A Numerical Implementation of the LASS Subtraction Scheme

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in collaboration with

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Motivation

Motivation

LASS : Local Analytic Sector Subtraction scheme arXiv:2212.11190, 2010.14493, 1806.09570

- Massless partons
- For final state radiation (i.e. $e^+ e^-$ collisions)

~~For details see Gloria's talk in this session!~~

Subtraction and slicing methods at NNLO QCD:

- Several methods for final/initial state both for massless and massive partons
- Several processes already calculated with several different methods
- Methods already started to be extended (easily/less painfully) to N^3LO

What is the point creating yet another?

Motivation

Motto:

Zoltán Nagy: ”We solve a math problem”

- Can have multiple solutions
- Can reach the same answer on a multitude of paths
- Physics and math is also about elegance
- Life will not stop at NNLO:
  - Can an $N^{n}LO$ scheme extended to $N^{n+1}LO$?
  - Can a scheme for $e^{+}e^{-}$ extended to pp collisions?
  - At what cost?

What about carbon-friendly computing?

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A way to check subtractions

A way to check subtractions

- Numerical checks are essential for validating both individual subterms and whole contributions:
  - Individual subterms:
    - Proper usage of mappings
    - Parameters are calculated correctly
    - Normalizations are correct
  - Full ensemble:
    - Parametrizations are synchronized (where need be)
    - Spurious singularities are happening, need to cancel:

$$\mathsf{K}^{(1)}\ \text{limits}:\quad \mathsf{K}^{(2)} \leftrightarrow \mathsf{K}^{(12)}\,,\quad \mathsf{K}^{(2)}\ \text{limits}:\quad \mathsf{K}^{(1)} \leftrightarrow \mathsf{K}^{(12)}$$

A way to check subtractions

"My CAS is Fortran" after Bob Pease

Idea: check everything numerically, but with arbitrary precision!

- `Fortran90` and `MPFUN20` by David Bailey to have arbitrary precision floating point numbers
- 50-60 digits suffice for most checks (only for checks, MC in double precision)
- Starting with an $\mathsf{n}+2$ or $\mathsf{n}+1$ parton PS point we create a sequence of PS points bringing partons to specific limits
- Ratio is monitored between SME and subterm/full contribution:

$$\mathsf{n}+2\,\text{parton line}: \quad \lim_{5||6||7} \frac{\left|\mathcal{M}^{\mathrm{RR}}\right|^2}{\mathcal{C}_{765}}, \quad \lim_{5\,,6\to 0} \frac{\mathcal{S}_{65}\mathcal{C}_{765}}{\mathcal{C}_{765}}, \quad \frac{\left|\mathcal{M}^{\mathrm{RR}}\right|^2}{\mathsf{K}^{(1)}+\mathsf{K}^{(2)}+\mathsf{K}^{(12)}}$$

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A way to check subtractions – example

- LASS currently is for final state radiation only ($e^+ e^-$ collisions)
- Three-jet production is the obvious choice (two-jet requires a special version due to number of final state partons)
- Most complicated subprocess: $e^+ e^- \to q\bar{q}\,ggg$
  - 187 subtraction terms
  - 49 singular regions

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A way to check subtractions – example

Ratio sequence for a single soft subtraction term and the RR SME:

$$\lim_{5\to 0}\frac{\left|\mathcal{M}^{\mathrm{RR}}\right|^2}{\mathcal{S}_5}$$

Ratio sequence for the complete $\mathsf{K}^{(1)}+\mathsf{K}^{(2)}+\mathsf{K}^{(12)}$ and the RR SME in a collinear limit:

$$\lim_{7||3}\frac{\left|\mathcal{M}^{\mathrm{RR}}\right|^2}{\mathsf{K}^{(1)}+\mathsf{K}^{(2)}+\mathsf{K}^{(12)}}$$

A way to check subtractions – example

```
Checking subtractions for ep em > d db g g g           Checking singular region (12/49)
Checking individual subterm (1/187)                    9.6594481743241900295619875419238279005187022510109e-1
1.3615262894238763826759820725791757465583666344146e1  1.0058615538802959144867884196860137973711485348134e0
1.0694493566690845438842431546992987055507713973236e0  9.9231059807099869356172685061099655988514093224880e-1
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```
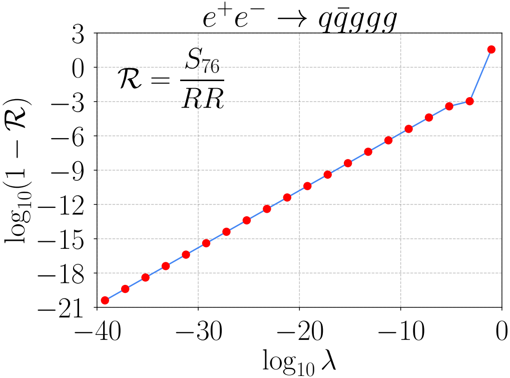
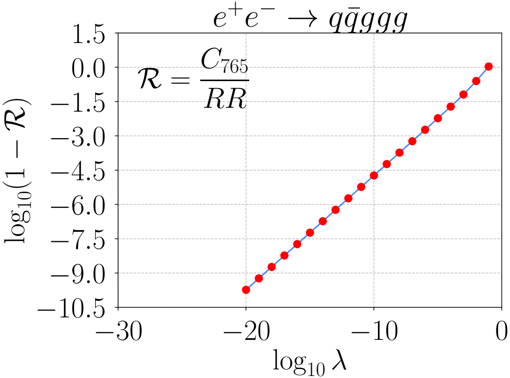

A way to check subtractions – example

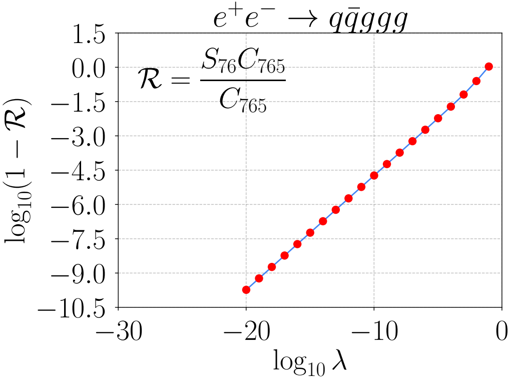
Physical, genuine NNLO limits:

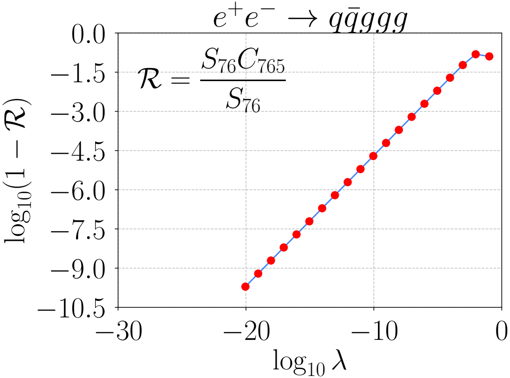




A way to check subtractions – example

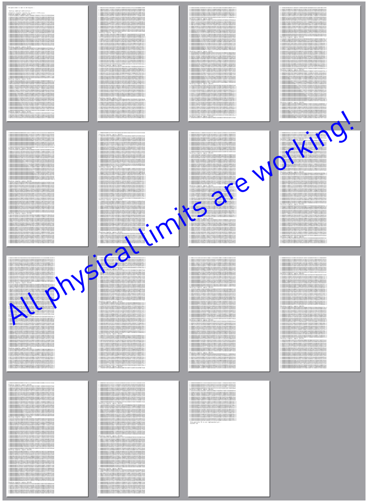
Cancellation of spurious singularities (K$^{(1)}$ $\leftrightarrow$ K$^{(12}$ or K$^{(2)}$ $\leftrightarrow$ K$^{(12}$):





A way to check subtractions – example

All separate subterms are working!



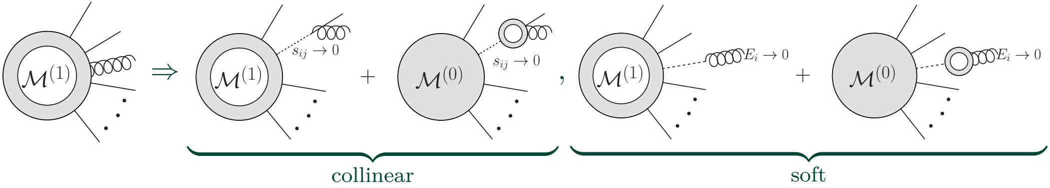
All physical limits are working!

A way to check subtractions – example

Contributions at NNLO:

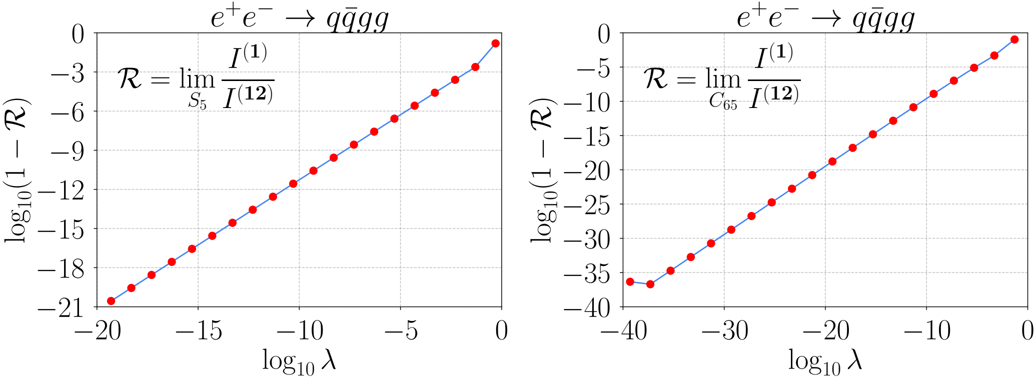
$$\mathrm{d}\sigma = \mathrm{d}\sigma^{\mathrm{RR}} + \mathrm{d}\sigma^{\mathrm{RV}} + \mathrm{d}\sigma^{\mathrm{VV}}$$

- $\mathrm{d}\sigma^{\mathrm{RR}}$ is the most difficult, double-unresolved radiation
- $\mathrm{d}\sigma^{\mathrm{RV}}$ only has single-unresolved radiation but with one-loop amplitudes!



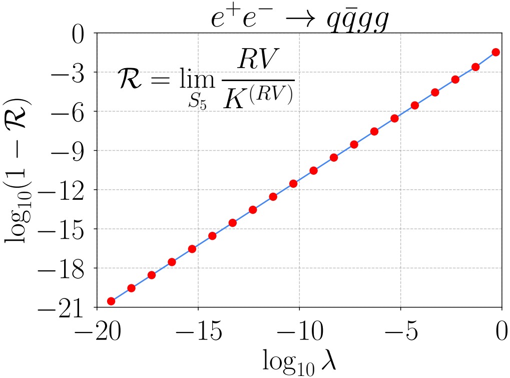
A way to check subtractions – example

Same checks are carried out for the $\mathrm{n}+1$ parton contribution (RV):
($\int_1 \mathrm{K}^{(1)} = \mathrm{I}^{(1)}$, $\int_1 \mathrm{K}^{(12)} = \mathrm{I}^{(12)}$)



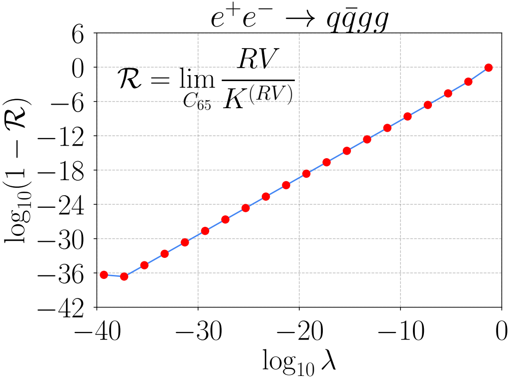
A way to check subtractions – example

Same checks are carried out for the $n + 1$ parton contribution (RV):



$e^+e^- \to q\bar{q}gg$

$$\mathcal{R} = \lim_{S_5} \frac{RV}{K^{(RV)}}$$



$e^+e^- \to q\bar{q}gg$

$$\mathcal{R} = \lim_{C_{65}} \frac{RV}{K^{(RV)}}$$

A first glimpse at numerical integrations

A first glimpse at numerical integrations

- Checking subterms and complete subtractions are important but thorough numerical checks are also needed
  - A sequence does not cover whole phase space
  - A sequence is too artificial
  - What happens close to two-jet limit?
  - ...
- One possible check is the saturation of cross section:

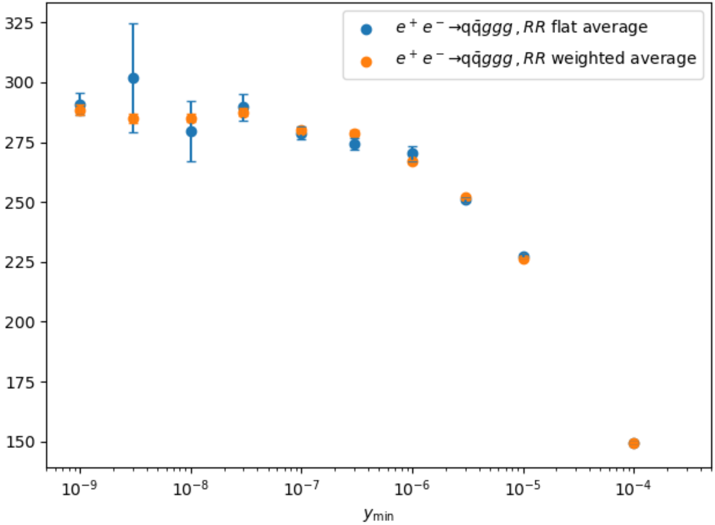
$$\min_{i,j} y_{ij} = \min_{i,j} \frac{s_{ij}}{s} \geq y_{\min}$$

- Cut at edge of phase space, than decrease the cut
- In case all singularities are regularized contribution should stabilize (saturate)

A first glimpse at numerical integrations

$n+2$ parton contribution (double-real):

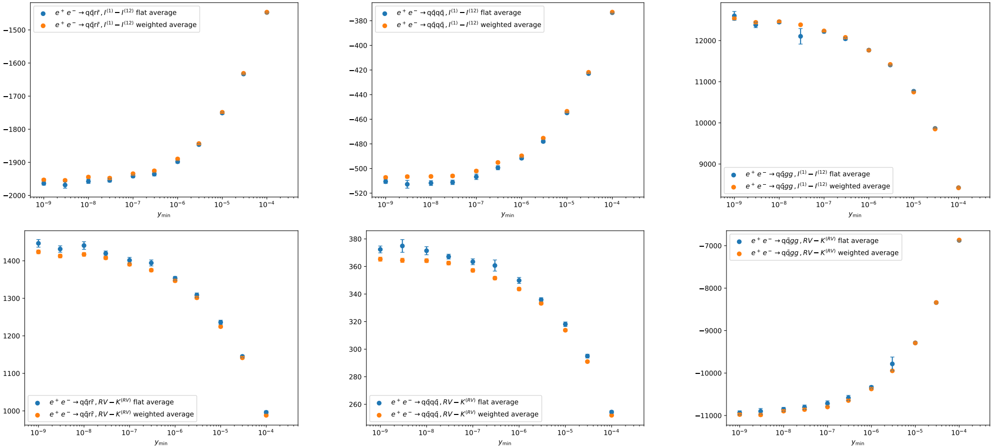
Saturation plot for $e^+e^- \to q\bar{q}\,ggg$



A first glimpse at numerical integrations

- The $\mathrm{n}+1$ parton contribution is NLO in nature
- Subterms are more complicated (and have more)
  - Kernels from one-loop factorization
  - Integrated subterms ($\mathrm{I}^{(1)}$, from $\mathrm{n}+2$ parton cont.) also diverge

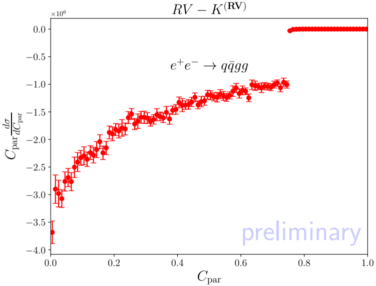
A first glimpse at numerical integrations

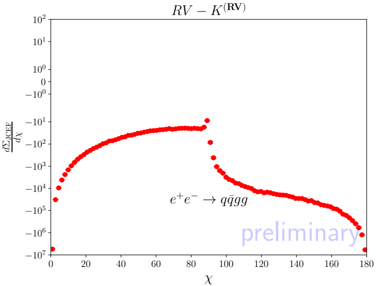


Saturation works for all contributions!

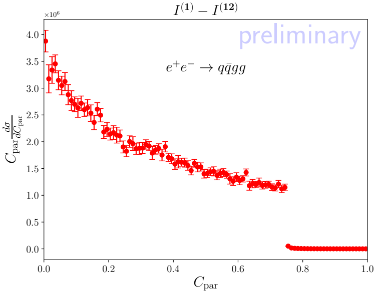
A first glimpse at numerical integrations

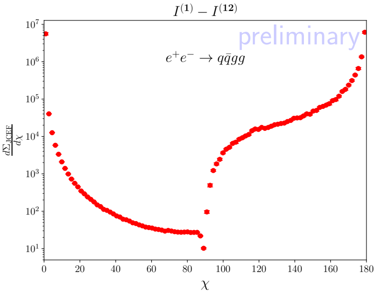
One-loop part:





Integrated subterms:





Summary

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Summary

- First numerical implementation of LASS in the making
- Numerically proved subterms regularize kinematic singularities
- Framework is created to generate subtractions automatically to arbitrary processes
- Applied to three-jet production in $e^+ e^-$
- Cross section contributions saturate in the $n+2$ and $n+1$ parton contributions
- Histograms can already be produced for the $n+1$ parton contribution

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Thank you for your attention!