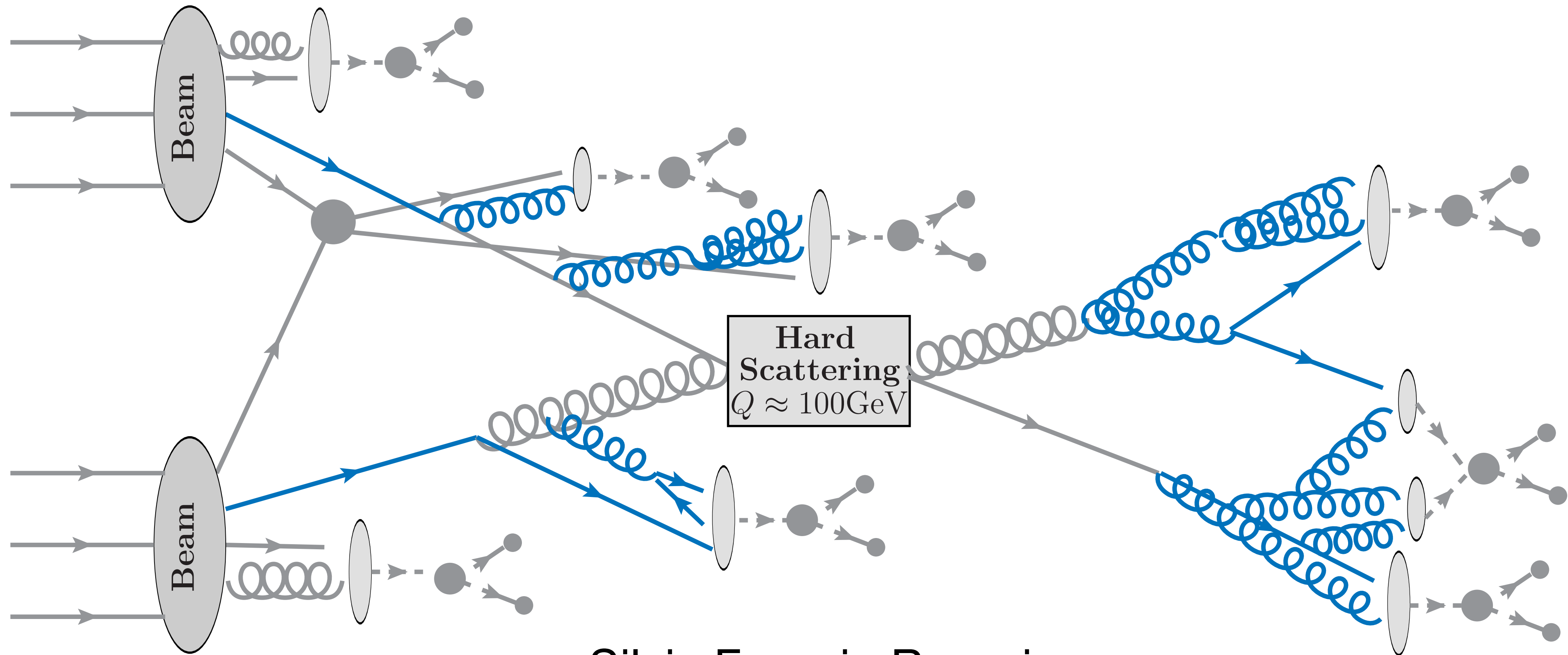


Parton Showers with higher logarithmic accuracy



Silvia Ferrario Ravasio

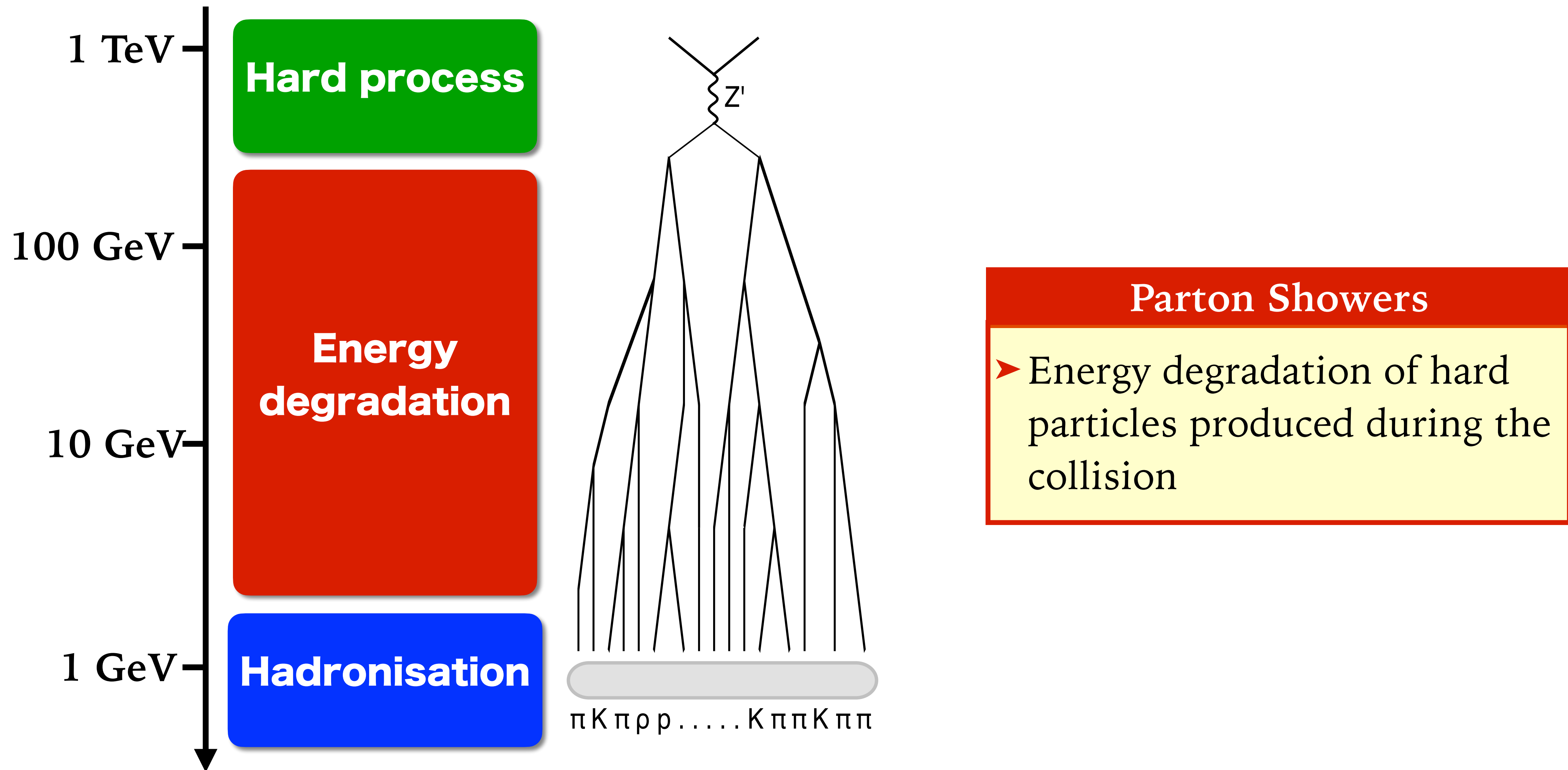
High Precision for Hard Processes

10th September 2024, University of Torino, Italy



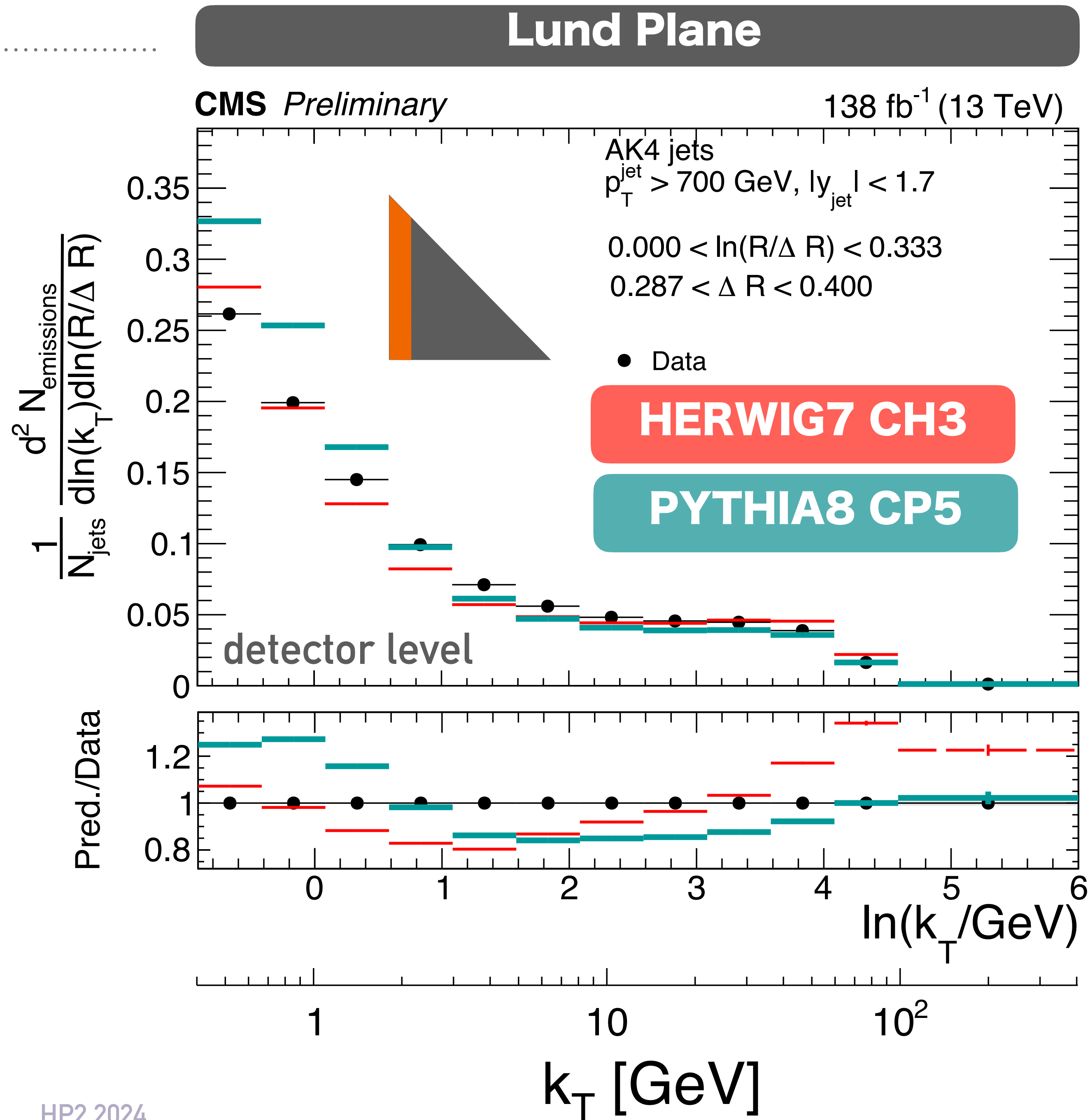
Shower Monte Carlo event generators

SHOWER MONTE CARLO EVENT GENERATORS = default tool for interpreting collider data

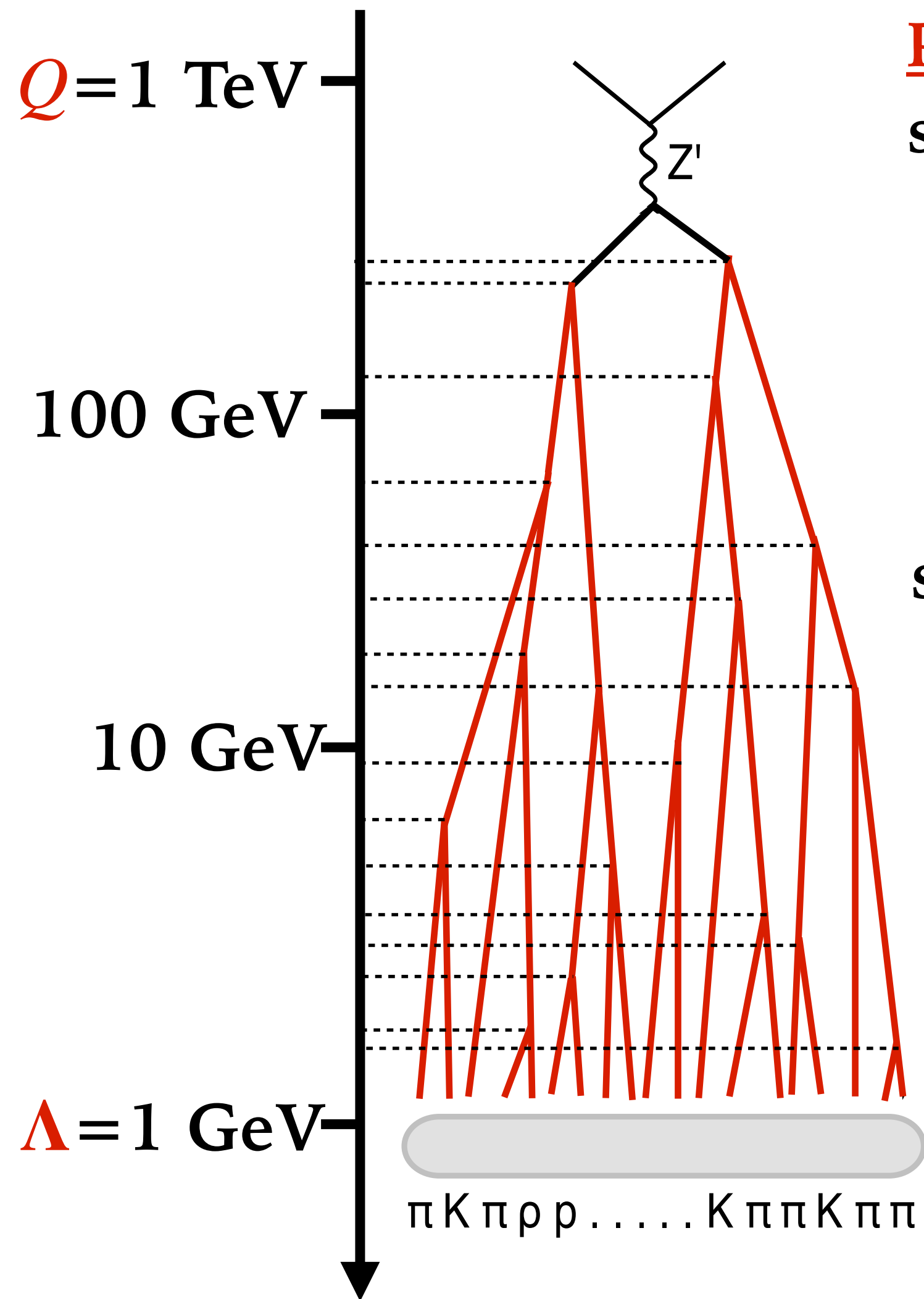


Are current showers good enough?

- showers do an amazing job on many observables for **LHC**
- various places see **10–30% discrepancies** between showers and data
- A lot of work is required to meet the **percent precision target!**



Logarithmically-accurate Parton Showers



PARTON SHOWERS = energy degradation via an iterated sequence of softer and softer emissions

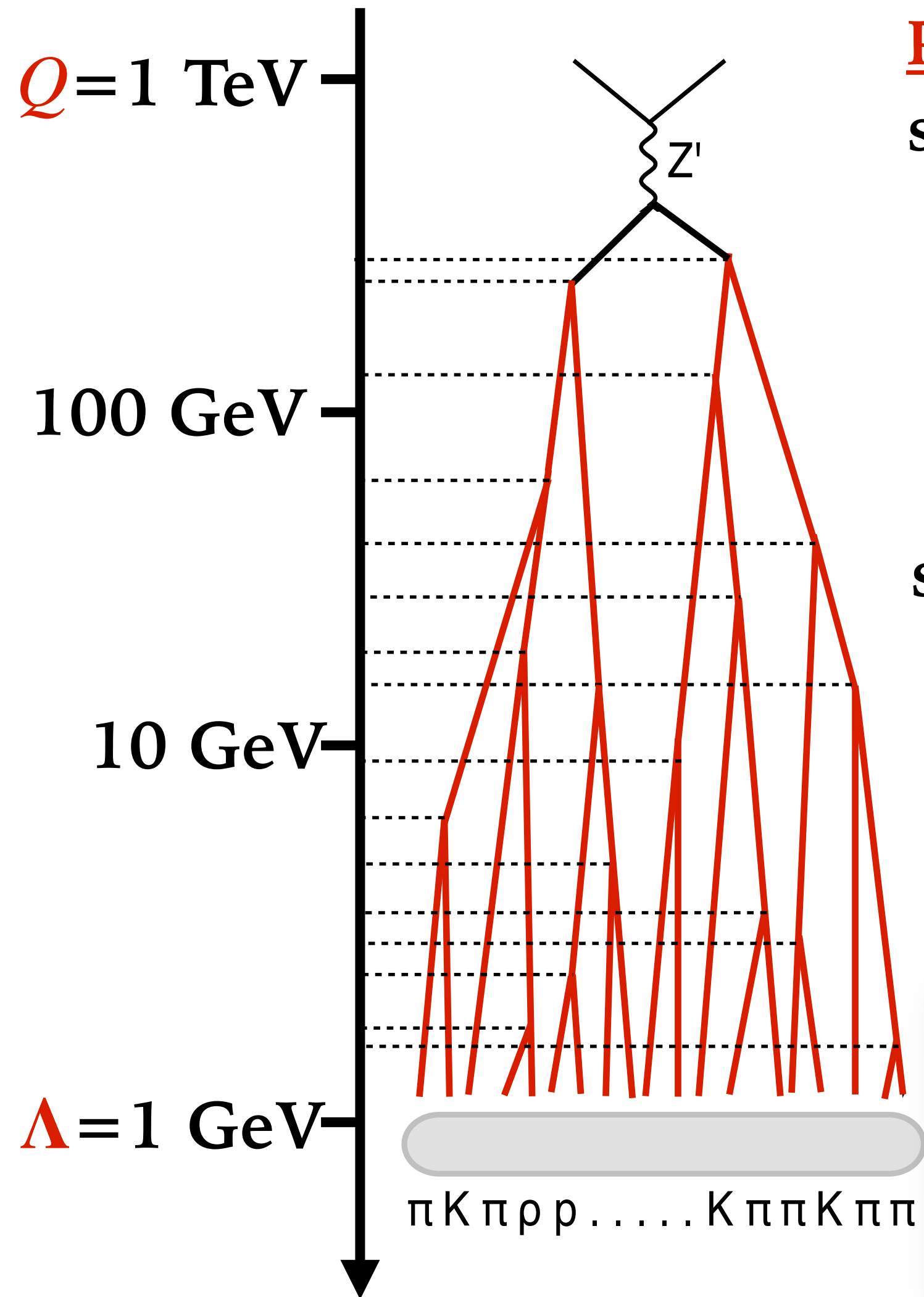
$$L = \ln \frac{Q}{\Lambda} \gg 1$$

simple algorithm to include the **dominant radiative corrections** at all orders for **any observable!**

$$\Sigma(O < e^{-L}) = \exp \left(-L g_{LL}(\beta_0 \alpha_s L) + \dots \right)$$

LL = leading logs

Logarithmically-accurate Parton Showers



PARTON SHOWERS = energy degradation via an iterated sequence of softer and softer emissions

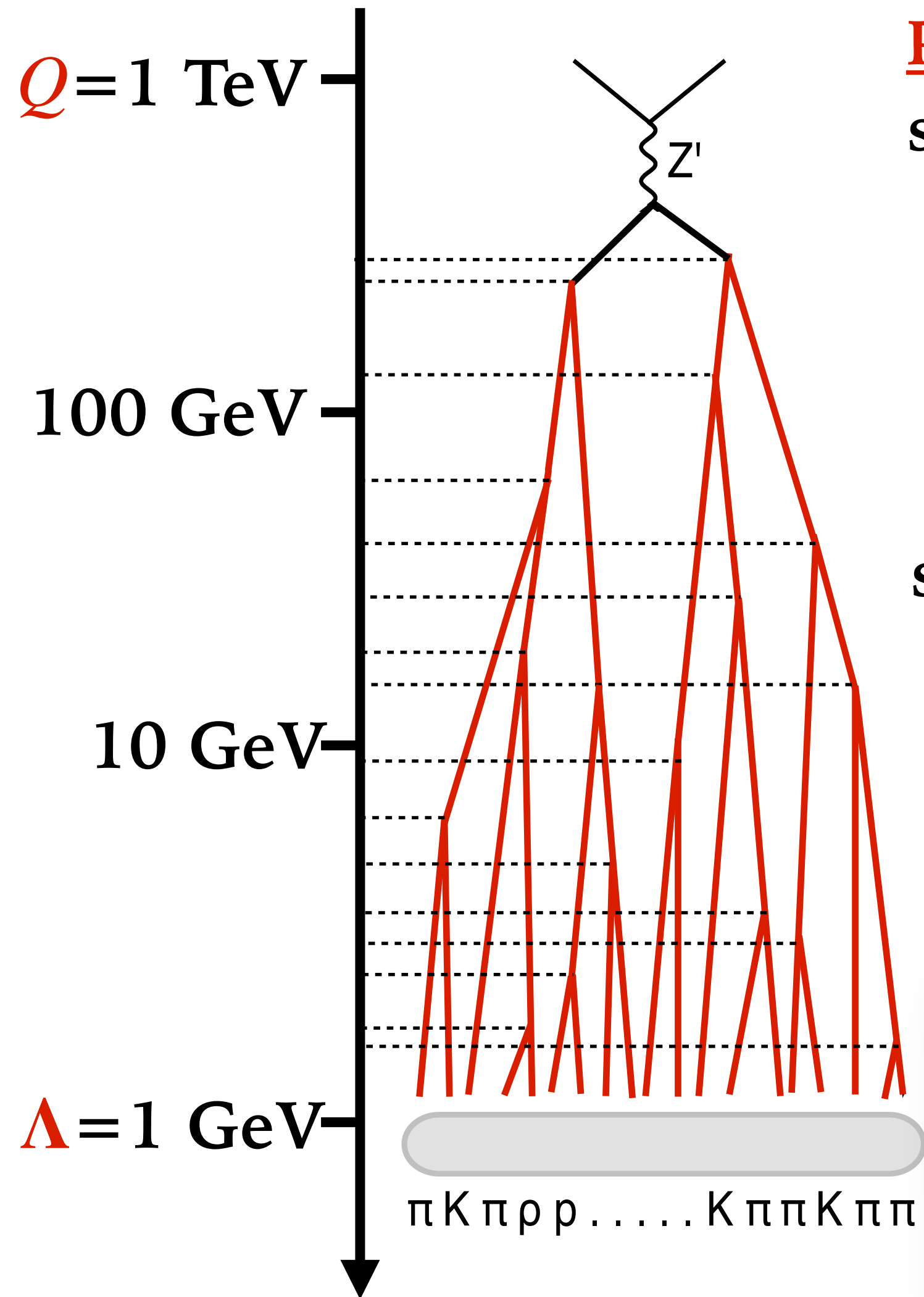
$$L = \ln \frac{Q}{\Lambda} \gg 1$$

simple algorithm to include the **dominant radiative corrections** at all orders for **any observable!**

$$\Sigma(O < e^{-L}) = \exp \left(-L g_{\text{LL}}(\beta_0 \alpha_s L) + \boxed{g_{\text{NLL}}(\beta_0 \alpha_s L)} + \dots \right)$$

For $Q \sim 50 - 10000 \text{ GeV}$, $\beta_0 \alpha_s L \sim 0.3 - 0.5$:
Next-to-Leading Logarithms needed for quantitative predictions!

Logarithmically-accurate Parton Showers



PARTON SHOWERS = energy degradation via an iterated sequence of softer and softer emissions

$$L = \ln \frac{Q}{\Lambda} \gg 1$$

simple algorithm to include the **dominant radiative corrections** at all orders for **any observable!**

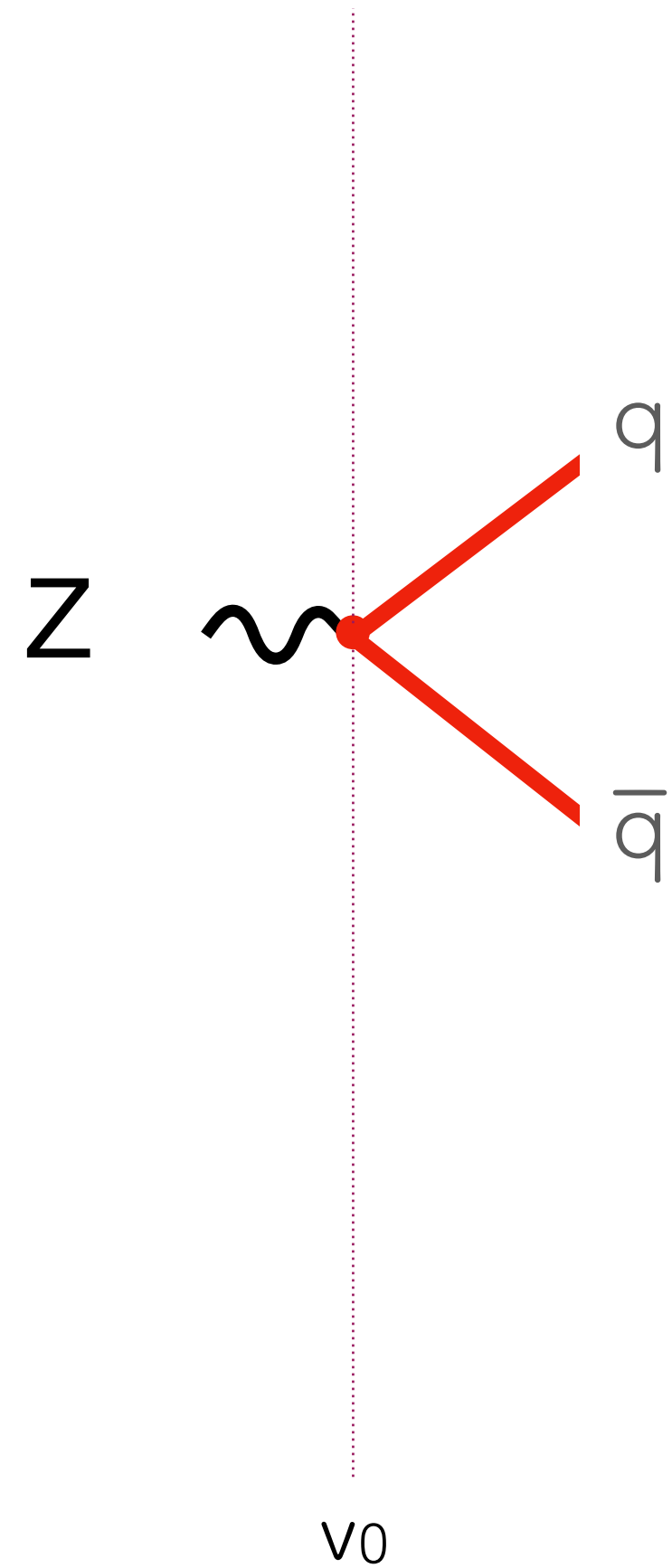
$$\Sigma(O < e^{-L}) = \exp \left(-L g_{LL}(\beta_0 \alpha_s L) + g_{NLL}(\beta_0 \alpha_s L) + \alpha_s g_{NNLL}(\beta_0 \alpha_s L) + \dots \right)$$

For $Q \sim 50 - 10000 \text{ GeV}$, $\beta_0 \alpha_s L \sim 0.3 - 0.5$:
Next-to-Next-to-Leading Logarithms needed for %-level

Parton Showers in a nutshell

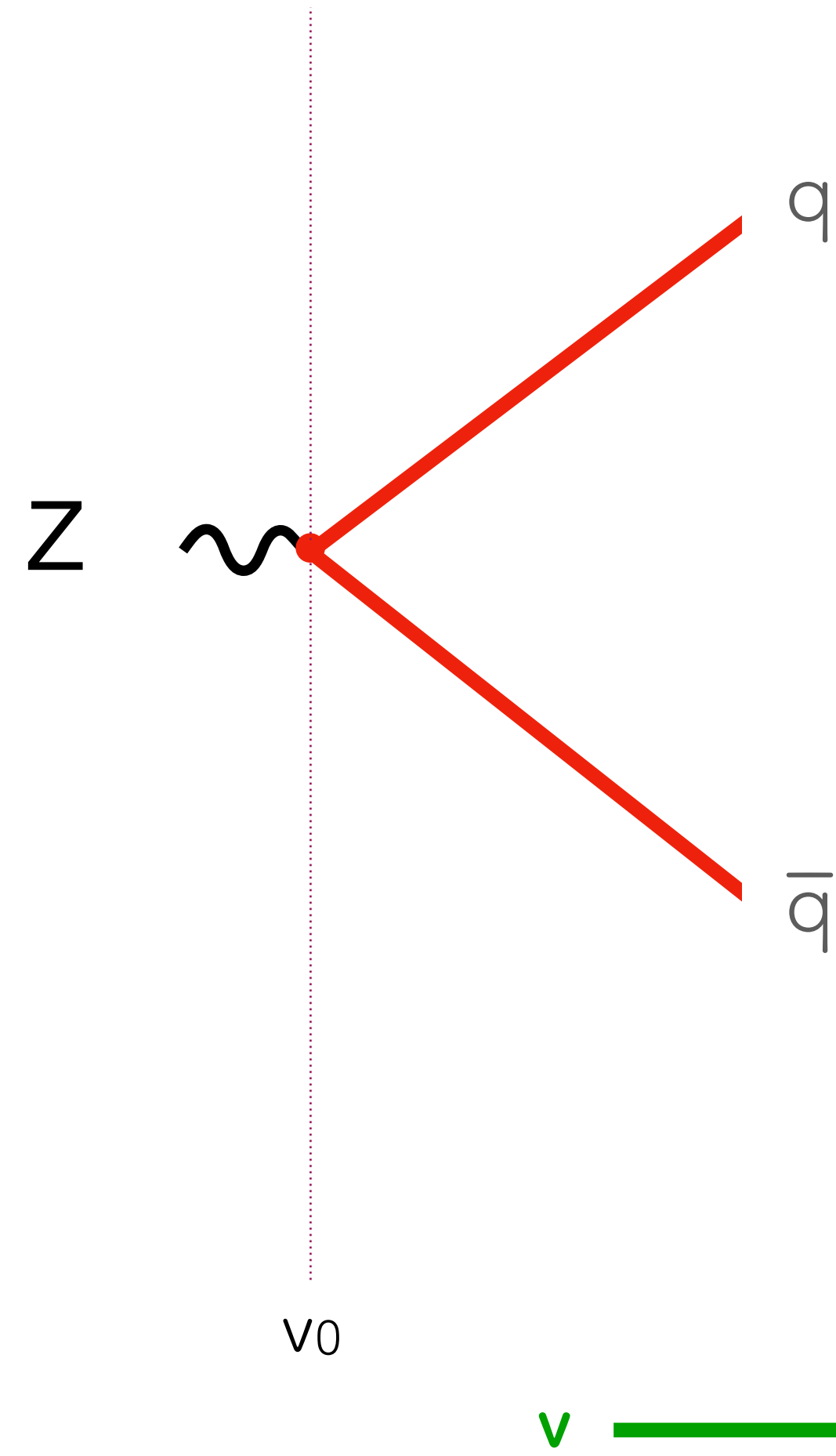
Dipole showers [Gustafson, Pettersson, '88] are the most used shower paradigm

Start with $q\bar{q}$ state produced at a hard scale v_0 .



Parton Showers in a nutshell

Dipole showers [Gustafson, Pettersson, '88] are the most used shower paradigm



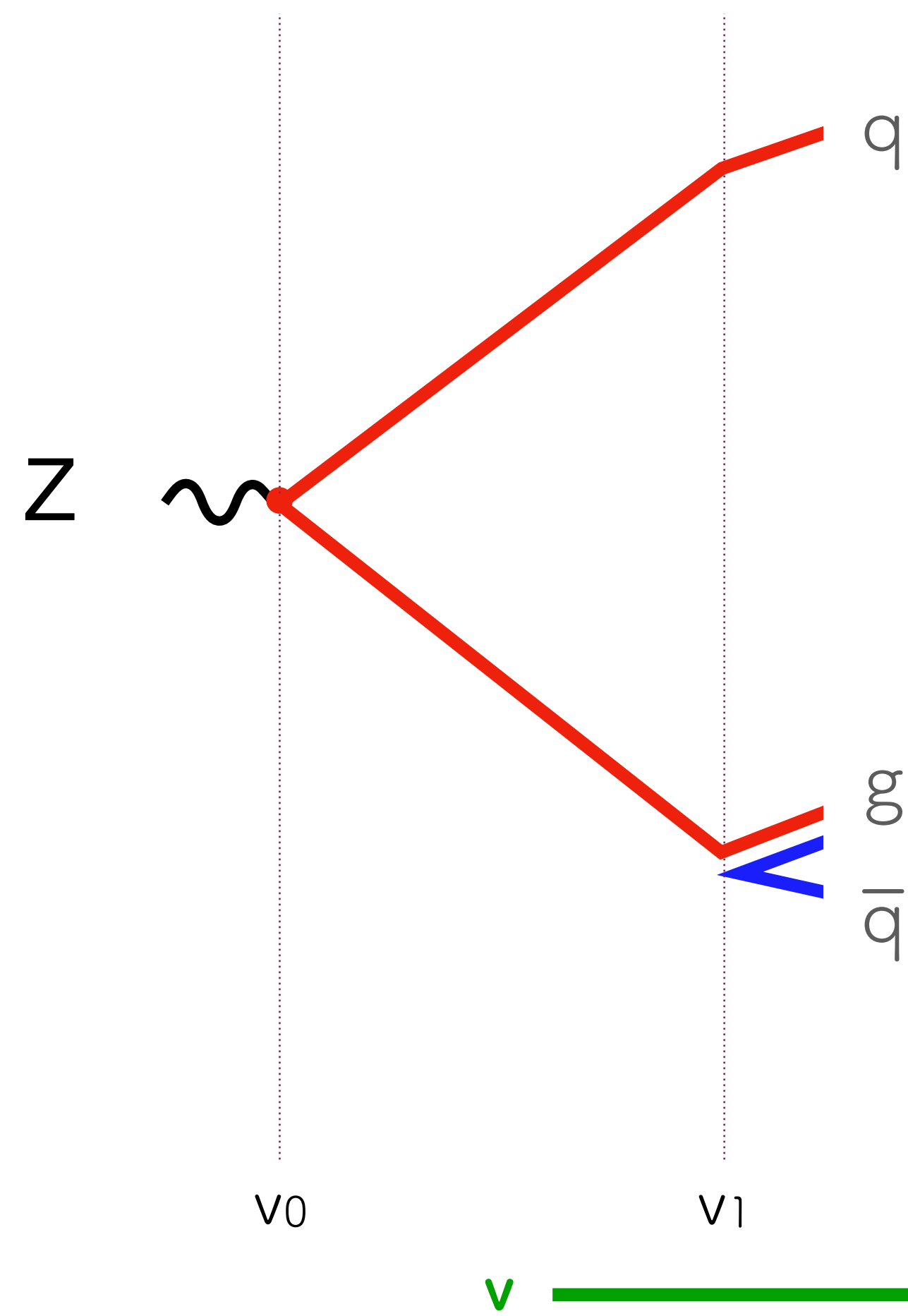
Start with $q\bar{q}$ state produced at a hard scale v_0 .

Throw a random number to determine down to what **scale** state persists unchanged

$$\Delta(v_0, v) = \exp \left(- \int_v^{v_0} dP_{q\bar{q}}(\Phi) \right)$$

Parton Showers in a nutshell

Dipole showers [Gustafson, Pettersson, '88] are the most used shower paradigm



Start with $q\bar{q}$ state produced at a hard scale v_0 .

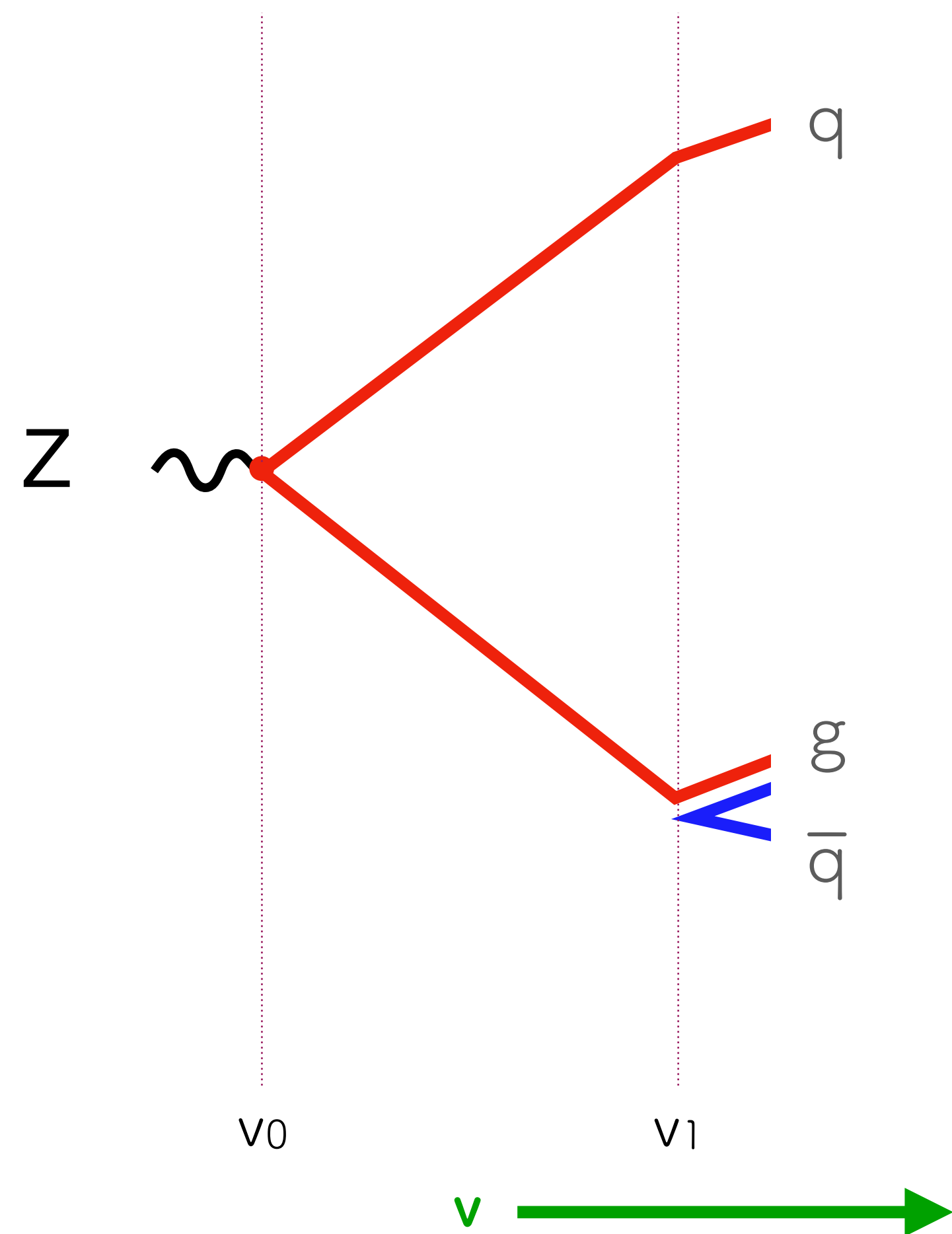
Throw a random number to determine down to what **scale** state persists unchanged

At some point, **state splits** ($2 \rightarrow 3$, i.e. emits gluon) at a scale $v_1 < v_0$. The kinematic (rapidity and azimuth) of the gluon is chosen according to

$$dP_{q\bar{q}}(\Phi(v_1)) \quad \Phi = \{v, \eta, \varphi\}$$

Parton Showers in a nutshell

Dipole showers [Gustafson, Pettersson, '88] are the most used shower paradigm



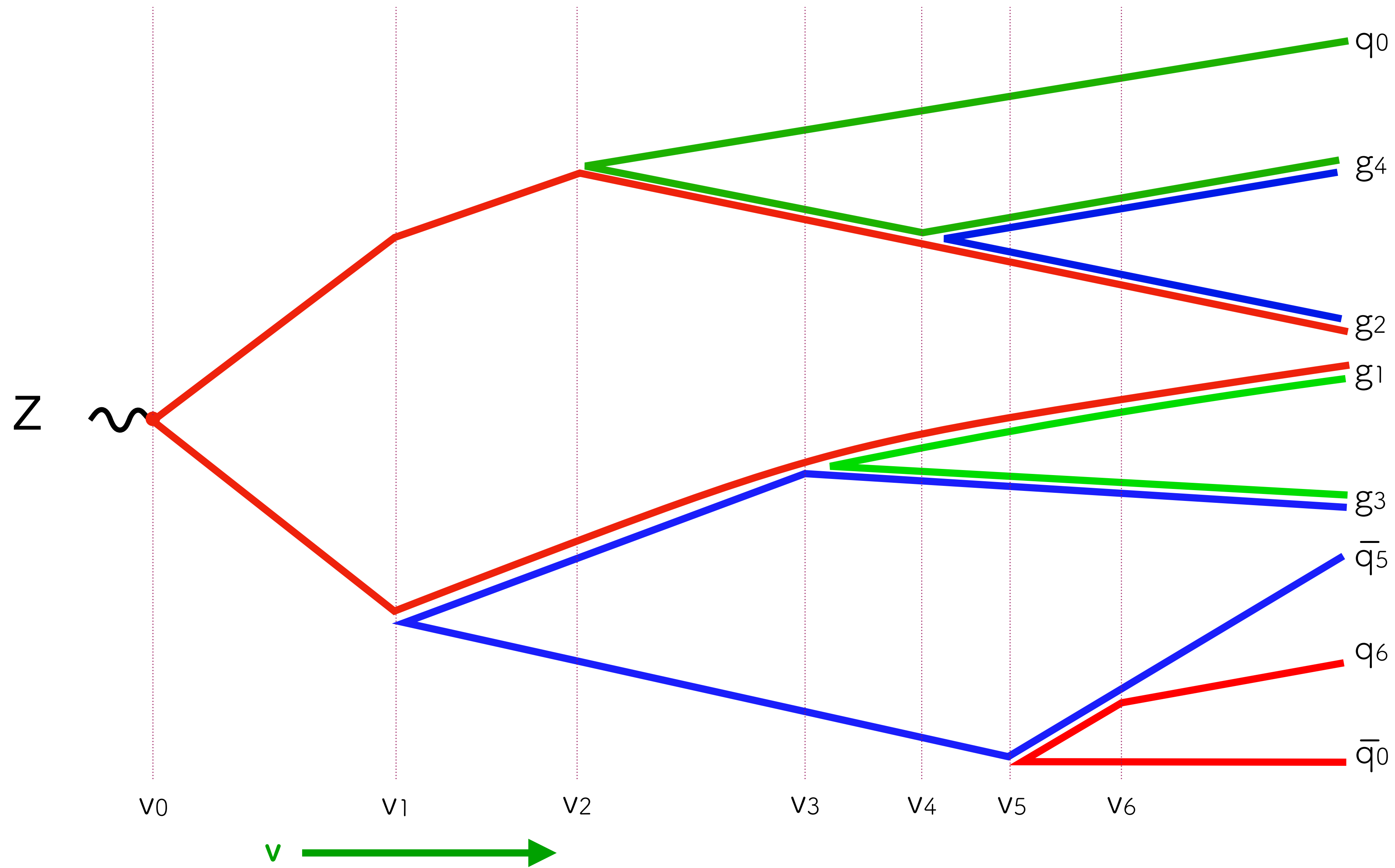
Start with $q\bar{q}$ state produced at a hard scale ν_0 .

Throw a random number to determine down to what **scale** state persists unchanged

At some point, **state splits** ($2 \rightarrow 3$, i.e. emits gluon) at a scale $\nu_1 < \nu_0$.

The gluon is part of two dipoles (qg) , $(g\bar{q})$.

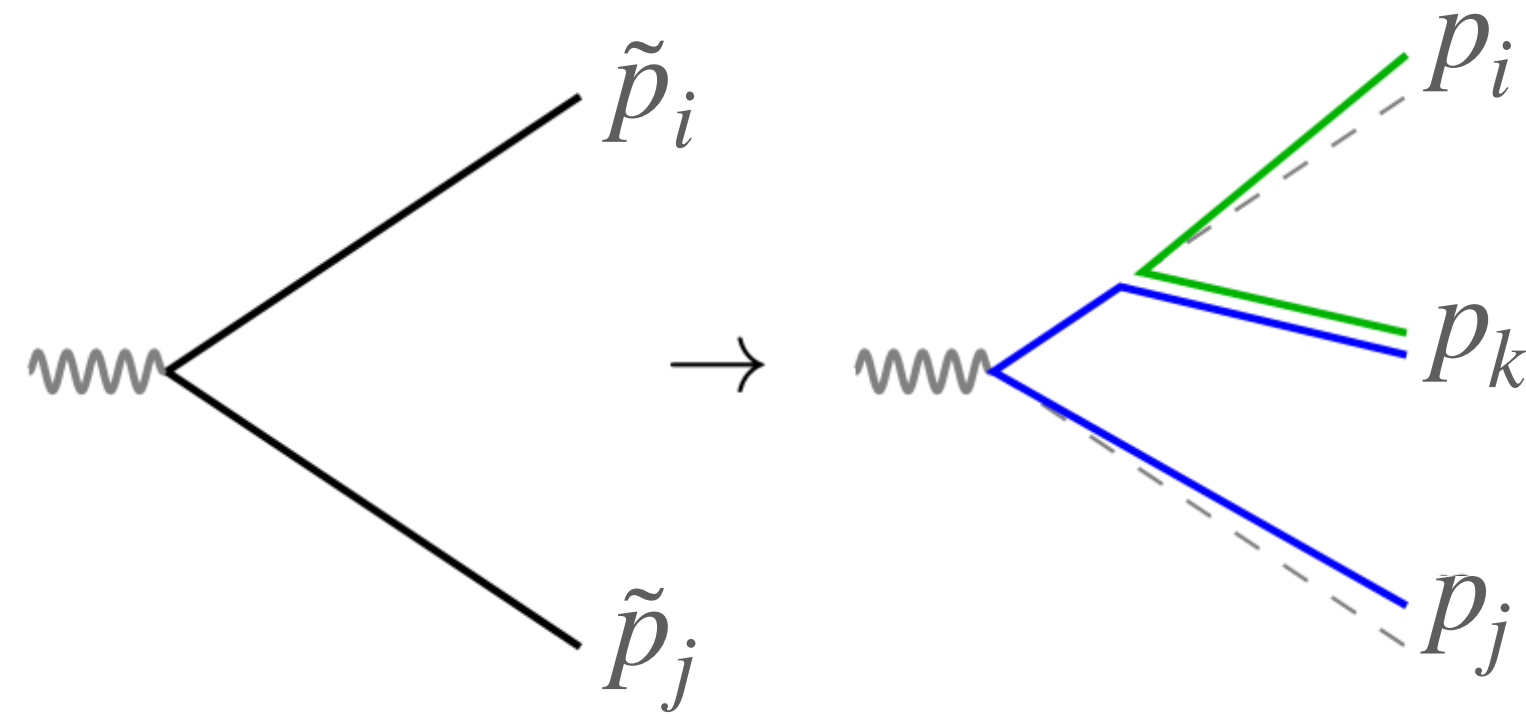
Iterate the above procedure for both dipoles independently, using ν_1 as starting scale.



self-similar
 evolution
 continues until it
 reaches a non-
 perturbative
 scale

Dissecting the parton shower emission probability

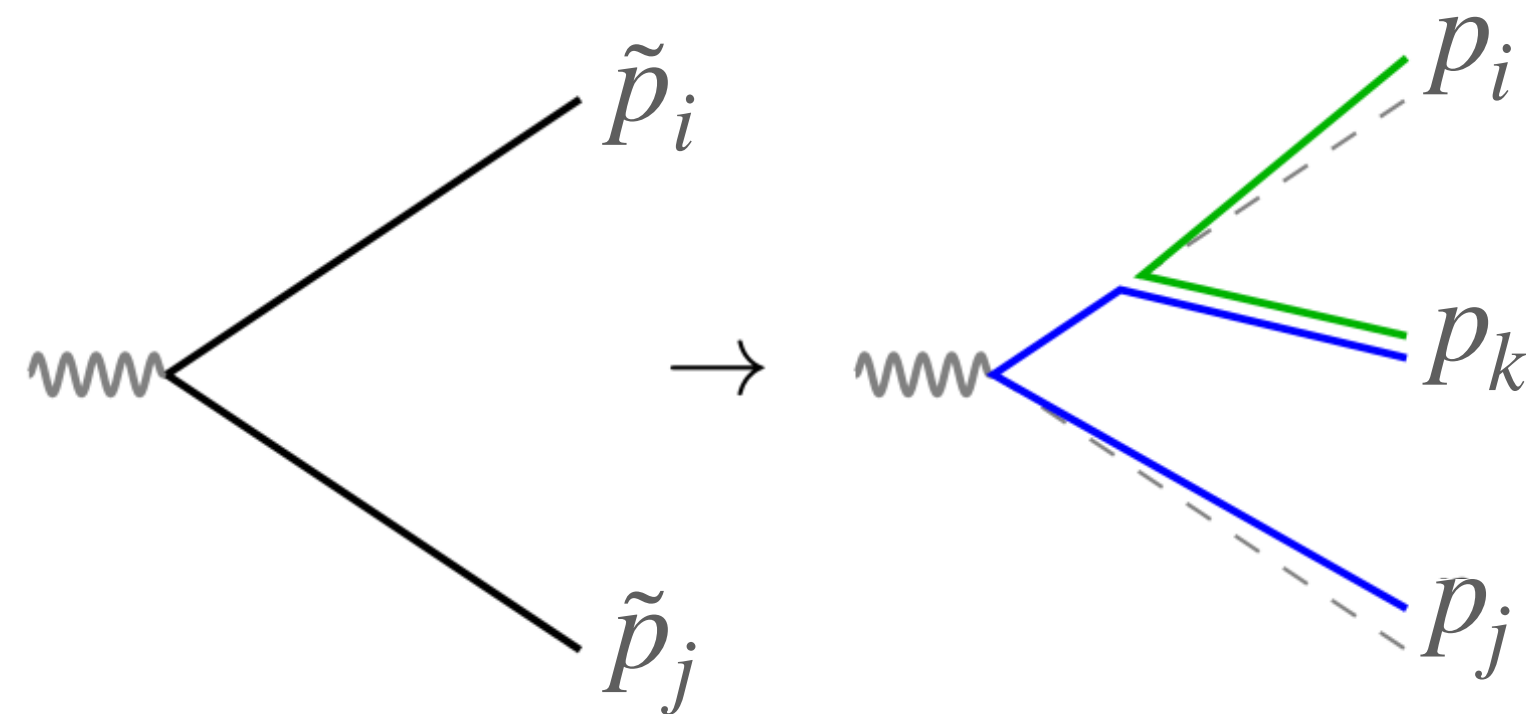
Starting from a $e^+e^- \rightarrow Z^* \rightarrow q\bar{q}$ system, what is the splitting probability?



$$d\mathcal{P}_{\tilde{i}\tilde{j} \rightarrow ijk} \sim \frac{dv^2}{v^2} d\bar{\eta} \frac{d\varphi}{2\pi} P_{\tilde{i},\tilde{j} \rightarrow i,j,k}(v, \bar{\eta}, \varphi)$$

Dissecting the parton shower emission probability

Starting from a $e^+e^- \rightarrow Z^* \rightarrow q\bar{q}$ system, what is the splitting probability?

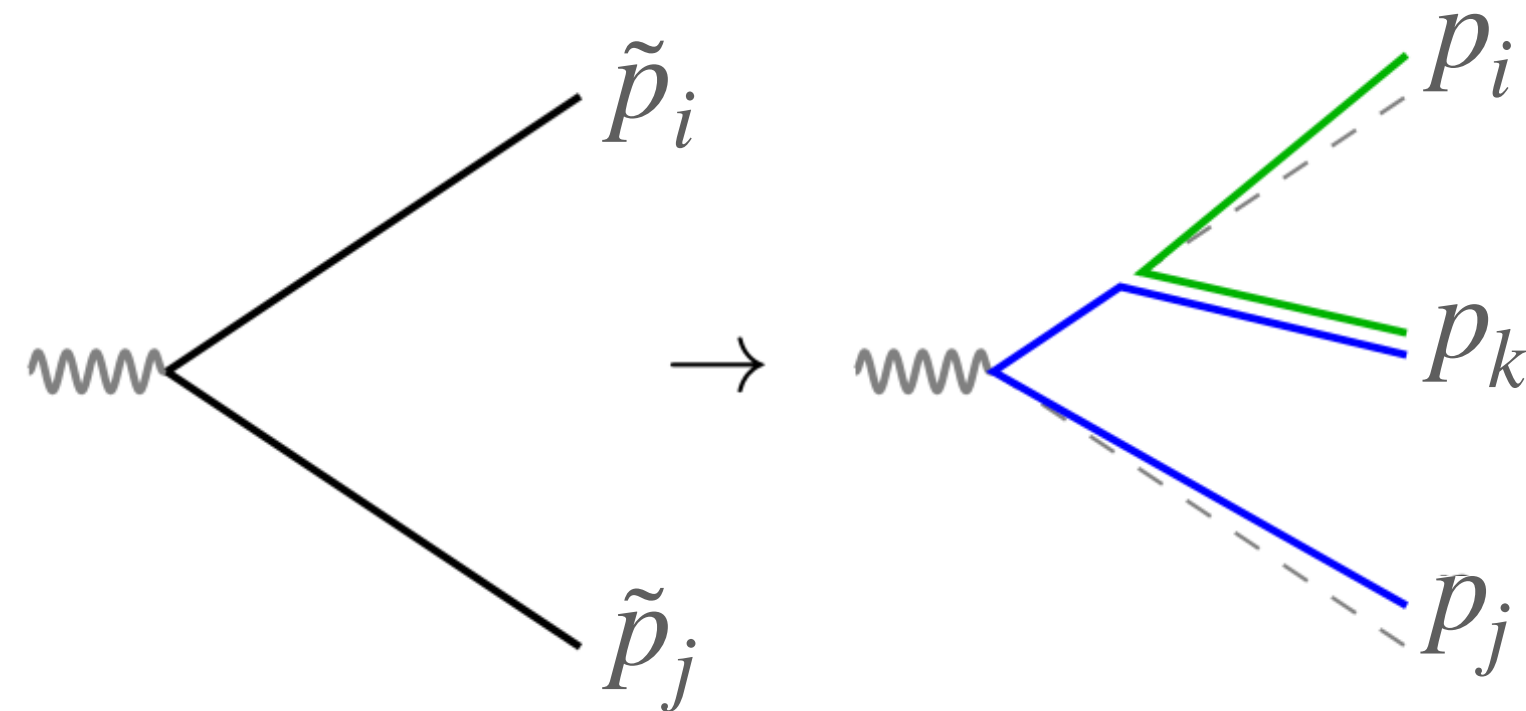


$$d\mathcal{P}_{\tilde{i}\tilde{j} \rightarrow ijk} \sim \frac{dv^2}{v^2} d\bar{\eta} \frac{d\varphi}{2\pi} P_{\tilde{i},\tilde{j} \rightarrow i,j,k}(v, \bar{\eta}, \varphi)$$

Matrix element for emitting a parton k from a parton i (or j)

Dissecting the parton shower emission probability

Starting from a $e^+e^- \rightarrow Z^* \rightarrow q\bar{q}$ system, what is the splitting probability?



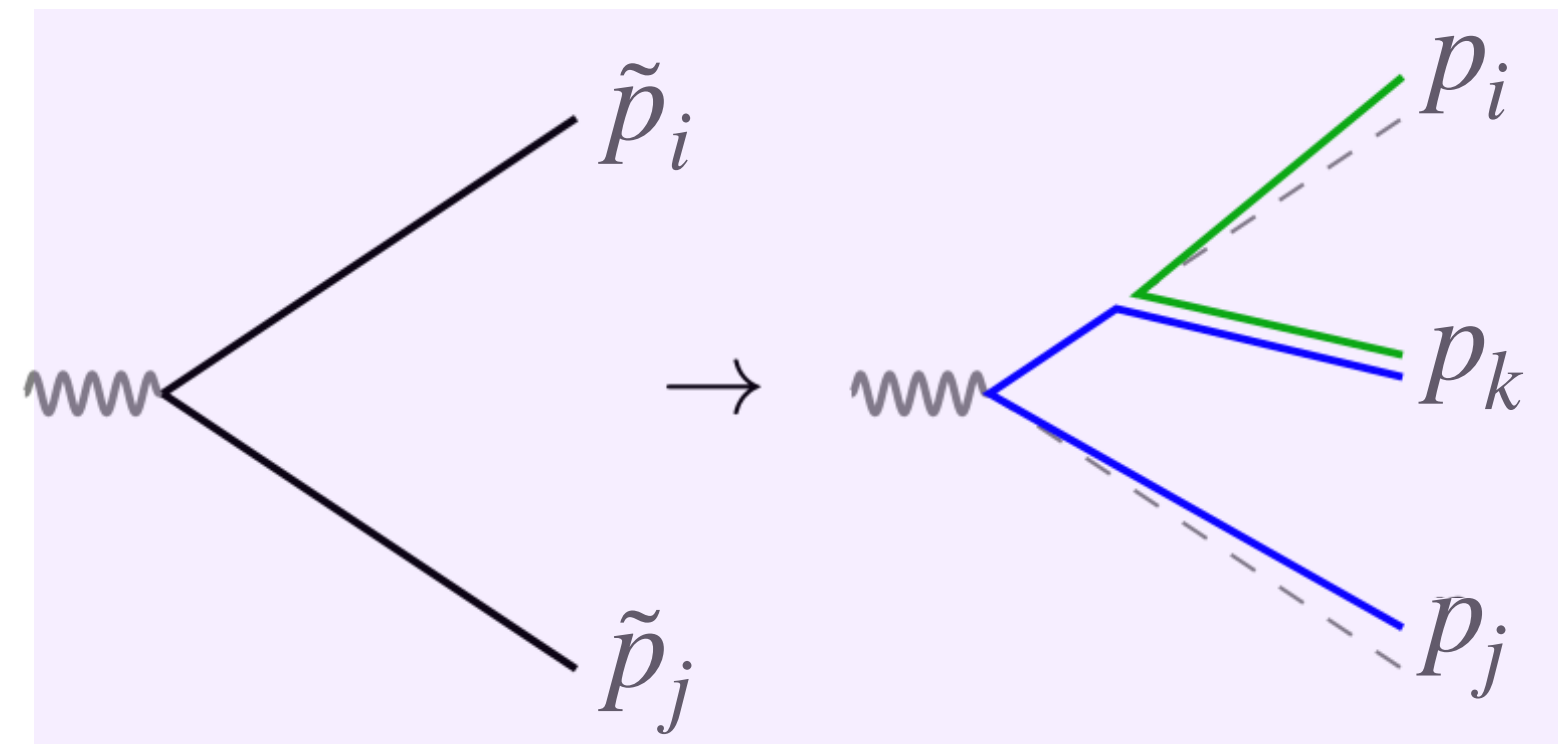
$$d\mathcal{P}_{\tilde{i}\tilde{j} \rightarrow ijk} \sim \frac{dv^2}{v^2} d\bar{\eta} \frac{d\varphi}{2\pi} P_{\tilde{i},\tilde{j} \rightarrow i,j,k}(v, \bar{\eta}, \varphi)$$

Evolution variable:
emissions are ordered
 $Q > v_1 > v_2 > \dots > \Lambda$

Matrix element for
emitting a parton k
from a parton i (or j)

Dissecting the parton shower emission probability

Starting from a $e^+e^- \rightarrow Z^* \rightarrow q\bar{q}$ system, what is the splitting probability?



Kinematic mapping:
how to reshuffle the momenta of i and j after the emission takes place

$$d\mathcal{P}_{\tilde{i}\tilde{j} \rightarrow ijk} \sim \frac{dv^2}{v^2} d\bar{\eta} \frac{d\varphi}{2\pi} P_{\tilde{i},\tilde{j} \rightarrow i,j,k}(v, \bar{\eta}, \varphi)$$

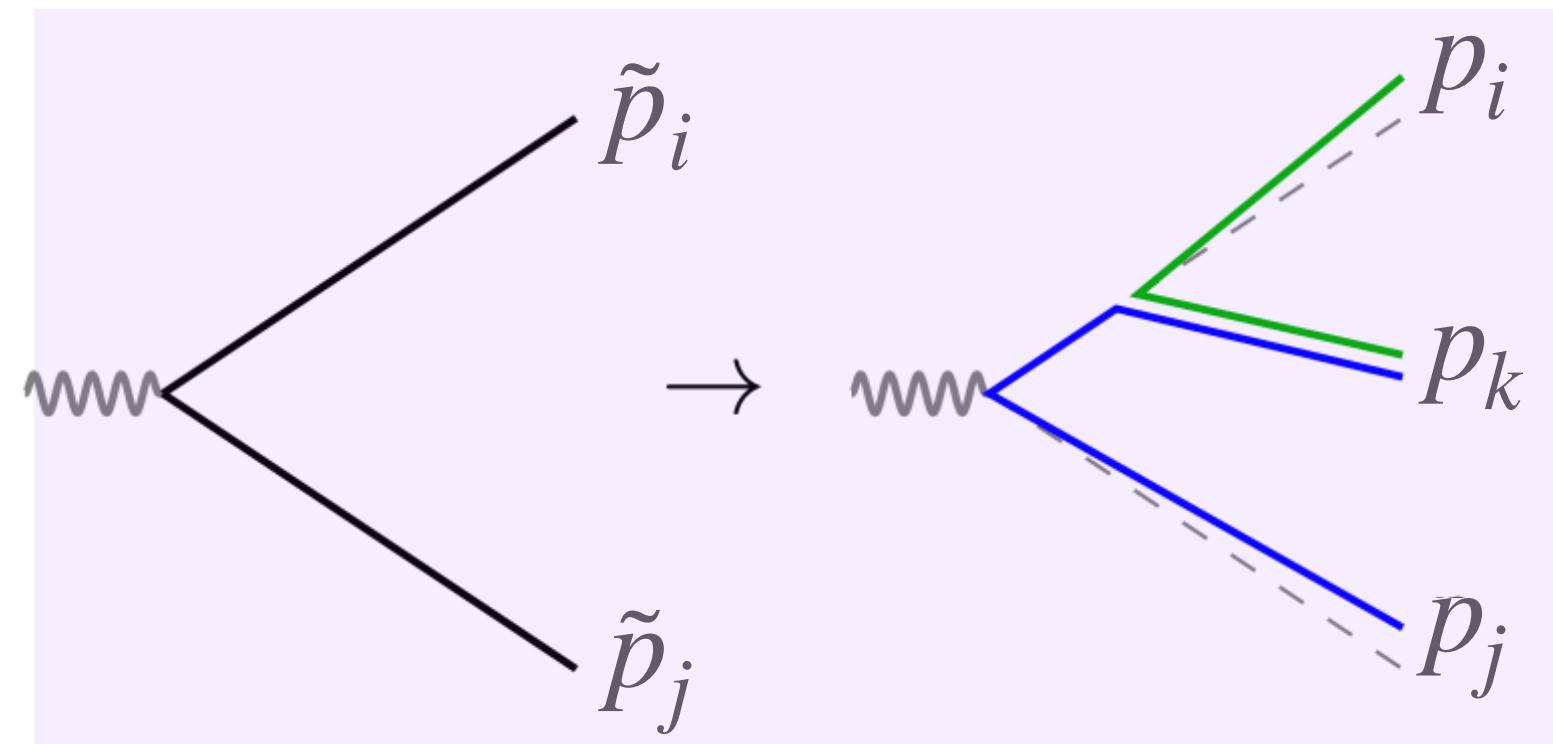
Evolution variable:
emissions are ordered
 $Q > v_1 > v_2 > \dots > \Lambda$

Matrix element for emitting a parton k from a parton i (or j)

Dissecting the parton shower emission probability

Starting from a $e^+e^- \rightarrow Z^* \rightarrow q\bar{q}$ system, what is the splitting probability?

Their interplay determines the shower **logarithmic accuracy**



Kinematic mapping: how to reshuffle the momenta of i and j after the emission takes place

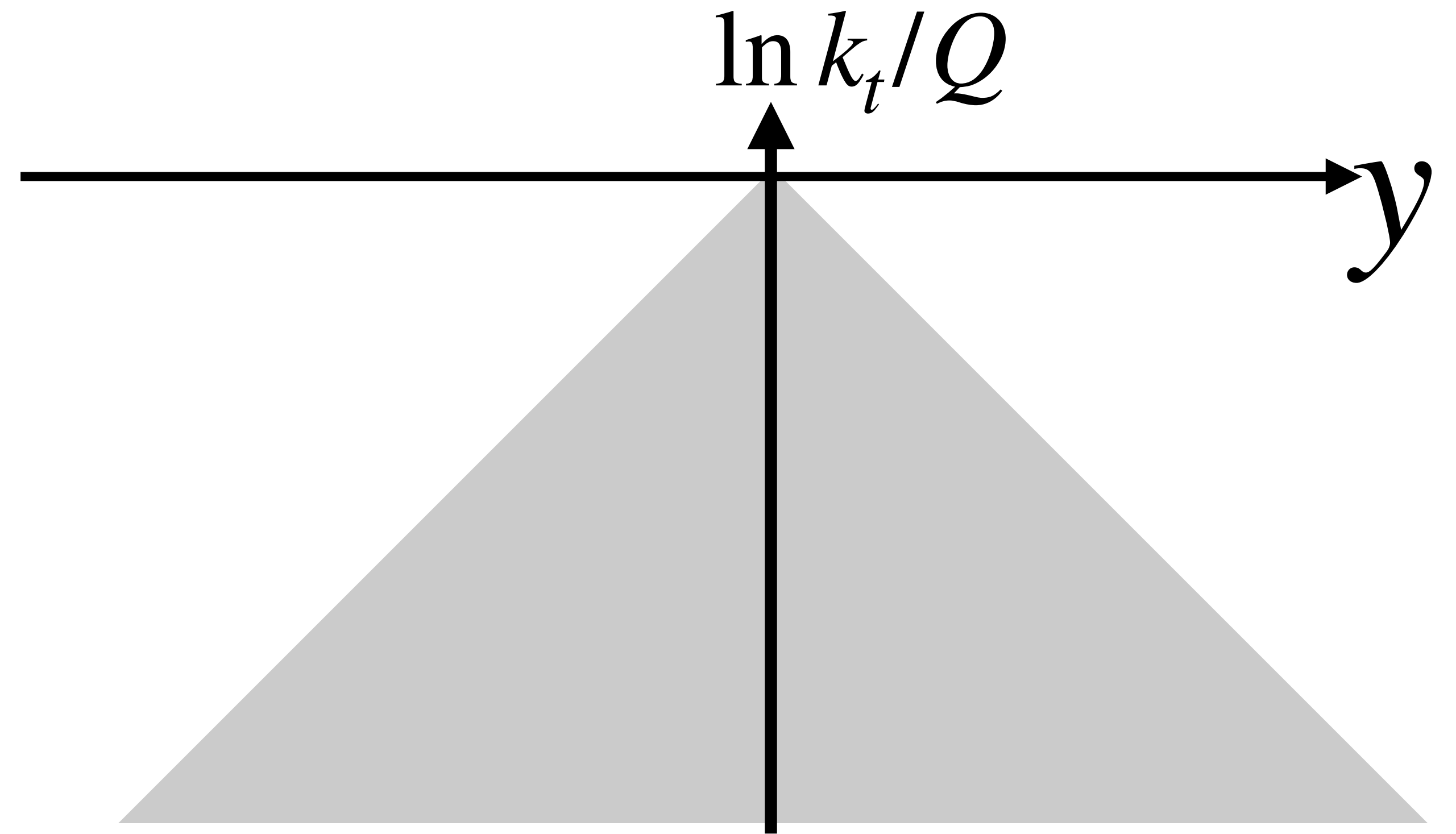
$$d\mathcal{P}_{\tilde{i}\tilde{j} \rightarrow ijk} \sim \frac{dv^2}{v^2} d\bar{\eta} \frac{d\varphi}{2\pi} P_{\tilde{i},\tilde{j} \rightarrow i,j,k}(v, \bar{\eta}, \varphi)$$

Evolution variable: emissions are ordered $Q > v_1 > v_2 > \dots > \Lambda$

Matrix element for emitting a parton k from a parton i (or j)

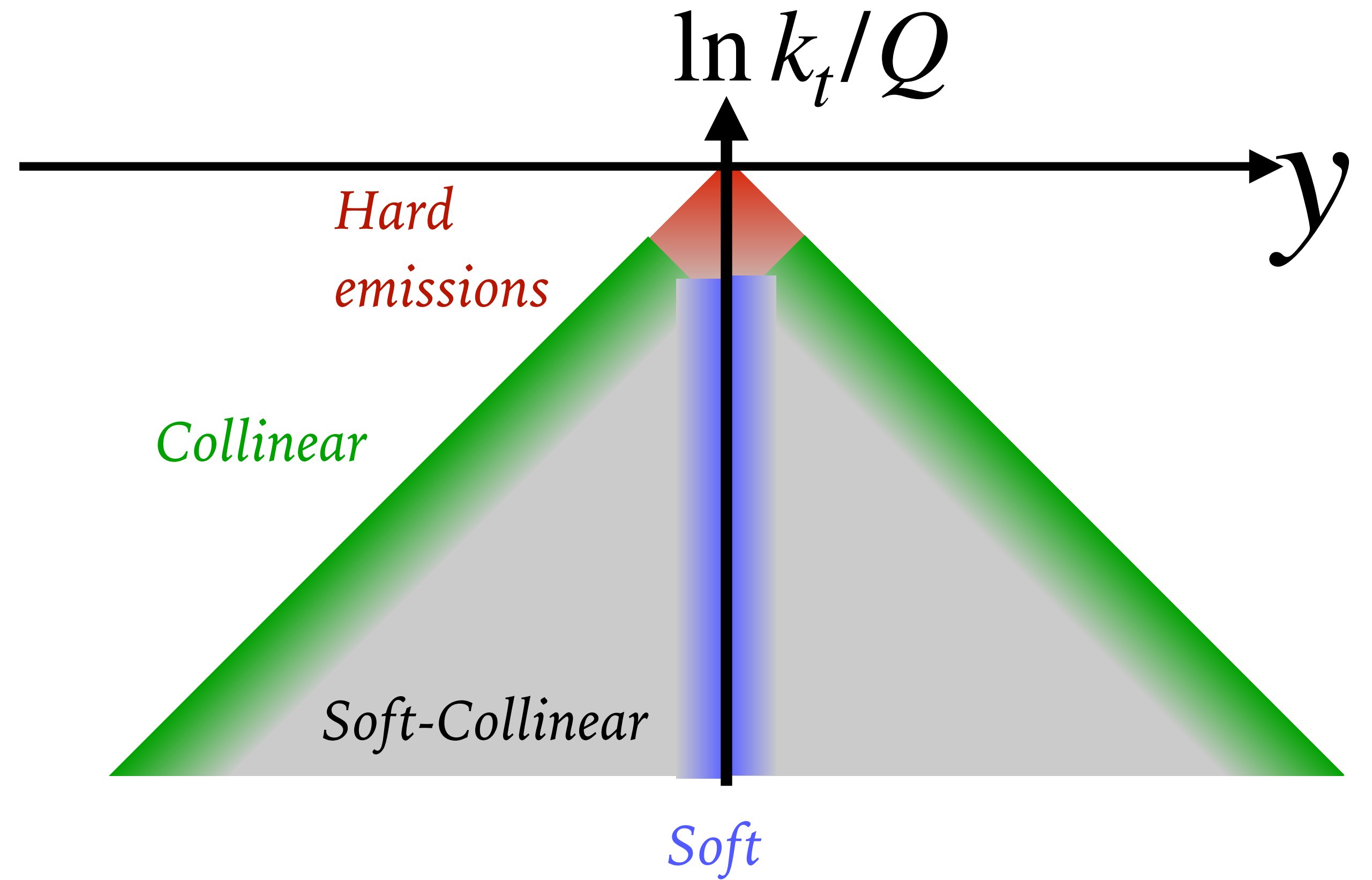
How to build a logarithmically-accurate parton shower?

- **The Lund plane:** diagnostic tools for resummation and parton showers



How to build a logarithmically-accurate parton shower?

- **The Lund plane:** diagnostic tools for resummation and parton showers

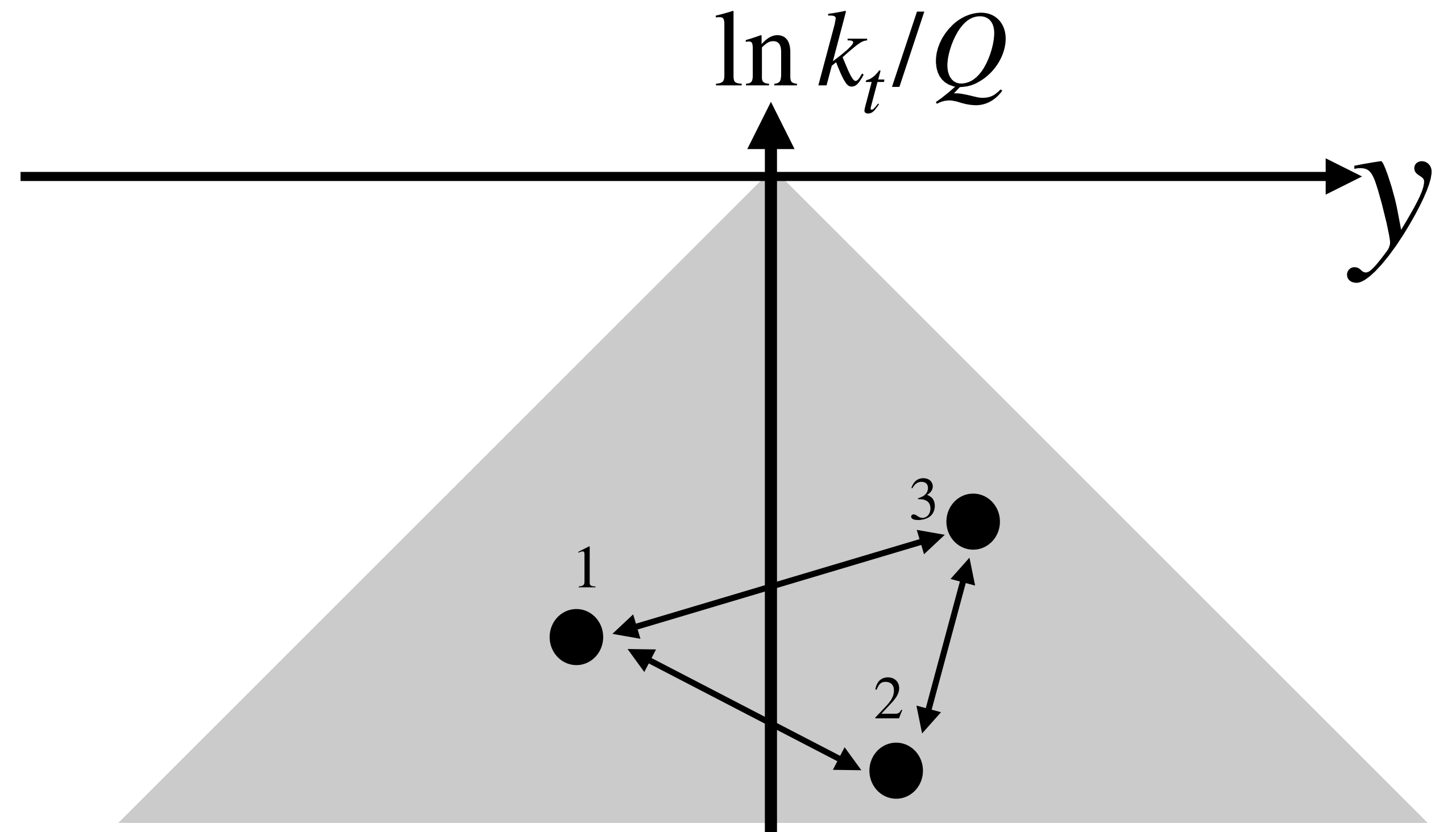


How to build a LL parton shower?

- The Lund plane: diagnostic tools for resummation and parton showers
- At Leading Logarithmic accuracy we only care about **soft-collinear emissions** very separated between each others

$$dP_i = \frac{\alpha_s(k_t)}{\pi} \frac{2C_F}{z} dz d \ln k_t$$

One-loop QCD coupling constant at $\mu_R = k_t$ *LO soft splitting function*



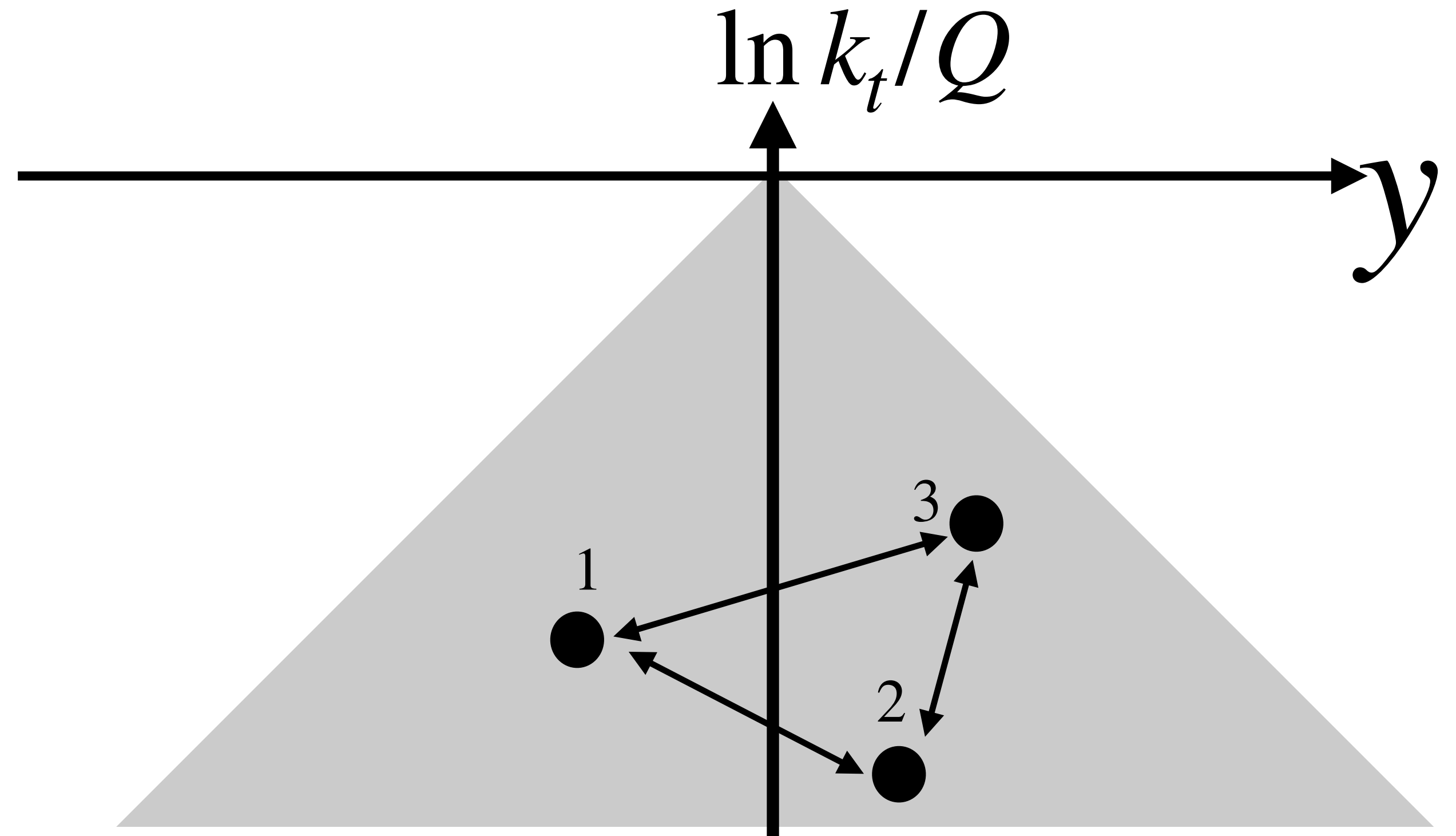
How to build a LL parton shower?

- The Lund plane: diagnostic tools for resummation and parton showers
- At Leading Logarithmic accuracy we only care about **soft-collinear emissions** very separated between each others

$$dP_i = \frac{\alpha_s(k_t)}{\pi} \frac{2C_F}{z} dz d \ln k_t$$

*One-loop QCD coupling
constant at $\mu_R = k_t$*

*LO soft splitting
function*



This tells us what **matrix element** should we use to generate a new emission

How to build a LL parton shower?

➤ The Lund plane: diagnostic tools for resummation and parton showers

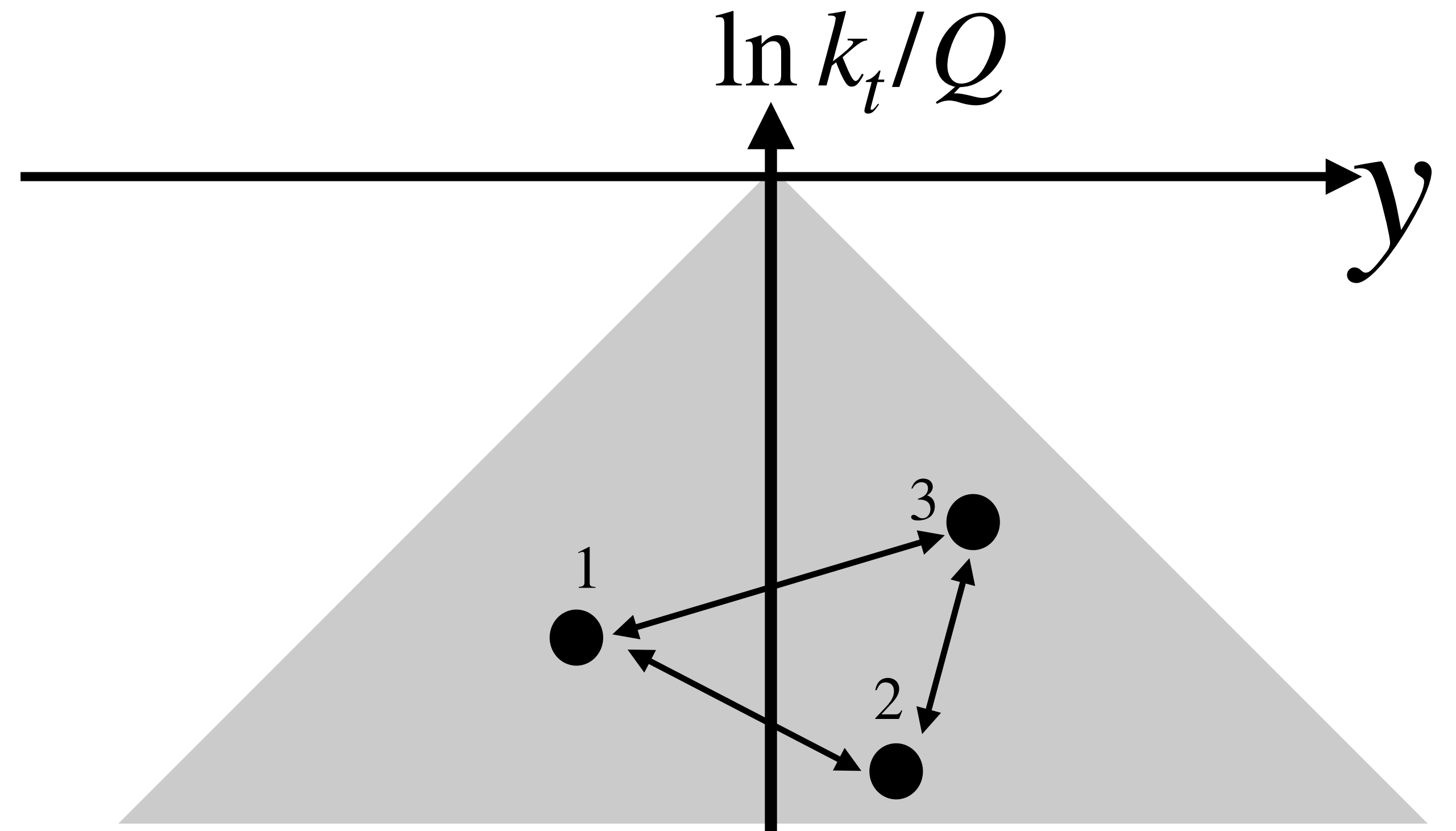
➤ At Leading Logarithmic accuracy we only care about

soft-collinear emissions very separated between each others

$$dP_i = \frac{\alpha_s(k_t)}{\pi} \frac{2C_F}{z} dz d \ln k_t$$

One-loop QCD coupling constant at $\mu_R = k_t$

LO soft splitting function



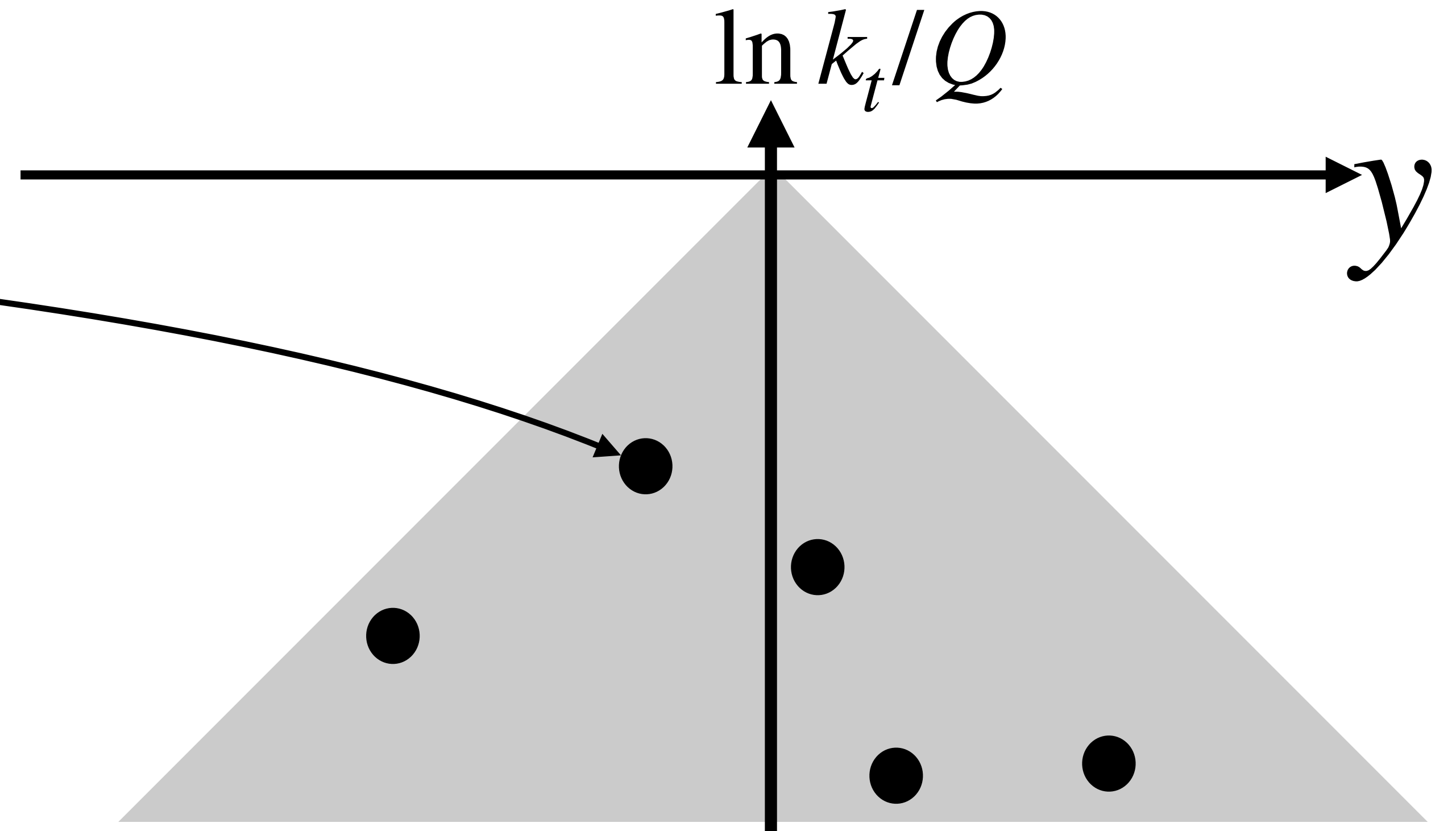
This constraints the **kinematic mapping** $\Phi_n \rightarrow \Phi_{n+1}$ and the **ordering variable** choice: emissions well separated in rapidity and transverse momentum are independent from each others

How to build a NLL parton shower?

At NLL accuracy:

- The rate for soft-collinear emissions must be correct at NLO

$$dP_i = \frac{\alpha_s(k_t)}{\pi} \left(1 + \frac{\alpha_s(k_t)}{2\pi} K_1 \right) \frac{2C_F}{z} dz d \ln k_t$$



How to build a NLL parton shower?

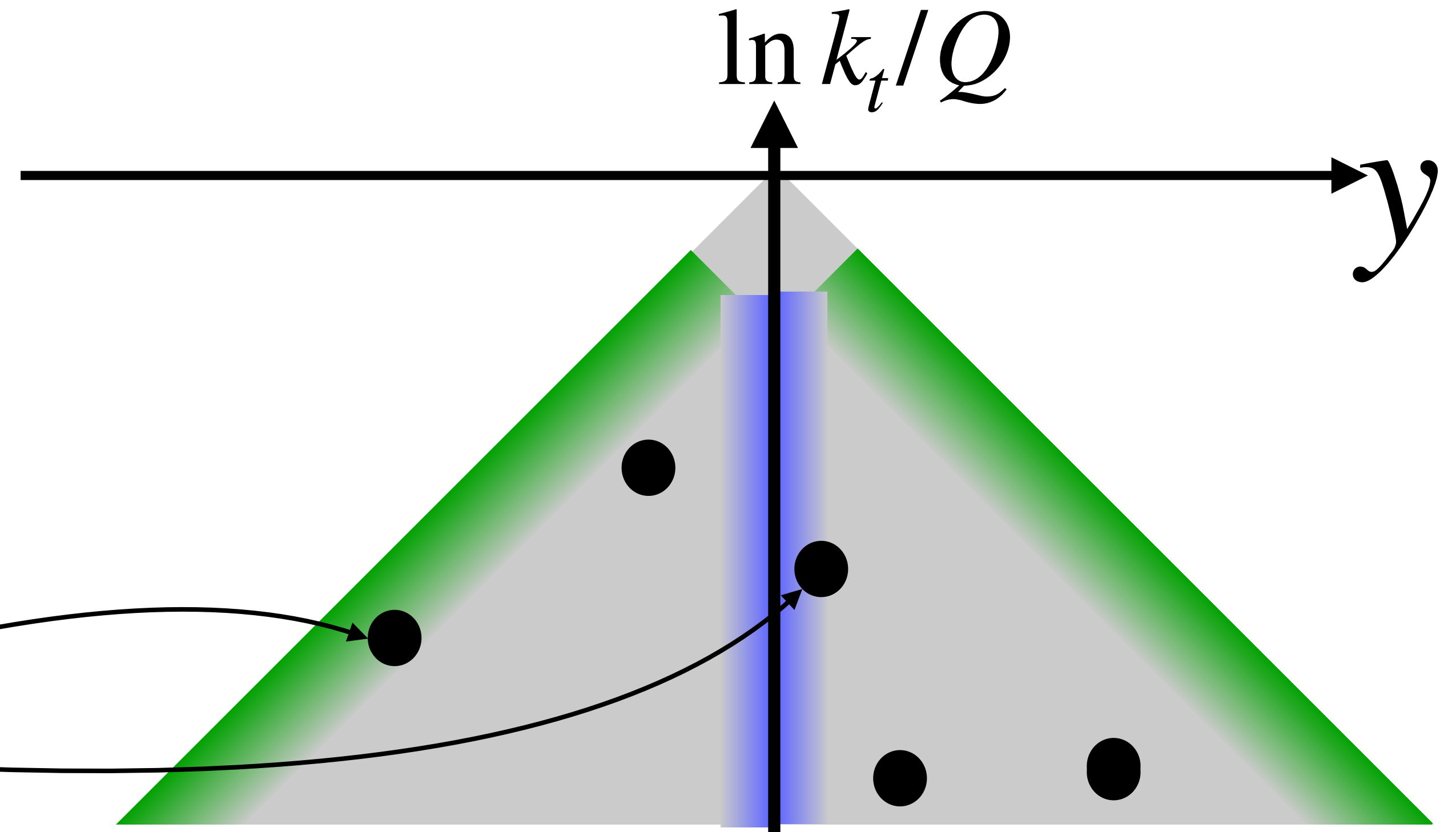
At NLL accuracy:

- The rate for soft-collinear emissions must be correct at NLO

$$dP_i = \frac{\alpha_s(k_t)}{\pi} \left(1 + \frac{\alpha_s(k_t)}{2\pi} K_1 \right) \frac{2C_F}{z} dz d \ln k_t$$

- We need to include soft and collinear contributions at LO

$$dP_i = \frac{\alpha_s(k_t)}{\pi} P(z) dz d \ln k_t$$



How to build a NLL parton shower?

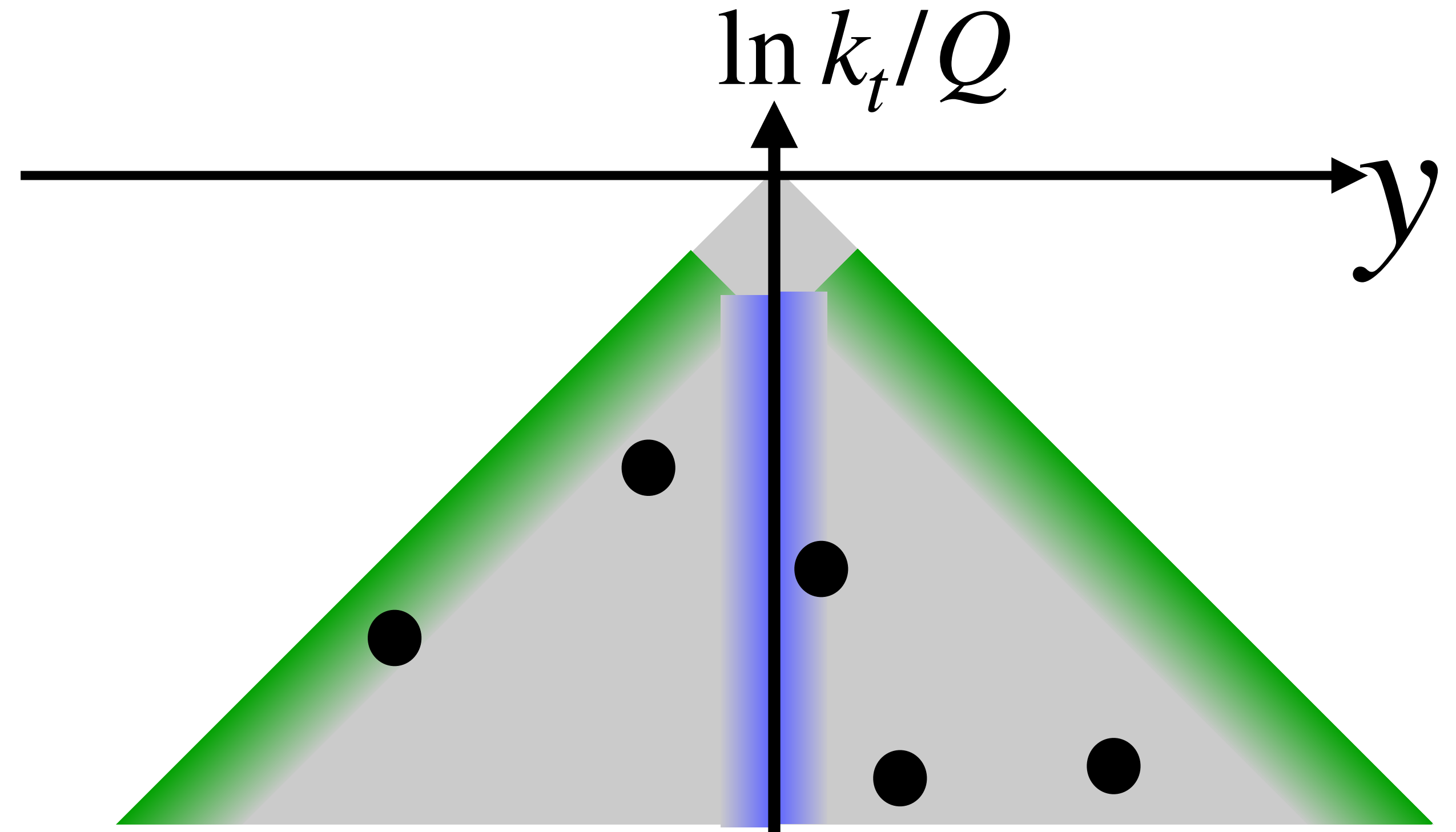
At NLL accuracy:

- The rate for soft-collinear emissions must be correct at NLO

$$dP_i = \frac{\alpha_s(k_t)}{\pi} \left(1 + \frac{\alpha_s(k_t)}{2\pi} K_1 \right) \frac{2C_F}{z} dz d \ln k_t$$

- We need to include soft and collinear contributions at LO

$$dP_i = \frac{\alpha_s(k_t)}{\pi} P(z) dz d \ln k_t$$



This tells us what **matrix element** should we use to generate a new emission

Catani, Marchesini, Webber '91

How to build a NLL parton shower?

At NLL accuracy:

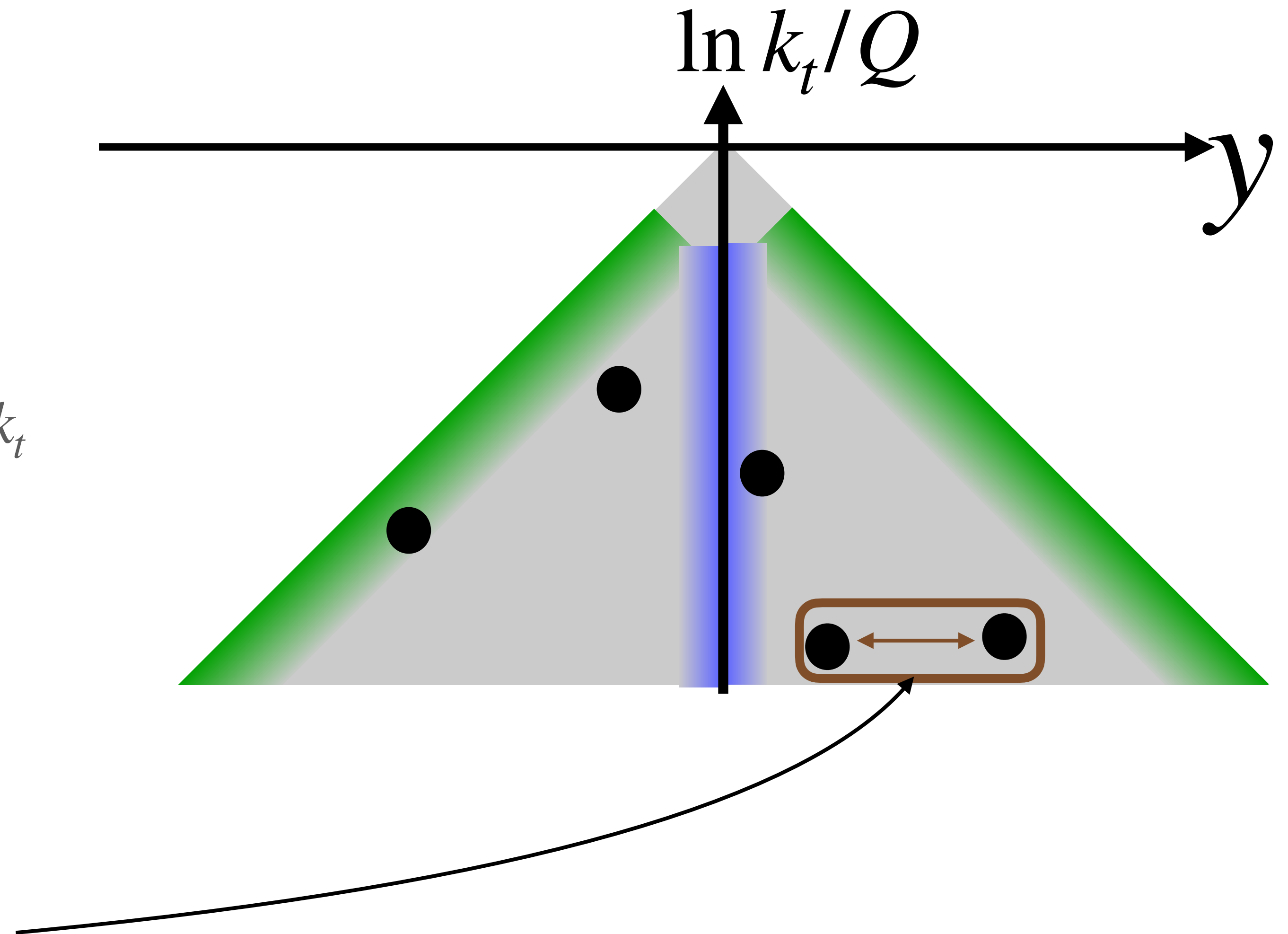
- ▶ The rate for soft-collinear emissions must be correct at NLO

$$dP_i = \frac{\alpha_s(k_t)}{\pi} \left(1 + \frac{\alpha_s(k_t)}{2\pi} K_1 \right) \frac{2C_F}{z} dz d \ln k_t$$

- ▶ We need to include soft and collinear contributions at LO

$$dP_i = \frac{\alpha_s(k_t)}{\pi} P(z) dz d \ln k_t$$

- ▶ Emissions separated in **just one direction** in the Lund plane enter at this order



How to build a NLL parton shower?

At NLL accuracy:

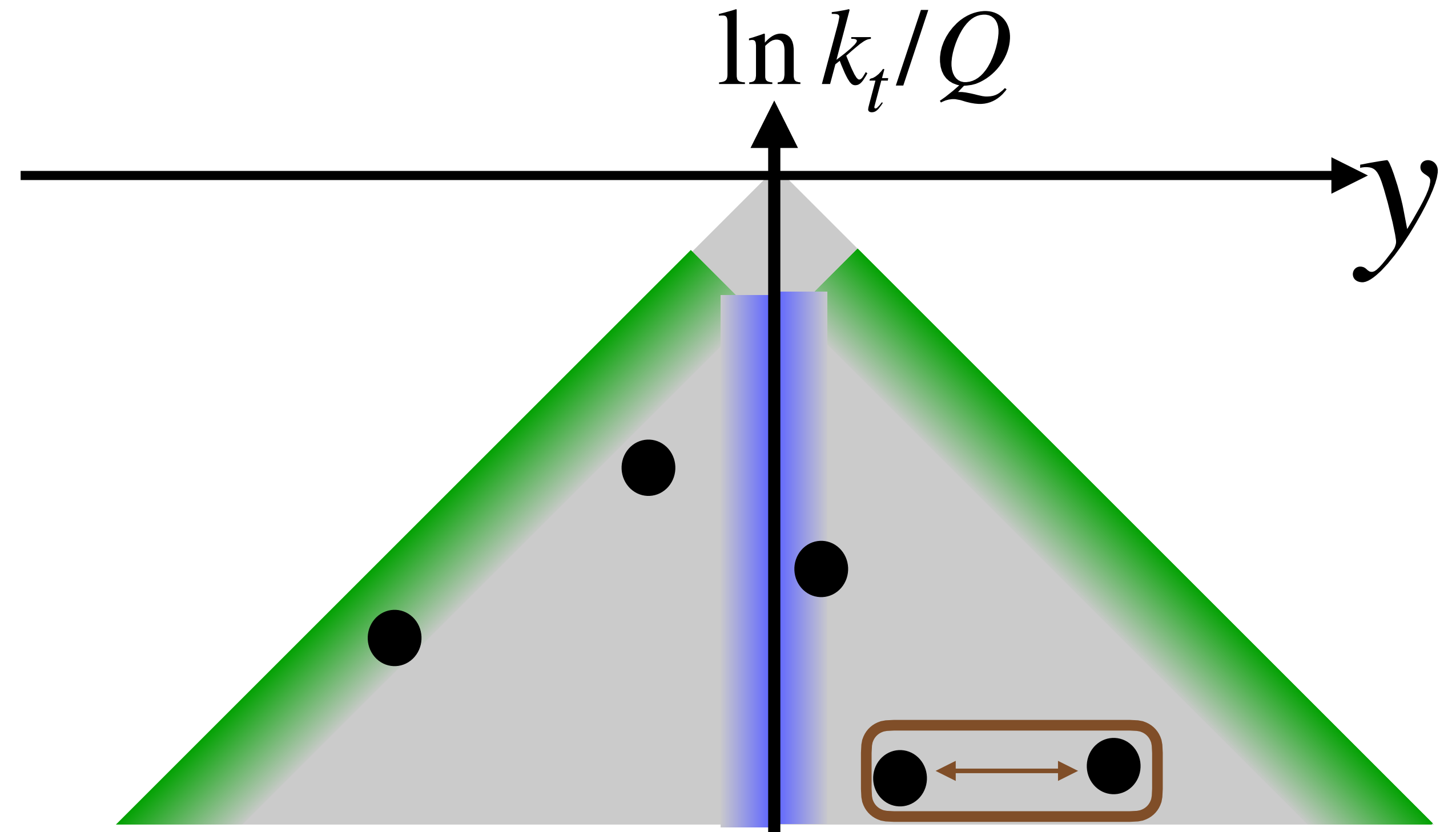
- The rate for soft-collinear emissions must be correct at NLO

$$dP_i = \frac{\alpha_s(k_t)}{\pi} \left(1 + \frac{\alpha_s(k_t)}{2\pi} K_1 \right) \frac{2C_F}{z} dz d \ln k_t$$

- We need to include soft and collinear contributions at LO

$$dP_i = \frac{\alpha_s(k_t)}{\pi} P(z) dz d \ln k_t$$

- Emissions separated in **just one direction** in the Lund plane enter at this order

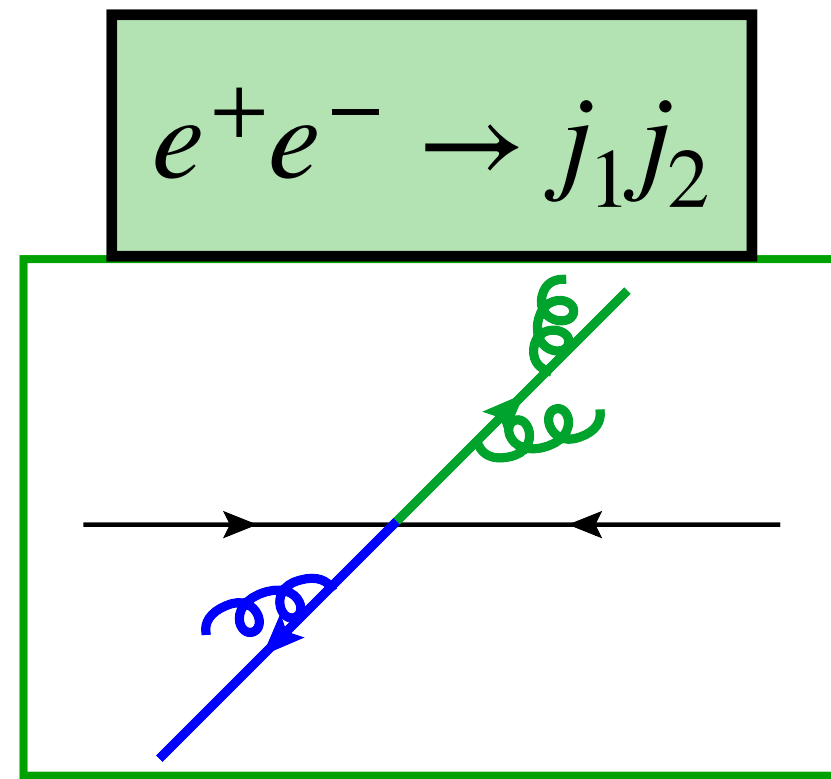


Constraints **kinematic mapping** $\Phi_n \rightarrow \Phi_{n+1}$ and **ordering variable**: emissions well separated in rapidity are independent from each other, even if they have similar transverse momentum

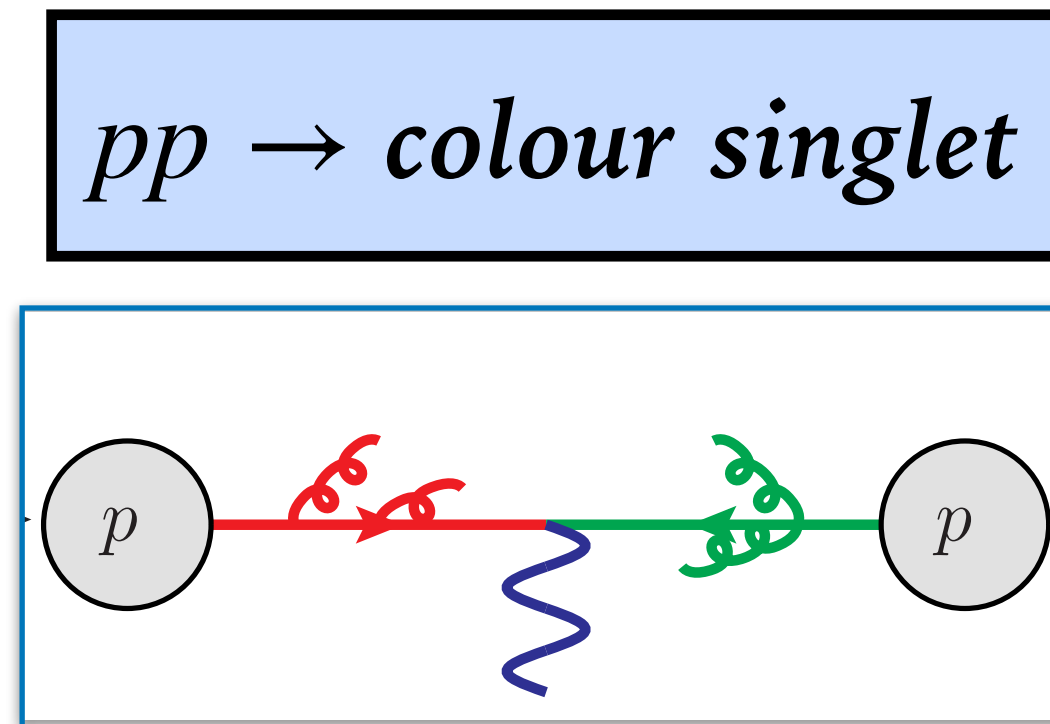
Dasgupta, Dreyer, Hamilton, Monni, Salam, 1805.09327 ;+ Soyez, 2002.11114

Status of NLL PanScales showers

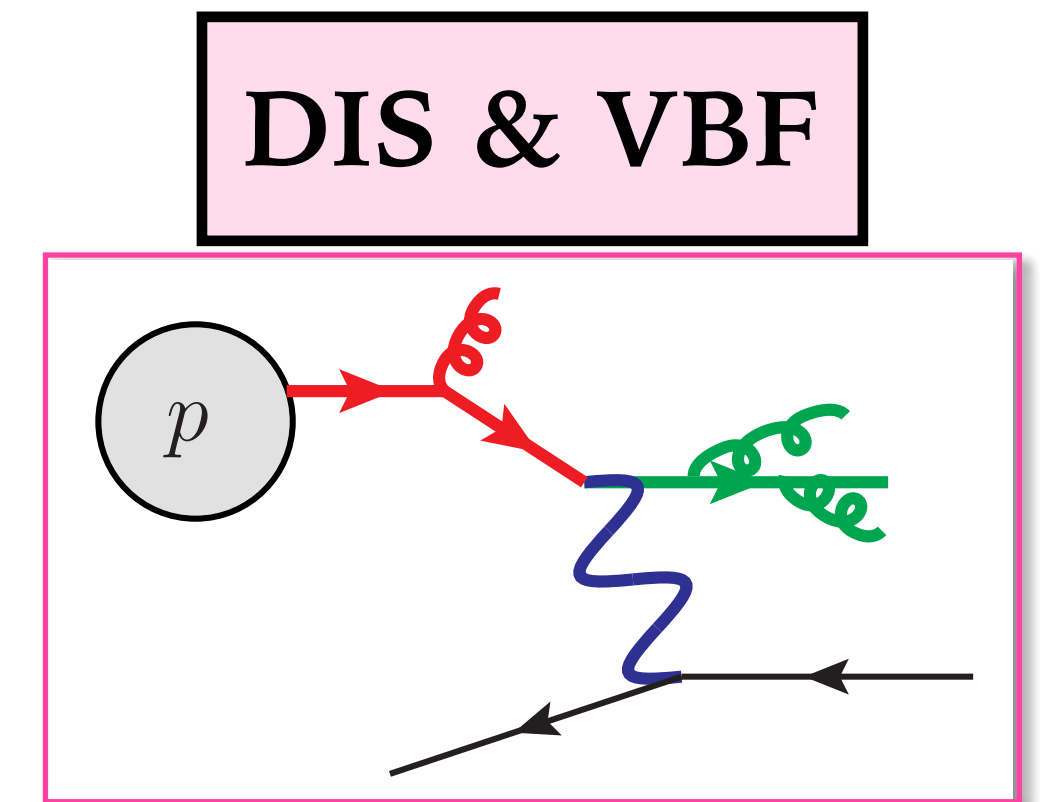
- This enabled the **PanScales** to devise the **first** showers with **general** NLL accuracy for



Dasgupta, Dreyer, Hamilton,
Monni, Salam, Soyez,
2002.11114



van Beekveld, SFR, Soto-Ontoso,
Salam, Soyez, Verheyen, 2205.02237,
+ Hamilton 2207.09467



van Beekveld, SFR,
2305.08645

...with **subleading colour** (2011.10054) and **spin correlations** (2103.16526, 2111.01161)

- Herwig7 angular-ordered shower for the same processes is NLL but only for **global event shapes** (Bewick, SFR, Richardson, Seymour, 1904.11866, 2107.04051)
- **Deductor** has been proven to be NLL at least for $e^+e^- \rightarrow j_1j_2$ (Nagy, Soper 2011.04777)
- **Alaric** is NLL at leading colour for $e^+e^- \rightarrow j_1j_2$ (2208.06057), recently extended to generic pp collisions (2404.14360) — expected to retain NLL accuracy for $pp \rightarrow \text{colour singlet}$

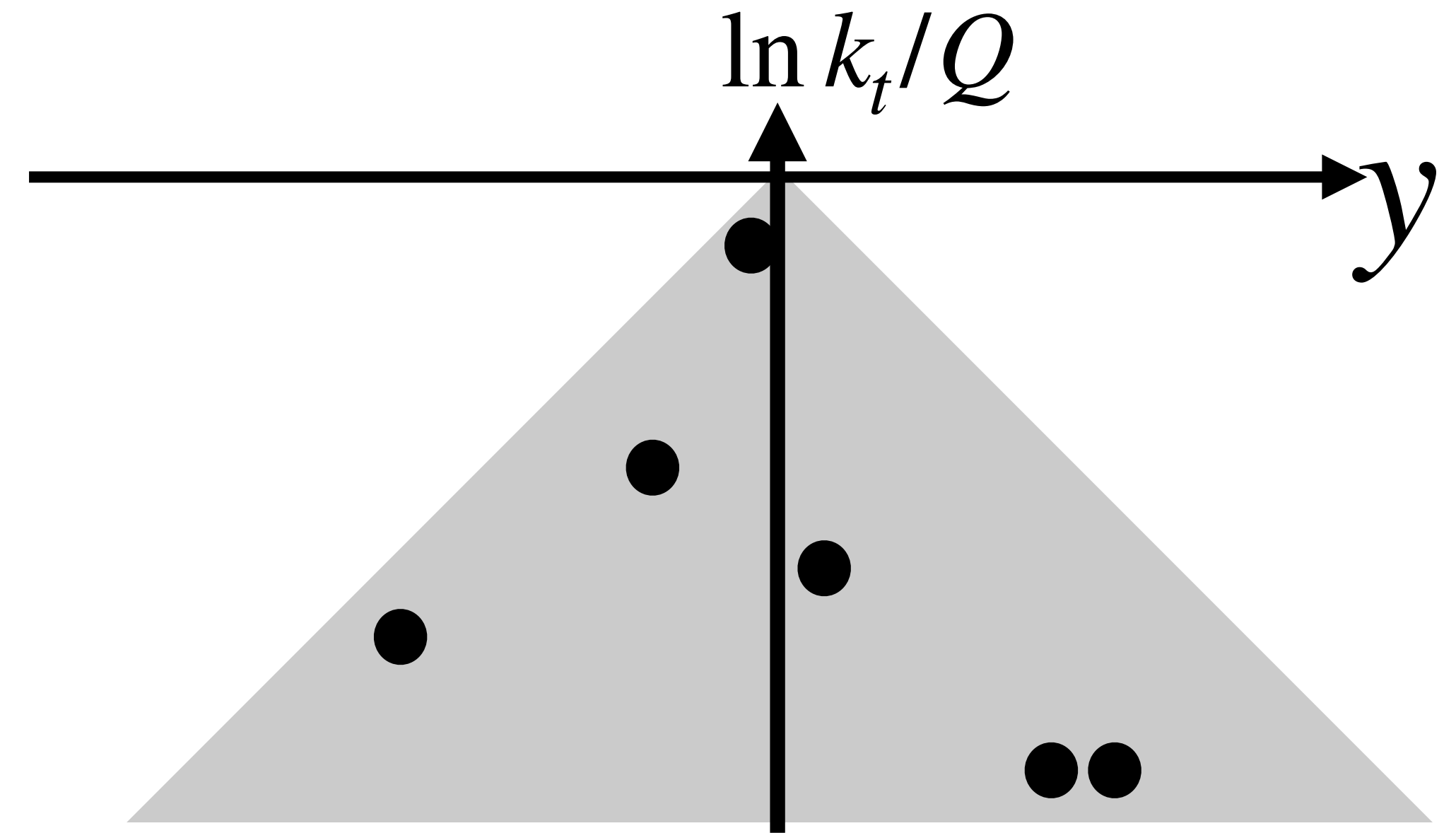
How to go beyond NLL in a parton shower?

[SFR, Hamilton, Karlberg, Salam, Scyboz, Soyez [2307.11142](#)]

Focus on soft emissions

- ✓ Soft-collinear emsns at NLO
- ✓ Soft (large angle) emsns at LO
- ✓ Correct rate for pair of emsns separated only in one Lund coordinate

NLL



How to go beyond NLL in a parton shower?

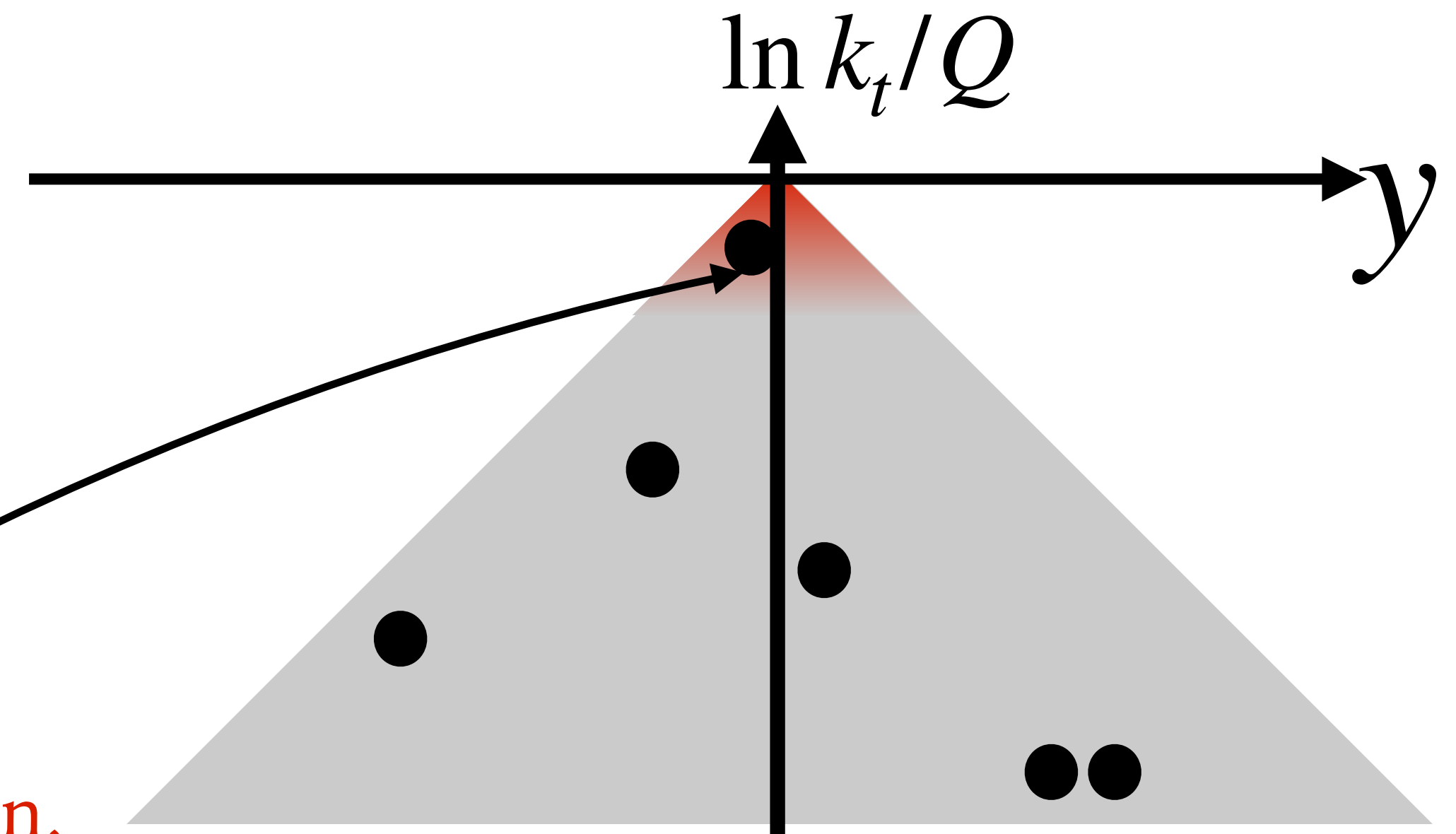
[SFR, Hamilton, Karlberg, Salam, Scyboz, Soyez [2307.11142](#)]

Focus on soft emissions

- ✓ Soft-collinear emsns at NLO
- ✓ Soft (large angle) emsns at LO
- ✓ Correct rate for pair of emsns separated only in one Lund coordinate

✓ **Hard** emissions at **LO**

[Hamilton, Karlberg, Scyboz, Salam, Verheyen, [2301.09645](#)]



See also S. Zanolì's talk!

NLL

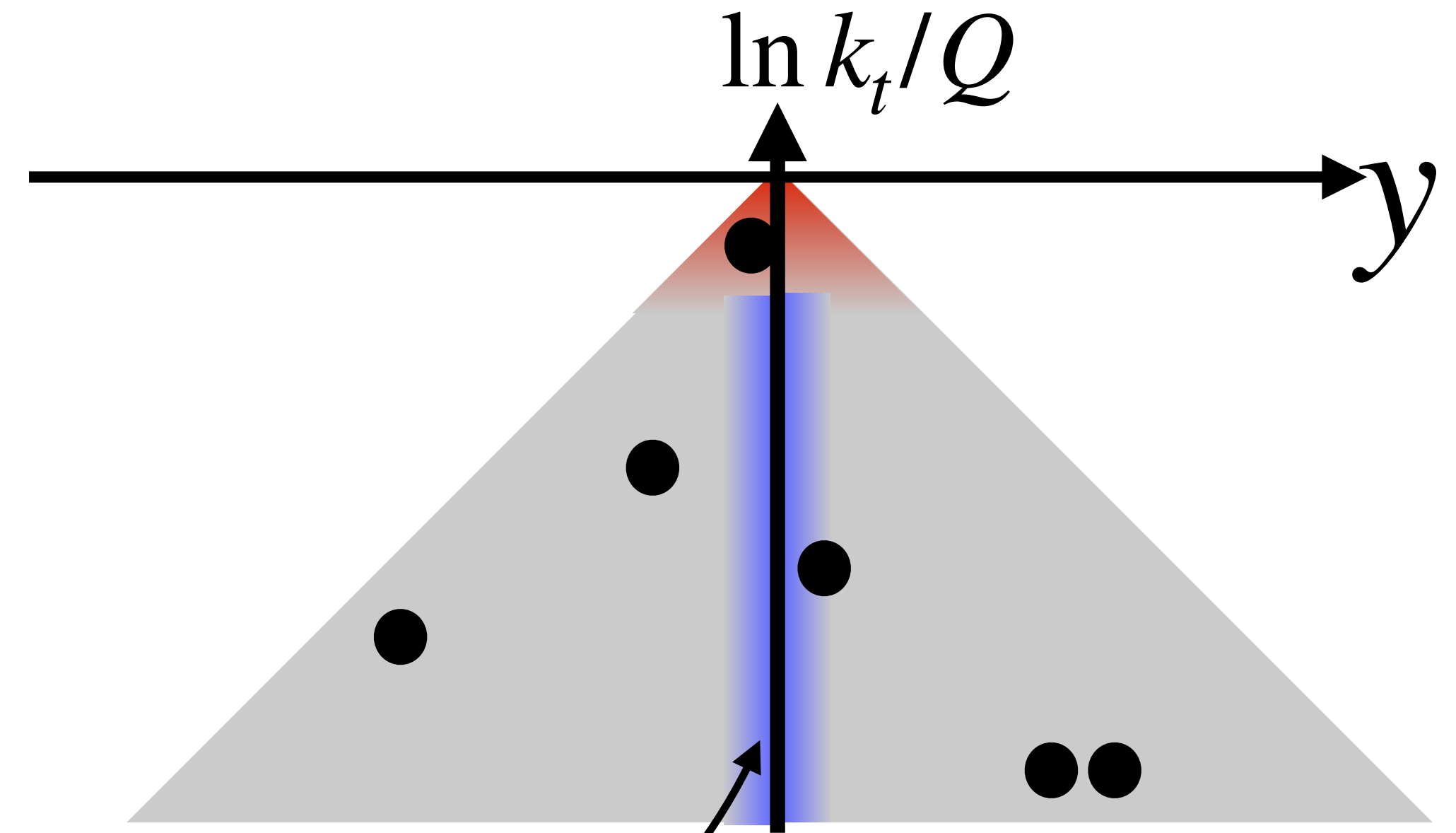
NNLL

How to go beyond NLL in a parton shower?

[SFR, Hamilton, Karlberg, Salam, Scyboz, Soyez [2307.11142](#)]

Focus on soft emissions

- ✓ Soft-collinear emsns at **NLO**
- ✓ Soft (large angle) emsns at **LO**
- ✓ Correct rate for pair of emsns separated only in **one Lund coordinate**



- ✓ **Hard** emissions at **LO**
- ✓ Soft (large angle) emsns at **NLO**

NLL

NNLL

How to go beyond NLL in a parton shower?

[SFR, Hamilton, Karlberg, Salam, Scyboz, Soyez [2307.11142](#)]

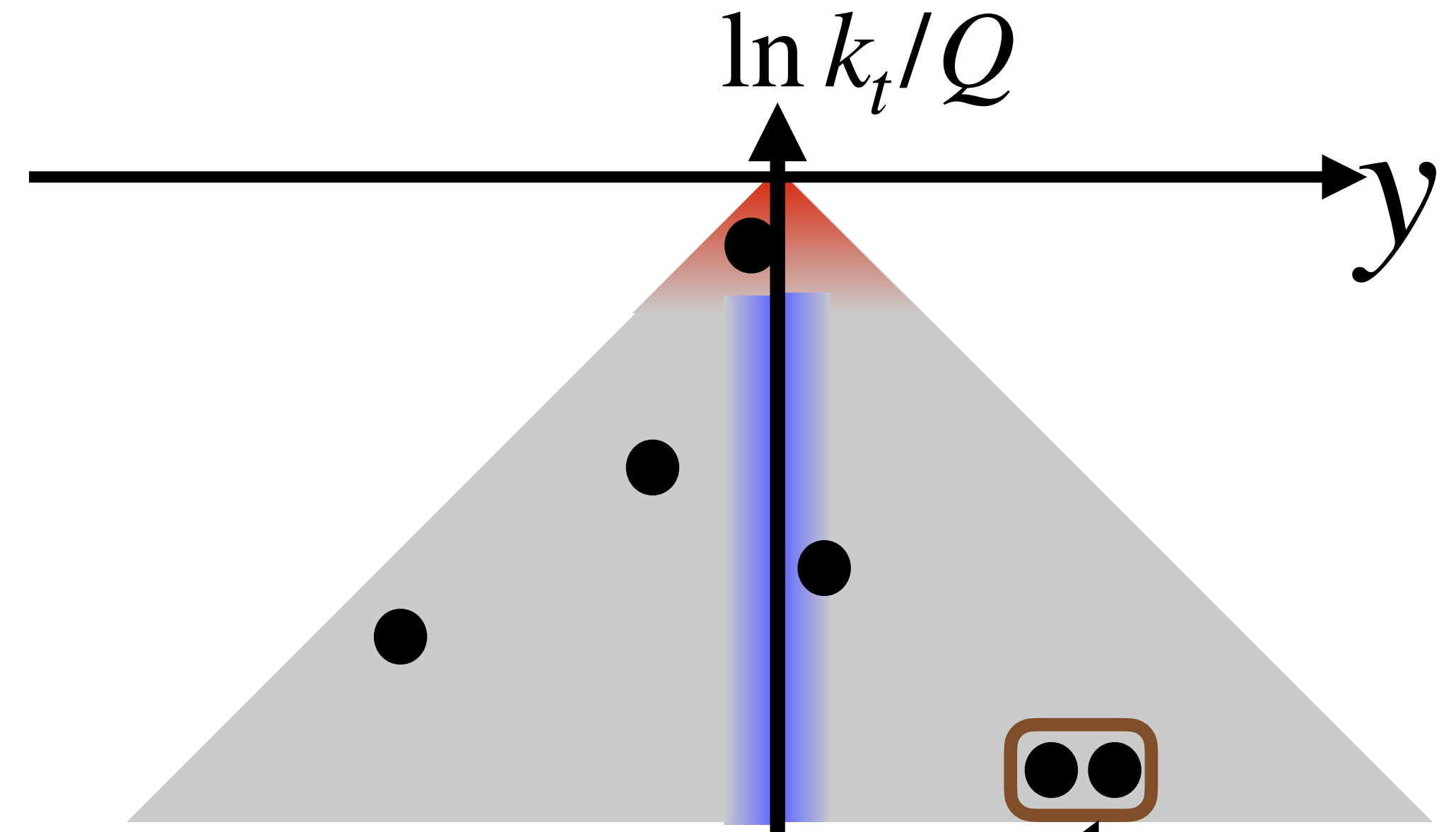
Focus on soft emissions

NLL

- ✓ Soft-collinear emsns at NLO
- ✓ Soft (large angle) emsns at LO
- ✓ Correct rate for pair of emsns separated only in **one Lund coordinate**

NNLL

- ✓ **Hard** emissions at LO
- ✓ Soft (large angle) emsns at NLO
- ✓ Correct rate for pair of emsns **close in the Lund plane**



How to go beyond NLL in a parton shower?

[SFR, Hamilton, Karlberg, Salam, Scyboz, Soyez [2307.11142](#)]

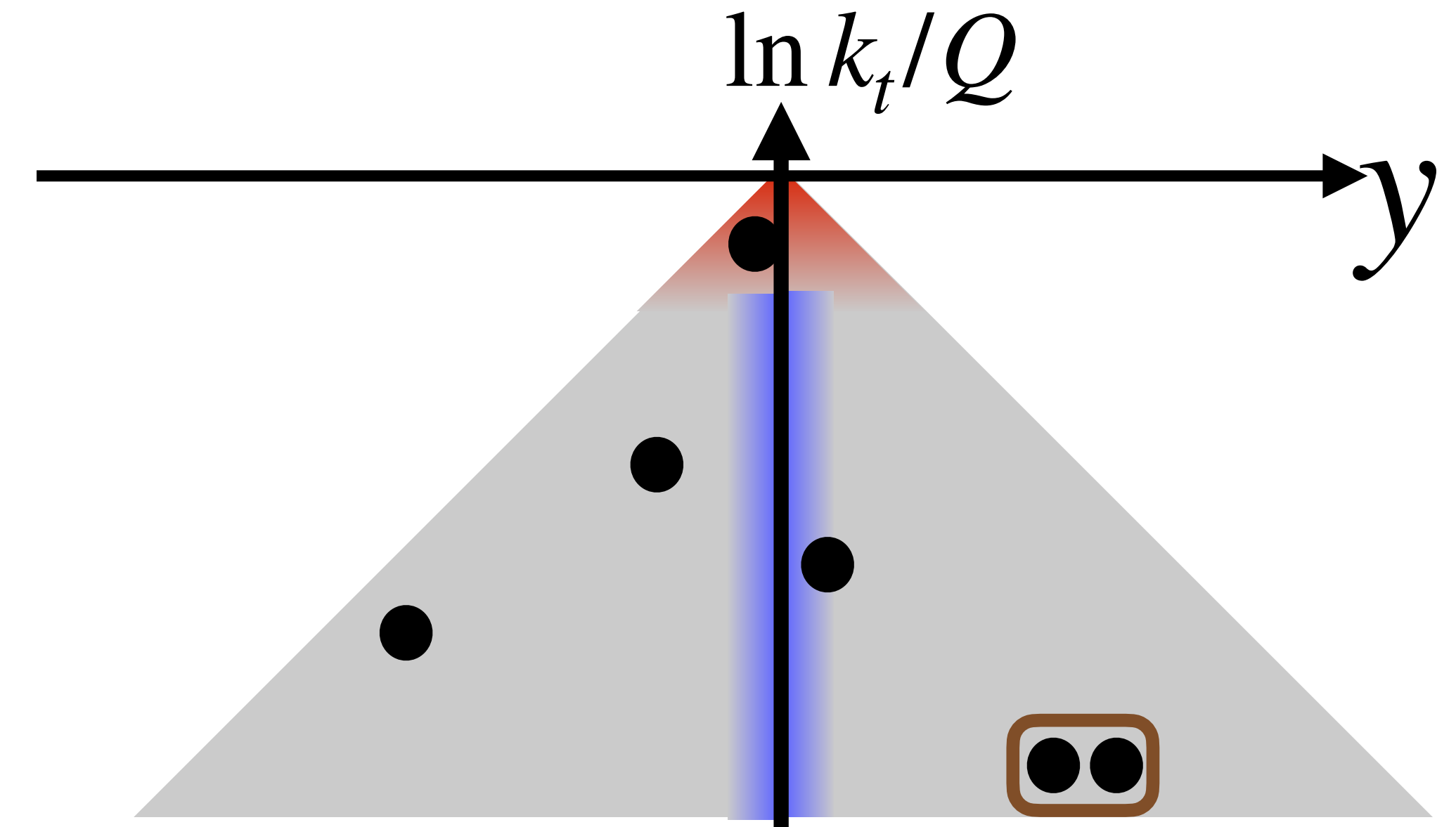
Focus on soft emissions

NLL

- ✓ Soft-collinear emsns at **NLO**
- ✓ Soft (large angle) emsns at **LO**
- ✓ Correct rate for pair of emsns separated only in **one Lund coordinate**

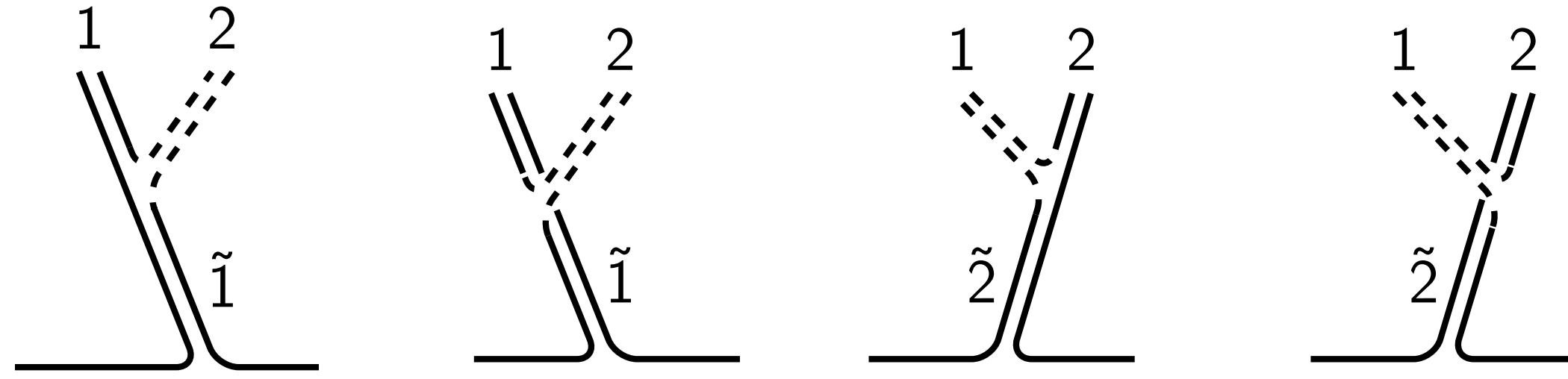
NNLL

- ✓ **Hard** emissions at **LO**
- ✓ Soft (large angle) emsns at **NLO**
- ✓ Correct rate for pair of emsns **close in the Lund plane**
- ✓ ...

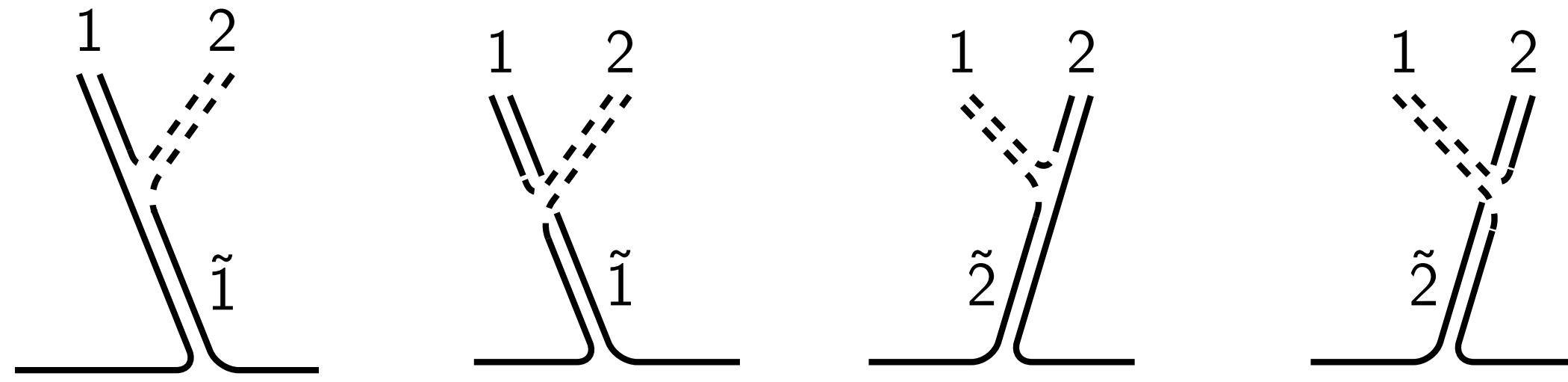


- **NNDL** for [subset] multiplicities, i.e. $\alpha_s^n L^{2n}$, $\alpha_s^n L^{2n-1}$, $\alpha_s^n L^{2n-2}$
- **Next-to-Single-Log (NSL)** for non-global logarithms, e.g. energy in a slice, all terms $\alpha_s^n L^n$ and $\alpha_s^n L^{n-1}$

Correct rate for **pairs** or soft emissions = **Real** corrections

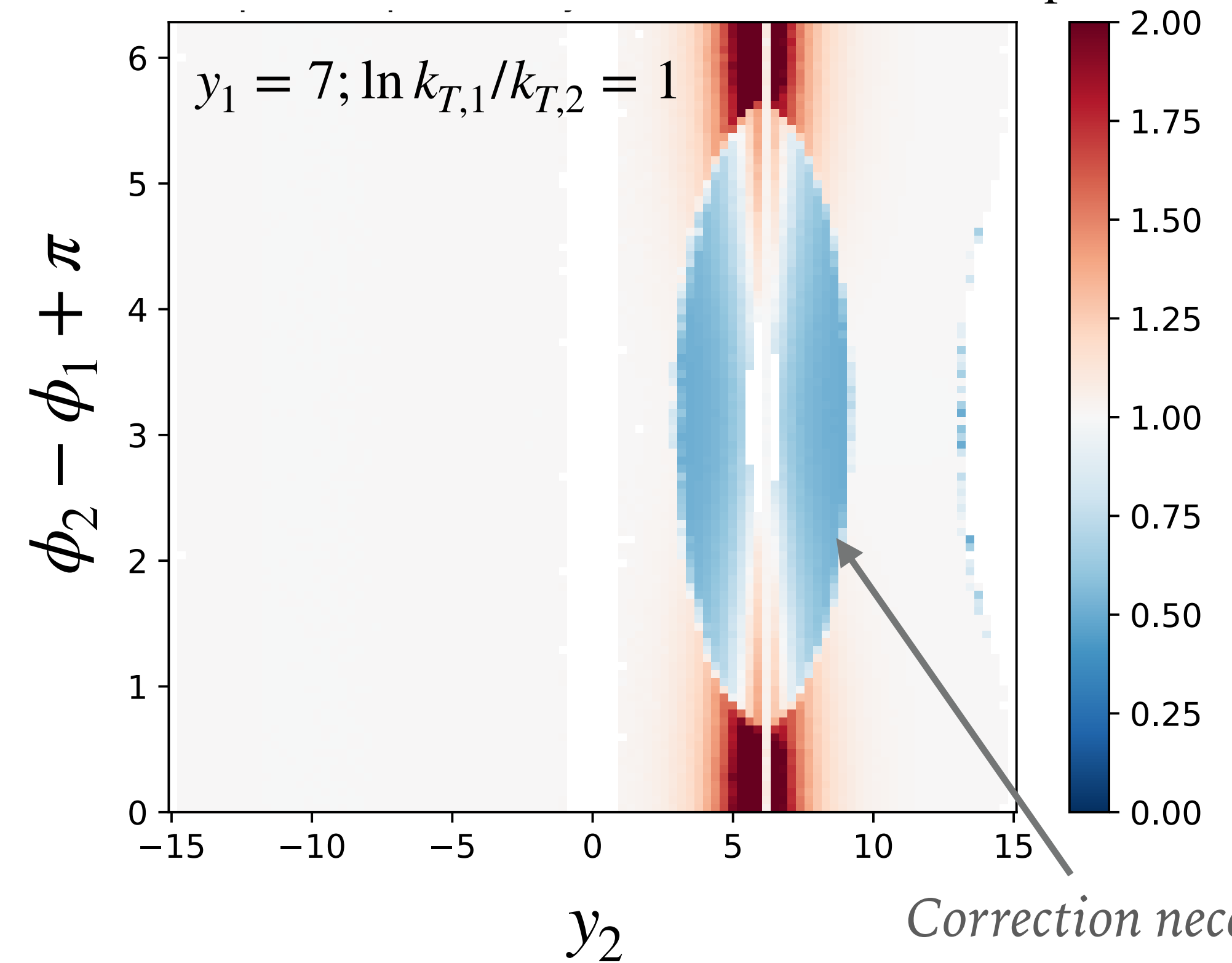


Correct rate for pairs or soft emissions = **Real** corrections



- a given two-emission configuration can come from several shower histories
- **accept a given emission with exact double-soft $M_{\text{exact}}^{(\text{DS})}$ divided by shower's effective double-soft matrix element summed over the histories h that could have produced that configuration**

Double-soft acceptance P_{accept}



Correction necessary only for neighbouring emsn as the shower is already NLL

$$P_{\text{accept}} = \frac{M_{\text{exact}}^{(\text{DS})}}{\sum_h M_{h,\text{PS}}^{(\text{DS})}}$$

NLO corrections to a single soft emission: standard behaviour

► For a soft emission

$$\text{Tree-level diagram} + \int_{y, p_{\perp} \text{ fixed}} \text{NLO diagram} = \frac{\alpha_s}{2\pi} K_1$$

► If this happens also in a **parton shower** simulation, we have the emission rate correct at $\mathcal{O}(\alpha_s^2)$

NLO corrections to a single soft emission: standard behaviour

► For a soft emission

$$\text{V} + \int_{y, p_{\perp} \text{ fixed}} \text{R} = \frac{\alpha_s}{2\pi} K_1$$

The diagram shows a blue circle labeled 'V' with a wavy line extending from it, representing a virtual correction. This is added to an integral over a green circle labeled 'R' with a wavy line, representing a real emission. The integral is taken over fixed variables y, p_{\perp} . The result is a yellow box containing the expression $\frac{\alpha_s}{2\pi} K_1$.

- If this happens also in a **parton shower** simulation, we have the emission rate correct at $\mathcal{O}(\alpha_s^2)$
- In a parton shower, **virtual corrections** are obtained by unitarity (=no emission probability)

$$\text{V}_{\text{PS}} \equiv - \int \text{R}_{\text{PS}}$$

*At fixed “shower variables”,
but the rapidity and p_{\perp} of
the jet can vary*

The diagram shows a blue circle labeled 'V_{PS}' with a wavy line, representing virtual corrections in a parton shower. This is equal to the negative of an integral over a green circle labeled 'R_{PS}' with a wavy line, representing real emissions in a parton shower. The integral is taken over shower variables. A pink note explains that at fixed shower variables, the rapidity and p_{\perp} of the jet can vary.

NLO corrections to a single soft emission: standard behaviour

► For a soft emission

$$V + \int_{y, p_{\perp} \text{ fixed}} R = \frac{\alpha_s}{2\pi} K_1$$

► If this happens also in a **parton shower** simulation, we have the emission rate correct at $\mathcal{O}(\alpha_s^2)$

► In a parton shower, **virtual corrections** are obtained by unitarity (=no emission probability)

$$V_{\text{PS}} = - \int_{\text{At fixed "shower variables", but the rapidity and } p_{\perp} \text{ of the jet can vary}} R_{\text{PS}}$$

► **Catani**, **Marchesini** and **Webber** defined the “CMW” scheme for the coupling in the shower

[*Nucl.Phys.B* 349 (1991) 635-654]

$$\alpha_s^{\text{CMW}} = \alpha_s \left(1 + \frac{\alpha_s}{2\pi} K_1 \right)$$

Additional virtual correction added directly to the splitting function

Ensures “on average”

$$V_{\text{PS}} + \int R_{\text{PS}} = \frac{\alpha_s}{2\pi} K_1$$

Revisiting **virtual** corrections to a single soft emission

- With our double soft acceptance we have $\mathbf{R}_{\text{PS}} = \mathbf{R}$. This yields

$$\text{Diagram 1} = \frac{\alpha_s}{2\pi} K_1 - \int \text{Diagram 2} \quad \text{Fixed shower variables}$$

The diagram on the left shows a blue circle labeled V_{PS} on a horizontal line, with a brown cone extending upwards and a wavy line inside it. The diagram on the right shows a pink cone on a horizontal line, with a green circle labeled \mathbf{R} inside it and wavy lines extending from the cone's edges.

Revisiting virtual corrections to a single soft emission

- ▶ With our double soft acceptance we have $\mathbf{R}_{\text{PS}} = \mathbf{R}$. This yields

$$V_{\text{PS}} = \frac{\alpha_s}{2\pi} K_1 - \int \text{Fixed shower variables}$$

- ▶ We modify the CMW scheme

$$K_1 \rightarrow K_1 + \Delta K_1(\Phi_{\text{PS}}^{(1)})$$

$$\frac{\alpha_s}{2\pi} \Delta K_1(\Phi_{\text{PS}}^{(1)}) = \int \text{Fixed shower variables} - \int \text{y, p}_\perp \text{ fixed}$$

Revisiting virtual corrections to a single soft emission

- With our double soft acceptance we have $\mathbf{R}_{\text{PS}} = \mathbf{R}$. This yields

$$\underbrace{\text{Diagram}}_{V_{\text{PS}}} = \frac{\alpha_s}{2\pi} K_1 - \int \underbrace{\text{Diagram}}_{\mathbf{R}} \text{ Fixed shower variables}$$

- We modify the CMW scheme

$$K_1 \rightarrow K_1 + \Delta K_1(\Phi_{\text{PS}}^{(1)})$$

$$\frac{\alpha_s}{2\pi} \Delta K_1(\Phi_{\text{PS}}^{(1)}) = \int \underbrace{\text{Diagram}}_{\mathbf{R}} \text{ Fixed shower variables} - \int \underbrace{\text{Diagram}}_{\mathbf{R}} \text{ } y, p_{\perp} \text{ fixed}$$

- ...so to have

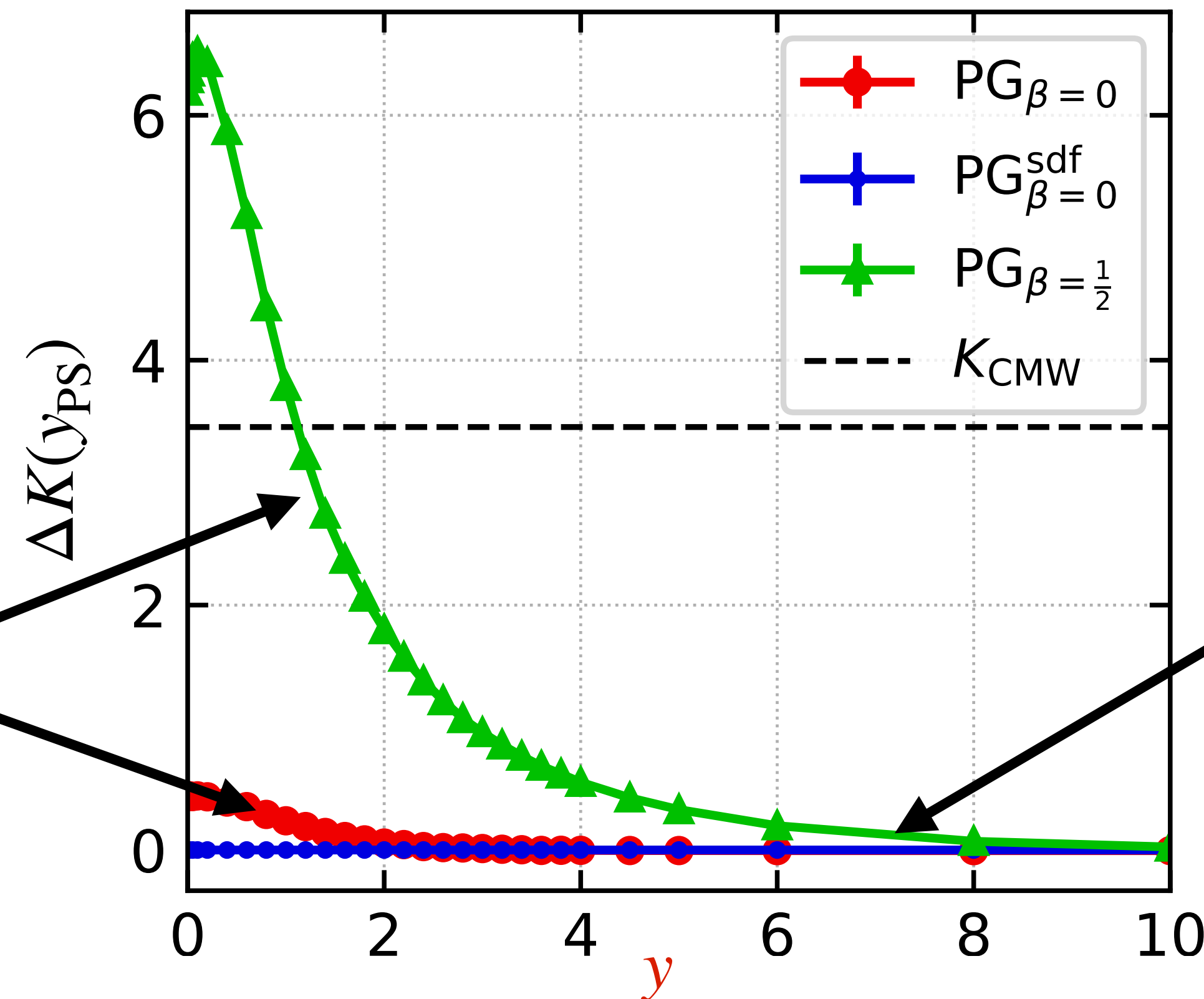
$$\underbrace{\text{Diagram}}_{V_{\text{PS}}} = \frac{\alpha_s}{2\pi} K_1 - \int \underbrace{\text{Diagram}}_{\mathbf{R}} \text{ } y, p_{\perp} \text{ fixed}$$

Virtual corrections to a single soft emission



$$= \frac{\alpha_s}{2\pi} \left(K_1 + \Delta K_1(\Phi_{PS}^{(1)}) \right) - \int \text{Fixed shower variables}$$


example ΔK_1 correction



Soft large-angle emissions can require a “large” ΔK_1

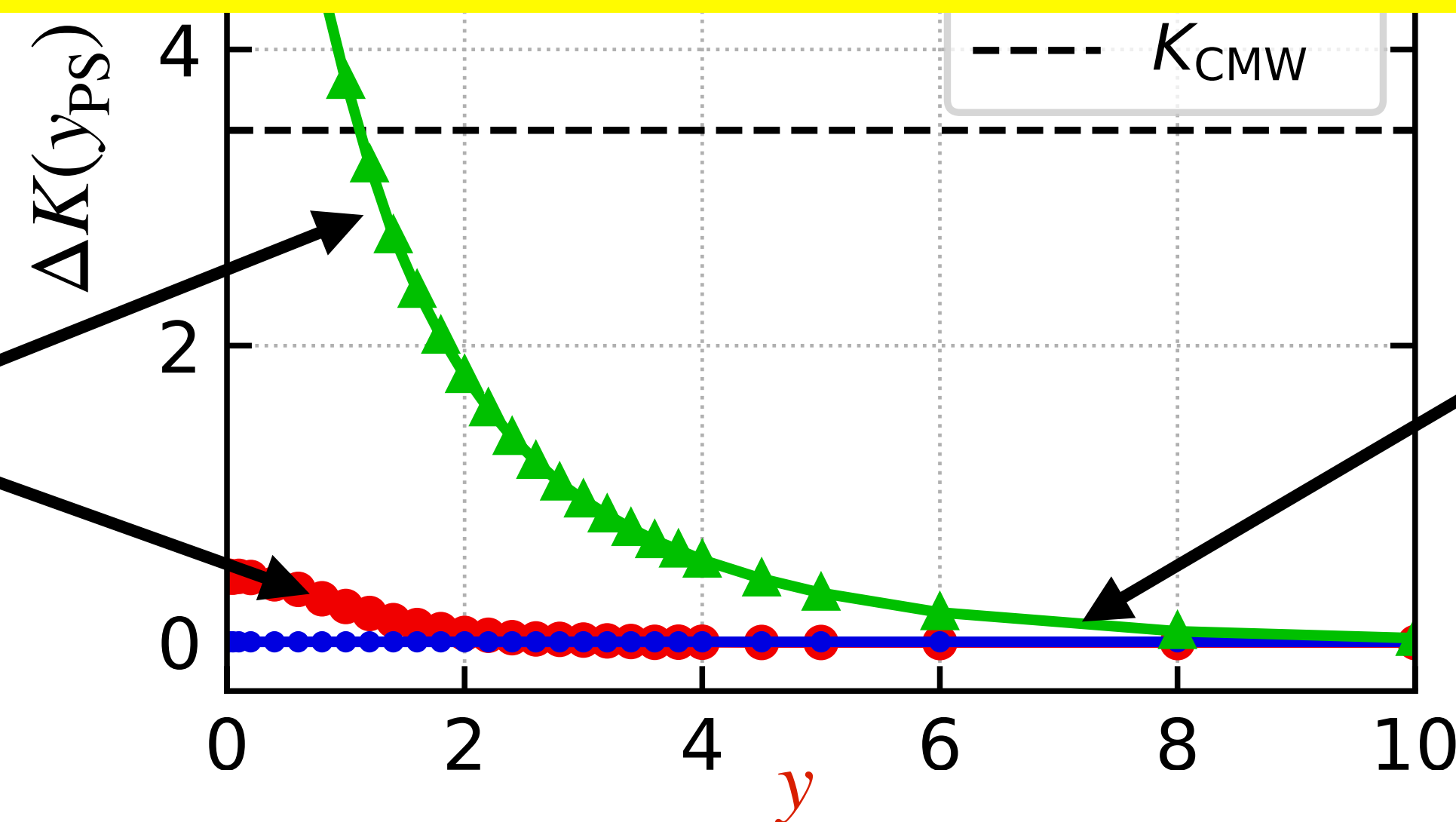
Soft-collinear emissions are already OK (because the shower is NLL)

Virtual corrections to a single soft emission

$$V_{PS} = \frac{\alpha_s}{2\pi} \left(K_1 + \Delta K_1(\Phi_{PS}^{(1)}) \right) - \int_{\mathcal{R}} \text{Fixed shower variables}$$

Augmenting the order of the splitting function used is not sufficient to achieve superior logarithmic accuracy!

Soft large-angle emissions can require a “large” ΔK_1



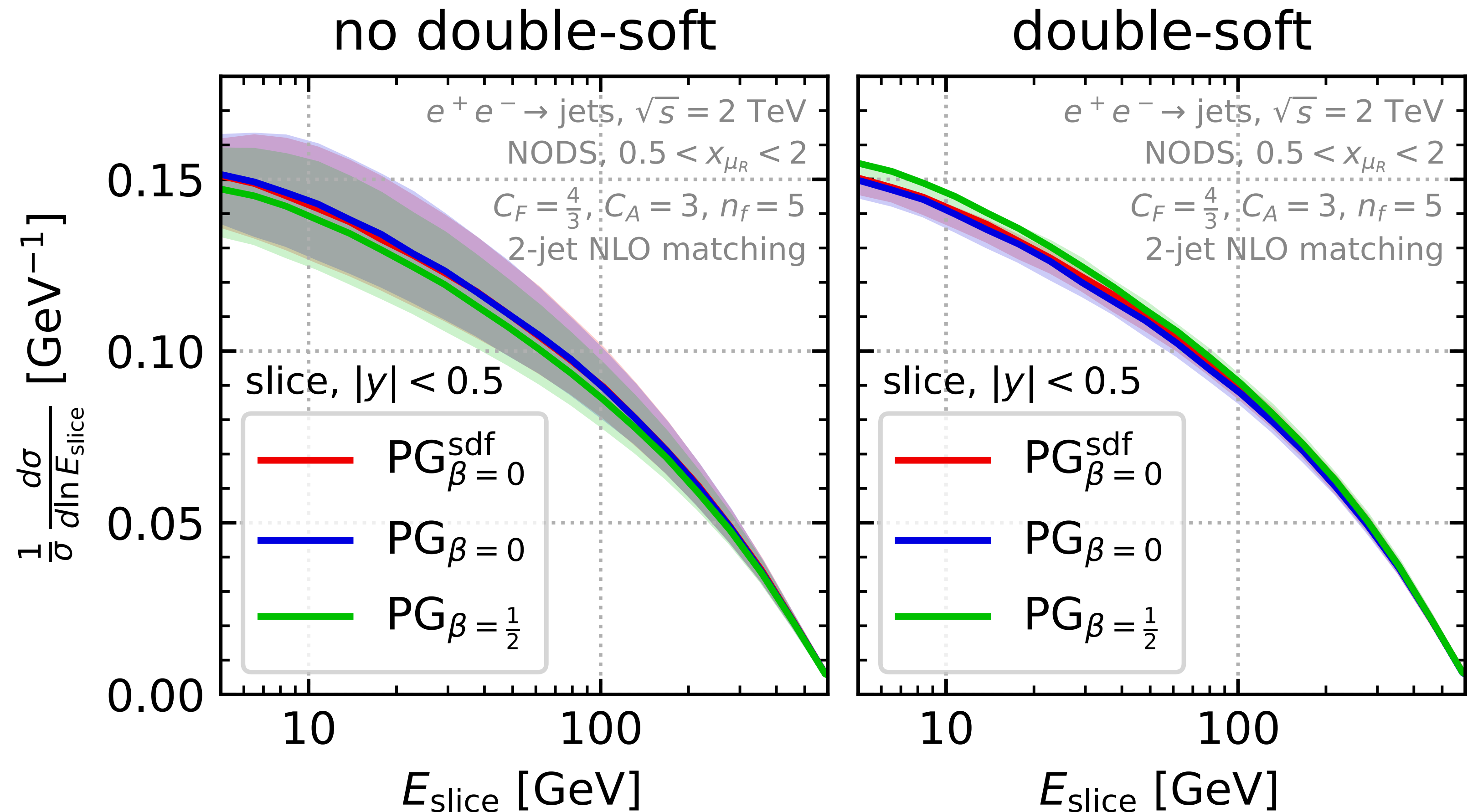
Soft-collinear emissions are already OK (because the shower is NLL)

NSL Pheno outlook

S.F.R., Hamilton,
Karlberg, Salam,
Scyboz, Soyez
[2307.11142](https://arxiv.org/abs/2307.11142)

- Energy flow in slice between two 1 TeV jets
- **Double-soft reduces uncertainty band**

Uncertainty here is estimated varying the renormalisation scale



$$\alpha_s^{\text{CMW}}(k_t; x_R) = \alpha_s(x_R k_t) \left(1 + \frac{\alpha_s(x_R k_t)}{2\pi} (K_1 + \Delta K_1(\Phi)) + 2\alpha_s(x_R k_t) b_0 (1-z) \ln x_R \right)$$

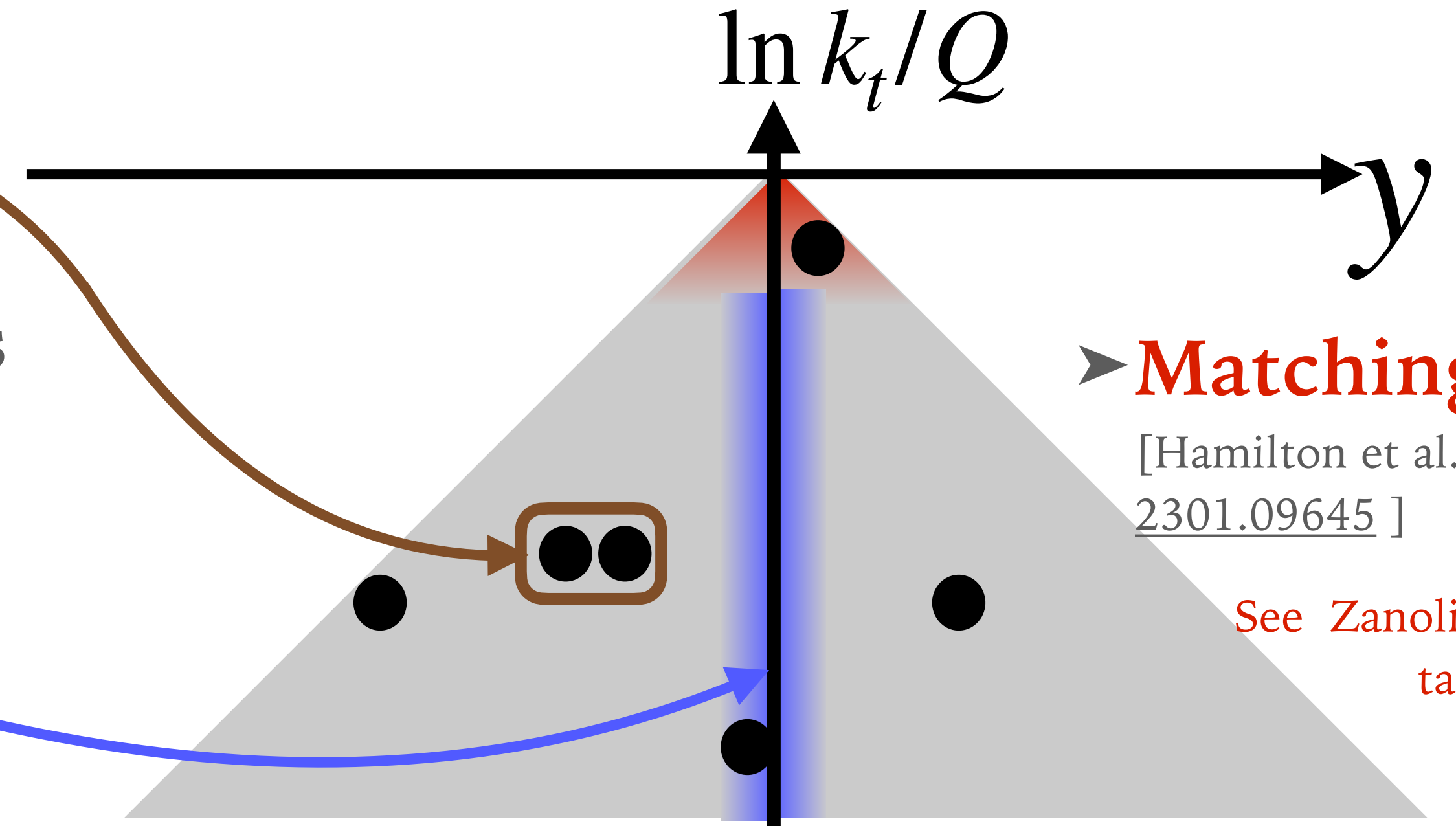
Building a NNLL shower

SFR et al, 2307.11142

- ▶ **Double-soft “reweighting”** for neighbouring soft-collinear emsns
- ▶ NLO corrections for soft, large-angle emissions

$$\alpha_s^{\text{eff}}(k_t) = \alpha_s(k_t) \left(1 + \frac{\alpha_s(k_t)}{2\pi} (K_1 + \Delta K_1) \right)$$

Catani, Marchesini,
Webber, '91



▶ **Matching**

[Hamilton et al.
2301.09645]

See Zanolli's
talk

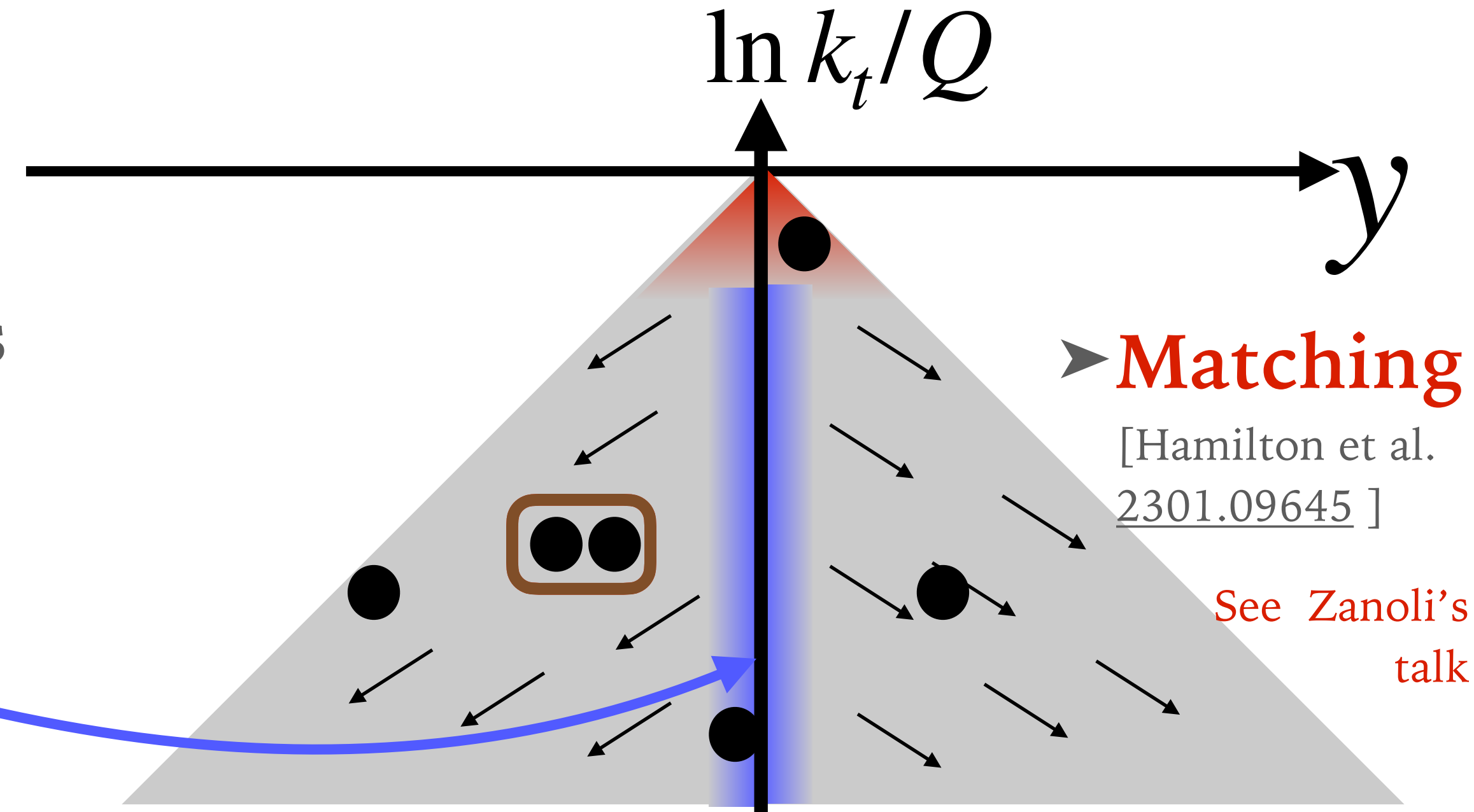
Building a NNLL shower

SFR et al, 2307.11142

- **Double-soft “reweighting”** for neighbouring soft-collinear emissions
- NLO corrections for soft, large-angle emissions

$$\alpha_s^{\text{eff}}(k_t) = \alpha_s(k_t) \left(1 + \frac{\alpha_s(k_t)}{2\pi} (K_1 + \Delta K_1) \right)$$

Catani, Marchesini, Webber, '91



Drift in rapidity of an emission when it further branches

$$\int 2C_F d\eta \Delta K_1(\eta) \propto \langle \Delta y \rangle$$

Building a NNLL shower

SFR et al, 2307.11142

- ▶ **Double-soft “reweighting”** for neighbouring soft-collinear emsns

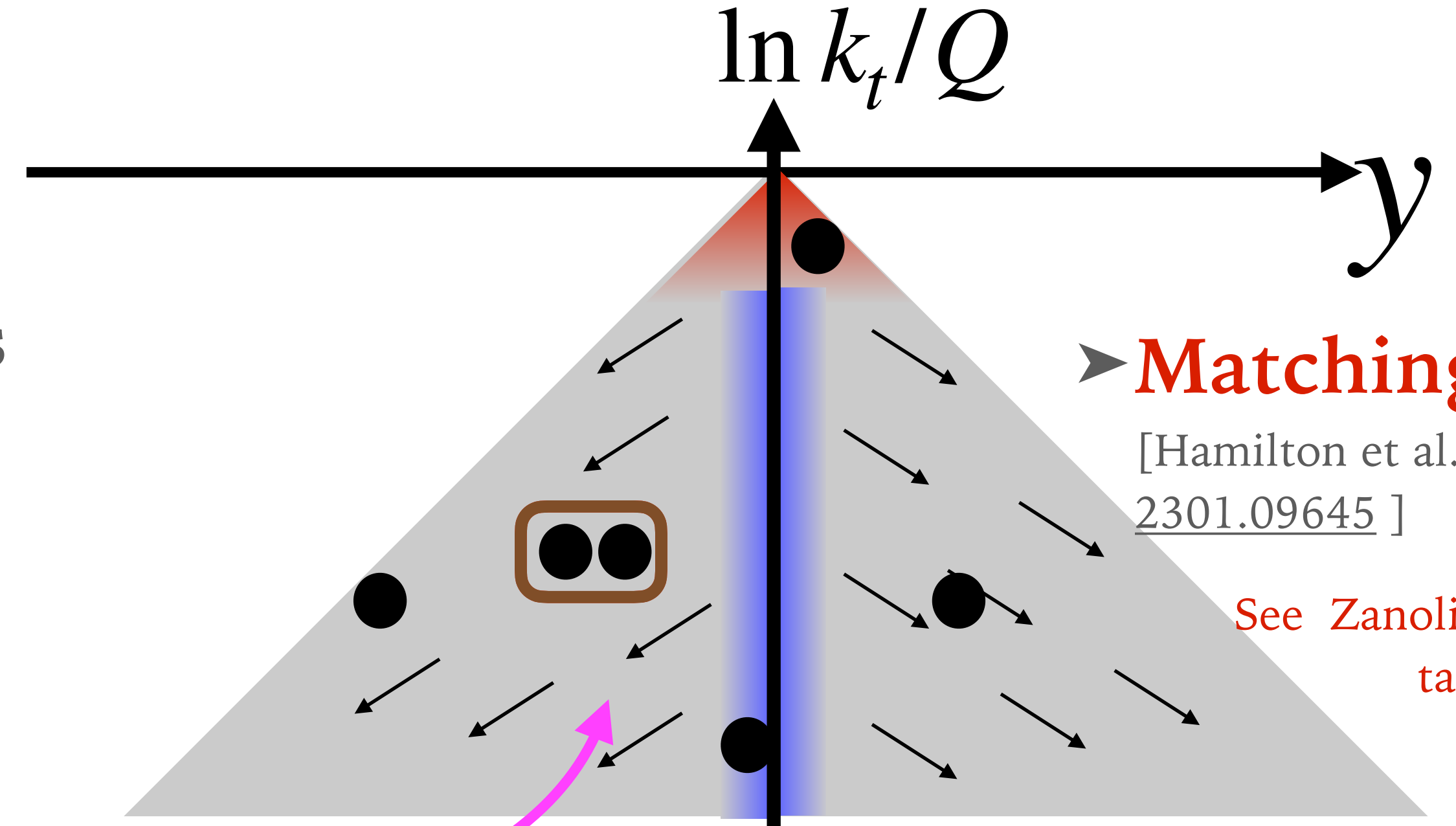
- ▶ NLO corrections for soft, large-angle emissions

$$\alpha_s^{\text{eff}}(k_t) = \alpha_s(k_t) \left(1 + \frac{\alpha_s(k_t)}{2\pi} (K_1 + \Delta K_1) \right)$$

- ▶ **NNLO corrections** for soft-collinear emsns

$$\alpha_s^{\text{eff}}(k_t) = \alpha_s(k_t) \left(\dots + \frac{\alpha_s^2(k_t)}{4\pi^2} (K_2 + \Delta K_2) \right)$$

Banfi, El-Menoufi,
Monni, 1807.11487



▶ **Matching**

[Hamilton et al.
2301.09645]

See Zanolì's
talk

Drift in $\ln k_t$ of an emission when it further branches

$$\Delta K_2 \propto \beta_0 \langle \Delta \ln k_t \rangle$$

Building a NNLL shower

SFR et al, 2307.11142

➤ **Double-soft “reweighting”** for neighbouring soft-collinear emsns

➤ NLO corrections for soft, large-angle emissions

$$\alpha_s^{\text{eff}}(k_t) = \alpha_s(k_t) \left(1 + \frac{\alpha_s(k_t)}{2\pi} (K_1 + \Delta K_1) \right)$$

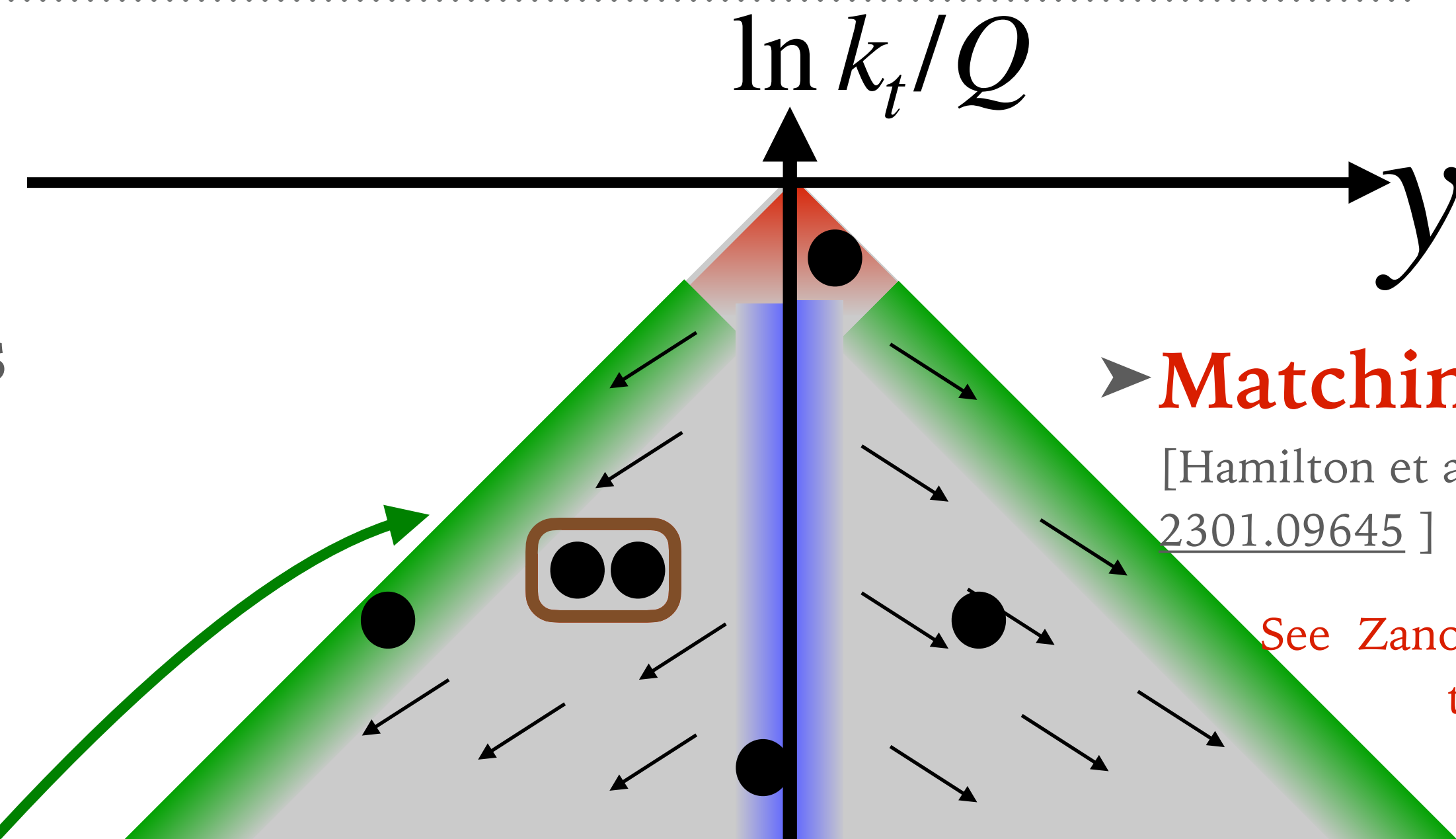
➤ **NNLO corrections** for soft-collinear emsns

$$\alpha_s^{\text{eff}}(k_t) = \alpha_s(k_t) \left(\dots + \frac{\alpha_s^2(k_t)}{4\pi^2} (K_2 + \Delta K_2) \right)$$

➤ **NLO corrections** for collinear emsns

$$d\mathcal{P}_{\text{coll}} \propto P(z) \left(1 + \frac{\alpha_s}{2\pi} (B_2(z) + \Delta B_2(z)) \right)$$

Dasgupta, El-Menoufi 2109.07496,
+ van Beekveld, Helliwell, Monni 2307.15734,
++ Karlberg 2402.05170



➤ **Matching**

[Hamilton et al. 2301.09645]

See Zanolì's talk

Drift in $\ln z = \ln k_t + y$ of an emission when it further branches

$$\int P(z) dz \Delta B_2(z) \propto - \langle \Delta \ln z \rangle$$

At this accuracy, it is sufficient to get the integral right, not the functional form of $\Delta B_2(z)$

A new standard for the logarithmic accuracy of parton showers

Melissa van Beekveld,¹ Mrinal Dasgupta,² Basem Kamal El-Menoufi,³ Silvia Ferrario Ravasio,⁴ Keith Hamilton,⁵ Jack Helliwell,⁶ Alexander Karlberg,⁴ Pier Francesco Monni,⁴ Gavin P. Salam,^{6,7} Ludovic Scyboz,³ Alba Soto-Ontoso,⁴ and Gregory Soyez⁸

We report on a major milestone in the construction of logarithmically accurate final-state parton showers, achieving next-to-next-to-leading-logarithmic (NNLL) accuracy for the wide class of observables known as event shapes. The key to this advance lies in the identification of the relation between critical NNLL analytic resummation ingredients and their parton-shower counterparts. Our analytic discussion is supplemented with numerical tests of the logarithmic accuracy of three shower variants for more than a dozen distinct event-shape observables in two final states. The NNLL terms are phenomenologically sizeable, as illustrated in comparisons to data.

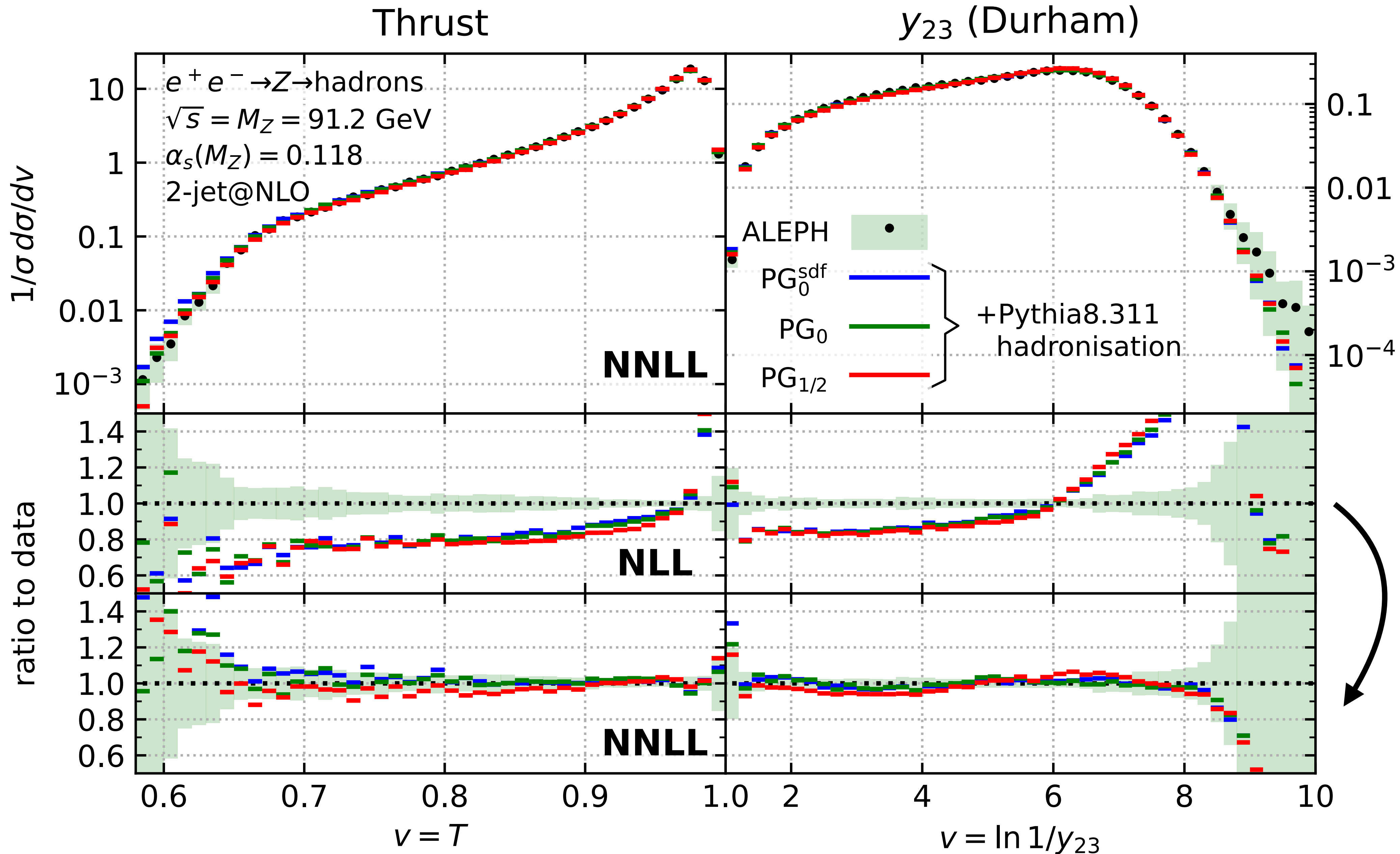
Dasgupta, El-Menoufi 2109.07496,
+ van Beekveld, Helliwell, Monni 2307.15734,
++ Karlberg 2402.05170

2406.02661

to get the integral
the junctional form of $\Delta B_2(z)$

NNLL showers vs NLL showers: pheno outlook

*The PanScales
collaboration,
2406.02661*



Agreement to
data
substantially
better when
 using **NNLL**
 showers

Conclusions

- **PanScales is first validated NLL shower**
 - All processes with **two colour legs** have been rigorously tested to be NLL for both global and non-global event shapes
 - benefits of **LL** → **NLL** include **reduced uncertainties** (reliable estimate)
 - NLO matching in place for some simple processes
- **Higher log accuracy is one of the next frontiers**
 - Double-soft (+ virtual) corrections: **NSL** accuracy for **non-global** event shapes, **NNDL** accuracy for subjet multiplicities.
 - **NNLL** accuracy for **global event shapes** in $e^+e^- \rightarrow j_1j_2$
- **Public code**
 - <https://gitlab.com/panscales/panscales-0.X>

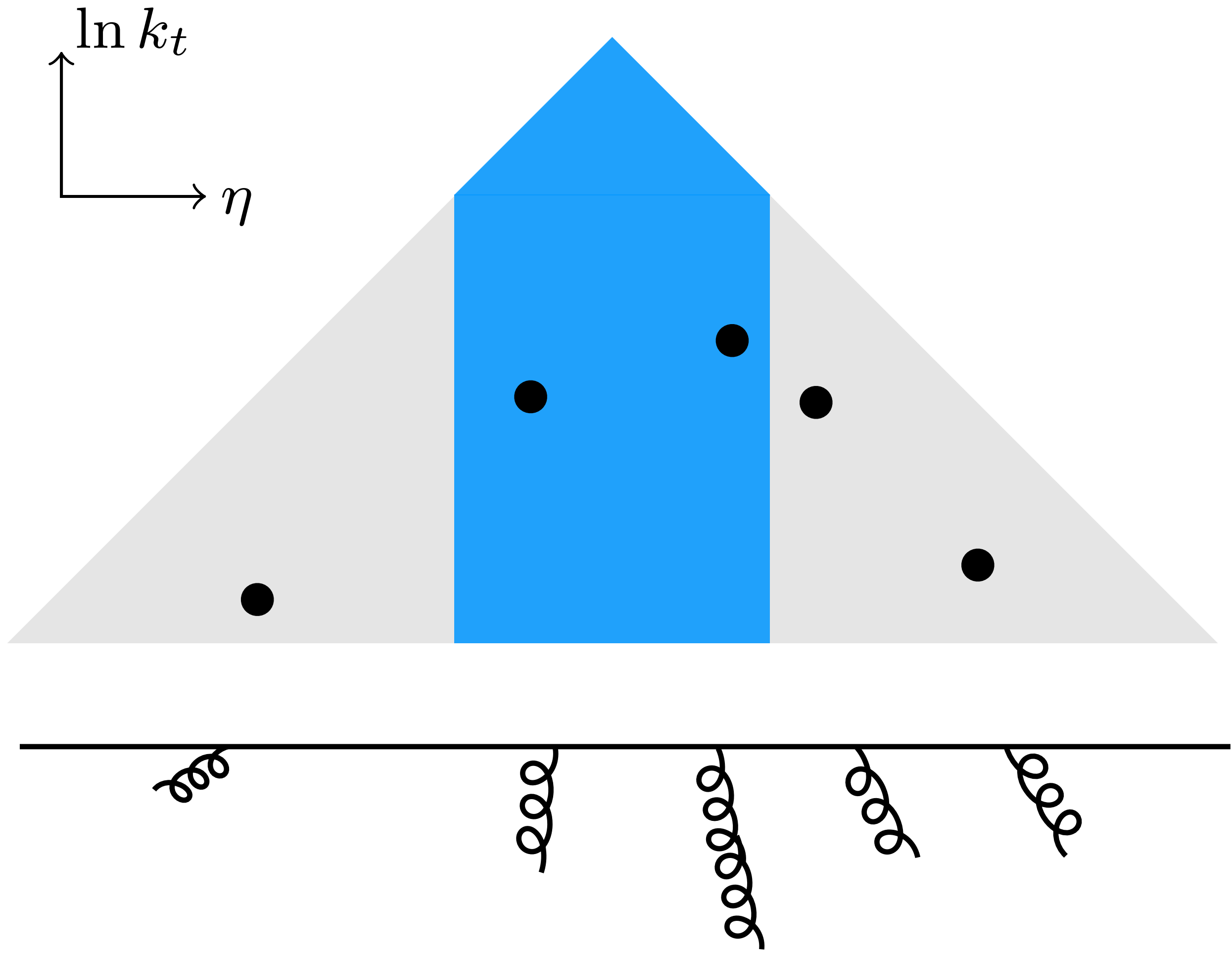
*The PanScales collaboration,
2312.13275*

Conclusions

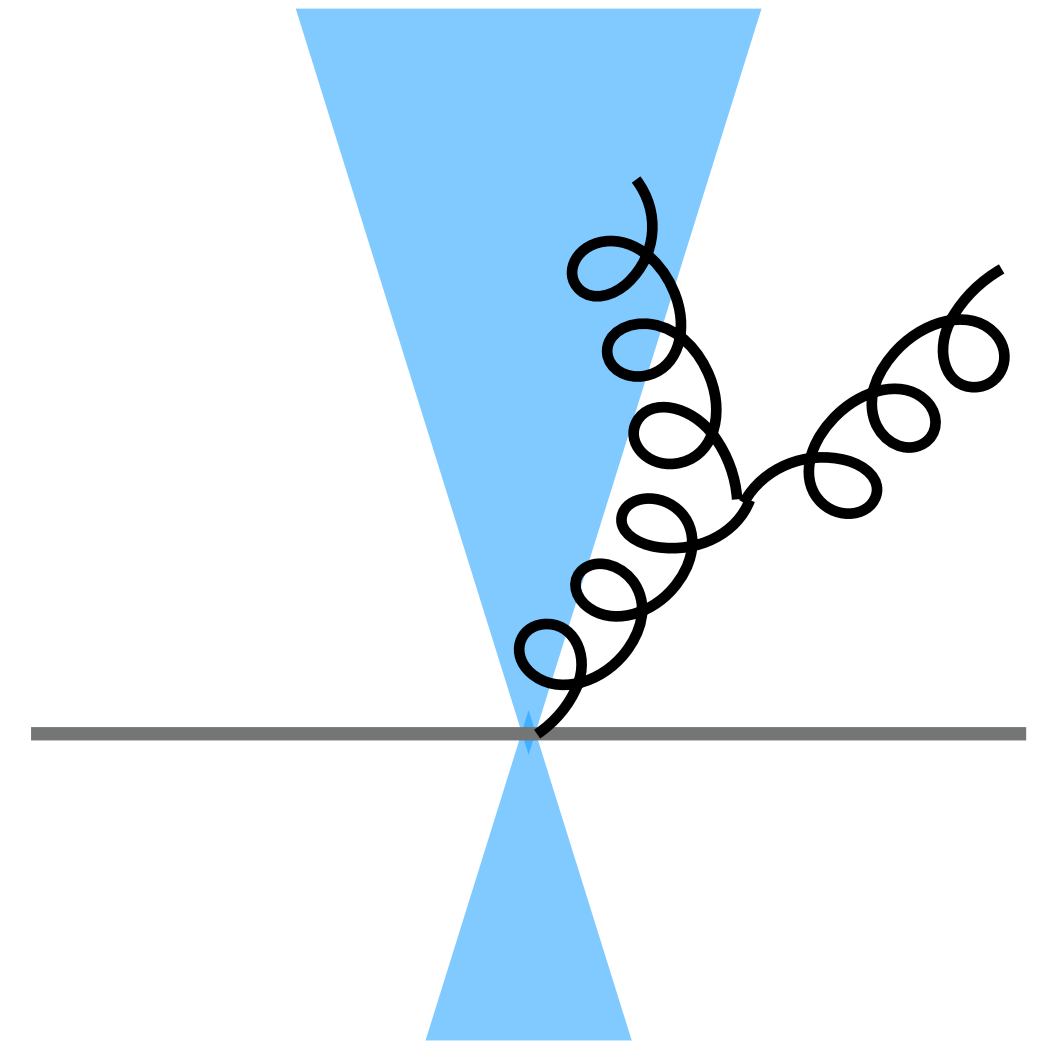
- **PanScales is first validated NLL shower**
 - All processes with **two colour legs** have been rigorously tested to be NLL for both global and non-global event shapes
 - benefits of **LL** → **NLL** include **reduced uncertainties** (reliable estimate)
 - **NLO matching in place for some simple processes** **Current matching schemes typically preserve at best the LL...**
See more in S. Zanolli's talk!!
- **Higher log accuracy is one of the next frontiers**
 - Double-soft (+ virtual) corrections: **NSL** accuracy for **non-global** event shapes, **NNDL** accuracy for subjet multiplicities.
 - **NNLL** accuracy for **global event shapes** in $e^+e^- \rightarrow j_1j_2$
- **Public code**
 - <https://gitlab.com/panscales/panscales-0.X>

*The PanScales collaboration,
2312.13275*

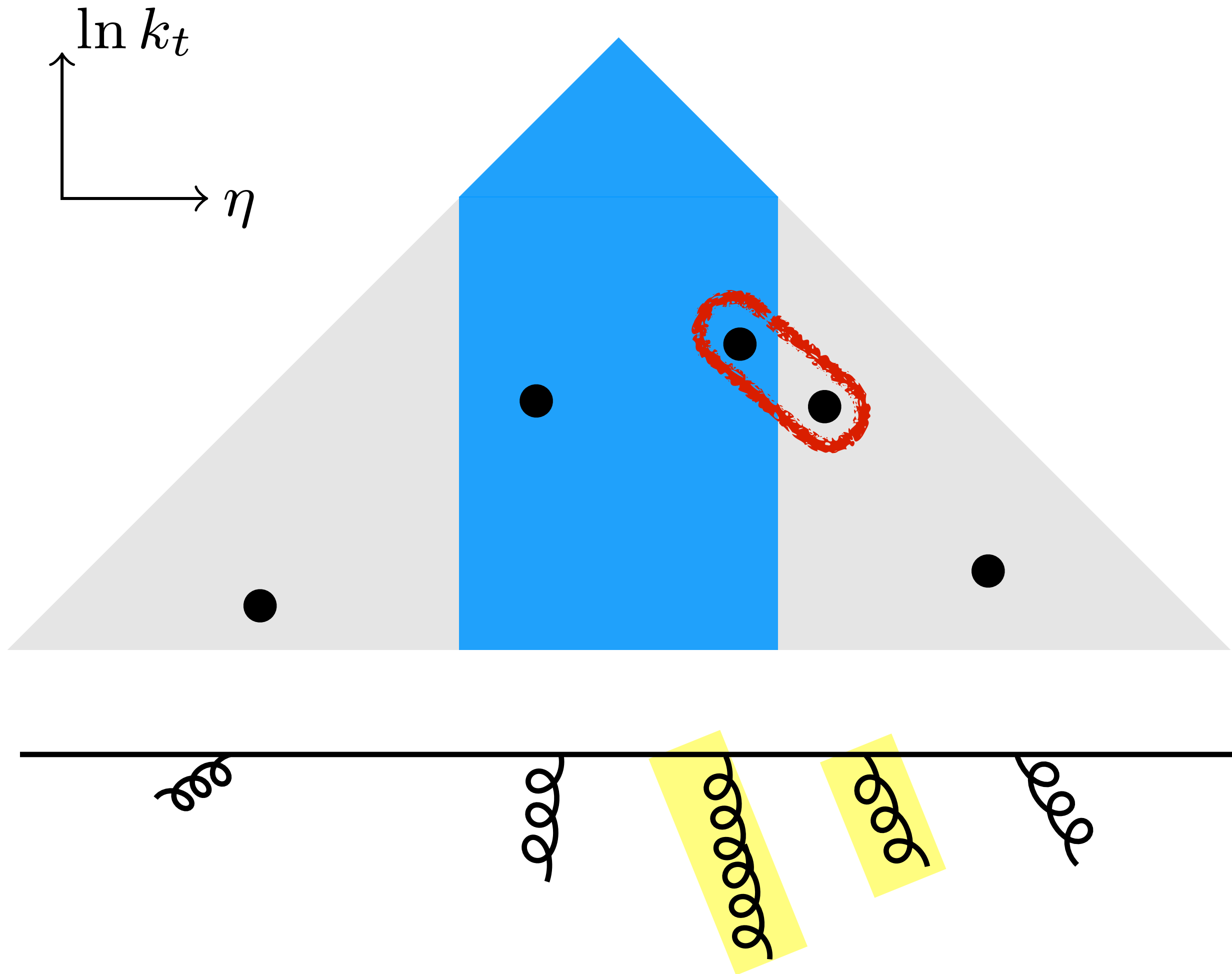
NSL for the energy flow in a rapidity slice



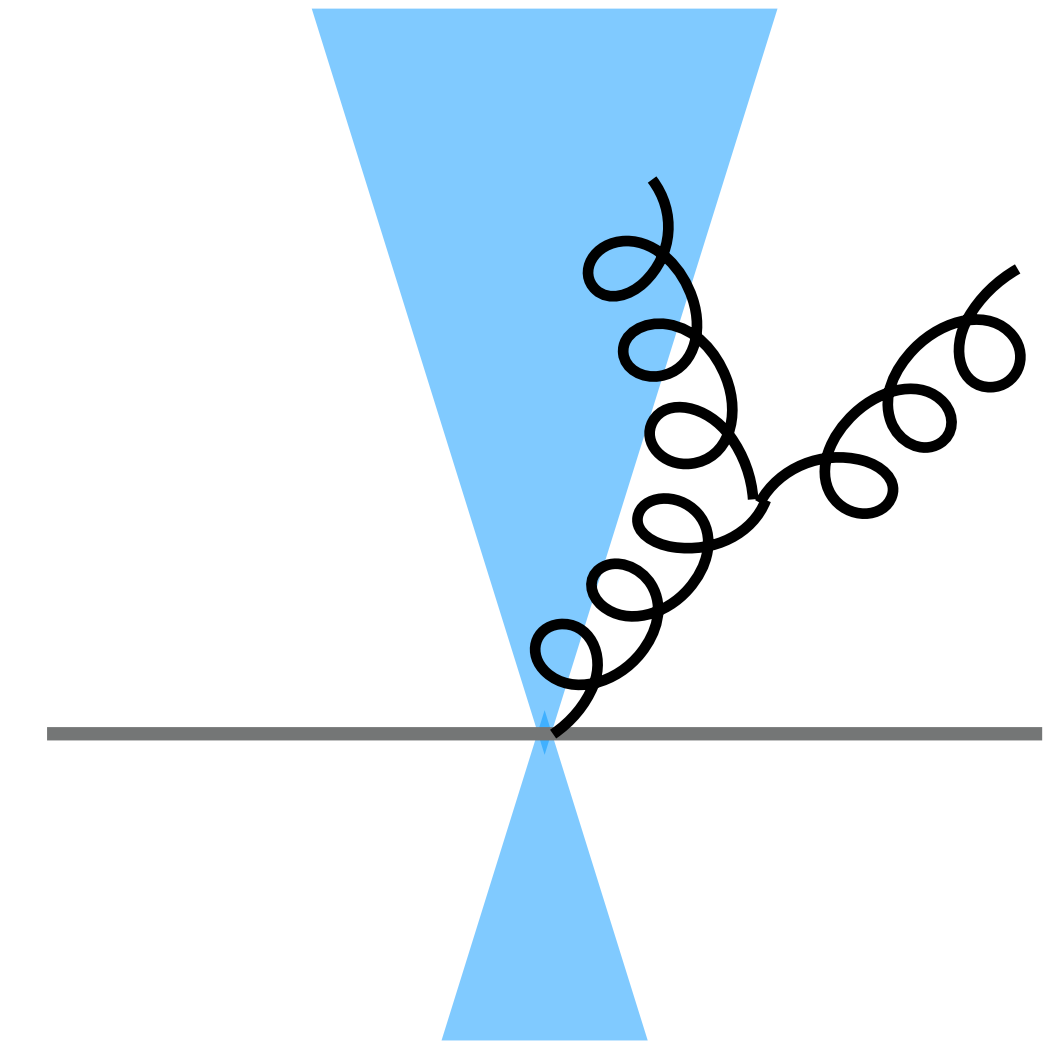
Non-global
observable



NSL for the energy flow in a rapidity slice

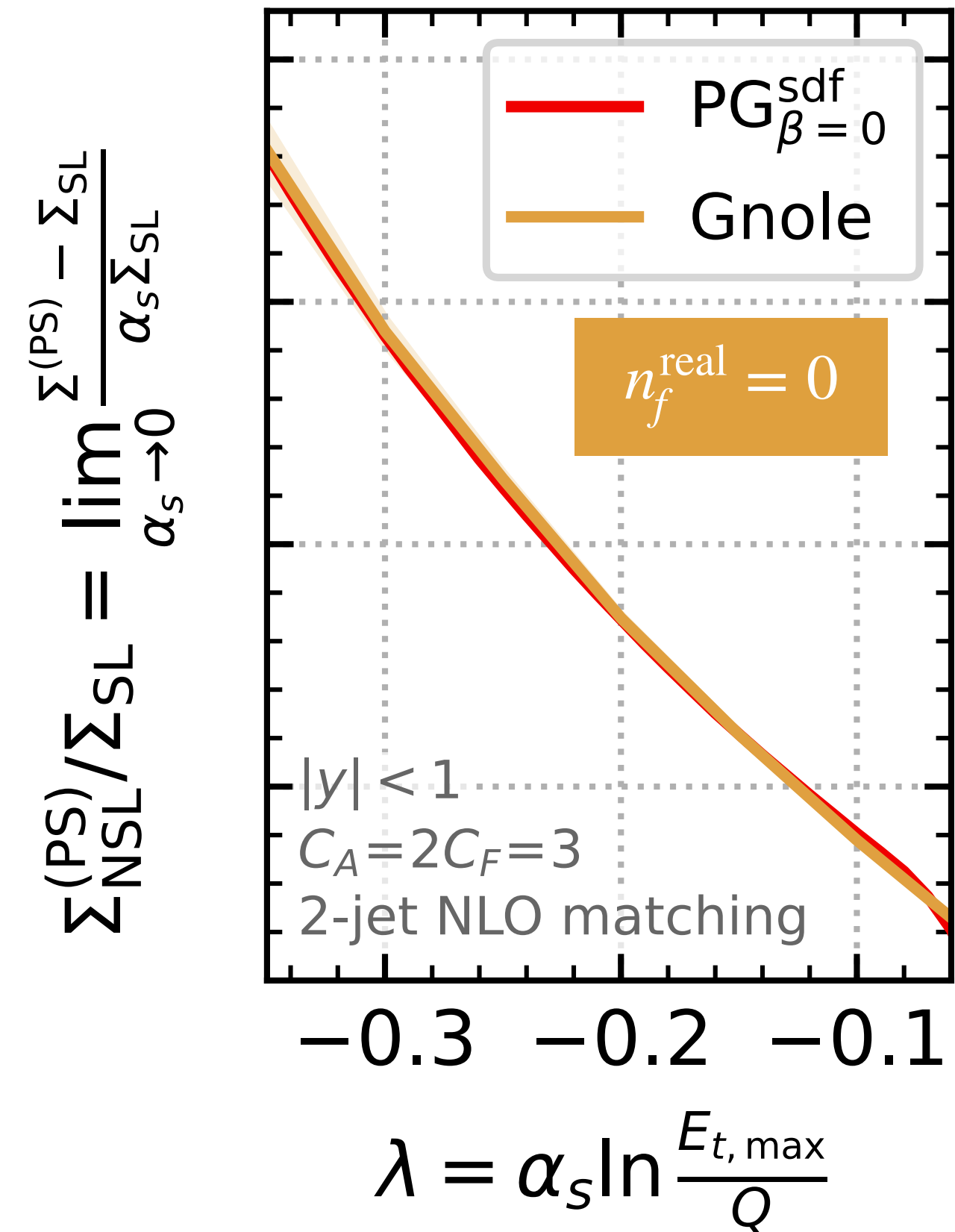


Non-global
observable



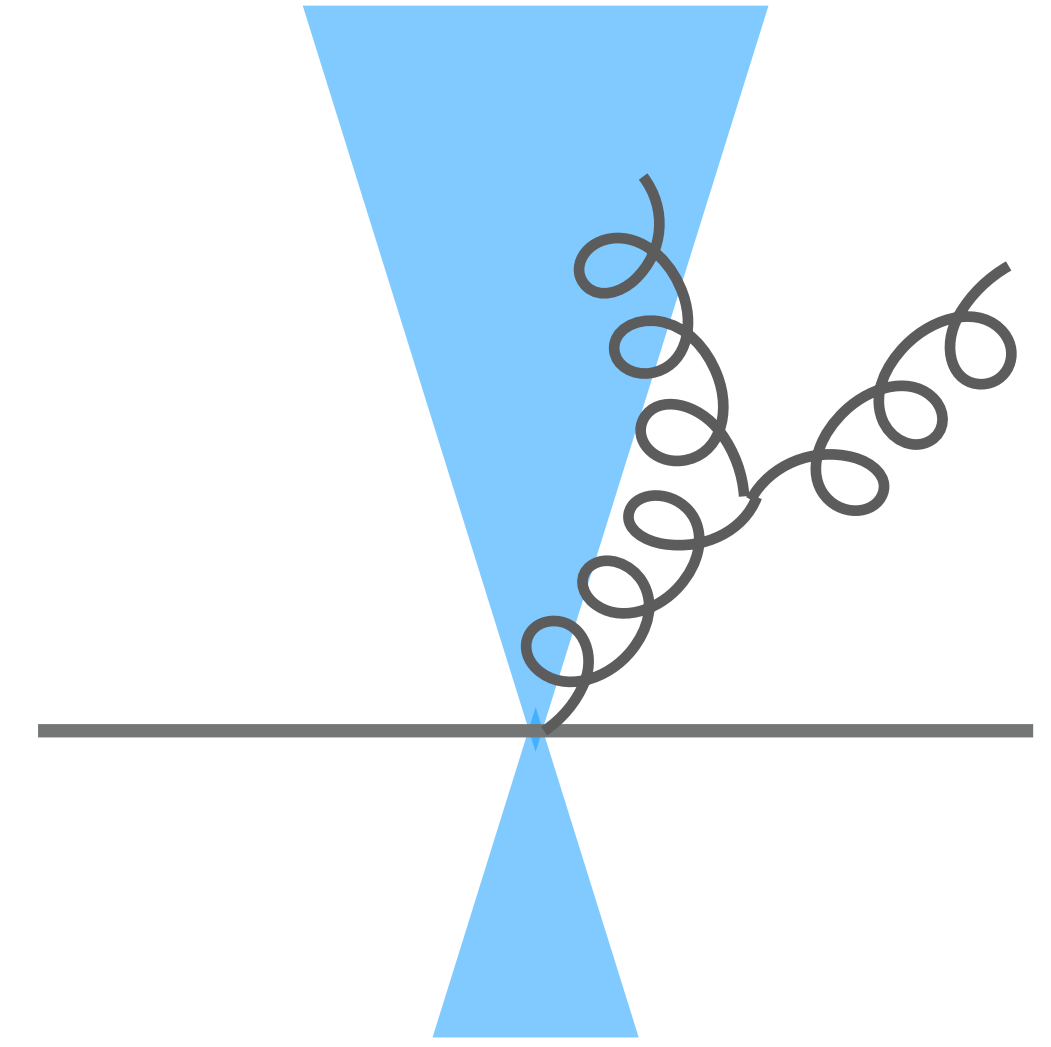
- **NSL** ($\alpha_s^n L^{n-1}$) analytic reference from Banfi, Dreyer, Monni, [2104.06416](#), [2111.02413](#) (“**Gnole**”)
[NB: see also Becher, Schalch, Xu, [2307.02283](#)]

NSL for the energy flow in a rapidity slice



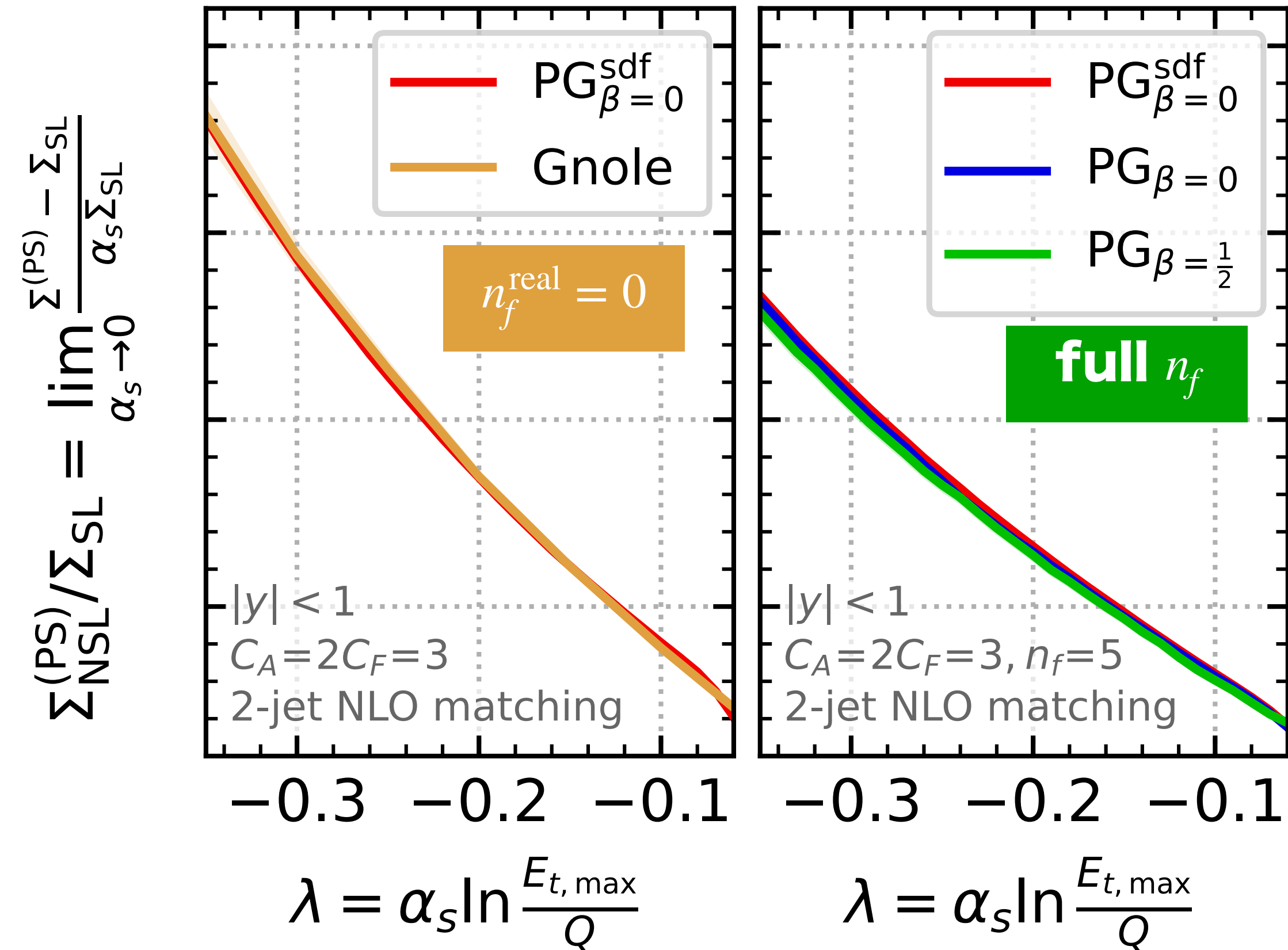
S.F.R., Hamilton, Karlberg, Salam,
Scyboz, Soyez [2307.11142](#)

Non-global
observable



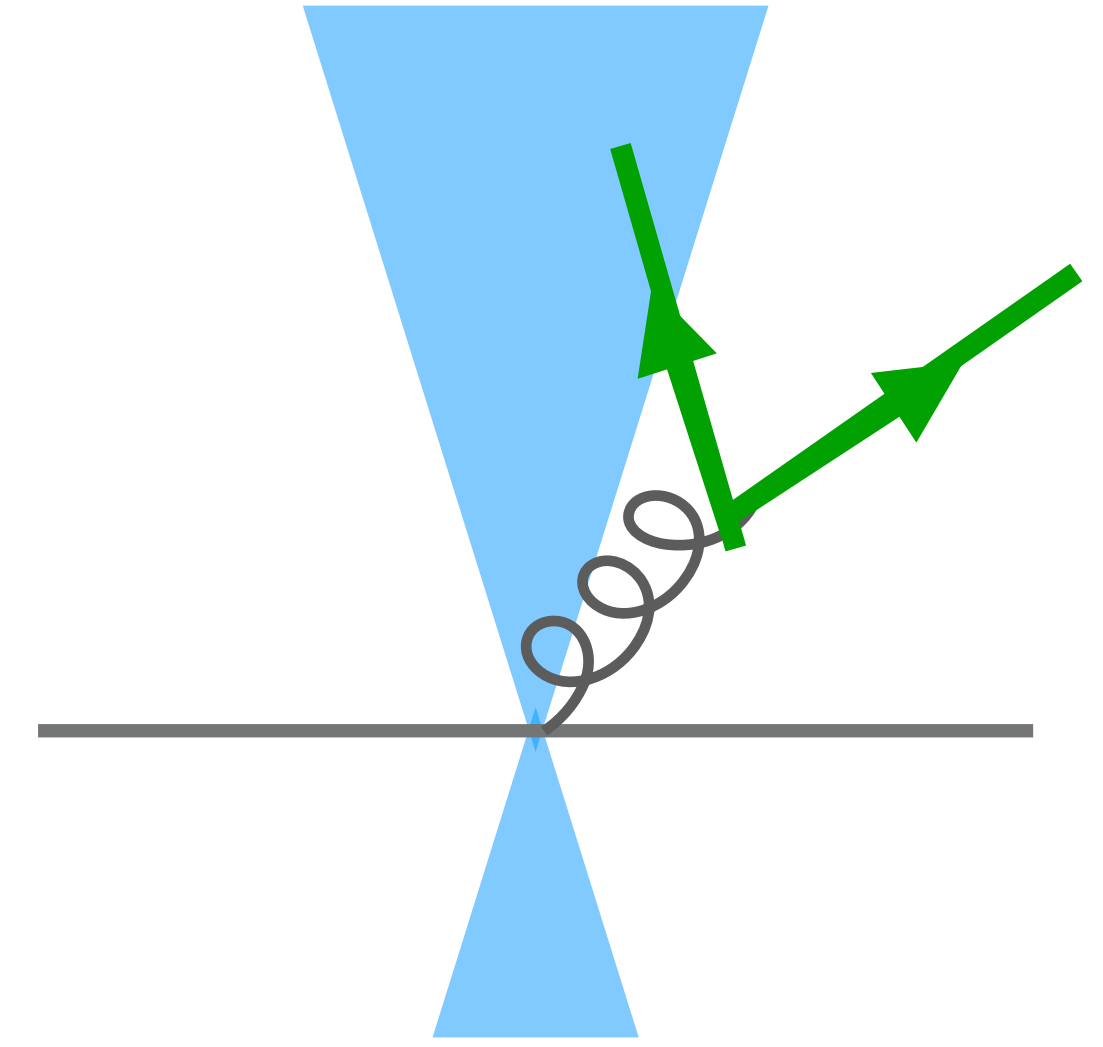
- **NSL** ($\alpha_s^n L^{n-1}$) analytic reference from Banfi, Dreyer, Monni, [2104.06416](#), [2111.02413](#) (“**Gnole**”) [NB: see also Becher, Schalch, Xu, [2307.02283](#)]

NSL for the energy flow in a rapidity slice



S.F.R., Hamilton, Karlberg, Salam,
Scyboz, Soyez [2307.11142](#)

Non-global
observable

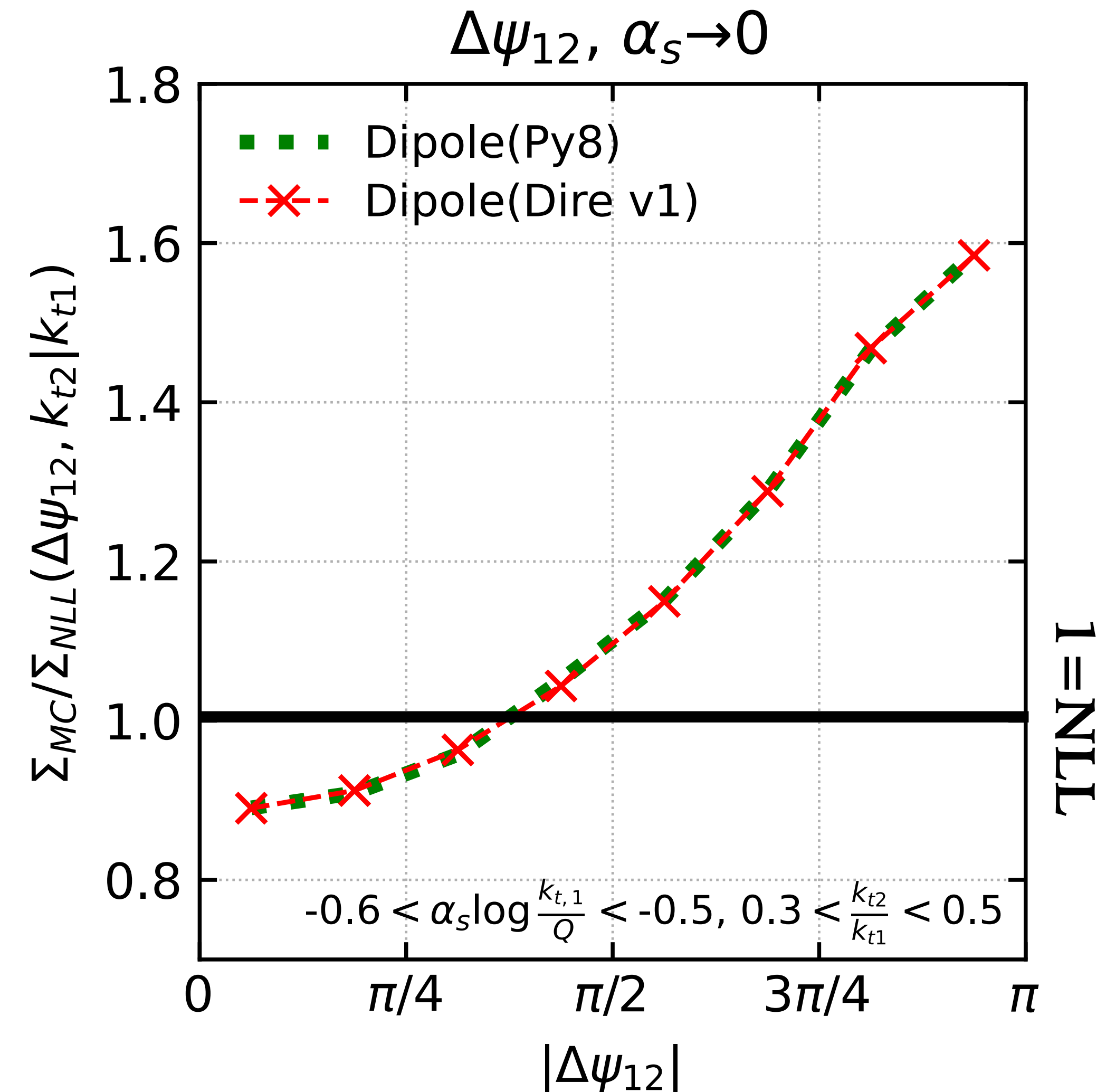


- **NSL** ($\alpha_s^n L^{n-1}$) analytic reference from Banfi, Dreyer, Monni, [2104.06416](#), [2111.02413](#) (“**Gnole**”) [NB: see also Becher, Schalch, Xu, [2307.02283](#)]
- First large- N_c **full- n_f** results for NSL non-global logs

What is available in Shower Monte Carlo generators?

- Showers routinely used to interpret LHC (and LEP) data are **not NLL!**

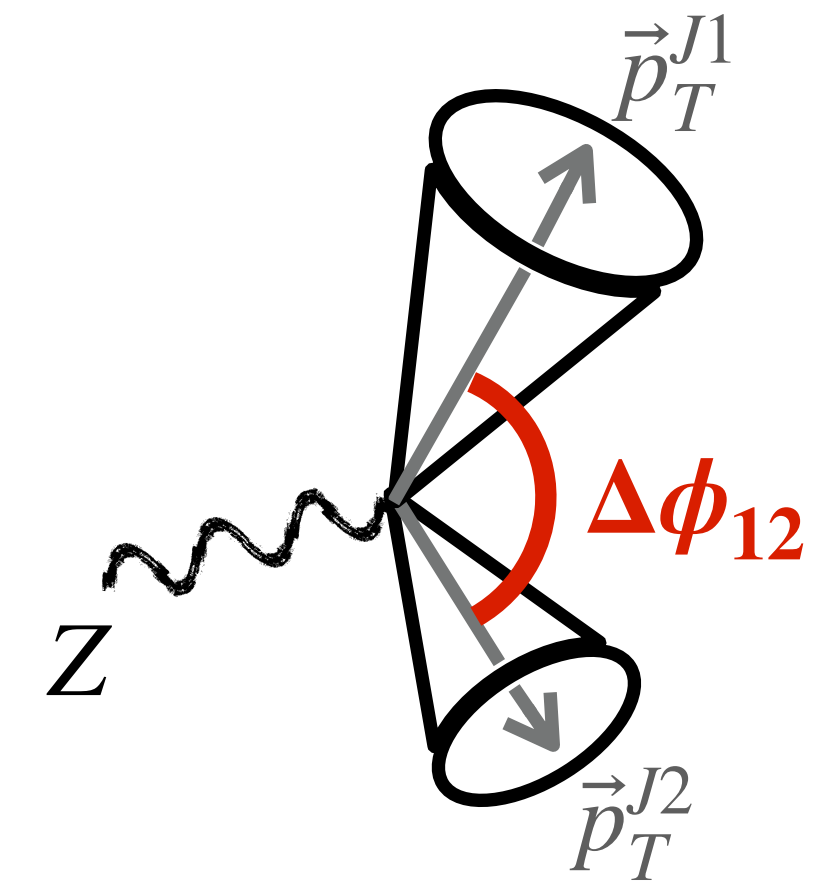
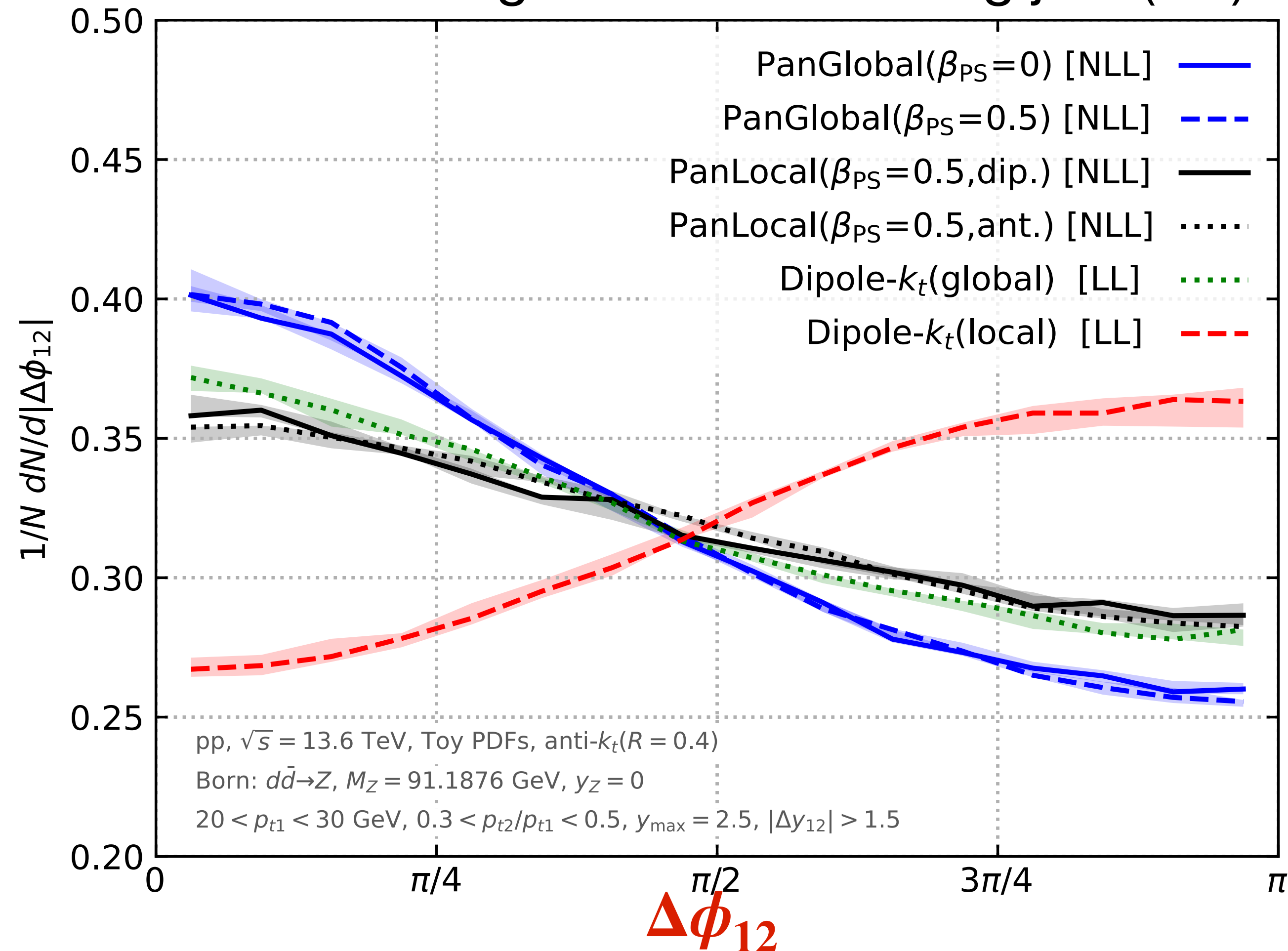
Dasgupta et al. [2002.11114](#)



Exploratory phenomenology for Drell-Yan at the LHC

$$m_{\ell\ell} = 91.2 \text{ GeV}$$

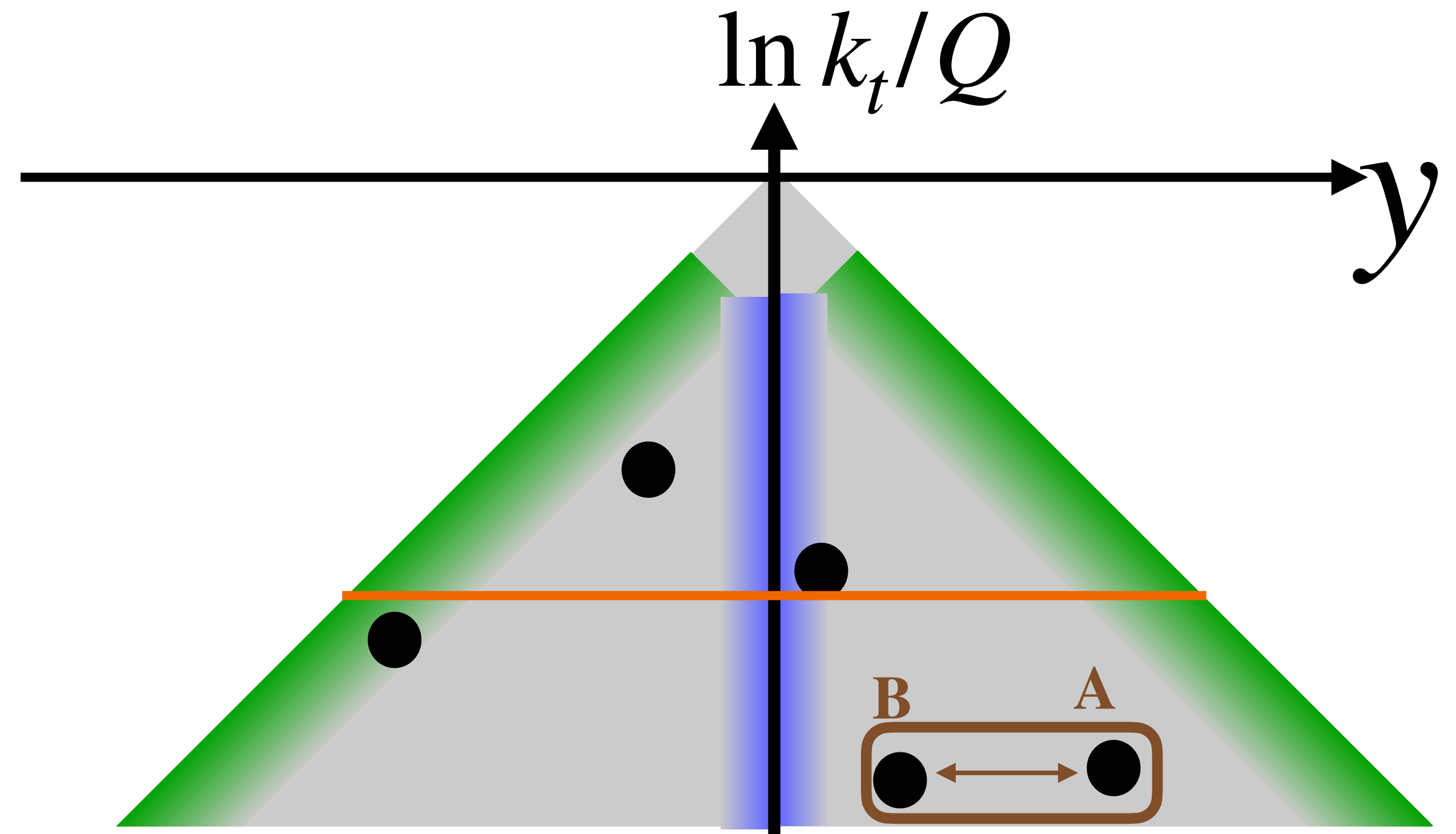
Azimuthal angle between leading jets (DY)



PanScales for $pp \rightarrow$
 colour singlet:
[2207.09467](https://arxiv.org/abs/2207.09467), van
 Beekveld, SFR,
 Hamilton, Salam
 Soto Ontoso, Soyez,
 Verheyen:

How to build a NLL parton shower?

- Standard showers implement **local transverse momentum k_t conservation** and **transverse momentum ordering**: emission **A** will change substantially after emission **B**!



Constraints **kinematic mapping** $\Phi_n \rightarrow \Phi_{n+1}$ and **ordering variable**: emissions well separated in **rapidity** are independent from each other, even if they have similar transverse momentum

How to build a NLL parton shower?

- Standard showers implement local transverse momentum k_t conservation and transverse momentum ordering: emission **A** will change substantially after emission **B**!

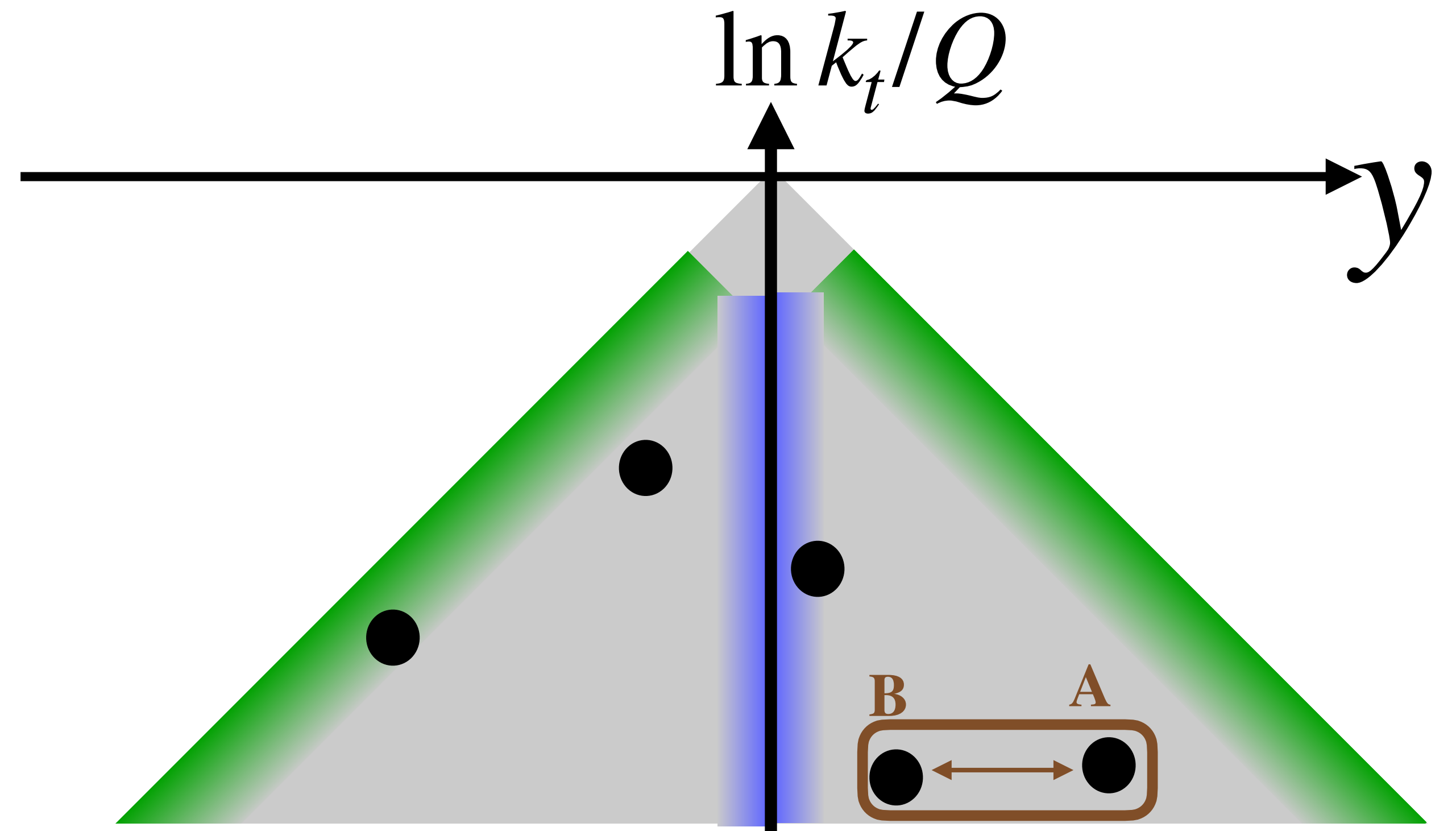
Global k_t
conservation

PanScales

FHP 2003.06400 ,

Alaric 2208.06057,

Apollo 2403.19452



Constraints **kinematic mapping** $\Phi_n \rightarrow \Phi_{n+1}$ and **ordering variable**: emissions well separated in **rapidity** are independent from each other, even if they have similar transverse momentum

How to build a NLL parton shower?

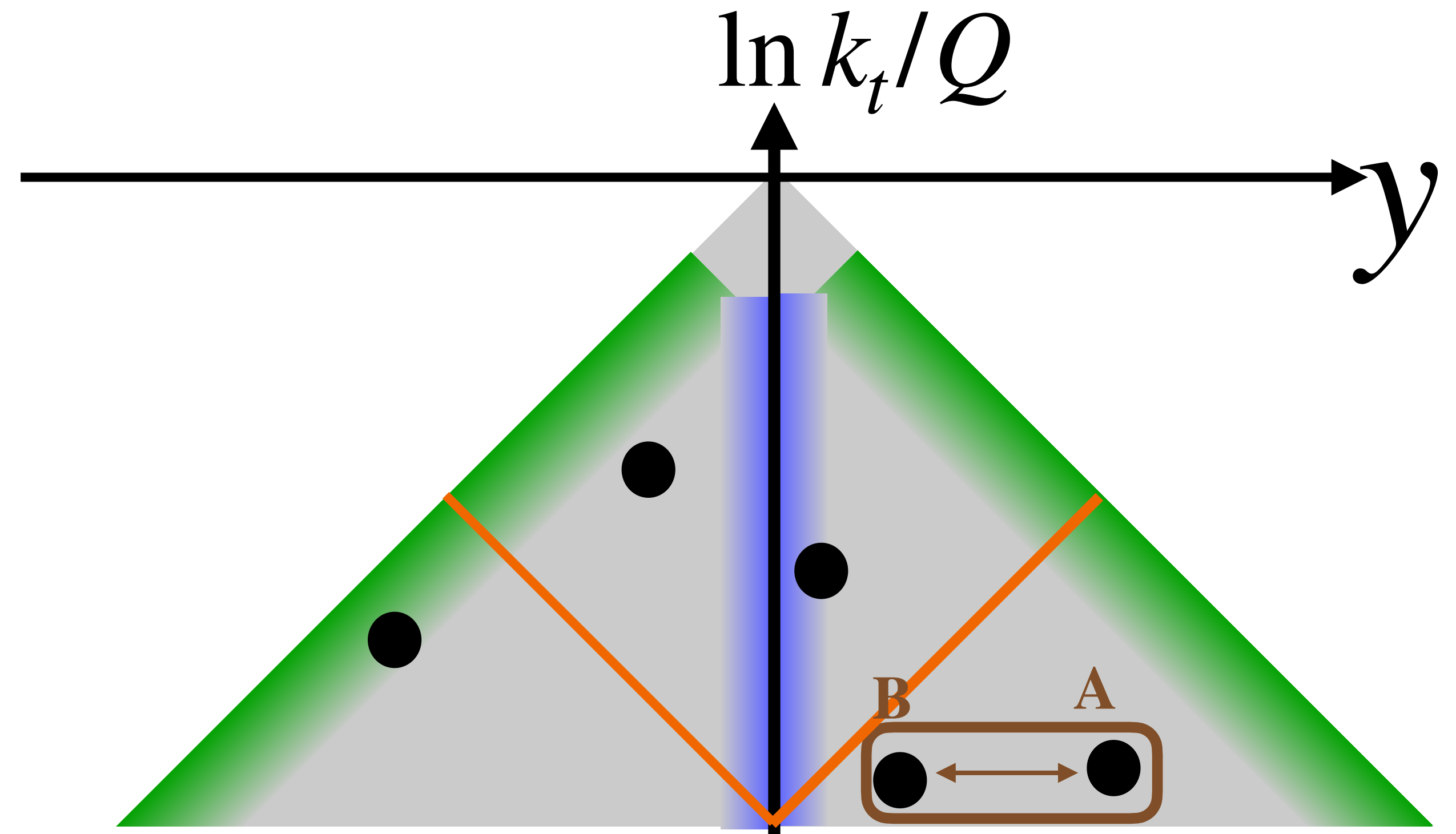
- Standard showers implement local transverse momentum k_t conservation and transverse momentum ordering: emission **A** will change substantially after emission **B**!

Global k_t conservation

PanScales
FHP 2003.06400 ,
Alaric 2208.06057,
Apollo 2403.19452

Ordering variable

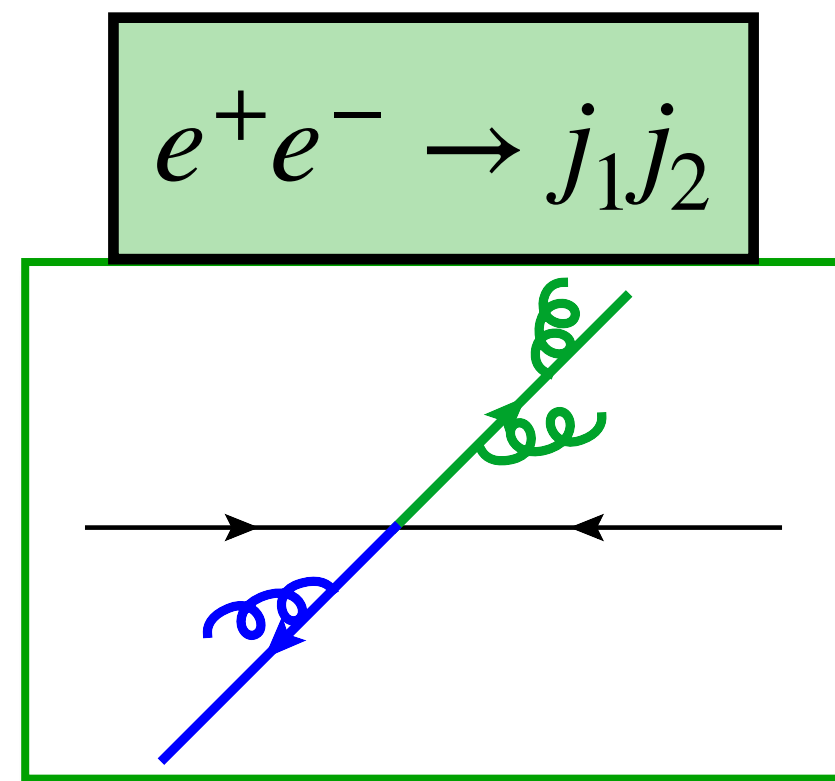
to enforce some angular ordering
Deductor
2011.04777,
PanScales



Constraints **kinematic mapping** $\Phi_n \rightarrow \Phi_{n+1}$ and **ordering variable**: emissions well separated in **rapidity** are independent from each other, even if they have similar transverse momentum

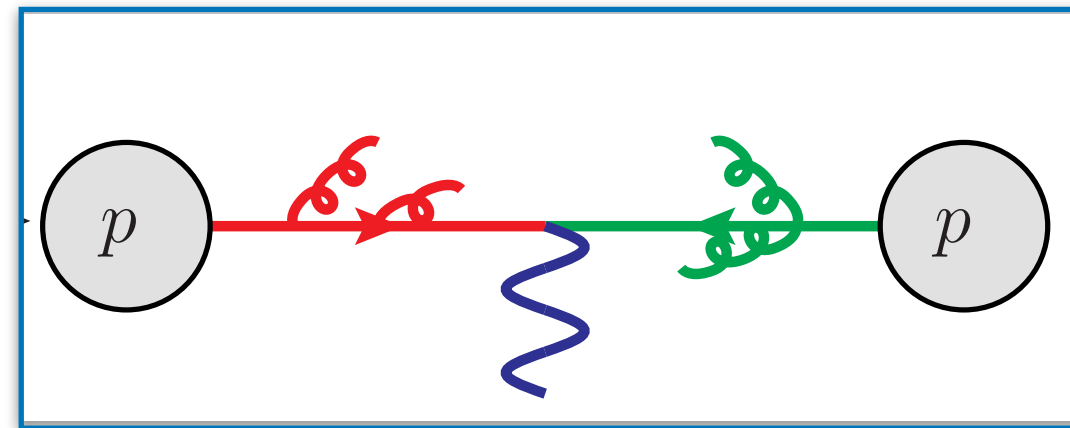
Status of NLL PanScales showers

- This enabled the PanScales to devise the first showers with **general** NLL accuracy for



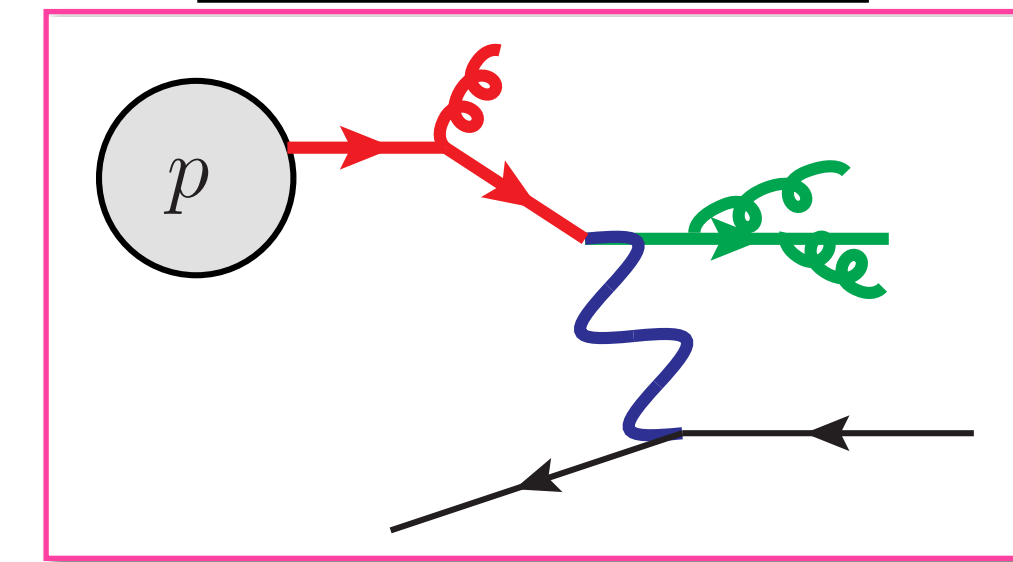
Dasgupta, Dreyer, Hamilton,
Monni, Salam, Soyez,
2002.11114

$pp \rightarrow$ colour singlet



van Beekveld, SFR, Soto-Ontoso,
Salam, Soyez, Verheyen, 2205.02237,
+ Hamilton 2207.09467

DIS & VBF



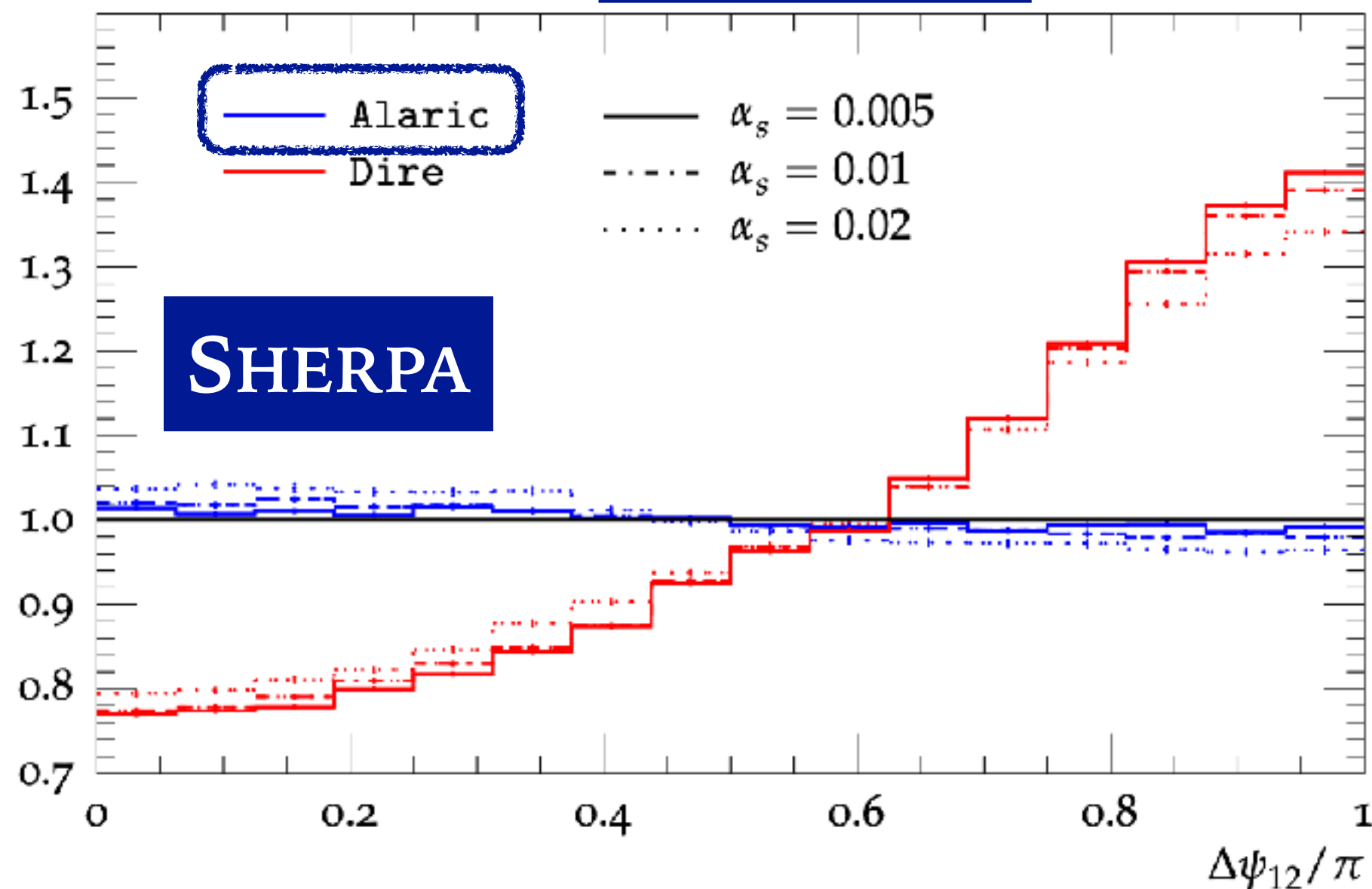
van Beekveld, SFR,
2305.08645

...with **subleading colour** (2011.10054) and
spin correlations (2103.16526, 2111.01161)

What can be available in Shower Monte Carlo generators?

- Showers routinely used to interpret LHC (and LEP) data are **not NLL**!
- **Many groups** are independently formulating new showers with **NLL accuracy for e^+e^-**

Herren et al. [2208.06057](#)



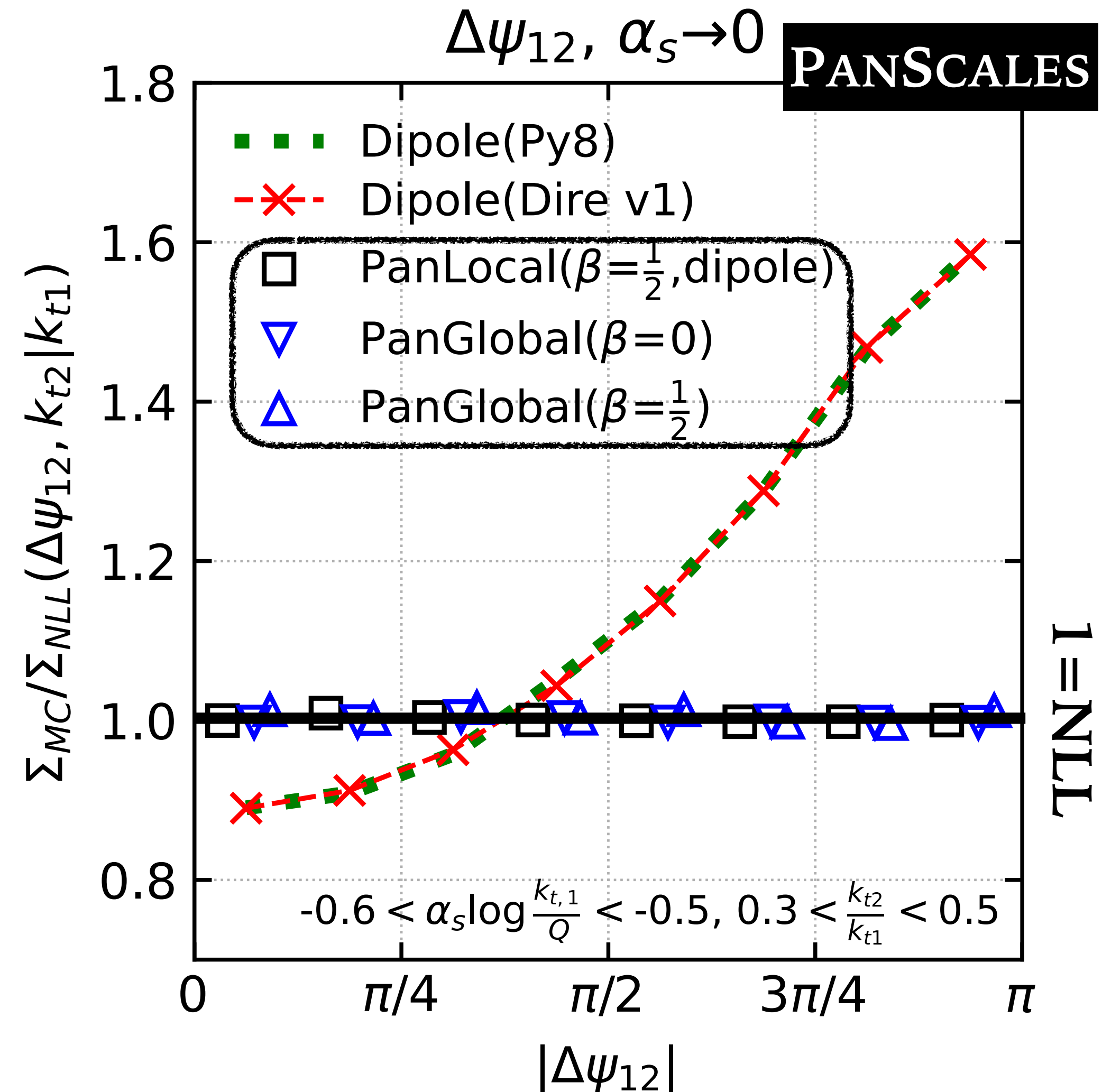
DEDUCTOR

Nagy&Soper,
[2011.04777](#)

CVOLVER

Forshaw et. al,
[2003.06400](#)

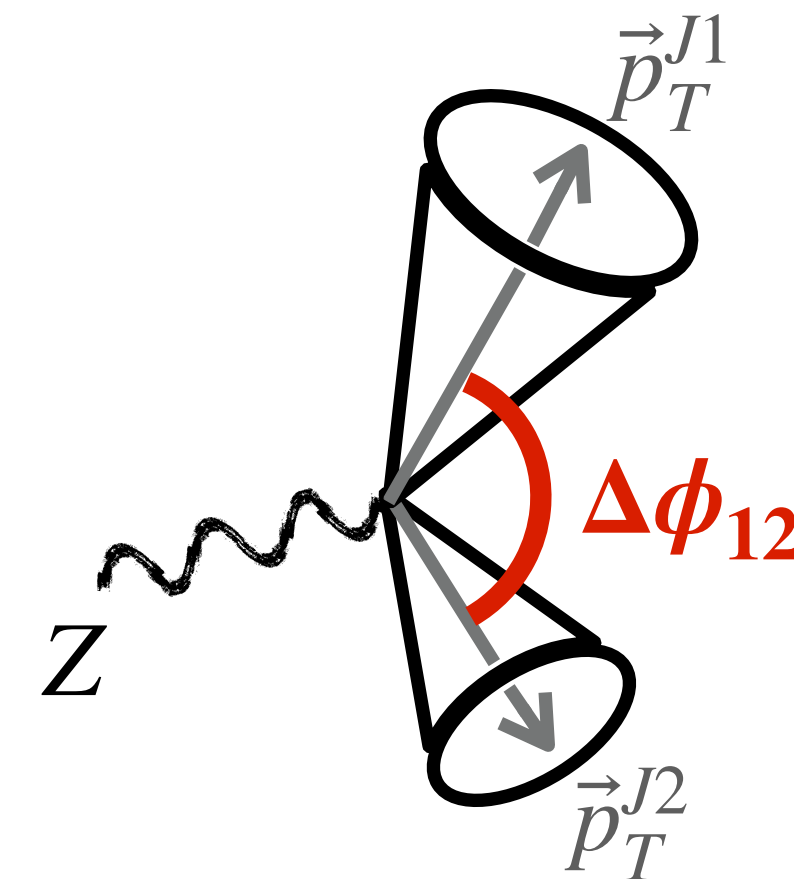
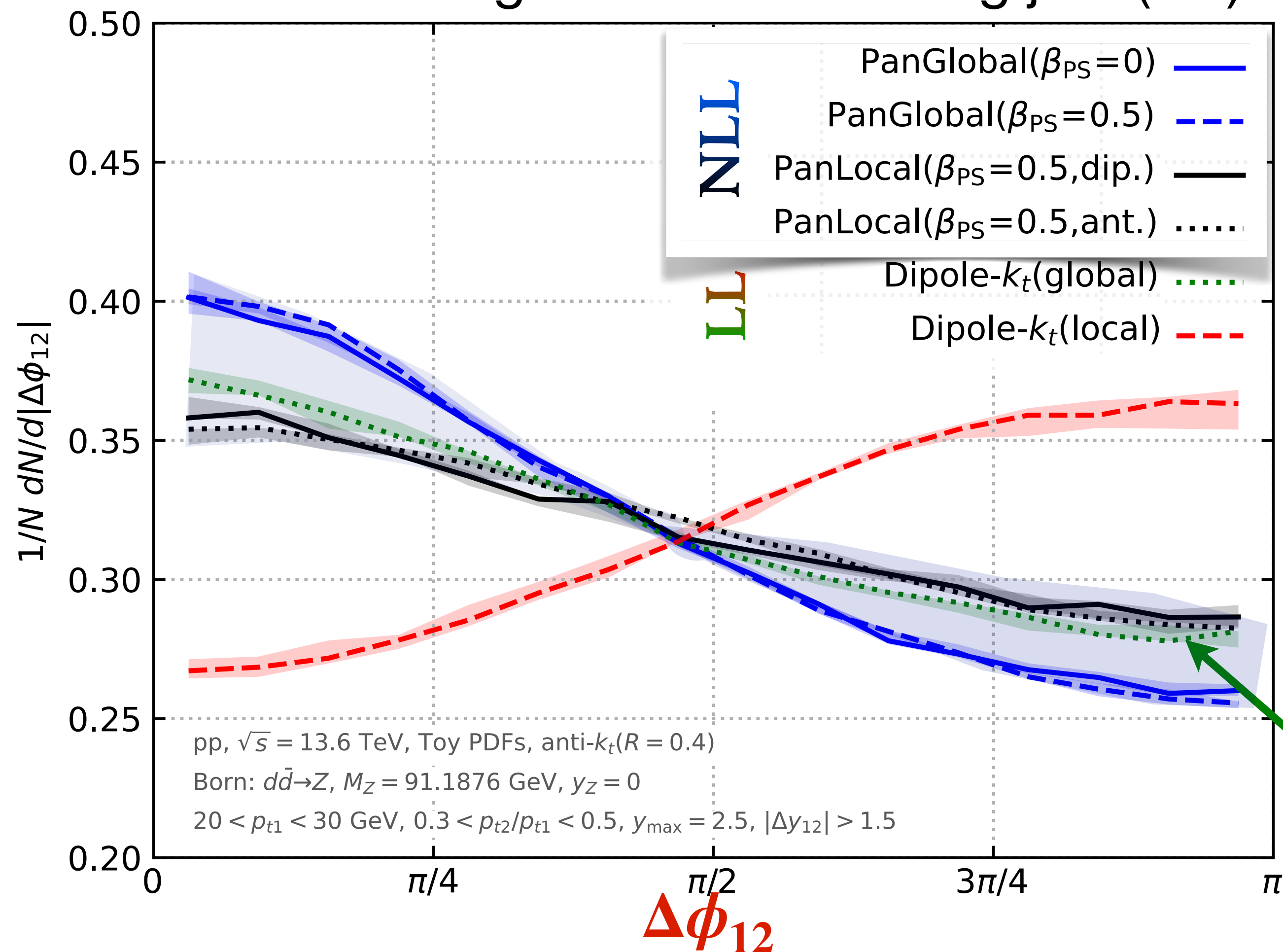
Dasgupta et al. [2002.11114](#)



Exploratory phenomenology for Drell-Yan at the LHC

$$m_{\ell\ell} = 91.2 \text{ GeV}$$

Azimuthal angle between leading jets (DY)



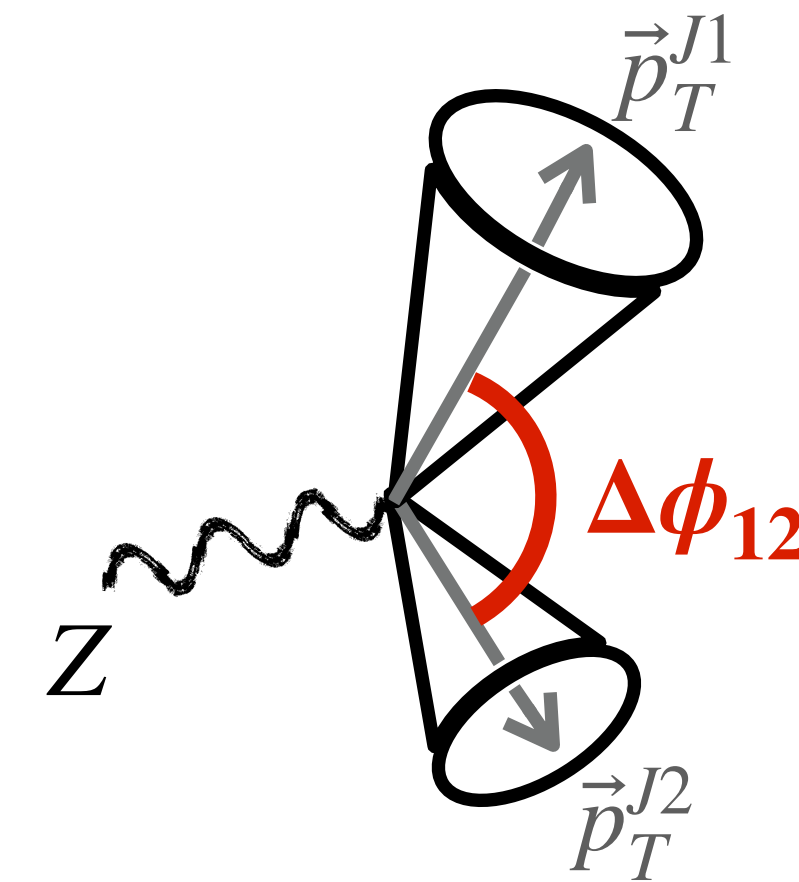
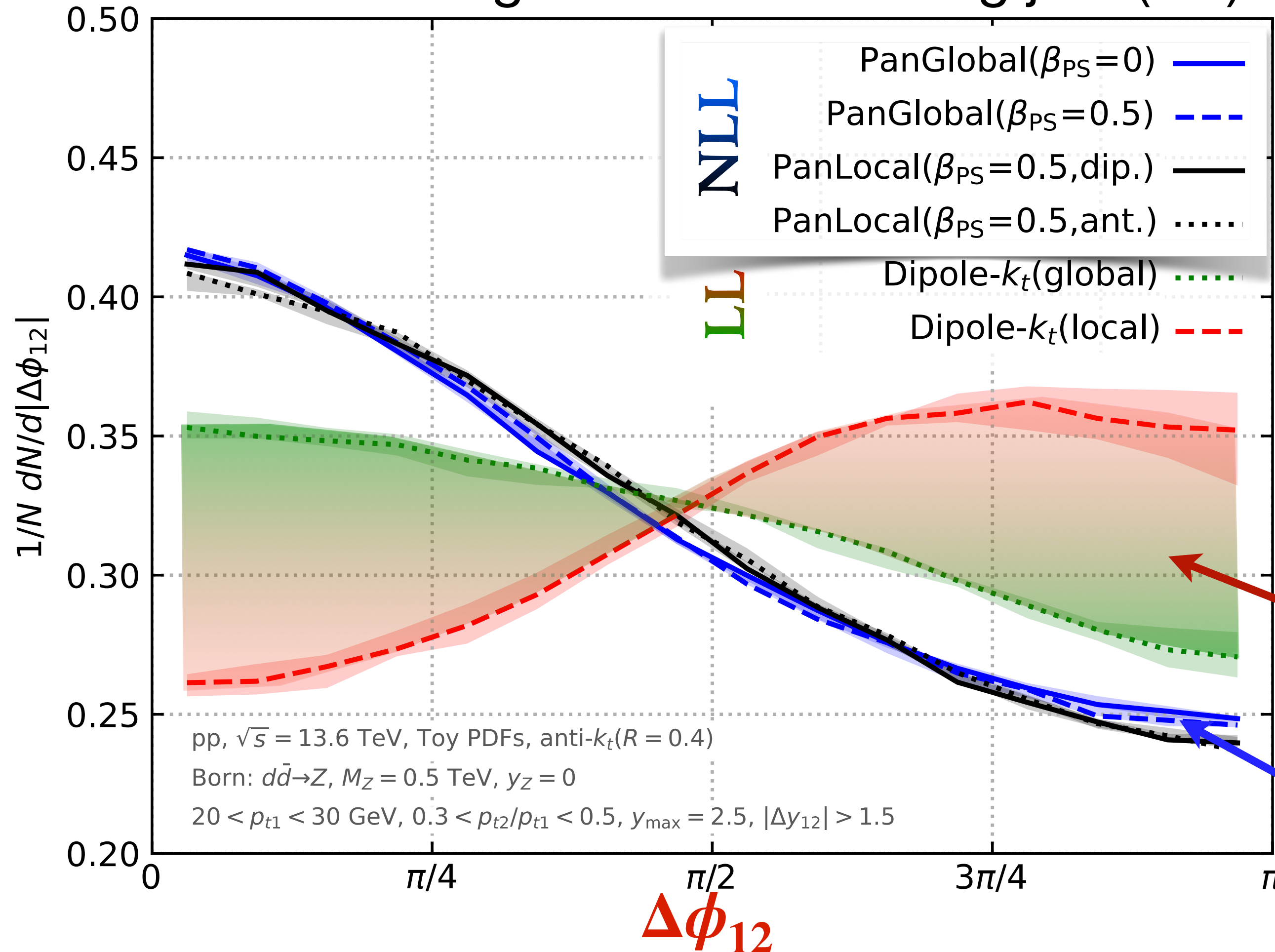
PanScales for $pp \rightarrow$
 colour singlet:
[2207.09467](https://arxiv.org/abs/2207.09467), van
 Beekveld, SFR,
 Hamilton, Salam
 Soto Ontoso, Soyez,
 Verheyen:

This LL shower
 lives within the
 span of the
 NLL showers

Exploratory phenomenology for high-mass Drell-Yan at the LHC

$$m_{\ell\ell} = 500 \text{ GeV}$$

Azimuthal angle between leading jets (DY)



PanScales for $pp \rightarrow$
 colour singlet:
[2207.09467](https://arxiv.org/abs/2207.09467), van
 Beekveld, SFR,
 Hamilton, Salam
 Soto Ontoso, Soyez,
 Verheyen:

NLL/LL discrepancies at larger scales

LL showers

NLL showers