Expansions for multi-scale two-loop processes

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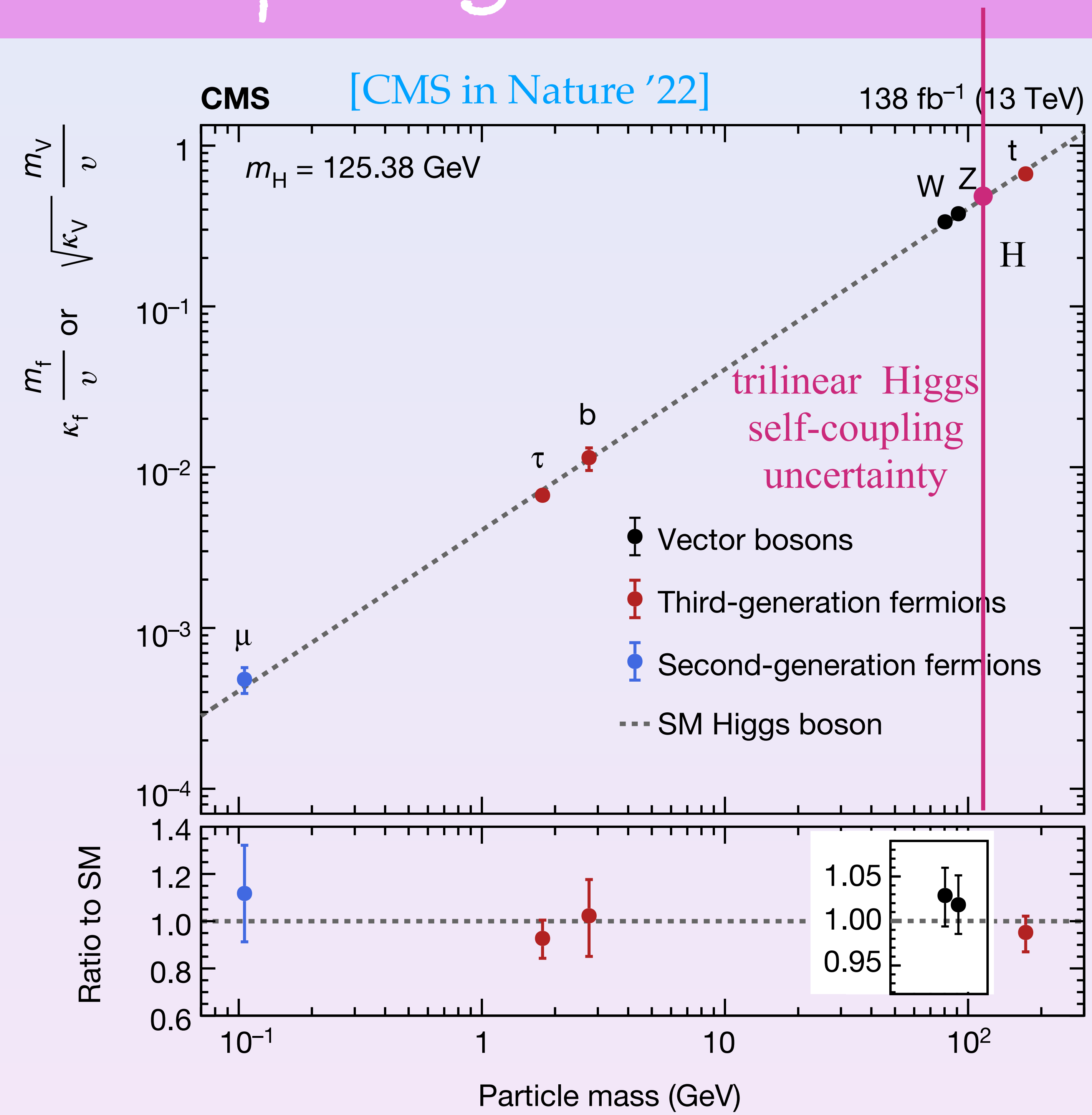
Higgs couplings

3rd generation fermion and gauge boson couplings to Higgs boson fairly good measured

2nd generation fermion couplings first results available

Higgs self-couplings?

First generation Yukawa couplings?



Higgs couplings

3rd generation fermion and gauge boson couplings to Higgs boson fairly good measured

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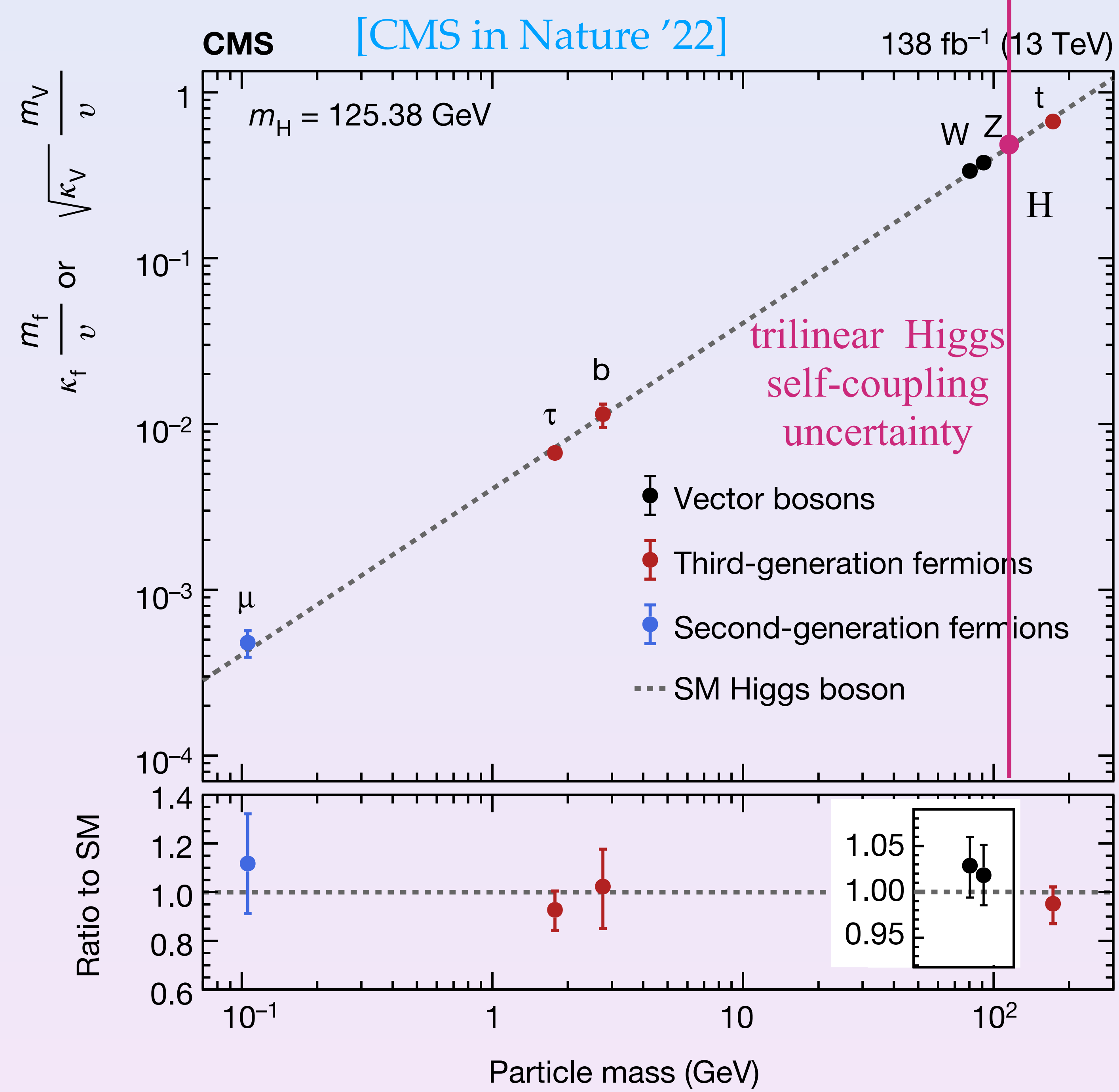
Higgs self-couplings?

First generation Yukawa couplings?

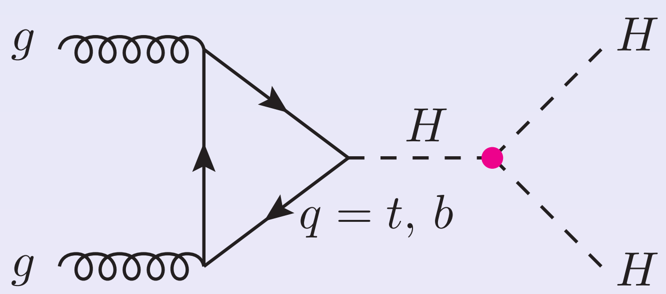
→ Still many things to be learned about the Higgs boson!

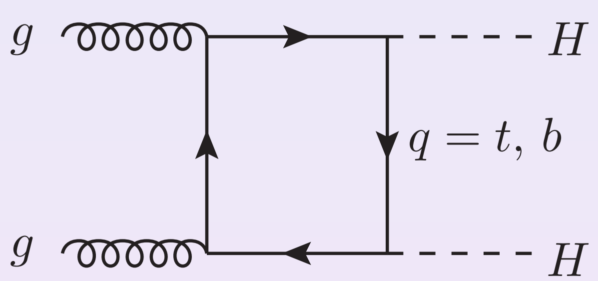


→ Many of those require to measure 2 -> 2 processes with Higgs bosons in gluon fusion

Higgs Pair Production

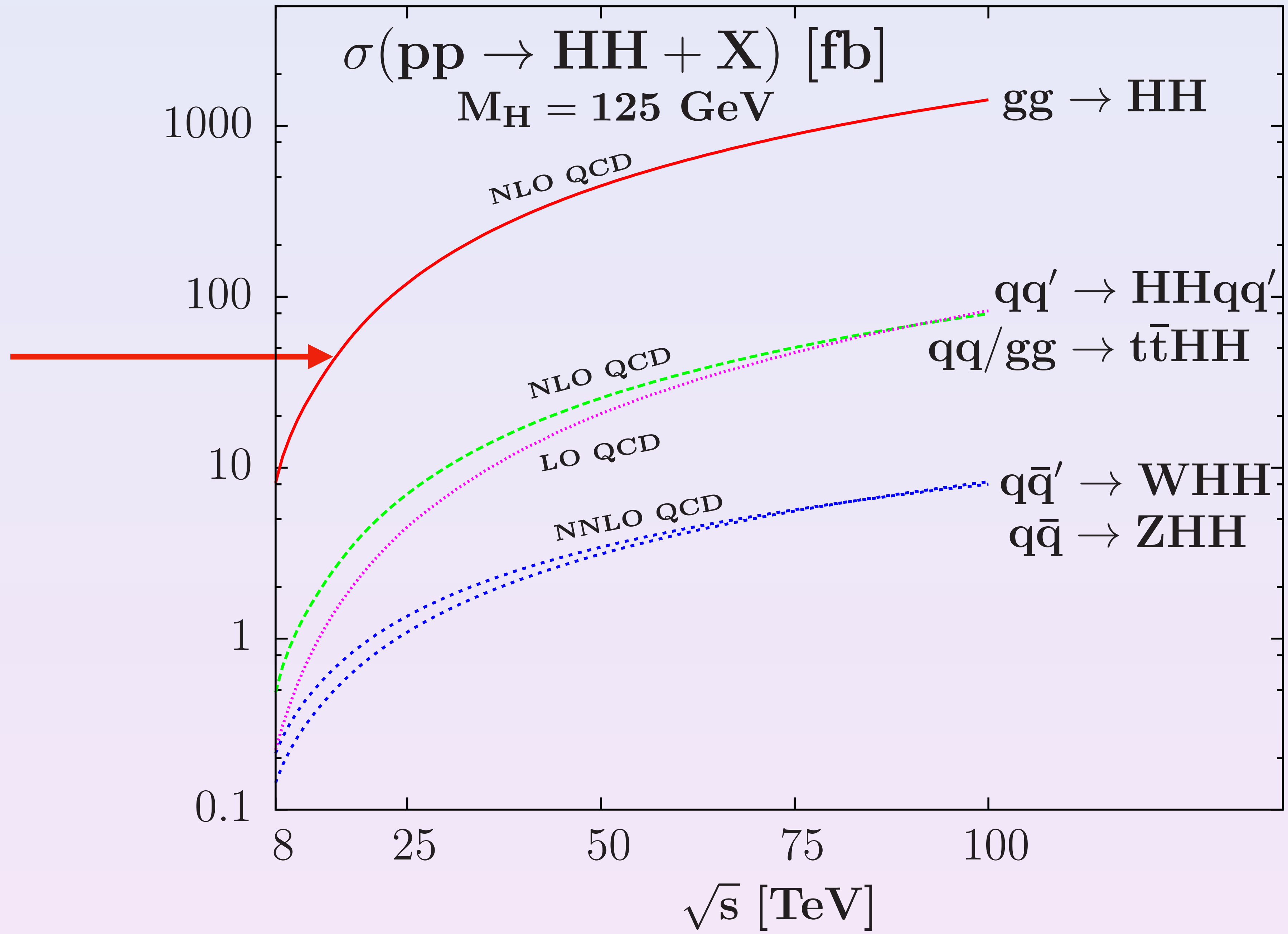
$\underline{gg \to HH}$





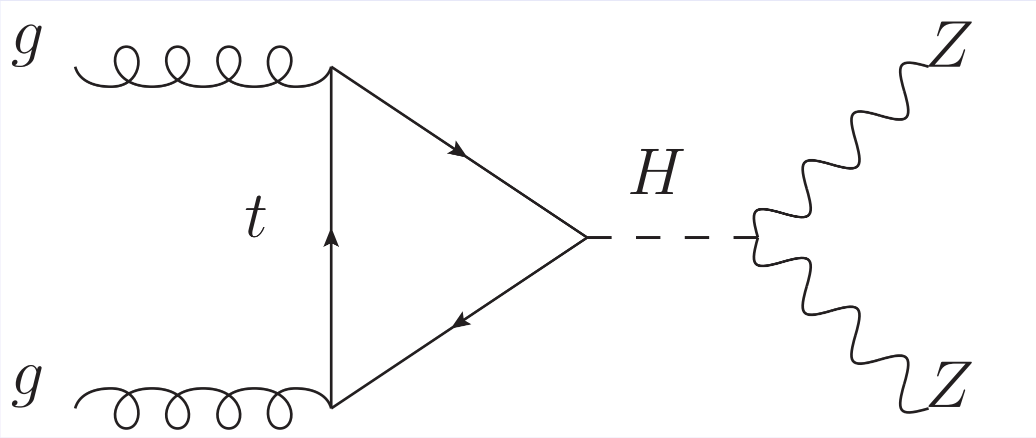
Dominant process is gluon fusion

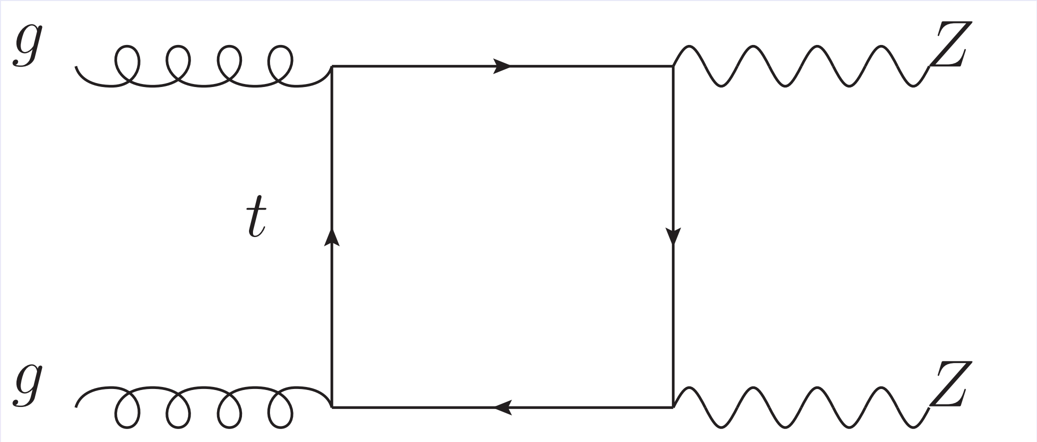
[Baglio, Djouadi, RG, Mühlleitner, Quevillon, Spira '12]



Other gluon fusion processes

Off-shell Higgs boson





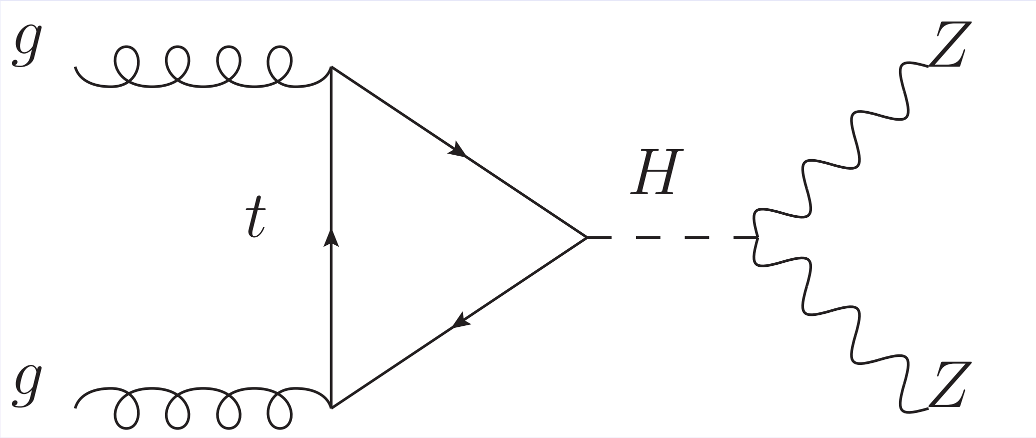
allows for Higgs width measurement [Kauer, Passarino '12, Caola, Melnikov '13, Campell, Ellis, Williams '13]

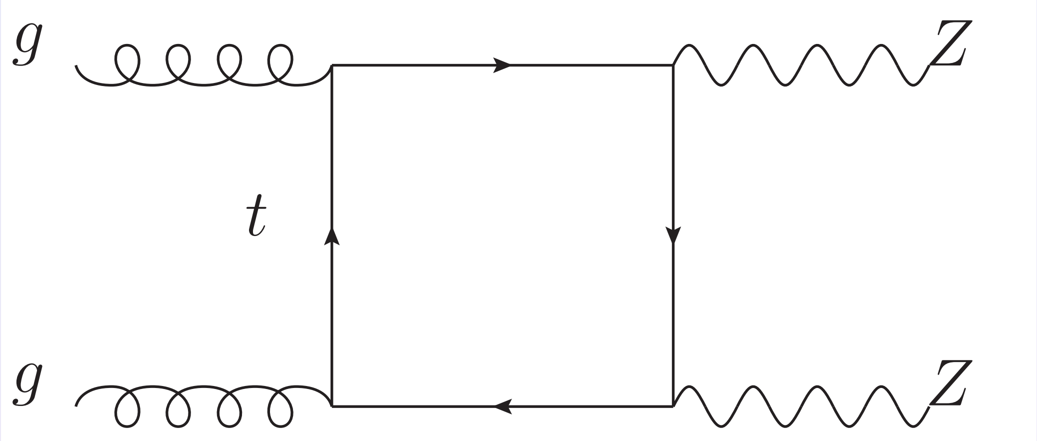
probes trilinear Higgs self-coupling [Haisch, Koole '21]

probes light quark Yukawa couplings [Balzani, RG, Vitti '23]

Other gluon fusion processes

Off-shell Higgs boson





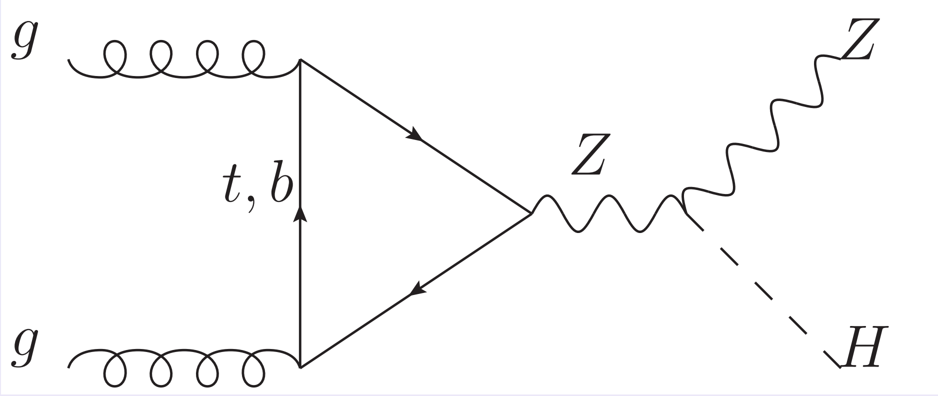
$\sqrt{s} = 13$ TeV	σ [fb]	$\sigma/\sigma_{\rm NLO} - 1$
LO	$36.848(1)\ ^{+7.1\%}_{-8.1\%}$	-24.8%
NLO	$48.990(2)\ ^{+3.1\%}_{-2.9\%}$	—
$q\bar{q}$NNLO	$52.07(4)\ ^{+1.4\%}_{-1.4\%}$	$+6.3\%$
	σ [fb]	$\sigma/\sigma_{\rm ggLO} - 1$
ggLO	$4.2967(3)\ ^{+25.6\%}_{-18.4\%}$	—
ggNLO	$7.80(2)\ ^{+17.1\%}_{-13.9\%}$	$+81.5\%$

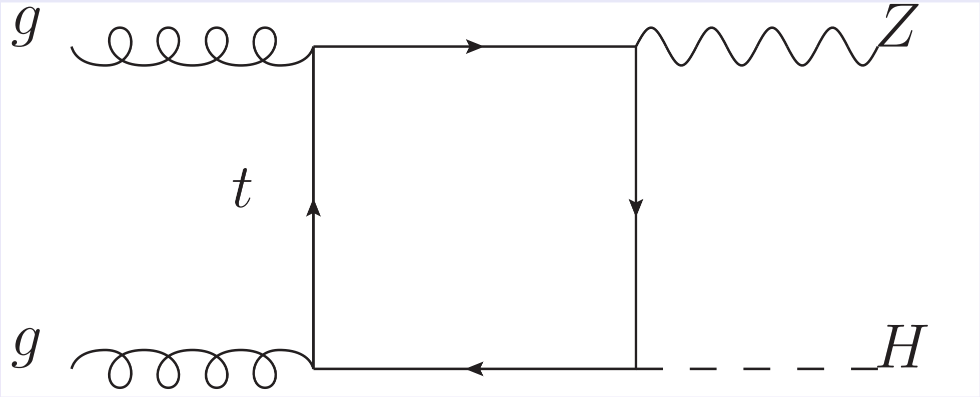
does not contain top loop
drives uncertainty at NNLO QCD

[Grazzini, Kallweit, Wiesemann, Yook '21]

Other gluon fusion processes

VH production





third largest cross section for Higgs production

measurement of $h \to b\bar{b}$

Gluon fusion @ 2 loop in QCD

Higgs pair production

computed in full top mass dependence numerically in [Borowka et al '16, Baglio et al '18]

large top mass renormalisation scheme dependence [Baglio et al '18]

implemented into POWHEG using a grid for virtual corrections [Heinrich et al '17]

gg -> ZH

computed in full top mass dependence numerically in [Chen et al '22]

gg -> ZZ

massless loops computed in [Manteuffel, Tancredi '15, Caola, (Dowling), Melnikov, Röntsch, Tancredi '16]

in MATRIX and MiNNLO$_{PS}$ [Grazzini, Kallweit, Wiesemann, Yook '18; Buonocore, Koole, Lombardi,Rottoli, Wiesemann Zanderighi '21]

top loop computed numerically in [Bronnum-Hansen, Wang '21; Agarwal, Jones, von Manteuffel '20, '24]

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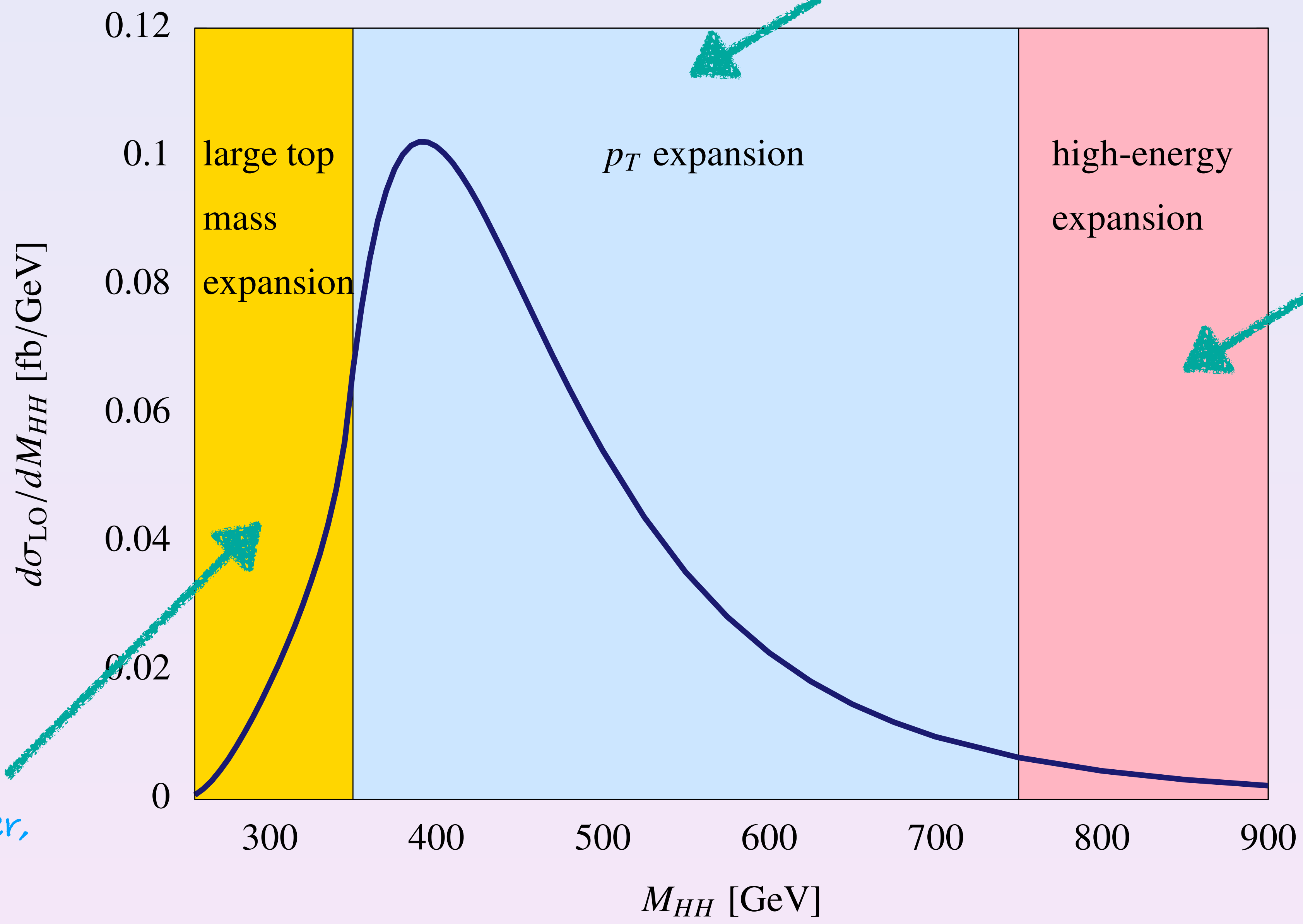
Can the virtual corrections be also computed analytically? → Monte Carlo → top mass renormalisation scheme uncertainty

Approximations

Higgs pair production

NLO QCD
[Bonciani, Degrassi, Giardino, RG' 18]



NLO QCD
[Davies, Mishima, Steinhauser, Wellmann, ' 18]

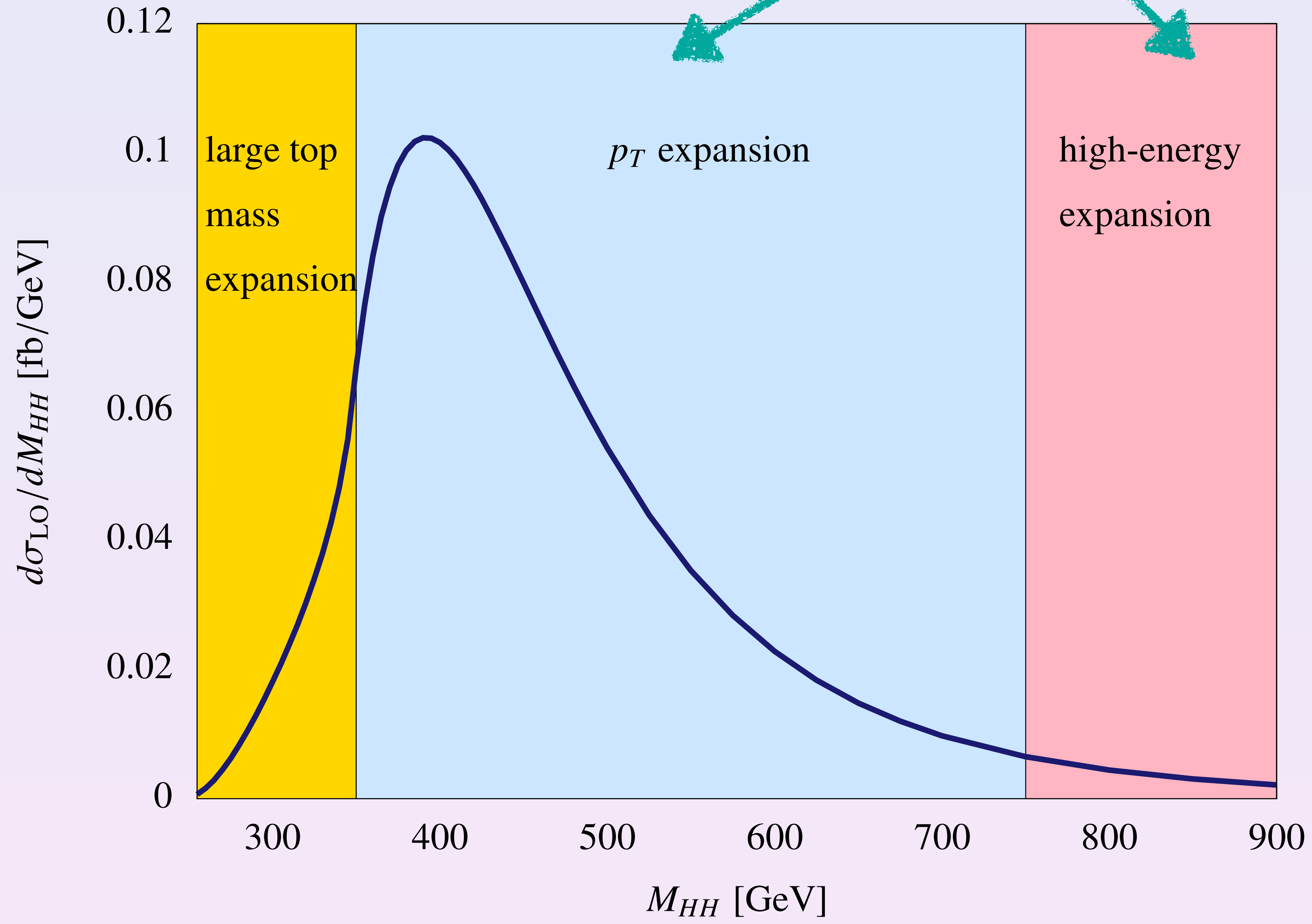
NLO QCD
[Dawson, Dittmaier, Spira ' 98]

NNNLO QCD
[L.-B. Chen, H. T. Li, H.-S. Shao and J. Wang '19]

Approximations

Higgs pair production

complementary, can be combined



[Bellafronte, Degrassi, Giardino, RG, Vitti '22; Davies, Mishima, Schönwald, Steinhauser '23]

Expansion

We can use $\hat{t} \sim 0 \longrightarrow p_T^2 = \frac{\hat{t}\hat{u} - m_H^4}{\hat{s}} \sim 0$

But $p_T^2 \sim 0 \not\longrightarrow \hat{t} \sim 0$

Expansion

We can use $\hat{t} \sim 0$ $\longrightarrow$ $p_T^2 = \dfrac{\hat{t}\hat{u} - m_H^4}{\hat{s}} \sim 0$

But $p_T^2 \sim 0$ $\longrightarrow$

- $\hat{t} \sim 0 \quad \hat{u} \sim -\hat{s}$
- $\hat{t} \sim -\hat{s} \quad \hat{u} \sim 0$

two cases

we are lucky though

$$\mathcal{M}^{\mu\nu}(\hat{t}, \hat{u}) = \mathcal{M}^{\mu\nu}(\hat{u}, \hat{t})$$

symmetric in t and u

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$\hat{t} \sim -\hat{s} \quad \hat{u} \sim 0$

two cases

we are lucky though

$$\mathcal{M}^{\mu\nu}(\hat{t}, \hat{u}) = \mathcal{M}^{\mu\nu}(\hat{u}, \hat{t})$$

symmetric in t and u

$$\sigma = \int_{t_-}^{t_+} d\hat{t} \frac{d\sigma}{d\hat{t}} \sim \int_{t_-}^{t_m} d\hat{t} \frac{d\sigma(\hat{t} \sim 0)}{d\hat{t}} + \int_{t_m}^{t_+} d\hat{t} \frac{d\sigma(\hat{t} \sim -\hat{s})}{d\hat{t}} = \int_{t_-}^{t_+} d\hat{t} \frac{d\sigma(\hat{t} \sim 0)}{d\hat{t}}$$

Expansion

Define

$r^\mu = p_1^\mu + p_3^\mu \longrightarrow r^2 = \hat{t}$

$$r^\mu = \frac{\hat{t} - m_h^2}{\hat{s}}(p_2^\mu - p_1^\mu) + r_T^\mu \quad \text{with} \quad r_T^2 = -p_T^2$$

Then we can expand

$$\hat{t} \sim 0 \longrightarrow r^\mu \sim 0 \longrightarrow p_1^\mu \sim -p_3^\mu$$

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Then we can expand

$$\hat{t} \sim 0 \longrightarrow r^\mu \sim 0 \longrightarrow p_1^\mu \sim -p_3^\mu$$

$$F_i = F_i\Big|_{p_3=-p_1} + r_\mu \frac{\partial F_i}{\partial p_3^\mu}\Bigg|_{p_3=-p_1} + r_\mu r_\nu \frac{\partial^2 F_i}{\partial p_3^\mu \partial p_3^\nu}\Bigg|_{p_3=-p_1} + \ldots$$

First order p_T expansion needs second order p_3 expansion

High-energy expansion

Valid for

$$\hat{s}, \hat{t}, \hat{u} \gg m_t^2 > m_{ext}^2$$

Difficulty: Imposing the boundary conditions to the loop integrals

[Mishima '18]

results depend on s, t and can be expressed in terms of harmonic polylogarithms up to weight 4

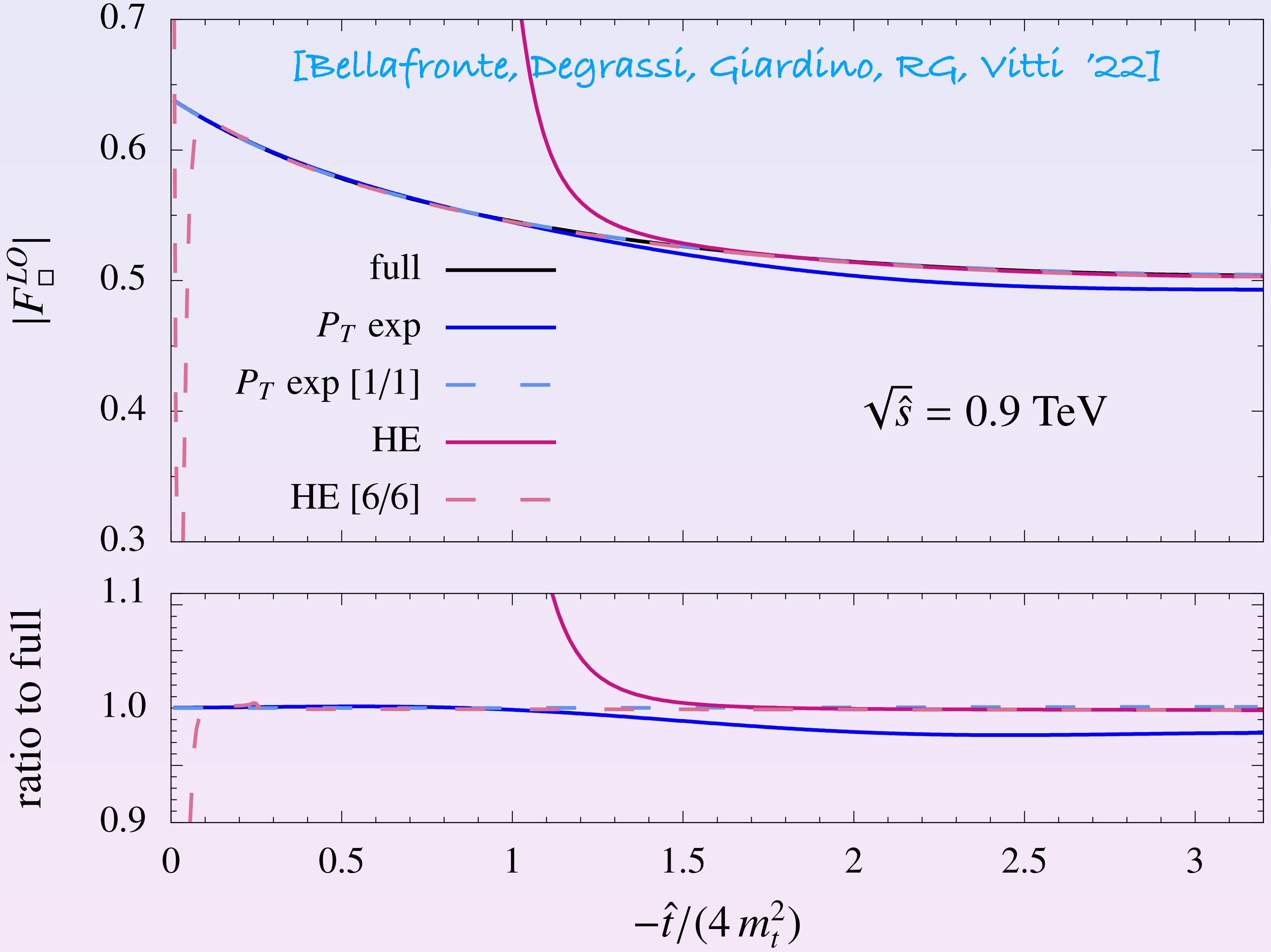
Results available up to high orders (16) in m_t^2

[HH: Davies, Mishima, Steinhauser, Wellmann '18; ZH: Davies, Mishima, Steinhauser, Wellmann '20; ZZ: Davies, Mishima, Steinhauser '21]

Padé approximants can push validity down to $p_T \sim 150$ GeV

Combination of expansions

Leading order form factor for Higgs pair production:

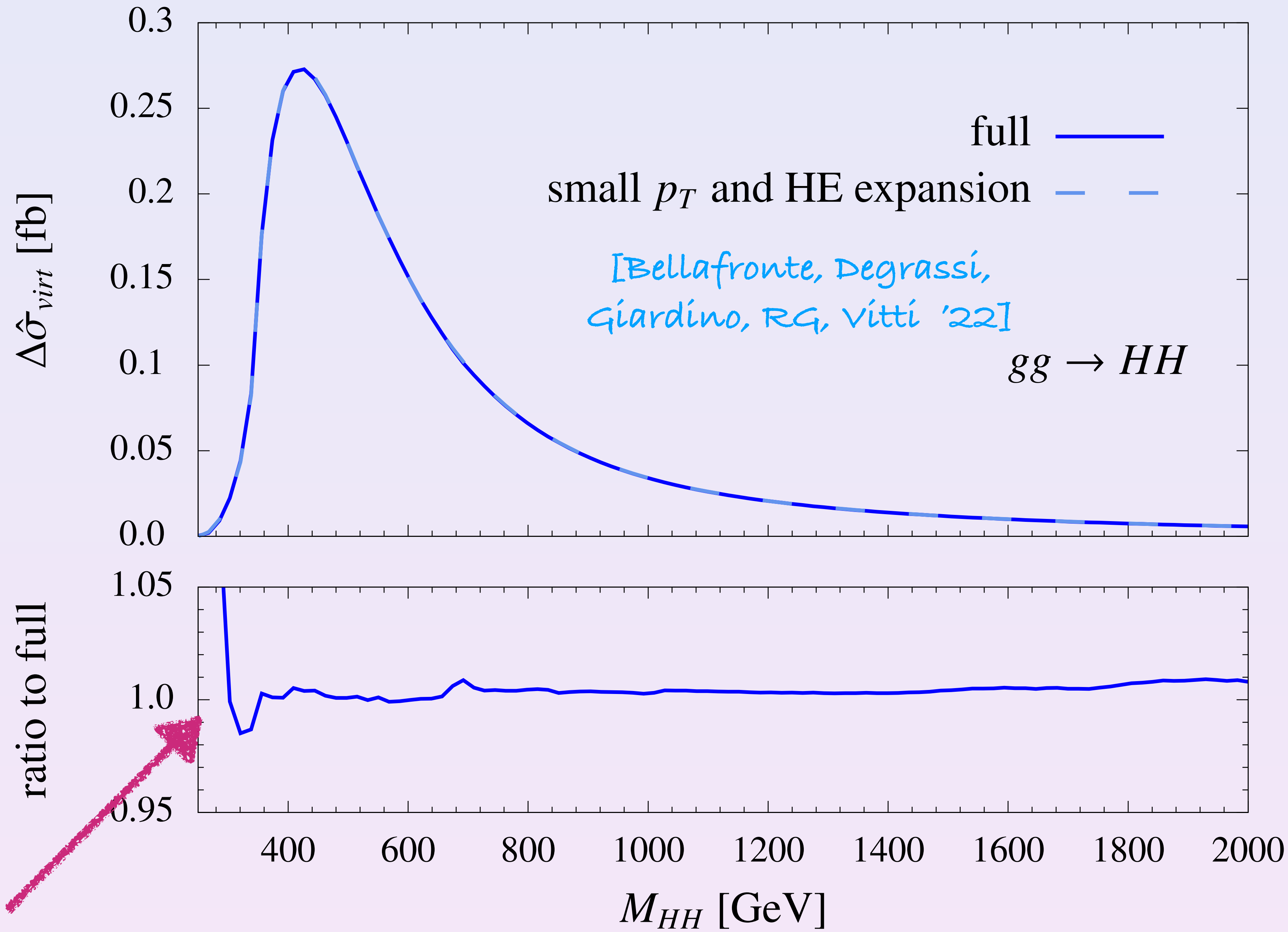


Padé approximant

$$[n/m] = \frac{a_0 + a_1 x + \ldots + a_n x^n}{1 + b_1 x + \ldots b_m x^m}$$

Expansions are complementary, using Padé approximants one can increase convergence

Combination of expansions



few phase space points in virtual grid of

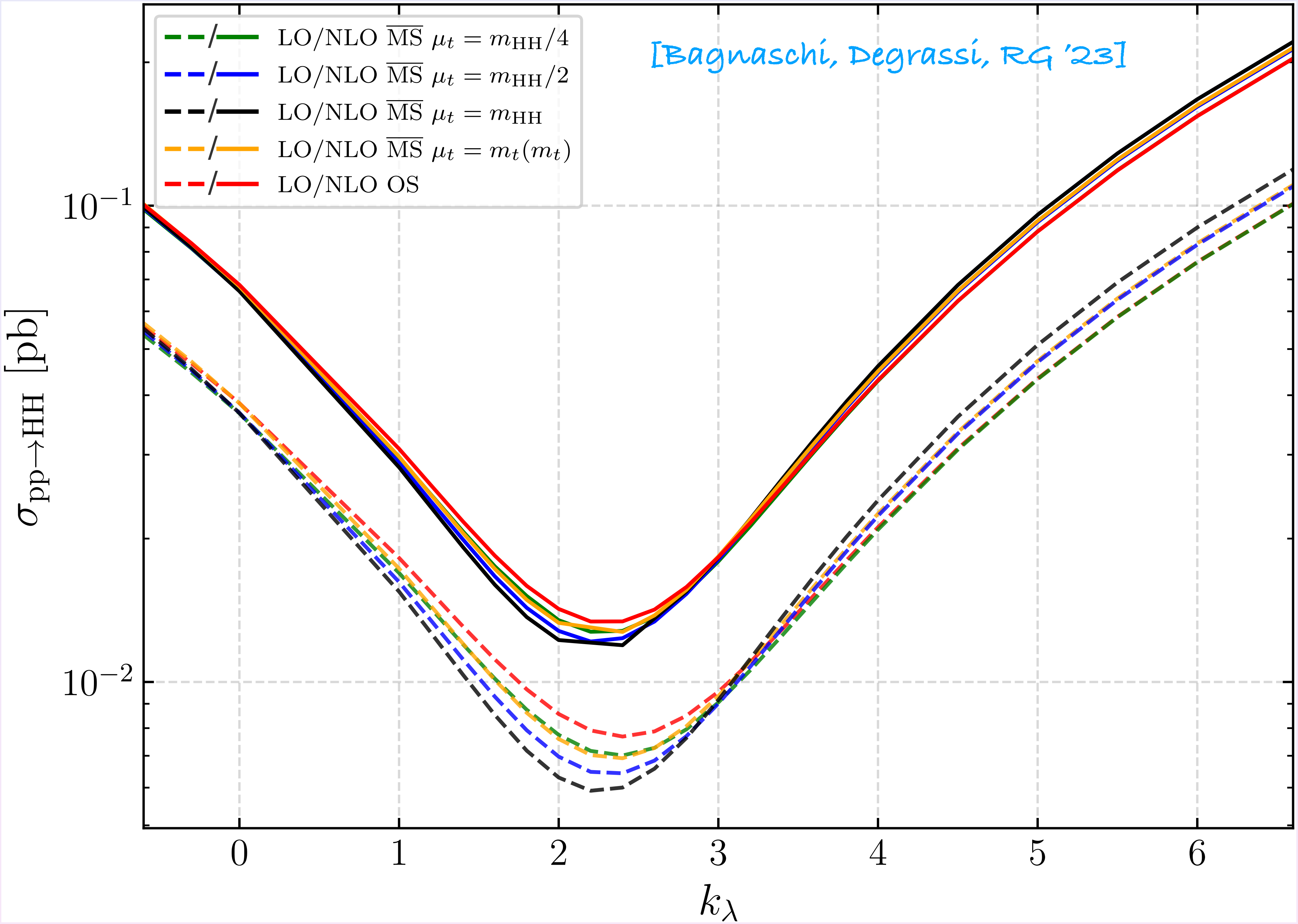
[Davies, Heinrich, Jones et al. '19]

Works incredibly well (difference < 1%)

New POWHEG implementation

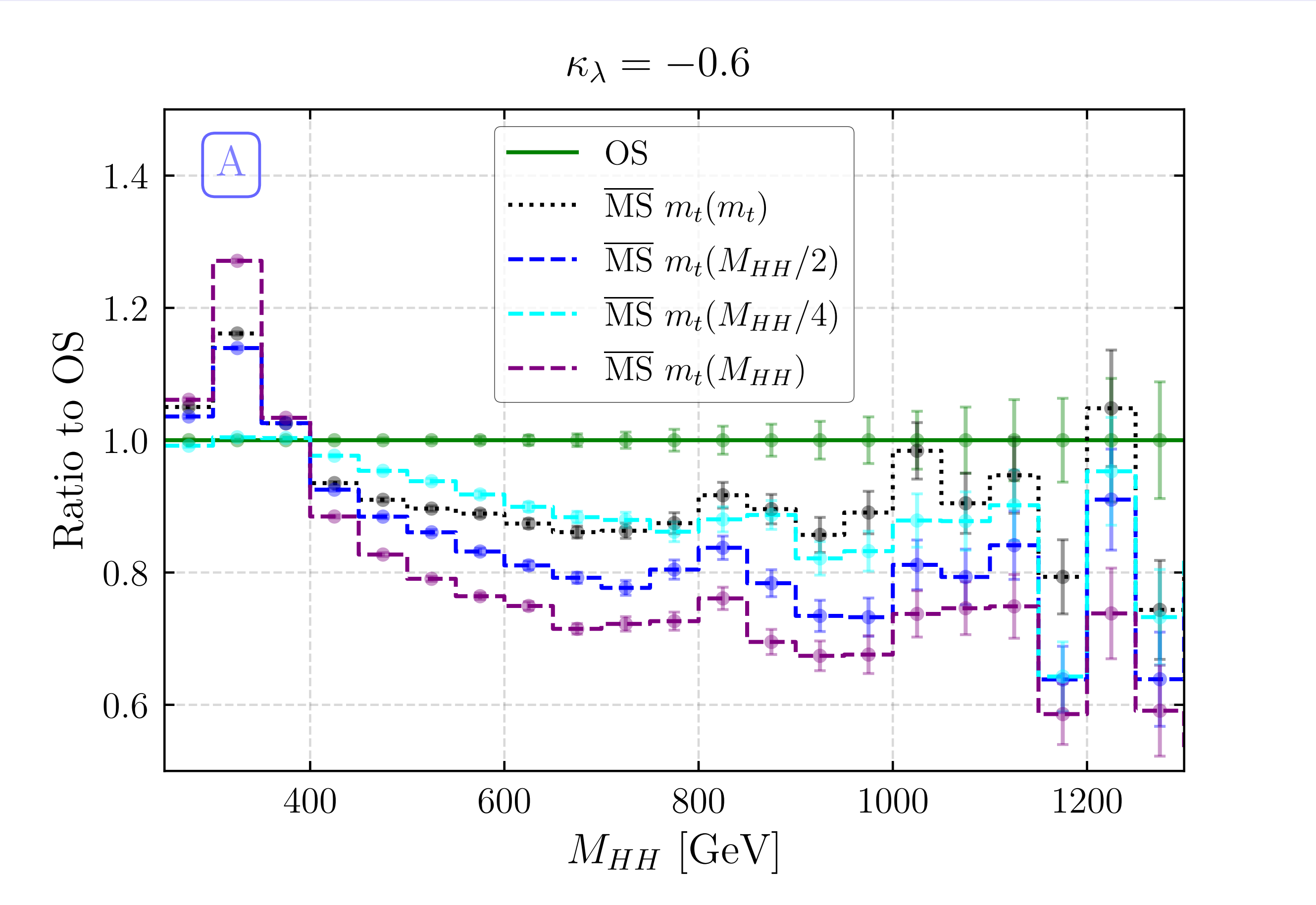
virtuals with expansion technique analytically

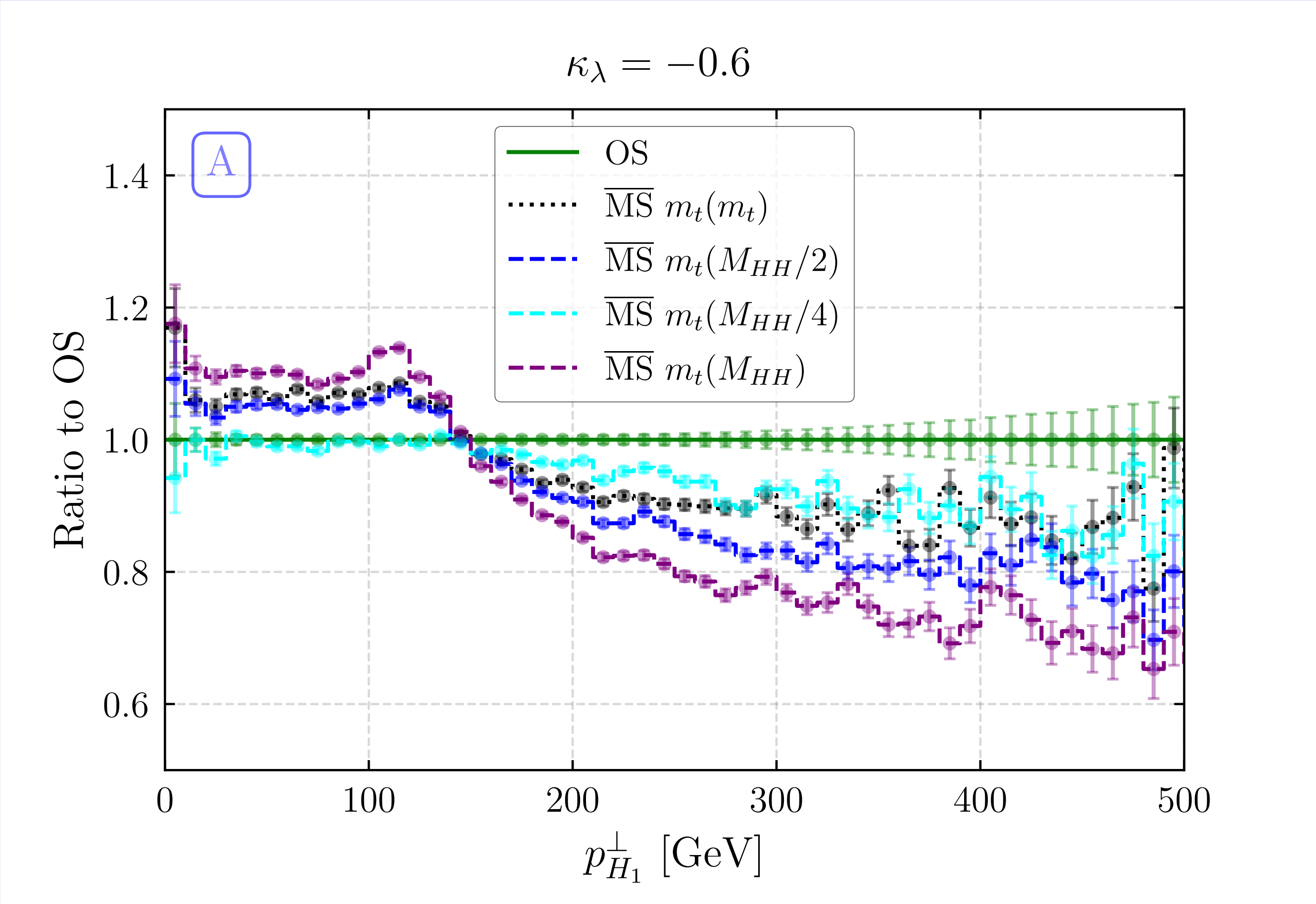
reals with MadLoop [Hirschi et al. '11]



New POWHEG implementation

[Bagnaschi, Degrassi, RG '23]





flexibility of analytic approach allows to vary top mass renormalisation scheme

gg-> ZZ

$$\hat{A}^{\mu\nu\rho\sigma}(p_1,p_2,p_3) = \sum_{i=1}^{20} S_i^{\mu\nu\rho\sigma} f_i(\hat{s},\hat{t},\hat{u},m_t,m_Z),$$

20 Lorentz structures

→

$$\hat{A}^{\mu\nu\rho\sigma}(p_1,p_2,p_3) = \sum_{i=1}^{20} \mathcal{P}_i^{\mu\nu\rho\sigma} A_i(\hat{s},\hat{t},\hat{u},m_t,m_Z),$$

20 projectors

16 with norm non equal zero

↓

p_T expansion

←

MIs expressed in terms of GPLs

two elliptic integrals

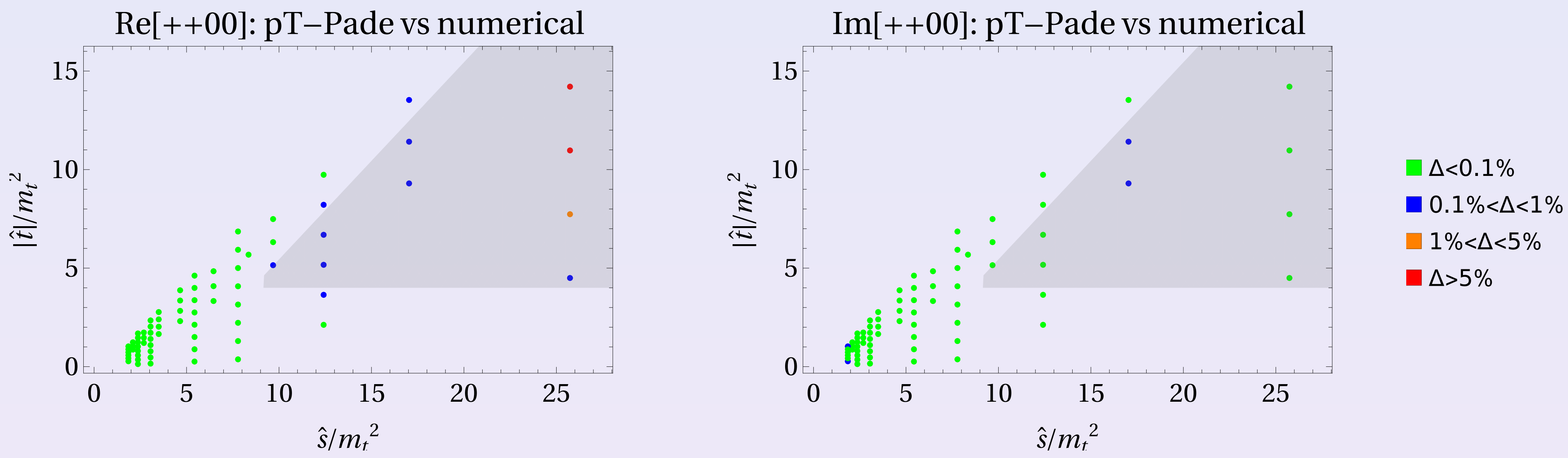
[von Manteuffel, Tancredi '17; Bonciani, Degrassi, Giardino, RG, '18]

↓

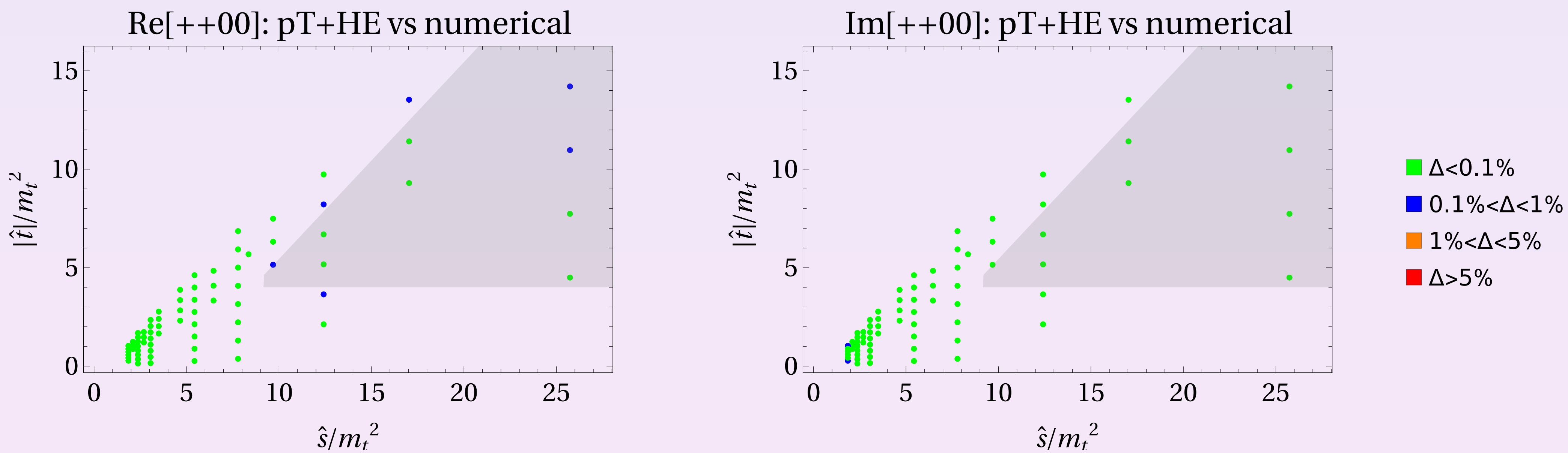
combination with results in high-energy limit via Padé approximants

[Davies, Mishima, Steinhauser, Wellmann '20]

Comparison with [Agarwal, Jones, von Manteuffel '20]



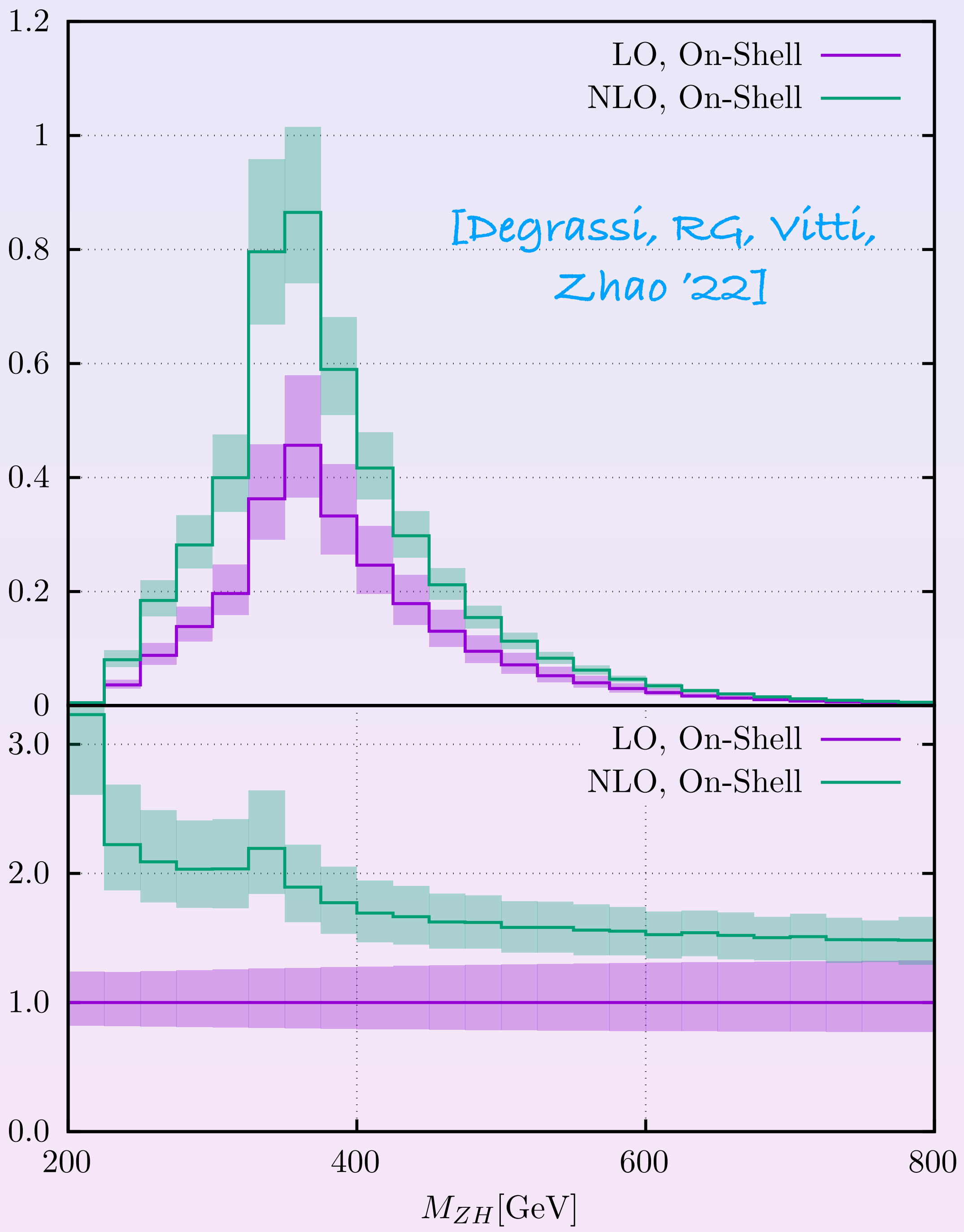
(a)



[Degrassi, RG, Vitti '24]

(b)

Top-mass scheme	LO [fb]	$\sigma_{LO}/\sigma_{LO}^{OS}$	NLO [fb]	$\sigma_{NLO}/\sigma_{NLO}^{OS}$	$K = \sigma_{NLO}/\sigma_{LO}$
On-Shell	$64.01^{+27.2\%}_{-20.3\%}$	-	$118.6^{+16.7\%}_{-14.1\%}$	-	1.85
$\overline{\text{MS}}, \mu_t = M_{ZH}/4$	$59.40^{+27.1\%}_{-20.2\%}$	0.928	$113.3^{+17.4\%}_{-14.5\%}$	0.955	1.91
$\overline{\text{MS}}, \mu_t = m_t^{\overline{\text{MS}}}(m_t^{\overline{\text{MS}}})$	$57.95^{+26.9\%}_{-20.1\%}$	0.905	$111.7^{+17.7\%}_{-14.6\%}$	0.942	1.93
$\overline{\text{MS}}, \mu_t = M_{ZH}/2$	$54.22^{+26.8\%}_{-20.0\%}$	0.847	$107.9^{+18.4\%}_{-15.0\%}$	0.910	1.99
$\overline{\text{MS}}, \mu_t = M_{ZH}$	$49.23^{+26.6\%}_{-19.9\%}$	0.769	$103.3^{+19.6\%}_{-15.6\%}$	0.871	2.10

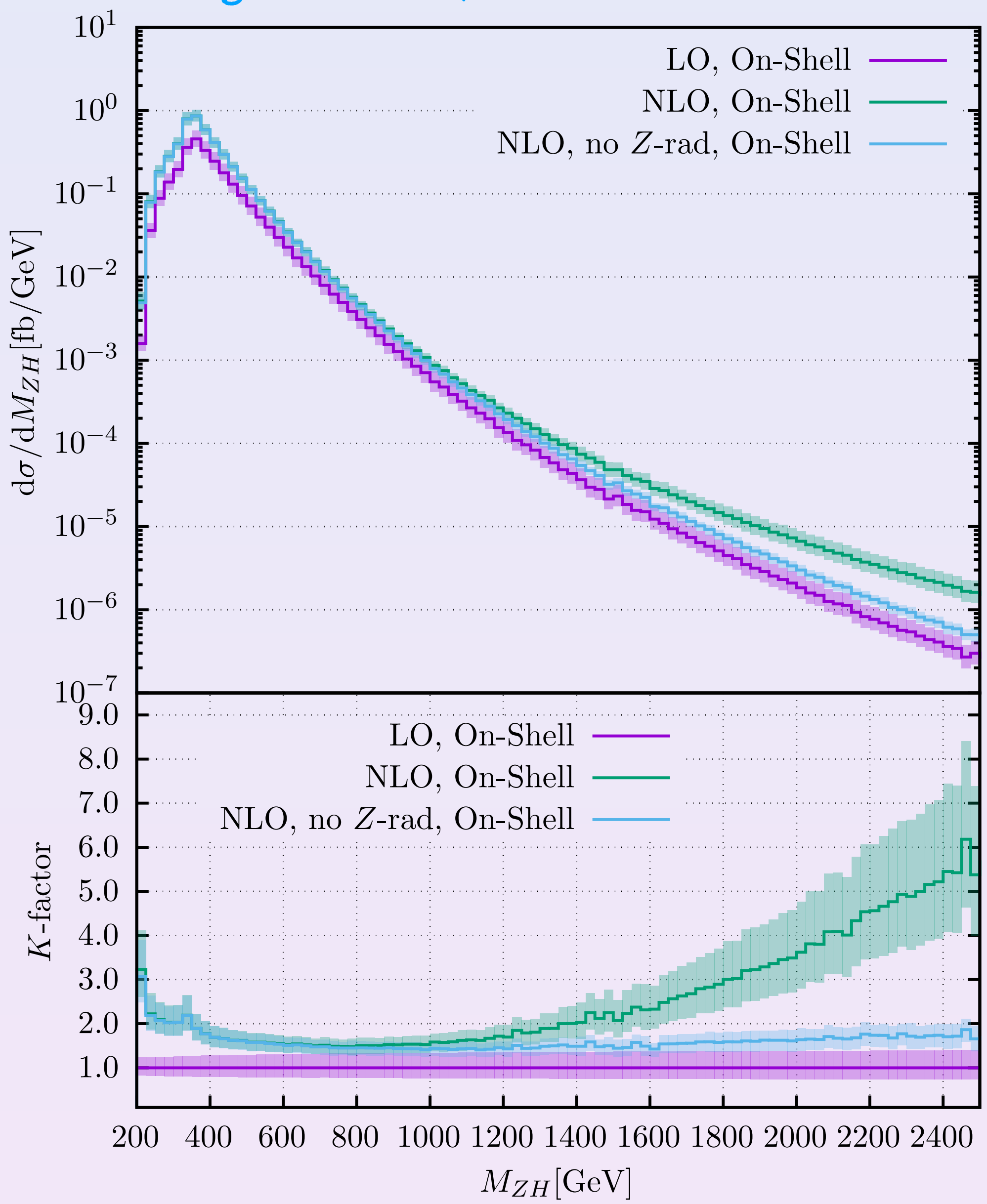


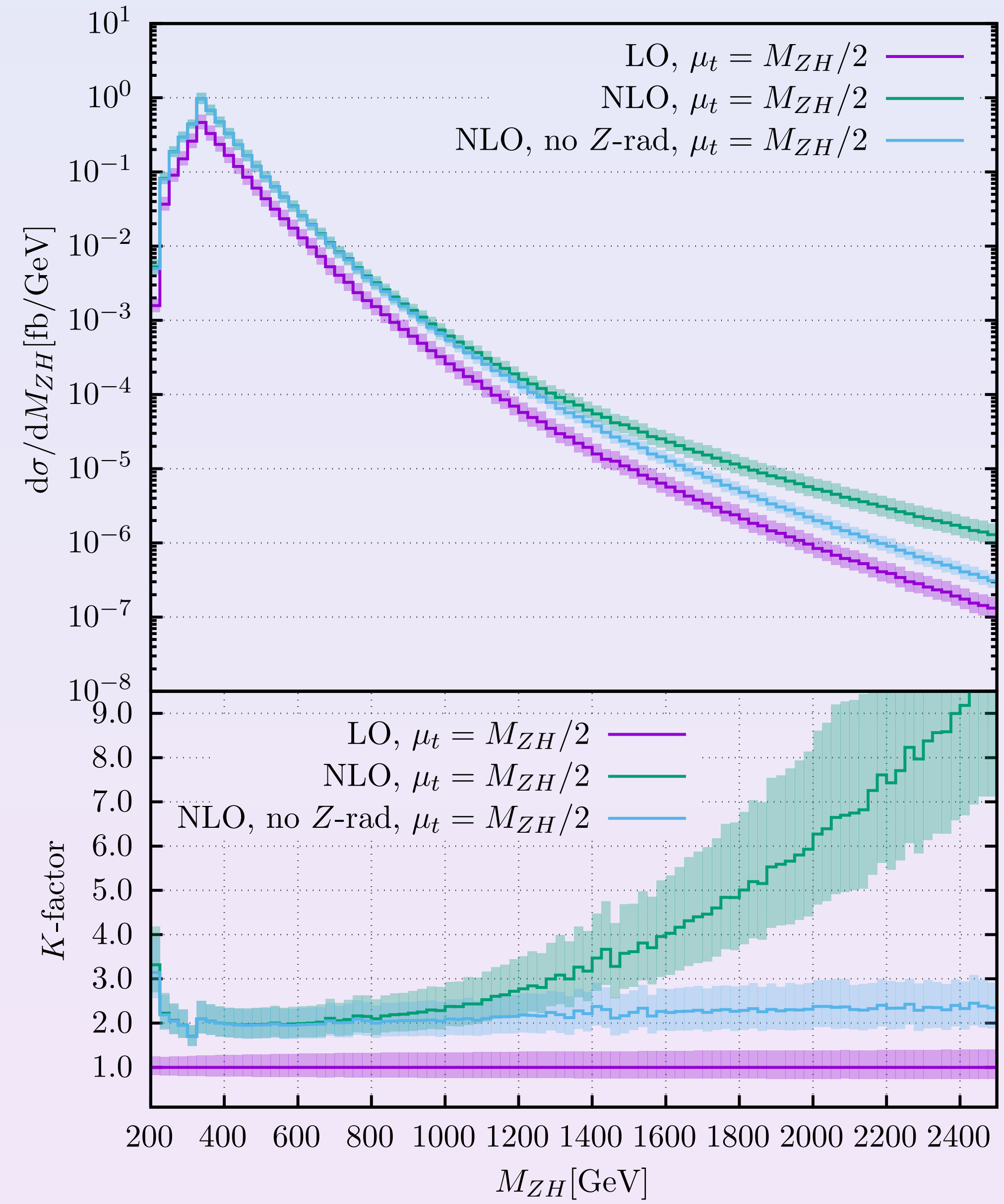
NLO QCD corrections important

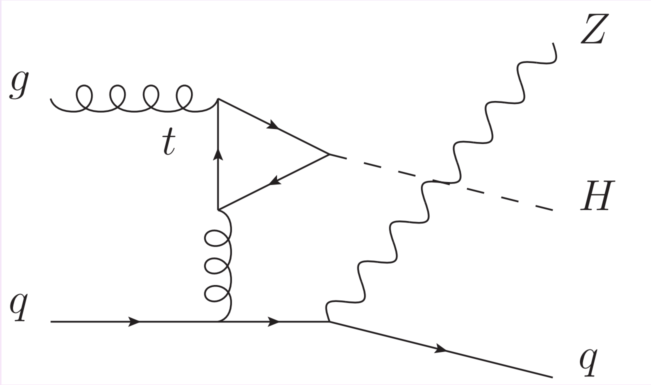
top mass renormalisation scheme dependence sizeable

gg-> ZH

[Degrassi, RG, Vitti, Zhao '22]







Z radiation: EW logs of type $\log^2(m_Z^2/M_{ZH}^2)$

Conclusion

- Precise predictions for 2 -> 2 gluon fusion processes important -> multi-scale problem
- analytically they can be computed using various expansions and combine them
- for Monte Carlo a analytic approach showed here is useful and can be sufficiently precise
- Monte Carlo with analytic approach is very flexible and can be easily extended to BSM
- to be explored: how can the idea of the expansion be applied to more complicate cases?

see for instance [Davies, Schönwald, Steinhauser (Vitti) '23 ('24)] for first efforts at NNNLO QCD

Thanks for your attention!

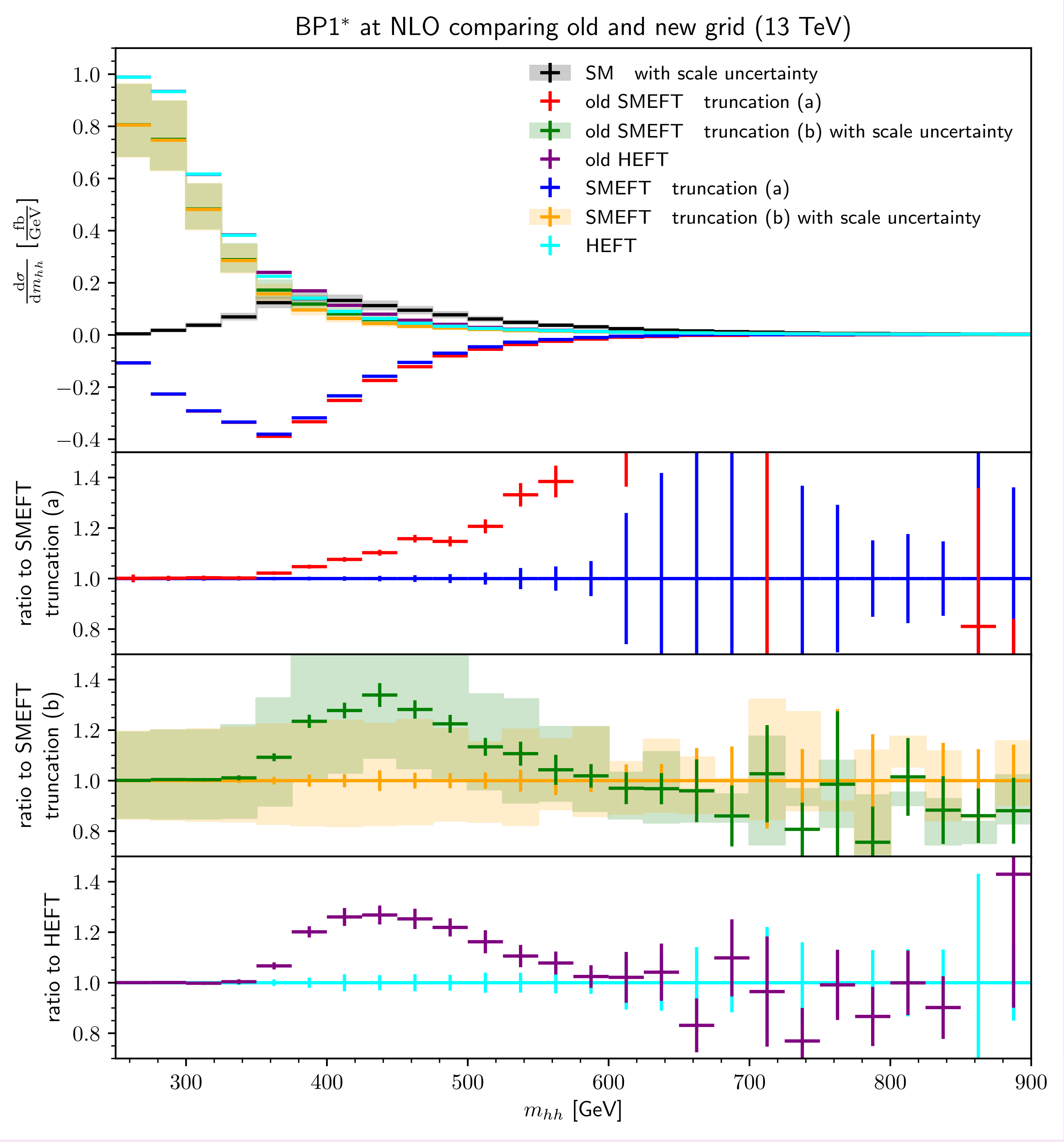
Backup

New POWHEG implementation

We had a discrepancy with respect to the POWHEG by [Heinrich et al '20 '22] when varying the trilinear Higgs self-coupling

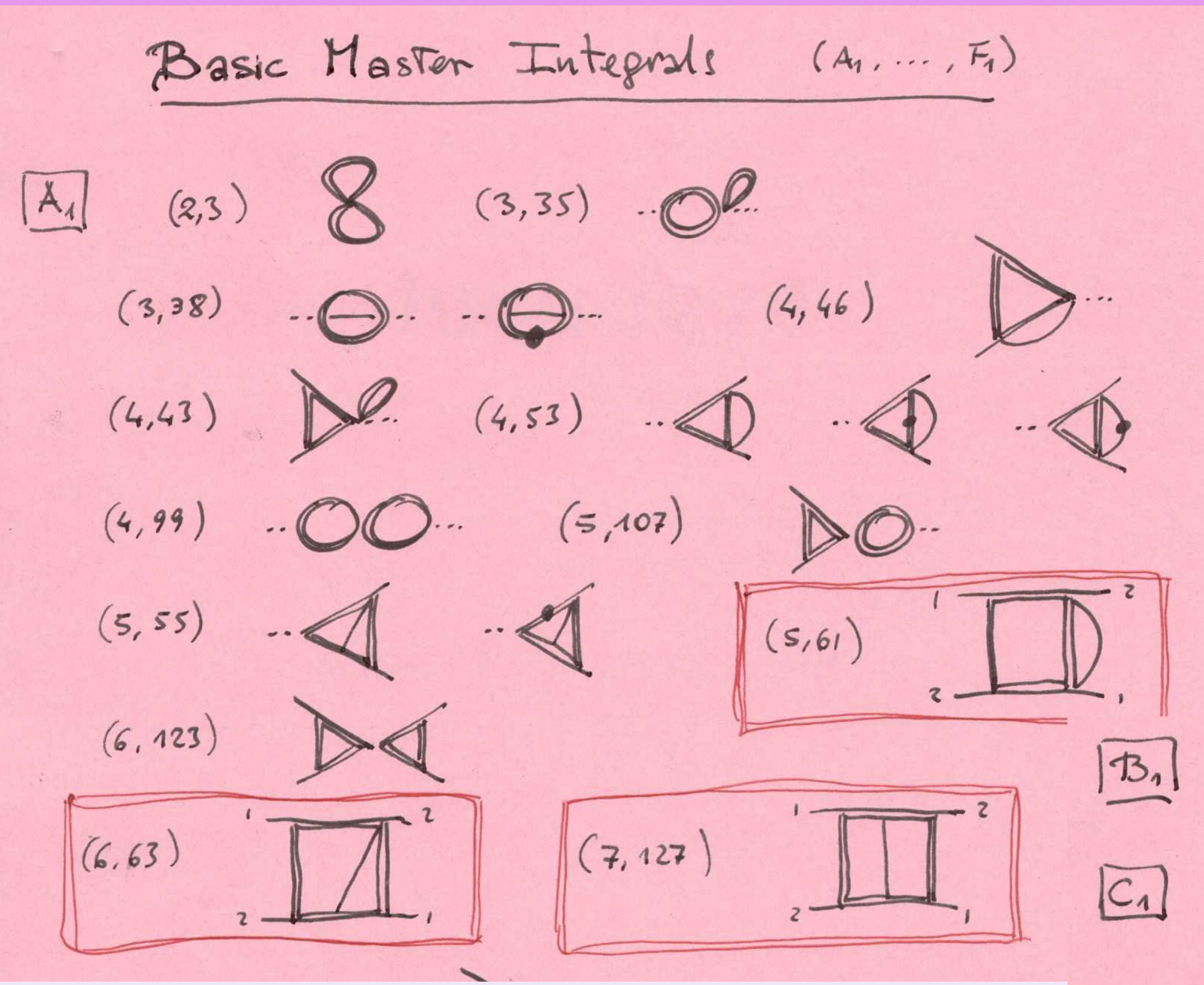
BP1:

$c_{hhh} \approx 5.1, c_t = 1.1$

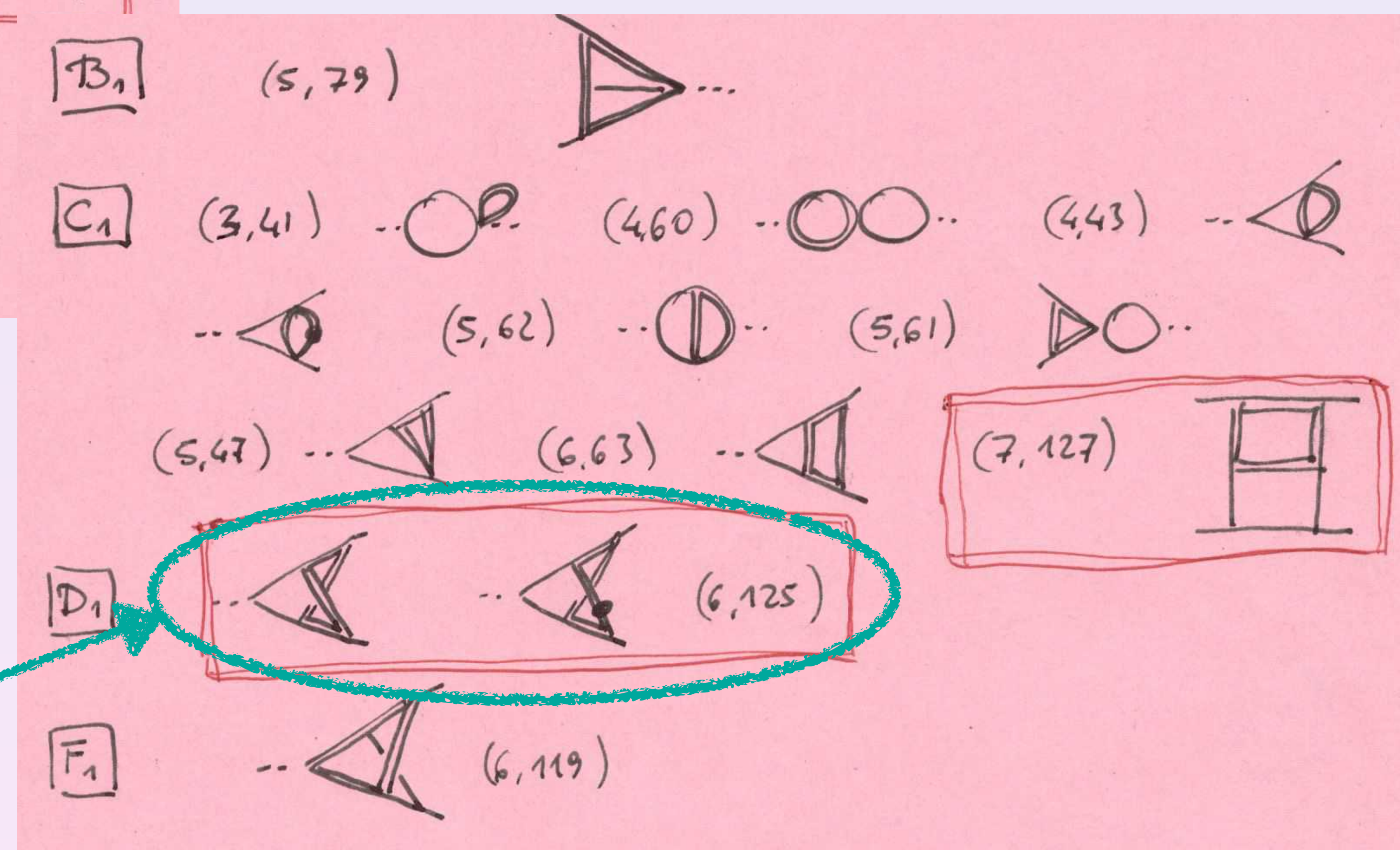


[Heinrich et al '22]

NLO expansion



- O(50) master integrals
- all of them known, though we needed to recompute some for the forward kinematics



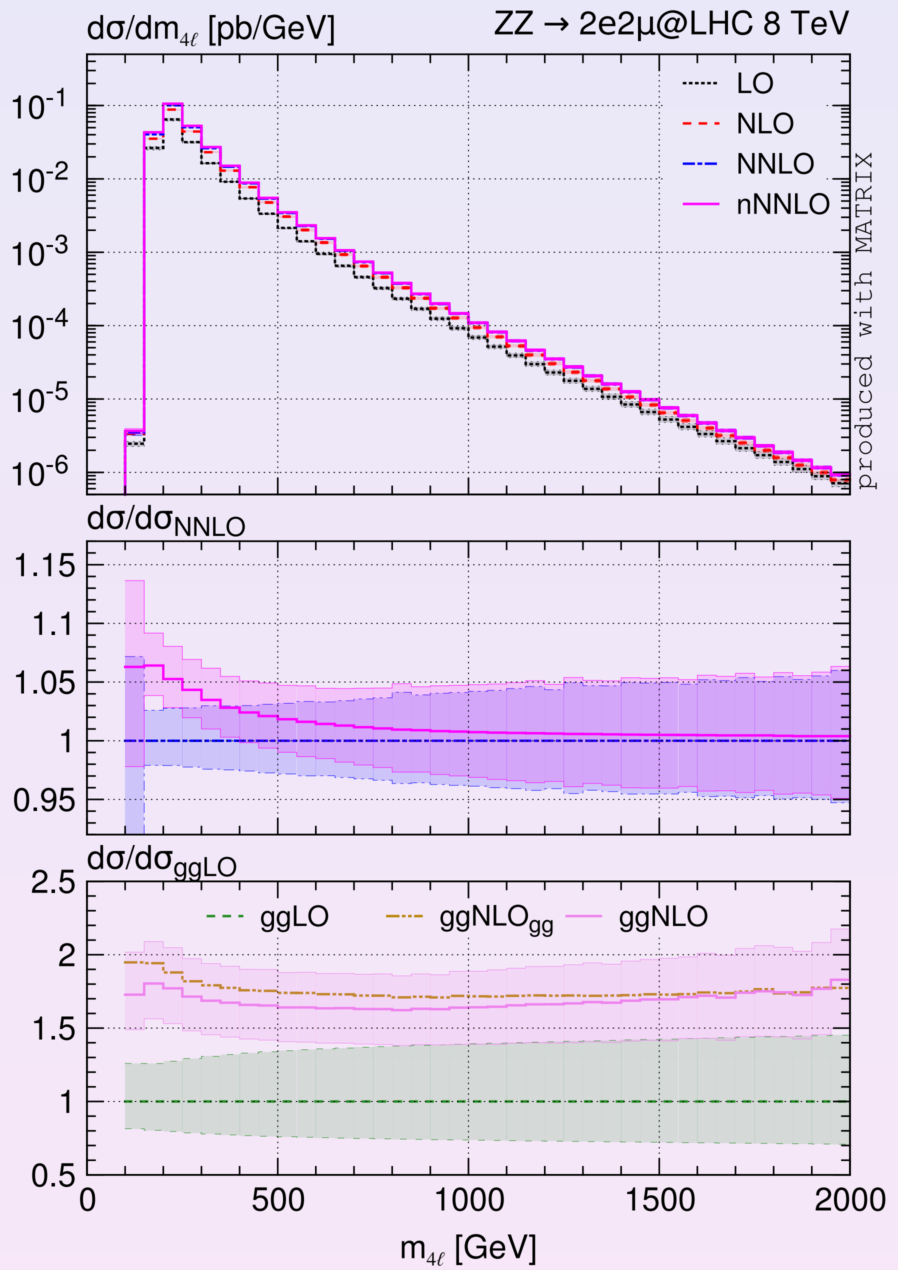
- everything fully analytic in terms of HPLs and GPLs
- But: the two elliptic integrals

Interplay: gg-> ZZ @ NLO

massless loops:

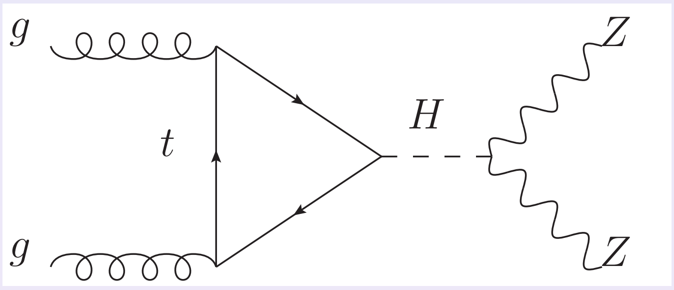
[Manteuffel, Tancredi '15, Caola, (Dowling), Melnikov, Röntsch, Tancredi '16]

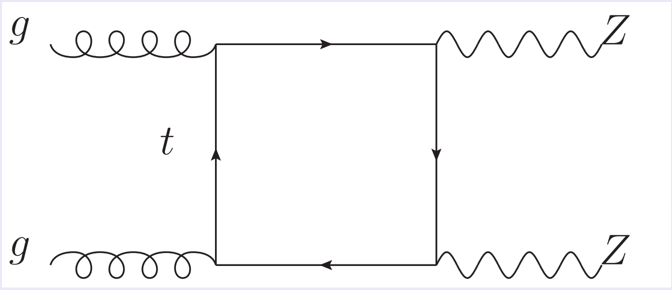
[Grazzini, Kallweit, Wiesemann, Yook '18]



also in MiNNLO$_{PS}$

[Buonocore, Koole, Lombardi, Rottoli, Wiesemann, Zanderighi '21]





for interference with Higgs important also massive loops

@ NLO QCD computed numerically in

[Agarwal, Von Manteuffel, Jones '20, Bronnum-Hansen, Wang '21]

no Monte Carlo yet