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Three-loop amplitudes for V+jet and H+jet production

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with

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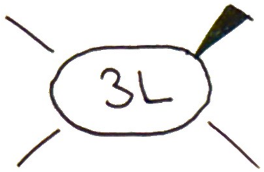
based on 2301.10845, 2306.10170, 2307.15405, 2410.xxxxx

MOTIVATION



amplitudes vital to collider programme of the next 2 decades

incarnations :

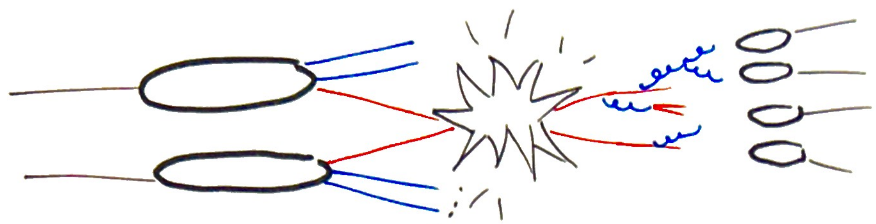
$e^+e^- \to 3$ jets @ N^3LO : clean, event shapes, α_s extraction

$pp \to V+$jet @ N^3LO : large xsection, leptonic decay - clear signature $\to$ PDF fitting

$pp \to H+$jet @ N^3LO : large p_T : resolve loop, light Yukawa couplings, new physics coupling to H

$gg \to H$ @ N^4LO differential : main production mechanism @ LHC

Progress in all parts of particle theory workflow required



AMPLITUDES 

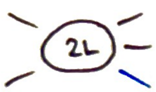
next order =

THEMES

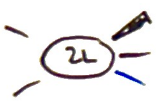
- comp. cost (IBPs)
- size of expressions
- function spaces
- efficient evaluation

⋮

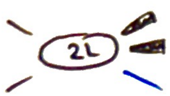
\# legs ↑



done [many ; review: 2207.12295]



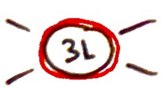
[Badger, Hartanto, Zoia, Kryś 2021-22]
[Abreu, F. Cordero, Ita, Klinkert, Page, Sotnikov '21]



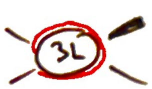
PL integrals [Abreu, Chicherin, Sotnikov, Zoia 2024]

&

\# loops ↑



[Caola, Chakraborty, Gambuti, von Manteuffel, Tancredi ; Bargiela 2021-22]



this talk!

(… and others: 4L FF; 6pt ; with internal masses …)

Common Workflow

1. Identify master integrals & derive DEs
2. Solve in terms of a class of <u>transcendental functions</u> $\left\{ \begin{array}{l} \text{HPLs} \\ \text{MPLs} \\ \text{elliptic polylogs} \\ \dots \end{array} \right.$
3. IBP-reduce amplitude (get <u>rational coefficients</u>) & insert masters

↯ integrals have repeated, spurious, evanescent structures

- contribute to intermediate swell
- cancel only at the end

traditional: **integrals** $\xrightarrow{\text{insert}}$ **amplitude** $\xrightarrow{\text{observe}}$ **physics**

"bootstrap": ~~**integrals**~~ **amplitude** $\xleftarrow{\text{enforce}}$ **physics**

e.g. [Dixon, Gurdogan, McLeod, Wilhelm '22]

this talk:

$$\begin{tikzcd} \textbf{integrals} \arrow[r, "\text{tie}"] & \cdot & \textbf{amplitude} \arrow[l] & \textbf{physics} \arrow[lll, bend left, "\text{impose}"] \end{tikzcd}$$

TOOL - Chen iterated integrals [Chen 1973]

$$I(\ell_1, \ell_2, \dots, \ell_n) = \int_0^1 dt\, \partial_t \ell_1\, I(\ell_2, \dots, \ell_n)\, (\underbrace{g(t)}_{\text{path}})$$

- "alphabet" - independent kernels ℓ_i = functions of kinematics
- "length/weight" - number of integrations

ENCODES ANALYTIC STRUCTURE

- For ε-factorised DE system $d\vec{J}(x_i;\varepsilon) = \varepsilon\, dA(x_i)\, \vec{J}(x_i;\varepsilon)$ connection matrix
- formal solution $\vec{J} = \sum_{n=0} \varepsilon^n \sum_{i=0}^{n} \tilde{A}^i\, \vec{V}_{n-i}$

boundary vector (weight $n-i$)

"EASY" TO OBTAIN

After shuffling, $I(\vec{\ell}_1)\, I(\vec{\ell}_2) = \sum_{\ell = \vec{\ell}_1 ⧢ \vec{\ell}_2} I(\vec{\ell})$

no hidden zeros

INDEPENDENT

→ **VECTOR SPACE** $\mathcal{J}_w$ of shuffled iterated integrals up to weight w over $\mathbb{Q}$

with basis $\quad b_{\mathcal{J}_w} = \{ \varepsilon^{-a} \zeta_n^b \, I(\ell_1, \dots, \ell_c; x) \} \qquad a + bn + c \leq 2L$

@ any rat. kinematic point : scattering amplitudes $\in \mathcal{J}_w$

master integrals $\in \mathcal{J}_w$

cast master integrals as a $\mathbb{Q}$ matrix (truncate @ $w = 2L$)

up to 2L functions

$$S = \begin{array}{cc} \begin{array}{c} J_1 \\ J_2 \\ \vdots \\ \\ J_{n_{MI}} \end{array} & \begin{pmatrix} & & & \\ & & \mathbb{Q} & \\ & & & \\ & & & \end{pmatrix} \\ & \begin{array}{cccc} \uparrow & \uparrow & & \uparrow \\ 1 & \varepsilon I(\ell_1) & \cdots & \varepsilon^2 I(\ell_1, \ell_2) \ \dots \end{array} \end{array}$$

coefficients of Chen integrals in MI

Master Integral Subspace $\mathcal{M}_w \subseteq \mathcal{I}_w$

spanned by the MI solution-vectors

= row space of matrix S

$$\#\,\text{columns}(S) = \dim(\mathcal{I}_w) \gg n_{MI} \geq \dim(\mathcal{M}_w) = \text{rank}(S)$$!

typically: thousands / millions of iterated integrals

but! # of indep. combinations bounded by # MIs

generous bound!

Sources of dependence:

- symmetries
- iteration
- truncation

All taken into account in Chen solutions

used for pentagon functions!
[Chicherin, Sotnikov '20]
[Chicherin, Sotnikov, Zoia '21]

Physical subspace $\mathcal{A}^{\parallel}_w$

$\mathcal{M}_w$ encompasses <u>any</u> amplitude with this topology

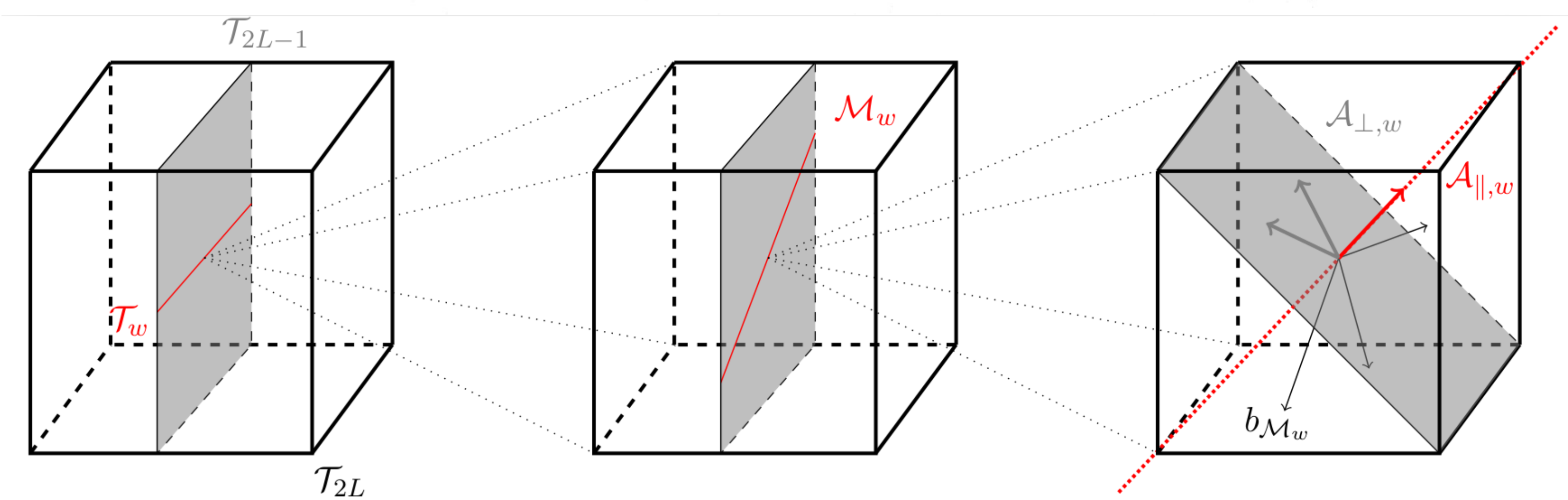
but for particular objects still too big   cf. Won Lim's talk

masters : <u>21</u> alphabet letters

FF of $\mathrm{Tr}(\phi_R^2)$: <u>6</u> letters, restrictions...

$$\mathcal{M}_w = \mathcal{A}^{\parallel}_w \oplus \mathcal{A}^{\perp}_w$$

physically favoured : alphabet, disc.'s, adjacencies, poles ...



CONSEQUENCES

Minimality over all relations manifest

UV & IR pole prediction from lower orders

restrictions on finite part of amp. without knowledge of integrand

Ex 1 I_1 and I_2 only appear in one minimal function

$$\Psi_i = 1 + \ldots + \varepsilon^n I_1 + \ldots + \varepsilon^{n+1} I_2 + \ldots \quad \rightarrow \quad \text{coeff. of } I_1 \text{ in } \tfrac{1}{\varepsilon} \text{ of amp.} \; \underset{\text{(known!)}}{} \; \stackrel{!}{=} \; \text{coeff. of } I_2 \text{ in } \varepsilon^0 \text{ of amp.}$$

Ex 2 Letter l_x appears for the first time @ this loop order

→ entire function Ψ_i must be suppressed by ε^{2L-n}

(no cancellation only vanishing due to minimality)

Generally: hundreds / thousands of restrictions on coefficients in finite part!

RECAP

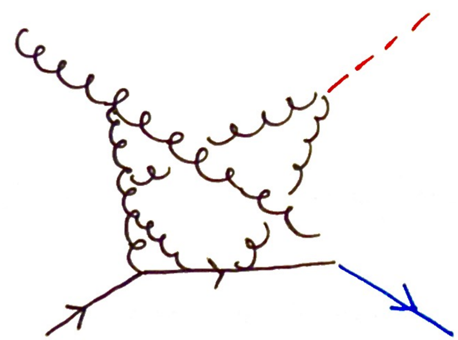
Chen iterated integrals:
- formal solution to DEs
- all (even unknown!) relations manifest
- independent (if indep. letters & shuffled)

⇒ suitable for building basis of functions: minimal, graded, tailored to an amplitude

Before amp. is known:
- insights about analytic structure,
- information poles → finite part

After amp. is known:
- no cancellations,
- DEs only for relevant functions → solve (choose fibration) & analytic continuation & evaluation
- finite-field: fewer coefficients with more restrictions

Application to X + jet amplitudes



COMPUTATIONAL SETUP

(1.) Lagrangian

- Z boson: vector (γ^μ) & $\underbrace{\text{axial-vector}}$ ($\gamma^\mu\gamma^5$) coupling (Larin)

  suppressed but in diff. observables NNLO (AV) $\approx$ N^{3}LO (V)

  Singlet contribution due to $m_b \ll m_t$; 2L amps

- H boson: infinite m_t limit

  [Gehrmann, Tancredi, Peraro : 2306.10170]

  $$\mathcal{L}_{HEFT} = -\frac{\lambda}{4} H G_a^{\mu\nu} G_{a,\mu\nu}$$

  coupling directly to gluons, no int. masses

(2.) Tensor structure following [Tancredi, Peraro '19-20]

- avoid evanescent terms from d-dimensional Dirac algebra
- zero (even) or one (odd) instance of $\varepsilon^{\mu\nu\rho\sigma}$ in each basis elm. $\rightarrow$ parity manifest
- # tensor structures = # helicity amplitudes $\rightarrow$ minimal

$Vq\bar{q}$: 6+6 hel. amp.

$Vggg$: 12+12 hel. amp.

$Hgq\bar{q}$: 1 hel. amp.

$Hggg$: 2 hel. amp.

3.) Colour decomposition

Assume $\underset{(3)}{N_c} \sim N \sim \underset{(5)}{N_f}$

and $N^2 = \mathcal{O}(10)$

$$\mathcal{A}^{(3L)} = \boxed{\bullet N_c^3 + \bullet N_c^2 N_f + \bullet N_c N_f^2 + \bullet N_f^3} + \bullet N_c + \bullet N_f + \bullet N_f^2/N_c + \bullet 1/N_c + \bullet N_f/N_c^2 + \bullet 1/N_c^3$$

leading colour

V+jet planar

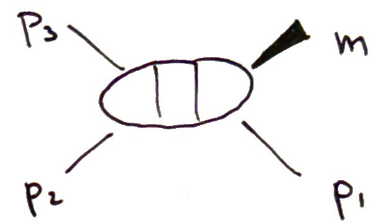
H+jet also <u>non-planar</u>

4.) Kinematics

2 indep. dimensionless variables

$(x+y+z=1)$

$$x = \frac{s_{12}}{m^2} \qquad y = \frac{s_{13}}{m^2} \qquad z = \frac{s_{23}}{m^2}$$



5.) Symbol alphabet

- 2L integrals [Gehrmann, Remiddi '01]
- 3L planar integrals [Di Vita, Mastrolia, Schubert, Yundin '14; Canko, Syrrakos '21]

$\{x, y, z, 1-x, 1-y, 1-z\}$

VS.

- 3L NPL integrals [Henn, Lim, T.-Bobadilla '23; in prep.]

3 new types:

$$\left\{ \frac{xy - \sqrt{xyz}}{xy + \sqrt{xyz}},\ x^2 + xy + y,\ x^2 - x + y \right\}$$

$(\times 6$ perm.$)$

REDUCTION

- tensor rank up to 6
- $\mathcal{O}$(1M) integrals at full colour
- top sectors ($t=10$) with up to 18 masters (8 @ L.C.)

prohibitively expensive

Fully determine analytic structure without complicated reduction !

(1.) Source of new quadratic & sqrt letters are NPL graphs with ≥ 8 propagators

easy reductions!

→ 7-particle cuts discard only functions with 2L letters & span the coefficients of all new functions

(2.) Expressed in minimal set of graded transc. functions (combinations of Chen integrals), all expected / conjectural properties are manifest

RESULTS - example

→ reductions exact in ε at several numerical points with FIRE 6 [Smirnov, Chukharev '19] and KIRA 2 [Klappert, Lange, Maierhöfer, Usovitsch '20]

- 1282 minimal function basis
- only 93 contain new letters (but 100s of MIs!)

Ex $H \to g^+ g^+ g^+$, N_c^3 colour layer, point $x = \frac{3}{13}$, $y = \frac{11}{17}$

new letters starting @ w4

$\psi_1 \to \mathcal{O}(\epsilon^3)$
$\psi_2 \to \mathcal{O}(\epsilon^3)$
$\psi_3 \to \mathcal{O}(\epsilon^3)$
$\psi_4 \to \frac{1746}{48641}\epsilon^2 + \mathcal{O}(\epsilon^3)$
$\psi_5 \to \mathcal{O}(\epsilon^3)$
$\psi_6 \to \frac{22}{289}\epsilon^2 + \mathcal{O}(\epsilon^3)$
$\psi_7 \to \frac{16}{169}\epsilon^2 + \mathcal{O}(\epsilon^3)$
$\psi_8 \to \mathcal{O}(\epsilon^3)$
$\psi_9 \to \mathcal{O}(\epsilon^3)$

starting @ w5

$\psi_{10} \to \mathcal{O}(\epsilon^2)$	$\psi_{25} \to -\frac{121}{63}\epsilon + \mathcal{O}(\epsilon^2)$
$\psi_{11} \to \mathcal{O}(\epsilon^2)$	$\psi_{26} \to \frac{11}{6}\epsilon + \mathcal{O}(\epsilon^2)$
$\psi_{12} \to \mathcal{O}(\epsilon^2)$	$\psi_{27} \to -\frac{121}{6}\epsilon + \mathcal{O}(\epsilon^2)$
$\psi_{13} \to \mathcal{O}(\epsilon^2)$	$\psi_{28} \to \frac{685}{63}\epsilon + \mathcal{O}(\epsilon^2)$
$\psi_{14} \to \mathcal{O}(\epsilon^2)$	$\psi_{29} \to \frac{3179}{126}\epsilon + \mathcal{O}(\epsilon^2)$
$\psi_{15} \to \mathcal{O}(\epsilon^2)$	$\psi_{30} \to \frac{55}{18}\epsilon + \mathcal{O}(\epsilon^2)$
$\psi_{16} \to \mathcal{O}(\epsilon^2)$	$\psi_{31} \to -\frac{11}{2}\epsilon + \mathcal{O}(\epsilon^2)$
$\psi_{17} \to \mathcal{O}(\epsilon^2)$	$\psi_{32} \to \frac{187}{6}\epsilon + \mathcal{O}(\epsilon^2)$
$\psi_{18} \to \mathcal{O}(\epsilon^2)$	$\psi_{33} \to \frac{85}{63}\epsilon + \mathcal{O}(\epsilon^2)$
$\psi_{19} \to \mathcal{O}(\epsilon^2)$	$\psi_{34} \to \frac{11}{6}\epsilon + \mathcal{O}(\epsilon^2)$
$\psi_{20} \to \mathcal{O}(\epsilon^2)$	$\psi_{35} \to -\frac{55}{126}\epsilon + \mathcal{O}(\epsilon^2)$
$\psi_{21} \to \mathcal{O}(\epsilon^2)$	$\psi_{36} \to \frac{11}{9}\epsilon + \mathcal{O}(\epsilon^2)$
$\psi_{22} \to \mathcal{O}(\epsilon^2)$	
$\psi_{23} \to \mathcal{O}(\epsilon^2)$	
$\psi_{24} \to \mathcal{O}(\epsilon^2)$	

starting @ w6

$\psi_{37-63} \to \mathcal{O}(\epsilon)$
$\psi_{64-93} \to \mathcal{O}(\epsilon)$

diquadratic, sqrt letters

only uniquadratic

RESULTS - summary

H+jet amplitudes do not exhibit complexity of integrals!

1.) **sqrt and diquadratic** letters drop out of L.C. amplitudes

2.) **maximum transcendentality** principle [Kotikov, Lipatov '04 etc.]

seemingly obeyed = no new letters in weight 6 functions in Hggg

↳ cf. Sudakov FF @ 3L [Gehrmann, Henn, Huber '12]

Hggg amp.s @ 2L [Brandhuber, Travaglini, Yang '12]

↳ same as $Tr(\phi^2)$ FF in planar $\mathcal{N}=4$ [see Won Lim's talk]

3.) **remarkable simplicity** & patterns, size of rational coefficients; at most one new uniquadratic letter / function

→ New avenues for bootstrapping approaches

SUMMARY

computed:

- 1L and 2L amplitudes extended to $\mathcal{O}(\varepsilon^2)$ [2301.10849]

  and with axial-vector coupling of Z [2306.10170]

- $\gamma^*/Z/W^\pm$ + jet 3L amplitudes @ L.C. [2307.15405]

- NPL master integrals for H+jet @ L.C. [2302.12776; in prep.]

- all X+jet integrands

$$\boxed{\begin{array}{c}\text{minimal graded transc. functions} \\ \text{for simpler amplitude \& evaluation of int.s \& early insight into structure}\end{array}}$$

coming:

- full analytic expressions for H+jet @ L.C.

- remaining NPL topologies