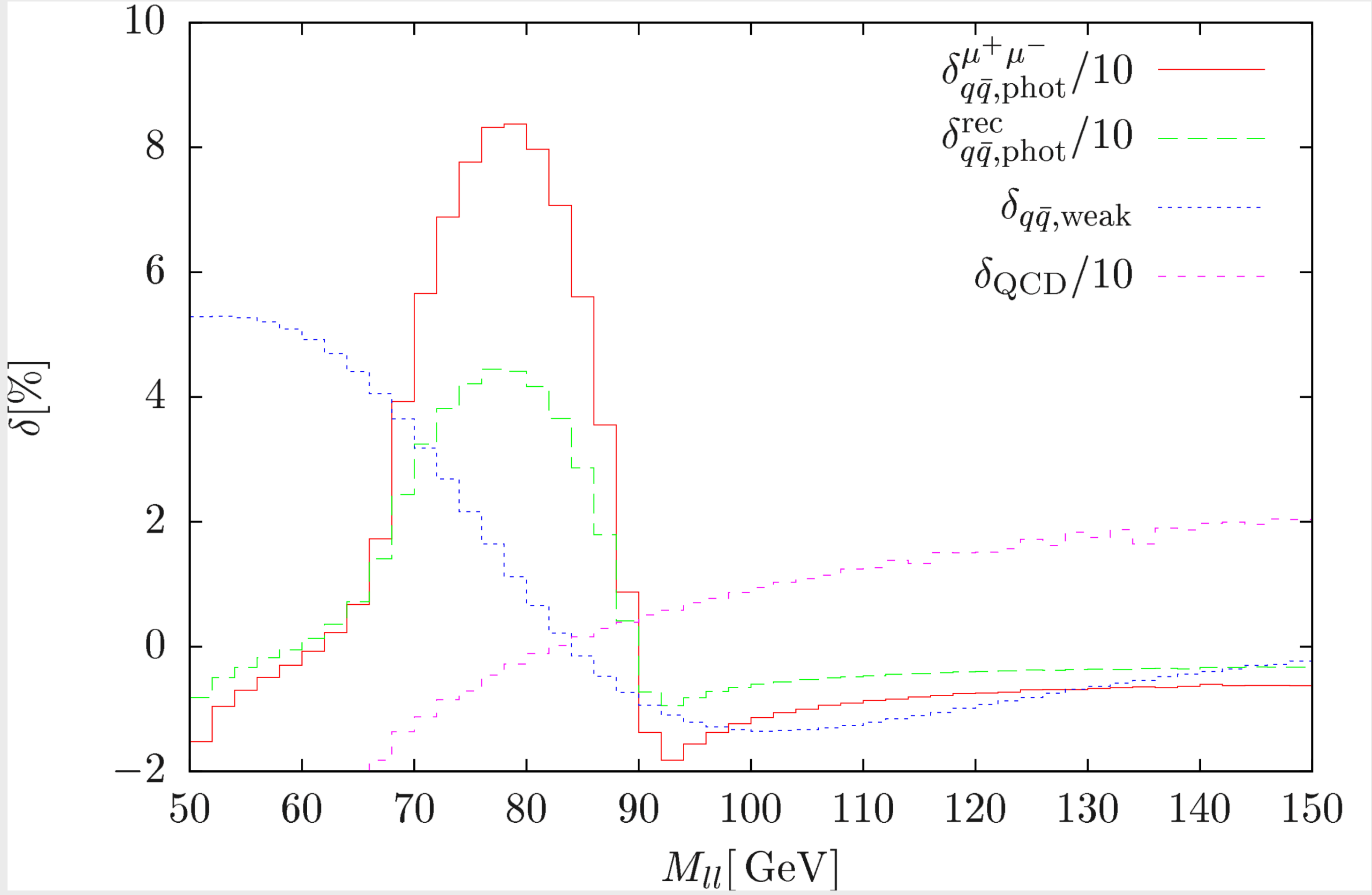


Why electroweak corrections?

- LHC precision measurements in the EW sector can validate the SM and get sensitivity to new physics

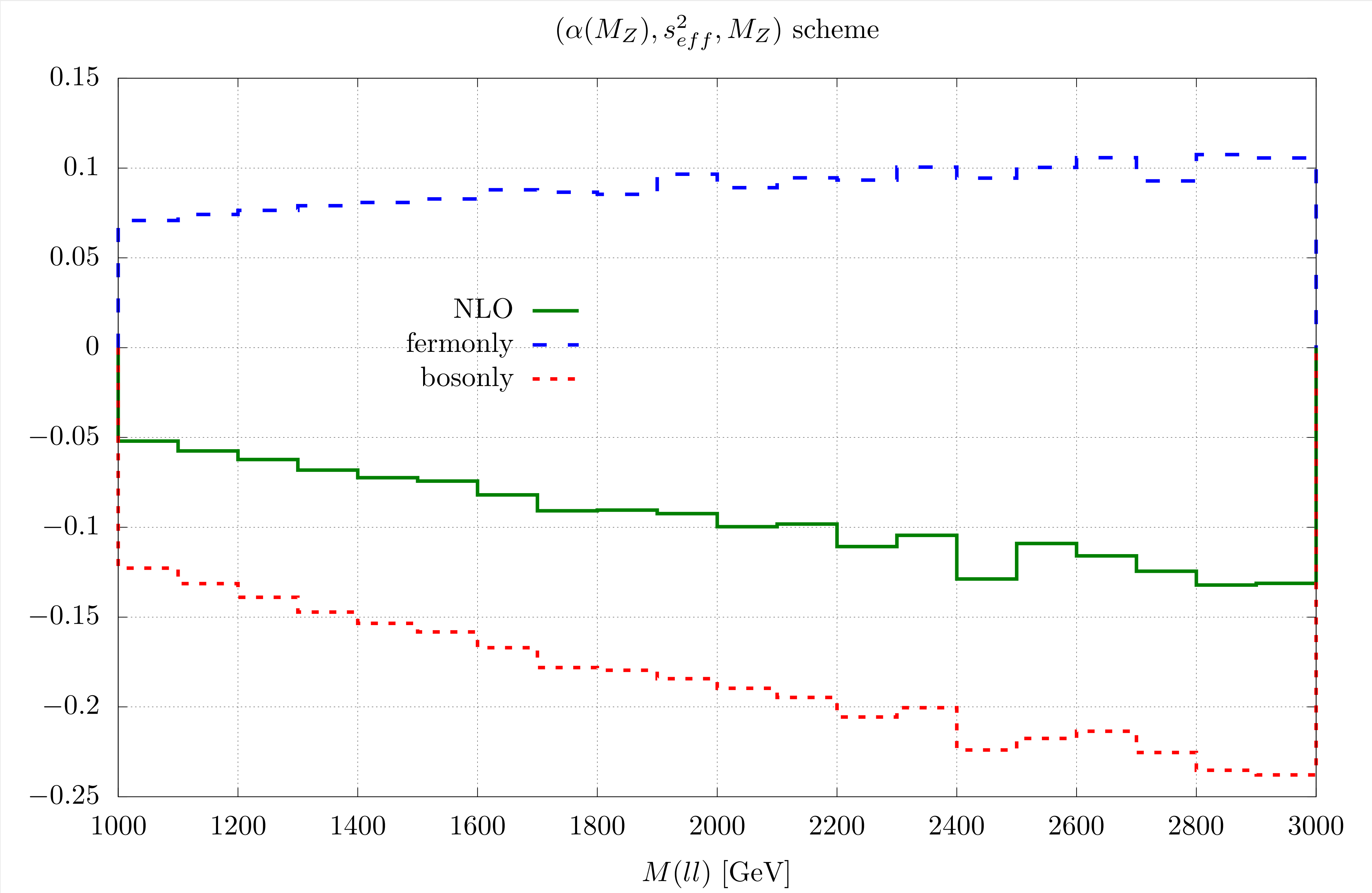
- EW corrections are not always at (sub)percent level

Radiative return of photon radiation in NC DY



Dittmaier, Huber, JHEP 1001:060,2010

Sudakov logarithms in NC DY

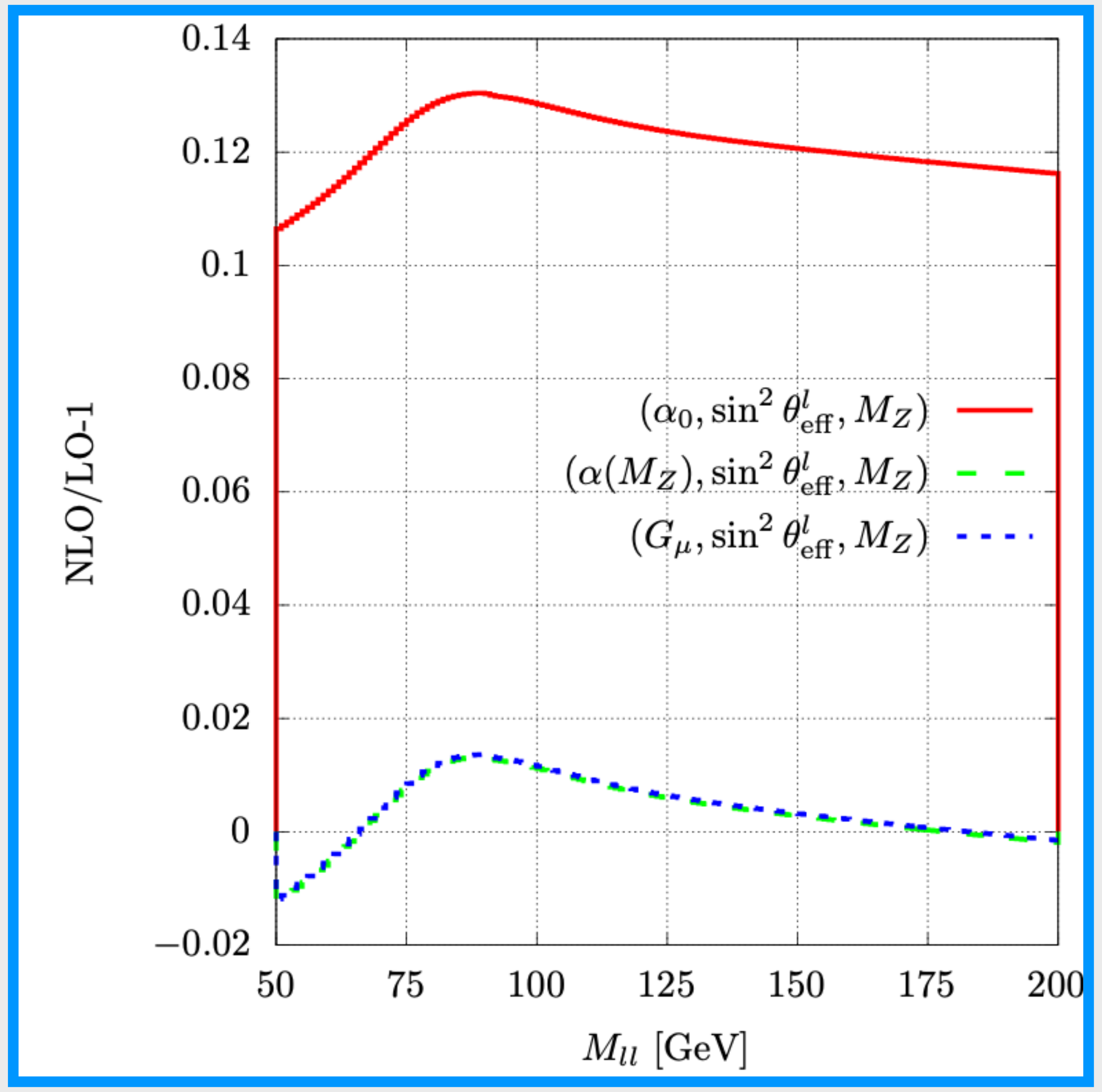


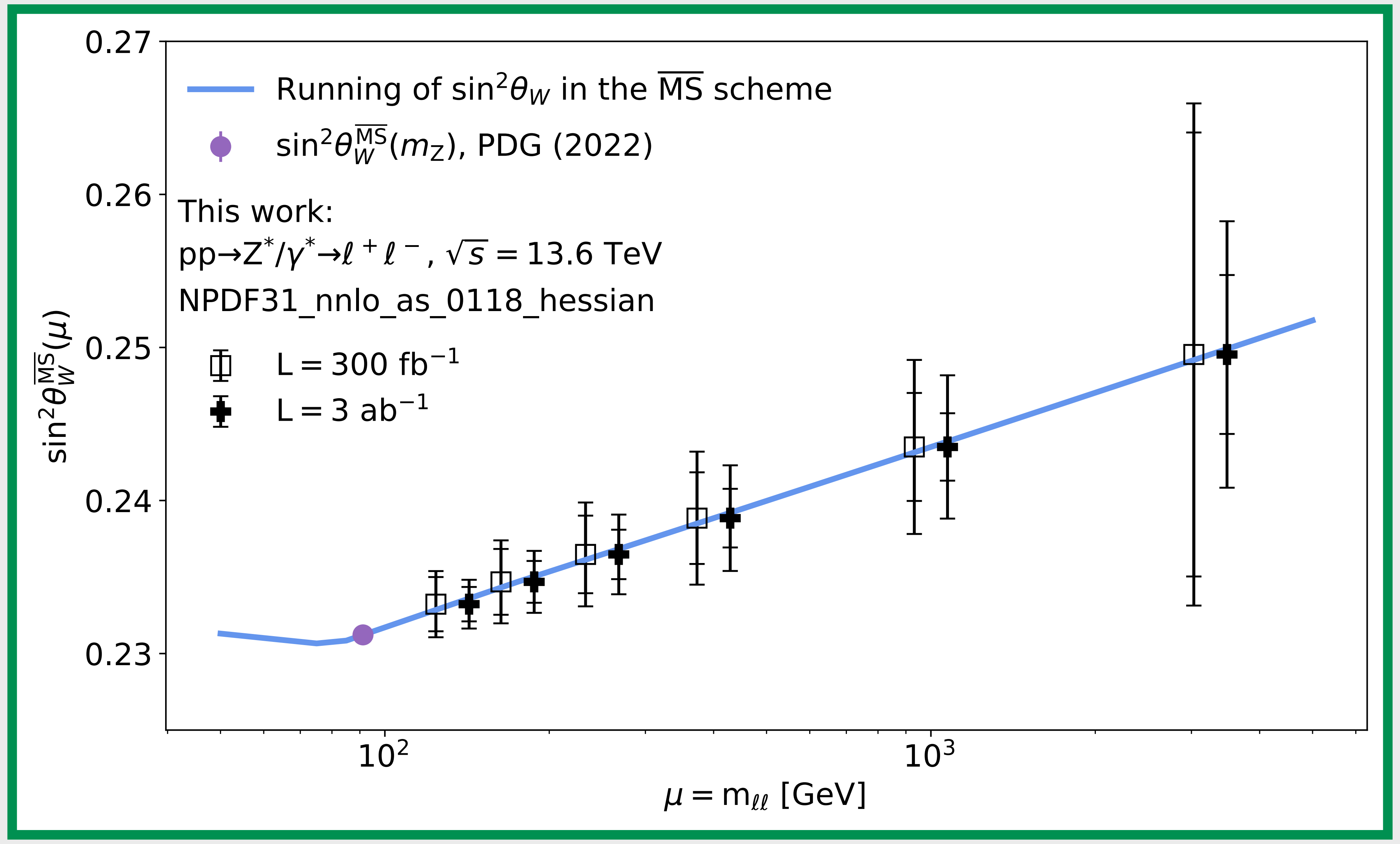
Why input parameter schemes?

- **intrinsic theoretical uncertainties** (size of radiative corrections at fixed order, convergence of the perturbative series)

- **parametric theoretical uncertainties** via choice of parameters

- **direct determination of parameters** in template fits

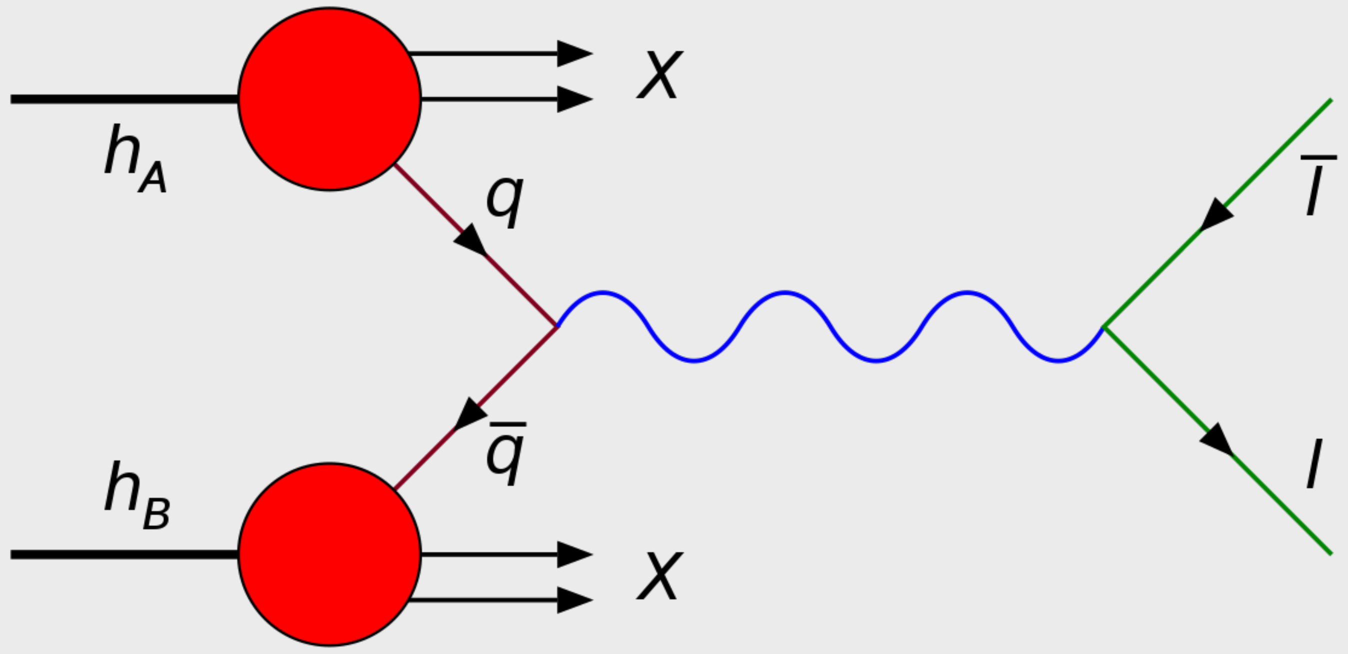




Amoroso et al., Phys. Lett. B 844 138103, 2023

The neutral-current Drell Yan

large cross section and clean experimental signature



precision tests of EW SM at high energy
→ determination e.g. $\sin^2\theta_w$

State-of-art: $< 10^{-2}$ experimental precision

NNLO QCD + NLO EW theoretical accuracy for fully exclusive cross section

Hamberg, van Neerven, Matsuura, Nucl.Phys.B 359 (1991) 343-405
Gavin et al., Comput.Phys.Commun. 182 (2011) 2388-2403
Gavin et al., Comput.Phys.Commun. 184 (2013) 208-214
Catani et al., Phys.Rev.Lett. 103 (2009) 082001
Melnikov, Petriello, Phys.Rev.D 74 (2006) 114017
Melnikov, Petriello, Phys.Rev.Lett. 96 (2006) 231803
Anastasiou et al., Phys.Rev.D 69 (2004) 094008
Baur, Keller, Sakumoto, Phys.Rev.D 57 (1998) 199-215
Zykunov, Eur.Phys.J. direct 3 1 (2001) 9
Baur et al., Phys.Rev.D 65 (2002) 033007

Dittmaier, Krämer, Phys.Rev.D 65 (2002) 073007
Baur, Wackeroth, Phys.Rev.D 70 (2004) 073015
Arbuzov et al., EPJC 46 (2006) 407-412
Carloni Calame et al., JHEP 12 (2006) 016
Zykunov, Phys.Rev.D 75 (2007) 073019
Carloni Calame et al., JHEP 10 (2007) 109
Arbuzov et al., EPJC 54 (2008) 451-460
Brensing et al., Phys.Rev.D 77 (2008) 073006
Dittmaier, Huber, JHEP 01 (2010) 060
Boughezal, Li, Petriello, Phys.Rev.D 89 3 (2014) 034030

The Drell-Yan process

More recent theoretical progress includes

see also T. Armadillo’s talk

- inclusive $\mathrm{N^3LO}$ QCD calculations
- resummation of large logs in QCD and QED and multi-photon emission
- mixed QCDxQED corrections

Ahmed et al., Phys.Rev.Lett. 113 11 (2014) 112002
Catani et al., Nucl.Phys.B 888 (2014) 75-91
Duhr, Dulat, Mistlberger, JHEP 11 (2020) 143
Duhr, Dulat, Mistlberger, Phys.Rev.Lett. 125 17 (2020) 172001
Baglio et al., JEHP 12 (2022) 066
Chen et al., Phys.Rev.Lett. 128 5 (2022) 052001
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Chen et al., Phys.Rev.Lett. 128 25 (2022) 252001
Neumann, Campbell, Phys.Rev.D 107 1 (2023) L011506

Guzzi, Nadolsky, Wang, Phys.Rev.D 90 1 (2014) 014030
Kulesza, Stirling, EPJC 20 (2001) 349-356
Catani et al., JHEP 12 (2015) 047
Balasz, Yuan, Phys.Rev.D 56 (1997) 5558-5583
Landry et al., Phys.Rev.D 67 (2003) 073016
Cieri, Codareschi, de Florian, JHEP 06 (2015) 185
Bozzi et al., Phys.Rev.B 696 (2011) 207-213
Mantry, Petriello, Phys.Rev.D 83 (2011) 053007

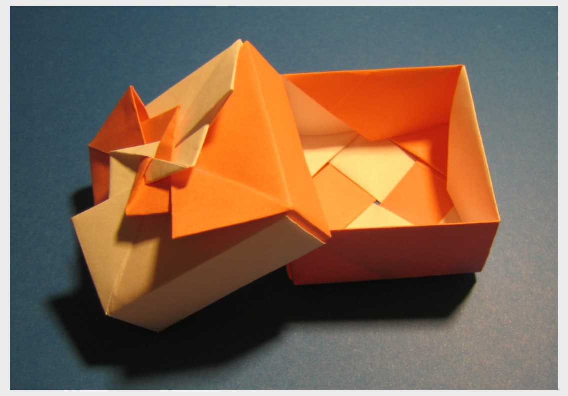
Becher, Neubert, Wilhelm, JHEP 02 (2012) 124
Bizón et al., JHEP 12 (2018) 132
Bizón et al., EPJC 79 10 (2019) 868
Re, Rottoli, Torrielli, JHEP 2021, 108 (2021)
Ju, Schönherr, JHEP 10 (2021) 088
Camarda, Cieri, Ferrera, Phys.Rev.D 104 11 (2021) L111503
Camarda, Cieri, Ferrera, Phys.Lett.B 845 (2023) 138125
Isaacson, Fu, Yuan, FERMILAB-PUB-23-322-T (2023)
Cieri, Ferrera, Sborlini, JHEP 07 (2023) 104
Autieri et al., JHEP 07 (2023) 104
Placzek, Jadach, EPJC 29 (2003) 325-339
Carloni Calame et al., Phys.Rev.D 69 (2004) 037301

Dittmaier, Huss, Schwinn, Nucl.Phys.B 885 (2014) 318-372
Dittmaier, Huss, Schwinn, Nucl.Phys.B 904 (2016) 216-252
Dittmaier, Huss, Schwarz, JHEP 05 (2024) 170
De Florian, Der, Fabre, Phys.Rev.D 98 9 (2018) 094008
Bonciani et al., Phys.Rev.D 101 3 (2020) 031301
Bonciani et al., JHEP 02 (2022) 095
Delto et al., JHEP 01 (2020) 043
Buccioni et al., Phys.Rev.B 811 (2020) 135969
Cieri et al., JHEP 09 (2020) 155
Behring et al., Phys.Rev.D 103 1 (2021) 031008
Bonciani et al., JHEP 09 (2016) 091
Heller, et al., Rhys.Rev.D 102 1 (2020) 016025
Hasan Schubert, JHEP 11 (2020) 107
Heller et al., JHEP 05 (2021) 213
Bonciani et al., Phys.Rev.Lett. 128 1 (2022) 012002
Armadillo et al., JHEP 05 (2022) 072
Armadillo et al., Comp.Phys.Commun. 282 (2023) 108545
Buccioni et al., JHEP 06 (2022) 022
Buonocore, Phys.Rev.D 103 (2021) 114012
Buonocore, Rottoli, Torrielli, JHEP 07 (2024) 193

POHWEG implementation of neutral-current Drell Yan

`Z_ew-BMNNPV` package of POWHEG-BOX-V2 svn 4067

NLO QCD + NLO EW + matching to QED and QCD PS



Chiesa, Del Pio, Piccinini, Eur. Phys. J. C 84 (2024) 5, 539
Amoroso et al., Phys. Lett. B 844 138103, 2023
Chiesa, Piccinini, Vicini, Phys. Rev. D 100 071302, 2019
Carloni Calame et al., Phys. Rev. D 96, 093005, 2017
Barzé et al., Eur.Phys.J. C73 (2013) 2474

here EW NLO + leading universal fermionic h.o. corrections related to $\Delta\alpha, \Delta\rho$

input parameter and renormalization schemes **comparison at the Z peak** and in the **high energy region**

Electroweak input schemes

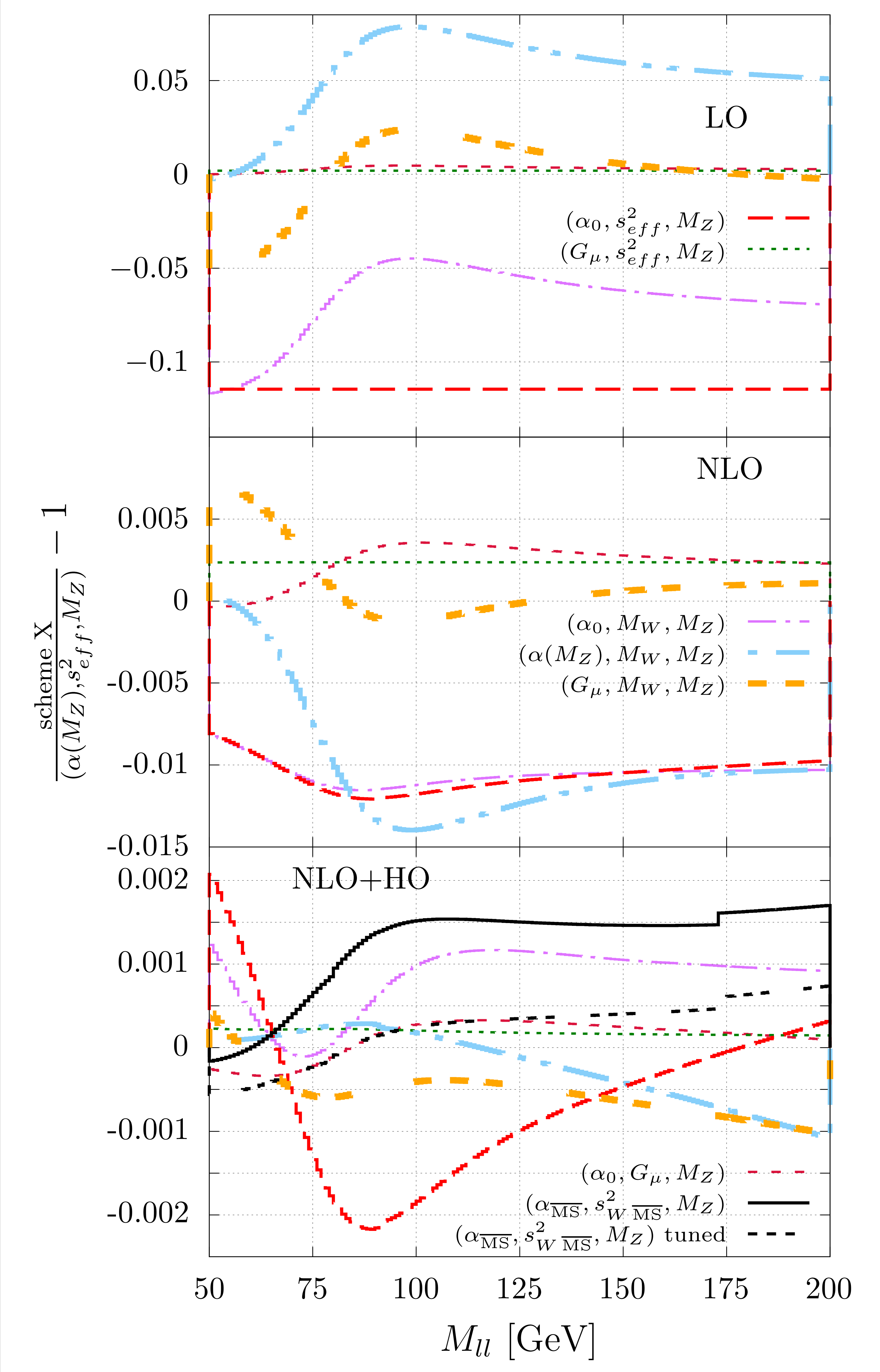
Higgs and fermion masses + 3 other independent parameters

1. $(\alpha_0\,/\,\alpha(M_Z^2)\,/\,G_\mu, M_W, M_Z)$: commonly used at the LHC
   Böhm, Denner, Joos, Gauge theories of the strong and electroweak interactions, 2001
2. $(\alpha_0\,/\,\alpha(M_Z^2)\,/\,G_\mu, s^2_{eff}, M_Z)$: for s^2_{eff} extraction
   Chiesa, Piccinini, Vicini, Phys. Rev. D 100 071302, 2019
3. (α, G_μ, M_Z): LEP1 scheme
   Bardin, Grünewald, Passarino, arXiv:hep-ph/9902452
4. $(\alpha(\mu^2), s^2_w(\mu^2), M_Z)$: for $s^2_{\overline{MS}}$ extraction
   Chiesa, Del Pio, Piccinini, Eur. Phys. J. C 84 (2024) 5, 539
   Amoroso et al., Phys. Lett. B 844 138103, 2023
   Erler, Ramsey-Musolf, Phys. Rev. D 72 073003, 2005
   Erler, Ferro-Hernández, JHEP 03 196, 2018

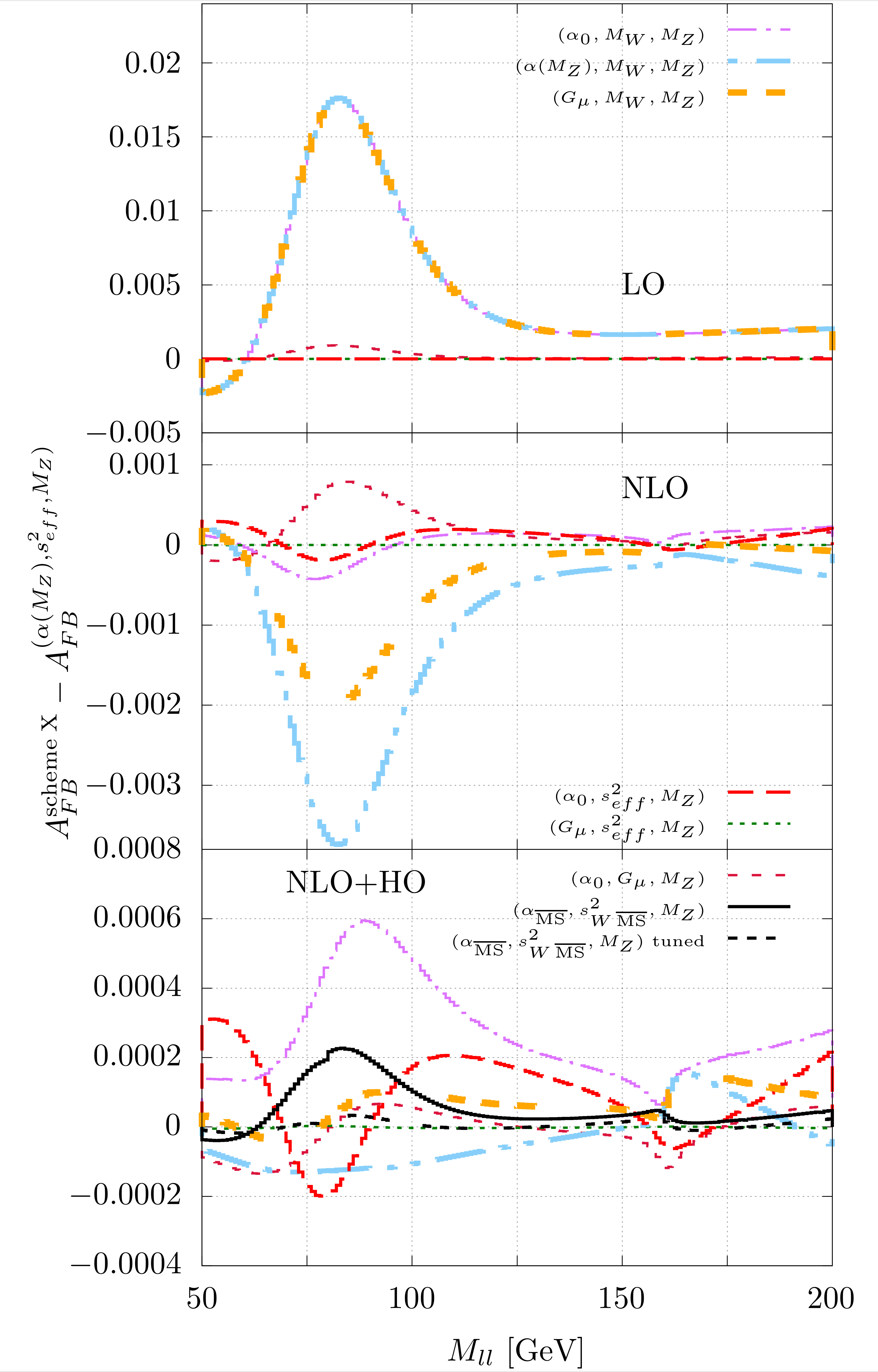
Parametric uncertainties	$\alpha_0 \quad G_\mu \quad M_Z$
Convergence of perturbative series	$\alpha_0\,/\,\alpha(M_Z^2)\,/\,G_\mu \quad M_W \quad M_Z$
Direct determinations of parameters	$\alpha_0\,/\,\alpha(M_Z^2)\,/\,G_\mu \quad \sin^2\theta^\ell_{eff} \quad M_Z$ $\alpha(\mu^2) \quad \sin^2\theta_w(\mu) \quad M_Z$

Scheme comparison 50-200 GeV

Cross section



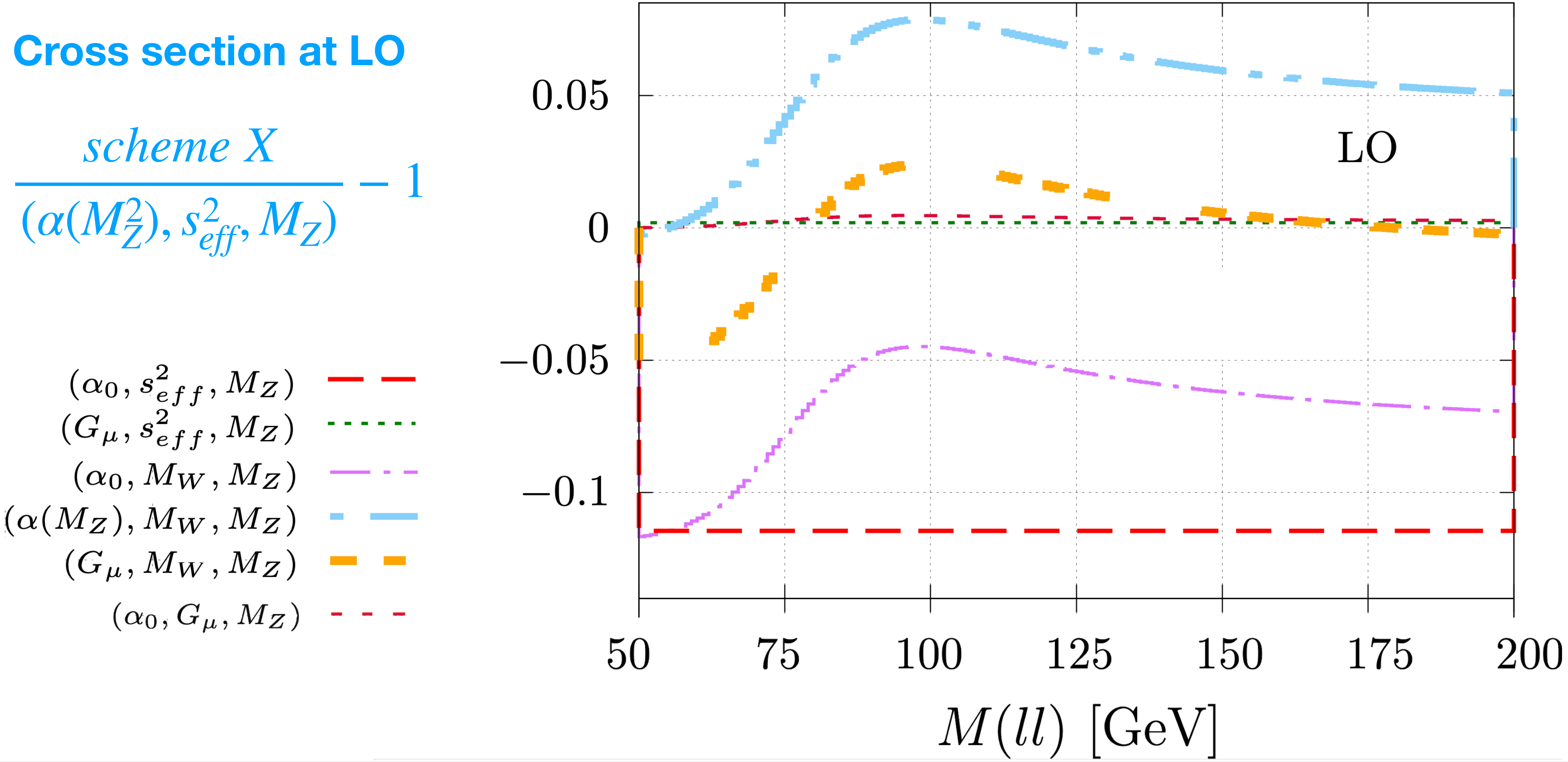
Asymmetry



Scheme comparison 50-200 GeV

Cross section at LO

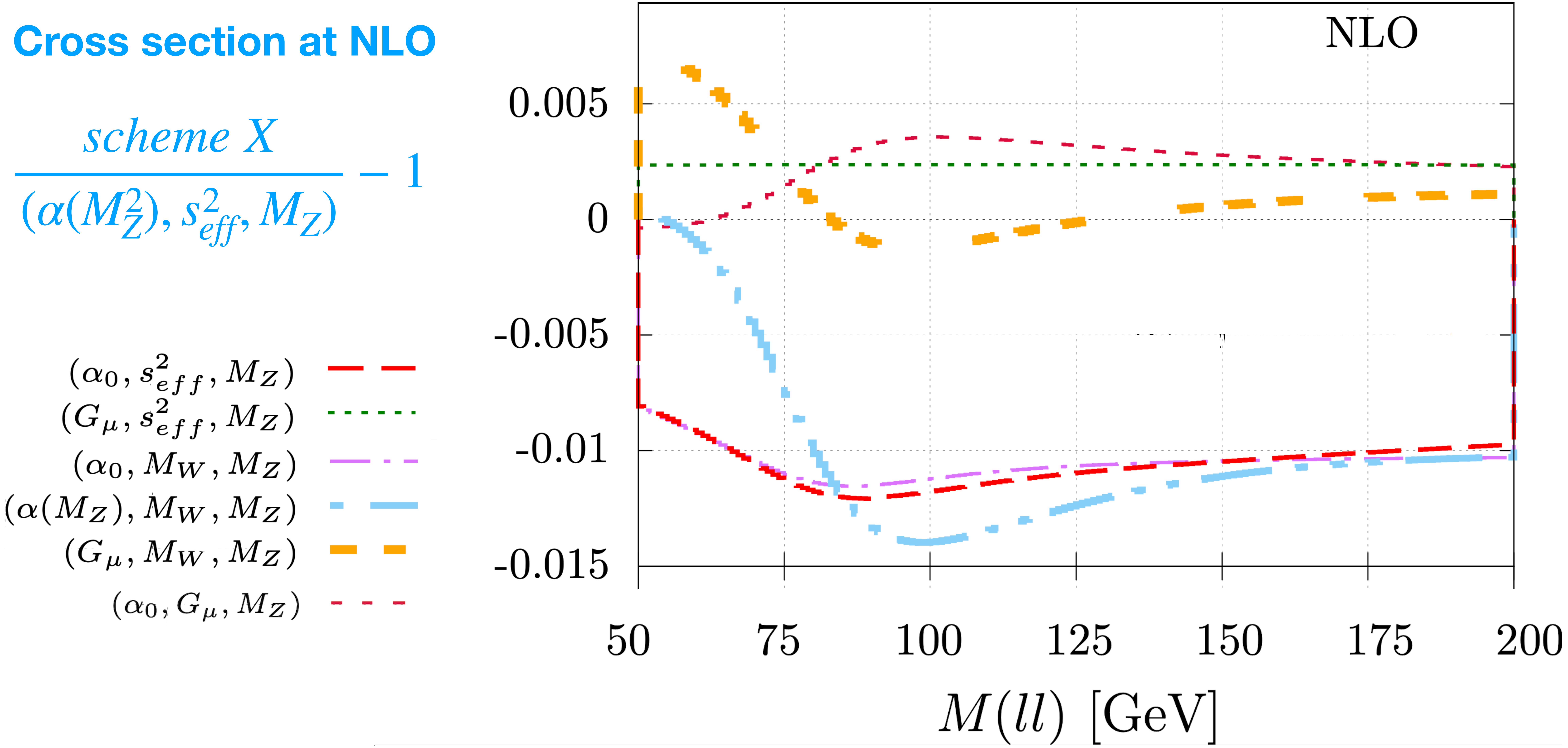
$$\frac{\textit{scheme } X}{(\alpha(M_Z^2), s^2_{eff}, M_Z)} - 1$$



Scheme comparison 50-200 GeV

Cross section at NLO

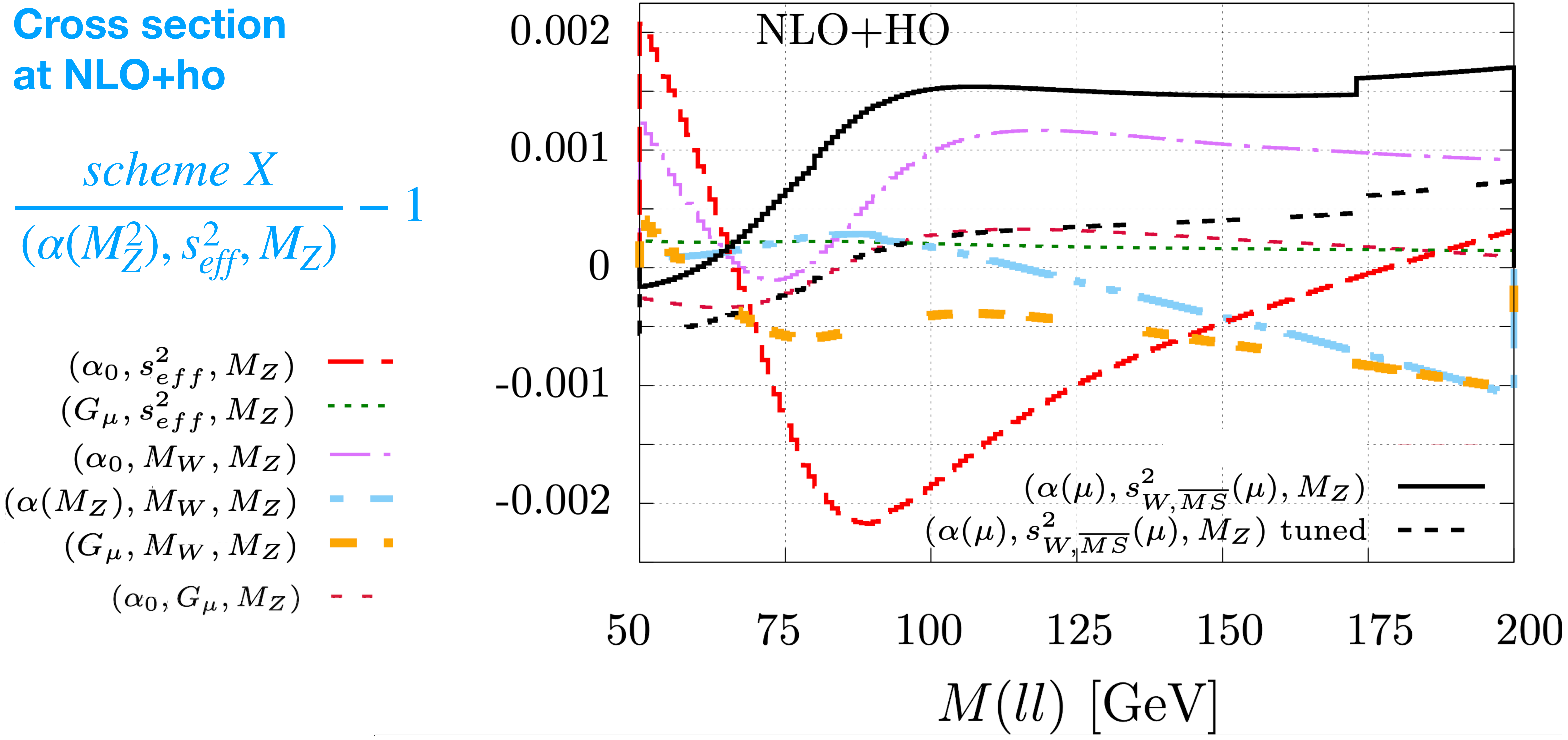
$$\frac{scheme\ X}{(\alpha(M_Z^2), s_{eff}^2, M_Z)} - 1$$



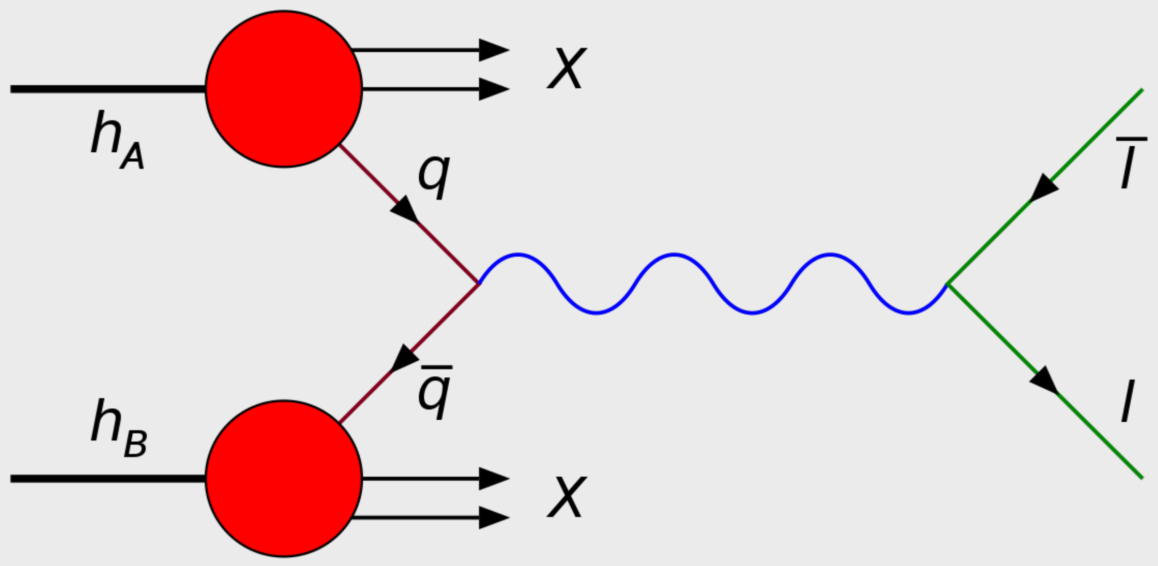
Scheme comparison 50-200 GeV

Cross section at NLO+ho

$$\frac{\textit{scheme } X}{(\alpha(M_Z^2), s^2_{eff}, M_Z)} - 1$$



Comparison with LEP1 theory predictions

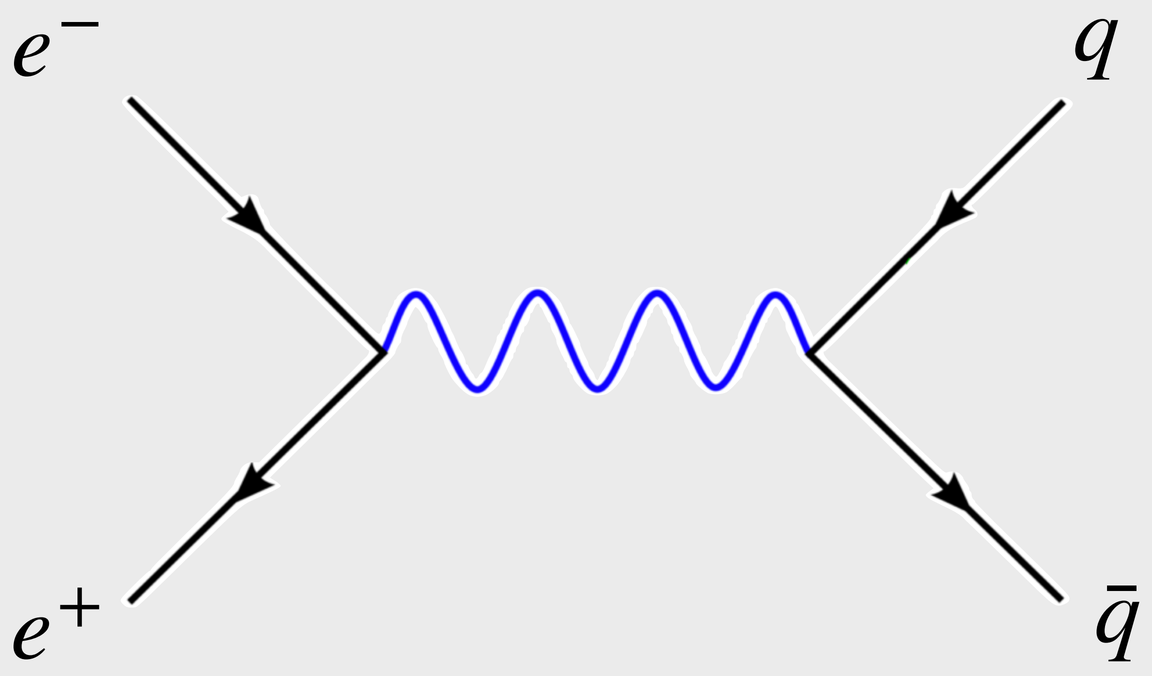


LHC: hadronic machine

Determine s^2_{eff} in Collins-Soper frame with template fit

independent input schemes

$\sim 10^{-3}$ agreement on x-section



LEP: leptonic machine at $\sqrt{s} \sim M_Z$

Extract s^2_{eff} from pseudo-observables

MC codes: all schemes are “tuned realisations” of α_0, G_μ, M_Z

10^{-4} agreement on $\Gamma_{Z\ell\bar{\ell}}$ and x-section

Comparison with LEP1 theory predictions

Tuning to the reference scheme (α_0, G_μ, M_Z)

$$\alpha(M_Z^2) = \frac{\alpha_0}{1 - \Delta\alpha}$$

$(\alpha_0, s^2_{eff}, M_Z)$

$$s^2_{eff}\big|_{G_\mu} = \frac{1}{2} - \sqrt{\frac{1}{4} - \frac{\pi}{\sqrt{2}G_\mu M_Z^2}\,\alpha(M_Z^2)\left(1 + \Delta\tilde{r}_{HO}\right)}$$

$$\Delta\tilde{r}_{rem} = \Delta\tilde{r}^{(1)} - \Delta\alpha + \Delta\rho^{(1)}$$

$$\Delta\tilde{r}_{HO} = \Delta\tilde{r}_{rem} - \Delta\rho$$

$$\Delta\rho = \Delta\rho^{(1)} + \Delta\rho^{(2)}$$

(α_0, M_W, M_Z)

$$M_W\big|_{G_\mu} = \frac{M_Z^2}{2}\left(1 + \sqrt{1 - \frac{\sqrt{8}\pi}{G_\mu M_Z^2}\,\alpha(M_Z^2)\,\frac{1 + \Delta r^{(1)} - \Delta\alpha + \frac{c_W^2}{s_W^2}\Delta\rho^{(1,X)}}{1 + \frac{c_W^2}{s_W^2}\Delta\rho^{(X)}}}\right)$$

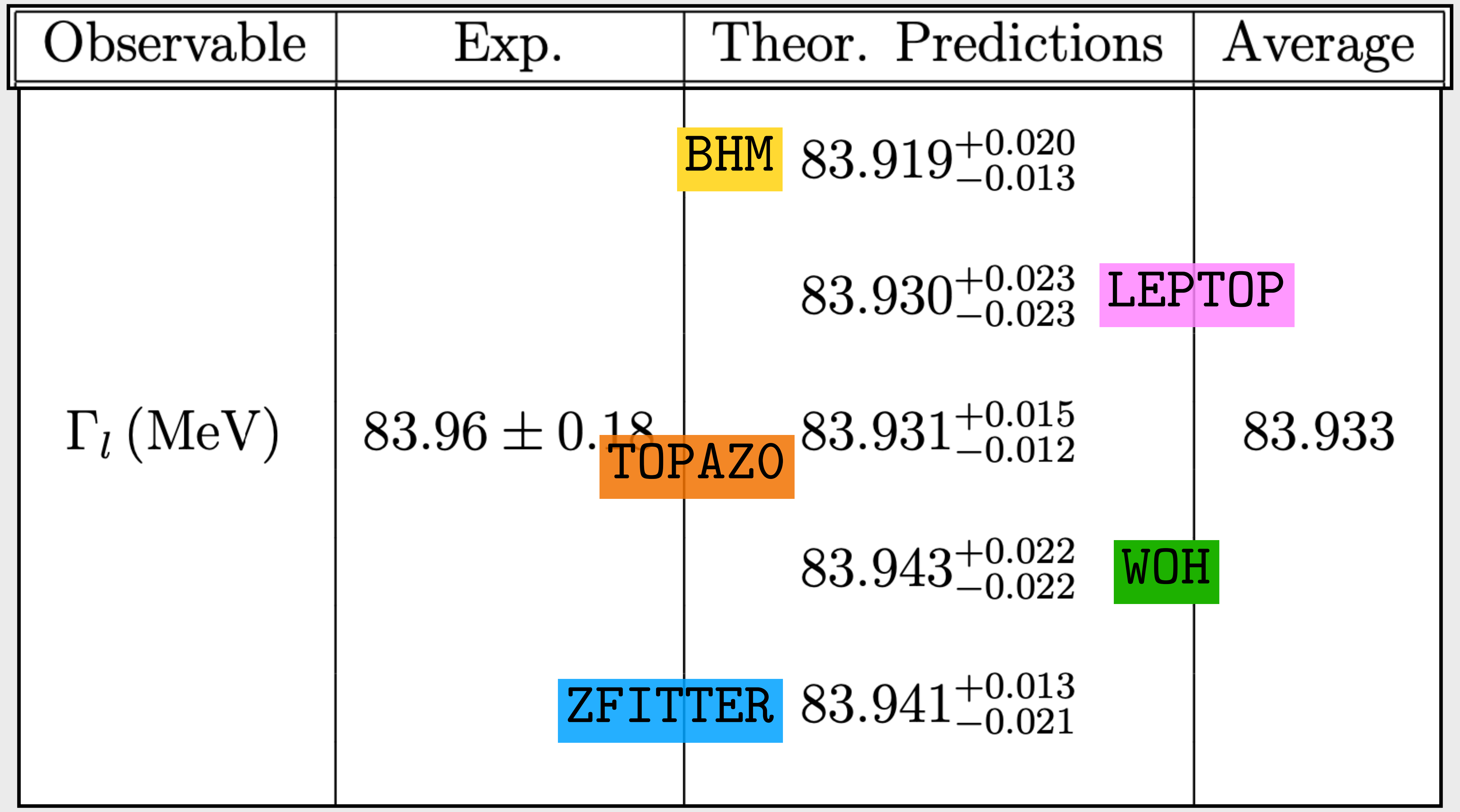
$$\Delta\rho^{(1,X)} = \frac{\Sigma_T^{ZZ}(M_Z)}{M_Z^2} - \frac{\Sigma_T^{W}(M_W)}{M_W^2}\bigg|_{fin,\,\mu_{dim}=M_Z}$$

$$\Delta\rho^{(X)} = \Delta\rho^{(1,X)} + \Delta\rho^{(2,X)}$$

N.B.: no subleading contributions $\mathcal{O}(\alpha^2 M_{\text{top}}^2)$ included

Bardin, Passarino, The standard model in the making, 1999
Degrassi Gambino, Vicini, Phys. Lett. B 393 2 (1996) 219-226
Degrassi, Gambino, Nucl.Phys.B 567 (2000)

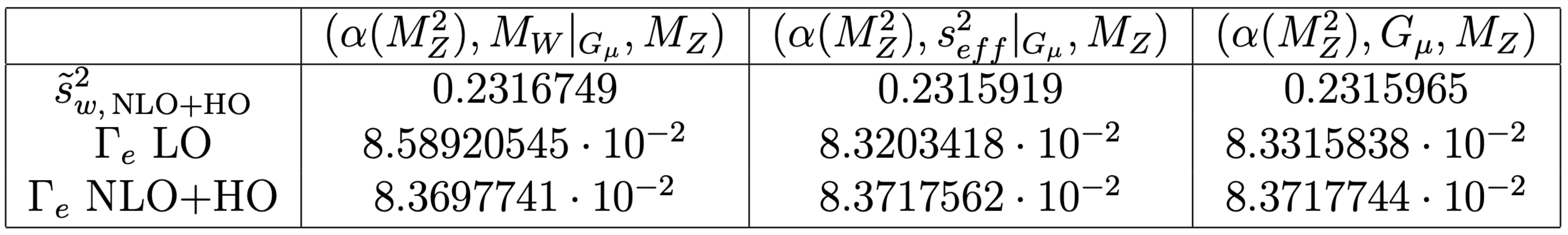
Results of tuning: $Z\ell\bar{\ell}$ width



Observable	Exp.	Theor. Predictions	Average
Γ_l (MeV)	83.96 ± 0.18	BHM $83.919^{+0.020}_{-0.013}$ $83.930^{+0.023}_{-0.023}$ LEPTOP TOPAZO $83.931^{+0.015}_{-0.012}$ $83.943^{+0.022}_{-0.022}$ WOH ZFITTER $83.941^{+0.013}_{-0.021}$	83.933

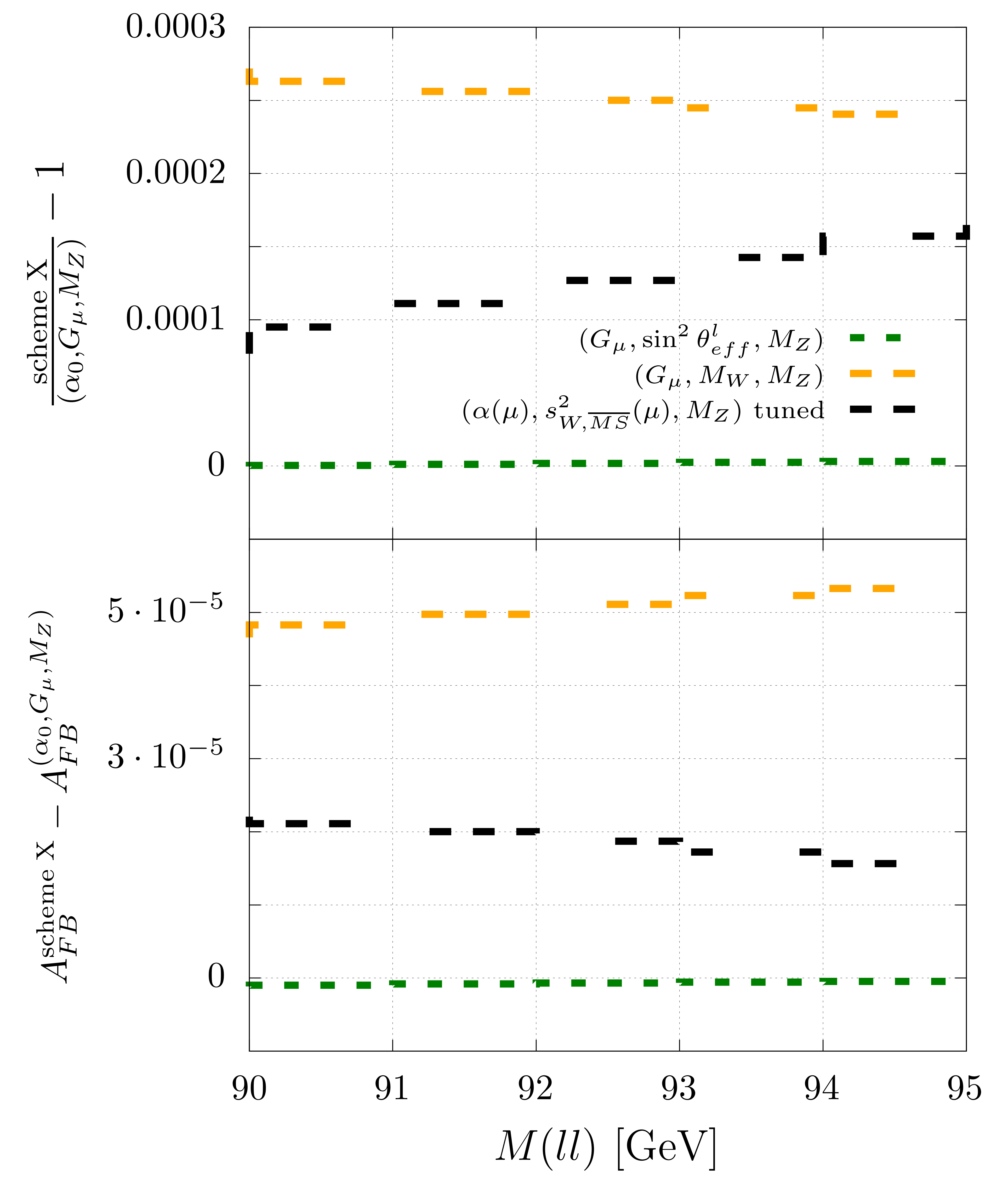
Bardin *et al.*, CERN 95-03, 1995

Agreement within 10^{-4}



	$(\alpha(M_Z^2), M_W\vert_{G_\mu}, M_Z)$	$(\alpha(M_Z^2), s^2_{eff}\vert_{G_\mu}, M_Z)$	$(\alpha(M_Z^2), G_\mu, M_Z)$
$\tilde{s}^2_{w,\,\mathrm{NLO+HO}}$	0.2316749	0.2315919	0.2315965
Γ_e LO	$8.58920545 \cdot 10^{-2}$	$8.3203418 \cdot 10^{-2}$	$8.3315838 \cdot 10^{-2}$
Γ_e NLO+HO	$8.3697741 \cdot 10^{-2}$	$8.3717562 \cdot 10^{-2}$	$8.3717744 \cdot 10^{-2}$

Results of tuning: cross section and A_{FB}



Comparison at higher energy

see also L. Mai's talk

Channel with d-quarks only: no PDFs dependence
→ no large unphysical distortions at high energies

Sudakov logs

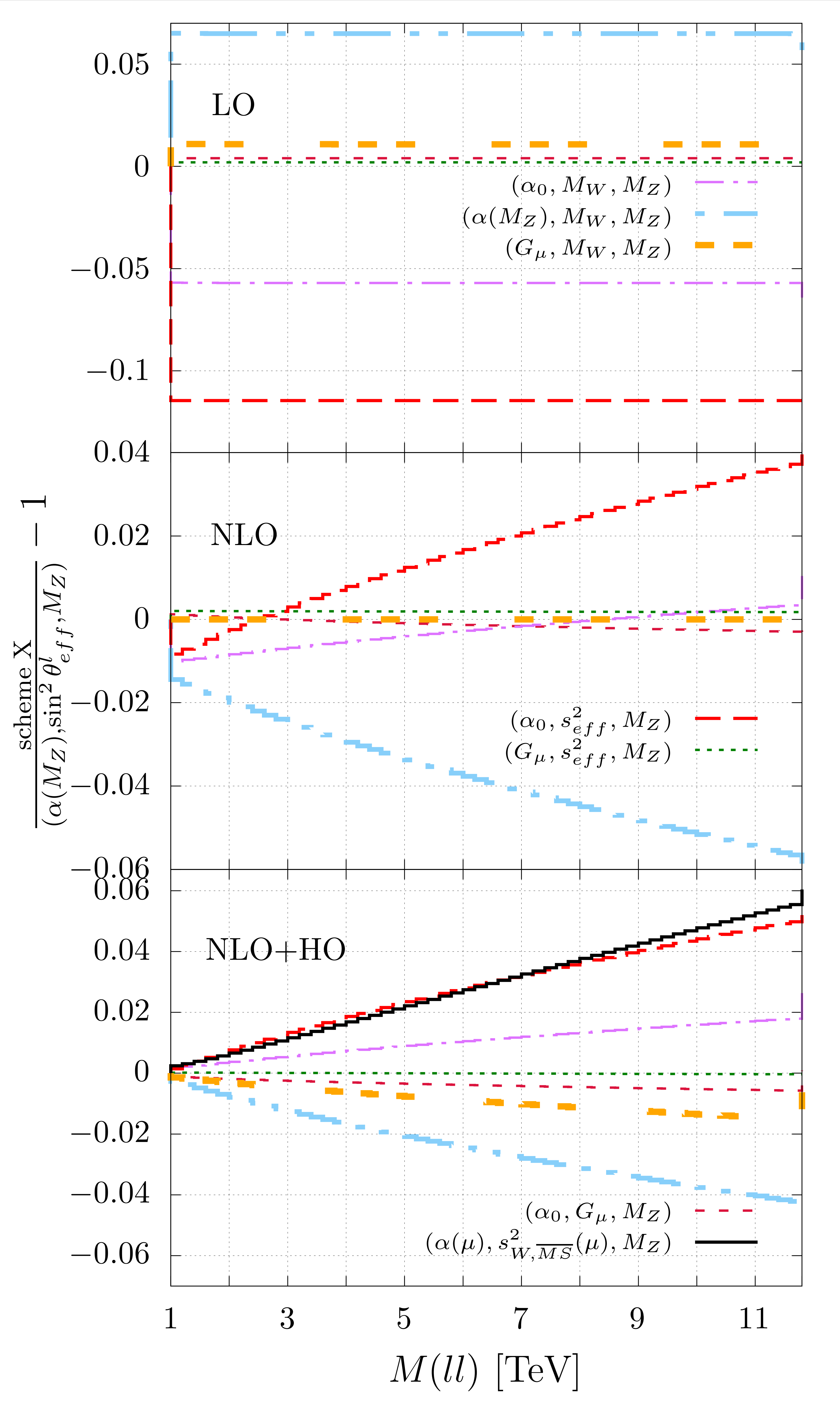
$$A(\alpha, s_w^2)\,\ln^2\frac{s}{M_Z^2} + B(\alpha, s_w^2)\,\ln\frac{s}{M_Z^2}$$

Parameter-renormalization logs

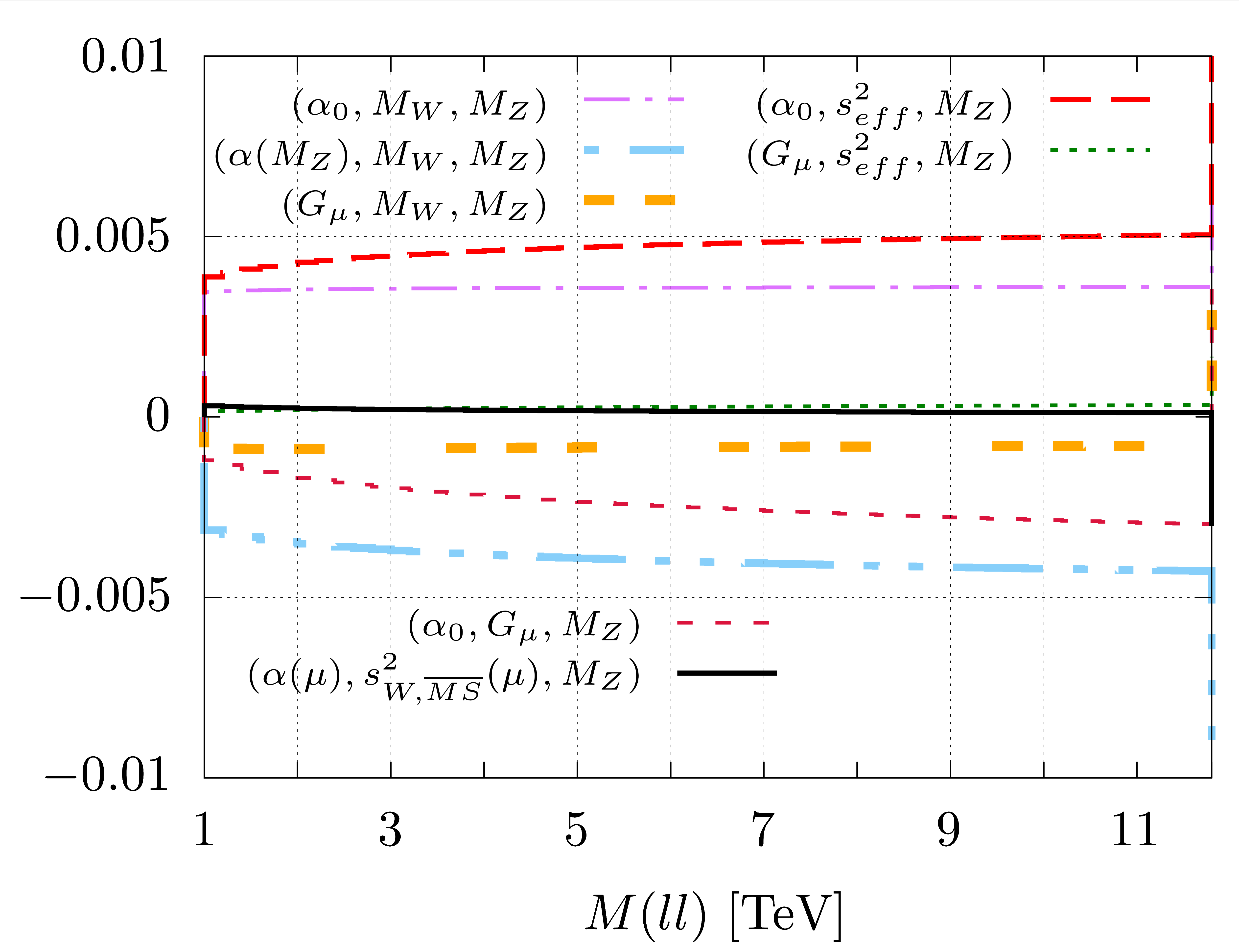
$$\frac{1}{\epsilon} - \ln\frac{r_{ct}^2}{\mu_{dim}^2} - \frac{1}{\epsilon} + \ln\frac{r_{bare}^2}{\mu_{dim}^2} = \ln\frac{r_{bare}^2}{r_{ct}^2} \sim \ln\frac{M_{ll}^2}{m^2}$$

Counterterms from parameter renorm.

Bare diagrams



Scheme comparison at higher energy

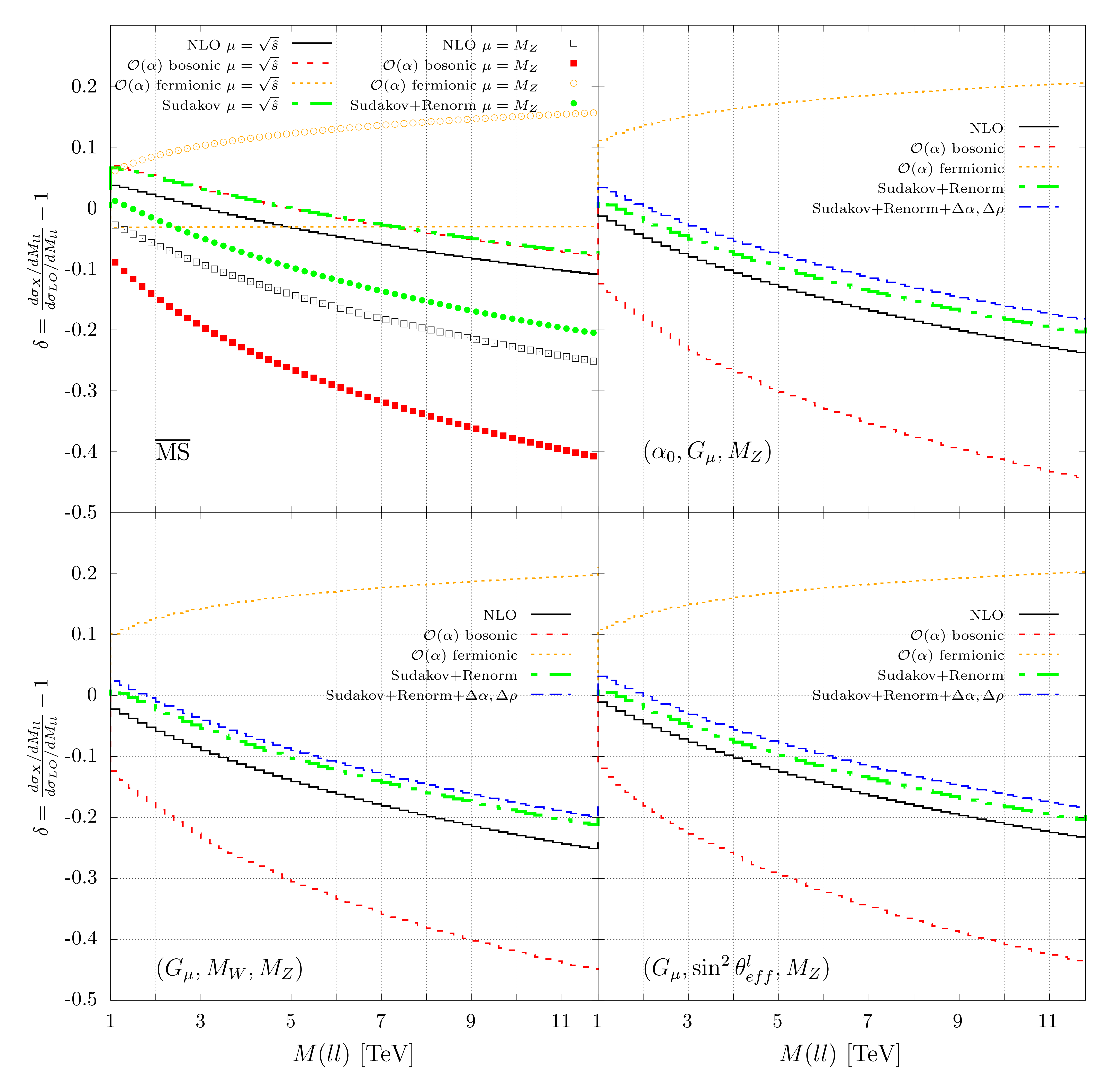


Cross section at NLO+ho - (Sudakov + param. renorm. logs)

$$\frac{scheme\ X}{(\alpha(M_Z^2), s^2_{eff}, M_Z)} - 1$$

Sudakov approx

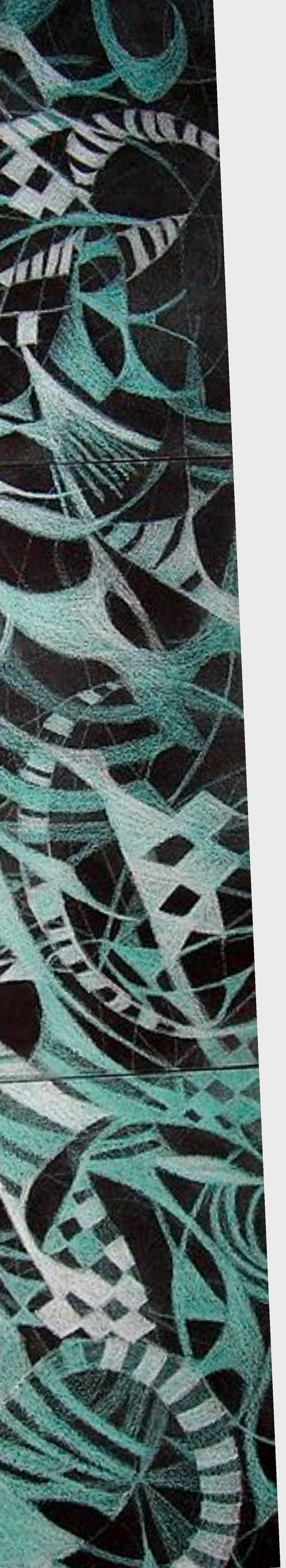
True NLO - approx. $\sim 5\,\%$



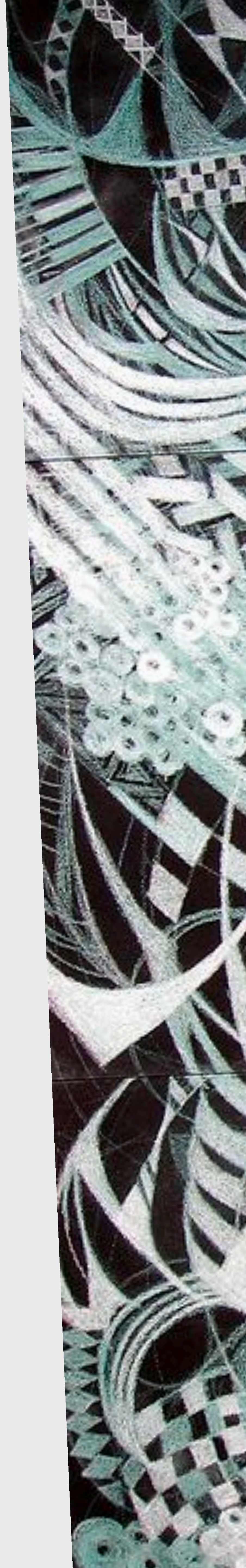
Conclusions

- Electroweak corrections are relevant at the LHC and measurements in the EW sector represent a **fundamental test of the Standard Model**
- The choice of an EW input parameter and renormalization scheme is important for the quantification of **theoretical uncertainties** and for **SM parameter determinations**
- Updated version of `Z_ew-BMNNPV` which includes input schemes suitable for direct determination of weak mixing angle in both on-shell and $\overline{MS}$ scheme

→ *Amoroso et al., Phys. Lett. B 844 138103, 2023*



Back-up



Tuning and implementation of higher orders

In tuned schemes, **higher orders** in Born Improved Approximation with $\alpha(M_Z^2)$ and s_{eff}^2

(α_0, G_μ, M_Z)

$$\tilde{s}^2_{w,\,\mathrm{LO}} = \frac{1}{2}\frac{-g_R}{g_L - g_R}$$

$$\tilde{s}^2_{w,\,\mathrm{NLO}} = \frac{1}{2}\frac{-g_R}{g_L - g_R} + \frac{1}{2}\frac{g_L g_R}{(g_L - g_R)^2}\mathrm{Re}\left(\frac{\delta g_L}{g_L} - \frac{\delta g_R}{g_R}\right)$$

(α_0, M_W, M_Z)

$(\alpha_0, s_{eff}^2, M_Z)$

Tuning and implementation of higher orders

In tuned schemes, **higher orders** in Born Improved Approximation with $\alpha(M_Z^2)$ and s_{eff}^2

(α_0, G_μ, M_Z)

$$\tilde{s}^2_{w,\,\mathrm{NLO+HO}} = \frac{1}{2} - \sqrt{\frac{1}{4} - \frac{\pi\alpha}{\sqrt{2}G_\mu M_Z^2}(1+\Delta\tilde{r}|_{s_w^2})}$$

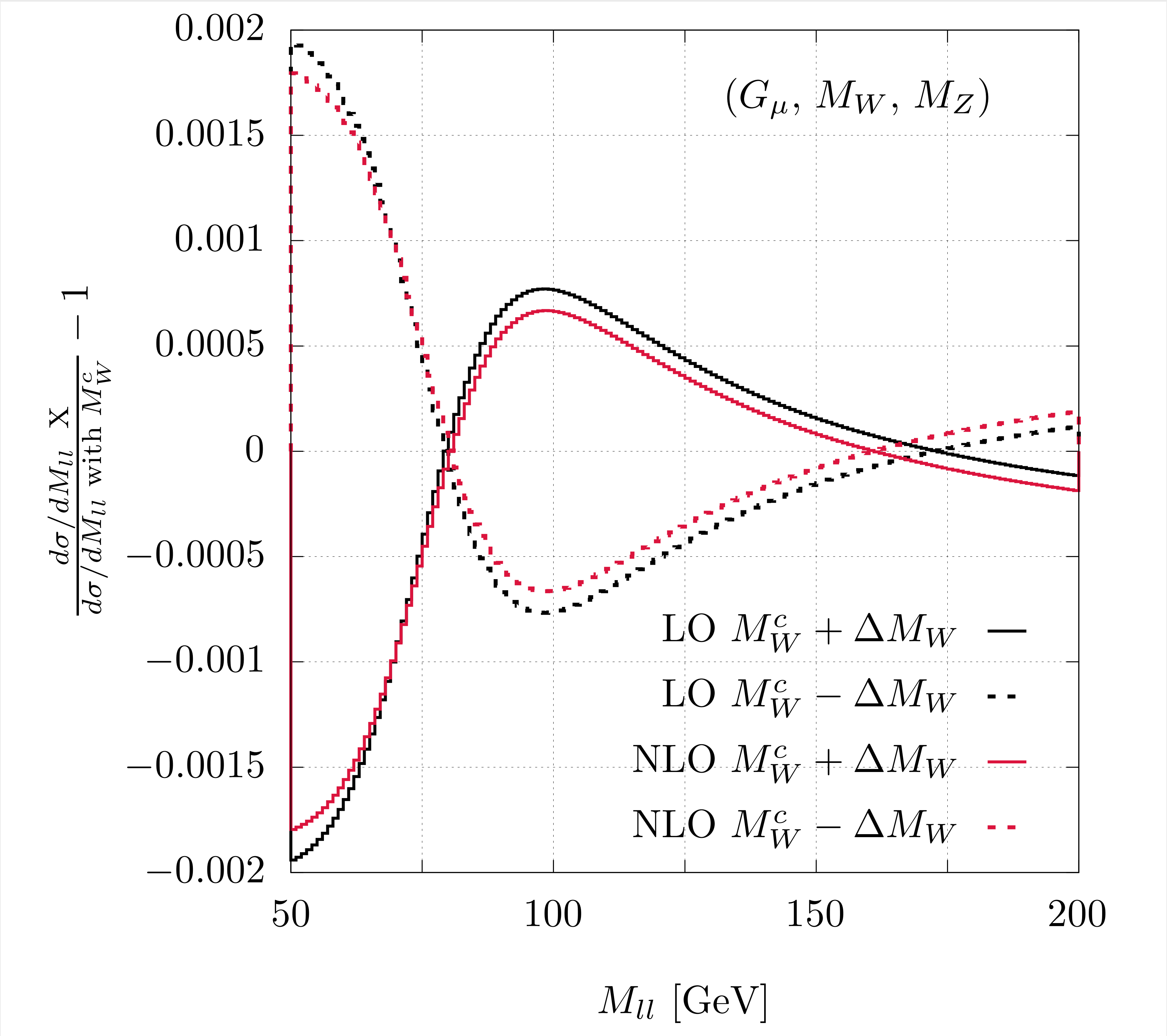
(α_0, M_W, M_Z)

$$\tilde{s}^2_{w,\,\mathrm{NLO+HO}} = s_w^2\left(1+\frac{c_w^2}{s_w^2}\Delta\rho^{(X)}\right)\left[1-\frac{c_w^2}{s_w^2}\Delta\rho^{(1,X)}+\frac{1}{s_w^2}\frac{1}{2}\frac{g_L g_R}{(g_L-g_R)^2}\mathrm{Re}\left(\frac{\delta g_L}{g_L}-\frac{\delta g_R}{g_R}\right)\right]$$

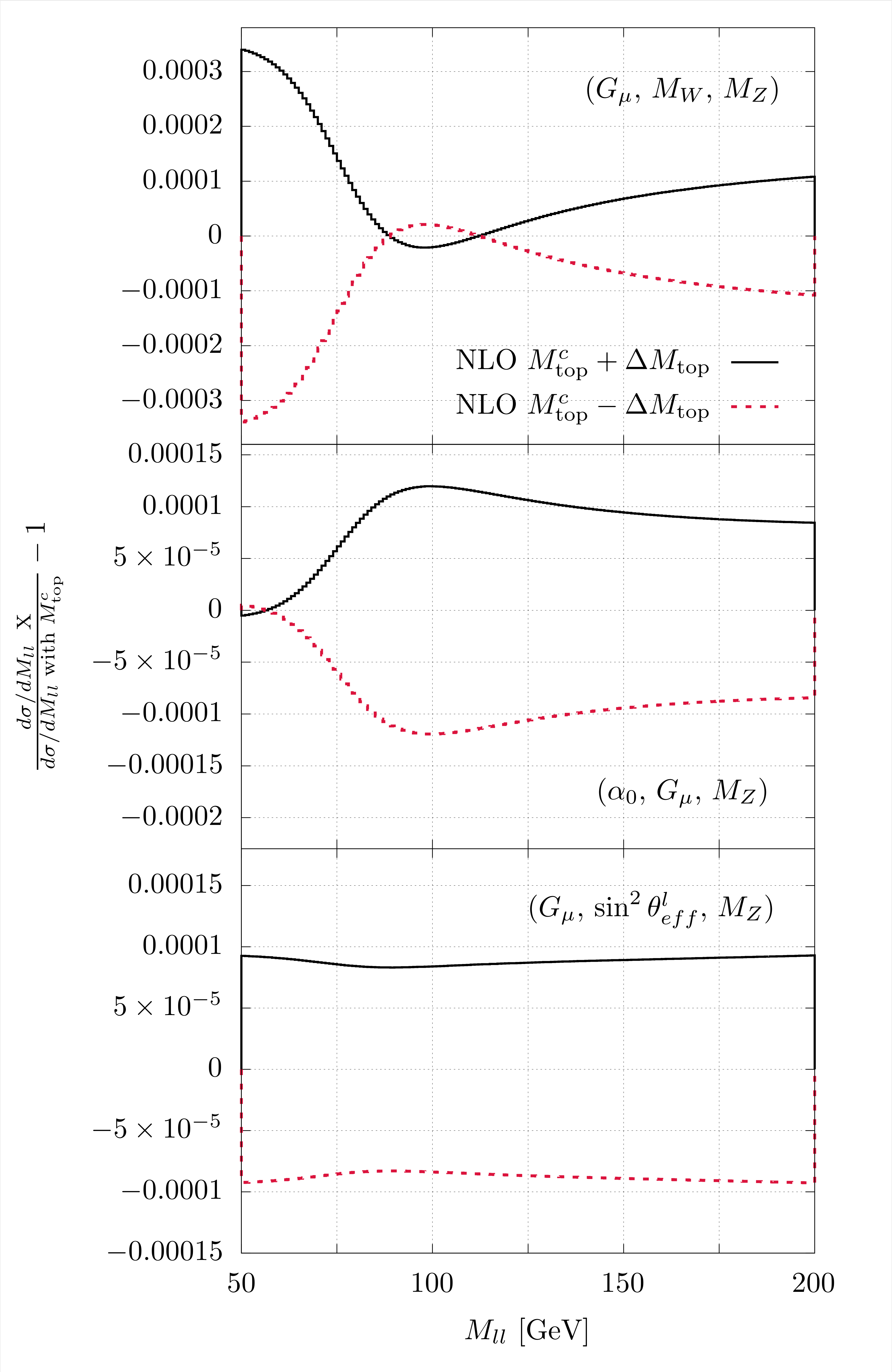
$(\alpha_0, s_{eff}^2, M_Z)$ already ok

Parametric uncertainties

M_W



M_{top}



Resonance treatment

Complex-mass scheme

Denner, Dittmaier, Roth, Wackeroth, Nucl. Phys. B 560 no. 1-3, 33–65, 1999
Denner, Dittmaier, Roth, Wieders, Nucl. Phys. B 724 no. 1-2, 247–294, 2005
Denner, Dittmaier, Nucl. Phys. B - Proceedings Supplements 160, 22–26, 2006

$$\mu_Z = M_Z - i\Gamma_Z M_Z \qquad \mu_W = M_W - i\Gamma_W M_W$$

Pole scheme

Stuart, Phys. Lett. B 262 no. 1, 113–119, 1991 - Sirlin, Phys. Lett. B 267 no. 2, 240–242, 1991
Gambino, Grassi, Phys. Rev. D 62 no. 7, 2000 - Grassi, Kniehl, Sirlin, Phys. Rev. D 65 no. 8, 2002
Stuart, Phys. Rev. Lett. 70, 3193–3196, 1993 - Dittmaier, Huber, JHEP 2010 no. 1, 2010)

$$\mathscr{M} = \frac{\tilde{R}(\mu_P^2)}{p^2 - \mu_P^2} + \frac{R(p^2) - R(M_P^2)}{p^2 - M_P^2} + \tilde{N}(p^2)$$

Factorization scheme

Argyres et al., Phys. Lett. B 358 no. 3-4, 339–346, 1995
Kurihara, Perret-Gallix, Shimizu, Phys. Lett. B 349 no. 3, 367–374, 1995
S. Dittmaier and M. Krämer, Phys. Rev. D 65 no. 7, 2002

$$f_P(p^2) = \frac{p^2 - M_P^2}{p^2 - \mu_P^2}$$