Catani's collinear factorization to all perturbative orders in QCD

arXiv: 2402.14749

High Precision for Hard Processes (HP2 2024), Turin, Italy

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13 September 2024

in collaboration with

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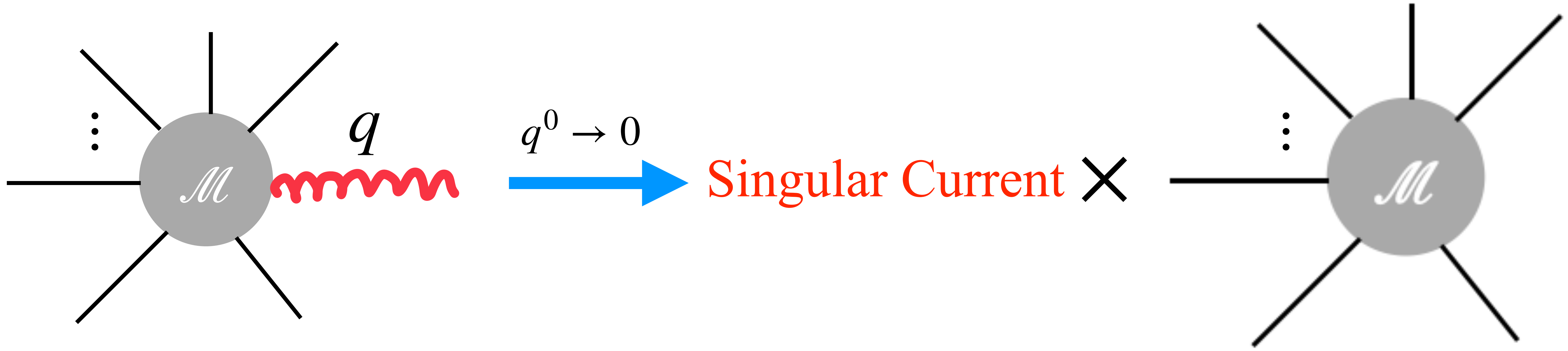
European Commission
Marie Skłodowska-Curie Actions

* Factorization in Massless QCD
    - Soft Factorization
    - Collinear Factorization
* Generalization of Collinear Factorization: Time-Like Vs Space-Like
* Catani's Generalization to All Perturbative Orders
* Squared Amplitudes & Cross Sections: Comments
* Future Outlook

Loop expansion

$$\mathscr{M}\left(p_1,\ldots,p_n\right)=\mathscr{M}^{(0)}\left(p_1,\ldots,p_n\right)+\sum_{i=1}\mathscr{M}^{(i)}\left(p_1,\ldots,p_n\right)$$

For a single **soft** gluon



quantitatively,

$$\langle a\,|\,\mathscr{M}^{(0)}(q,p_1,\ldots,p_m)\rangle = g_S\,\mu_0^{\epsilon}\,\varepsilon^{\mu}(q)\,J_{\mu}^{a\,(0)}(q)\,|\,\mathscr{M}^{(0)}(p_1,\ldots,p_m)\rangle + \cdots$$

Power suppressed contributions

where the current $$\boldsymbol{J}^{\mu\,(0)}(q)=\sum_{i=1}^{m}\boldsymbol{T}_i\,\frac{p_i^{\mu}}{p_i\cdot q}$$

$$q^{\mu}\boldsymbol{J}_{\mu}^{(0)}(q)\,|\,\mathscr{M}^{(0)}(p_1,\ldots,p_m)\rangle=\sum_{i=1}^{m}\boldsymbol{T}_i\,|\,\mathscr{M}^{(0)}(p_1,\ldots,p_m)\rangle=0$$

Leading power contribution

$$\mathscr{M}\left(\lambda s, h\right)\Big|_{\lambda\to 0} \sim \frac{1}{\lambda^m}\text{mod}\left(\ln^r \lambda\right) + \cdots$$

Set of hard particles

$$|\,\mathscr{M}(s,h)\rangle \simeq \boldsymbol{J}(s;h)\,|\,\mathscr{M}(h)\rangle$$

Set of soft particles

Reduced amplitude

Loop expansion: $\boldsymbol{J}(s;h) = \boldsymbol{J}^{(0)}(s;h) + \sum_{i=1} \boldsymbol{J}^{(i)}(s;h)$

Berends-Giele, Campbell-Glover (9710255), Catani-Grazzini (9908523), Bern-Chalmers (9503236), Bern-Del Duca-Kilgore-Schmidt (9810409, 9903516), Catani-Grazzini (0007142), Bierenbaum-Czakon-Mitov (1107.4384), Feige-Schwartz (1403.6472), Catani-Colferai-Torrini (1908.01616), Del Duca-Duhr-Haindl-Liu (2206.01584), Catani-Cieri-Colferai-Coradeschi (2210.09397), Zhu (2009.08919), Catani-Cieri (2108.13309), Czakon-Eschment-Schellenberger (2211.06465), Li-Zhu (1309.4391), Duhr-Gehrmann (1309.4393), Dixon-Herrmann-Yan-Zhu (1912.09370), Chen-Luo-Yang-Zhu (2309.03832), Herzog-Ma-Mistlberger-Suresh (2309.07874)

Sudakov parametrization

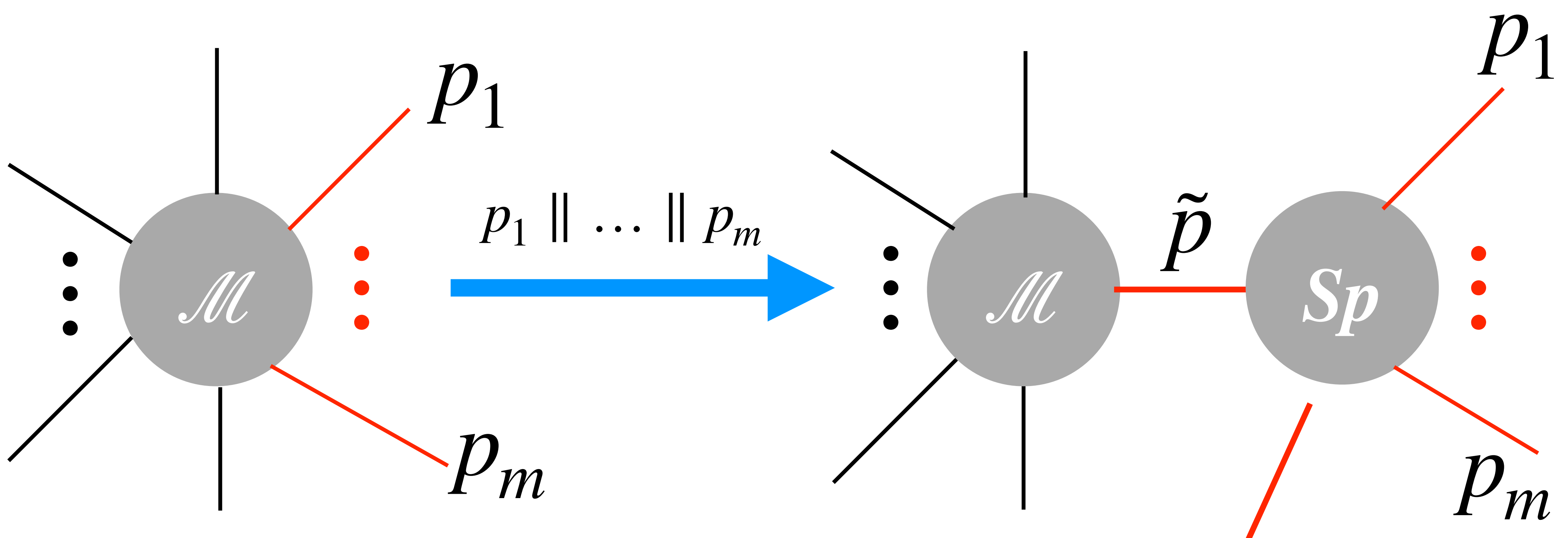
$$p_i^\mu = x_i\, \tilde{p}^\mu + k_{\perp i}^\mu - \frac{k_{\perp i}^2}{x_i}\frac{n^\mu}{2n\cdot\tilde{p}} \qquad i \in c \equiv \{1,\ldots,m\}$$

$n^2 = 0$

where the on-shell vector

$$\tilde{p}^\mu = p_{1\ldots m}^\mu - \frac{s_{1\ldots m}}{2n\cdot p_{1\ldots m}} n^\mu$$

$\tilde{p}^2 = 0$, $s_{1\ldots m} = p_{1\ldots m}^2$, $p_{1\ldots m}^\mu = \sum_{i=1}^{m} p_i^\mu$



Singular splitting amplitude

Boost invariant quantities: $z_i = \dfrac{x_i}{\sum_i x_i}$

$$\sum_i z_i = 1$$

$$\tilde{k}_{\perp i}^\mu = k_{\perp i}^\mu - z_i \sum_i k_{\perp i}^\mu$$

$$\sum_i \tilde{k}_{\perp i}^\mu = 0$$

Tree-level factorization

$$\mathscr{M}^{(0)}(p_1, \ldots, p_m, \ldots, p_n)\rangle = \boldsymbol{Sp}^{(0)}_{a_1\ldots a_m}(p_1, \ldots, p_m; \tilde{p}) \,|\, \mathscr{M}^{(0)}(\tilde{p}, p_{m+1}, \ldots, p_n)\rangle + \cdots$$

Example: $1 \to 2$ splitting

$$\boldsymbol{Sp}^{(0)}_{q_1 g_2} = \frac{g_S \mu_0^{\epsilon}}{s_{12}} \boldsymbol{t}^{c_2}_{c_1 c}\, \bar{u}(p_1) \not{\varepsilon}(p_2) u(\tilde{p}) \qquad \boldsymbol{Sp}^{(0)}_{q_1 \bar{q}_2} = \frac{g_S \mu_0^{\epsilon}}{s_{12}} \boldsymbol{t}^{c}_{c_1 c_2}\, \bar{u}(p_1) \not{\varepsilon}^{*}(\tilde{p}) v(p_2)$$

To all perturbative orders

$$|\, \mathscr{M}(c, h)\rangle \simeq \boldsymbol{Sp}(c; \tilde{p}) \,|\, \mathscr{M}(\tilde{p}, h)\rangle$$

Set of collinear particles

$$\boldsymbol{Sp}(c; \tilde{p}) = \sum_{i=0} \boldsymbol{Sp}^{(i)}(c; \tilde{p})$$

Campbell-Glover (9710255), Catani-Grazzini (9908523), Catani-Grazzini (9810389), Bern-Del Duca-Kilgore-Schmidt (9810409, 9903516), Kosower-Uwer (9903515), Catani-de Florian-Rodrigo (1112.4405), Sborlini-de Florian-Rodrigo (1310.6841), Del Duca-Frizzo-Maltoni (9909464), Birthwright-Glover-Khoze-Marquard (0503063, 0505219), Del Duca-Duhr-Haindl-Lazopoulos (1912.06425, 2007.05345), Catani-de Florian-Rodrigo (0312067), Badger-Buciuni-Peraro (1507.05070), Czakon-Sapeta (2204.11801), Bern-Dixon-Kosower (0404293), Badger-Glover (0405236), Duhr-Gehrmann-Jaquier (1411.3587), Guan-Herzog-Ma-Mistlberger-Suresh (2408.03019)

Space-Like: collinear partons are in both physical initial and final states

Factorization is not exact:

$$|\mathcal{M}(c,h)\rangle \simeq \boldsymbol{Sp}(c;\tilde{p};h)\,|\mathcal{M}(\tilde{p},h)\rangle$$

Dependence on hard non-collinear partons

And the loop expansion

$$\boldsymbol{Sp}(c;\tilde{p};h) = \boldsymbol{Sp}^{(0)}(c;\tilde{p}) + \sum_{i=1} \boldsymbol{Sp}^{(i)}(c;\tilde{p};h)$$

Forshaw-Kyrieleis-Seymour (0604094, 0808.1269), Collins-Qiu (0705.2141), Keates-Seymour (0902.0477), Rogers-Mulders (1001.2977), Echevarria-Idilbi-Scimemi (1111.4996, 1211.1947), Rogers (1304.4251), Feige-Schwartz (1403.6472), Rothstein-Stewart (1601.04695), Schwartz-Yan-Zhu (1703.08572), Dixon-Herrmann-Yan-Zhu (1912.09370), Becher-Neubert-Shao (2107.01212), Henn-Ma-Xu-Yan-Zhang-Zhu (2406.14604), Becher-Hager-Jaskiewicz-Neubert-Schwienbacher (2408.10308)

Double collinear splitting ($\tilde{p} \to p_1 + p_2$) at 1-loop

Time-like splitting process: momentum fractions z_1 and z_2 are both positive

$$\delta(p_1, p_2; \tilde{p}) = -\frac{1}{\epsilon}(C_{12} + C_2 - C_1) f(\epsilon; z_2)$$

$$f\left(\epsilon; \frac{1}{x}\right) = \frac{1}{\epsilon}\left[{}_2F_1(1, -\epsilon, 1-\epsilon, 1-x) - 1\right]$$

Space-like splitting process: one of the momentum fractions is negative, say z_2

$$\boldsymbol{\delta}(p_1, p_2; p_3, \ldots, p_n) = +\frac{2}{\epsilon}\sum_{j=3}^{n} \boldsymbol{T}_2 \cdot \boldsymbol{T}_j\, f(\epsilon; z_2 - i\,0\, s_{j2})$$

$s_{j2} \to (p_j + p_2)^2$

$$= \delta_R(p_1, p_2; \tilde{p}) + i\,\boldsymbol{\delta}_I(p_1, p_2; p_3, \ldots, p_n)$$

$$\delta_R(p_1, p_2; \tilde{p}) = -\frac{1}{\epsilon}(C_{12} + C_2 - C_1) f_R(\epsilon; z_2)$$

$$\boldsymbol{\delta}_I(p_1, p_2; p_3, \ldots, p_n) = -\frac{2}{\epsilon}\boldsymbol{T}_2 \cdot \left[\sum_{j=3}^{n} \boldsymbol{T}_j\, \mathrm{sign}(s_{j2})\right] f_I(\epsilon; z_2)$$

Color conservation: $\sum_{j=3}^{n} \boldsymbol{T}_j = -(\boldsymbol{T}_1 + \boldsymbol{T}_2)$

Catani-de Florian-Rodrigo (1112.4405)

Double collinear splitting ($\tilde{p} \to p_1 + p_2$) at 2-loops

Space-like splitting — No DIS

$$\Delta_C^{(2)} \sim \pi f_{abc} \sum_{i=1,2} \sum_{\substack{j,k=3 \\ j \neq k}}^{n} T_i^a T_j^b T_k^c \, \Theta(-z_i) \, \text{sign}(s_{ij}) \, \Theta(-s_{jk})$$

$$\times \ln\left(-\frac{s_{j\tilde{p}} \, s_{k\tilde{p}} \, z_1 \, z_2}{s_{jk} \, s_{12}} - i\,0 \right) \left[-\frac{1}{2\epsilon^2} + \frac{1}{\epsilon} \ln\left(\frac{-z_i}{1-z_i} \right) \right]$$

Non-abelian

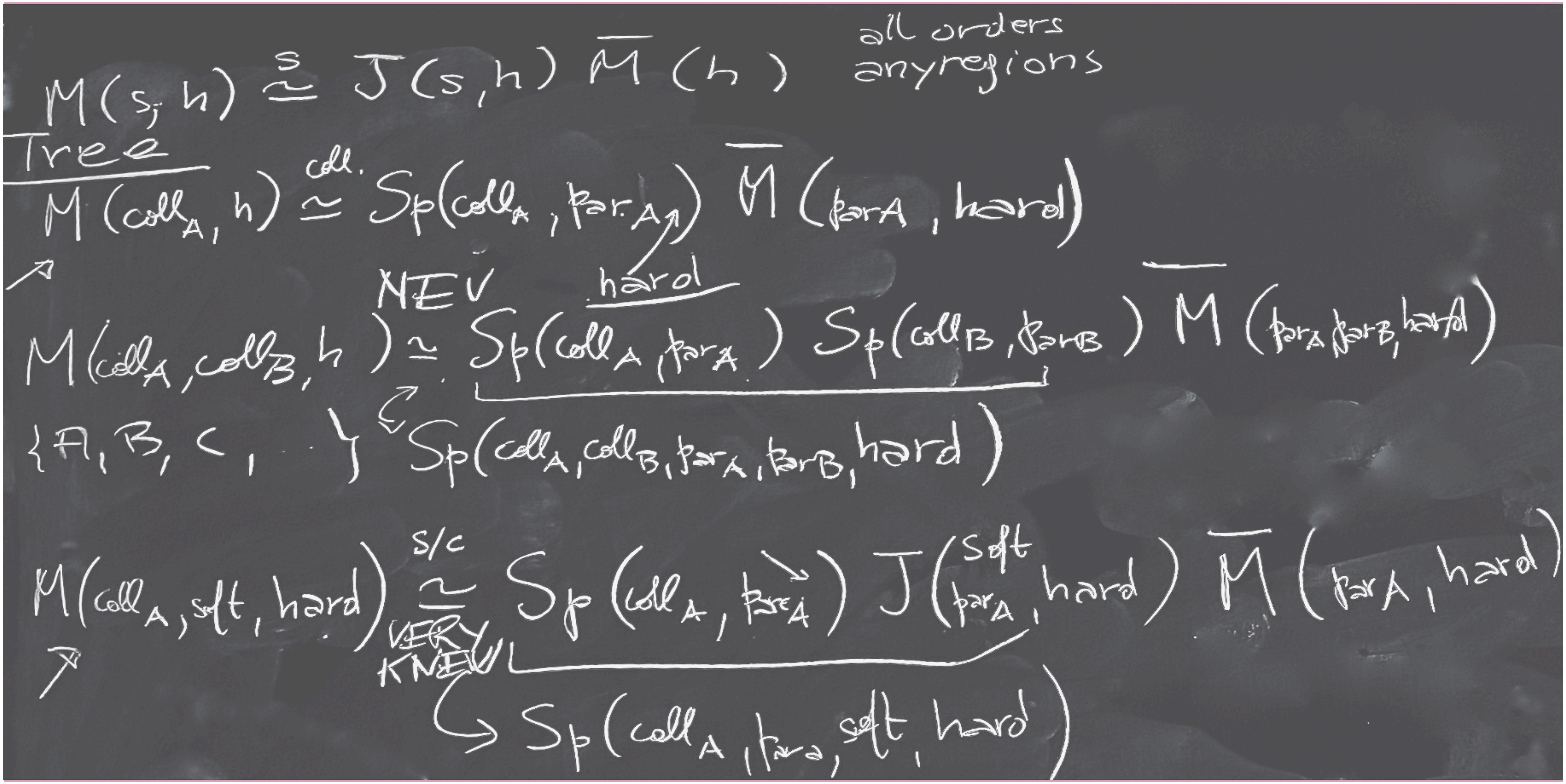
Real and positive

* $\mathcal{O}(\epsilon^0)$ contributions in $\mathcal{N} = 4$ SYM have been recently computed.

Henn-Ma-Xu-Yan-Zhang-Zhu (2406.14604)

ALL ORDER GENERALIZATION PROPOSED BY STEFANO CATANI

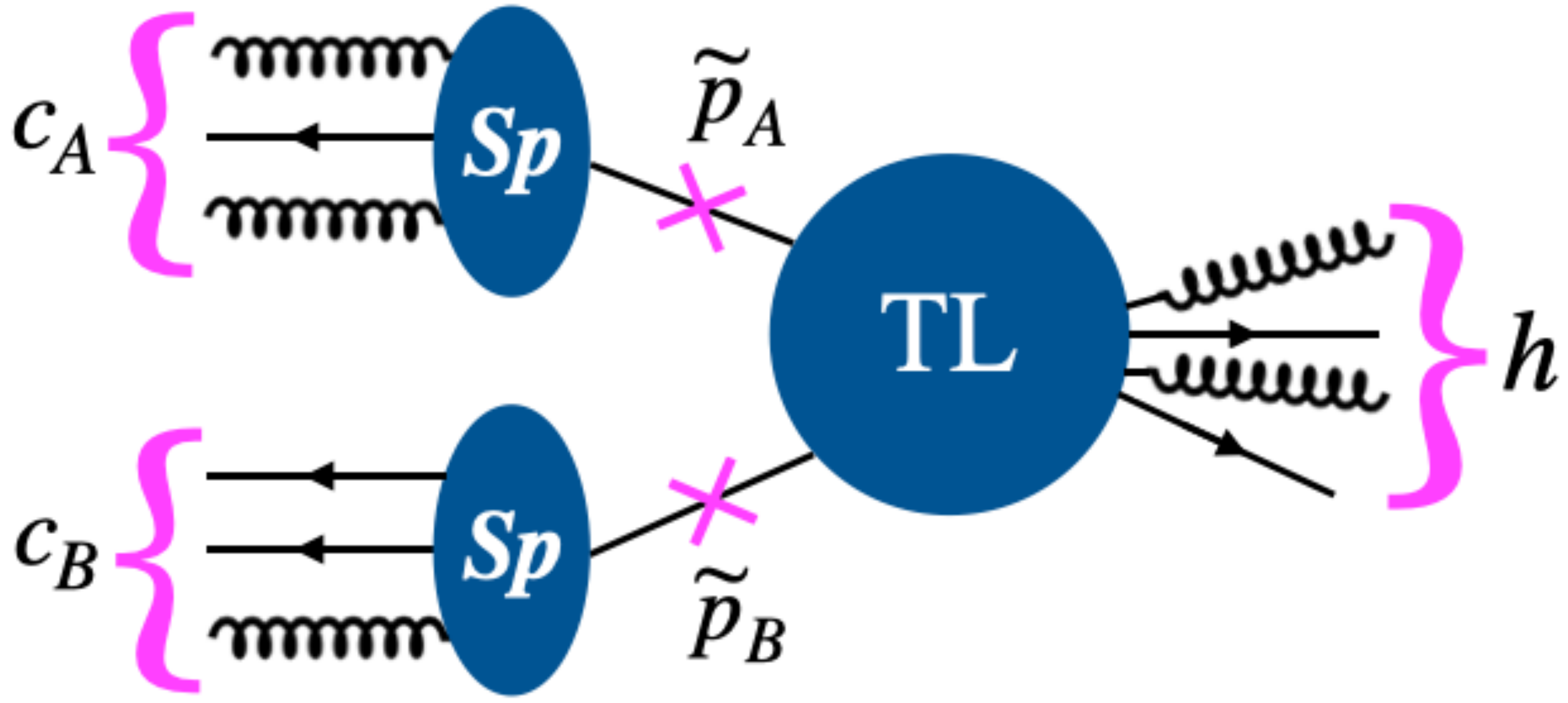
Cieri-PKD-Rodrigo (2402.14749)



Time-like

$$|\mathscr{M}(c_A, c_B, h)\rangle \simeq \boldsymbol{Sp}(c_A; \tilde{p}_A)\, \boldsymbol{Sp}(c_B; \tilde{p}_B)\, |\mathscr{M}(\tilde{p}_A, \tilde{p}_B, h)\rangle$$

Factorizable



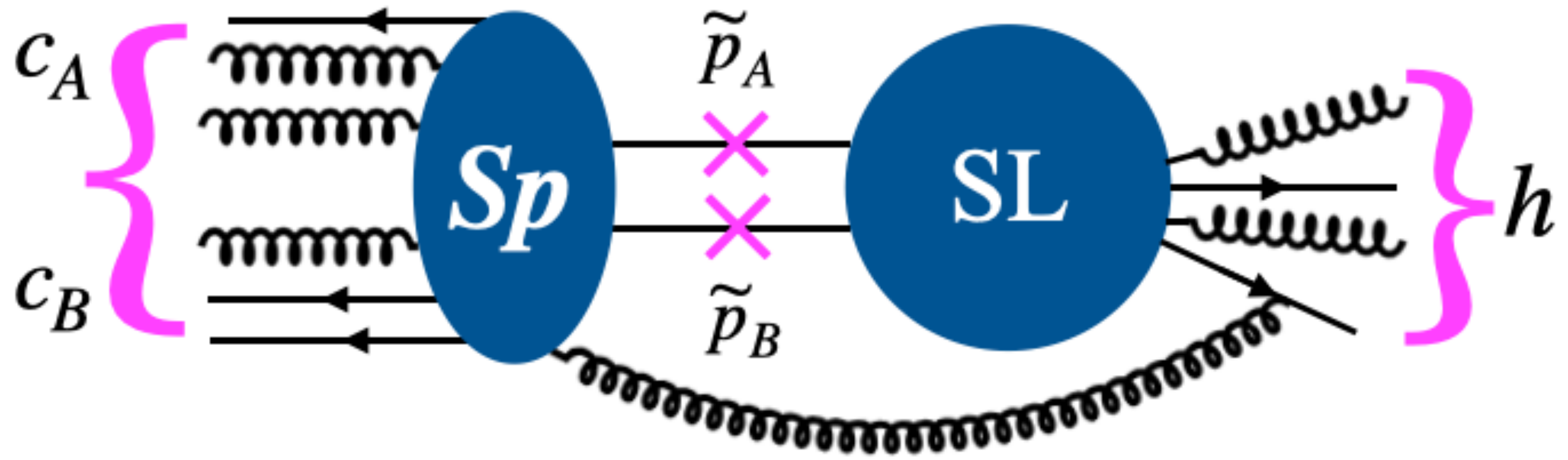
Space-like

Non-factorizable

Hard degrees of freedom

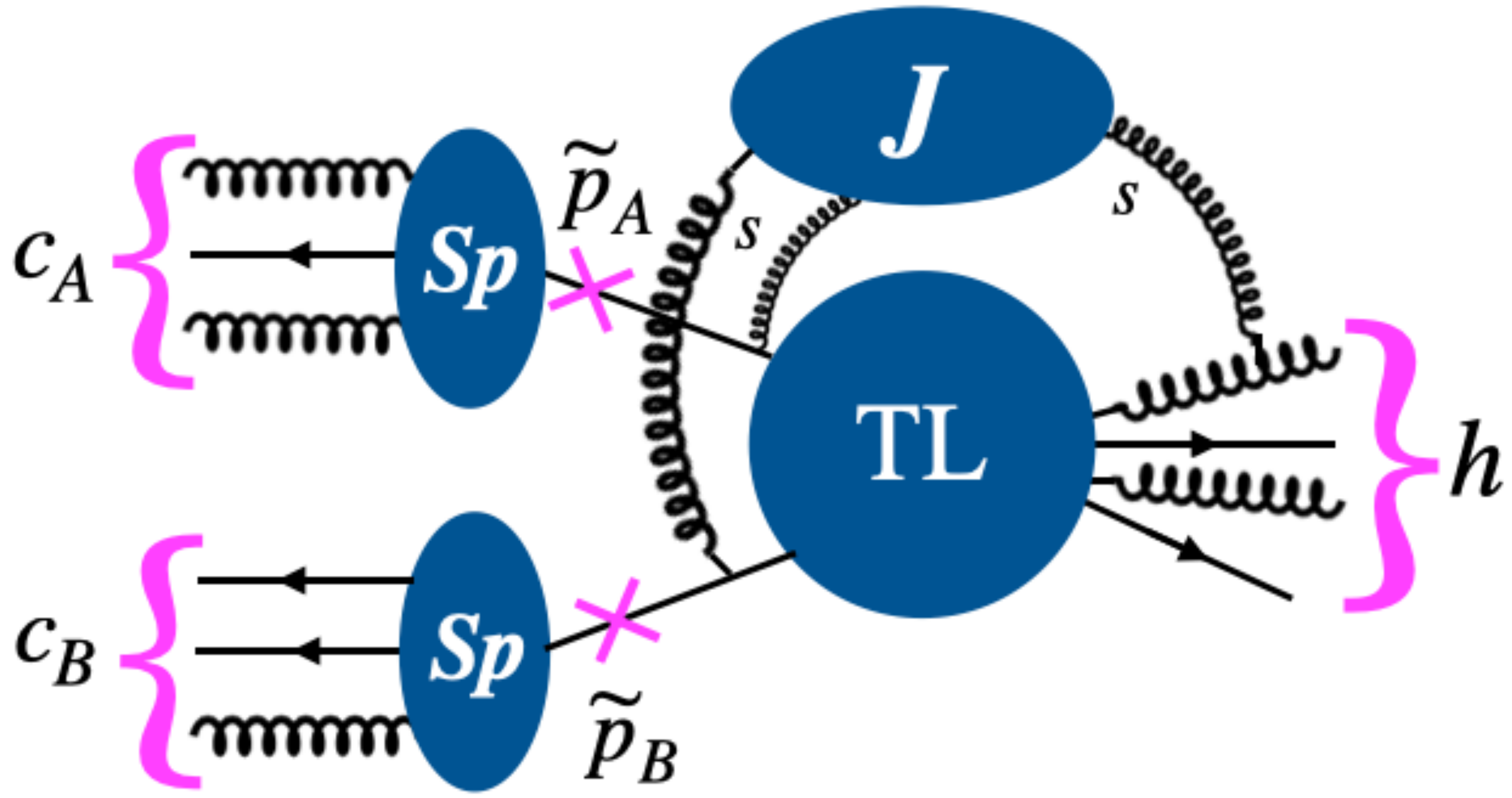
$$|\mathscr{M}(c_A, c_B, h)\rangle \simeq \boldsymbol{Sp}(c_A, c_B; \tilde{p}_A, \tilde{p}_B; h)\, |\mathscr{M}(\tilde{p}_A, \tilde{p}_B, h)\rangle$$

$$\boldsymbol{Sp}^{(0)}(c_A, c_B; \tilde{p}_A, \tilde{p}_B; h) = \boldsymbol{Sp}^{(0)}(c_A; \tilde{p}_A)\, \boldsymbol{Sp}^{(0)}(c_B; \tilde{p}_B)$$



Time-like

$$|\mathscr{M}(s,c,h)\rangle \simeq \boldsymbol{Sp}(c;\tilde{p})\,\boldsymbol{J}(s;\tilde{p};h)\,|\mathscr{M}(\tilde{p},h)\rangle$$



$$|\mathscr{M}(s,c_A,c_B,h)\rangle \simeq \boldsymbol{Sp}(c_A;\tilde{p}_A)\,\boldsymbol{Sp}(c_B;\tilde{p}_B)\,|\,\boldsymbol{J}(s;\tilde{p}_A,\tilde{p}_B;h)\,\mathscr{M}(\tilde{p}_A,\tilde{p}_B,h)\rangle$$

Factorizable

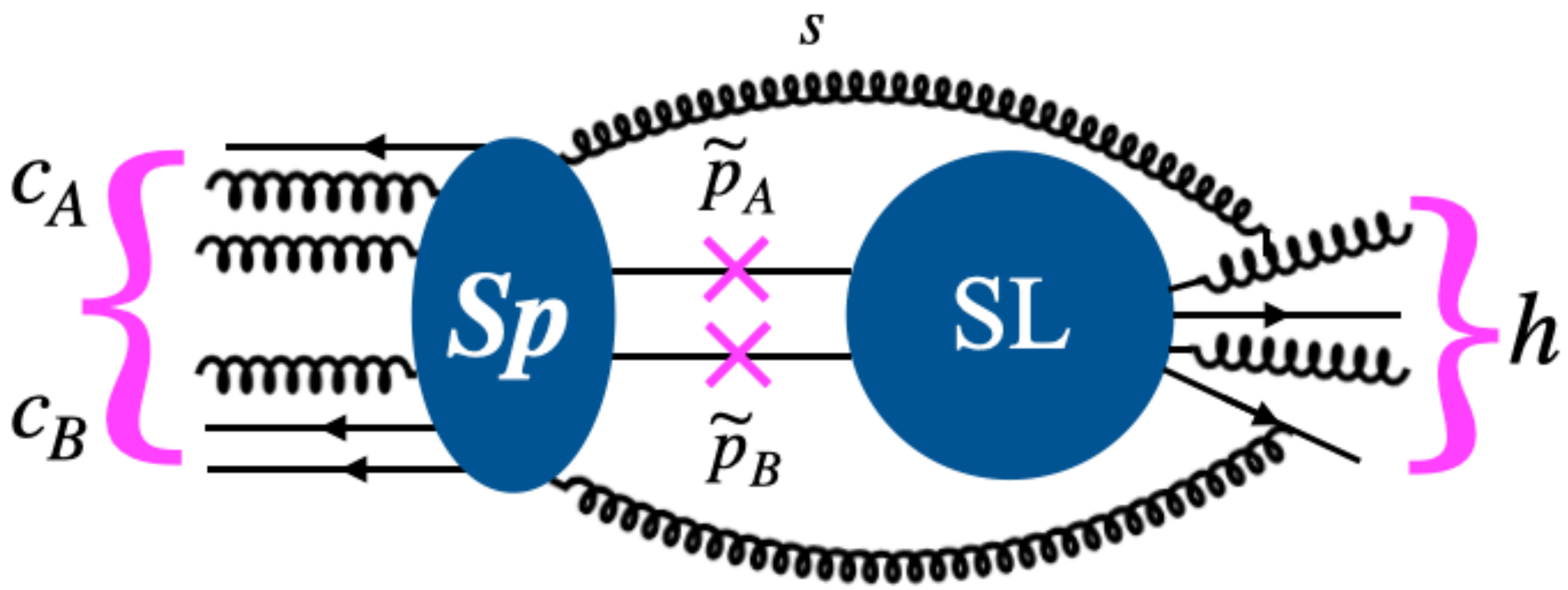
Space-like

Non-factorizable

Hard degrees of freedom

$$|\mathscr{M}(s,c_A,c_B,h)\rangle \simeq \boldsymbol{Sp}(c_A,c_B;\tilde{p}_A,\tilde{p}_B;s;h)\,|\mathscr{M}(\tilde{p}_A,\tilde{p}_B,h)\rangle$$

$$\boldsymbol{Sp}^{(0)}(c_A,c_B;\tilde{p}_A,\tilde{p}_B;s;h) = \boldsymbol{Sp}^{(0)}(c_A;\tilde{p}_A)\,\boldsymbol{Sp}^{(0)}(c_B;\tilde{p}_B)\,\boldsymbol{J}^{(0)}(s;\tilde{p}_A,\tilde{p}_B;h)$$



$$|\mathscr{M}|^2 \simeq \langle \mathscr{M}(\tilde{p}, h) | \boldsymbol{Sp}^\dagger(c; \tilde{p}; h)\, \boldsymbol{Sp}(c; \tilde{p}; h) | \mathscr{M}(\tilde{p}, h) \rangle$$

Purely imaginary contributions drops away

- Factorization breaking effects for double collinear splitting processes at 1-loop is purely imaginary, and hence gets cancelled. Catani-de Florian-Rodrigo (1112.4405)

- Factorization breaking effects for IR pole parts of double collinear splitting processes at 2-loops gets cancelled for tree level amplitudes generated in pure QCD.
  Catani-de Florian-Rodrigo (1112.4405), Forshaw-Seymour-Siodmok(1206.6363), Dixon-Herrmann-Yan-Zhu (1912.09370)

- Factorization breaking effects for double collinear splitting processes up to 2-loops gets cancelled in $\mathcal{N} = 4$ SYM theory. Henn-Ma-Xu-Yan-Zhang-Zhu (2406.14604)

Dedicated studies are required beyond this point!!!

* Factorization breaking effects are observed in mostly for non-inclusive (+ non-global such as gap-between-jets) observables. Still conclusive statements are yet to be made!

  Bacchetta-Bomhof-Mulders-Pijlman (0406099, 0505268, 0601171), Forshaw-Kyrieleis-Seymour (0604094, 0808.1269), Collins-Qiu (0705.2141), Gaunt (1405.2080), Zeng (1507.01652)

* Super-leading logarithms in gap-between-jets are one of the implications of factorization breaking.

  Forshaw-Kyrieleis-Seymour (0604094, 0808.1269), Becher-Neubert-Shao-Stillger (2107.01212, 2307.06359), Boer-Hager-Neubert-Stillger-Xu (2405.05305)

* Very recently, it has been found a contribution at 3-loops that has the required form to turn the double super leading logarithms to single-logarithmic evolution.

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  See very nice talk by Thomas Becher

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In summary, collinear factorization breaking doesn't necessarily imply PDF factorization breaking!!!

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Thank You Very Much!