Antenna Subtraction beyond NNLO

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IP3

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HP2 2024, Turin

11/09/2024

THE ROYAL SOCIETY Newton International Fellowship

ongoing work with **Xuan Chen**, **Petr Jakubčík** and **Giovanni Stagnitto**

Outline

- Introduction: N^3LO antenna functions;
- Phase-space sectors for antenna mapping;
- *Lite* antenna subtraction at N^3LO (RRR);
- Summary and conclusions;

INTRODUCTION

Antenna subtraction is a well established method for NNLO calculations involving jets.

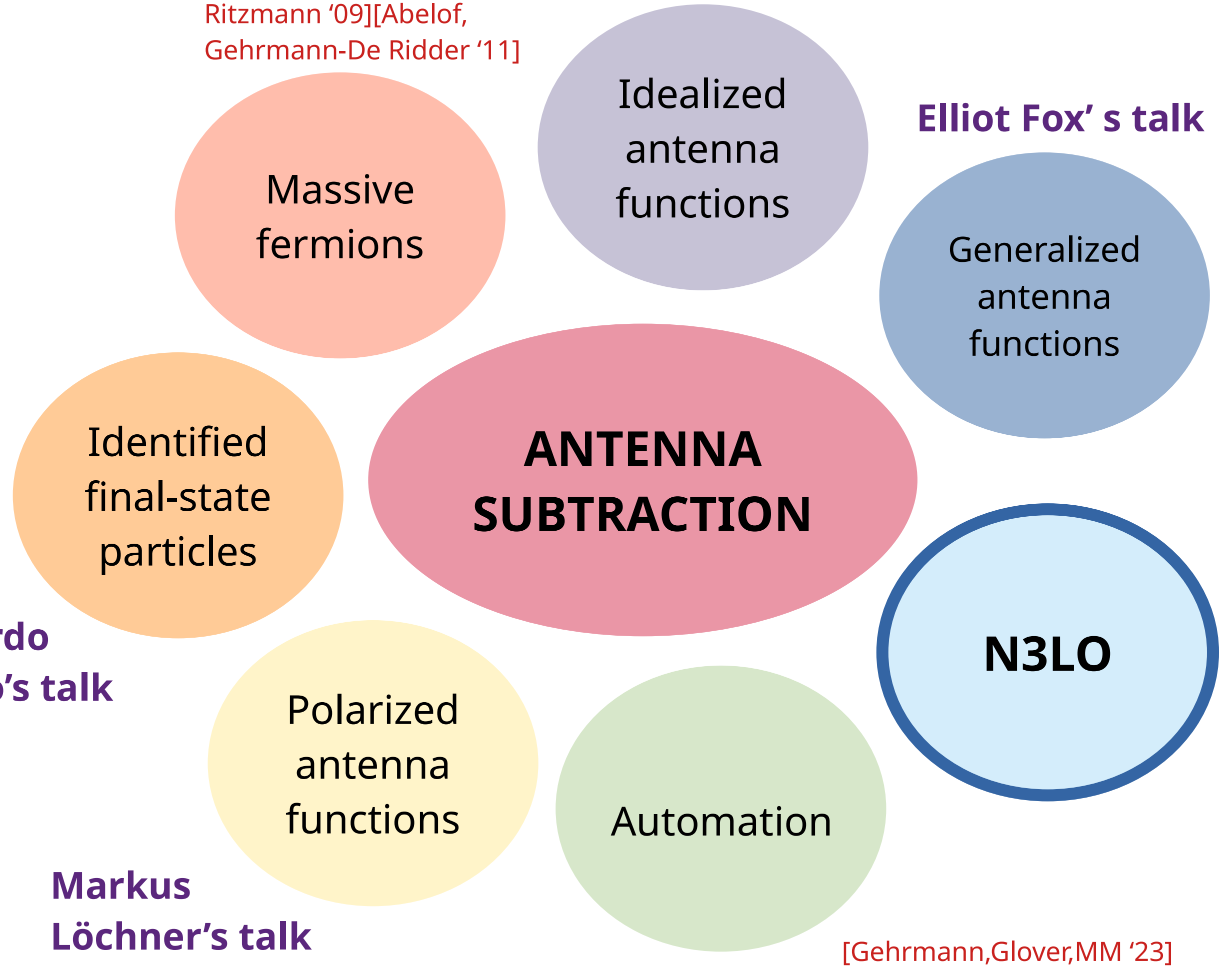
[Gehrmann,Gehrmann-De Ridder,Glover '05]
[Currie,Glover,Wells '13]

Implemented in the ***NNLOJET*** Monte Carlo framework:

- ee→jj, ee→jjj
- ep→j, ep→jj
- pp→jj
- pp→V, pp→V+j
- pp→H, pp→H+j
- pp→ɣ+j
- pp→ɣɣ
- VBFH
- …

[Gehrmann-De Ridder, Ritzmann ‘09][Abelof, Gehrmann-De Ridder ‘11]

[Braun-White,Glover,Preuss ‘23]
[Fox,Glover ‘23]



Elliot Fox’ s talk

[Gehrmann,Stagnitto ‘22]
[Bonino,Gehrmann,MM, Schürmann,Stagnitto ‘24]

Leonardo Bonino’s talk

Markus Löchner’s talk

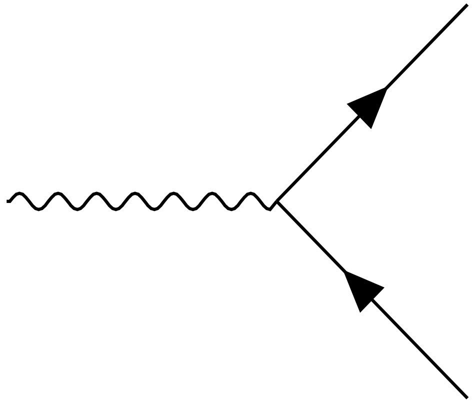
[Gehrmann,Glover,MM ‘23]
[Chen,Gehrmann,Glover,Huss,MM ‘22]

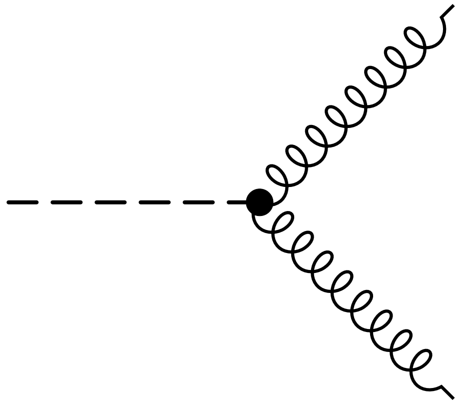
FF antenna functions: colour-singlet decay MEs

Photon decay: $\gamma^* \to q\bar{q}$

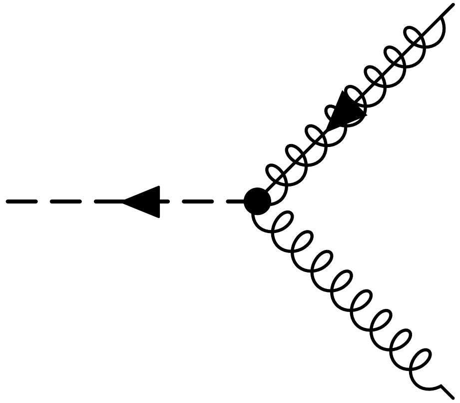
Higgs decay: $H \to gg$

Neutralino decay: $\chi \to \tilde{g}g$





$$\mathcal{L} = -\frac{\lambda}{4} H F^{a,\mu\nu} F^{a}_{\mu\nu}$$



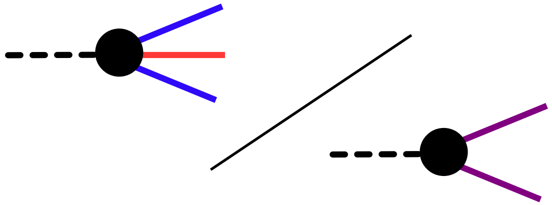
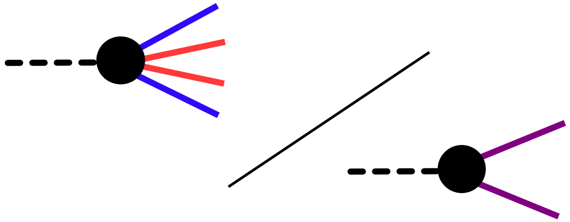
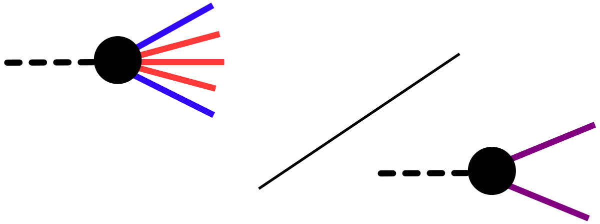
$$\mathcal{L} = i\eta \bar{\psi}^{a}_{\tilde{g}} \sigma^{\mu\nu} \psi_{\tilde{\chi}} F^{a}_{\mu\nu} + \text{h.c.}$$

Quark-Antiquark
antenna functions

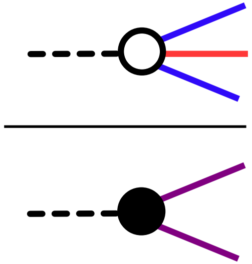
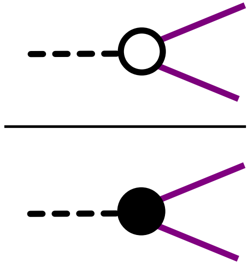
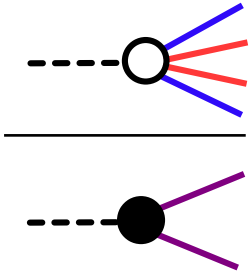
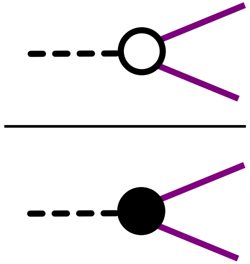
Gluon-Gluon
antenna functions

Quark-Gluon
antenna functions

Tree-level antenna functions

		Antenna	Integrated antenna
NLO:		$X_3^0 = \frac{M_3^0}{M_2^0}$	$\mathcal{X}_3^0 \propto \int \mathrm{d}\Phi_3 \, X_3^0$
NNLO:		$X_4^0 = \frac{M_4^0}{M_2^0}$	$\mathcal{X}_4^0 \propto \int \mathrm{d}\Phi_4 \, X_4^0$
N³LO:		$X_5^0 = \frac{M_5^0}{M_2^0}$	$\mathcal{X}_5^0 \propto \int \mathrm{d}\Phi_5 \, X_5^0$

One-loop antenna functions

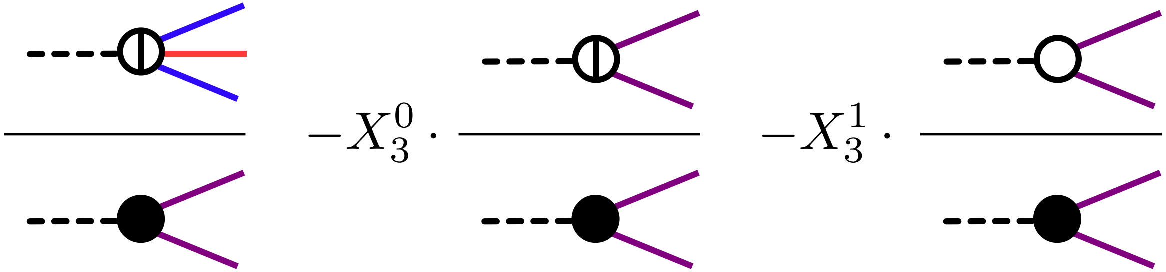
	Antenna	Integrated antenna
NLO: ❌		
NNLO:  $-X_3^0 \cdot$ 	$X_3^1 = \frac{M_3^1}{M_2^0} - X_3^0 \frac{M_2^1}{M_2^0}$	$\mathcal{X}_3^1 \propto \int \mathrm{d}\Phi_3 \, X_3^1$
N³LO:  $-X_4^0 \cdot$ 	$X_4^1 = \frac{M_4^1}{M_2^0} - X_4^0 \frac{M_2^1}{M_2^0}$	$\mathcal{X}_4^1 \propto \int \mathrm{d}\Phi_4 \, X_4^1$

Two-loop antenna functions

NLO: ❌

NNLO: ❌

N³LO:



Antenna $$X_3^2 = \frac{M_3^2}{M_2^0} - X_3^0 \frac{M_2^2}{M_2^0} - X_3^1 \frac{M_2^1}{M_2^0}$$

Integrated antenna $$\mathcal{X}_3^2 = \int \mathrm{d}\Phi_3 X_3^2$$

Analytic integration

Integration of **renormalized matrix elements** for colour-singlet decay over the **fully inclusive phase space**:

$$\int \mathrm{d}\Phi_5 M_5^0, \quad \int \mathrm{d}\Phi_4 M_4^1, \quad \int \mathrm{d}\Phi_3 M_3^2, \quad \int \mathrm{d}\Phi_2 M_2^3$$

[Jakubcik,MM,Stagnitto '22]
[Chen,Jakubcik,MM,Stagnitto '23]

Two-parton three-loop, for validation:

$$\int \mathrm{d}\Phi_5 M_5^0 + \int \mathrm{d}\Phi_4 M_4^1 + \int \mathrm{d}\Phi_3 M_3^2 + \int \mathrm{d}\Phi_2 M_2^3 =$$ finite N³LO inclusive XS

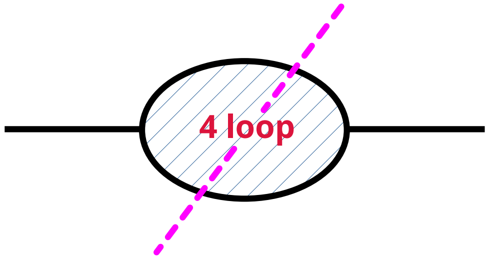
Master integrals from

[Gituliar,Magerya,Pikelner '18]
[Magerya,Pikelner '19]

Reverse unitarity: $$2\pi i \delta^+(p^2) \to \frac{1}{p^2 - i0} - \frac{1}{p^2 + i0}$$

[Cutkosky '60]
[Anastasiou, Melnikov '02,'03]

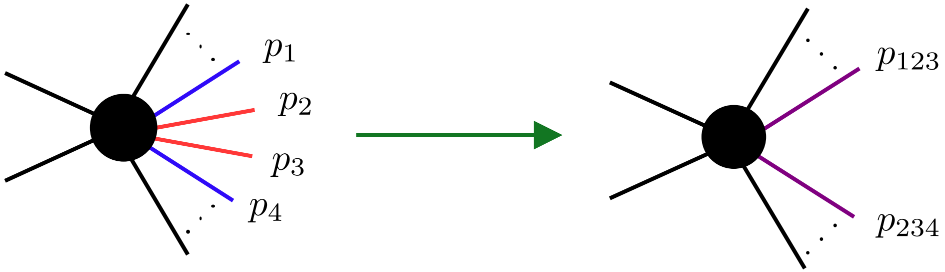
- Phase space and (genuine) loop integrals addressed simultaneously;
- **Systematic treatment of all four layers** within a common framework;



ANTENNA MAPPING AT N^{3}LO

work with Xuan Chen

Antenna mapping at NNLO



- momentum conservation;
- on-shellness conditions;
- IR limits;

2 unresolved partons: 4 → 2 mapping

$$\{p_1^h, p_2, p_3, p_4^h\} \to \{p_{123}^h, p_{234}^h\}$$

$$p_{123} = xp_1 + r_1 p_2 + r_2 p_3 + z p_4$$

$$p_{234} = (1-x)p_1 + (1-r_1)p_2 + (1-r_2)p_3 + (1-z)p_4$$

The mapping assumes the **(1-2-3-4) ordering**:

✅ colour-ordered emissions;

❌ colour-unordered (photon-like) emissions;

$$r_1 = \frac{s_{23}+s_{24}}{s_{12}+s_{23}+s_{24}}$$

$$r_2 = \frac{s_{34}}{s_{13}+s_{23}+s_{34}}$$

$$x = \frac{1}{2(s_{12}+s_{13}+s_{14})}\Big[(1+\rho)s_{1234} - r_1(s_{23}+2s_{24}) - r_2(s_{23}+s_{34}) + (r_1-r_2)\frac{s_{12}s_{34}-s_{13}s_{24}}{s_{14}}\Big]$$

$$z = \frac{1}{2(s_{14}+s_{24}+s_{34})}\Big[(1-\rho)s_{1234} - r_1(s_{23}+2s_{12}) - r_2(s_{23}+s_{13}) - (r_1-r_2)\frac{s_{12}s_{34}-s_{13}s_{24}}{s_{14}}\Big]$$

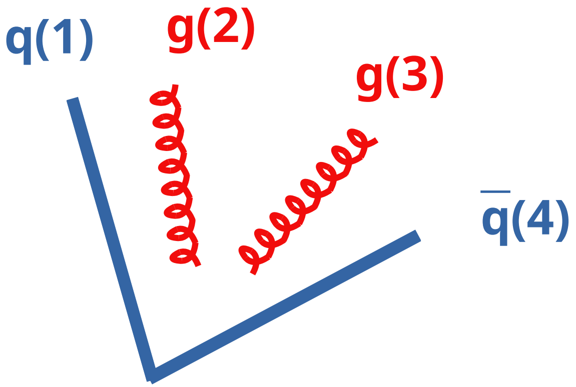
$$\rho^2 = 1 + \frac{(r_1-r_2)^2}{s_{14}^2 s_{1234}^2}\lambda(s_{12}s_{34}, s_{14}s_{23}, s_{13}s_{24}) + \frac{1}{s_{14}s_{1234}}\Big[2\big(r_1(1-r_2)+r_2(1-r_1)\big)(s_{12}s_{34}+s_{13}s_{24}-s_{23}s_{14}) + 4r_1(1-r_1)s_{12}s_{24} + 4r_2(1-r_2)s_{13}s_{34}\Big]$$

$$\lambda(u,v,w) = u^2+v^2+w^2-2(uv+uw+vw).$$

[Kosower '02]

Antenna mapping at NNLO

Prototype antenna function: $A_4^0\left(q, g, g, \bar{q}\right)$

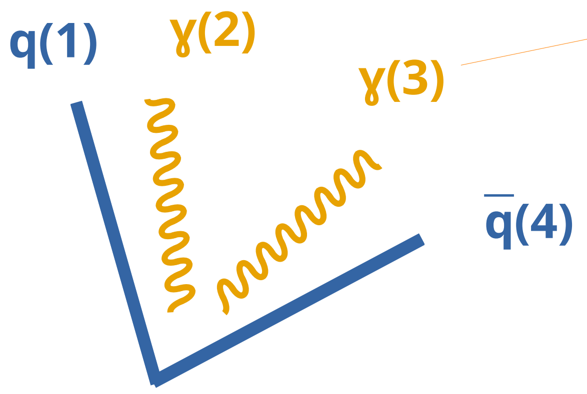


Colour-ordered: **does not** contain the **1//3** and **2//4** collinear limits.

Ordered mapping works fine.

Antenna mapping at NNLO

Prototype antenna function: $\widetilde{A}_4^0\left(q,\gamma,\gamma,\bar{q}\right)$



Colour-unordered: **does** contain the **1//3** and **2//4** collinear limits.

Ordered mapping does not work!

Traditional solution: **partial fractioning**:

$$\widetilde{A}_4^0\left(q,\gamma,\gamma,\bar{q}\right) = \widetilde{a}_4^0\left(q,\gamma_2,\gamma_3,\bar{q}\right) + \widetilde{a}_4^0\left(q,\gamma_3,\gamma_2,\bar{q}\right)$$

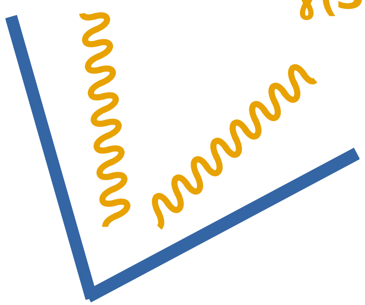
1//2, 3//4 $\quad\left\{p_1^h,p_2,p_3,p_4^h\right\}$

1//3, 2//4 $\quad\left\{p_1^h,p_3,p_2,p_4^h\right\}$

$$\frac{1}{s_{12}s_{13}} = \frac{1}{s_{12}\left(s_{12}+s_{13}\right)} + \frac{1}{s_{13}\left(s_{12}+s_{13}\right)}$$

- harder to integrate (typically not done);
- split soft limits: potential numerical instabilities;
- poor scaling with number of unresolved emissions;

Phase-space sectors for NNLO mapping



Sector solution: keep the **full antenna** but split the phase space into **two sectors** isolating the problematic limits.

The appropriate **colour-ordered mapping** is then used in each sector.

$$\mathcal{S}_a : \; s_{12}s_{34} \leq s_{13}s_{24}$$

contains only **1//2** and **3//4**
1-2-3-4 mapping is used

$$\mathcal{S}_b : \; s_{13}s_{24} < s_{12}s_{34}$$

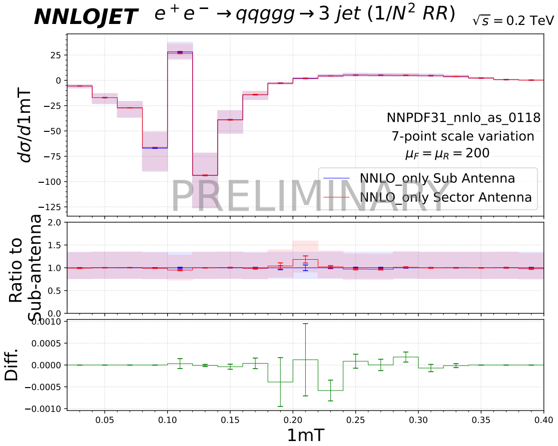
contains only **1//3** and 2**//4**
1-3-2-4 mapping is used

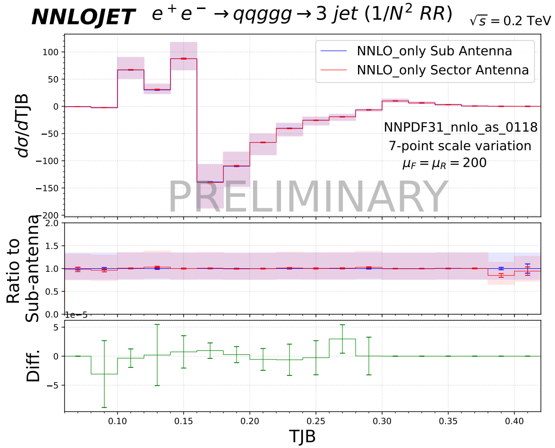
- used for $\widetilde{A}_4^0$, $\widetilde{D}_4^0(D_{4,c}^0)$, $\widetilde{F}_4^0\,(F_{4,b}^0)$;
- **avoid partial fractioning**;
- **easily generalizable beyond NNLO**;

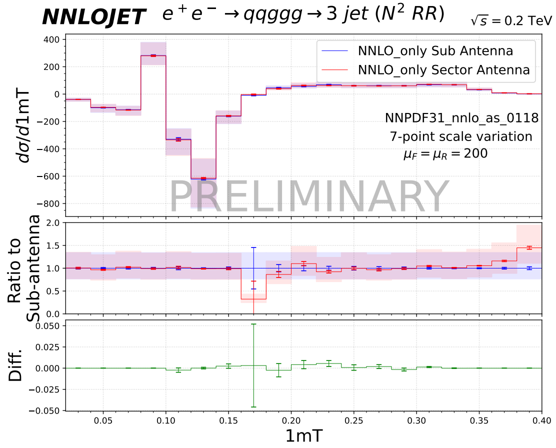
Phase-space sectors for NNLO mapping

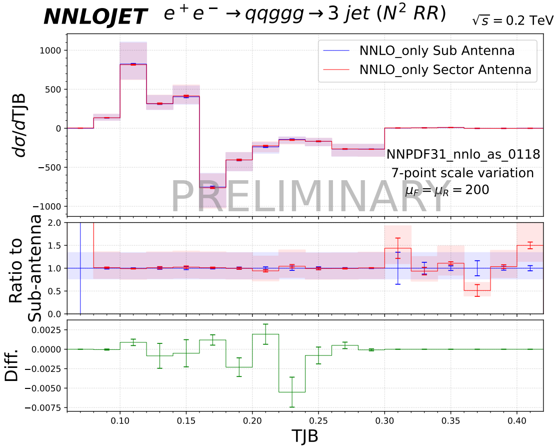
We compared the RR contribution to event-shapes in $e^+e^- \to jjj$

- with partial-fractioned **sub-antennae**;
- with phase-space **sectors** for antenna mapping;

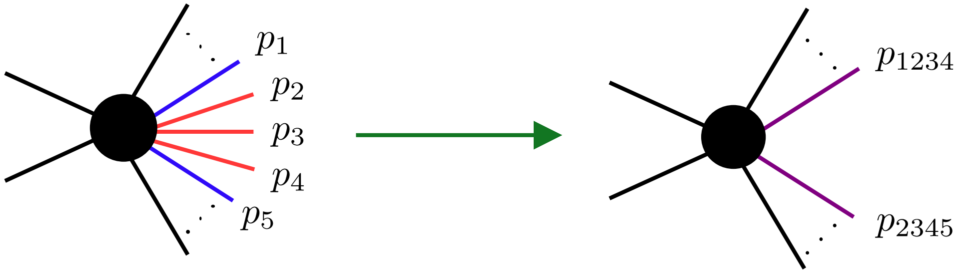








Antenna mapping at N³LO



3 unresolved partons: 5 → 2 mapping

$$\{p_1^h, p_2, p_3, p_4, p_5^h\} \to \{p_{1234}^h, p_{2345}^h\}$$

$$p_{1234} = xp_1 + r_1 p_2 + r_2 p_3 + r_3 p_4 + z p_5$$

$$p_{2345} = (1-x)p_1 + (1-r_1)p_2 + (1-r_2)p_3 + (1-r_3)p_4 + (1-z)p_5$$

The mapping assumes the **(1-2-3-4-5) ordering**:

✅ colour-ordered emissions;

❌ colour-unordered (photon-like) emissions;

$$
\begin{aligned}
r_1 &= \frac{s_{23}+s_{24}+s_{25}}{s_{12}+s_{23}+s_{24}+s_{25}} \\
r_2 &= \frac{s_{34}+s_{35}}{s_{13}+s_{23}+s_{34}+s_{35}} \\
r_3 &= \frac{s_{45}}{s_{14}+s_{24}+s_{34}+s_{45}}
\end{aligned}
$$

$$
\begin{aligned}
x = \frac{1}{2s_{15}(s_{12}+s_{13}+s_{14}+s_{15})} \Big[& r_3 s_{14}(s_{25}+s_{35}) - r_3 s_{45}(s_{12}+s_{13}) + s_{23}s_{15} \\
& + s_{15}(s_{24} - r_3 s_{24} + s_{12} + s_{25} + s_{34} + s_{13} + s_{35} + s_{14} + s_{45} \\
& + \rho(s_{23}+s_{24}+s_{12}+s_{25}+s_{34}+s_{13}+s_{35}+s_{14}+s_{45}) - r_3(s_{34}+2s_{45})) \\
& + (1+\rho)s_{15}^2 + r_2(- s_{35}(s_{12}+s_{14}) + s_{13}(s_{25}+s_{45}) - (s_{23}+s_{34}+2s_{35})s_{15}) \\
& - r_1(- s_{12}(s_{35}+s_{45}) + (s_{23}+s_{24})s_{15} + s_{25}(s_{13}+s_{14}+2s_{15})) \Big]
\end{aligned}
$$

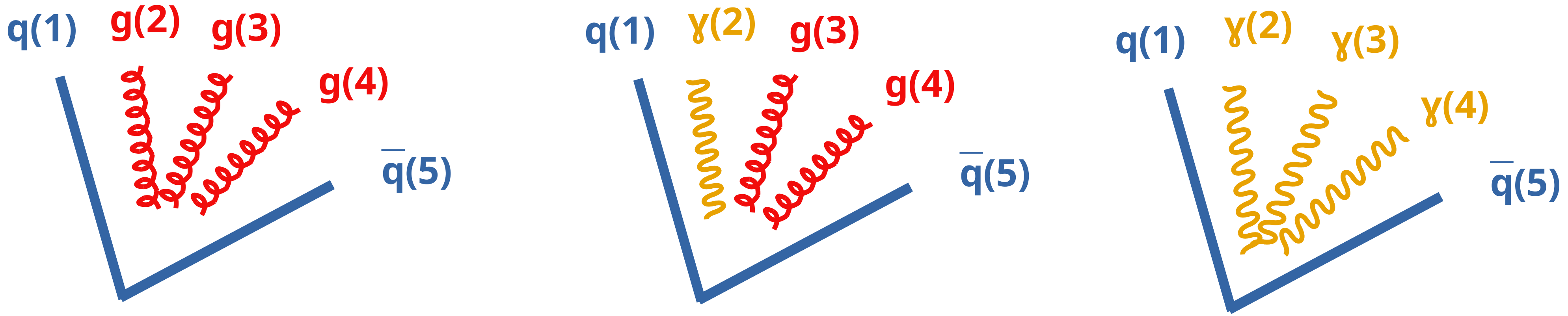
$$
\begin{aligned}
z = \frac{1}{2s_{15}(s_{25}+s_{35}+s_{45}+s_{15})} \Big[& - r_3 s_{14}(s_{25}+s_{35}) + r_3 s_{45}(s_{12}+s_{13}) + s_{23}s_{15} \\
& + s_{15}(s_{24} - r_3 s_{24} + s_{12} + s_{25} + s_{34} + s_{13} + s_{35} + s_{14} + s_{45} \\
& - \rho(s_{23}+s_{24}+s_{12}+s_{25}+s_{34}+s_{13}+s_{35}+s_{14}+s_{45}) - r_3(s_{34}+2s_{14})) \\
& + (1-\rho)s_{15}^2 + r_1(s_{25}(s_{13}+s_{14}) - s_{12}(s_{35}+s_{45}) - (s_{23}+s_{24}+2s_{12})s_{15}) \\
& - r_2(- s_{35}(s_{12}+s_{14}) + (s_{23}+s_{34})s_{15} + s_{13}(s_{25}+s_{45}+2s_{15})) \Big]
\end{aligned}
$$

$$
\begin{aligned}
\rho^2 = 1 + \frac{2}{s_{15}s_{12345}} \Big[& - 2r_1^2 s_{12}s_{25} - 2r_2^2 s_{13}s_{35} + r_1 \{ s_{25}(s_{13} - 2r_2 s_{13} + s_{14} - 2r_3 s_{14}) \\
& + s_{12}(2s_{25} + s_{35} - 2r_2 s_{35} + s_{45} - 2r_3 s_{45}) + s_{15}(-s_{23} + 2r_2 s_{23} - s_{24} + 2r_3 s_{24}) \} \\
& + r_2 \{ s_{25}s_{13} + s_{12}s_{35} + s_{35}s_{14} - 2r_3 s_{35}s_{14} + s_{13}(2s_{35} + s_{45} - 2r_3 s_{45}) + 2r_3 s_{34}s_{15} \\
& - (s_{23}+s_{34})s_{15} \} + r_3 \{ (s_{12}+s_{13})s_{45} + s_{14}(s_{25}+s_{35} - 2(-1+r_3)s_{45}) \\
& - (s_{24}+s_{34})s_{15} \} \Big] + \frac{1}{s_{15}^2 s_{12345}^2} \Big[r_1^2 \{ (s_{25}(s_{13}+s_{14}) - s_{12}(s_{35}+s_{45}))^2 \\
& - 2s_{15}(- 2s_{12}s_{25}s_{34} + (s_{23}+s_{24})s_{25}(s_{13}+s_{14}) + (s_{23}+s_{24})s_{12}(s_{35}+s_{45})) \\
& + (s_{23}+s_{24})^2 s_{15}^2 \} + r_2^2 \{ (s_{35}(s_{12}+s_{14}) - s_{13}(s_{25}+s_{45}))^2 \\
& - 2s_{15}(s_{25}s_{34}s_{13} + s_{12}s_{34}s_{35} - 2s_{24}s_{13}s_{35} + s_{34}s_{35}s_{14} + s_{23}s_{35}(s_{12}+s_{14}) \\
& + s_{34}s_{13}s_{45} + s_{23}s_{13}(s_{25}+s_{45})) + (s_{23}+s_{34})^2 s_{15}^2 \} \\
& + r_3^2 \{ ((s_{25}+s_{35})s_{14} - (s_{12}+s_{13})s_{45})^2 - 2s_{15}((s_{24}+s_{34})(s_{25}+s_{35})s_{14} \\
& + (s_{24}+s_{34})(s_{12}+s_{13})s_{45} - 2s_{23}s_{14}s_{45}) + (s_{24}+s_{34})^2 s_{15}^2 \} \\
& - 2r_1 \{ (r_3(s_{25}+s_{35})s_{14} - r_2 s_{35}(s_{12}+s_{14}) - r_3(s_{12}+s_{13})s_{45} + r_2 s_{13}(s_{25}+s_{45})) \\
& \times (s_{25}(s_{13}+s_{14}) - s_{12}(s_{35}+s_{45})) + s_{15}(2r_2 s_{12}s_{25}s_{34} + 2r_3 s_{12}s_{25}s_{34} \\
& + r_3 s_{25}(-s_{24}+s_{34})s_{13} - r_2 s_{25}(2s_{23}+s_{24}+s_{34})s_{13} + r_2 s_{25}(-s_{23}+s_{34})s_{14} \\
& - r_3 s_{25}(s_{23}+2s_{24}+s_{34})s_{14} - r_2 s_{35}(s_{12}s_{34} + s_{24}(s_{12} - 2s_{13} - s_{14}) + s_{23}(2s_{12}+s_{14})) \\
& + r_3 s_{35}(s_{12}s_{34} + s_{23}s_{14} - s_{24}(s_{12} + 2s_{13} + s_{14})) \\
& - r_3(s_{12}s_{34} + s_{24}(2s_{12}+s_{13}) + s_{23}(s_{12} - s_{13} - 2s_{14})) s_{45} \\
& + r_2(s_{12}s_{34} + s_{24}s_{13} - s_{23}(s_{12}+s_{13}+2s_{14}))s_{45}) \\
& + s_{15}^2 ((s_{23}+s_{24})(r_2 s_{23} + r_3 s_{24}) + (r_2 - r_3)(s_{23}-s_{24})s_{34}) \} \\
& + 2r_2 r_3 \{ s_{25}^2 s_{13}s_{14} + s_{12}^2 s_{35}s_{45} - s_{12}(s_{35}-s_{45})(s_{35}s_{14} - s_{13}s_{45}) - (s_{35}s_{14} - s_{13}s_{45})^2 \\
& + s_{15}(s_{24}s_{35}s_{14} + (s_{23}+2s_{34})s_{35}s_{14} + 2s_{34}s_{13}s_{45} + s_{23}(s_{13} - 2s_{14})s_{45} \\
& + s_{24}s_{13}(-2s_{35}+s_{45})) + s_{12}(- s_{24}s_{35} - s_{23}s_{45} + s_{34}(s_{35}+s_{45}))s_{15} \\
& + (s_{23}(s_{24}-s_{34}) - s_{34}(s_{24}+s_{34}))s_{15}^2 + s_{25}(- s_{35}s_{14}(s_{12} - s_{13} + s_{14}) \\
& - s_{13}(s_{12}+s_{13}-s_{14})s_{45} + (-s_{24}s_{13} - s_{23}s_{14} + s_{34}(2s_{12}+s_{13}+s_{14}))s_{15}) \} \Big].
\end{aligned}
$$

[Kosower '02]

Antenna mapping at N³LO

Prototype antenna functions: $A_5^0\,(q,g,g,g,\bar{q})\,,\;\widetilde{A}_5^0\,(q,\gamma,g,g,\bar{q})\,,\;\tilde{\tilde{A}}_5^0\,(q,\gamma,\gamma,\gamma,\bar{q})$

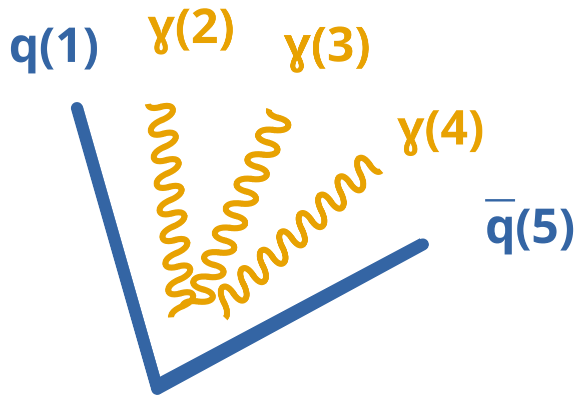


Colour-ordered mapping works fine

Colour-ordered mapping does not work

Phase-space sectors for N^{3}LO mapping

Prototype antenna functions: $\tilde{\tilde{A}}_5^0\,(q,\gamma,\gamma,\gamma,\bar{q})$



12 different sectors
(6 different mappings)

1) find

$$s_{\min} = \min_{\substack{i\in\text{unresolved},\\ h\in\text{hard}}} \{\, s_{ih} \,\}$$

2) put photon **i** adjacent to **h** and apply **NNLO-like sectors** to the remaining two photons;

Example:

$$s_{\min} = s_{12}$$

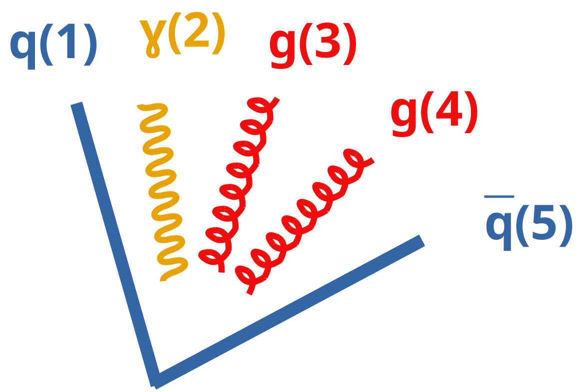
$$\{p_1^h,\, p_2,\, \ldots\, , p_5^h\}$$

$$s_{13}s_{45} < s_{14}s_{35}$$

$$\{p_1^h,\, p_2,\, p_3,\, p_4,\, p_5^h\}$$

Phase-space sectors for N³LO mapping

Prototype antenna functions: $\tilde{A}_5^0\,(q,\gamma,g,g,\bar{q})$



find $s_{\min} = \min_{ij}\{\,s_{ij}\,\}$

$s_{\min} < s_{34}$

$s_{\min} = s_{2h}$

(e.g. h=1)

$\{p_1^h,\, p_2,\, p_3,\, p_4,\, p_5^h\}$

$s_{\min} \neq s_{2h}$

$\{p_1^h,\, p_3,\, p_2,\, p_4,\, p_5^h\}$

$s_{\min} = s_{34}$

apply NNLO-like sectors with

$s_{1(3+4)}s_{25},\;\; s_{12}s_{(3+4)5}$

e.g. $s_{1(3+4)}s_{25} < s_{12}s_{(3+4)5}$

$\{p_1^h,\, p_3,\, p_4,\, p_2,\, p_5^h\}$

12 different sectors
(3 different mappings)

N^{3}LO SUBTRACTION FOR $e^+e^- \to jj$

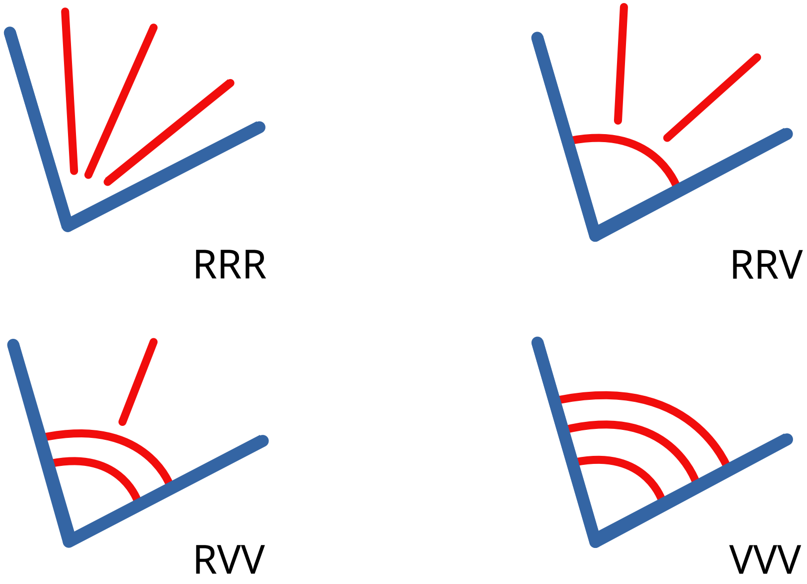
Motivation: you have to start somewhere

Simplifications:

- only **q-$\bar{\text{q}}$** N^{3}LO antenna functions;
- only **dipole-like correlations** at N3LO (two hard legs);

Goals:

- definition of **N3LO antenna functions**;
- removal of double- and single-unresolved limits;



Two-jet production rate computed at N^{3}LO in [Gerhrmann De-Ridder,Gehrmann,Glover,Heinrich '08]
Interesting to compute: **forward-backward asymmetry**, sensitive to the weak mixing angle

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

$$\sigma_F = \int_0^1 \mathrm{d}\cos\theta \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}$$

$$\sigma_B = \int_{-1}^0 \mathrm{d}\cos\theta \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}$$

angle between beam and **flavoured jet** axis

NNLO study in [Altarelli,Lampe '93] [Ravindran,van Nerveen '98] [Catani,Seymour '98] [Weinzierl '06]

M^0_{n+3} triple-real matrix element

M_{n+3}^0 triple-real matrix element

$- \sum X_3^0 \cdot M_{n+2}^0$ removes **SU** of

NLO

M_{n+3}^0 triple-real matrix element

$-\sum X_3^0 \cdot M_{n+2}^0$ removes **SU** of

NLO

$-\sum X_4^0 \cdot M_{n+1}^0$ removes **DU** of

M^0_{n+3} triple-real matrix element

$-\sum X^0_3 \cdot M^0_{n+2}$ removes **SU** of

$-\sum X^0_4 \cdot M^0_{n+1}$ removes **DU** of

has **SU** limits $\sum X^0_3 \cdot M^0_{n+1}$

has **SU** limits $\sum X^0_3 \cdot X^0_3$

$$M_{n+3}^{0}$$ triple-real matrix element

$$-\sum X_3^0 \cdot M_{n+2}^{0}$$ removes **SU** of

NLO

$$-\sum X_4^0 \cdot M_{n+1}^{0}$$ removes **DU** of

$$+\sum X_3^0 \cdot X_3^0 \cdot M_{n+1}^{0}$$ fixes

NNLO

M^0_{n+3} triple-real matrix element

$-\sum X^0_3 \cdot M^0_{n+2}$ removes **SU** of

NLO

$-\sum X^0_4 \cdot M^0_{n+1}$ removes **DU** of

$+\sum X^0_3 \cdot X^0_3 \cdot M^0_{n+1}$ fixes

NNLO

$-\sum X^0_5 \cdot M^0_{n}$ removes **TU** of

M_{n+3}^0 triple-real matrix element

$-\sum X_3^0 \cdot M_{n+2}^0$ removes **SU** of

$-\sum X_4^0 \cdot M_{n+1}^0$ removes **DU** of

$+\sum X_3^0 \cdot X_3^0 \cdot M_{n+1}^0$ fixes

$-\sum X_5^0 \cdot M_n^0$ removes **TU** of

has **DU** limits $\sum X_4^0 \cdot M_n^0$

has **SU** limits $\sum X_3^0 \cdot M_n^0$

has **SU** limits $\sum X_3^0 \cdot X_4^0$

and

has **DU** limits $\sum X_4^0 \cdot X_3^0$

$$M_{n+3}^0$$ triple-real matrix element

$$-\sum X_3^0 \cdot M_{n+2}^0$$ removes **SU** of

NLO

$$-\sum X_4^0 \cdot M_{n+1}^0$$ removes **DU** of

$$+\sum X_3^0 \cdot X_3^0 \cdot M_{n+1}^0$$ fixes

NNLO

$$-\sum X_5^0 \cdot M_n^0$$ removes **TU** of

$$+\sum X_3^0 \cdot X_4^0 \cdot M_n^0$$

$$+\sum X_4^0 \cdot X_3^0 \cdot M_n^0$$

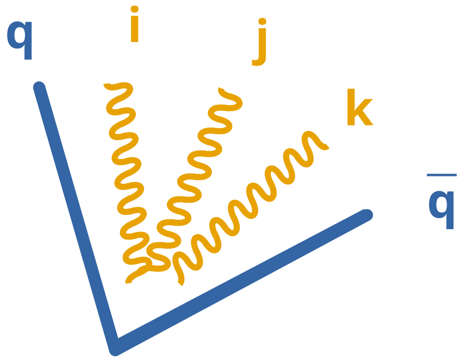
$$-\sum X_3^0 \cdot X_3^0 \cdot X_3^0 \cdot M_n^0$$

fixes

N^{3}LO

RRR subtraction term for abelian case

$$
\begin{aligned}
& M_{n+3}(q,i,j,k,\bar{q})J_5^2 \\
- & \sum_{(i,j,k)\in Z_3} A_3^0(q,i,\bar{q})M_{n+2}(\{q,i\},j,k,\{\bar{q},i\})J_4^2 \\
- & \sum_{(i,j,k)\in Z_3} \tilde{A}_4^0(q,i,j,\bar{q})M_{n+1}(\{q,i,j\},k,\{\bar{q},i,j\})J_3^2 \\
+ & \sum_{(i,j,k)\in S_3} A_3^0(q,i,\bar{q})A_3^0(\{q,i\},j,\{\bar{q},i\})M_{n+1}(\{\{q,i\},j\},k,\{\{\bar{q},i\},j\})J_2^2 \\
- & \tilde{\tilde{A}}_5^0(q,i,j,k,\bar{q})M_n(\{q,i,j,k\},\{\bar{q},i,j,k\})J_2^2 \\
+ & \sum_{(i,j,k)\in Z_3} \tilde{A}_4^0(q,i,j,\bar{q})A_3^0(\{q,i,j\},k,\{\bar{q},i,j\})M_n(\{\{q,i,j\},k\},\{\{\bar{q},i,j\},k\})J_2^2 \\
+ & \sum_{(i,j,k)\in Z_3} A_3^0(q,i,\bar{q})\tilde{A}_4^0(\{q,i\},j,k,\{\bar{q},i\})M_n(\{\{q,i\},j,k\},\{\{\bar{q},i\},j,k\})J_2^2 \\
- & \sum_{(i,j,k)\in S_3} A_3^0(q,i,\bar{q})A_3^0(\{q,i\},j,\{\bar{q},i\})A_3^0(\{\{q,i\},j\},k,\{\{\bar{q},i\},j\})M_n(\{\{\{q,i\},j\},k\},\{\{\{\bar{q},i\},j\},k\})J_2^2
\end{aligned}
$$



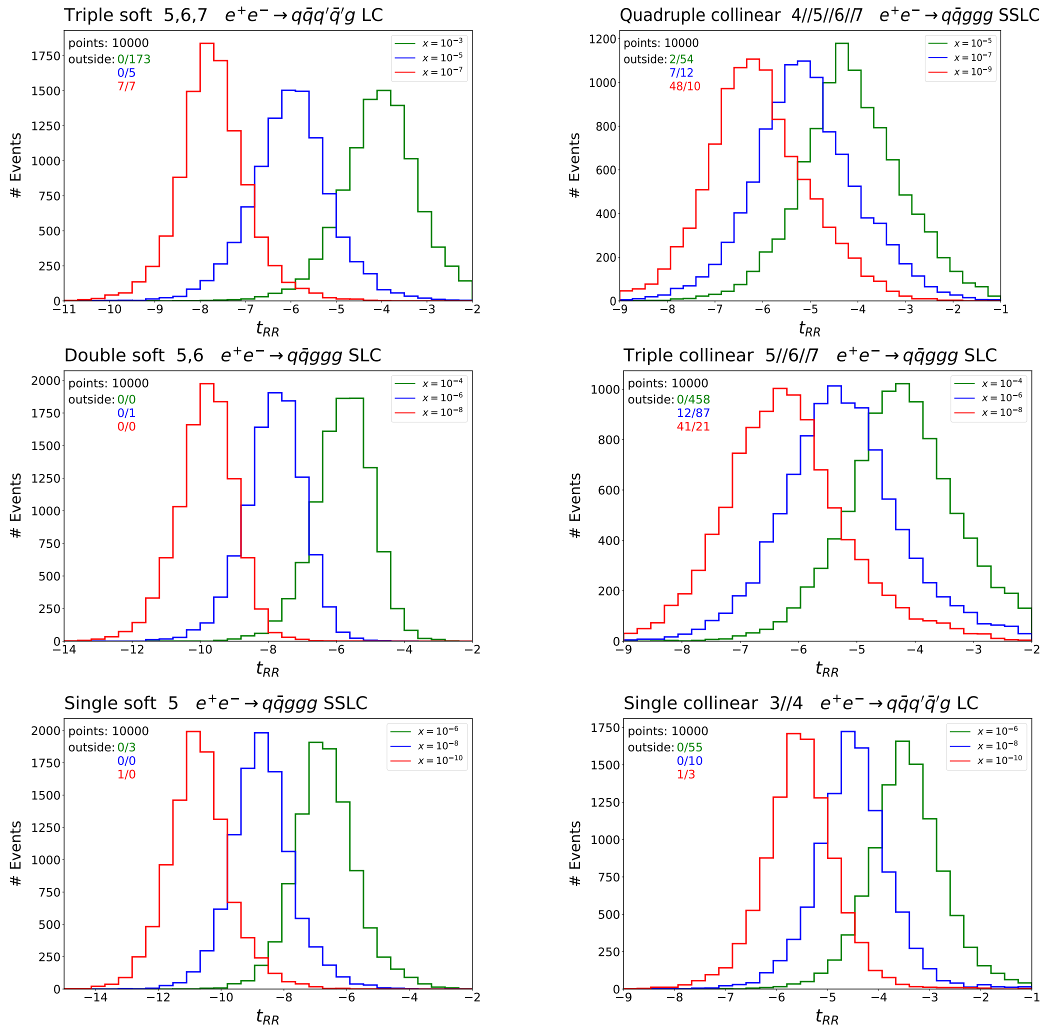
J_n^m

jet algorithm:
selects **m** jets
from **n** momenta

Numerical tests

Fully working subtraction terms for all RRR partonic channels:

- two quarks:
  - $e^+e^- \to q\,\bar{q}\,g\,g\,g$ LC
  - $e^+e^- \to q\,\bar{q}\,g\,g\,g$ SLC
  - $e^+e^- \to q\,\bar{q}\,g\,g\,g$ SSLC
- four quarks, different flavour:
  - $e^+e^- \to q\,\bar{q}\,q'\,\bar{q}'\,g$ LC
  - $e^+e^- \to q\,\bar{q}\,q'\,\bar{q}'\,g$ SLC
- four quarks, same flavour:
  - $e^+e^- \to q\,\bar{q}\,q\,\bar{q}\,g$ LC
  - $e^+e^- \to q\,\bar{q}\,q\,\bar{q}\,g$ SLC



RRV, RVV, VVV subtraction: work in progress ...

$$-\sum X_3^0 \cdot M_{n+2}^0$$

$$-\sum X_4^0 \cdot M_{n+1}^0$$

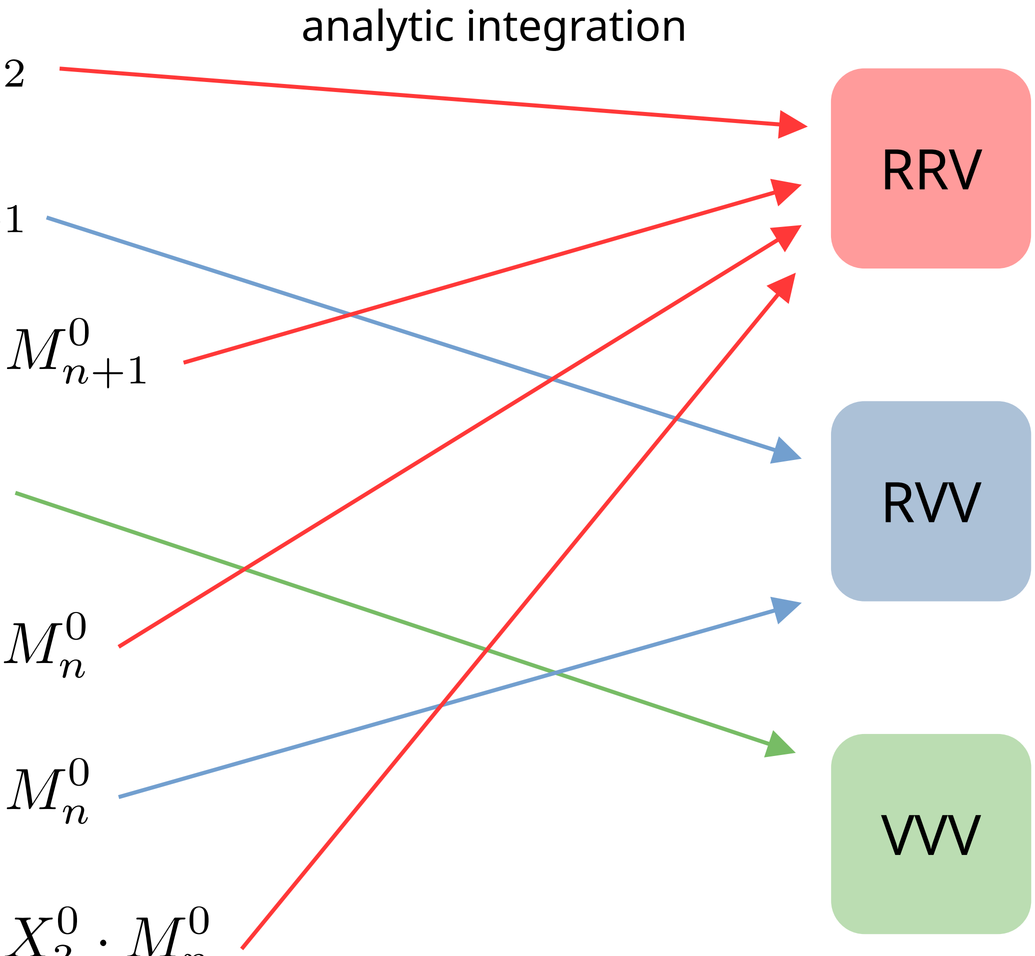
$$+\sum X_3^0 \cdot X_3^0 \cdot M_{n+1}^0$$

$$-\sum X_5^0 \cdot M_n^0$$

$$+\sum X_3^0 \cdot X_4^0 \cdot M_n^0$$

$$+\sum X_4^0 \cdot X_3^0 \cdot M_n^0$$

$$-\sum X_3^0 \cdot X_3^0 \cdot X_3^0 \cdot M_n^0$$



Problem: **numerical stability** of loop matrix elements and antenna functions:

- algebraic manipulation;
- expansions;
- quad-precision;

SUMMARY AND CONCLUSIONS

- **Antenna subtraction** for NNLO calculations is sufficienly well-established to allow for **extension at N^{3}LO**;

- The **analytical ingredients** and **numerical techniques** to apply antenna subtraction at N^{3}LO for **simple processes with no initial-state hadrons** are in place;

- We have **fully working RRR** subtracion terms for **ee→jj**. Work on RRV, RVV and VVV is in progress to finalize the first proof-of-principle application at N^{3}LO;

Thank you for your attention!

Backup Slides

Momentum mapping

- kinematics of real (R, RV, RR, ...) correction: **n**;
- kinematics of the subtraction term for **u** unresolved partons: **n-u**;

$$R\left(\{p\}_n\right)$$

real correction

$$S\left(\{p\}_n\right) = X\left(\{p\}_{(u+h)\in n}\right) B\left(\{\hat{p}\}_{n-u}\right)$$

subtraction term

unresolved factor (antenna)

h = hard radiators

reduced ME

$$\hat{p} = \hat{p}\left(\{p\}_n\right)$$

- momentum conservation;
- on-shellness conditions;
- IR limits;

factorization of the phase-space

$$\int \mathrm{d}\Phi_n\, S\left(\{p\}_n\right) = \int \mathrm{d}\Phi_X\, X\left(\{p\}_{(u+h)\in n}\right) \int \mathrm{d}\Phi_{n-u}\, B\left(\{\hat{p}\}_{n-u}\right)$$

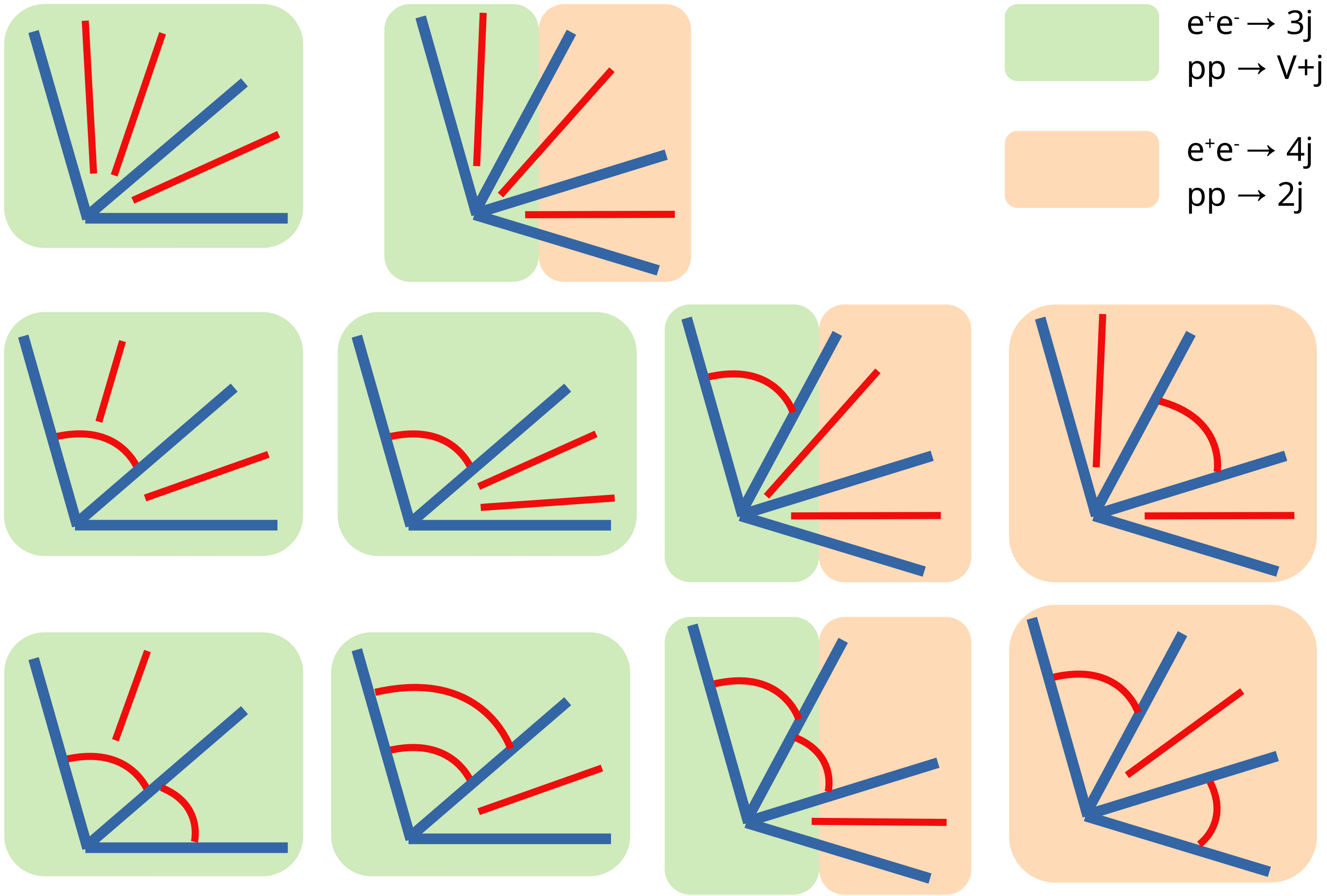
integrated analytically

Structures we will NOT be able to probe

RRR:

RRV:

RVV:



- q-g and g-g antennae needed too
- N^3LO LAST or equivalent high-multiplicty antennae

Elliot's talk