Final result

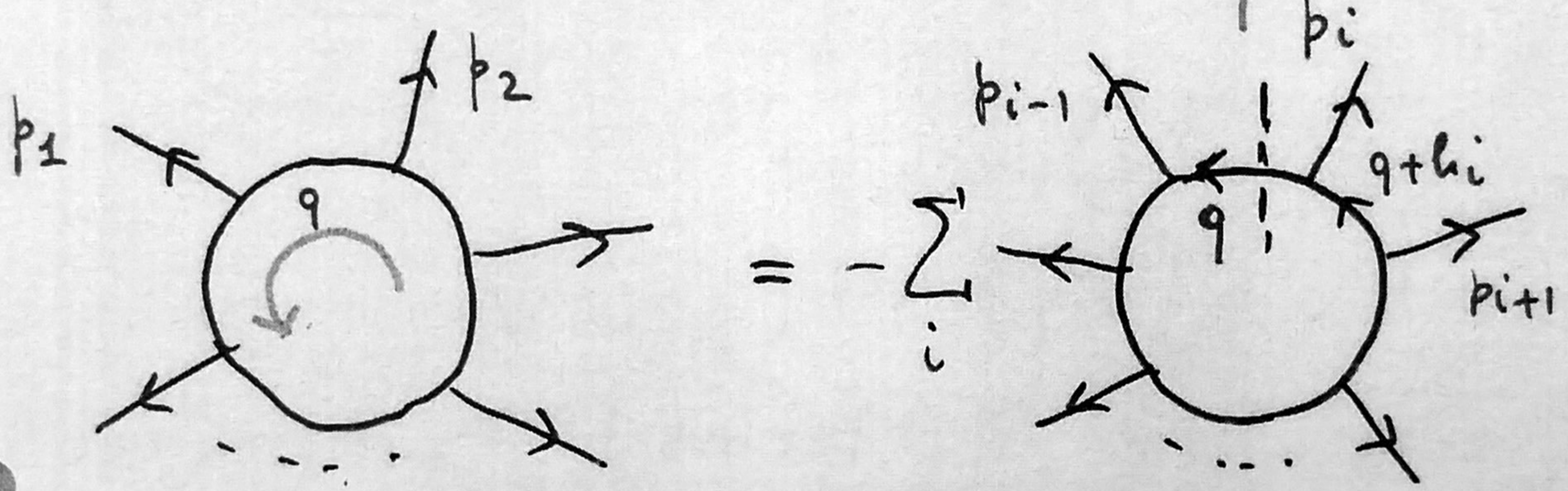
phase space integral

$$L^{(N)}(p_1,\dots,p_N) = -\sum_{i=1}^{N} I^{(N-1)}(k_i,\dots,k_{i+N-2})$$

loop integral

i : link between p_{i-1} and p_i

single cut diagram $2\pi\,\delta_+(q^2)$



$$k_i = p_i$$

$$k_{i+j} = \sum_{k=i}^{j} p_k$$

Hand written by Stefano CATANI, HP2 2006

$$I^{(n)}(k_1,\dots,k_n) = (\mu)^{4-d}\int \frac{d^dq}{(2\pi)^{d-1}}\,\delta_+(q^2)\left[\prod_{j=1}^{n} \frac{1}{2k_j\cdot q + k_j^2}\right]$$

phase space integral,

named: DUAL INTEGRAL*

* "mathematical object" more elementary than loop integral

Hand written by Stefano CATANI, HP2 2006

The solution is in the vacuum

Germán RODRIGO

IFIC INSTITUT DE FÍSICA CORPUSCULAR | CSIC | VNIVERSITAT DE VALÈNCIA

Financiado por la Unión Europea NextGenerationEU | GOBIERNO DE ESPAÑA MINISTERIO DE CIENCIA, INNOVACIÓN Y UNIVERSIDADES | Plan de Recuperación, Transformación y Resiliencia | GENERALITAT VALENCIANA Conselleria de Educación, Universidades y Empleo | GVANEXT

ASFAE2022/009 | PID2020-114473GB-I00 | PROMETEO/2021/071

the dream of reason produces monsters | Francisco de Goya, 1799

Loop-tree duality (LTD)

S. Catani, T. Gleisberg, F. Krauss, G. Rodrigo and J. C. Winter, “***From loops to trees by-passing Feynman's theorem,***” JHEP 0809 (2008) 065 [0804.3170].

I. Bierenbaum, **S. Catani**, P. Draggiotis and G. Rodrigo, “***A Tree-Loop Duality Relation at Two Loops and Beyond,***” JHEP 1010 (2010) 073 [1007.0194].

Gell-Mann suggested Feynman to study massless Yang-Mills theory: a toy unrealistic theory at that moment

R. P. Feynman, Closed Loop And Tree Diagrams. (talk),

Magic Without Magic: J. A. Wheeler, A Collection of Essays in Honor of his Sixtieth Birthday. Edited by John R. Klauder, **1972**, p.355, In *Brown, L.M. (ed.): Selected papers of Richard Feynman* 867-887

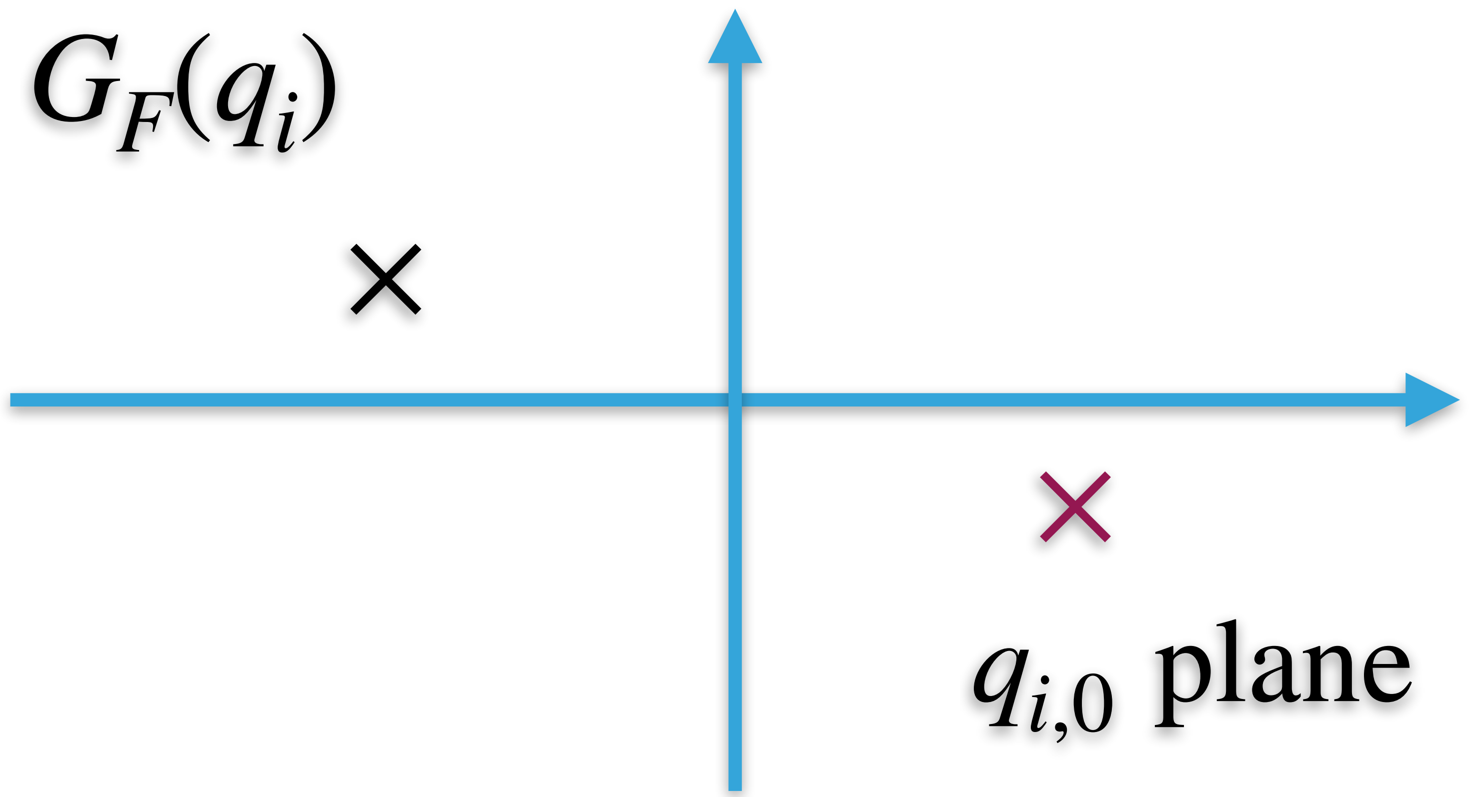
We shall show that any diagram with closed loops can be expressed in terms of sums (actually integrals) of tree diagrams. In each of these tree diagrams there is, in addition to the external particles of the original closed loop diagram, certain particles in the initial and in the final state of the tree diagram. These

[Catani et al. JHEP 0809 (2008) 065]

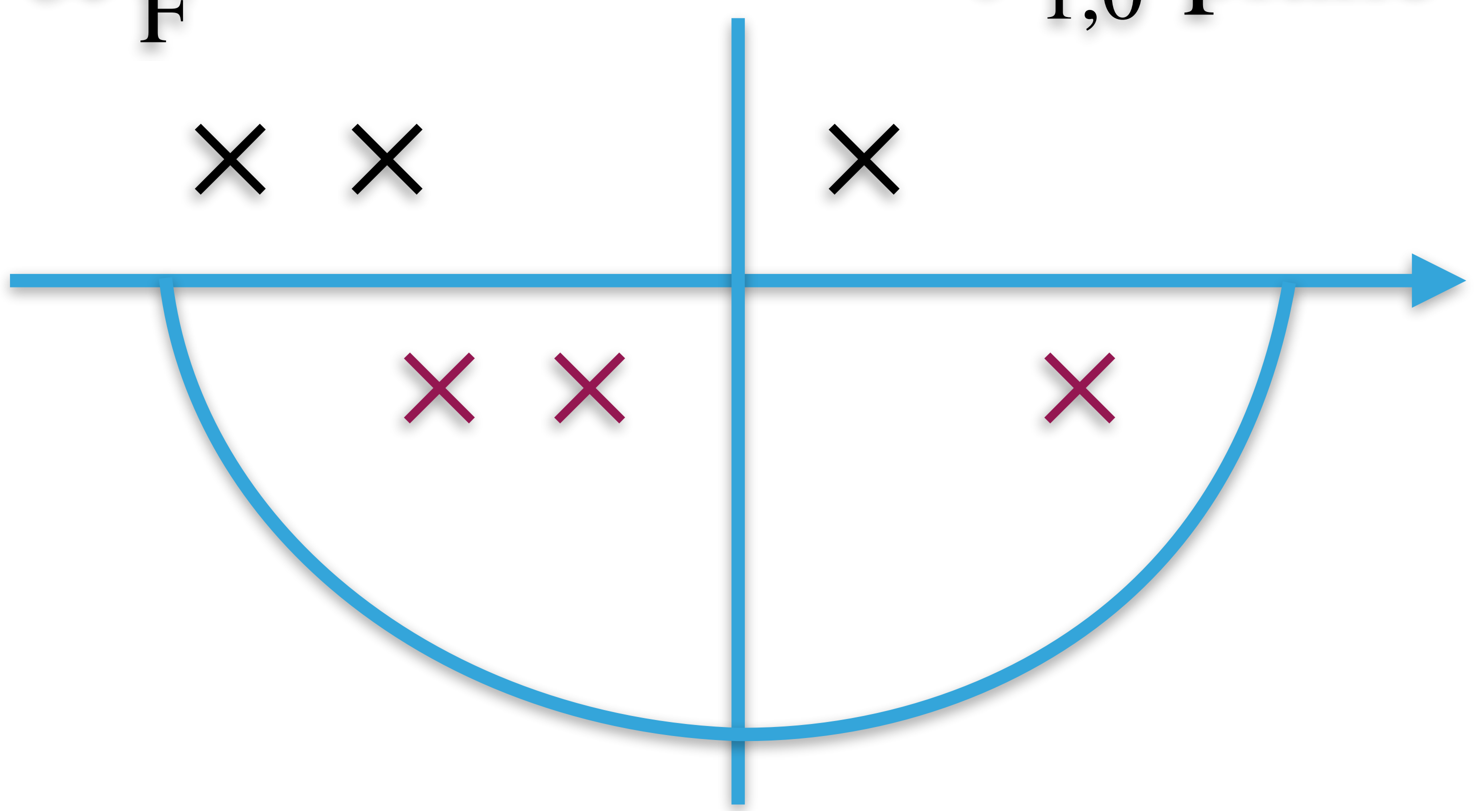
[see also Soper PRL 81 (1998) 2638]

The loop-tree duality (LTD)

$$G_F(q_i) = \frac{1}{q_i^2 - m_i^2 + i0} = \frac{1}{(q_{i,0} - q_{i,0}^{(+)})(q_{i,0} + q_{i,0}^{(+)})}$$



Cauchy residue theorem
in the loop energy complex plane



At fixed three-momentum, **each Feynman Propagator** has **two on-shell modes** with on-shell energies:

$$q_{i,0} \to \pm\, q_{i,0}^{(+)} = \pm \sqrt{\mathbf{q}_i^2 + m_i^2 - i0}$$

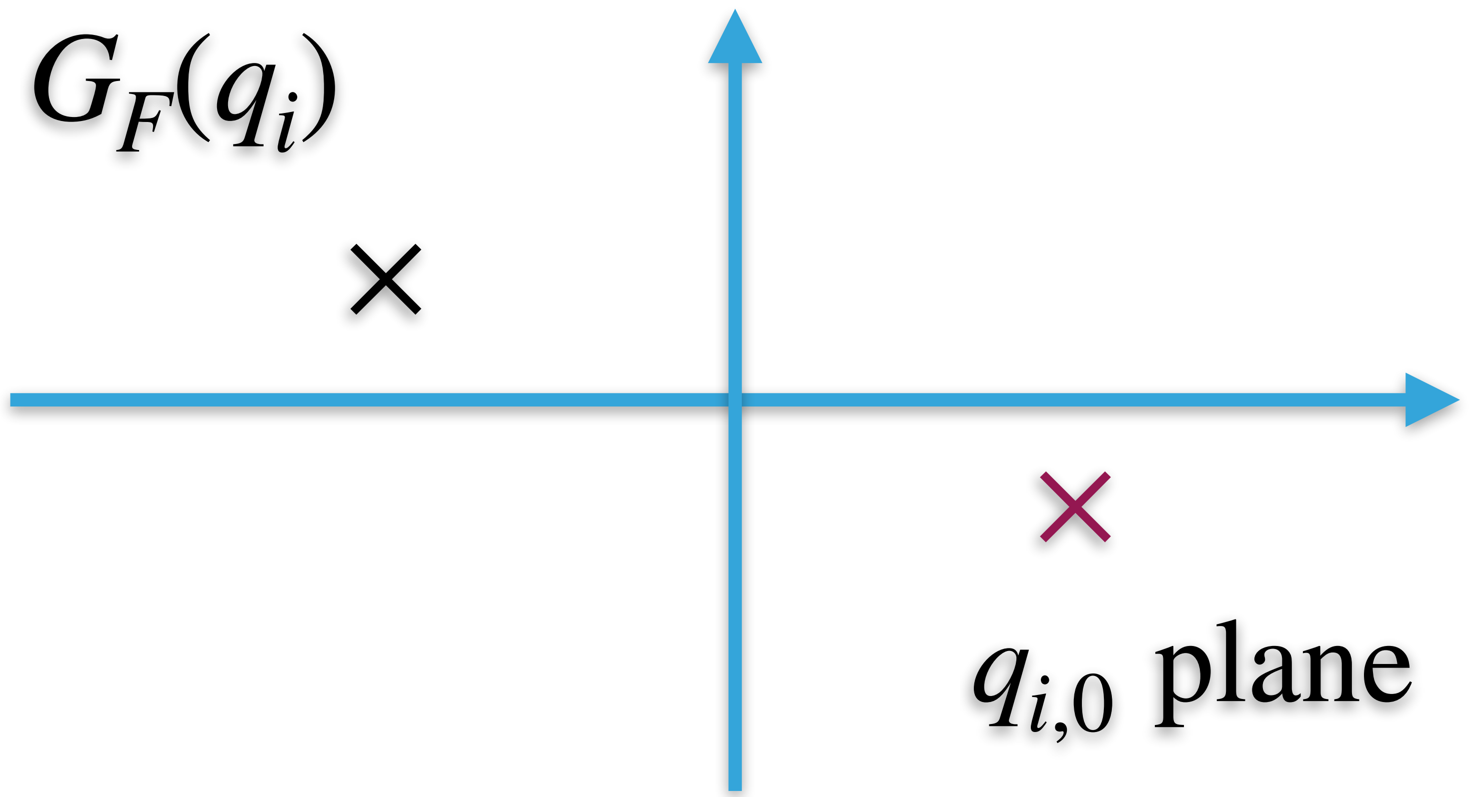
selects residues with definite **positive energy and negative imaginary part**

[Catani et al. JHEP 0809 (2008) 065]

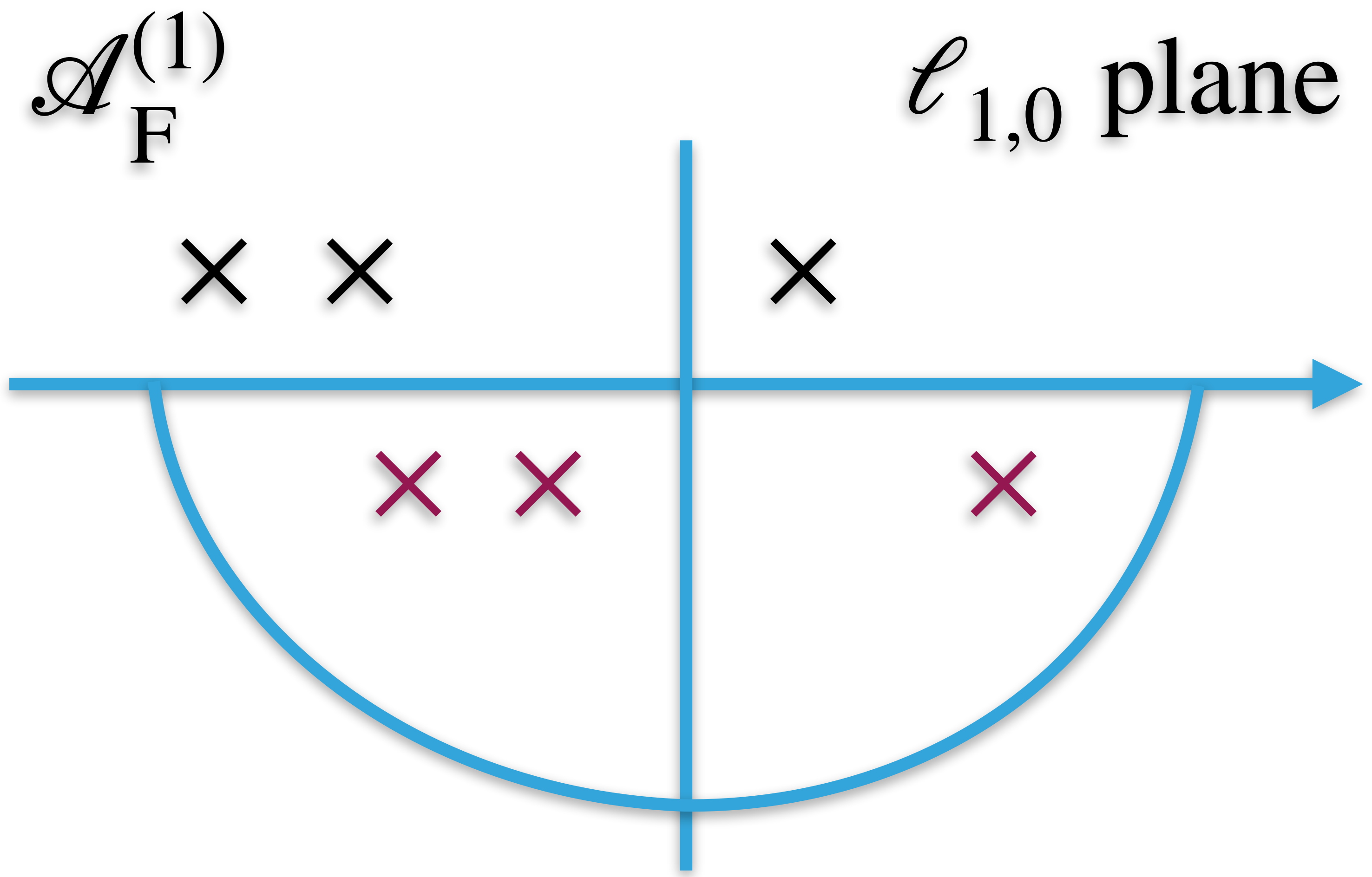
[see also Soper PRL 81 (1998) 2638]

The loop-tree duality (LTD)

$$G_F(q_i) = \frac{1}{q_i^2 - m_i^2 + i0} = \frac{1}{(q_{i,0} - q_{i,0}^{(+)})(q_{i,0} + q_{i,0}^{(+)})}$$



Cauchy residue theorem
in the loop energy complex plane



At fixed three-momentum, **each Feynman Propagator** has **two on-shell modes** with on-shell energies:

$$q_{i,0} \to \pm q_{i,0}^{(+)} = \pm\sqrt{\mathbf{q}_i^2 + m_i^2 - i0}$$

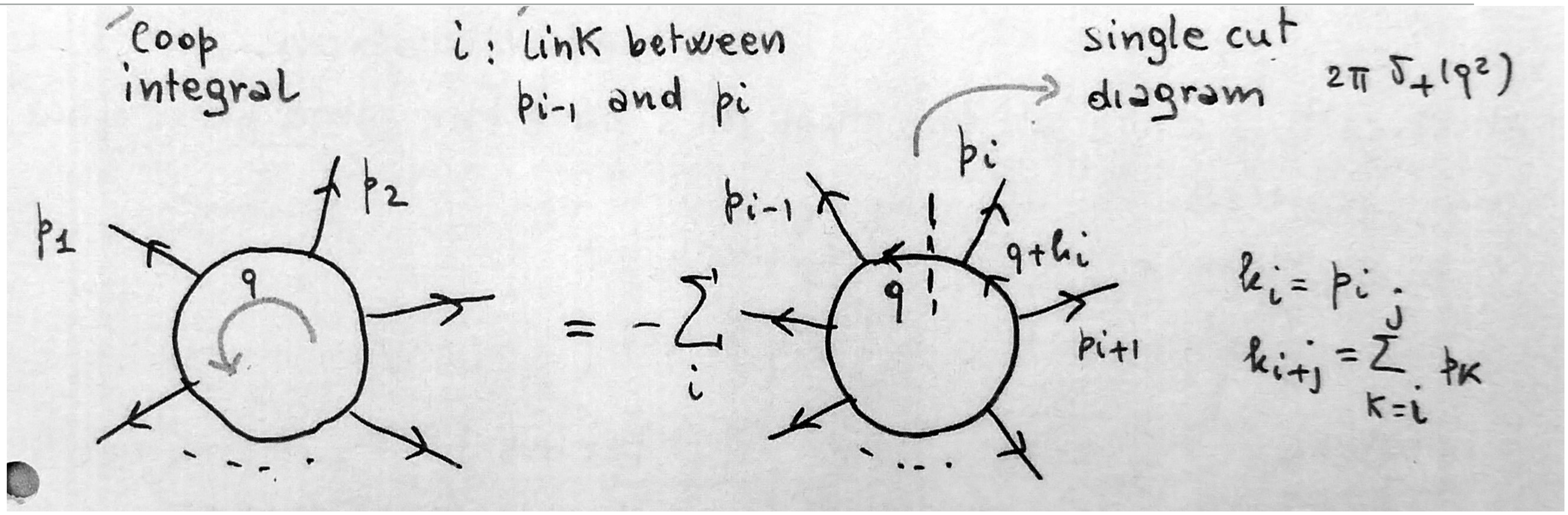
selects residues with definite **positive energy and negative imaginary part**

in an arbitrary coordinate system: reduce the dimension of the integration domain by one unit

[Catani et al. JHEP 0809 (2008) 065]

Loop-tree duality (LTD)

One-loop amplitudes in any relativistic, local and unitary QFT represented as a linear combination of N **single-cut phase-space/dual amplitudes** | **connected trees**



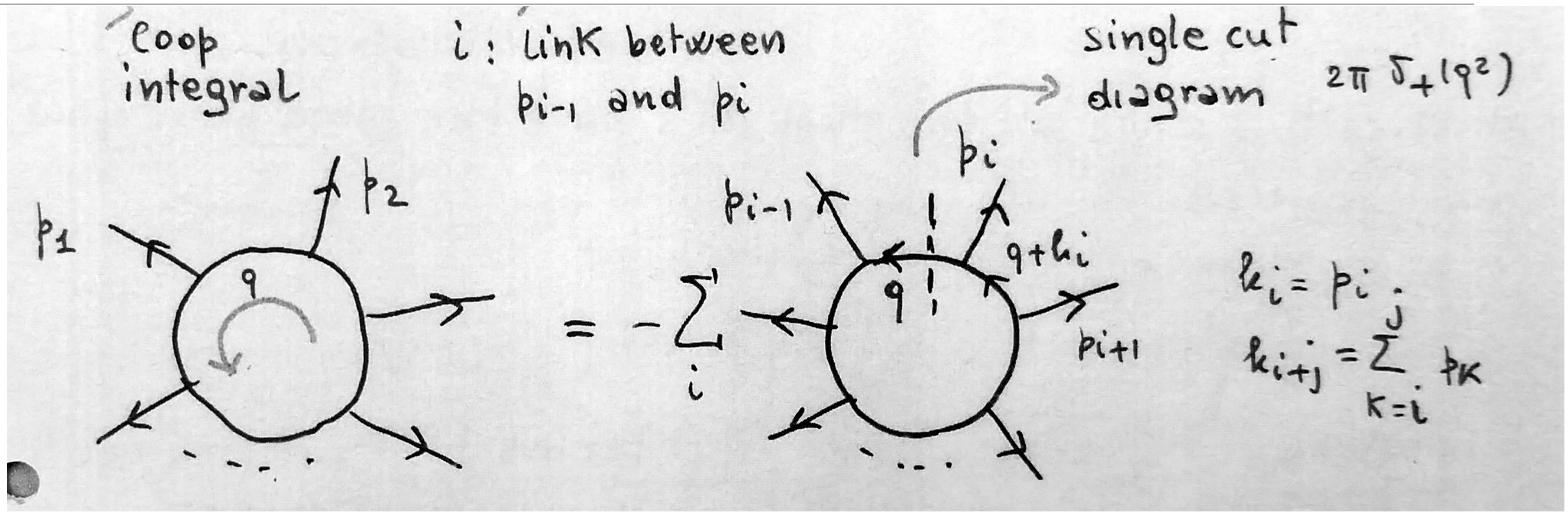
$$\mathscr{A}_N^{(1)} = \int_{\ell_1} \mathscr{N}(\ell_1) \prod G_F(q_i) = - \int_{\ell_1} \mathscr{N}(\ell_1) \otimes \sum \tilde{\delta}(q_i) \prod_{i \neq j} G_D(q_i; q_j)$$

- $\tilde{\delta}(q_i) = \imath 2\pi\, \theta(q_{i,0})\, \delta(q_i^2 - m_i^2)$ on-shell positive energy mode
- **dual propagator** $G_D(q_i; q_j) = \dfrac{1}{q_j^2 - m_j^2 - \imath 0\, \eta k_{ji}}$ $\qquad k_{ji} = q_j - q_i$
- LTD realised by **modifying the customary $+\imath 0$ prescription** of the Feynman propagators
- The **Catani's covariant complex prescription** encodes in a compact way the effect of **multiple-cut** contributions that appear in the **Feynman's Tree Theorem.** Convinient choice $\eta^\mu = (1,\mathbf{0})$: energy component integrated out, remaining integration in **Euclidean space**

[Catani et al. JHEP 0809 (2008) 065]

Loop-tree duality (LTD)

One-loop amplitudes in any relativistic, local and unitary QFT represented as a linear combination of N **single-cut phase-space/dual amplitudes** | **connected trees**

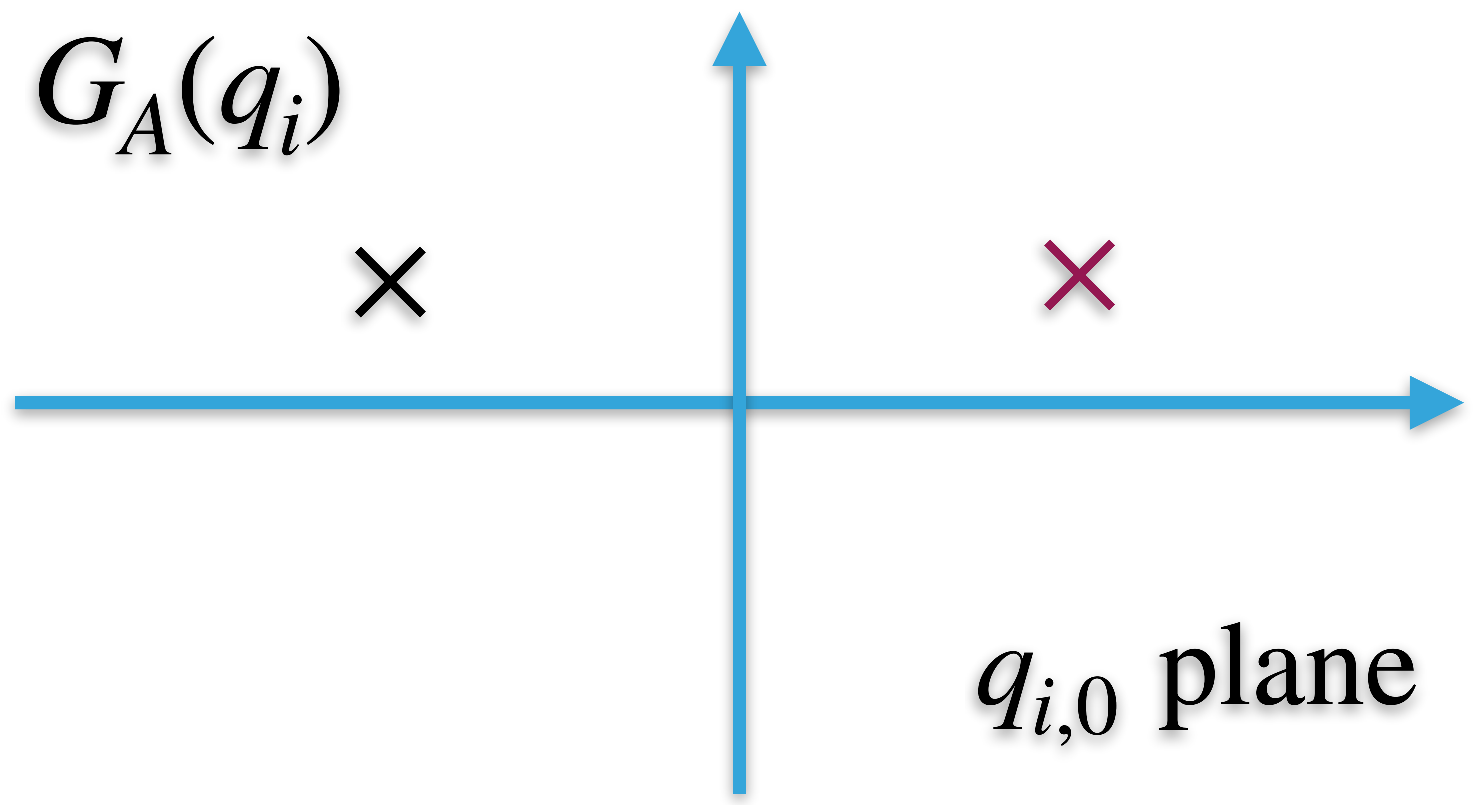


$$\mathscr{A}_N^{(1)} = \int_{\ell_1} \mathcal{N}(\ell_1) \prod G_F(q_i) = - \int_{\ell_1} \mathcal{N}(\ell_1) \otimes \sum \tilde{\delta}(q_i) \prod_{i \neq j} G_D(q_i; q_j)$$

- $\tilde{\delta}(q_i) = \imath 2\pi\, \theta(q_{i,0})\, \delta(q_i^2 - m_i^2)$ on-shell positive energy mode
- **dual propagator** $G_D(q_i; q_j) = \dfrac{1}{q_j^2 - m_j^2 - \imath 0\, \eta k_{ji}}$ $\quad k_{ji} = q_j - q_i$
- LTD realised by **modifying the customary** $+\imath 0$ **prescription** of the Feynman propagators
- The **Catani's covariant complex prescription** encodes in a compact way the effect of **multiple-cut** contributions that appear in the **Feynman's Tree Theorem.** Convinient choice $\eta^\mu = (1,\mathbf{0})$: energy component integrated out, remaining integration in **Euclidean space**

[Catani et al. JHEP 0809 (2008) 065]

The Feynman Tree Theorem



$$G_A(q_i) = \frac{1}{q_i^2 - m_i^2 - i0 q_{i,0}}$$

$$\int_{\ell_1} G_A(\alpha_k) = 0 = \int_{\ell_1} \left[G_F(\alpha_k) + \sum^{1} \cdots \tilde{\delta}(q_i) + \sum \cdots \tilde{\delta}(q_i)\tilde{\delta}(q_j) + \sum \cdots \tilde{\delta}(q_i)\tilde{\delta}(q_j)\tilde{\delta}(q_k) + \ldots \right]$$

- Some identities relating Feynman, Advanced/Retarded and Dual propagators

$$G_A(q_i) = G_F(q_i) + \tilde{\delta}(q_i) \qquad G_R(q_i) = G_F(q_i) + \tilde{\delta}(-q_i) \qquad G_A(-q_i) = G_R(q_i)$$

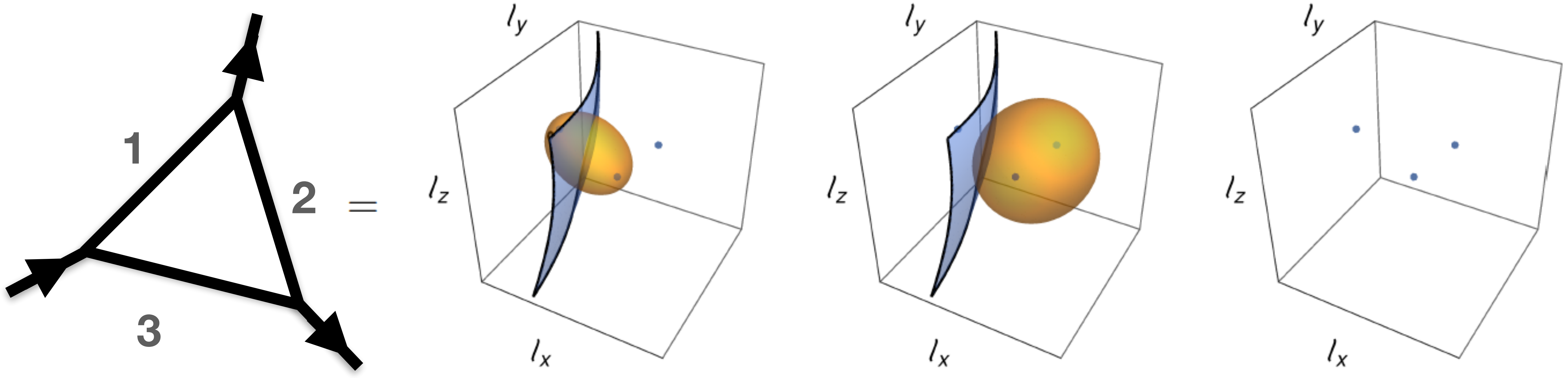
$$\tilde{\delta}(q_i)\, G_D(q_i; q_j) = \tilde{\delta}(q_i) \left[G_F(q_j) + \theta(\eta(q_j - q_i))\, \tilde{\delta}(q_j) \right]$$

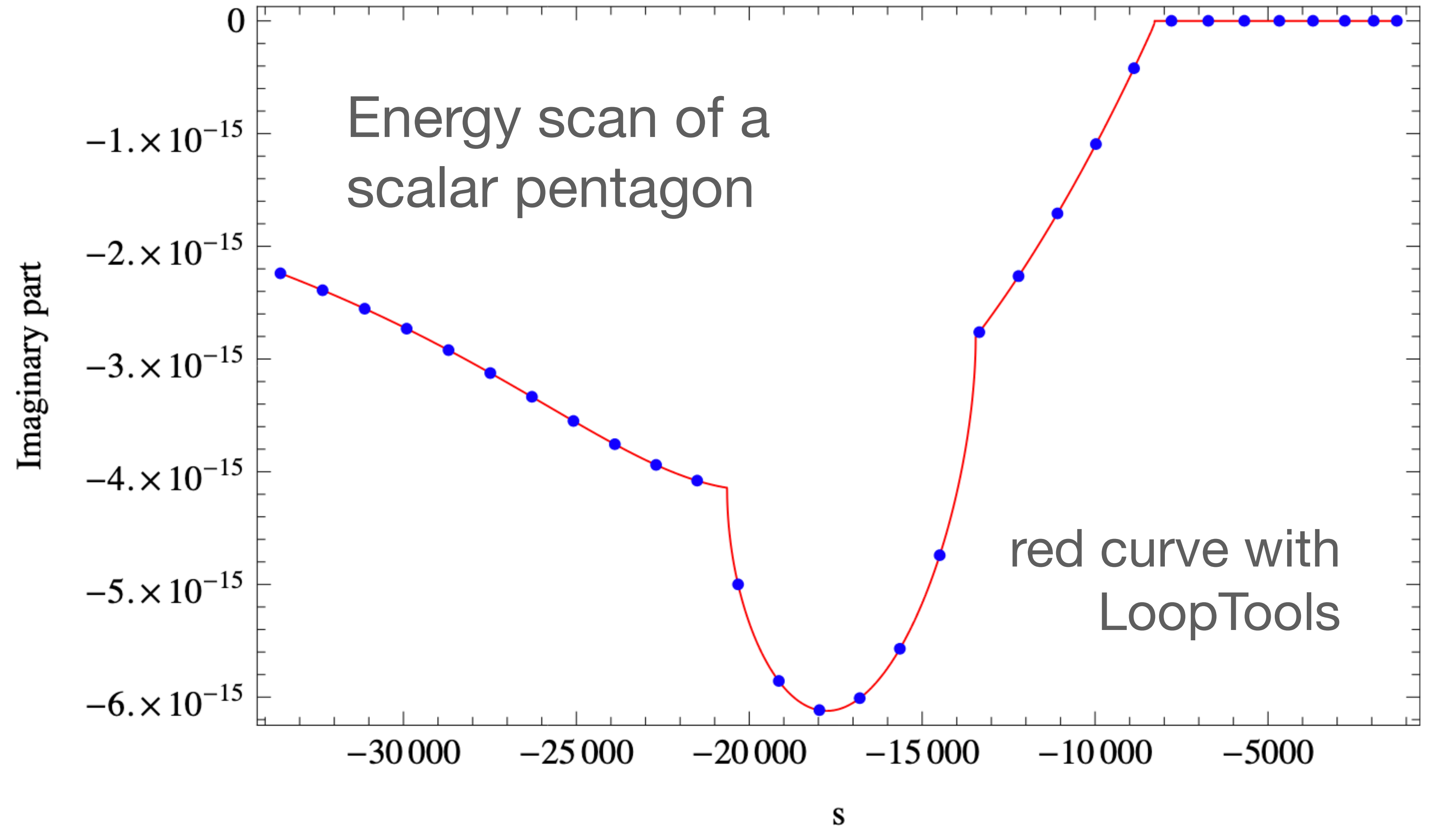
$$G_A(\alpha_k) = G_F(\alpha_k) + G_D(\alpha_k)$$

[Buchta PhD2015, Buchta, et al. Numerical implementation of the loop–tree duality method, EPJC **77** (2017) 274]

[See also Kermanschah's talk]

LTD singular structure of single cuts





- $\frac{1}{q_{i,0}^{(+)} - q_{j,0}^{(+)} \pm k_{ji,0}}$: noncausal singularities, $k_{ji}^2 - (m_i - m_j)^2 \leq 0$, undergo **dual cancellations** among dual pairs
  [at two loops JHEP12(2019)163
- $\frac{1}{q_{i,0}^{(+)} + q_{j,0}^{(+)} - k_{ji,0}}$: causal singularities, $k_{ji}^2 - (m_i + m_j)^2 \geq 0$, bounded to a **compact region** of size the **hard scale**
- causal singularities collapse to a finite segment for **infrared singularities**
- Numerical integration in the **Euclidean** space of the loop three-momenta, CPU/GPU time do not scale significantly with the number of legs

Causality in Feynman loop diagrams

- A Feynman propagator describes a **quantum superposition** of propagation in both directions

$$G_F(q_i) = \frac{1}{\sqrt{2}} \left(|0\rangle + |1\rangle \right)$$

- A Feynman diagram is a superposition of 2^n states

Causality in Feynman loop diagrams

- A Feynman propagator describes a **quantum superposition** of propagation in both directions

$$G_F(q_i) = \frac{1}{\sqrt{2}} \left(|0\rangle + |1\rangle \right)$$

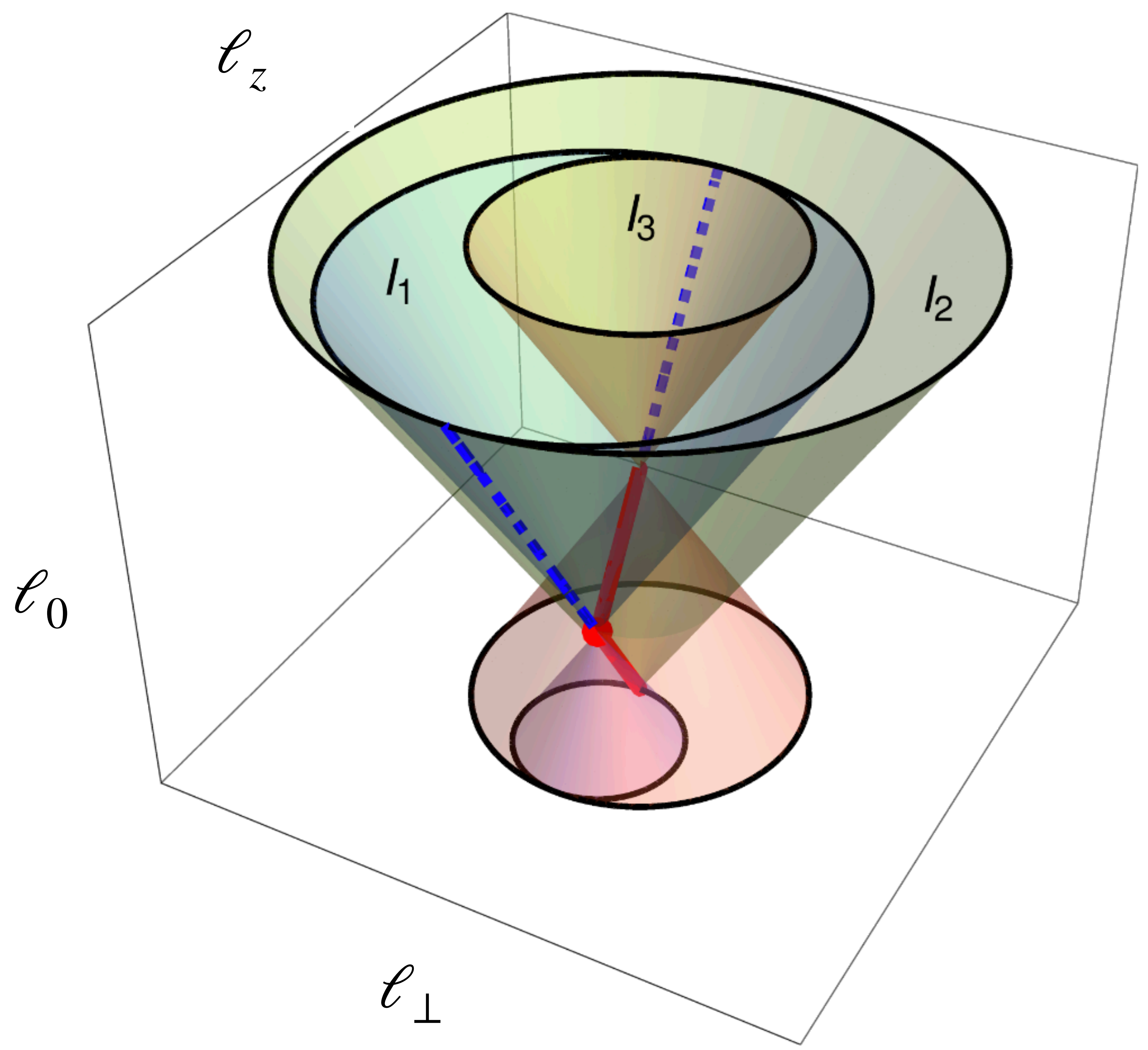
use case for quantum computing applications [Ramírez,Rentería, Sborlini, Vale Silva, Crippa, Clemente, Jansen, GR, 2021-2024]

- A Feynman diagram is a superposition of 2^n states

- If a particle returns to the point of emission: it **travels back in time** and thus **breaks causality** ≡ cyclic configurations are nonphysical

- Causal configurations of Feynman diagrams are **directed acyclic graphs (DAG)** in graph theory

Causality in Feynman loop diagrams



$$\mathcal{A}_N^{(L)} = \int_{\ell_1 \cdots \ell_L} \mathcal{A}_\mathrm{F}^{(L)}$$

- The Feynman representation is very compact but this apparent simplicity comes at a price
- The integrand of the Feynman representation in the Minkowski space of the loop four-momenta inevitably contains nonphysical singularities due to cyclic configurations
- It is **not manifestly causal**

Gell-Mann suggested Feynman to study massless Yang-Mills theory: a toy unrealistic theory at that moment

R. P. Feynman, Closed Loop And Tree Diagrams. (talk), Magic Without Magic: J. A. Wheeler, A Collection of Essays in Honor of his Sixtieth Birthday. Edited by John R. Klauder, 1972, p.355, In *Brown, L.M. (ed.): Selected papers of Richard Feynman* 867-887

If there is more than one loop in the original diagram, the loops may be opened in succession. Choose any one loop; that is, integration over any one virtual momentum k, leaving the others to integrate later. Then this loop can be opened. What results is a diagram sum and integral over diagrams with extra particles, but which still has loops remaining in it. However, there is now one less loop, and in each remaining loop all the propagators are I_+ (if equation 10 is used). Therefore, a remaining loop may be treated in the same way, thus reducing the number of loops still further, until there are none left.

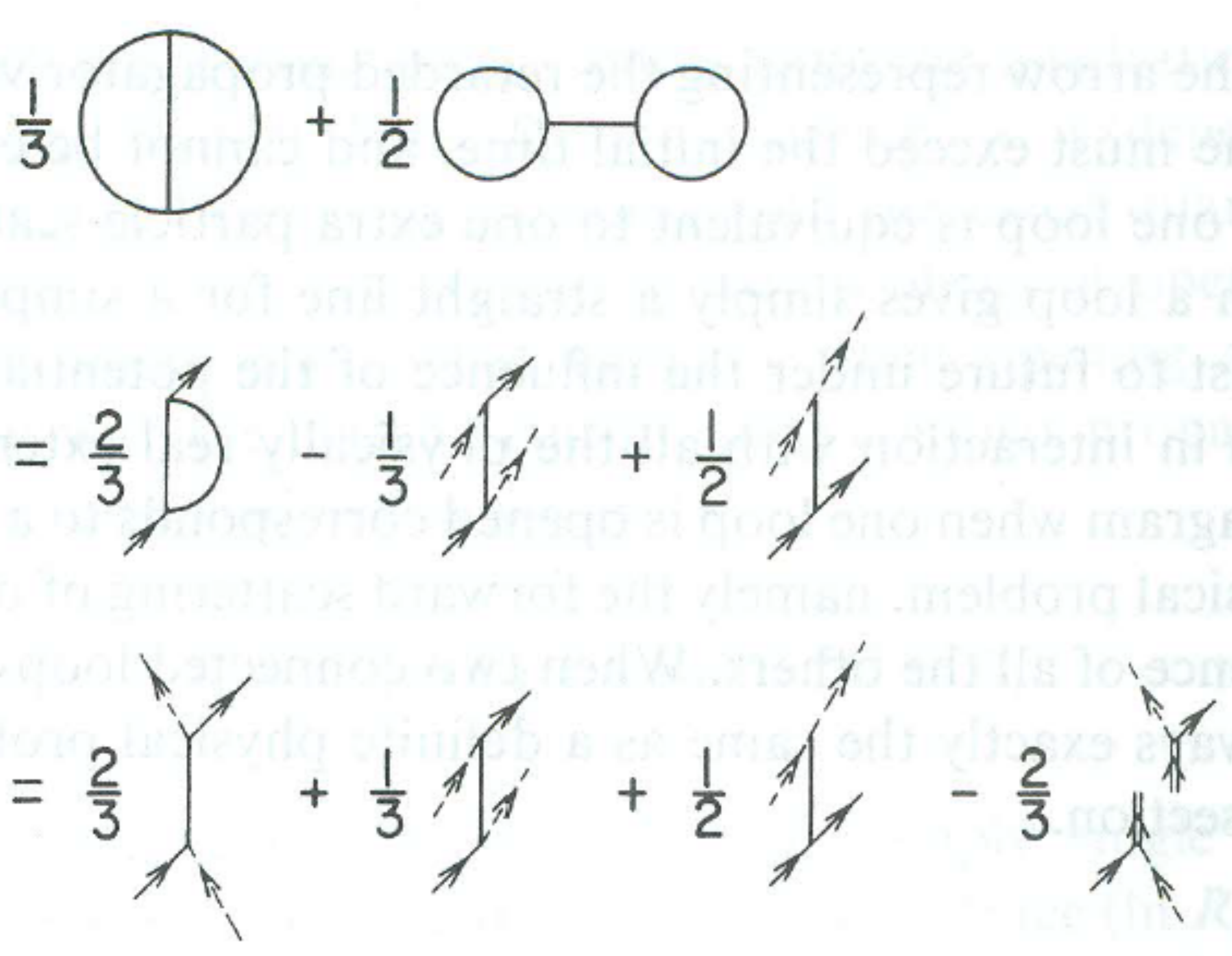


FIGURE 8.
Reduction of the Figure 6 diagrams to trees.

Gell-Mann suggested Feynman to study massless Yang-Mills theory: a toy unrealistic theory at that moment

R. P. Feynman, Closed Loop And Tree Diagrams. (talk),

Magic Without Magic: J. A. Wheeler, A Collection of Essays in Honor of his Sixtieth Birthday. Edited by John R. Klauder, **1972**, p.355, In *Brown, L.M. (ed.): Selected papers of Richard Feynman* 867-887

If there is more than one loop in the original diagram, the loops may be opened in succession. Choose any one loop; that is, integration over any one virtual momentum k, leaving the others to integrate later. Then this loop can be opened. What results is a diagram sum and integral over diagrams with extra particles, but which still has loops remaining in it. However, there is now one less loop, and in each remaining loop all the propagators are I_+ (if equation 10 is used). Therefore, a remaining loop may be treated in the same way, thus reducing the number of loops still further, until there are none left.

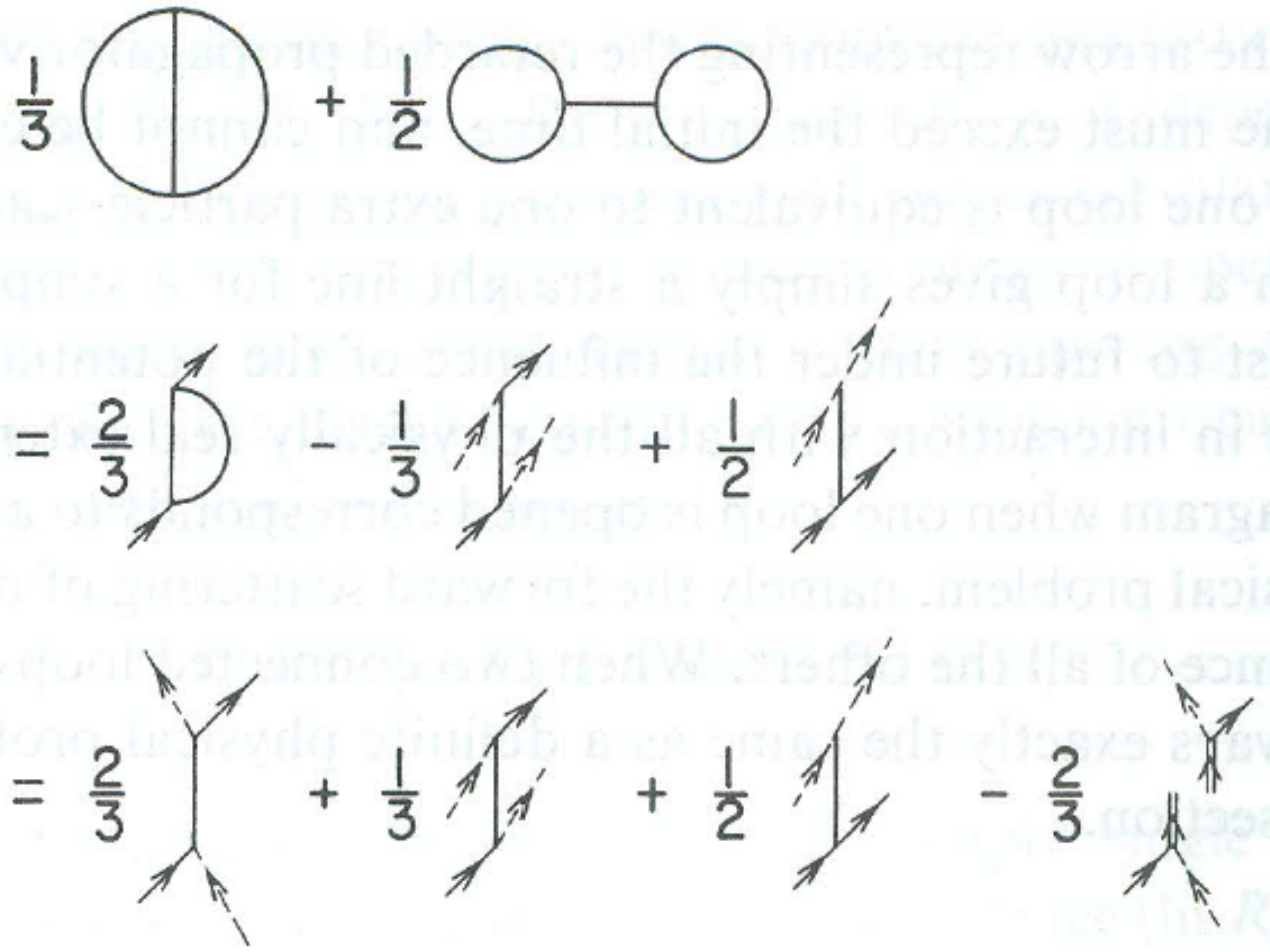


FIGURE 8.
Reduction of the Figure 6 diagrams to trees.

I. Bierenbaum, **S. Catani**, P. Draggiotis and G. Rodrigo, "***A Tree-Loop Duality Relation at Two Loops and Beyond***" JHEP 1010 (2010) 073 [1007.0194]

Aguilera, Driencourt, Hernández, Plenter, Ramírez, Rentería, Rodrigo, Sborlini, Torres, Tracz, PRL 124 (2020) 211602

LTD to all perturbative orders

[see also Runkel, Stör, Vesga, Weinzierl, PRL 122 (2019) 111603
Capatti, Hirschi, Kermanschah, Ruijl, PRL 123 (2019) 151602]

- **Multi-loop scattering amplitude:** n sets of internal momenta, each set depends on a specific linear combination of the L loop momenta. Starting from the integrand in the Feynman representation

$$\mathscr{A}_{\rm F}^{(L)}(1,\ldots,n) = \mathscr{N}(\{\ell_i\}_L, \{p_j\}_N)\, G_F(1,\ldots,n) \qquad G_F(1,\ldots,n) = \prod_{i\in 1\cup\ldots\cup n} \left(G_F(q_i)\right)^{a_i}$$

Aguilera, Driencourt, Hernández, Plenter, Ramírez, Rentería, Rodrigo, Sborlini, Torres, Tracz, PRL 124 (2020) 211602

LTD to all perturbative orders

[see also Runkel, Stör, Vesga, Weinzierl, PRL 122 (2019) 111603
Capatti, Hirschi, Kermanschah, Ruijl, PRL 123 (2019) 151602]

- **Multi-loop scattering amplitude:** n sets of internal momenta, each set depends on a specific linear combination of the L loop momenta. Starting from the integrand in the Feynman representation

$$\mathcal{A}_{\rm F}^{(L)}(1,\ldots,n) = \mathcal{N}(\{\ell_i\}_L, \{p_j\}_N)\, G_F(1,\ldots,n) \qquad G_F(1,\ldots,n) = \prod_{i\in 1\cup\ldots\cup n} \left(G_F(q_i)\right)^{a_i}$$

- The LTD representation is written in terms of **nested residues**

$$\mathcal{A}_{\rm D}^{(L)}(1;2,\ldots,n) = -2\pi\imath \sum_{i_1\in 1} \mathrm{Res}\left(\mathcal{A}_{\rm F}^{(L)}(1,\ldots,n), \mathrm{Im}(\eta q_{i_1}) < 0\right)$$

$$\mathcal{A}_{\rm D}^{(L)}(1,\ldots,r;r+1,\ldots) = -2\pi\imath \sum_{i_r\in r} \mathrm{Res}\left(\mathcal{A}_{\rm D}^{(L)}(1,\ldots,r-1;r,\ldots), \mathrm{Im}(\eta q_{i_r}) < 0\right)$$

Aguilera, Driencourt, Hernández, Plenter, Ramírez, Rentería, Rodrigo, Sborlini, Torres, Tracz, PRL 124 (2020) 211602

LTD to all perturbative orders

[see also Runkel, Stör, Vesga, Weinzierl, PRL 122 (2019) 111603
Capatti, Hirschi, Kermanschah, Ruijl, PRL 123 (2019) 151602]

- **Multi-loop scattering amplitude:** n sets of internal momenta, each set depends on a specific linear combination of the L loop momenta. Starting from the integrand in the Feynman representation

$$\mathcal{A}_{\rm F}^{(L)}(1,\ldots,n) = \mathcal{N}(\{\ell_i\}_L, \{p_j\}_N)\, G_F(1,\ldots,n) \qquad G_F(1,\ldots,n) = \prod_{i\in 1\cup\ldots\cup n} \left(G_F(q_i)\right)^{a_i}$$

- The LTD representation is written in terms of **nested residues**

$$\mathcal{A}_{\rm D}^{(L)}(1;2,\ldots,n) = -2\pi\imath \sum_{i_1\in 1} \mathrm{Res}\left(\mathcal{A}_{\rm F}^{(L)}(1,\ldots,n), \mathrm{Im}(\eta q_{i_1})<0\right)$$

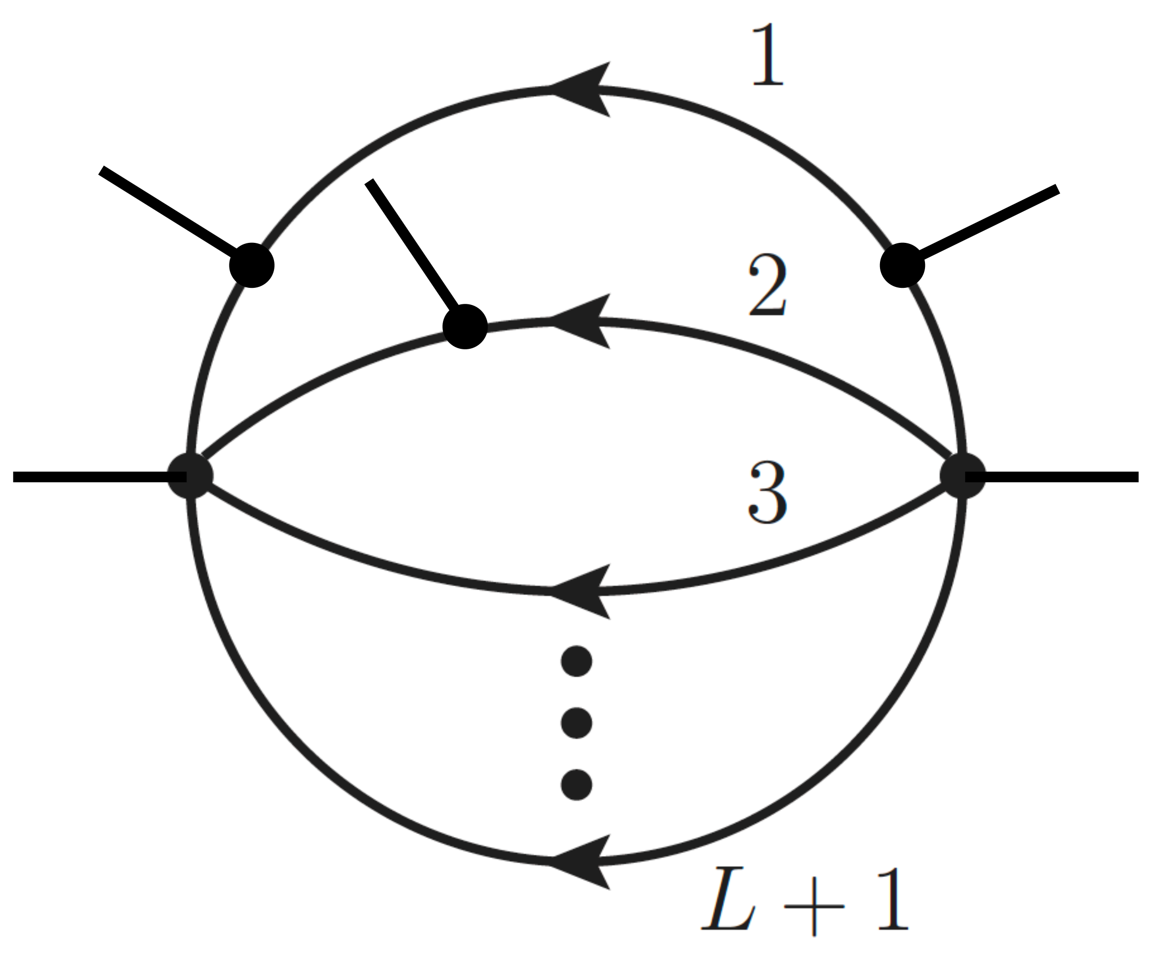
$$\mathcal{A}_{\rm D}^{(L)}(1,\ldots,r;r+1,\ldots) = -2\pi\imath \sum_{i_r\in r} \mathrm{Res}\left(\mathcal{A}_{\rm D}^{(L)}(1,\ldots,r-1;r,\ldots), \mathrm{Im}(\eta q_{i_r})<0\right)$$

- Determined by the **Catani's covariant complex prescription**. If $\eta = (1,\mathbf{0})$ the energy components are integrated out and the integration domain is **Euclidean**
- Requires to reverse some momenta to keep a coherent momentum flow: $s \to \bar{s} \quad (q_{i_s} \to -q_{i_s})$

Aguilera, Driencourt, Hernández, Plenter, Ramírez, Rentería, Rodrigo, Sborlini, Torres, Tracz, PRL 124 (2020) 211602

The maximal loop topology (MLT)

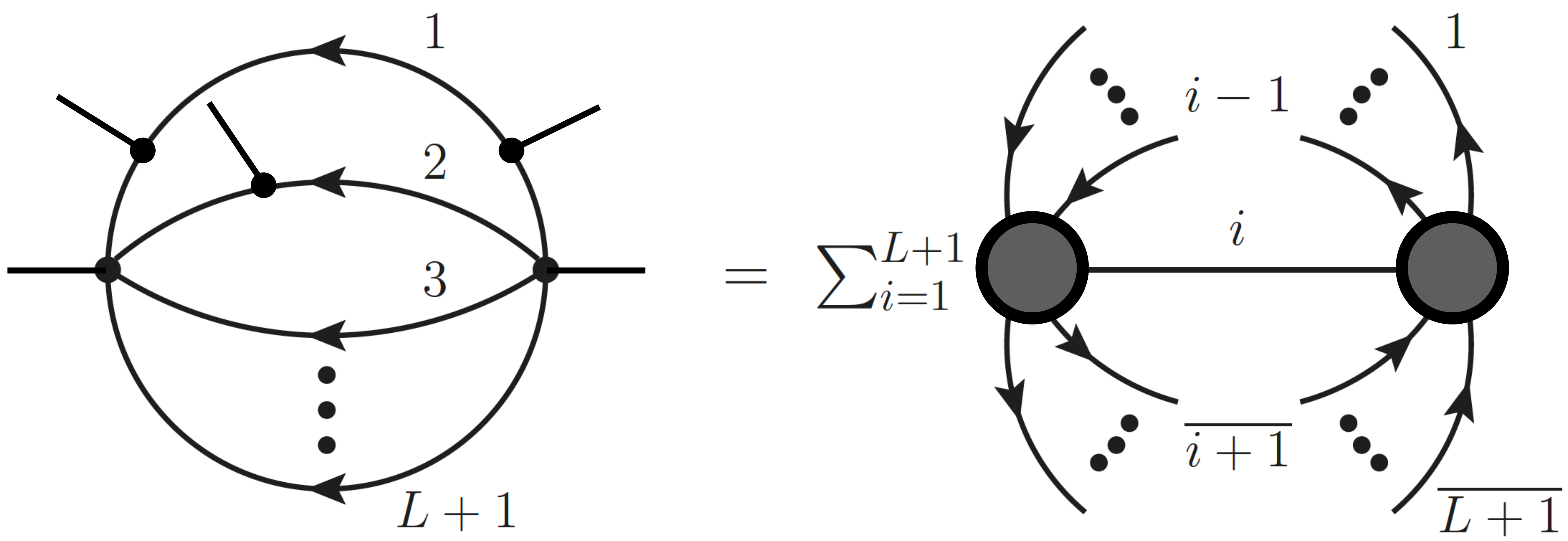
Σ 



- $L+1$ sets of momenta, arbitrary number of external particles and arbitrary number of raised propagators
- The only topology needed at **two loops**

Aguilera, Driencourt, Hernández, Plenter, Ramírez, Rentería, Rodrigo, Sborlini, Torres, Tracz, PRL 124 (2020) 211602

The maximal loop topology (MLT)

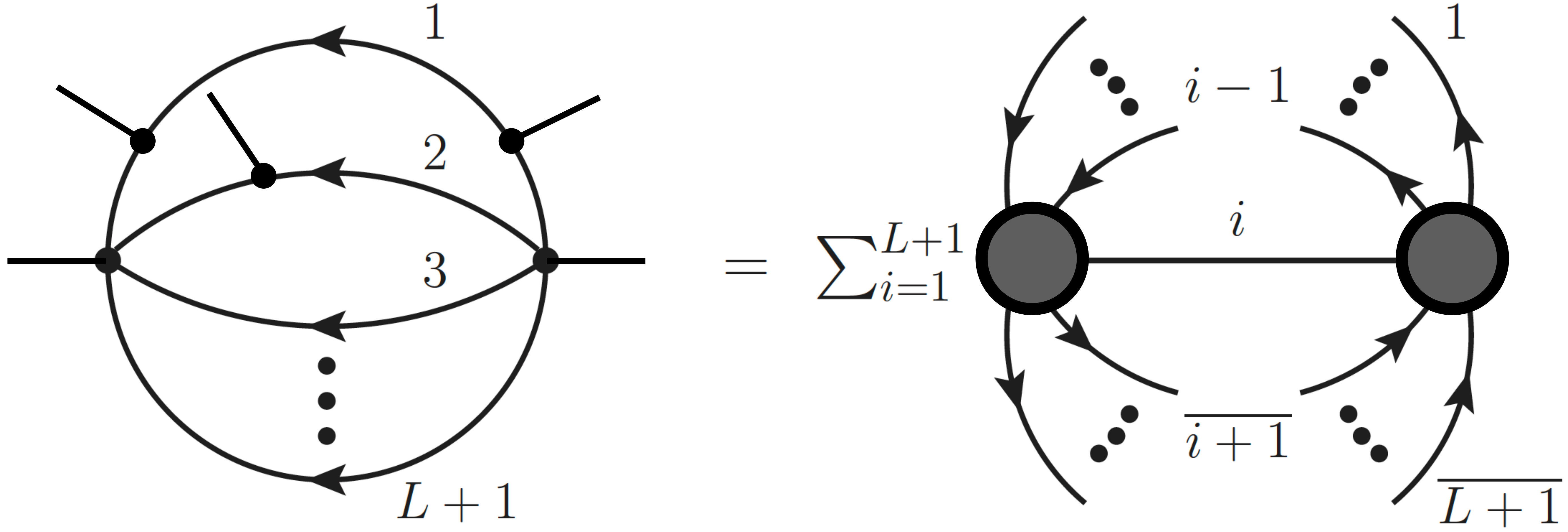


- $L+1$ sets of momenta, arbitrary number of external particles and arbitrary number of raised propagators
- The only topology needed at **two loops**

$$\mathscr{A}^{(L)}_{\rm MLT} = \int_{\vec\ell_1\ldots\vec\ell_L} \sum_{i=1}^{L+1} \mathscr{A}_{\rm D}(1,\ldots,i-1,\overline{i+1},\ldots,\overline{L+1};i)$$

Aguilera, Driencourt, Hernández, Plenter, Ramírez, Rentería, Rodrigo, Sborlini, Torres, Tracz, PRL 124 (2020) 211602

The maximal loop topology (MLT)



- $L+1$ sets of momenta, arbitrary number of external particles and arbitrary number of raised propagators
- The only topology needed at **two loops**

$$\mathcal{A}^{(L)}_{\rm MLT} = \int_{\vec{\ell}_1 \ldots \vec{\ell}_L} \sum_{i=1}^{L+1} \mathcal{A}_{\rm D}(1,\ldots,i-1,\overline{i+1},\ldots,\overline{L+1};i)$$

Lotty
THE LOOP-TREE DUALITY AUTOMATION
Torres Bobadilla

Aguilera, Driencourt, Hernández, Plenter, Ramírez, Rentería, Rodrigo, Sborlini, Torres, Tracz, PRL 124 (2020) 211602

The maximal loop topology (MLT)

Σ 

- If we sum up all the nested residues, e.g. scalar integral with one external momentum

$$\mathcal{A}^{(L)}_{\mathrm{MLT}} = \int_{\vec{\ell}_1 \ldots \vec{\ell}_L} \frac{1}{\prod 2 q^{(+)}_{i,0}} \left(\frac{1}{\lambda^{+}_{1,n}} + \frac{1}{\lambda^{-}_{1,n}} \right), \qquad \lambda^{\pm}_{1,n} = \sum q^{(+)}_{i,0} \pm k_{(1,n),0} \qquad q^{(+)}_{i,0} = \sqrt{\boldsymbol{q}_i^2 + m_i^2 - \imath 0}$$

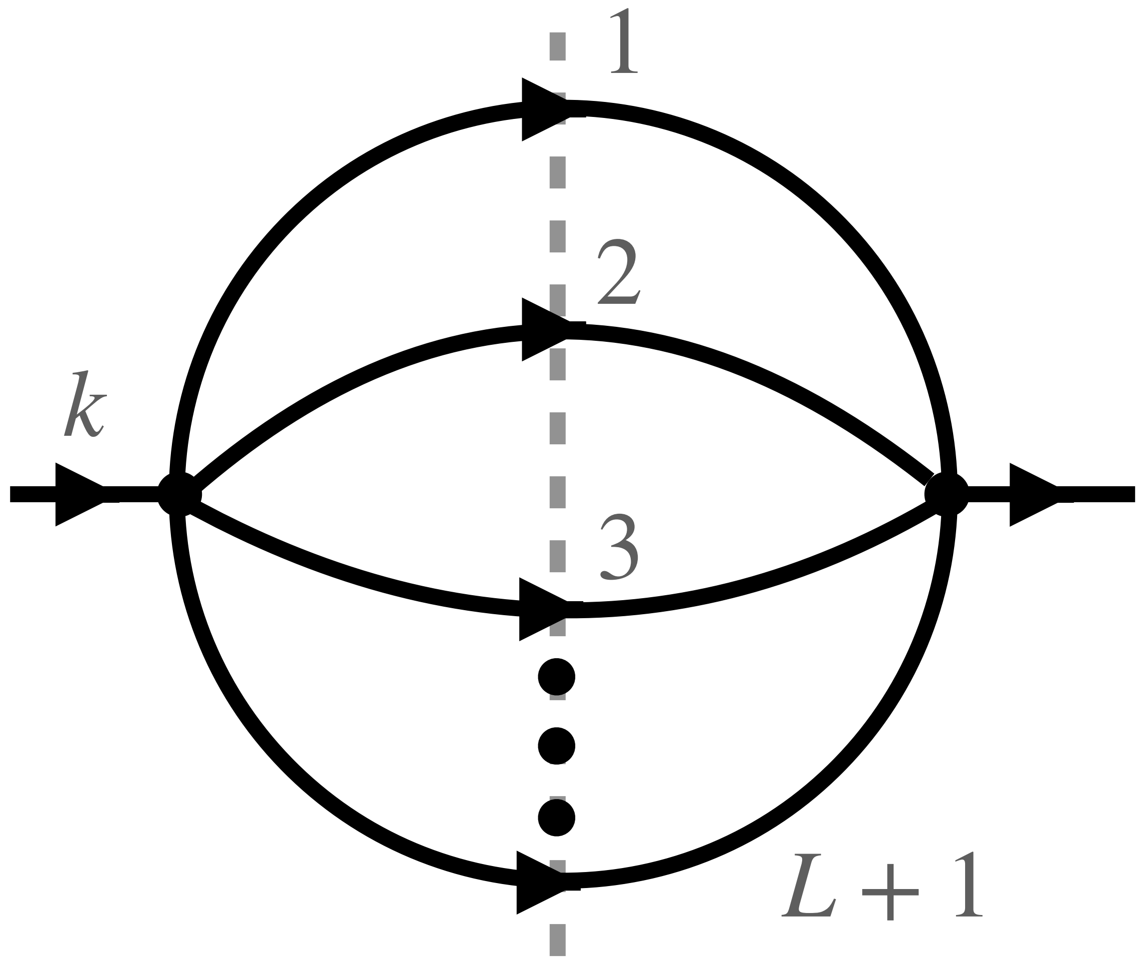
Aguilera, Driencourt, Hernández, Plenter, Ramírez, Rentería, Rodrigo, Sborlini, Torres, Tracz, PRL 124 (2020) 211602

The maximal loop topology (MLT)

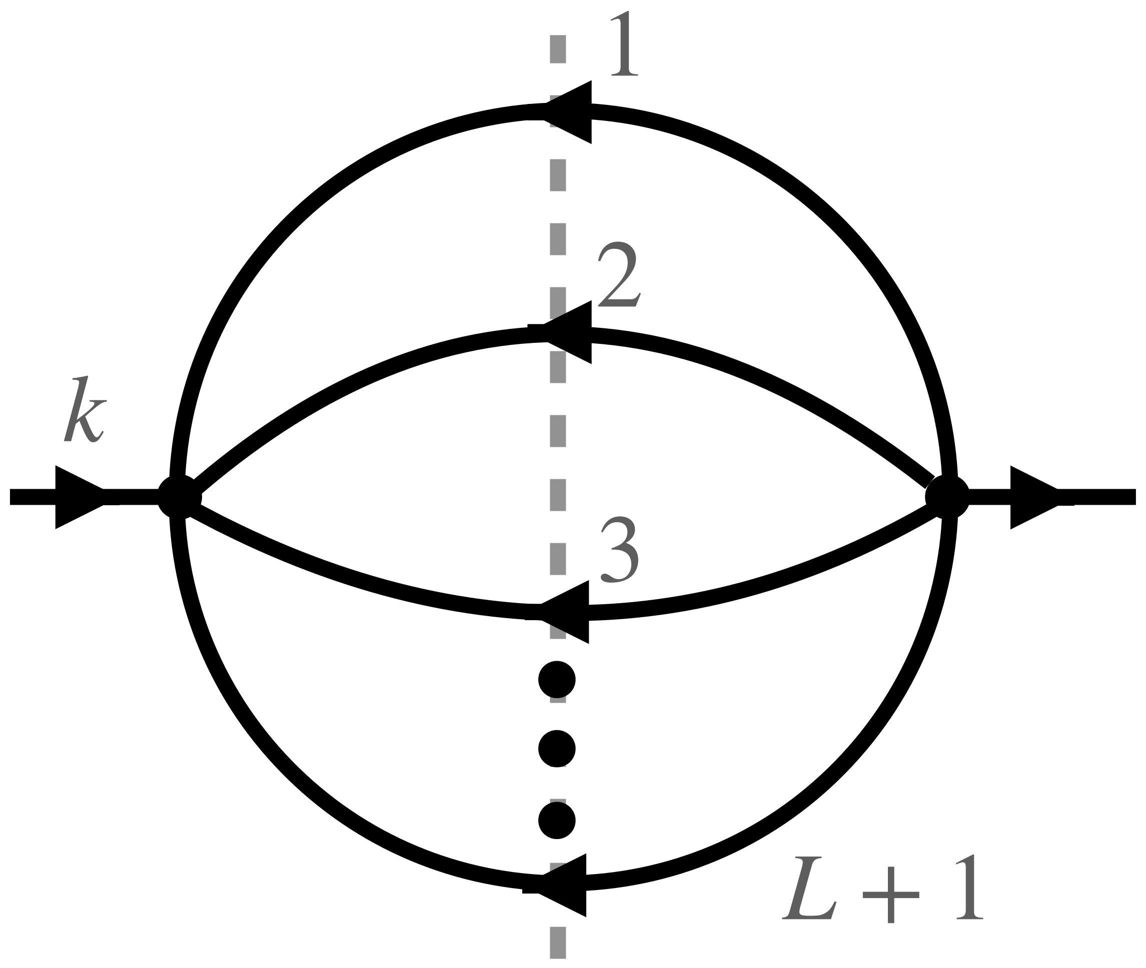
- If we sum up all the nested residues, e.g. scalar integral with one external momentum

$$\mathscr{A}^{(L)}_{\rm MLT} = \int_{\vec{\ell}_1 \ldots \vec{\ell}_L} \frac{1}{\prod 2 q^{(+)}_{i,0}} \left(\frac{1}{\lambda^{+}_{1,n}} + \frac{1}{\lambda^{-}_{1,n}} \right), \qquad \lambda^{\pm}_{1,n} = \sum q^{(+)}_{i,0} \pm k_{(1,n),0} \qquad q^{(+)}_{i,0} = \sqrt{\boldsymbol{q}_i^2 + m_i^2 - \imath 0}$$

$=$

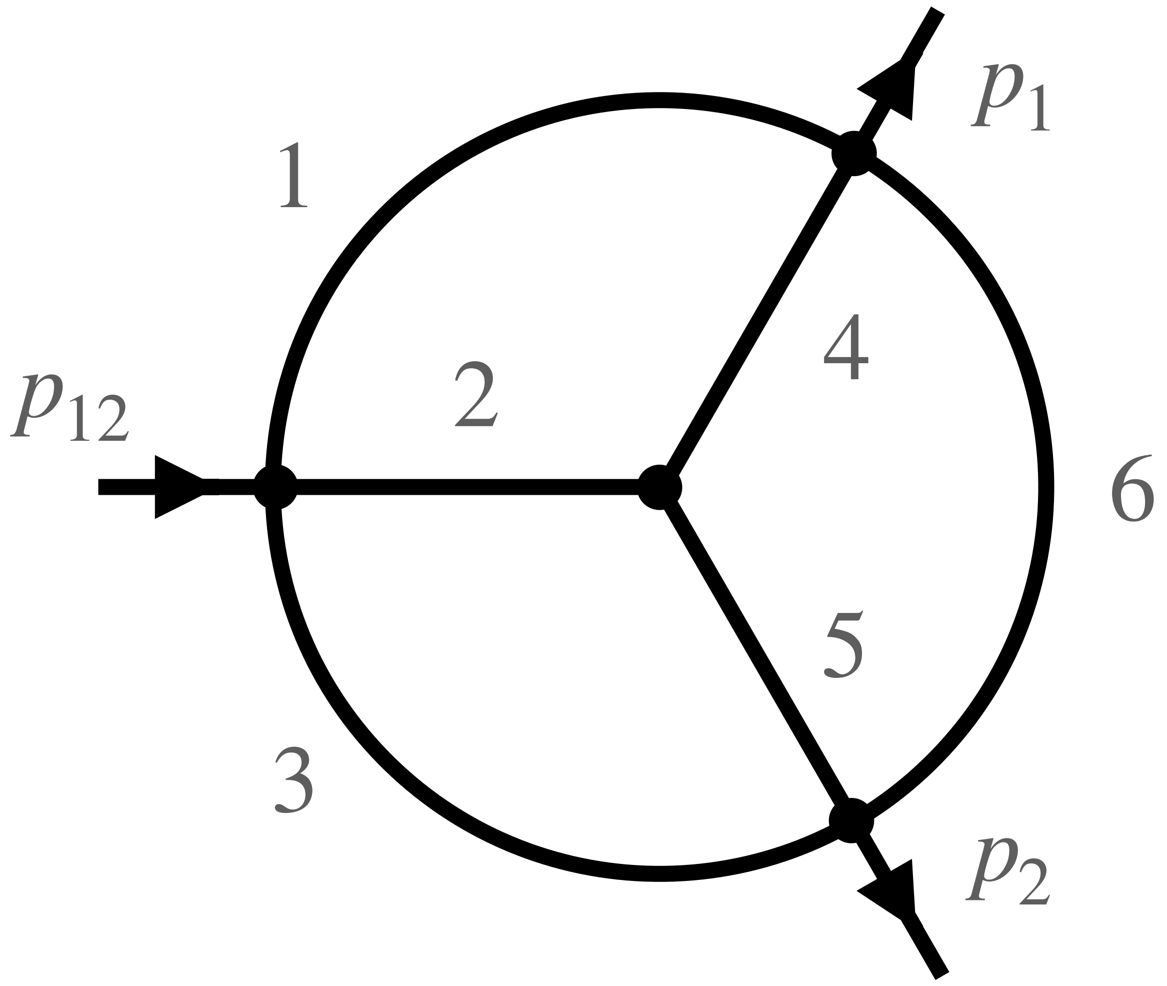


$+$

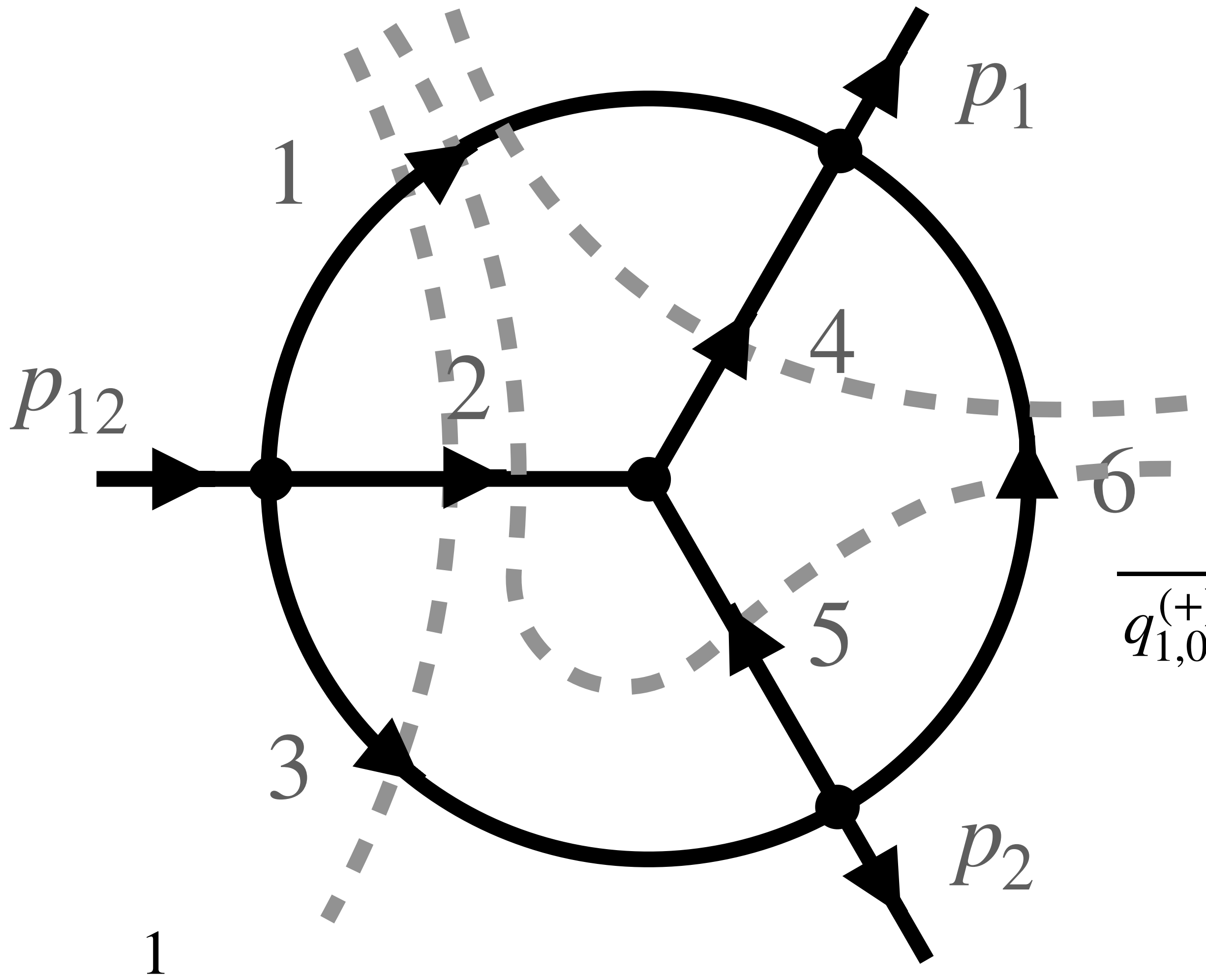


- Independent of the initial momentum flow assignments
- **Only causal configurations (DAG) survive**

LTD is manifestly causal



$= \sum$



$$\frac{1}{q_{1,0}^{(+)} + q_{4,0}^{(+)} + q_{5,0}^{(+)} - p_{1,0}}$$

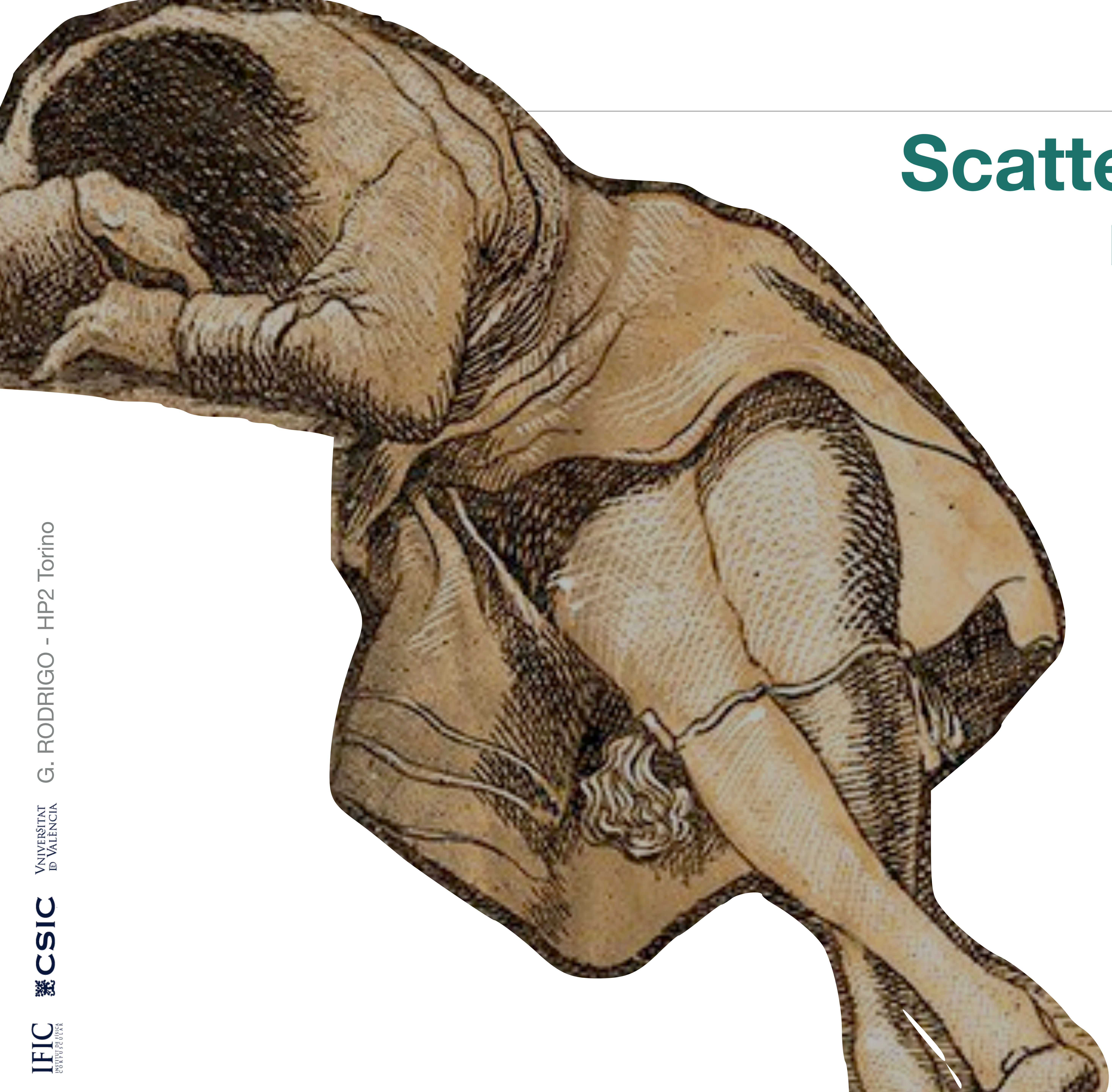
$$\frac{1}{q_{1,0}^{(+)} + q_{2,0}^{(+)} + q_{5,0}^{(+)} + q_{6,0}^{(+)} - p_{1,0}}$$

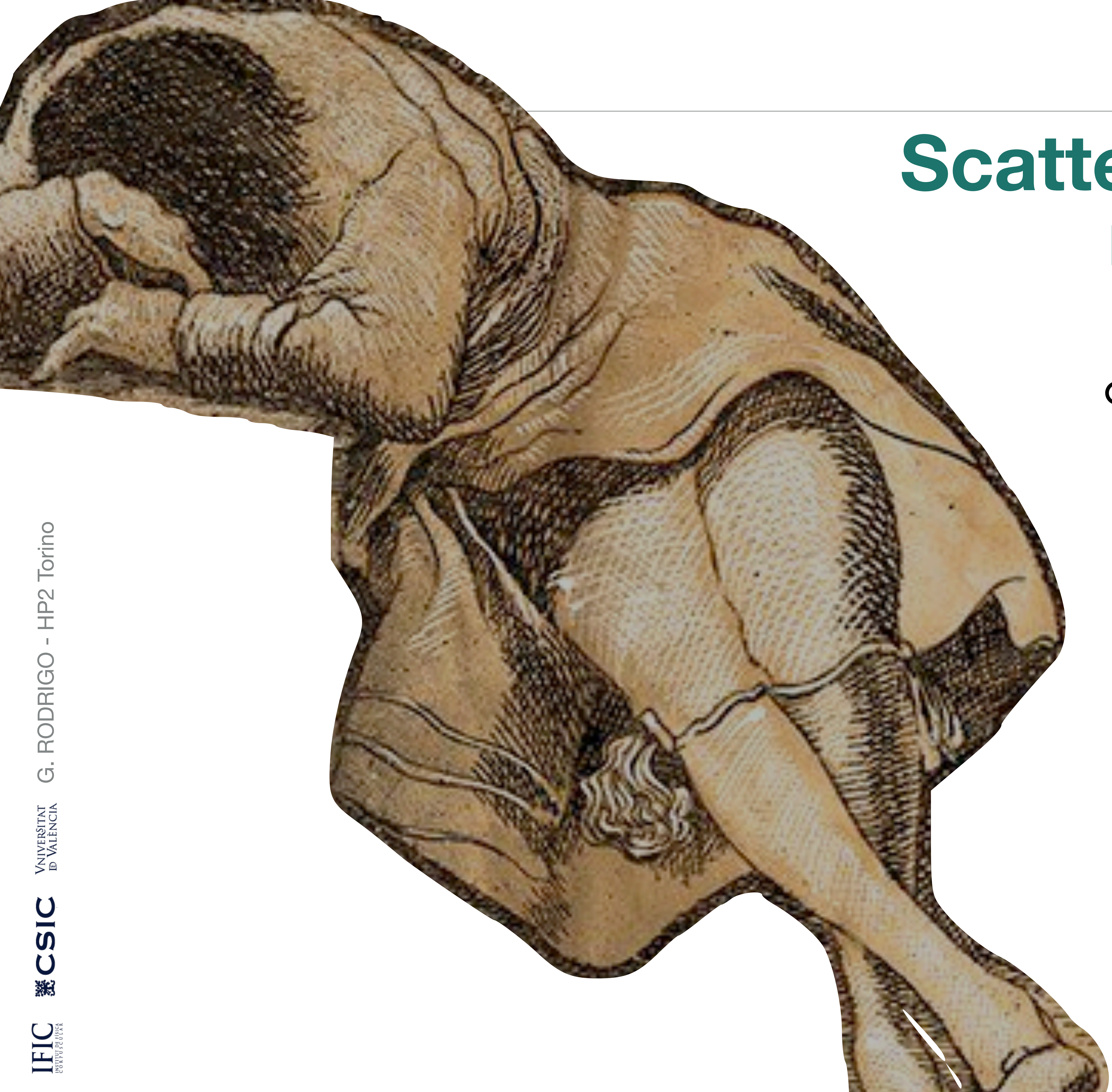
$$\frac{1}{q_{1,0}^{(+)} + q_{2,0}^{(+)} + q_{3,0}^{(+)} - p_{12,0}}$$

$$\mathscr{A}_{\rm LTD}^{(L)} = \int_{\vec{\ell}_1 \ldots \vec{\ell}_L} \frac{1}{\prod 2q_{i,0}^{(+)}} \sum_{\sigma\in\Sigma} \frac{\mathcal{N}_{\sigma(i_1,\ldots,i_{n-L})}}{\lambda_{\sigma(i_1)}^{h_1} \cdots \lambda_{\sigma(i_{n-L})}^{h_{n-L}}} + (\lambda_p^+ \leftrightarrow \lambda_p^-)$$

- Each combination of compatible causal propagators in Σ fixes the momentum flows of all the internal momenta
- Conversely, if we fix the causal momentum flows we can **bootstrap the causal LTD representation**
- **A causal propagator is singular when all internal particles involved are on shell, and divides the amplitude into two subamplitudes**
- geometric interpretation: Sborlini PRD **104** (2021) 036014

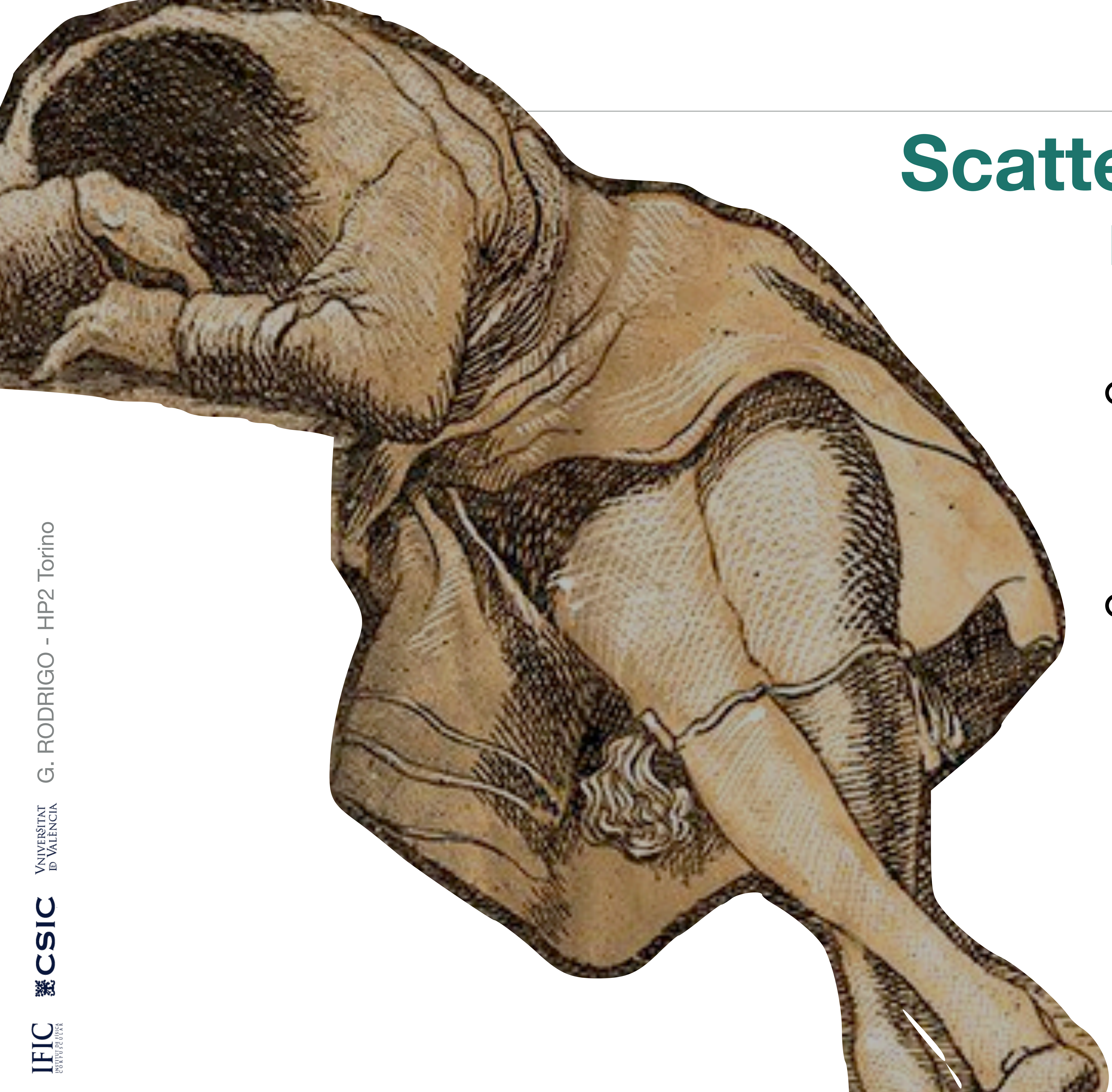
Scattering amplitudes are not physical objects





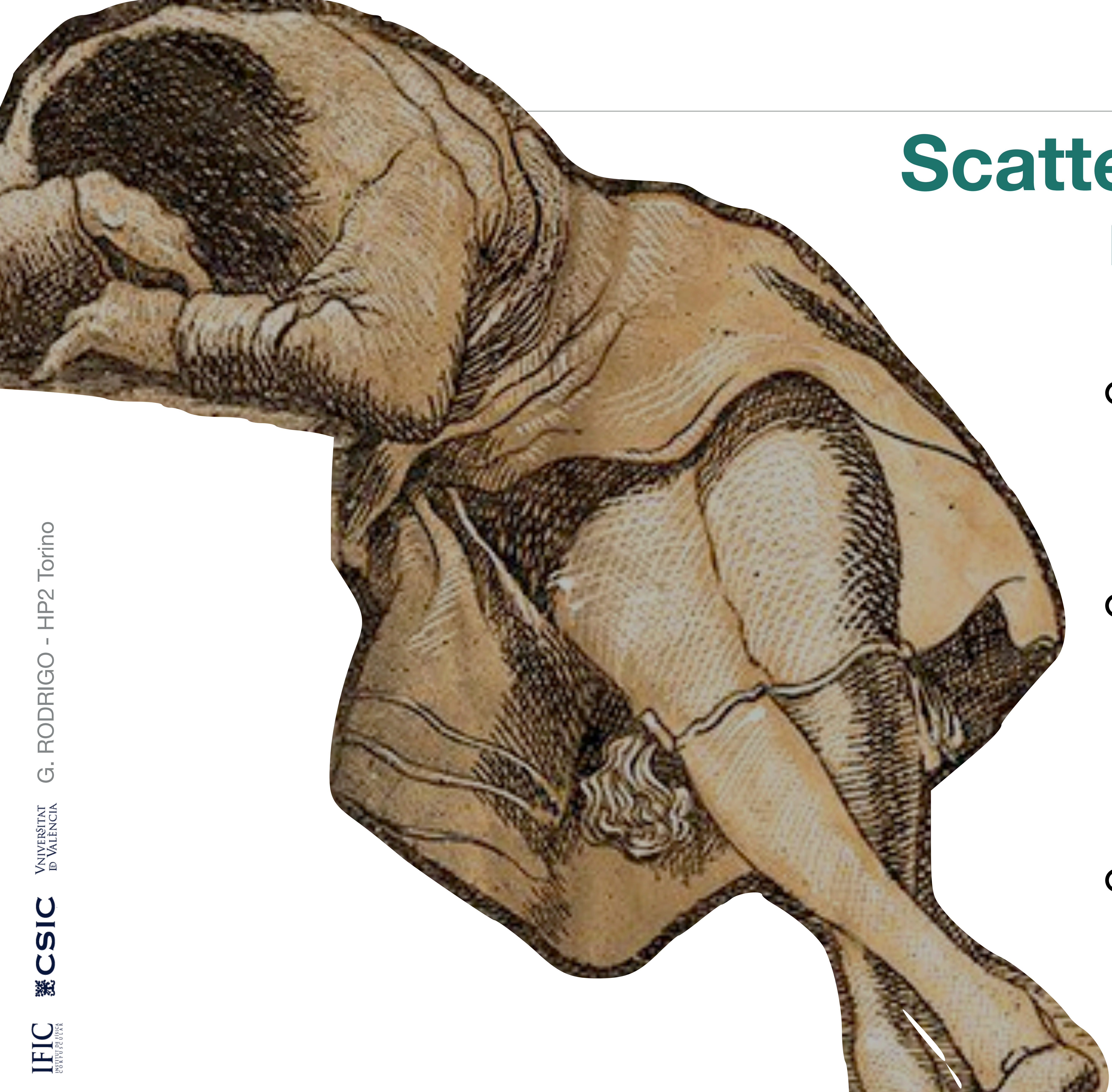
Scattering amplitudes are not physical objects

- Theoretical predictions at high-energy colliders from squared amplitudes



Scattering amplitudes are not physical objects

- Theoretical predictions at high-energy colliders from squared amplitudes
- They are defined for a **fixed number of external particles**, i.e. artificially separating loop and tree-level contributions



Scattering amplitudes are not physical objects

- Theoretical predictions at high-energy colliders from squared amplitudes
- They are defined for a **fixed number of external particles**, i.e. artificially separating loop and tree-level contributions
- Requires to work in an **unphysical spacetime** (DREG)

SVBLATA CAVSA TOLLITVR EFFECTVS

- If the number of external particles is the problem, let's drop external particles
- The solution is in the **vacuum**

muerto el perro, se acabó la rabia
dead dogs don't bite
tote Hunde beissen nicht

Starting Hypothesis

- The most efficient building blocks for assembling theory predictions are **vacuum amplitudes**, i.e., scattering amplitudes without external particles

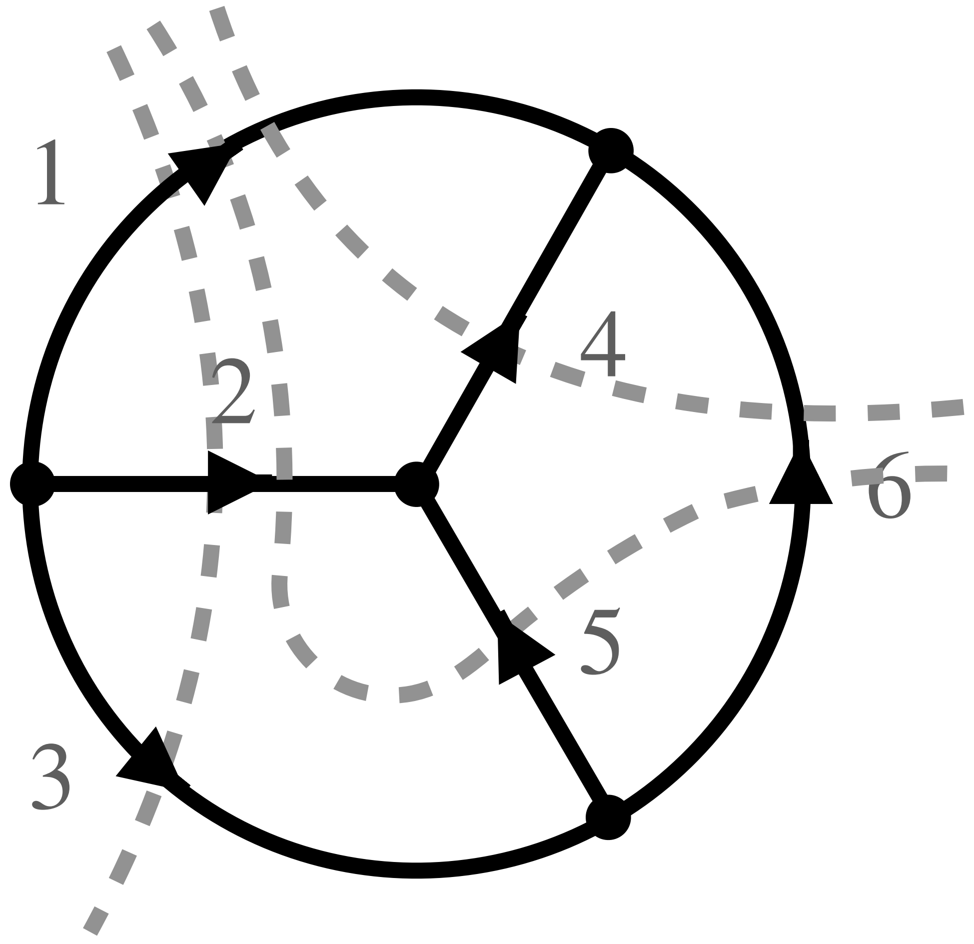
+

- **Loop-tree duality (LTD)** is the most suitable and powerful framework to implement this idea because it provides a **manifestly causal representation**

- S. Ramírez Uribe, P.K. Dhani, G.F.R. Sborlini, GR, "***Rewording theoretical predictions at colliders with vacuum amplitudes***," 2404.05491
- S. Ramírez Uribe, A.E. Rentería Olivo, D.F. Rentería Estrada, J.J. Martínez de Lejarza, P.K. Dhani, L. Cieri, R.J. Hernández Pinto, G.F.R. Sborlini, W.J. Torres Bobadilla, GR, "***Vacuum amplitudes and time-like causal unitary in the loop-tree duality***, 2404.05492
- J.J. Martínez de Lejarza, D.F. Rentería Estrada, M. Grossi, GR, "***Quantum integration of decay rates at second order in perturbation theory***,"

Vacuum amplitudes in LTD

$= \sum$

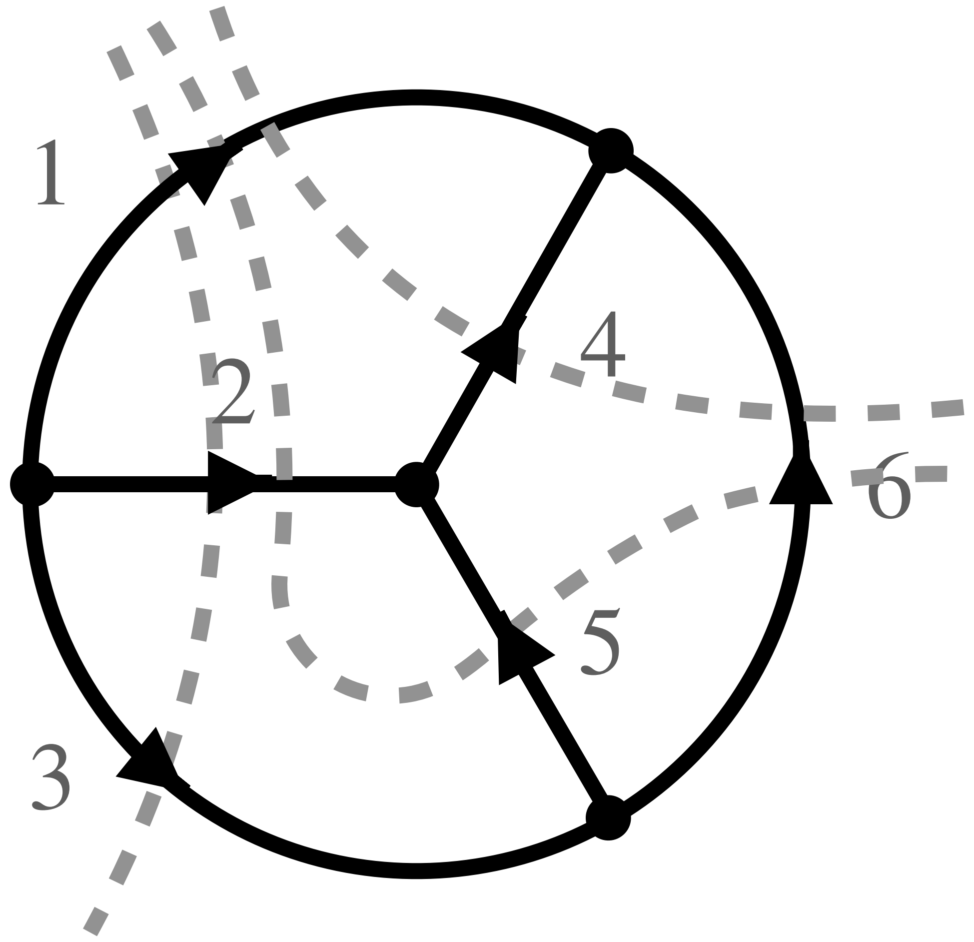


- Feynman propagators are substituted by **causal propagators** of the form

$$\frac{1}{\lambda_{i_1 i_2 \cdots i_n}} = \frac{1}{\sum_{s=1}^{n} q_{i_s,0}^{(+)}} , \qquad q_{i_s,0}^{(+)} = \sqrt{\boldsymbol{q}_{i_s}^2 + m_{i_s}^2 - \imath 0} \ .$$

Vacuum amplitudes in LTD

$= \sum$

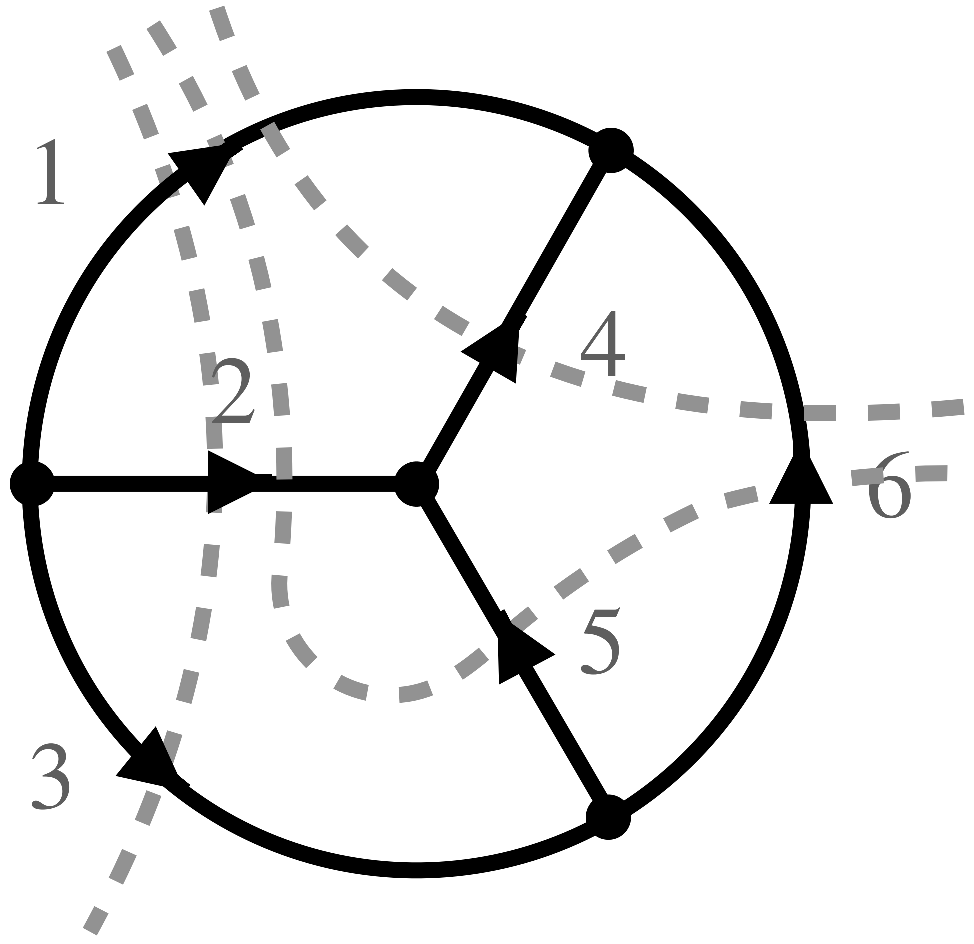


- Feynman propagators are substituted by **causal propagators** of the form

$$\frac{1}{\lambda_{i_1 i_2 \cdots i_n}} = \frac{1}{\sum_{s=1}^{n} q_{i_s,0}^{(+)}} , \qquad q_{i_s,0}^{(+)} = \sqrt{\boldsymbol{q}_{i_s}^2 + m_{i_s}^2 - \imath 0} \ .$$

- Each causal propagator involves a set of internal particles that divide the amplitude in **two subamplitudes**, with the momentum flow of all particles in the set aligned in the same direction

Vacuum amplitudes in LTD

$= \sum$ 

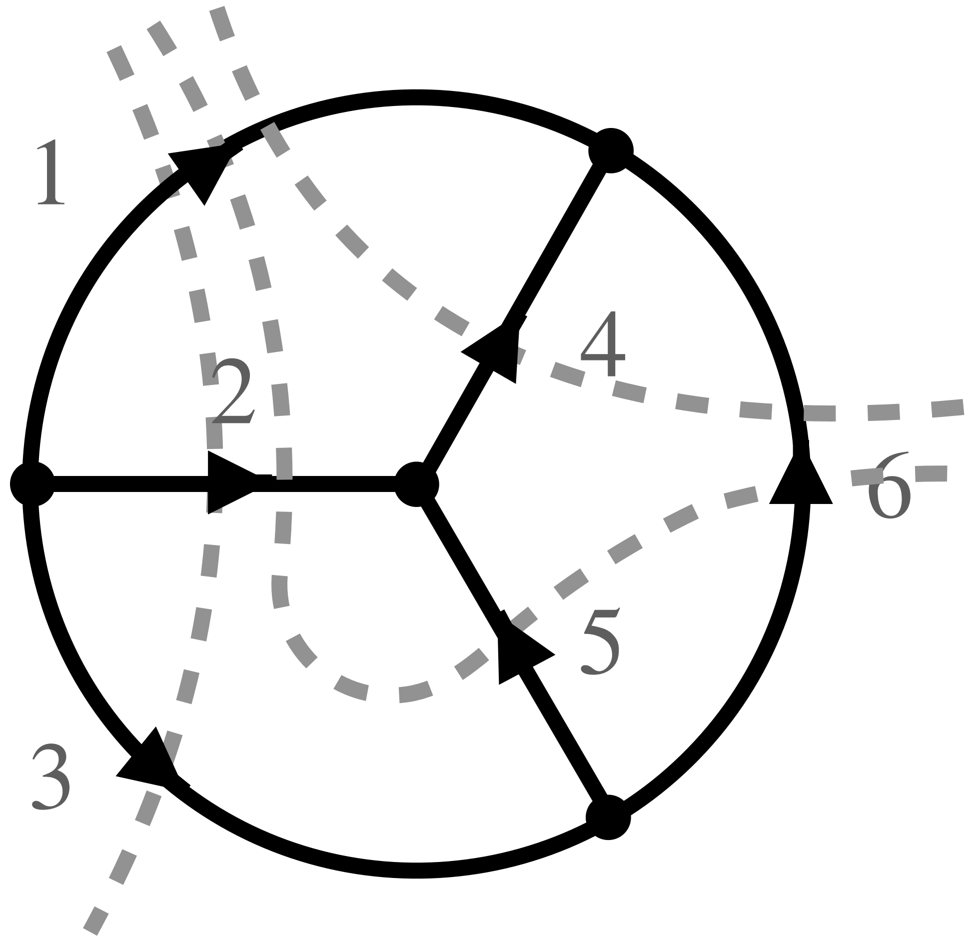
- Feynman propagators are substituted by **causal propagators** of the form

$$\frac{1}{\lambda_{i_1 i_2 \cdots i_n}} = \frac{1}{\sum_{s=1}^{n} q_{i_s,0}^{(+)}} \,, \qquad q_{i_s,0}^{(+)} = \sqrt{\boldsymbol{q}_{i_s}^2 + m_{i_s}^2 - \imath 0} \,.$$

- Each causal propagator involves a set of internal particles that divide the amplitude in **two subamplitudes**, with the momentum flow of all particles in the set aligned in the same direction
- If $\lambda_{i_1 i_2 \cdots i_n} = 0$, all particles in the set would become on shell and the vacuum amplitude would represent an **interference**, but $\lambda_{i_1 i_2 \cdots i_n}$ cannot vanish !!!

Vacuum amplitudes in LTD

$= \sum$



- Feynman propagators are substituted by **causal propagators** of the form

$$\frac{1}{\lambda_{i_1 i_2 \cdots i_n}} = \frac{1}{\sum_{s=1}^{n} q_{i_s,0}^{(+)}} , \qquad q_{i_s,0}^{(+)} = \sqrt{\boldsymbol{q}_{i_s}^2 + m_{i_s}^2 - \imath 0} \, .$$

- Each causal propagator involves a set of internal particles that divide the amplitude in **two subamplitudes**, with the momentum flow of all particles in the set aligned in the same direction
- If $\lambda_{i_1 i_2 \cdots i_n} = 0$, all particles in the set would become on shell and the vacuum amplitude would represent an **interference**, but $\lambda_{i_1 i_2 \cdots i_n}$ cannot vanish !!!
- **Generate all final states from residues on causal propagators after analytic continuation to negative values of those in the initial state: phase-space residues**

$$\mathscr{A}_{\rm D}^{(\Lambda,{\rm R})}(i_1 \cdots i_n ab) = {\rm Res}\left(\frac{x_{ab}}{2} \mathscr{A}_{\rm D}^{(\Lambda)}, \lambda_{i_1 \cdots i_n ab} \right) - \mathscr{A}_{\rm UV/C}^{(\Lambda)}(i_1 \cdots i_n ab)$$

$x_{ab} = 4 q_{a,0}^{(+)} q_{b,0}^{(+)}$

local UV renormalisation and local subtraction of initial-state collinear

without momentum mappings [FDU 2016; Prisco, Tramontano 2021; Pozzorini's talk]
or momentum rescaling [Soper; Local unitarity]

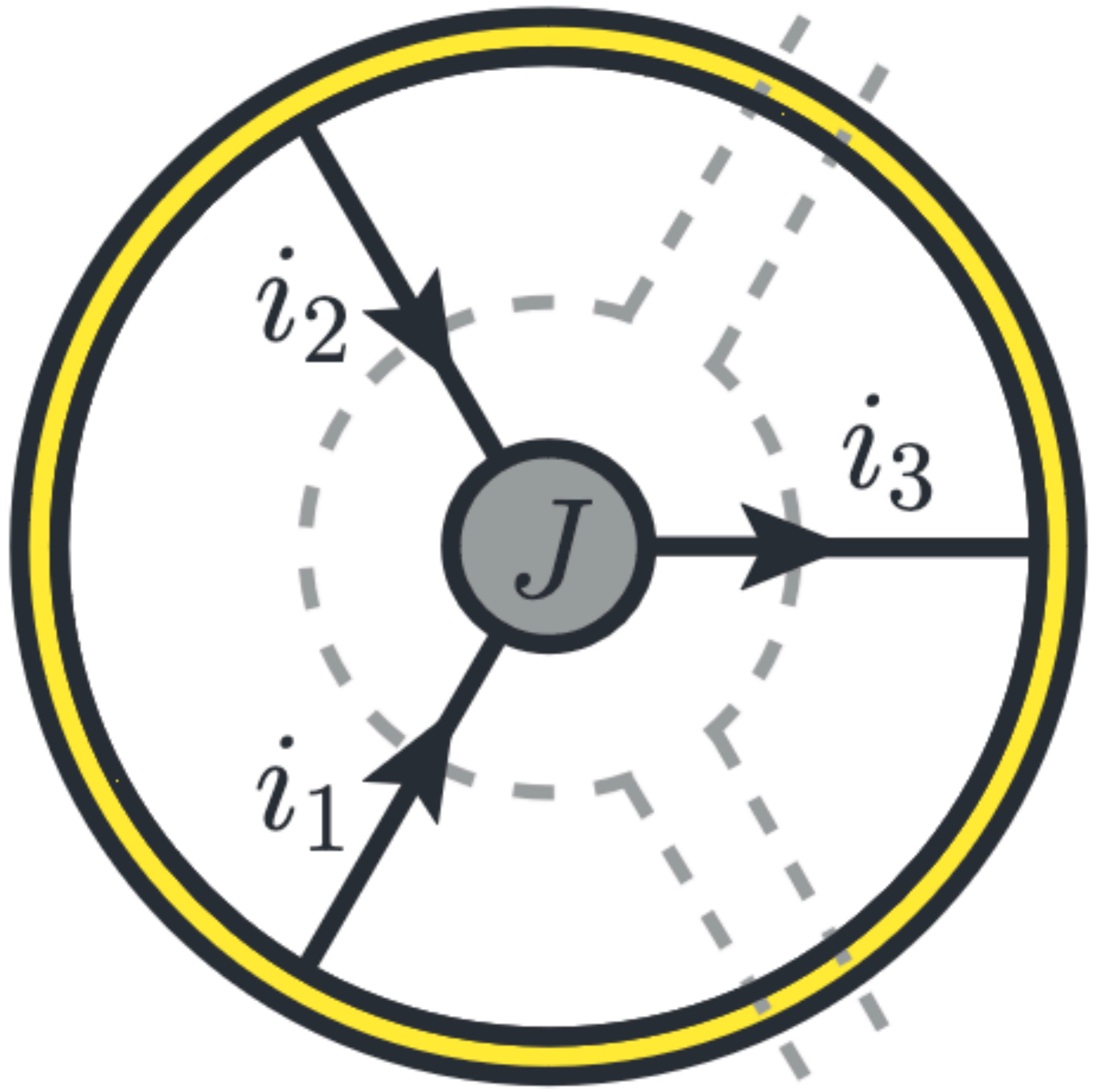
Master representation of differential observables

- From a **vacuum amplitude** $\mathscr{A}_{\rm D}^{(\Lambda)}$ in the LTD representation that depends on Λ loop momenta

$$d\sigma_{\rm N^kLO} = \frac{d\Lambda}{2s} \sum_{(i_1\cdots i_n ab)\in\Sigma} \mathscr{A}_{\rm D}^{(\Lambda,{\rm R})}(i_1\cdots i_n ab)\, \mathcal{O}_{i_1\cdots i_n}\, \tilde\Delta_{i_1\cdots i_n\bar a\bar b}$$

- One common integration measure $d\Lambda = \prod_{j=1}^{\Lambda-2} d\Phi_{\ell_j} = \prod_{j=1}^{\Lambda-2} \mu^{4-d} \frac{d^{d-1}\ell_j}{(2\pi)^{d-1}}$
- Specific observable, if $\mathcal{O}_{i_1\cdots i_n} = \mathbf{1}$, total cross section/decay rate
- Energy conservation $\tilde\Delta_{i_1\cdots i_n\bar a\bar b} = 2\pi\,\delta(\lambda_{i_1\cdots i_n\bar a\bar b})$
- Vacuum amplitudes are singular only in the UV, so soft, collinear and threshold singularities should **locally match** in the coherent sum over all phase-space residues (initial-state collinear limited by kinematics): well defined in the **four physical dimensions of the spacetime**

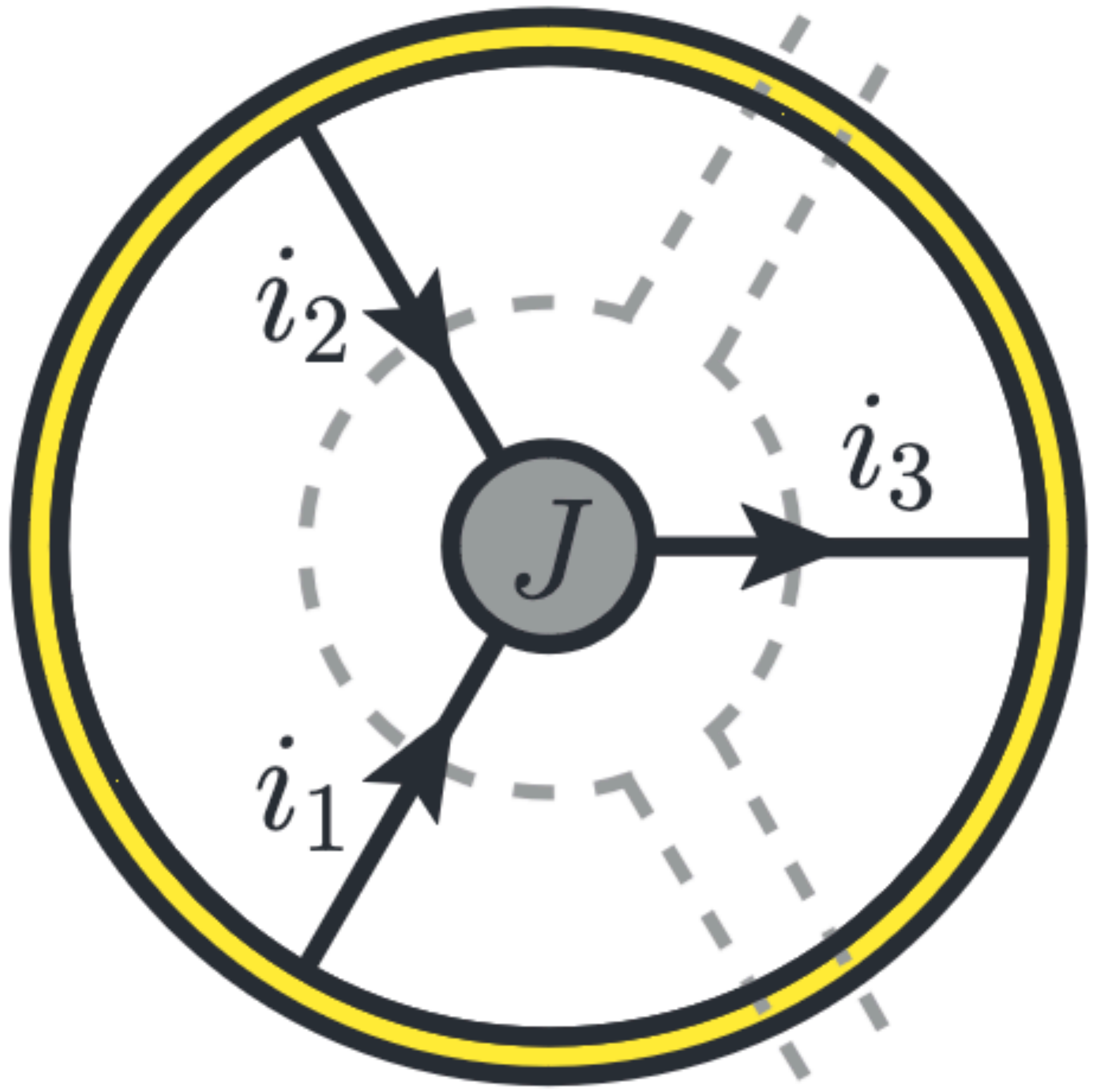
Double-collinear configuration



- A vacuum amplitude with the insertion of a trivalent interaction (it could be a multiloop subdiagram or an effective operator). The LTD vacuum amplitude is proportional to

$$\mathscr{A}_{\rm D}^{(\Lambda)} \sim \frac{1}{\lambda_{i_1 i_2 \cdots ab}\, \lambda_{i_3 \cdots ab}} \,,$$

Double-collinear configuration



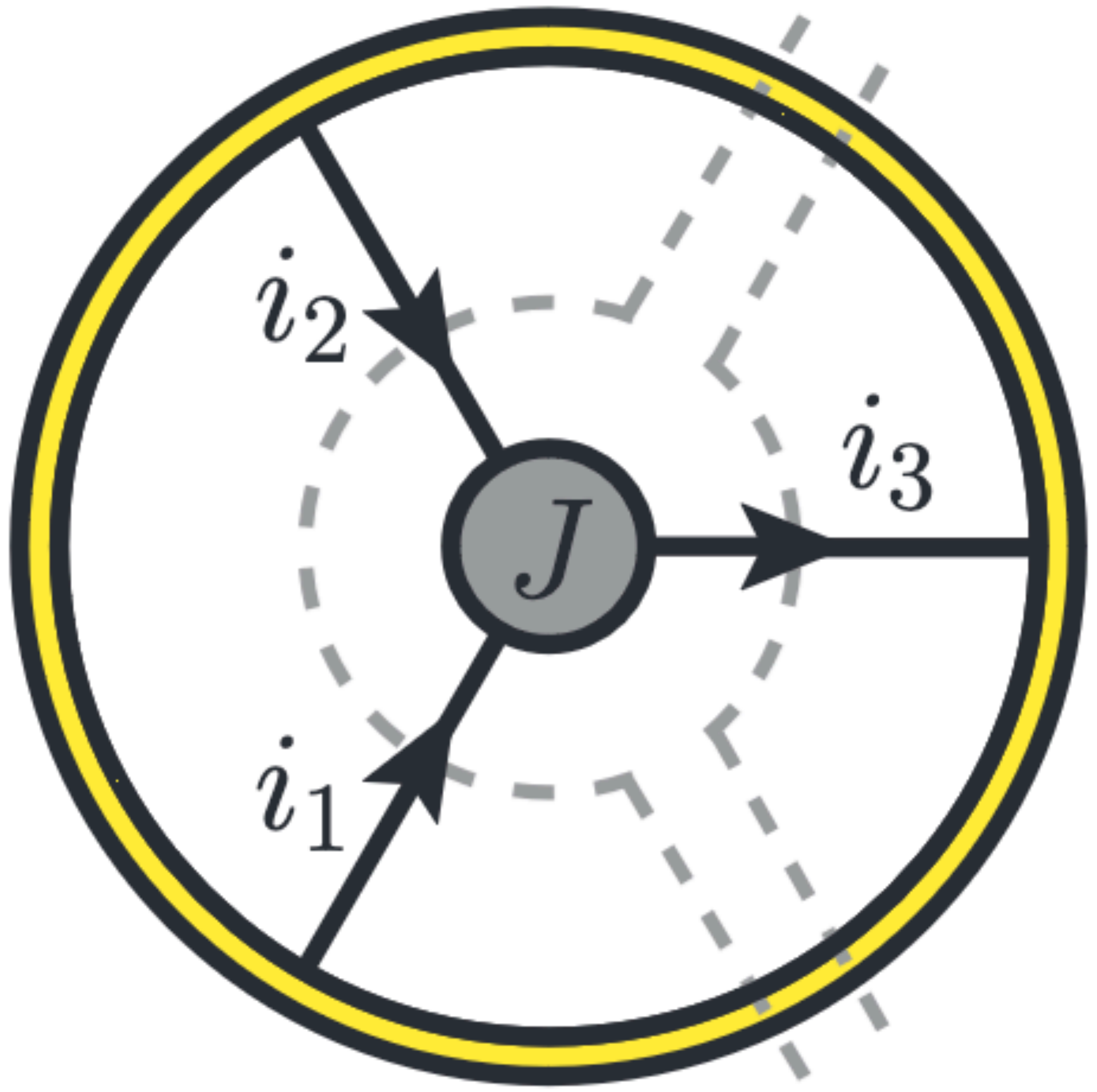
- A vacuum amplitude with the insertion of a trivalent interaction (it could be a multiloop subdiagram or an effective operator). The LTD vacuum amplitude is proportional to

$$\mathscr{A}_{\rm D}^{(\Lambda)} \sim \frac{1}{\lambda_{i_1 i_2 \cdots ab} \, \lambda_{i_3 \cdots ab}} \, ,$$

- Each phase-space residue is singular for $\lambda_{i_1 i_2 \bar{i}_3} = q_{i_1,0}^{(+)} + q_{i_2,0}^{(+)} - q_{i_3,0}^{(+)} \to 0,$ due to the following identities

$$\left. \frac{1}{\lambda_{i_1 i_2 \cdots ab}} \right|_{\lambda_{i_3 \cdots ab}=0} = \frac{1}{\lambda_{i_1 i_2 \bar{i}_3}} \, , \qquad \left. \frac{1}{\lambda_{i_3 \cdots ab}} \right|_{\lambda_{i_1 i_2 \cdots ab}=0} = - \frac{1}{\lambda_{i_1 i_2 \bar{i}_3}} \, ,$$

Double-collinear configuration



- A vacuum amplitude with the insertion of a trivalent interaction (it could be a multiloop subdiagram or an effective operator). The LTD vacuum amplitude is proportional to

$$\mathscr{A}_{\rm D}^{(\Lambda)} \sim \frac{1}{\lambda_{i_1 i_2 \cdots ab}\, \lambda_{i_3 \cdots ab}} ,$$

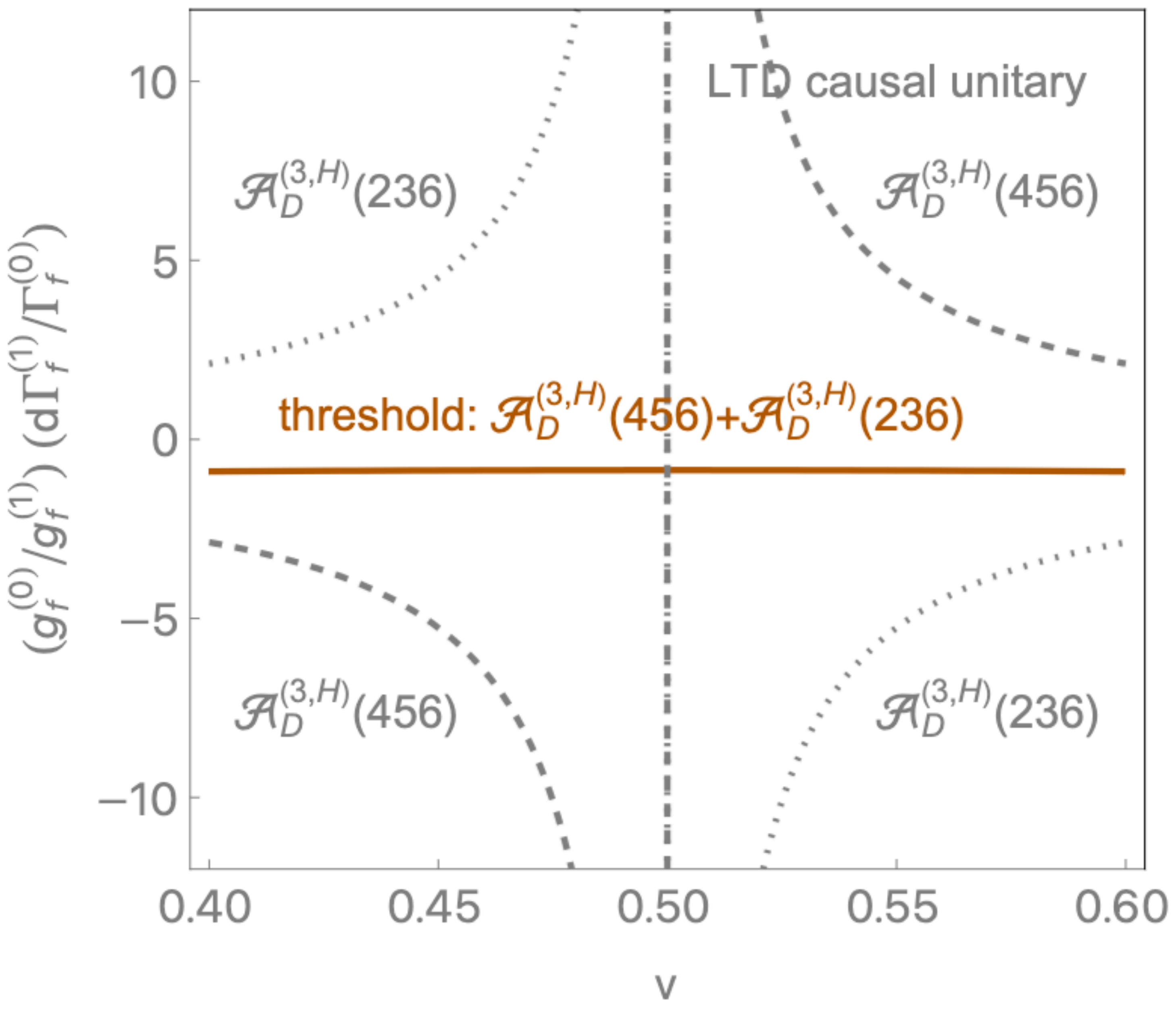
- Each phase-space residue is singular for $\lambda_{i_1 i_2 \bar{i}_3} = q_{i_1,0}^{(+)} + q_{i_2,0}^{(+)} - q_{i_3,0}^{(+)} \to 0$, due to the following identities

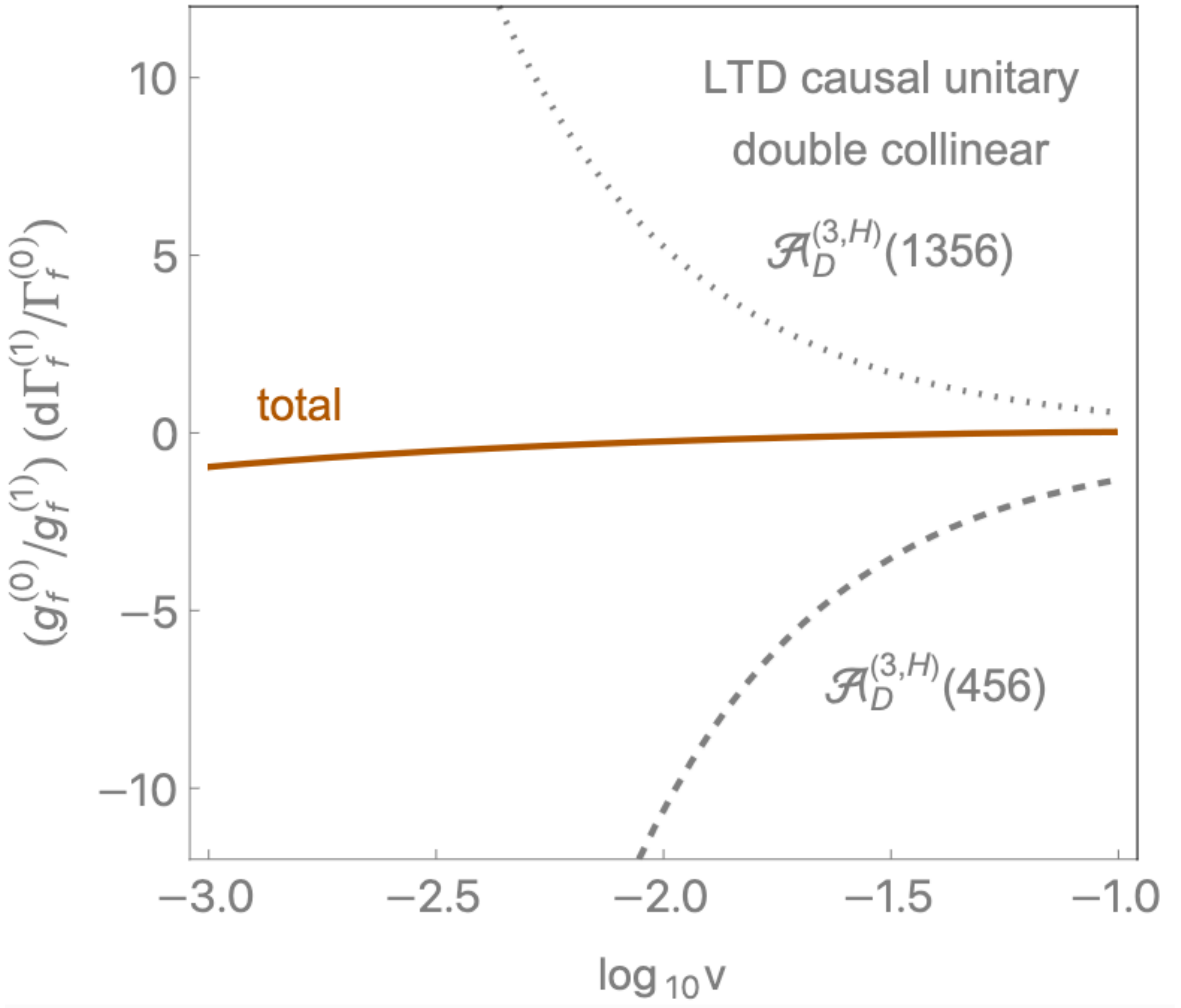
$$\left. \frac{1}{\lambda_{i_1 i_2 \cdots ab}} \right|_{\lambda_{i_3 \cdots ab}=0} = \frac{1}{\lambda_{i_1 i_2 \bar{i}_3}} , \qquad \left. \frac{1}{\lambda_{i_3 \cdots ab}} \right|_{\lambda_{i_1 i_2 \cdots ab}=0} = -\frac{1}{\lambda_{i_1 i_2 \bar{i}_3}} ,$$

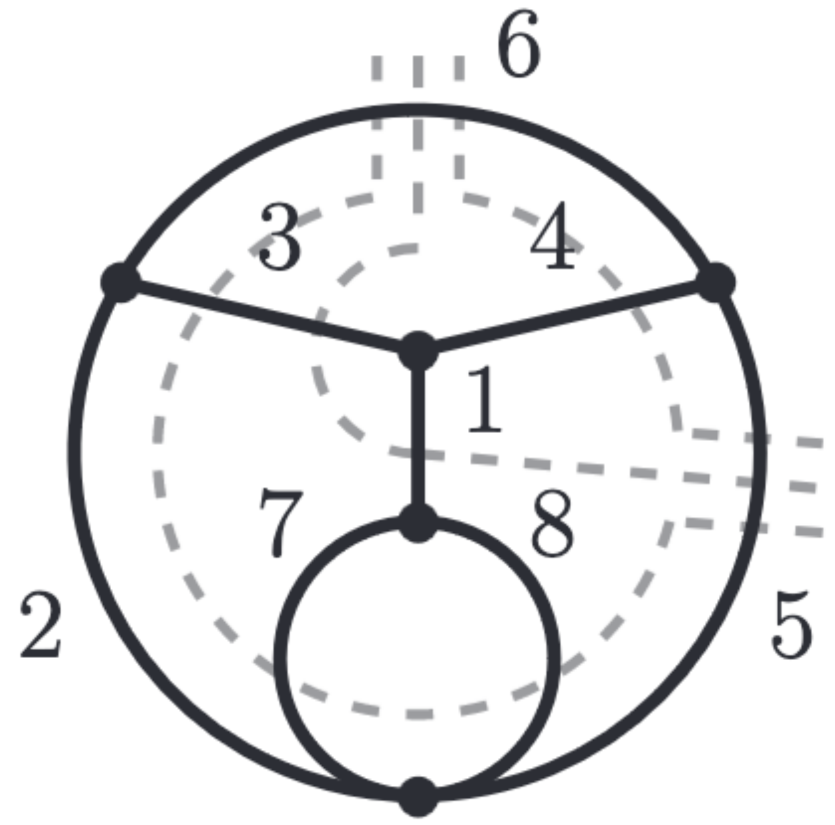
- Then, the sum of phase-space residues is finite in that limit:

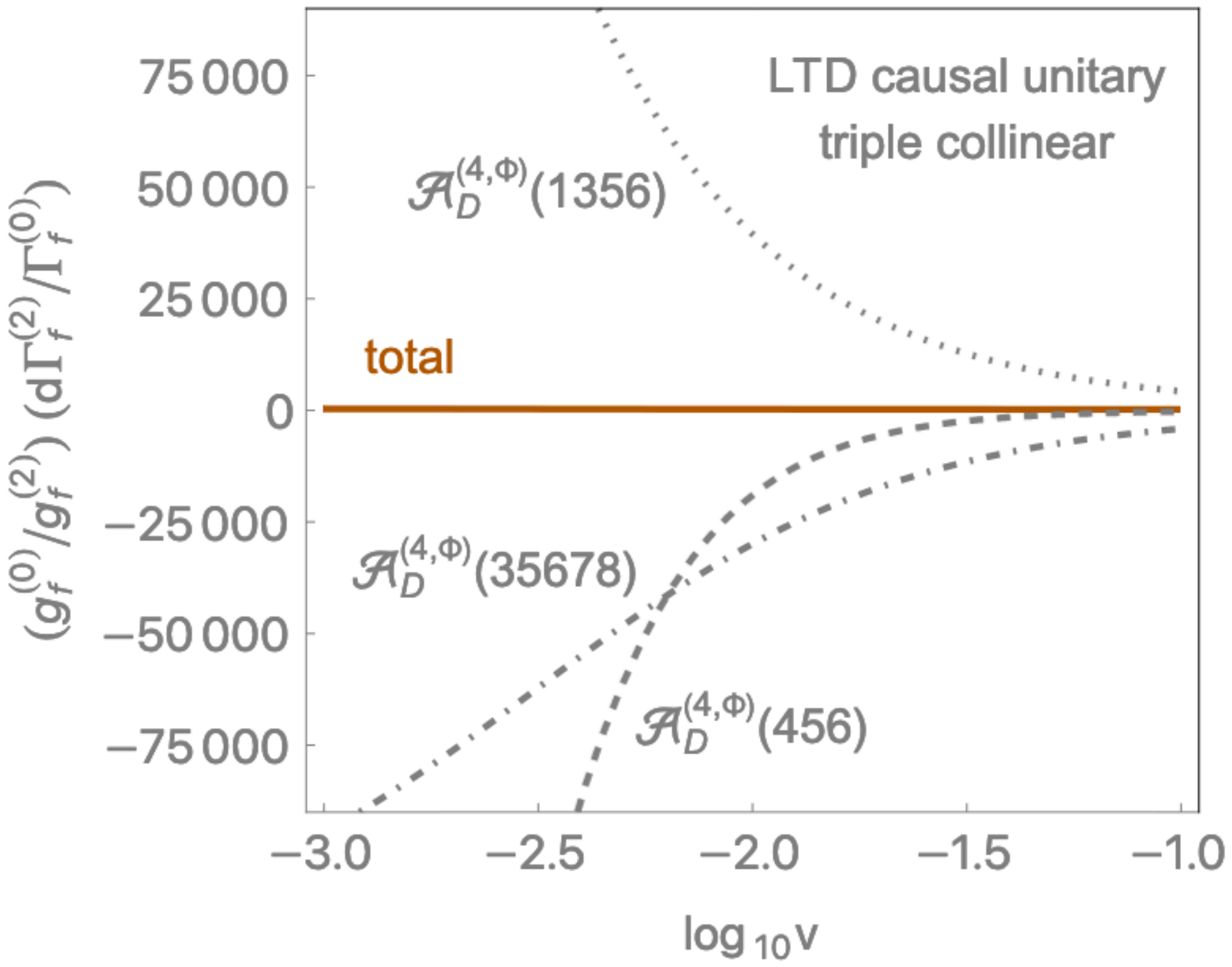
$$\lim_{\lambda_{i_1 i_2 \bar{i}_3} \to 0} \left(\mathscr{A}_{\rm D}^{(\Lambda)}(i_1 i_2 \cdots ab)\, \tilde{\Delta}_{i_1 i_2 \cdots \bar{a}\bar{b}} + \mathscr{A}_{\rm D}^{(\Lambda)}(i_3 \cdots ab)\, \tilde{\Delta}_{i_3 \cdots \bar{a}\bar{b}} \right) = \mathcal{O}(\lambda_{i_1 i_2 \bar{i}_3}^0) .$$

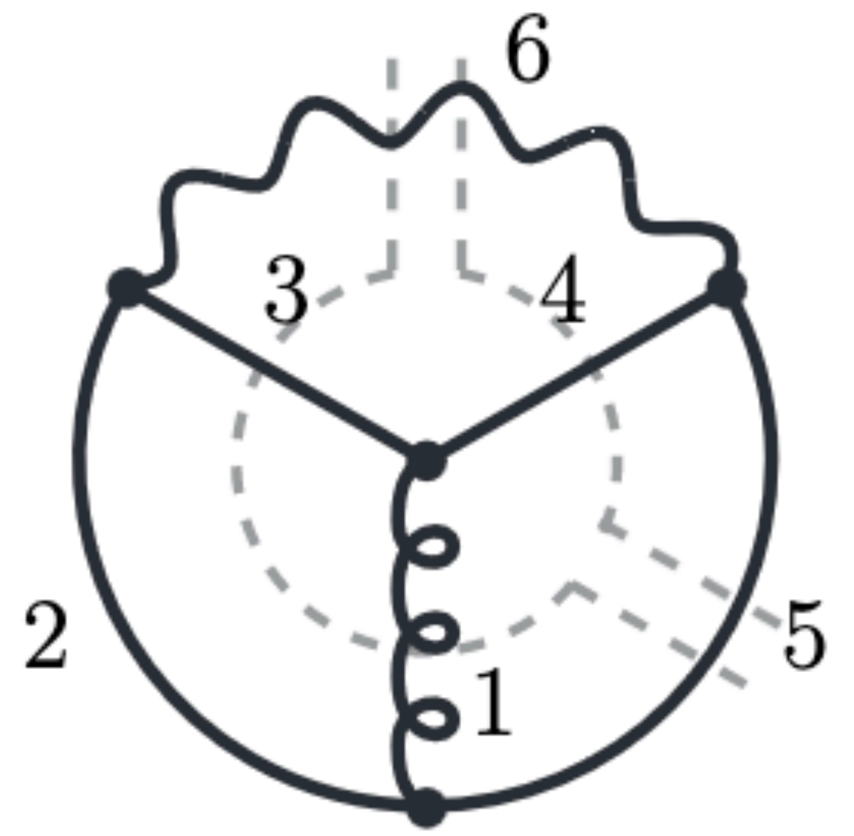
Proof of concept NLO and NNLO: flat integrands

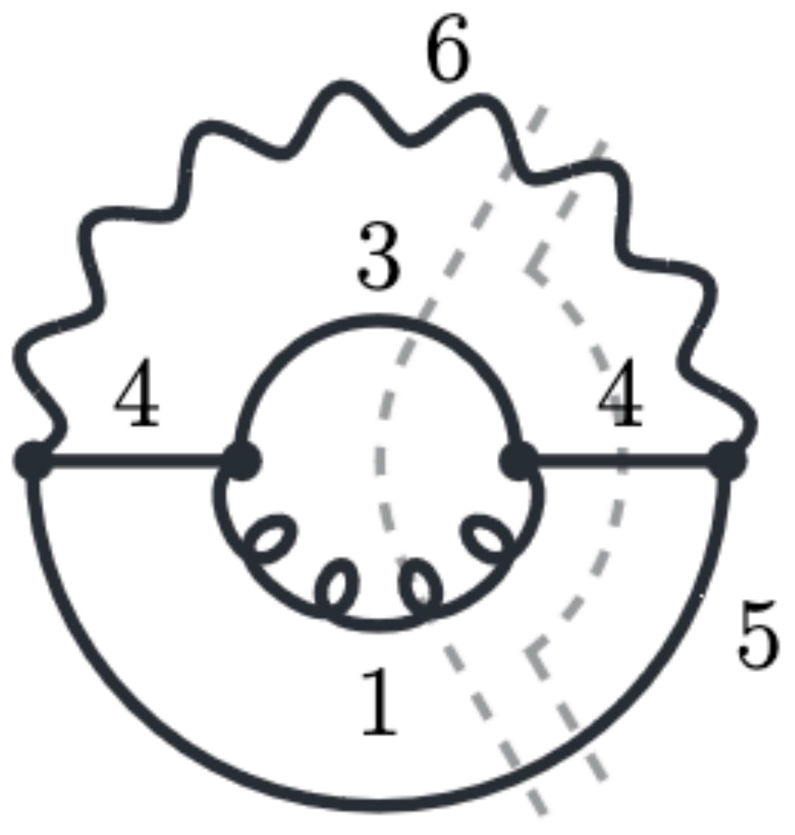












[See also Rios, Sborlini, 2402.13995]

Local UV renormalisation

- The wave function for external particles is also IR, selfenergy insertions required for a full local cancellation of IR singularities
- LTD causal unitary accounts properly for the wave function renormalisation of external legs

$$\mathscr{A}_{\rm UV}^{(3,H)}(456) = \frac{g_H^{(1)}}{x_{45}} \left[\Delta Z_H^{\rm (UV)} \overline{|\mathscr{M}_{H\to q\bar{q}}^{(0)}|^2} - \Delta Z_m^{\rm (UV)} 8m^2 \left(1+\beta^2\right) + \mathbf{\Delta}_H^{\rm (UV)} \right] ,$$

- Wave function + vertex interaction

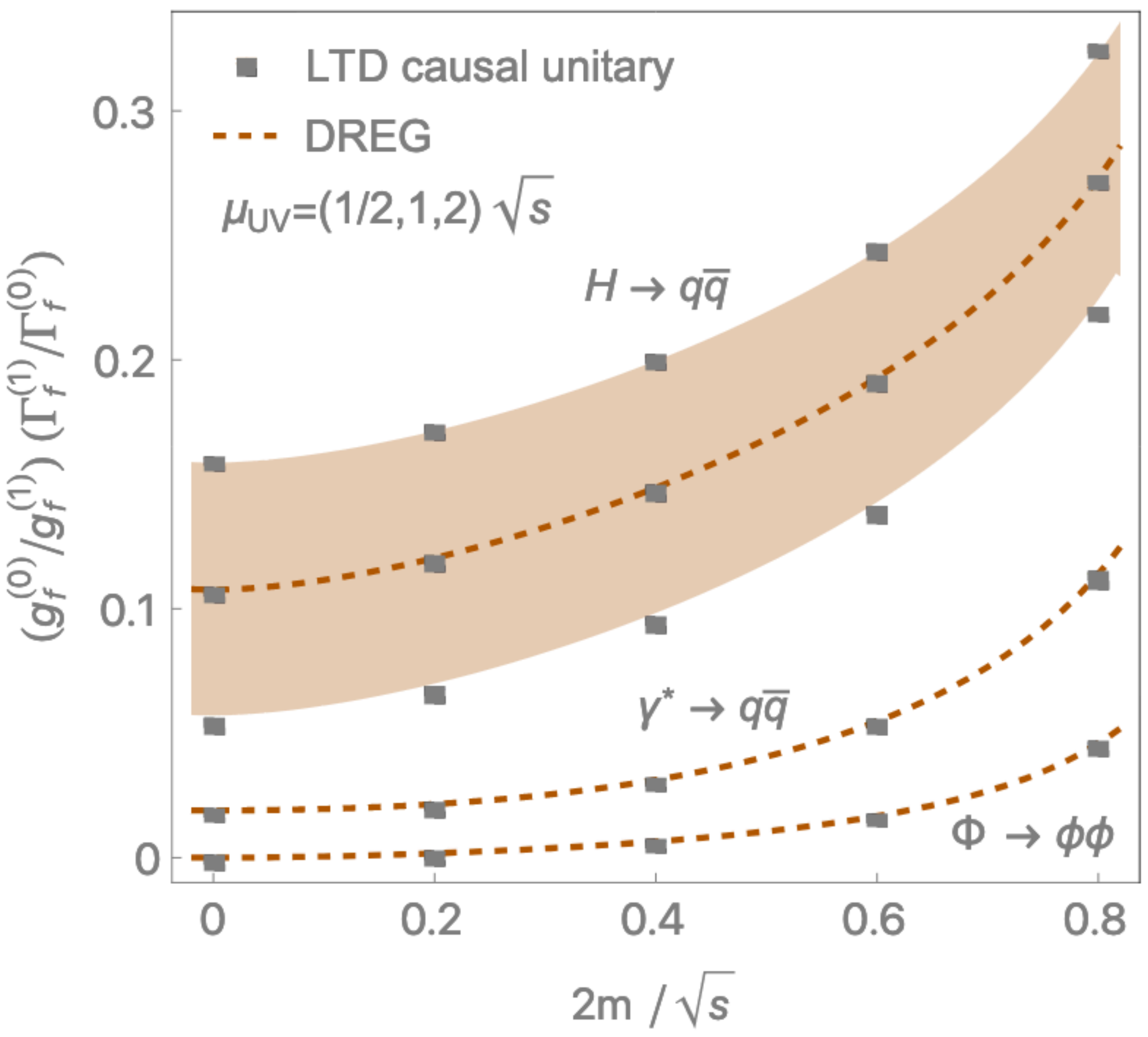
$$\Delta Z_H^{\rm (UV)} = \frac{1}{4\lambda_{\rm UV}^3} \left(c_H^{\rm (UV)} - c_\gamma^{\rm (UV)} + \frac{3\mu_{\rm UV}^2}{2\lambda_{\rm UV}^2} \right) , \qquad \lambda_{\rm UV} = \sqrt{\boldsymbol{\ell}_1^2 + \mu_{\rm UV}^2 - \imath 0}$$

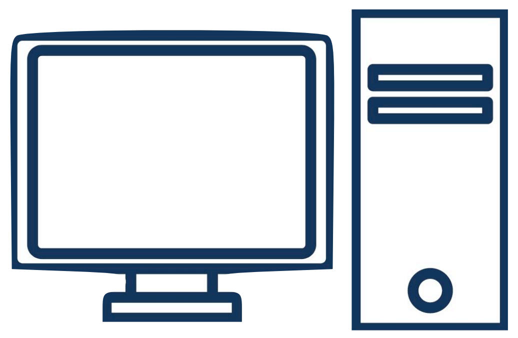
- Mass renormalisation

$$\Delta Z_m^{\rm (UV)} = \frac{1}{4\lambda_{\rm UV}^3} \left(c_H^{\rm (UV)} - c_\gamma^{\rm (UV)} + \frac{15\mu_{\rm UV}^2}{2\lambda_{\rm UV}^2} \right) \Bigg|_{\mu_{\rm UV}=m} .$$

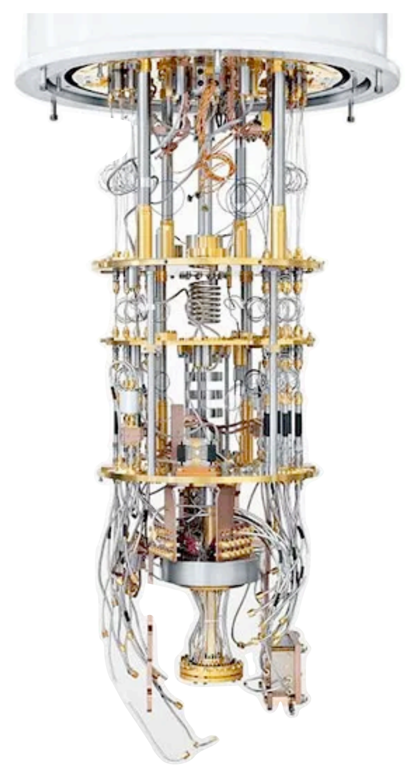
- The factor $\mathbf{\Delta}_H^{\rm (UV)}$ subtracts up to quadratic UV singularities that integrate to zero
- Conserved or partially conserved currents: e.g. $\Delta Z_\gamma^{\rm (UV)} = 0$

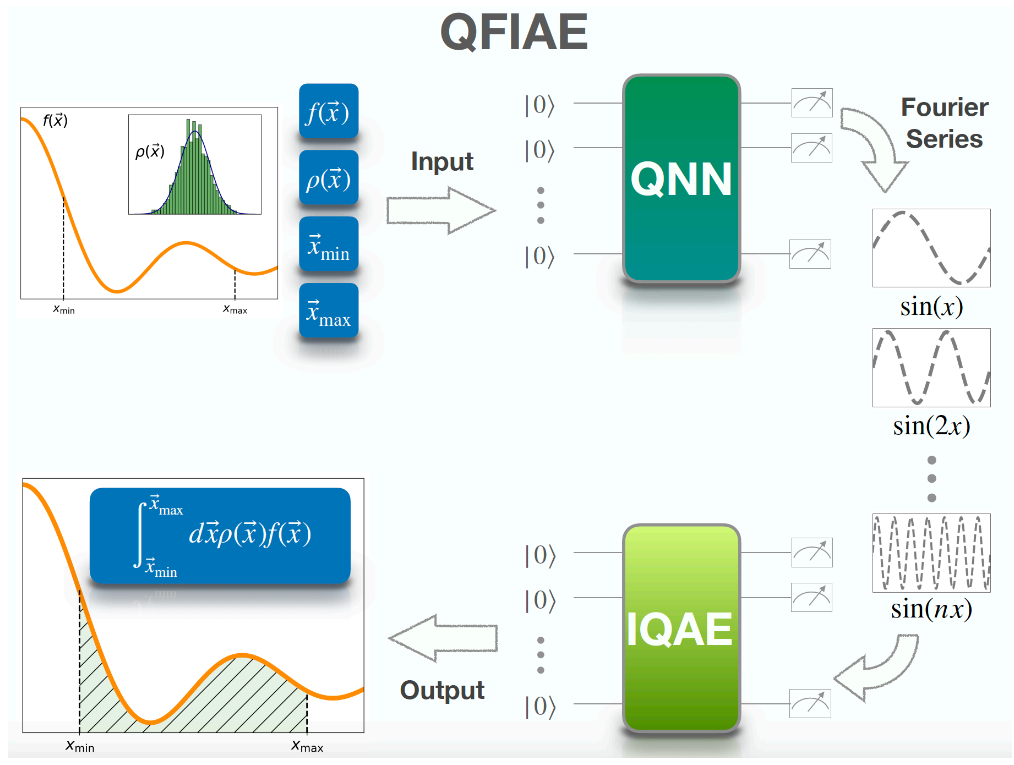
Proof of concept NLO: total decay rates





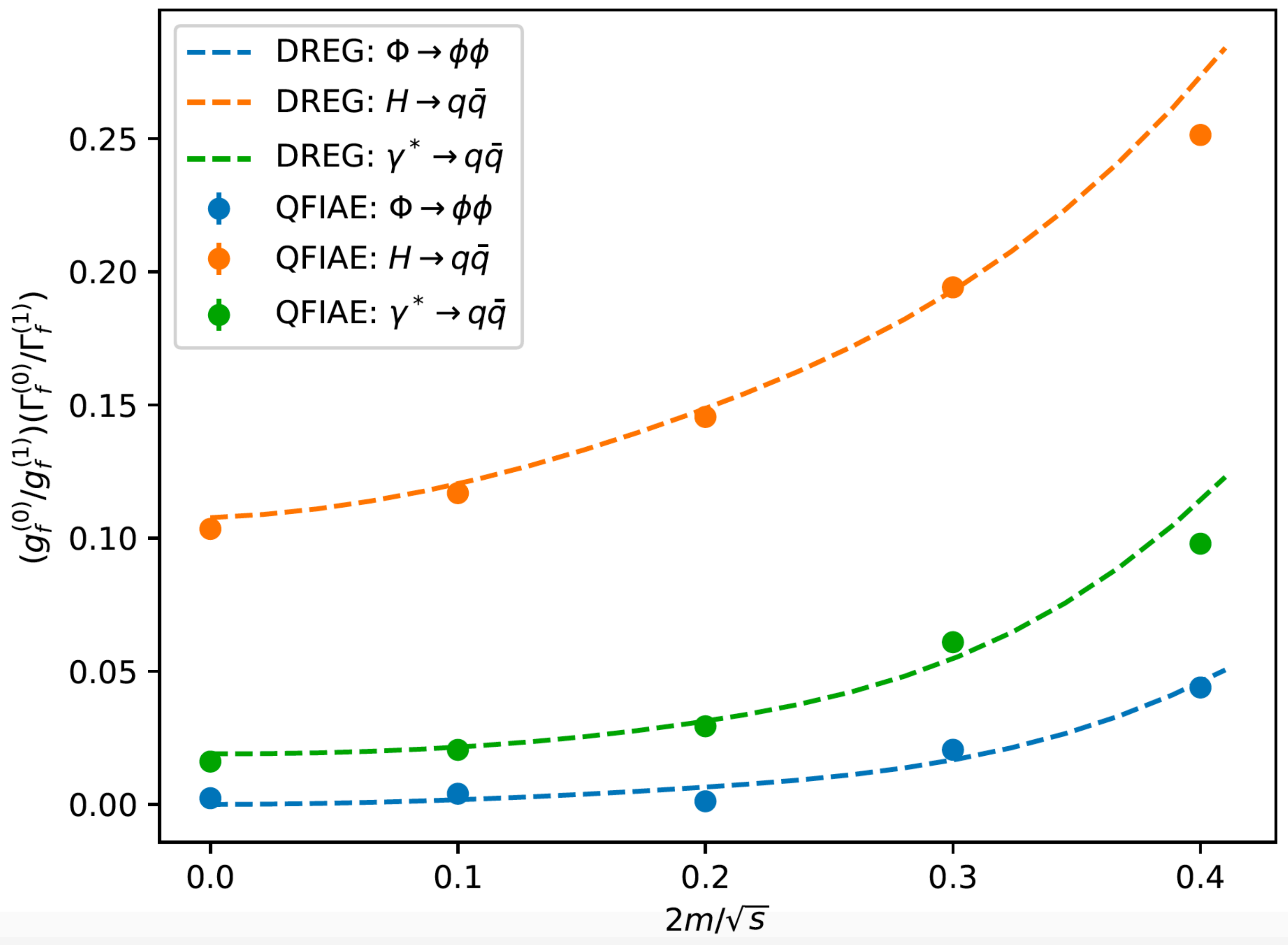
- Smooth massless limit: quasicollinear logs properly accounted for locally



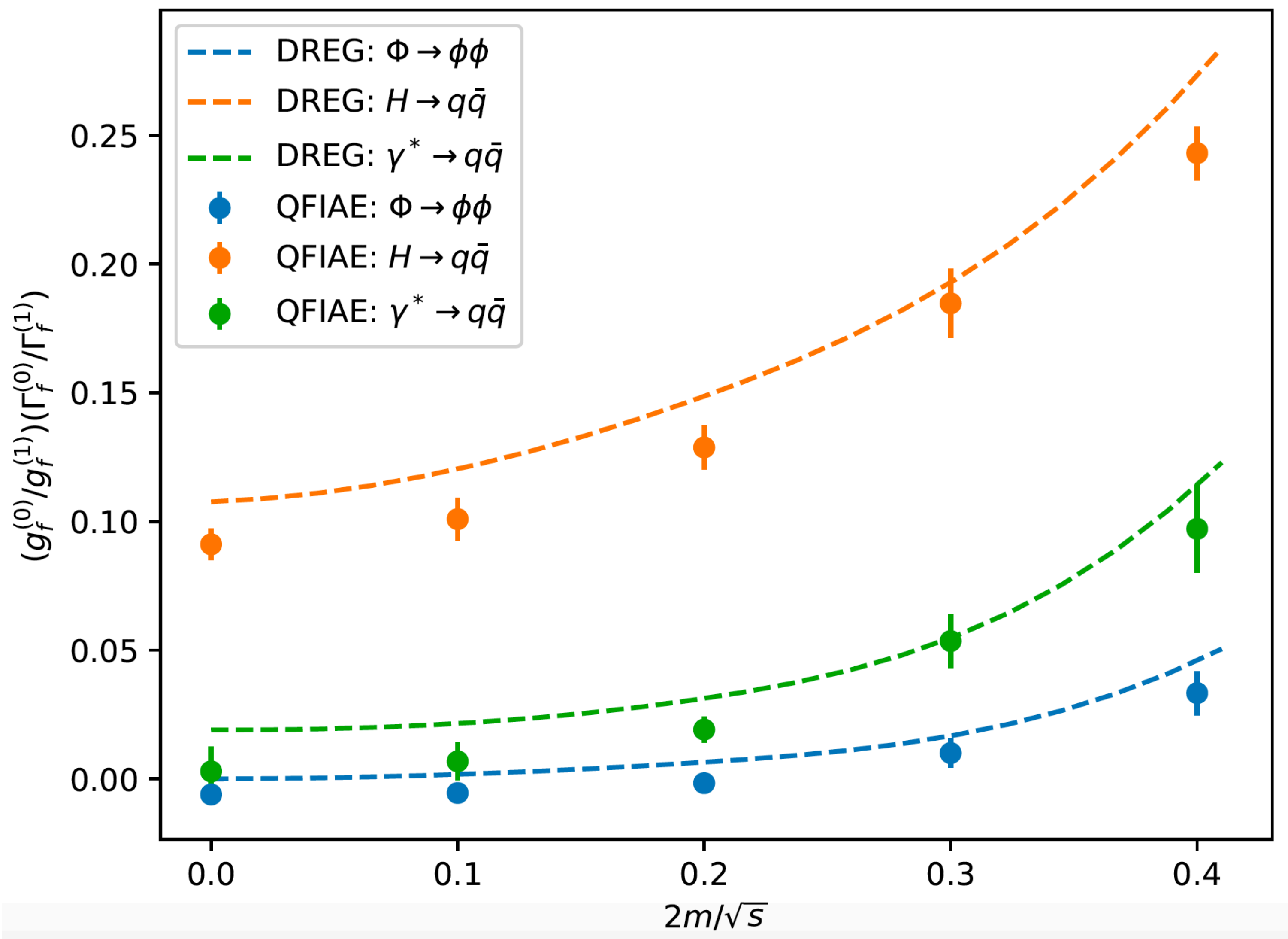


Quantum Fourier Iterative Amplitude Estimation

J.J. Martínez de Lejarza, M. Grossi, L. Cieri, GR



- In a quantum simulator (noiseless)



- Partly in quantum hardware

Conclusions

- **Fundamental initial boost on LTD by Stefano CATANI**
- LTD is more general than just integrating out the loop energy components thanks to the **CATANI's covariant complex prescription** and manifestly causal properties
- Many talks about in this conference [Kermanschah, Pozzorini, Bertolotti]
- Vacuum amplitudes acting as a kernel that generates all final states contributing to a scattering or decay process through **residues in the on-shell energies** of internal particles, after analytic continuation to negative values of those in the initial state
- The **sum** over all phase-space residues ensures the preservation of the competitive advantage of the vacuum amplitude: local matching of soft, collinear and threshold singularities … A novel representation of differential observables, which is well defined in the **four physical dimensions** of the spacetime
- First proof-of-concept results presented, more to come