Semi-Inclusive DIS at NNLO in QCD

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This talk is based on:

- *NNLO QCD corrections to polarized semi-inclusive DIS*
  S. Goyal, R.N. Lee, S. M., V. Pathak, N. Rana and V. Ravindran
  arXiv:2404.09959

- *Next-to-Next-to-Leading Order QCD Corrections to Semi-Inclusive Deep-Inelastic Scattering*
  S. Goyal, S. M., V. Pathak, N. Rana and V. Ravindran
  arXiv:2312.17711

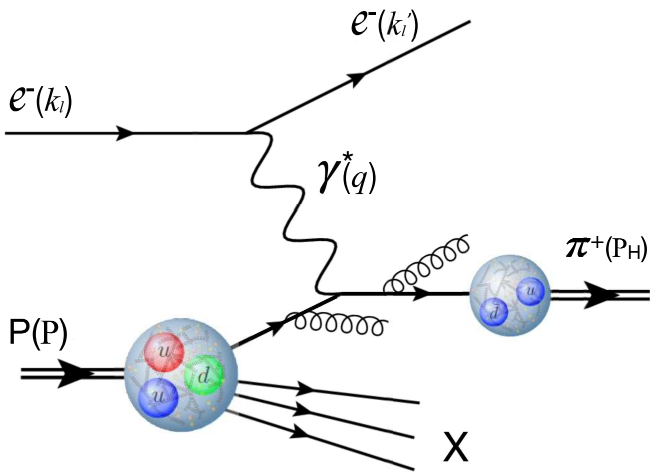
Related work:

- *Polarized semi-inclusive deep-inelastic scattering at NNLO in QCD*
  L. Bonino, T. Gehrmann, M. Löchner, K. Schönwald and G. Stagnitto
  arXiv:2404.08597

- *Semi-Inclusive Deep-Inelastic Scattering at Next-to-Next-to-Leading Order in QCD*
  L. Bonino, T. Gehrmann, and G. Stagnitto arXiv:2401.16281

Semi-inclusive deep-inelastic scattering

- SIDIS
  - production of identified hadrons in DIS
  - multiple hadron species: π, K, D, p, n, Λ, . .
  - probe of hadron structure in broad kinematic range



- QCD factorization at scale μ^2

$$\sigma_{\gamma H \to H'} = \sum_{ij} f_{i/H}(\mu^2) \otimes \hat{\sigma}_{\gamma i \to j}\left(\alpha_s(\mu^2), Q^2, \mu^2\right) \otimes D_{H'/j}(\mu^2)$$

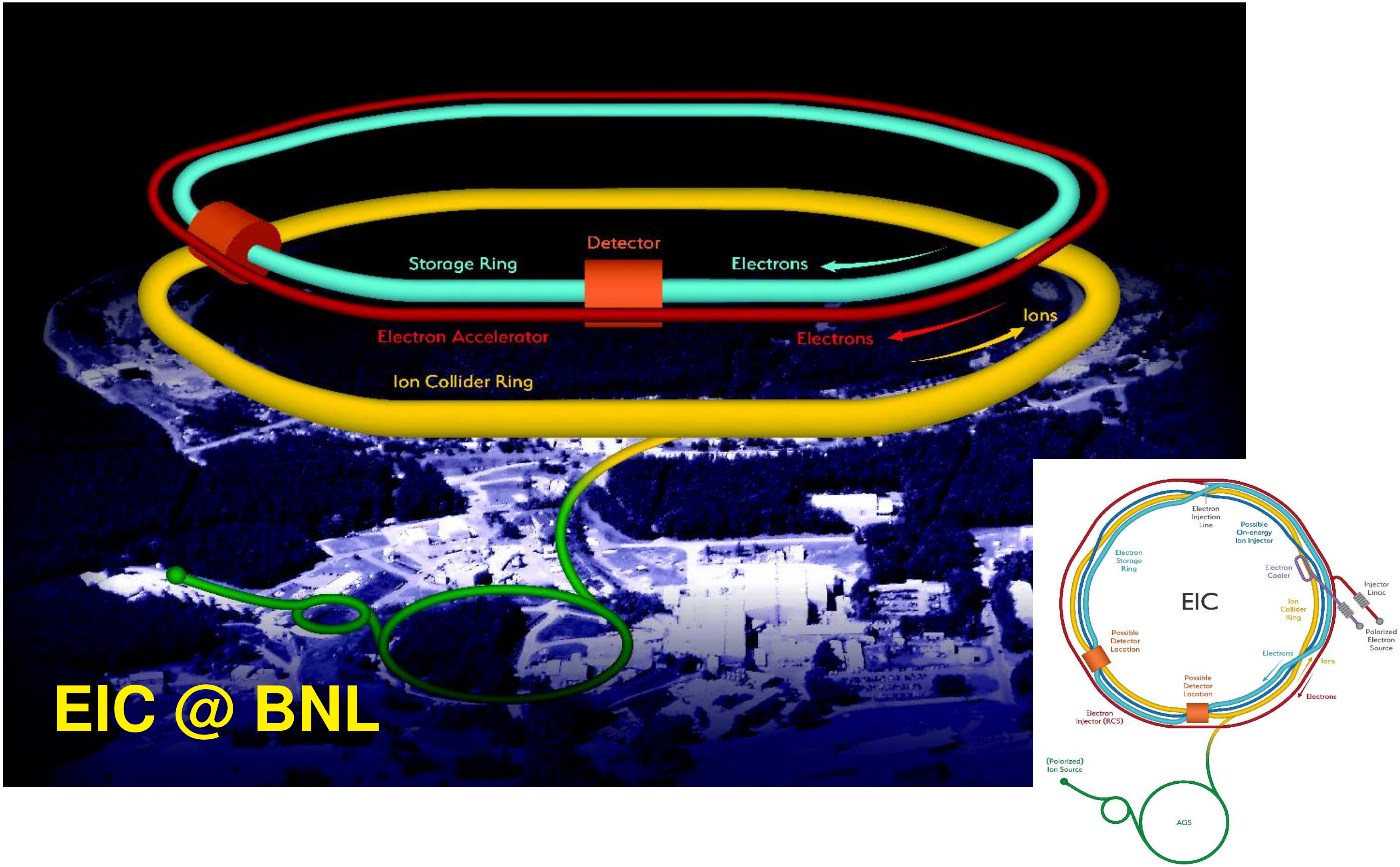
  - parton distribution function (PDF) $f_{i/H}(x, \mu^2)$
  - parton-to-hadron fragmentation function (FF) $D_{H'/j}(z, \mu^2)$
- Perturbative QCD
  - hard scattering cross section $\hat{\sigma}_{\gamma i \to j}\left(x, z, \alpha_s(\mu^2), Q^2, \mu^2\right)$ computed to NNLO

Once upon a time . . .

- HERA: deep structure of proton at highest Q^2 and smallest x

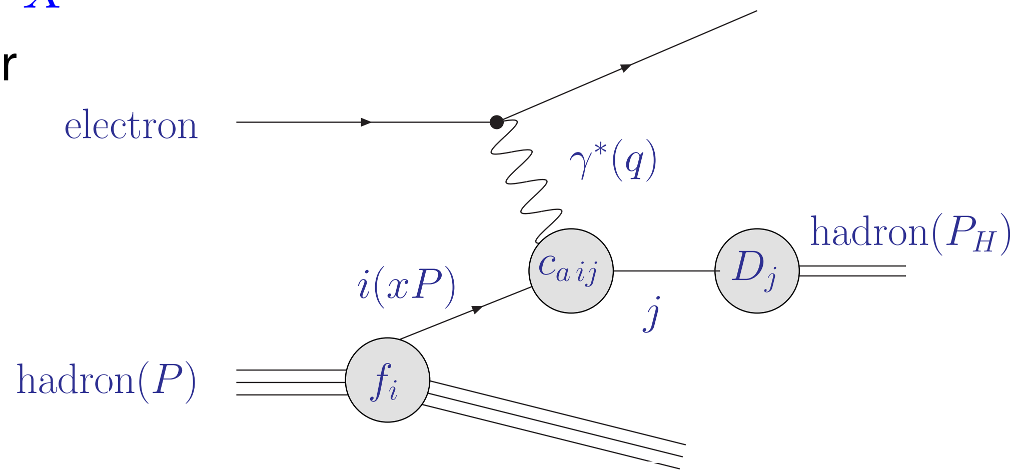
Bright future for precision hadron physics

- Electron-Ion Collider
  A machine that will unlock the secrets of the strongest force in Nature



SIDIS process

- $l(k_l) + H(P) \to l(k_l') + H'(P_H) + X$
  - space-like momentum transfer $q = k_l - k_l'$ with $Q^2 = -q^2$
  - Bjorken variable $x = \frac{Q^2}{2P\cdot q}$
  - inelasticity $y = \frac{P\cdot q}{P\cdot k_l}$
  - fragmenting hadron variable $z = \frac{P\cdot P_H}{P\cdot q}$



- Cross sections parametrized through structure functions
  - unpolarized SIDIS $\sigma = \frac{1}{4}\sum_{s_l,S,s_l',S_H} \sigma^{s_l',S_H}_{s_l,S}$

$$\frac{d^3\sigma}{dxdydz} = \frac{4\pi\alpha_e^2}{Q^2}\left[yF_1(x,z,Q^2) + \frac{(1-y)}{y}F_2(x,z,Q^2)\right]$$

  - polarized SIDIS $\Delta\sigma = \frac{1}{2}\sum_{s_l',S_H}\left(\sigma^{s_l',S_H}_{s_l=\frac{1}{2},S=\frac{1}{2}} - \sigma^{s_l',S_H}_{s_l=\frac{1}{2},S=-\frac{1}{2}}\right)$

$$\frac{d^3\Delta\sigma}{dxdydz} = \frac{4\pi\alpha_e^2}{Q^2}(2-y)g_1(x,z,Q^2)$$

Structure functions in perturbative QCD

- QCD factorization for structure function F_2 (up to order $\mathcal{O}(1/Q^2)$)

$$x^{-1} F_2(x, z, Q^2) \;=\; \sum_{ij} \int_x^1 \frac{dx'}{x'} \int_x^1 \frac{dz'}{z'} \, f_{i/H}(z', \mu^2) \, C_{2,ij}\left(\frac{x}{x'}, \frac{z}{z'}, \alpha_s(\mu^2), \frac{\mu^2}{Q^2}\right) D_{H'/j}(z', \mu^2)$$

  - coefficient functions $C_{a,ij} = \alpha_s^n \left(c^{(0)}_{a,ij} + \alpha_s c^{(1)}_{a,ij} + \alpha_s^2 c^{(2)}_{a,ij} + \dots \right)$
- Analogous for $g_1(x, z, Q^2)$ with polarized PDFs $\Delta f_{i/H}(z', \mu^2)$ and coefficient functions $\Delta C_{1,ij}$

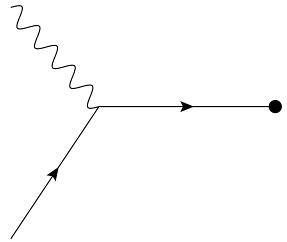
Parton evolution

$$\frac{d}{d \ln \mu^2} f_{i/H}(x, \mu^2) \;=\; \sum_j \left[P_{ij}(\alpha_s(\mu^2)) \otimes f_{j/H}(\mu^2) \right](x)$$

- Splitting functions $P_{ij} = \alpha_s P^{(0)}_{ij} + \alpha_s^2 P^{(1)}_{ij} + \alpha_s^3 P^{(2)}_{ij} + \alpha_s^4 P^{(3)}_{ij} + \dots$
  - space-like splitting functions for PDFs $f_{i/H}(x, \mu^2)$
  - time-like splitting functions for FFs $D_{H'/j}(z, \mu^2)$

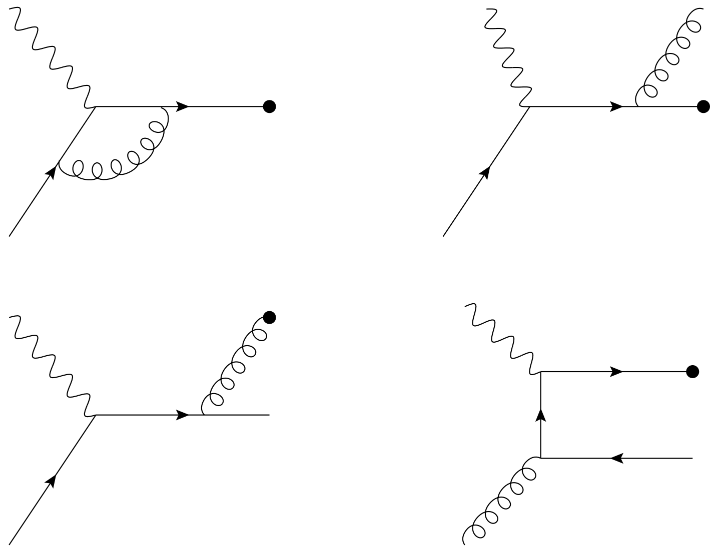
Coefficient functions (1)

- Leading order
- Born process $q(\bar{q}) + \gamma^* \to q(\bar{q})$
- $\mathcal{C}^{(0)}_{2,qq}(x', z') = \delta(1 - x')\,\delta(1 - z')$



- Next-to-leading order
- Real and virtual processes

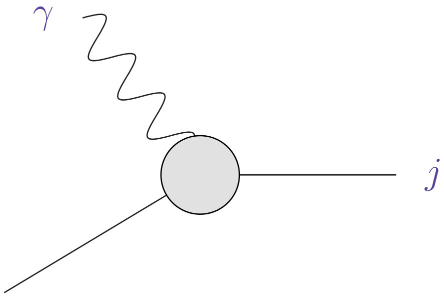
$$
\begin{aligned}
q(\bar{q}) + \gamma^* &\to q(\bar{q}) + \text{one loop} \\
q(\bar{q}) + \gamma^* &\to q(\bar{q}) + g \\
g + \gamma^* &\to q + \bar{q}
\end{aligned}
$$



- $\mathcal{C}^{(1)}_{a,ij}(x', z')$ known since long time
  Altarelli, Ellis, Martinelli, Pi ‘79; de Florian, Stratmann, Vogelsang ‘97

Coefficient functions (2)

$$\mathcal{C}^{(2)}_{a,ij}(x',z') \sim \mathcal{P}^{\mu\nu}_a \int \mathrm{dPS}_{X+j}\, \overline{\Sigma}|M_{ij}|^2_{\mu\nu}\, \delta\Big(z' - \frac{p_i \cdot p_j}{p_i \cdot q}\Big)$$



- Squared (projected) matrix elements
  - Feynman diagram with **Qgraf** Nogueira ‘91
  - symbolic manipulation with **Form**

    Vermaseren ‘00; Kuipers, Ueda, Vermaseren, Vollinga ‘12
  - UV and IR regularizazion in D dimensions
  - phase space integrals with kinematical constraint
- Reverse Unitarity method (Cutkosky rule)
  - phase-space integrals mapped to loop integrals

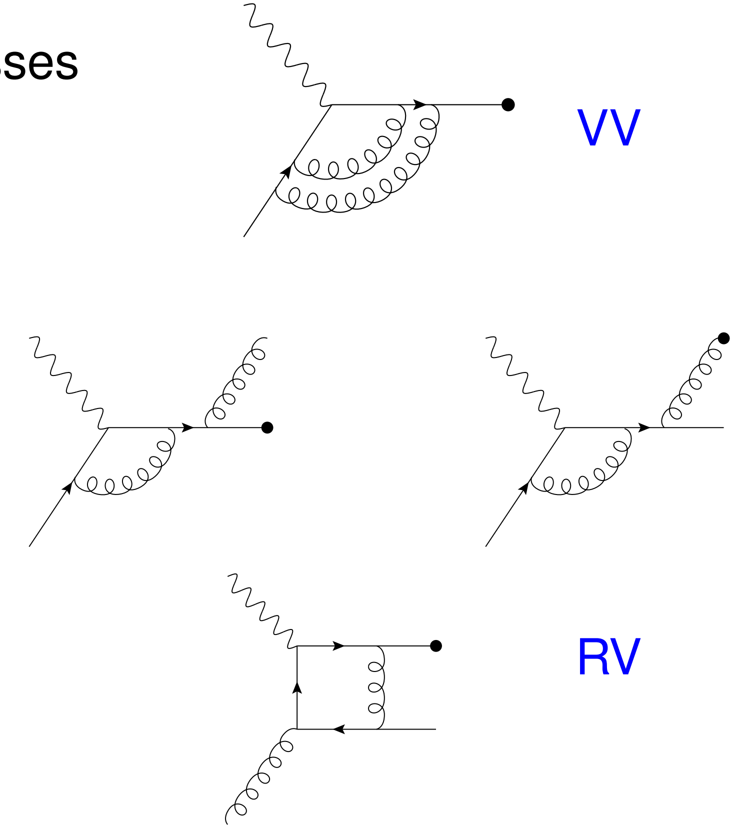
$$(2\pi i)\delta(p^2) \quad = \quad \frac{1}{p^2 + i\epsilon} + \text{cc.}$$

- Standard reduction with integration-by-parts to master integrals
- Phase-space master integrals computed through differential equations

Coefficient functions (3)

- Next-to-next-to-leading order
- Double-real, real-virtual and virtual processes

$$
\begin{aligned}
q(\bar{q}) + \gamma^* &\quad\rightarrow\quad q(\bar{q}) + \text{two loops} \\
q(\bar{q}) + \gamma^* &\quad\rightarrow\quad q(\bar{q}) + g + \text{one loop} \\
g + \gamma^* &\quad\rightarrow\quad q + \bar{q} + \text{one loop} \\
q(\bar{q}) + \gamma^* &\quad\rightarrow\quad q(\bar{q}) + g + g \\
g + \gamma^* &\quad\rightarrow\quad g + q + \bar{q} \\
q(\bar{q}) + \gamma^* &\quad\rightarrow\quad q(\bar{q}) + q' + \bar{q}' \\
q(\bar{q}) + \gamma^* &\quad\rightarrow\quad q(\bar{q}) + q + \bar{q}
\end{aligned}
$$



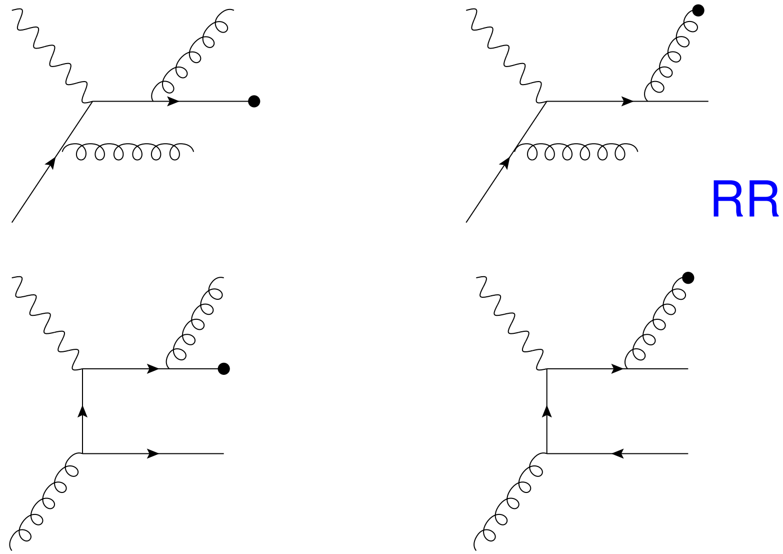
- VV contributions: massless two-loop form factor

Hamberg, van Neerven, Matsuura ‘88

- RV contributions with box integrals $\sim\ {}_2F_1(-\epsilon, -\epsilon, 1-\epsilon, f(x', z'))$
  - care needed for analyticity in the physical domain of x', z'

Coefficient functions (4)

- Double-real emissions
- RR requires
  three body phase space integrals
- 21 RR master integrals, functions of x', z'
  - differential equations in x', z'
  - boundary conditions by integration over z' from inclusive RR integrals (DIS coefficient functions)



- RR phase-space integrals in $D = 4 - 2\varepsilon$

$$(1-x')^{-1-a\varepsilon}(1-z')^{-1-b\varepsilon} f(x', z', \varepsilon)$$

  - regular functions $f(x', z', \varepsilon)$ in threshold limits $x' \to 1$ and/or $z' \to 1$
- IR divergences can be isolated ($w = x', z'$) with 'plus'-distributions

$$(1-w)^{-1+n\varepsilon} = \frac{1}{n\varepsilon}\delta(1-w) + \sum_{k=0}^{\infty} \frac{(n\varepsilon)^k}{k!} \left[\frac{\log^k(1-w)}{(1-w)}\right]_+$$

Coefficient functions (5)

Threshold resummation

- Coefficient functions $\mathcal{C}^{(n)}_{a,ij}(x', z') \sim \alpha_s^n \left[\frac{\log^k(1-w)}{(1-w)}\right]_+$
  - threshold logarithms in $w = x', z'$ and $k \leq 2n - 1$
- Prediction of threshold enhanced logarithms from resummation for SIDIS in Mellin variables $(x' \rightarrow) N$ and $(z' \rightarrow) M$
  - bears much resemblance with Drell-Yan rapidity distribution $z = Q^2/\hat{s} \rightarrow N$ and $\sqrt{z}\,\exp(\pm y) \rightarrow M$
- Useful approach to derive approximations at higher orders

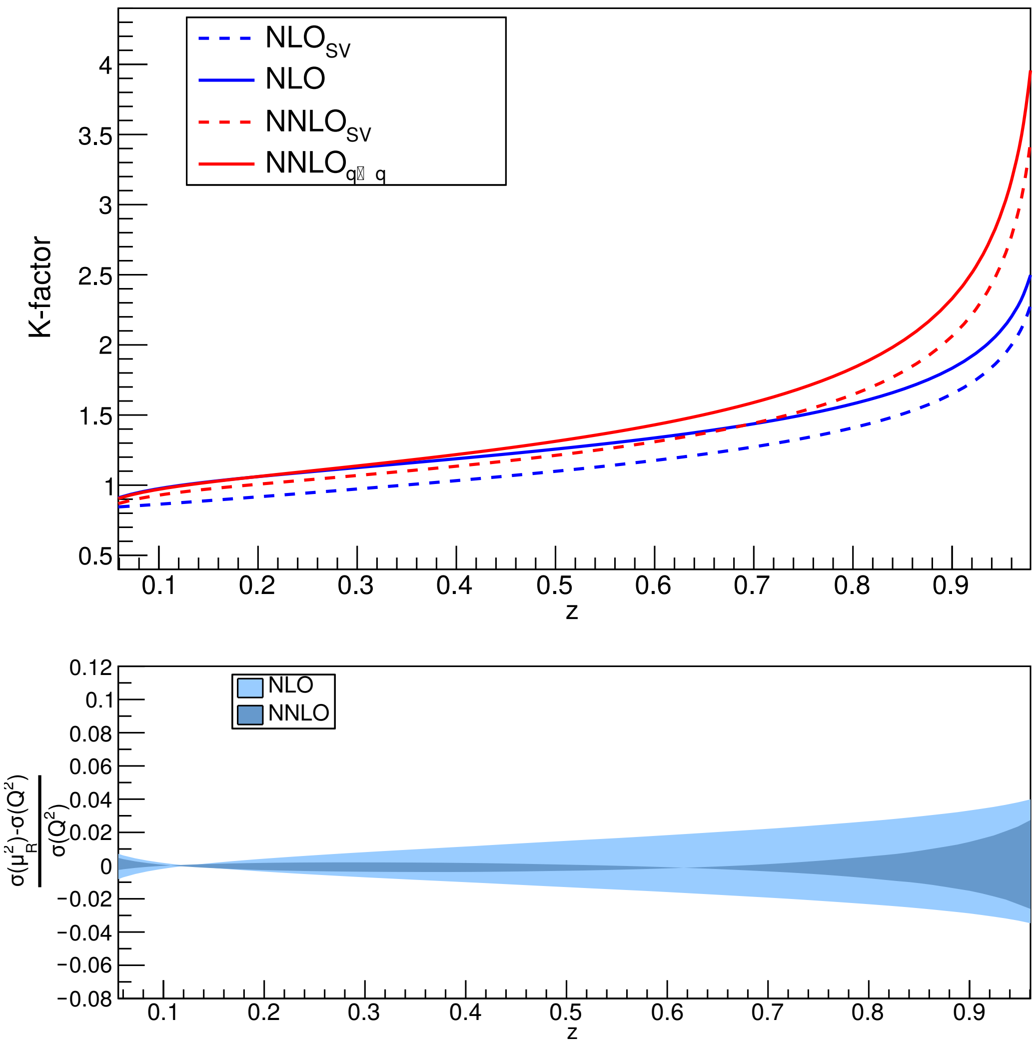
  Abele, de Florian, Vogelsang '21; '22
  - approximate NNLO and N^{3}LO QCD corrections
  - threshold resummation at N^{3}LL accuracy

Check

- Full agreement of exact computation with NNLO SV terms

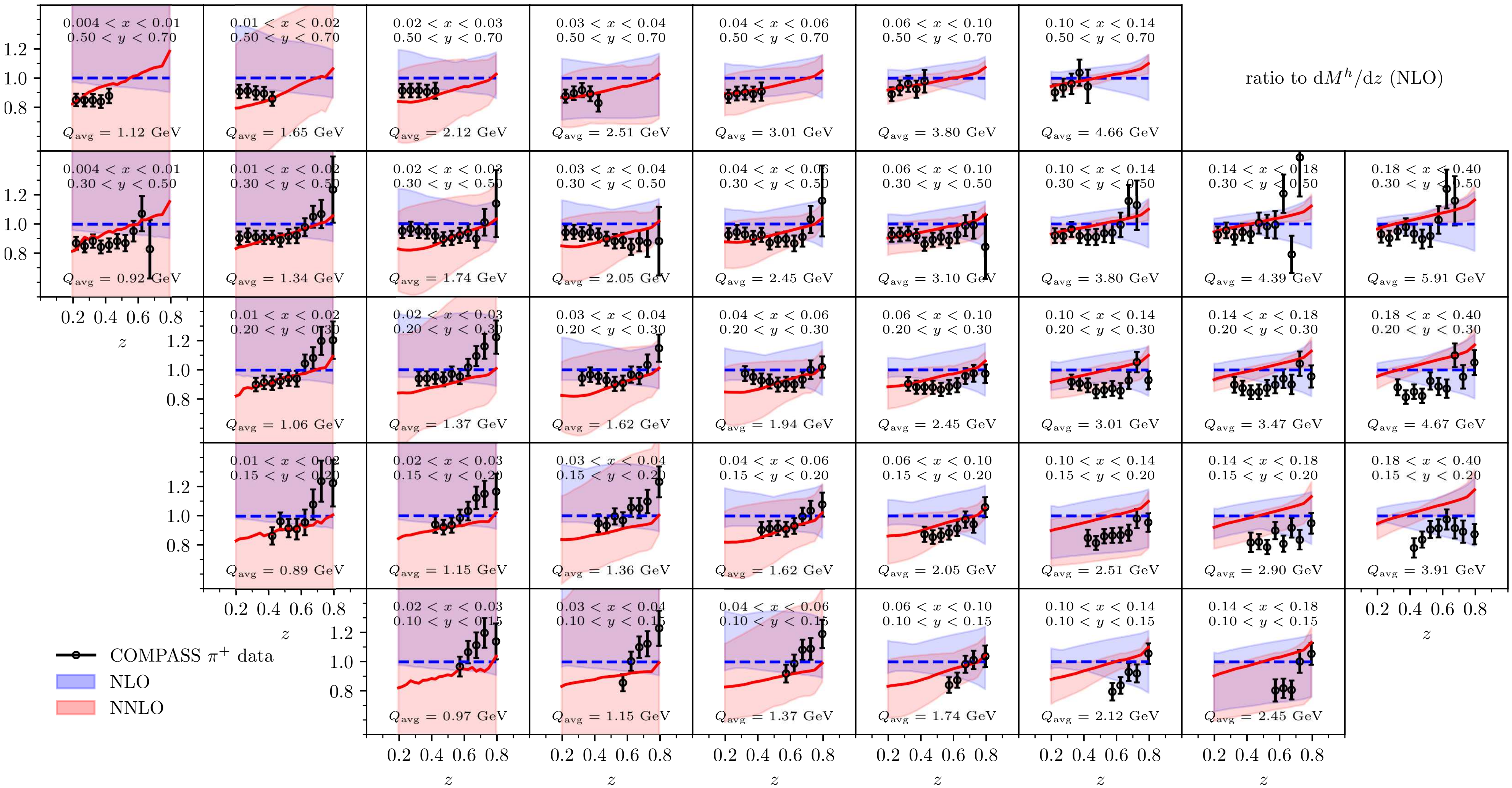
Results

- Unpolarized non-singlet coefficient function $\mathcal{C}^{(2)}_{2,qq}$



- K-factor as function of z for EIC with $\sqrt{s} = 140$ GeV
  - SV terms at NLO (blue dashed) and NNLO (red dashed)
  - full NLO (blue solid) and (non-singlet, leading color) NNLO (red solid)
- Uncertainty from renormalization scale variation $\mu_R^2 \in [Q^2/2, 2Q^2]$

Pion multiplicity



- Pion multiplicity $\dfrac{dM_{\pi^*}}{dz} = \dfrac{d^3\sigma^{\pi^+}}{dxdydz} \Big/ \dfrac{d^3\sigma^{\text{DIS}}}{dxdydz}$ compared to COMPASS data taken at $\sqrt{s} = 17.4$ GeV at NLO and NNLO

Bonino, Gehrmann, Stagnitto '24

Polarized SIDIS

- Polarized coefficient functions
  - appearance of γ_5 in vertex and spin projections
  - use Larin scheme $\gamma_5 \gamma_\mu = \frac{i}{6} \epsilon_{\mu\nu\rho\sigma} \gamma^\nu \gamma^\rho \gamma^\sigma$ Larin '93
- Structure function in Larin scheme

$$g_1(x,z) = \sum_{i,j} \Delta f_i{}^L(\mu_F^2) \otimes_{x'} \Delta \mathcal{C}_{1,ij}{}^L(\mu_F^2) \otimes_{z'} D_j(\mu_F^2)$$

- Scheme transformation (finite) from Larin to $\overline{\text{MS}}$ scheme
  - PDFs

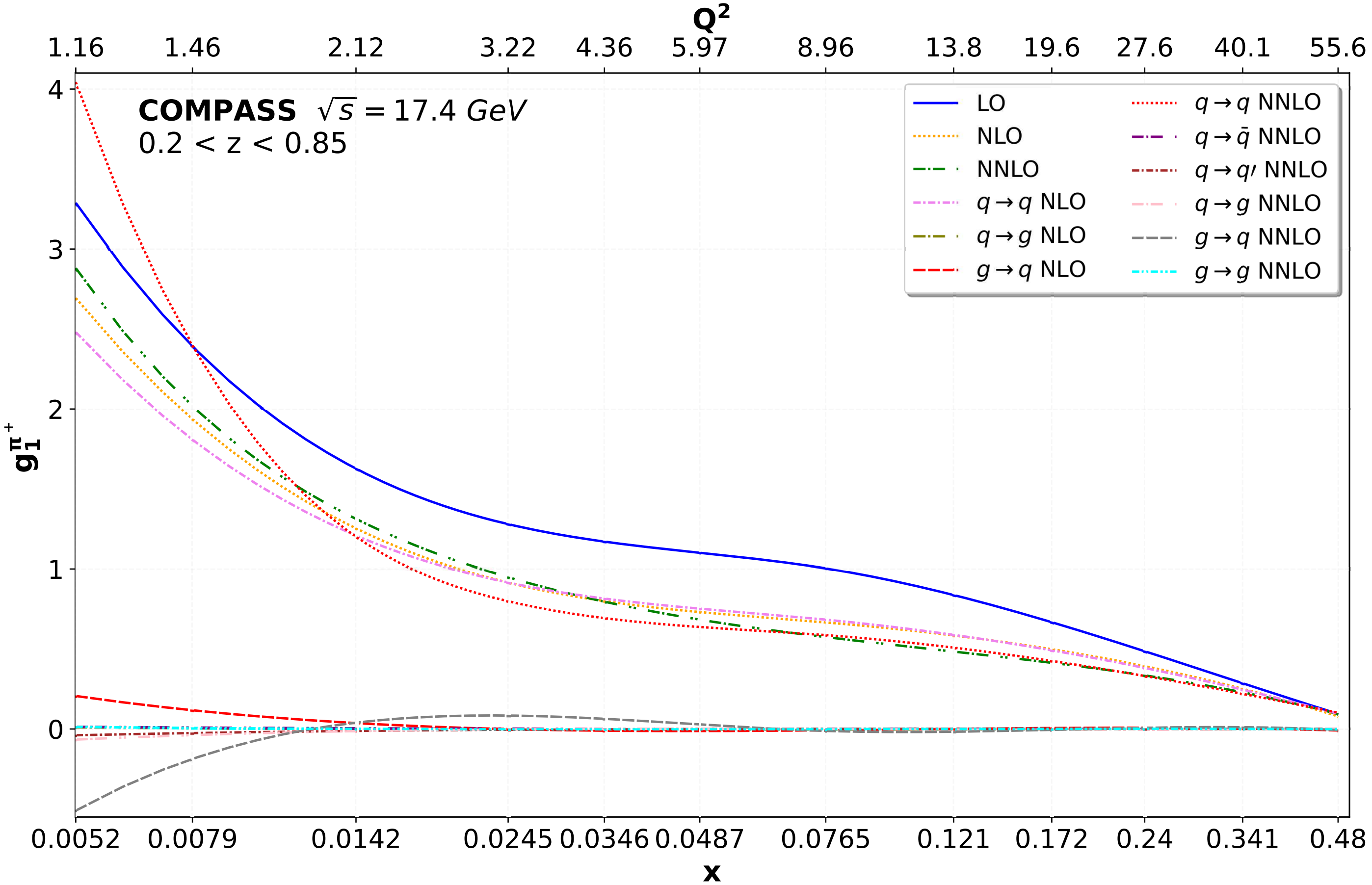
$$\Delta f_k(\mu_F^2) = Z_{ki}(\mu_F^2) \otimes \Delta f_i{}^L(\mu_F^2)$$

  - coefficient functions

$$\Delta \mathcal{C}_{1,ij}(\mu_F^2) = (Z^{-1}(\mu_F^2))_{ik} \otimes \Delta \mathcal{C}_{1,kj}{}^L(\mu_F^2)$$

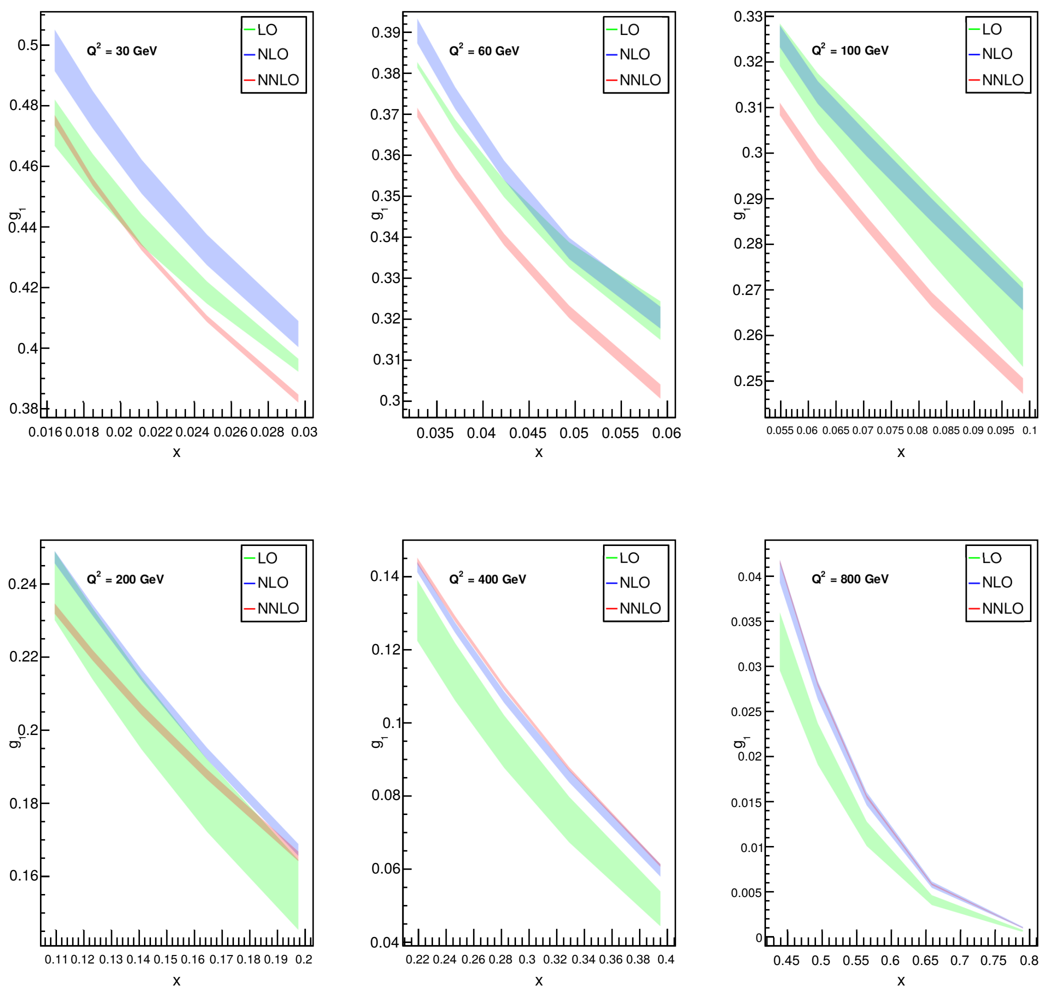
- Z_{ki} known to NNLO Matiounine, Smith, van Neerven '98

Polarized structure function (1)



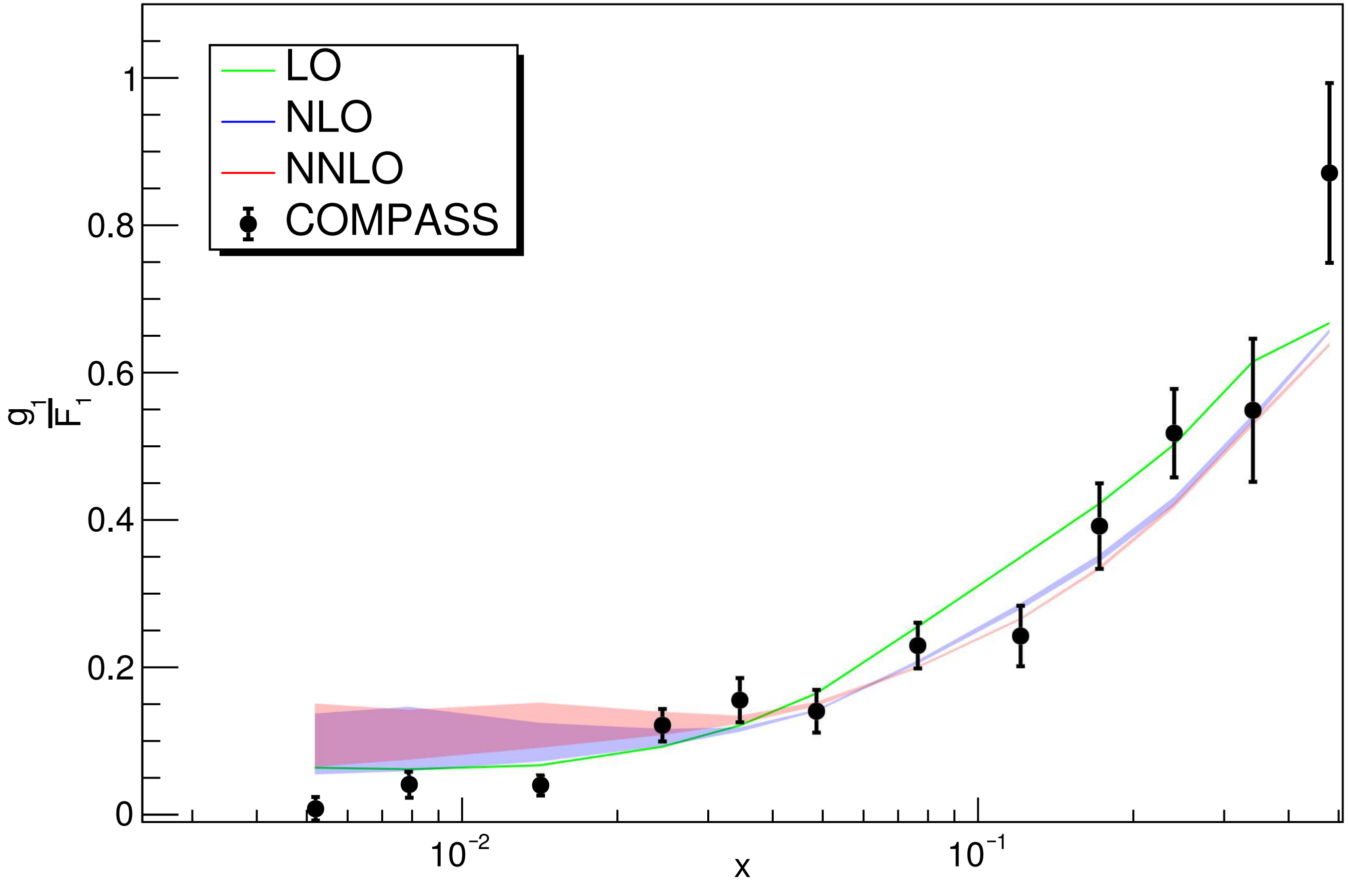
- Contributions from all partonic channels to $g_1^{\pi^+}(x)$ for COMPASS energy $\sqrt{s} = 17.4$ GeV
  - polarized PDFs from MAPPDF10 Bertone, Chiefa, Nocera '24
  - FFs from NNFF10 Bertone, Carrazza, Hartland, Nocera, Rojo '17

Polarized structure function (2)



- Scale dependence of $g_1^{\pi^+}(x)$ at various values of Q^2 in 7-point variation of μ_R and μ_F
  - polarized PDFs from MAPPDF10 Bertone, Chiefa, Nocera '24
  - FFs from MAPFF10 Abdul Khalek, Bertone, Khoudli, Nocera '22

Spin asymmetry



- Ratio of $g_1^{\pi^+}(x)/F_1^{\pi^+}(x)$ for COMPASS energy $\sqrt{s} = 17.4$ GeV with 7-point scale variation
  - polarized PDFs from MAPPDF10 Bertone, Chiefa, Nocera '24
  - unpolarized PDFs from NNPDF3.1 NNPDF '17
  - FFs from MAPFF10 Abdul Khalek, Bertone, Khoudli, Nocera '22

Summary

- Deep-inelastic scattering
  - Upcoming EIC will probe perturbative QCD in large range of kinematics
  - State-of-the-art detector can aim at experimental precision of $\lesssim 1\%$
- Polarized beams at EIC offer vast opportunities
  - new interest in large class of spin dependent observables
- Precision studies of hadron structure requires higher orders in perturbative QCD
  - theoretical predictions at NNLO in QCD nowadays standard
- Furhter improvements for SIDIS
  - Joint resummation beyond N^{3}LL accuracy
  - N^{3}LO QCD corrections within reach of current technologies