Heavy quarks mass corrections to threshold resummation

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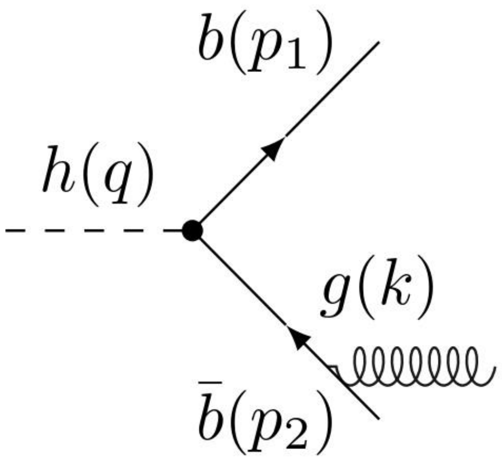
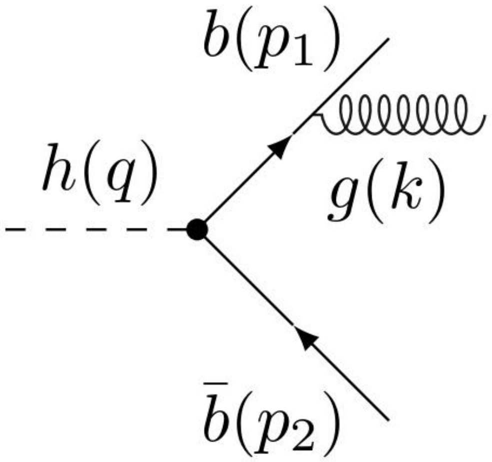
Based on works in collaboration with M. Cacciari, S.Marzani and G.Ridolfi

Introduction

We consider heavy flavour production from the decay of a colour singlet particle





- We want to compute the differential decay rate over $x = \frac{2\, p_1 \cdot q}{q^2}$ in the large x limit.
- x tends to 1 in the soft limit $(k \to 0)$ when considering massive quarks. In the case of masless particle, x tends to 1 in the collinear and soft limit

Why resummation is needed?

At every order, the perturbative coefficients are affected by the presence of large logs in the limit $x \to 1$:

$$c_k = \left(\frac{\log^{p-1}(1-x)}{1-x}\right)_+ + \dots, \quad p = \begin{cases} k & \text{massive} \\ 2k & \text{massless} \end{cases}$$

- These contributions are a remnant of the cancellation of the soft singularities.
- The resummation of these terms is achieved in Mellin space

  $(\log(1-x) \leftrightarrow \log N)$

$$\tilde{c}_k = \int_0^1 \mathrm{d}x \left(\frac{\log^{p-1}(1-x)}{1-x}\right)_+ x^{N-1} = \frac{1}{p}\log^p \frac{1}{N} + \dots$$

Massless vs Massive Scheme Approach

Massless Scheme:

- Quark mass used as a regulator
- Cross section computed as a convolution of a coefficient function times a fragmentation function
- Logs of $\xi = \frac{m^2}{q^2}$ resummed through DGLAP

Massive Scheme:

- All mass dependence taken into account
- Kinematics treated correctly at every order
- Large logs of the mass spoil the convergence of the series.

Problems with the merging

We want to merge the two different calculations of the differential decay rate resumming logs of N in the large N limit $(x \to 1)$ and the log of the mass.

Massless Scheme

- Double logs of N with mass independent coefficients (M.Cacciari, S. Catani; F. Maltoni, G. Ridolfi, M. Ubiali, M. Zaro)

Massive Scheme

- Single logs of N with mass dependent coefficient
- If we perform the limit $\xi \to 0$ after the large N limit, we do not recover the massless case

Different logarithmic structure in the threshold limit in the two cases

Problem with the merging

The soft resummation formula in the massive scheme is the product of a coefficient function times a soft function (E. Laenen, G. Oderda, G. Sterman)

$$\widetilde{\Gamma}(N,\xi) = C(\xi,\alpha_{\mathrm{s}})e^{-2\int_0^1 \mathrm{d}x\, \frac{x^{N-1}-1}{1-x}\gamma_{\mathrm{soft}}\left(\beta,\alpha_{\mathrm{s}}\left((1-x)^2q^2\right)\right)}$$

where γ_{soft} is the soft anomalous dimension.

If we perform the massless limit of the first order coefficient $C^{(1)}$ we find:

$$C^{(1)}(\xi) = C_{\mathrm{F}}\left(\frac{1}{2}\log^2\xi + \log\xi + \frac{\pi^2}{2} + \mathcal{O}(\xi)\right)$$

This term is not predicted by DGLAP!

Naive strategy

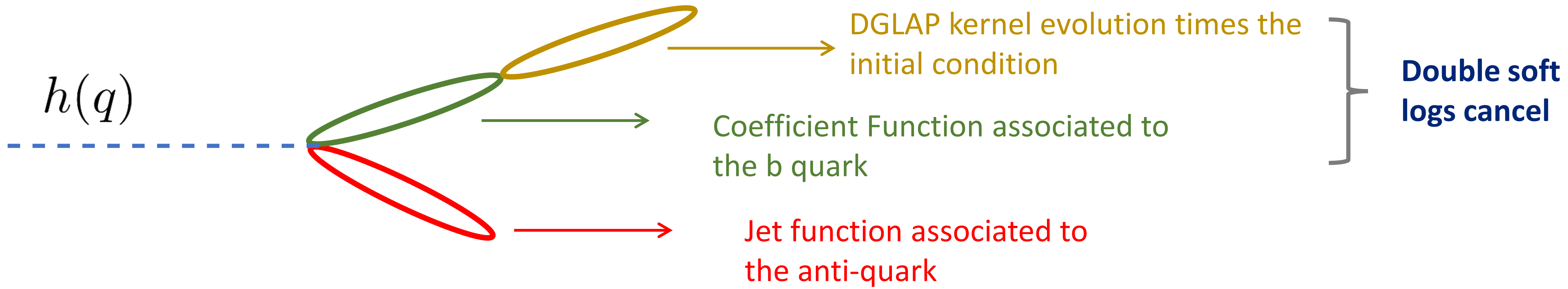
In the same spirit of FONLL, we would like to define a matching scheme merging togheter:

$$\widetilde{\Gamma}^{(4)}(N,\xi) = \widetilde{\Gamma}_k^{(4)}(N,\xi) + \widetilde{\Gamma}_{\ell_1}^{(4,\mathrm{res})}(N,\xi) - \text{double counting},$$
$$\widetilde{\Gamma}^{(5)}(N,\xi) = \widetilde{\Gamma}_\ell^{(5)}(N,\xi) + \widetilde{\Gamma}_{\ell\ell_2}^{(5,\mathrm{res})}(N,\xi) - \text{double counting}.$$

- We cannot identify an all-order subtraction term.
- We start from the resummed massless scheme expression taking into account mass effects in the regime in which $1 - x \ll \xi$.

The origin of the problem

At NLL one can see the threshold resumed expression in the massless framework as a product of two independent jet functions:



In the measured leg the double logarithmic structure cancel between the quark jet function and the fragmentation function.

Towards a Solution

- The tagged jet function is computed considering the b massless above the 5/4 flavor threshold, and massive below ⟶ double logs cancel.

- The recoiling jet function exhibit double log since in the massless approach is the $\bar{b}$ is always retained to be massless.

⬇

We have to consider also the recoiling jet function in the quasi-collinear limit so that:

1. When $1 - x \gg \xi$ we recover Cacciari-Catani formula.
2. When $1 - x \ll \xi$ we recover the resummed calculation made by Laenen-Oderda-Sterman

Resummation of the cumulative distribution in momentum space

- The resummation of the cumulative distribution related to the observable x coincide with the computation of the jet functions provided that we identify $1 - x = \frac{1}{N\, e^{\gamma_E}}$.
- We employ the quasi-collinear limit keeping $\xi \simeq \theta^2$.

Emission quasi collinear to the b

$$j(1-x,\xi) = -\int_0^1 \mathrm{d}z_1 \int_{z_1^2 m^2}^{q^2} \frac{\mathrm{d}k_t^2}{k_t^2} \frac{\alpha_s^{\mathrm{CMW}}(k_t^2)}{2\pi} P_{gb}(z_1, k_t^2 - z_1^2 m^2)\Theta\left(\eta_1\right)\Theta\left(z_1 - (1-x)\right),$$

$$\bar{j}(1-x,\xi) = -\int_0^1 \mathrm{d}z_2 \int_{z_2^2 m^2}^{q^2} \frac{\mathrm{d}k_t^2}{k_t^2} \frac{\alpha_s^{\mathrm{CMW}}(k_t^2)}{2\pi} P_{gb}(z_2, k_t^2 - z_2^2 m^2)\Theta\left(\eta_2\right)\Theta\left(\frac{k_t^2}{q^2 z_2} - (1-x)\right).$$

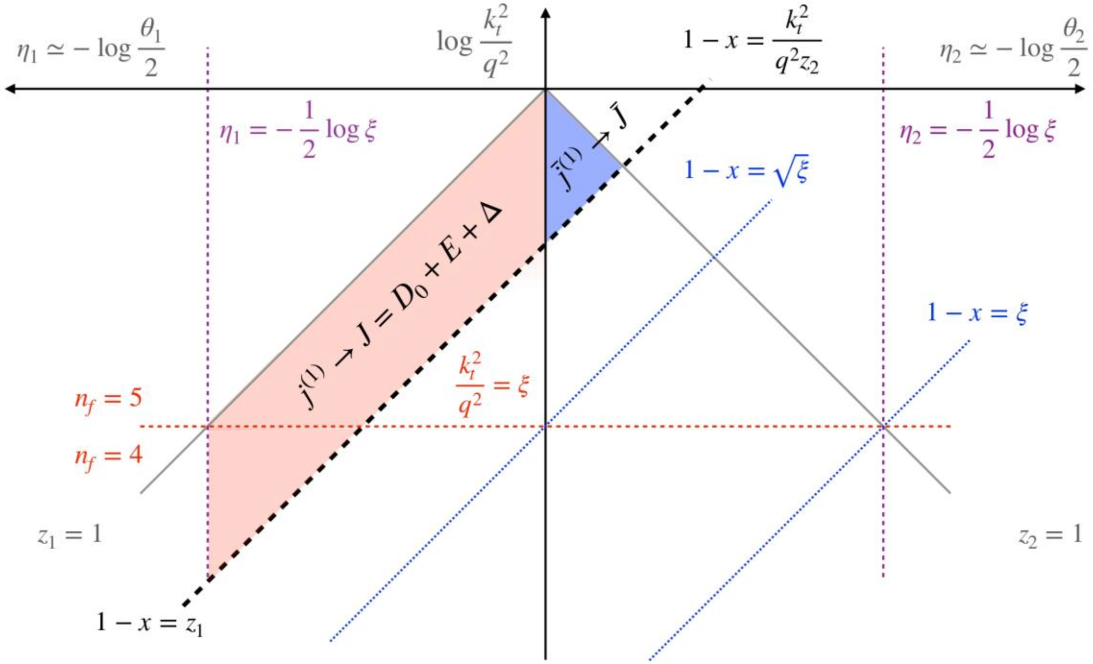
Emission quasi collinear to the $\bar{b}$

- Decoupling scheme is employed for the running coupling.
- The result can also be expressed as the product of a modified coeff. function times the perturbative fragmentation function in the non singlet approx

Region 1: $1 - x > \sqrt{\xi}$

- The observable $1 - x$ has two different parametrization in the two collinear regions: $V = z_1$ (gluon energy fraction) $\bar{V} = z_2\,\bar{\theta}^2$ (jet invariant mass).

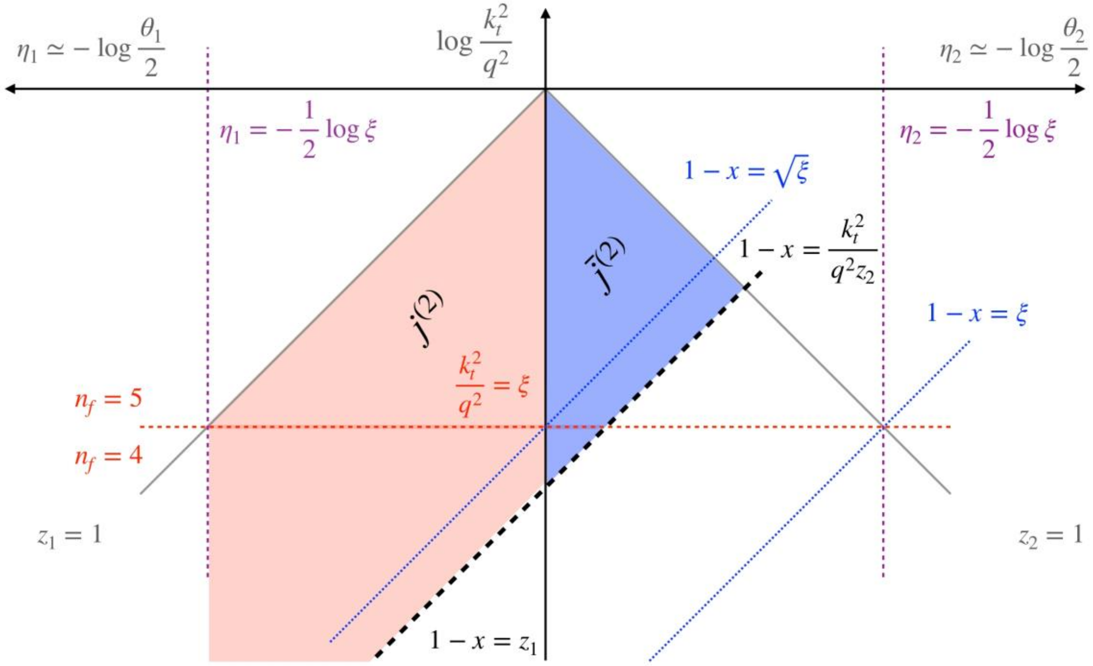
- When $1 - x > \sqrt{\xi}$, we recover the computation done by Cacciari and Catani



Region 2: $\sqrt{\xi} > 1 - x > \xi$

- When $\sqrt{\xi} > 1 - x > \xi$ we enter a transition region that interpolates the 4 flavor calculation with the 5 flavor one.

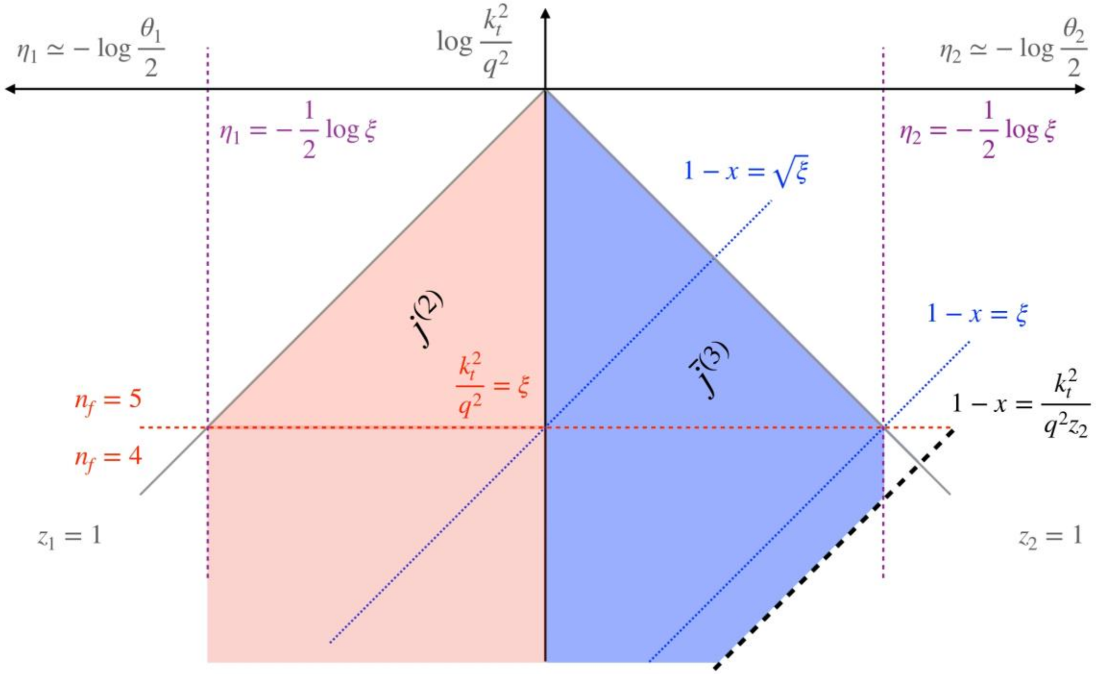
- Mass effects starts to become relevant also in the coefficient function.



Region 3: $\xi > 1 - x$

- When $1 - x < \xi$ we recover the massive calculation with the mass logs exponentiated.

- At $\mathcal{O}(\alpha_S)$: $\bar{J}^{(3)} = \frac{\alpha_S C_F}{2\pi} \log^2 \xi + \cdots$



Charm Fragmentation

We now want to exploit this technology to study the charm-ratio:

$$R = \frac{\Sigma\left(N, Q_A^2\right)}{\Sigma\left(N, Q_B^2\right)}, \quad \Sigma\left(N, Q^2\right) = \sigma_h\left(N, Q^2\right) D^{\mathrm{NP}}_{h\to D^{*+}}(N)$$

- σ_h is the perturbative cross section for the production of a charm quark c

$$\sigma_h\left(N, Q^2\right) = \sum_{i=q,g,h} C_i(N, Q^2, \mu^2) D_{i\to h}\left(N, \mu^2, \mu_0^2\right) + \mathcal{O}\left(\frac{m_h^2}{Q^2}\right)$$

- $D^{NP}_{h\to D^{*+}}$ is the non perturbative fragmentation function that must be fitted to experimental data

- Q_A and Q_B represents the center of mass energies at ALEPH ($Q_A = \mathrm{m_z} \simeq 91\ GeV$) and CLEO ($Q_B = \mathrm{m}_{\Upsilon(4S)} \simeq 11\ GeV$)

Charm Ratio

We expect the contributions from low-scale physics, i.e. the initial condition and its non-perturbative correction, to largely cancel in the ratio (non-singlet approx.):

$$R_{\text{pert}} = \frac{C_h\left(N, Q_A^2, \mu_A^2\right)}{C_h\left(N, Q_B^2, \mu_B^2\right)} E(N, \mu_A^2, \mu_B^2).$$

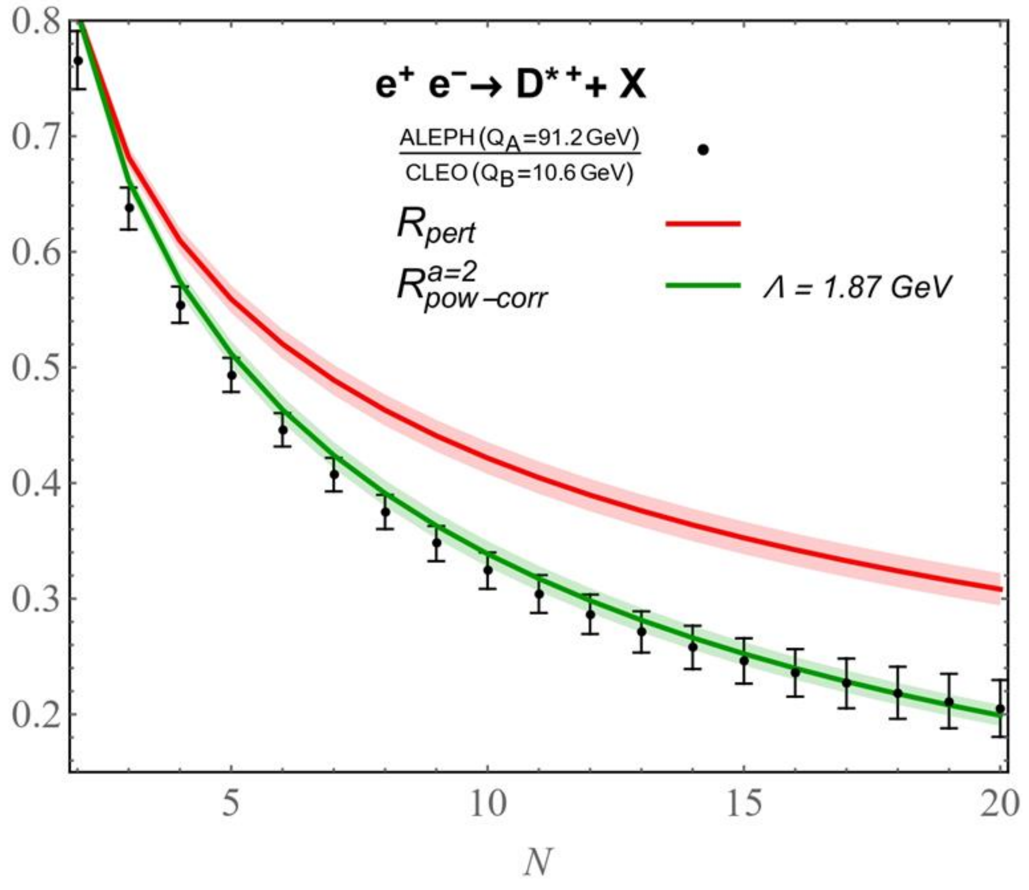
- In this approximation, the ratio provides us a direct test of perturbative QCD
- First studies at NLO+NLL (M. Cacciari, P. Nason and C. Oleari)
- Recently at NNLO+ NNLL (L. Bonino, M. Cacciari and G. Stagnitto), using the MELA code for DGLAP evolution (V. Bertone, S. Carrazza, E. Nocera)

NP corrections

Large discrepancy with the data at large N, ascribed to non-perturbative power corrections to the coefficient functions (B. Webber, M. Dasgupta)

$$R^a_{\text{pow-corr}} = R_{\text{pert}} \times \frac{1 + \frac{\Lambda^a}{Q^a_A}\mathcal{C}(N)}{1 + \frac{\Lambda^a}{Q^a_B}\mathcal{C}(N)}.$$

Renormalon analysis: $a = 2,\ \mathcal{C}(1) = 0$,
with $\mathcal{C}$ growing linearly with N



Fitted value of $\Lambda \simeq 1.87\ GeV$, larger than expected

Inclusion of mass effects

We now want to include the effects originated by the interplay between $\log N$ and $\log \xi$:

- Dead cone effect for emission collinear to the un-resolved anti-quark leg becomes relevant only when $N > \frac{m^2_{\Upsilon(4S)}}{m_c^2} e^{-\gamma_E} \simeq 30$ ($N \leq 20$ in this study)

- Heavy quark thresholds in the resummed coefficient function (extra thresholds due to b, c masses)

The framework described is valid only at NLL, and in this approximation, the ratio reads:

$$R_{\mathrm{thr}} = \frac{C^{(1)}_{\mathrm{thr}}(N, Q_A^2, \mu_A^2)\Theta_{A,1} + C^{(2)}_{\mathrm{thr}}(N, Q_A^2, \mu_A^2)\Theta_{A,2}}{\sum_{i=1}^{4} C^{(i)}_{\mathrm{thr}}(N, Q_B^2, \mu_B^2)\Theta_{B,i}} = \frac{C^{(1)}_{\mathrm{thr}}(N, Q_A^2, \mu_A^2)}{C^{(1)}_{\mathrm{thr}}(N, Q_B^2, \mu_B^2)} \times K_{\mathrm{thr}}$$

Matching to NNLO+NNLL

- We observe that $C_{thr}^{(i)}$ are computed at NLL while C_h at NNLO+NNLL. We need to access NNLL for $C_{thr}^{(i)}$ to do a complete matching.

- However, we notice that the calculation of the charm ratio in the first region coincides with C_h , albeit at a lower accuracy: multiplicative matching

$$\widetilde{R}_{\mathrm{pert}} = R_{\mathrm{pert}} \times K_{\mathrm{thr}}$$

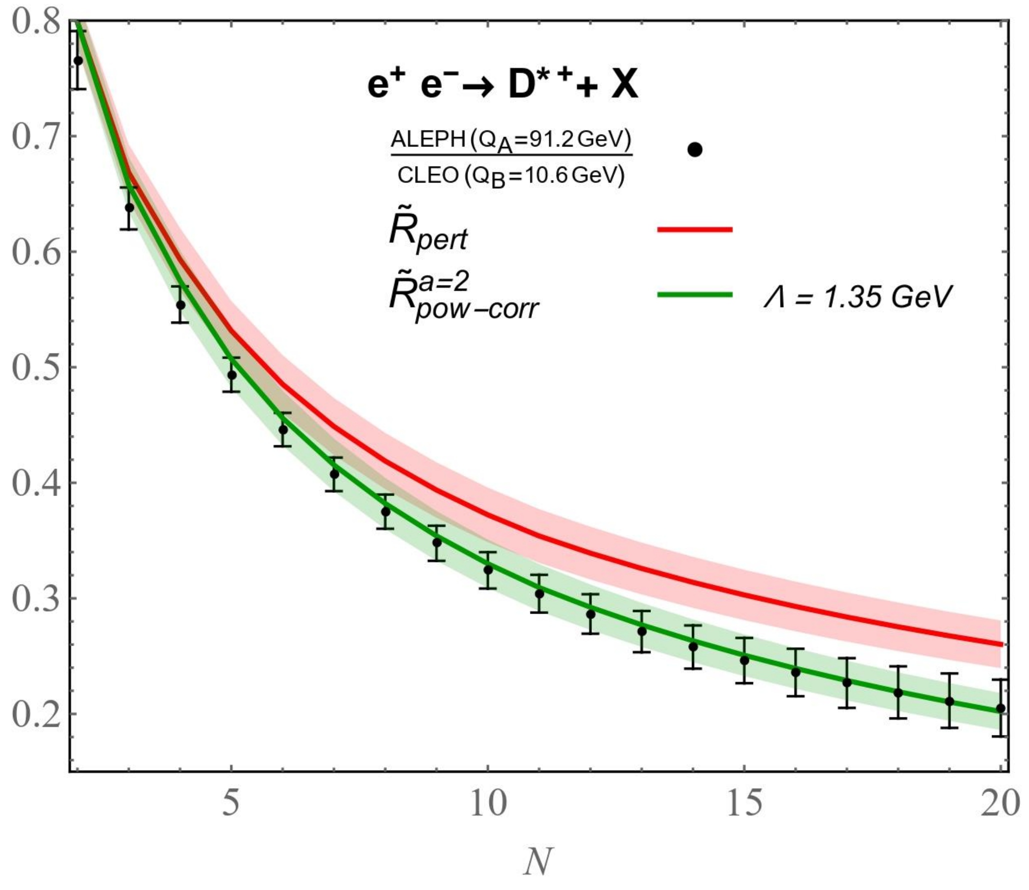
Correction to the coefficient function

Improved description of charm fragmentation data

We repeat the study about NP power corrections with our improved perturbative predictions

$$\widetilde{R}^{a}_{\text{pow-corr}} = \widetilde{R}_{\text{pert}} \times \frac{1 + \frac{\Lambda^a}{Q_A^a}\mathcal{C}(N)}{1 + \frac{\Lambda^a}{Q_B^a}\mathcal{C}(N)}.$$

- Reduced discrepancy to the experimental data due to the refined treatment of heavy quarks thresholds.
- The fitted NP parameter Λ is reduced to $1.35\ GeV$



Conclusions

- The merging of the massive and massless calculation is far from trivial because of the fact that the massless and soft limit do not commute.

- We build a joint resummation in such a way that if we are in the regime in which if $1 - x < \xi$ we recover the massive scheme resummation and if $1 - x > \xi$ we have the resummed expression obtained in the massless scheme at NLL accuracy.

- Same problem addressed by U. Aglietti, G. Ferrera et al using a different formalism

- Visible effects in the charm ratio observable: reduced gap between theory and experiments

Thanks for your attention !!!

Backup slides

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FONLL

Matching resummed scheme with fixed order calculations gives better predictions

in the study of differential decay rate in various regions of ξ:

$$\widetilde{\Gamma}(N,\xi) = \underbrace{\widetilde{\Gamma}_k^{(4)}(N,\xi)}_{\xi = \mathcal{O}(1)} + \underbrace{\widetilde{\Gamma}_\ell^{(5)}(N,\xi)}_{\xi \ll 1} - \text{double counting}$$

- k is the accuracy of the fixed order calculation, ℓ the logarithmic accuracy
- The double counting is the expansion of $\widetilde{\Gamma}_\ell^{(5)}$ to order k
- In the following we will restrict to the case $\ell = 1$ (M.Cacciari, M. Greco, P. Nason)

Resummed formula in x space

The full resumed calculation is given by:

$$\frac{1}{\Gamma_0}\frac{\mathrm{d}\Gamma}{\mathrm{d}x} = \int_{c-i\infty}^{c+i\infty}\frac{\mathrm{d}N}{2\pi i}\,x^{-N}\begin{cases}\widetilde{\Gamma}^{(1)}(N,\xi), & \text{if } 1-x>\sqrt{\xi},\\ \widetilde{\Gamma}^{(2)}(N,\xi), & \text{if } \xi<1-x<\sqrt{\xi},\\ \widetilde{\Gamma}^{(3)}(N,\xi), & \text{if } 1-x<\xi,\end{cases}$$

- The matching conditions for $\tilde{\Gamma}^{(1)}, \tilde{\Gamma}^{(3)}$ are determined by comparing our result with the massless and massive scheme calculations.

- At this accuracy level, arbitrariness for the overall constant of $\tilde{\Gamma}^{(2)}$ (see also U. Aglietti- G. Ferrera).