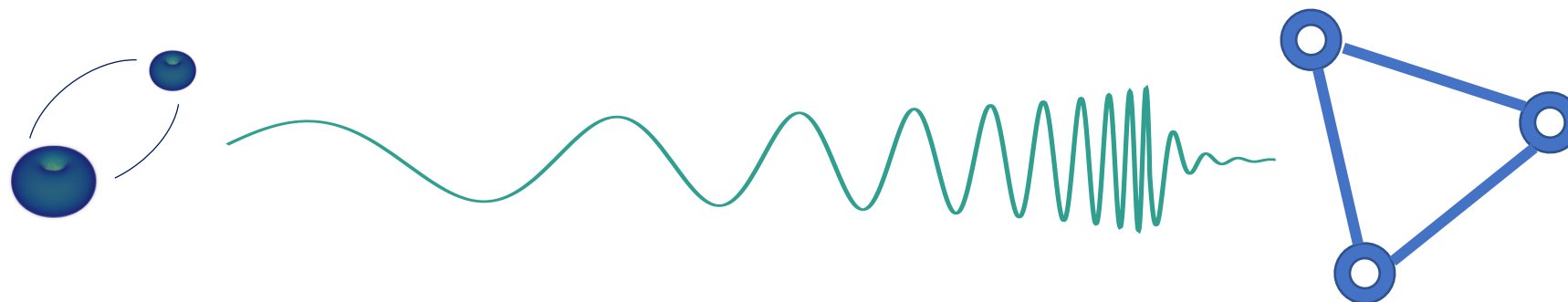


BOSON STARS AS TARGETS OF GRAVITATIONAL-WAVE SEARCHES AND PROBES OF NEW FUNDAMENTAL FIELDS



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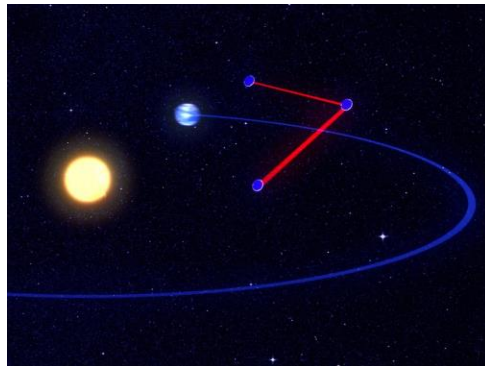


Gravitational-wave astronomy era

- Gravitational waves from binary systems are routinely observed by LIGO/Virgo.
- In the near future:

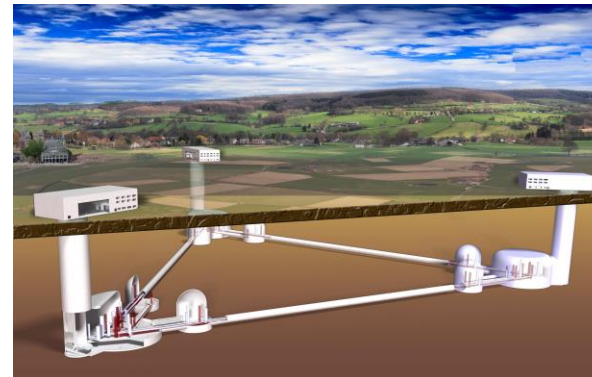
LISA (Laser Interferometer Space Antenna) – Launch expected in 2034

- Cluster of 3 spacecrafts in heliocentric orbit
- Equilateral triangle with $5 \cdot 10^6 km$ arm length
- Sensitivity band $\sim 0.1 \text{ mHz} - 1 \text{ Hz}$



ET (Einstein Telescope) – Construction will start in 2026

- Ground-based, triangle-shaped interferometer
- 10 km arm length (LIGO-Virgo $\sim 3 \text{ km}$)
- Sensitivity band $\sim 1 \text{ Hz} - 10^4 \text{ Hz}$



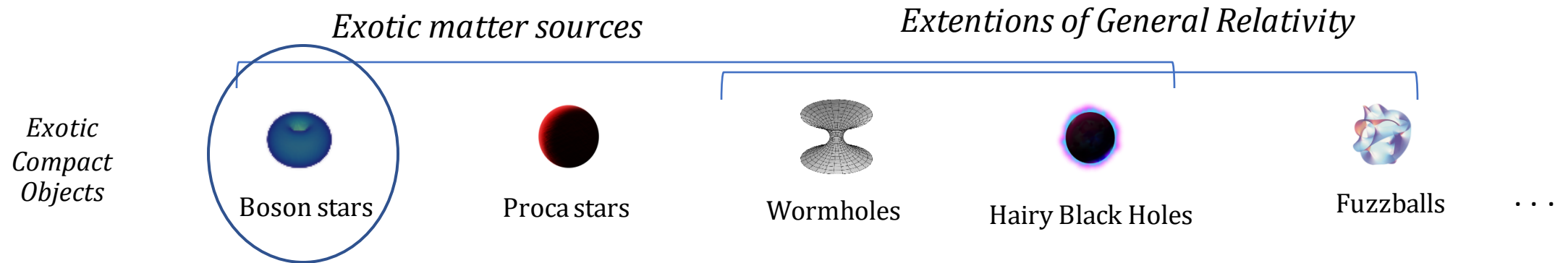
Detection of thousands of mergers of binary systems with a mass range from few to 10^7 solar masses
Opportunity to test the nature of compact objects with unprecedented accuracy

Exotic Compact Objects in the Universe?

- The actual paradigm is that an astrophysical compact object, which is heavier than few solar masses, is a Black Hole (BH).

Exotic Compact Object: An object which is neither a BH or a Neutron Star

- The observation of an ECO would imply some new physics:



*Astrophysical processes are insensitive to the geometry near the horizon: **we need Gravitational Waves!***

Properties of Boson Stars

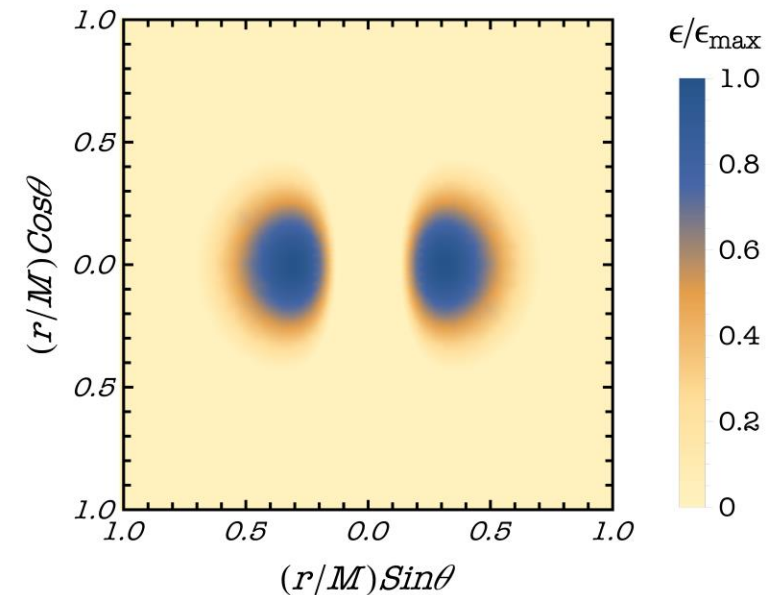
- Boson stars (BSs) are stationary configurations of a massive, complex scalar field, bound by gravity and could represent astrophysical objects which:
 - Can be almost as compact as Black Holes and can mimick their phenomenology
 - Are regular (no singularity at the center, no event horizon)
 - Could stand for some fraction of the dark matter content of the Universe
- Can be described in classical field theory:

$$G_{ab} = 8\pi T_{ab}, \quad \frac{1}{\sqrt{-g}} \partial_a (\sqrt{-g} g^{ab} \partial_b \phi) = \frac{dV(|\phi|^2)}{d|\phi|^2} \phi$$

Einstein equations

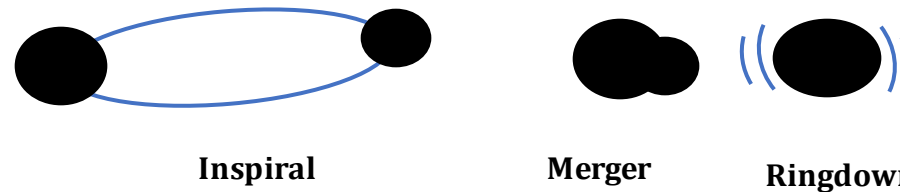
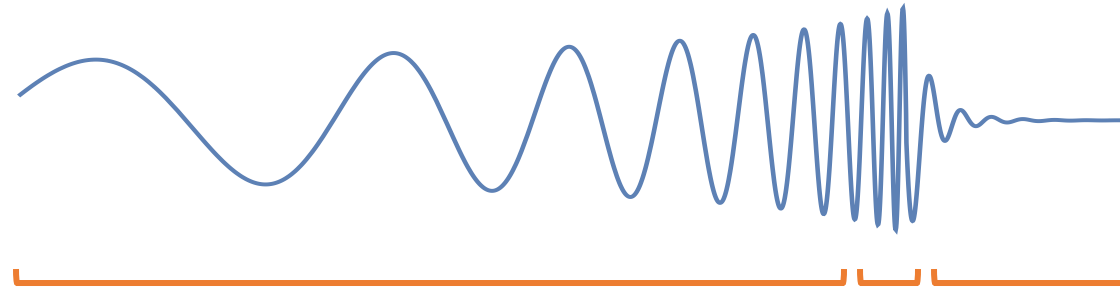
Klein-Gordon equation

Energy-density plot of a boson star



Stages of a binary coalescence

- The gravitational waveform from a binary coalescence can be thought as divided in three stages



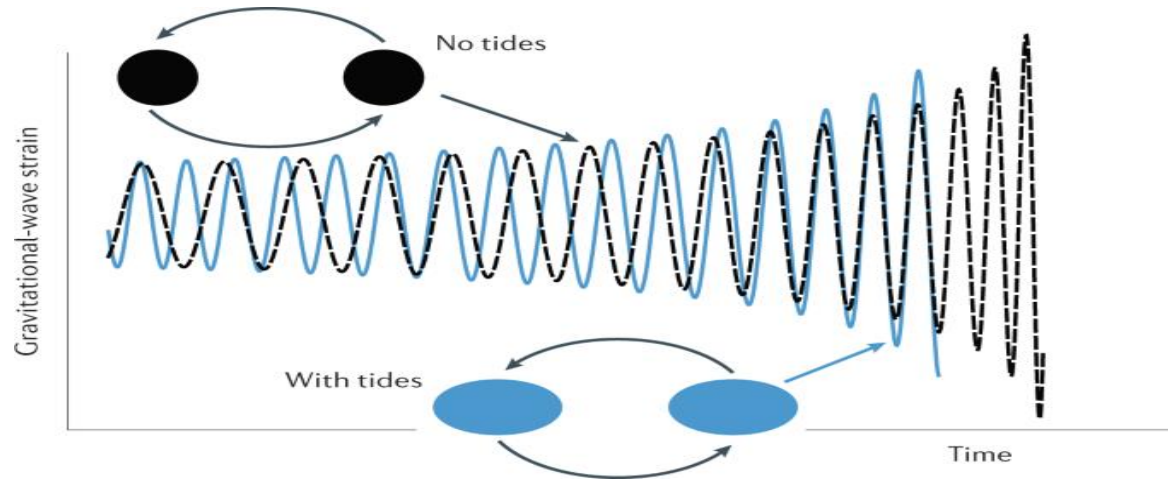
Post-Newtonian expansion in weak gravity and small velocities

Numerical Relativity 3+1 non-linear simulations

Description of the final remnant as an oscillating object with characteristic frequencies

A coherent inspiral waveform model

- Post-Newtonian (PN) expanded waveform in $x = \frac{1}{c^2} (\pi M f)^{\frac{2}{3}} \sim (v/c)^2$ including finite size effects:



$$h \sim \mathcal{A}(f) e^{i\psi(f)} \rightarrow \psi(f) = \psi_{pp}(f) + \psi_{tidal}(f) + \psi_{quad}(f) \propto x^{-\frac{5}{2}} \left(\sum_{n=0}^7 \alpha_n(f) x^{\frac{n}{2}} \right)$$

Point particle relative terms
 $\propto x^0 \dots x^{\frac{7}{2}}$ (0PN, 3.5PN)

C.K. Mishra et.al, Phys. Rev. D, 93, 8 (2016), 084054

Tidal corrections $\propto x^5, x^6$ (5PN, 6PN)

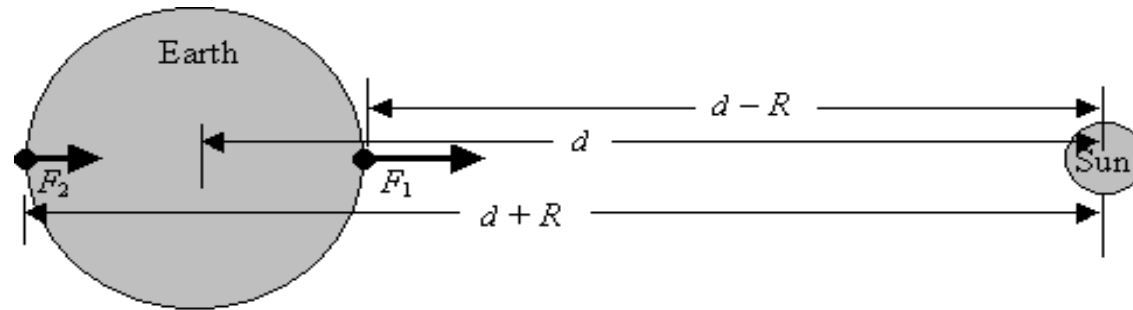
Lackey and L. Wade, Phys. Rev. D, 91, (2015) 4 043002

Quadrupole relative corrections
 $\propto x^2, x^3$ (2PN, 3PN)

Krishnendu et.al, Phys. Rev. Lett., 119, 9 (2017) 091101

Tidal deformability

- The presence of the companion induces a quadrupole moment in the star as response to the external tidal field:



$$F_2 = \frac{GMm}{(d+R)^2} \quad F_1 = \frac{GMm}{(d-R)^2} \quad \Delta F = F_2 - F_1 = \frac{GMm}{d^2} \left[\left(1 + \frac{R}{d}\right)^{-2} - \left(1 - \frac{R}{d}\right)^{-2} \right]$$

- The same happens in General Relativity and can be studied as a perturbation problem of the spherically symmetric background spacetime:

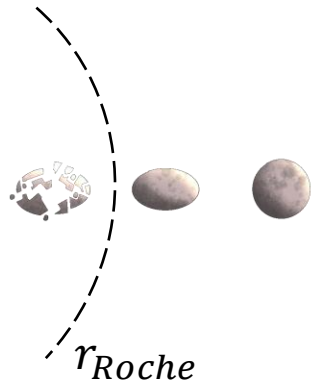
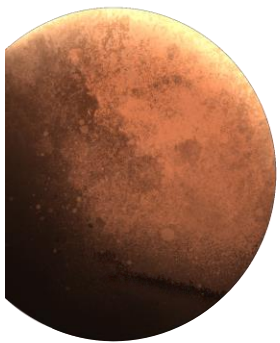
$$g_{00} = -1 + \frac{2M}{r} + \frac{3Q_{ij}}{r^3} \left(n_i n_j - \frac{\delta_{ij}}{3} \right) + O\left(\frac{1}{r^4}\right) - \varepsilon_{ij} x_i x_j + O(r^3)$$

$$Q_{ij} = -\lambda_T \varepsilon_{ij} \quad \lambda_T \text{ is the tidal deformability}$$

Tidal deformability of boson stars

- Dimensionless tidal $\Lambda = \lambda_T/M^5$ for BSs (*N. Sennett et al., Phys. Rev. D, 96, 2 (2017) 024002*)

$$\frac{M}{M_B} = \frac{\sqrt{2}}{8\sqrt{\pi}} \left[-0.828 + \frac{20.99}{\log \Lambda} - \frac{99.1}{(\log \Lambda)^2} + \frac{149.7}{(\log \Lambda)^3} \right] \longrightarrow \boxed{\Lambda = \Lambda\left(\frac{M}{M_B}\right)}$$



We fixed the maximum frequency of the waveform to that corresponding to the Roche radius:

$$f_{Roche} = \frac{c^3}{\pi G M M_{\odot}} \sqrt{\frac{1}{q^2} + q + 3 \left(1 + \frac{1}{q}\right) \left(\frac{C_2}{\gamma}\right)^{\frac{3}{2}}} \text{ Hz}$$

C_2 = Compactness of the secondary

γ = Numerical factor $\gamma \sim 2$

Multipole moments in General Relativity

- The Newtonian potential satisfy the Laplace equation $\nabla^2 V = 0$:

$$V(R, \theta, \varphi) = \sum_{k=0}^{\infty} \sum_{l=-k}^k \frac{M_{k,l} Y_k^l(\theta, \varphi)}{R^{k+1}} \xrightarrow{R \rightarrow 1/r} \tilde{V}(r, \theta, \varphi) = \sum_{k=0}^{\infty} \sum_{l=-k}^k M_{k,l} Y_k^l(\theta, \varphi) r^k$$

Consider now the Taylor expansion of \tilde{V} :
$$\tilde{V}(x^a) = \sum_{k=0}^{\infty} \frac{x^{a_1} \dots x^{a_k}}{k!} \partial_{a_1} \dots \partial_{a_k} \tilde{V} \Big|_{r=0}$$

- $Y_k^l(\theta, \varphi) r^k$ polinomial of degree k in $(x^1, x^2, x^3) = (r \sin \theta \cos \varphi, r \sin \theta \sin \varphi, r \cos \theta) \Rightarrow$

$$M_{k,l} \leftrightarrow \partial_{a_1} \dots \partial_{a_k} \tilde{V} \Big|_{r=0}$$

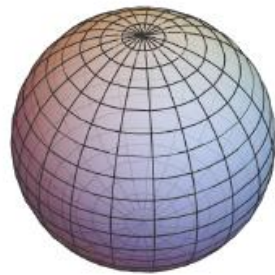
There is a correspondence between multipole moments and these quantities at infinity

- This definition have been generalized to stationary and asimptotically flat spacetimes in General Relativity (Geroch 1970, Hansen 1974).

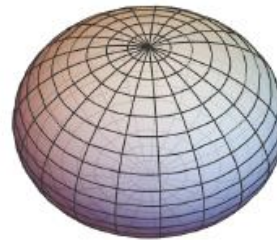
Multipole moments of Boson Stars

- Stationary **axysymmetric** spacetime \Rightarrow
 scalar mass moments M_0, M_2, \dots and current moments S_1, S_3, \dots

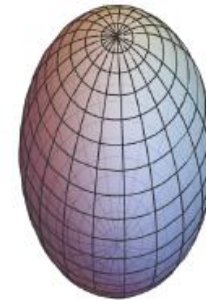
$M = M_0$
 $J = S_1$
 $Q = M_2$
 quadrupole
 moment
 ...



$$M_2 = 0$$



$$M_2 < 0$$



$$M_2 > 0$$

For a Kerr black hole

$$M_l + iS_l = M^{l+1} (i\chi)^l$$

$$\chi = \frac{J}{M^2},$$

Not true for a generic compact object!

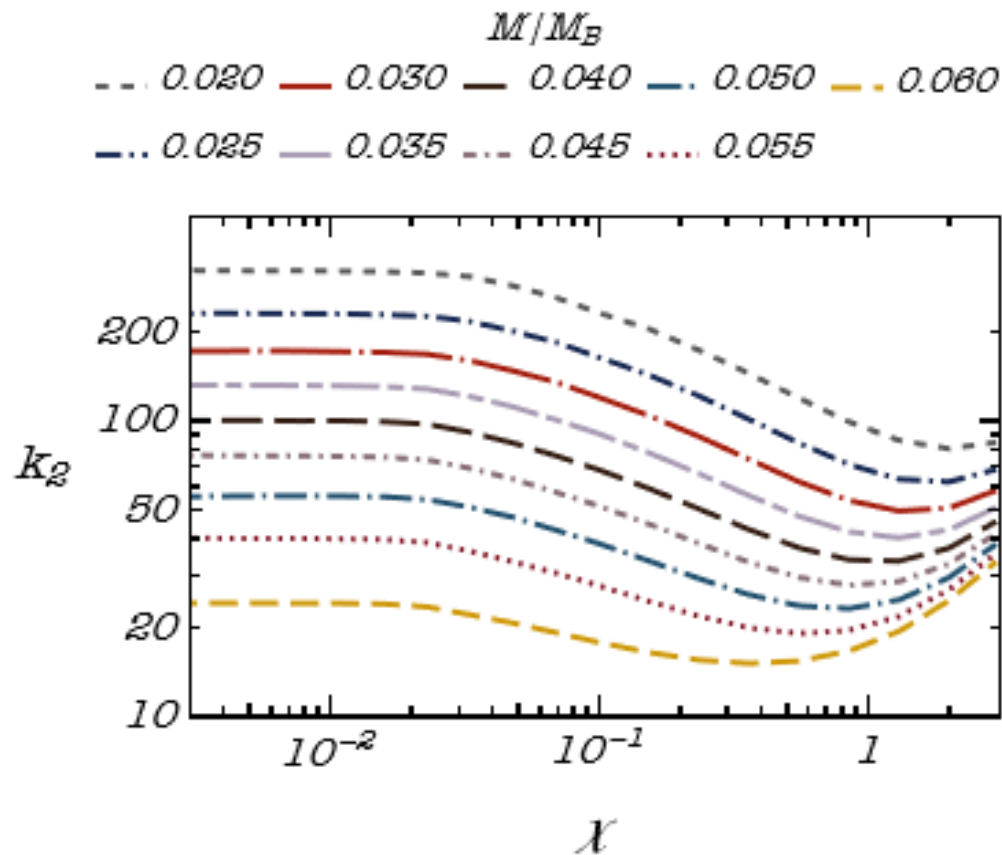
Multipole moments of Boson Stars

Kerr black hole

$$M_2 = -\chi^2 M^3$$

Boson Star

$$M_2 = -\kappa_2(\chi, M/M_B)\chi^2 M^3$$



$$\frac{\sqrt{\lambda\hbar}}{m_S^2} M_P^3$$

$$M_{max} \approx 0.06 M_B \approx 10^5 M_\odot \sqrt{\lambda\hbar} \left(\frac{\text{MeV}}{m_S}\right)^2$$

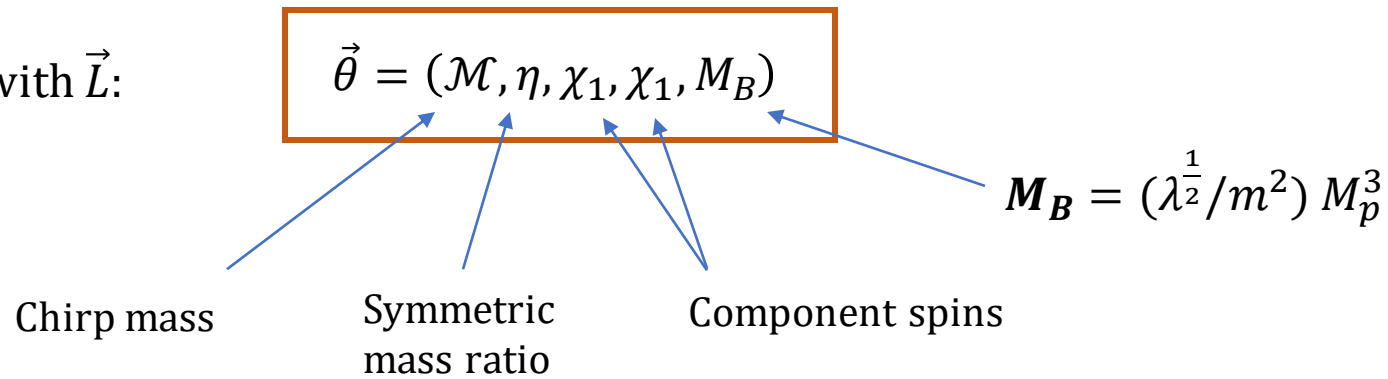
The multipolar structure of fast rotating boson stars: *M. Vaglio, C. Pacilio, A. Maselli, P. Pani, arXiv:2203.07442 (2022)*

Parameter estimation - Setting

- Bayesian parameter estimation on injected signals

$$\text{posterior } p(\vec{\theta}|d) = \frac{\text{prior } \pi(\vec{\theta}) \text{ likelihood } \mathcal{L}(d|\vec{\theta}, \mathcal{H})}{\text{evidence } \int d^m \theta \pi(\vec{\theta}) \mathcal{L}(d|\vec{\theta}, \mathcal{H})}$$

- Spins aligned with \vec{L} :



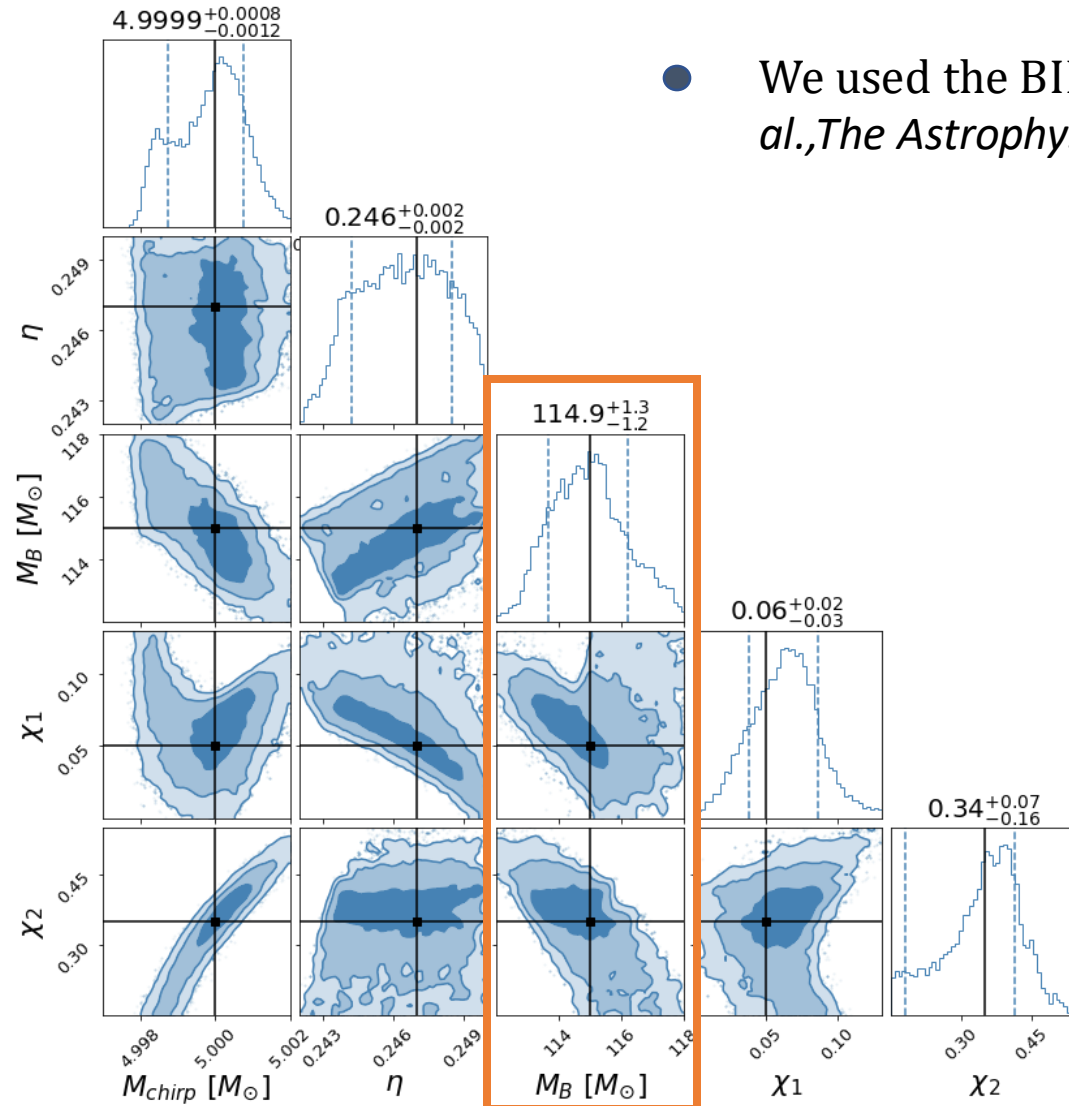
Fixed parameters:

ra, dec sky localization angles
ι system inclination angle
ψ wave polarization angle

Marginalized parameters:

dL Luminosity distance
t_c, φ_c time and phase at coalescence

Parameter estimation - Results

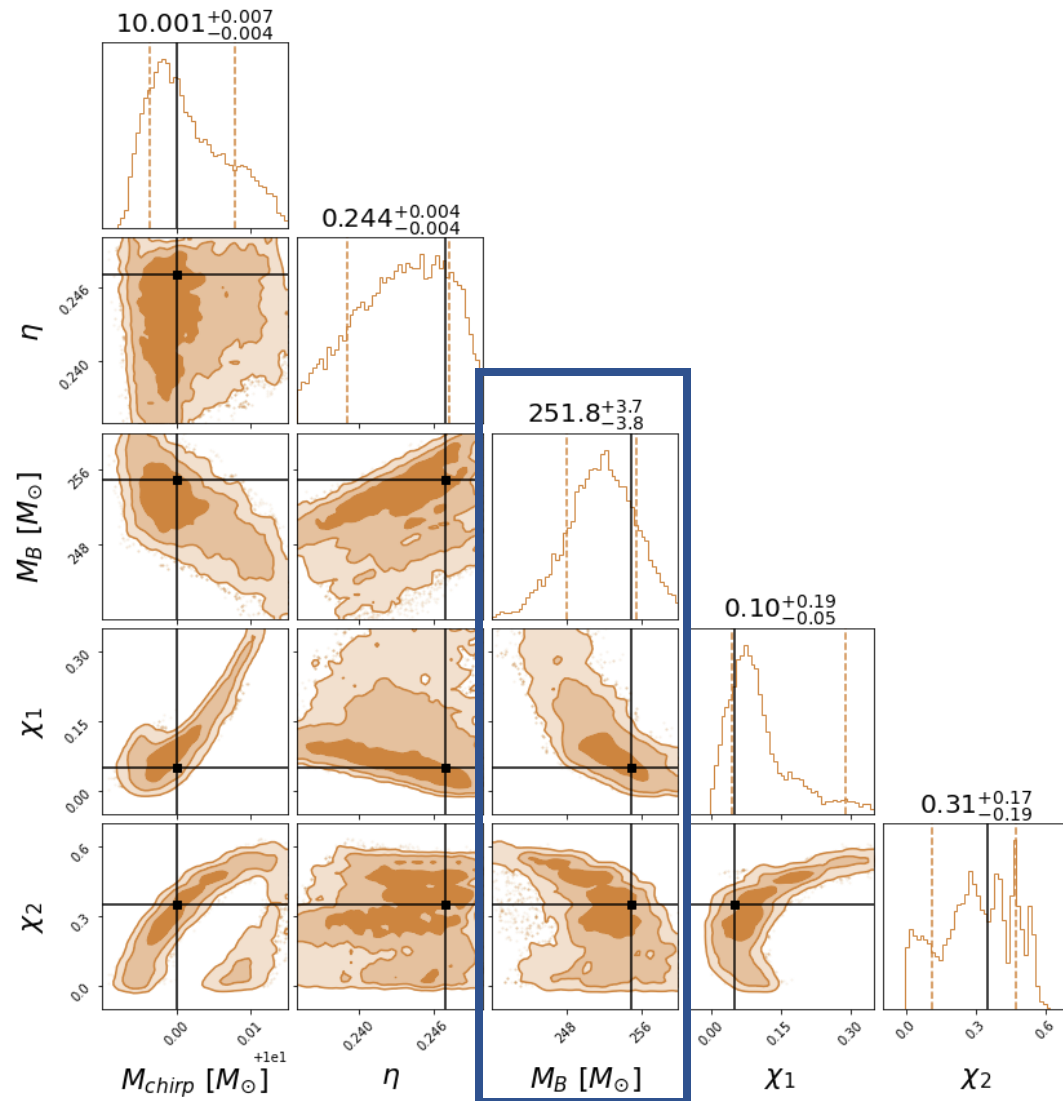


- We used the BILBY Bayesian Inference Library (Ashton, Gregory *et al.*, *The Astrophysical Journal Supplement Series*, voln.241,2019).

Injection and recovery of a signal with the Einstein Telescope (SNR = 130):

- $\mathcal{M} = 5M_{\odot}$
- $q = 0.8$
- $M_B = 115M_{\odot}$
- $\chi_1 = 0.05$
- $\chi_2 = 0.35$
- $f_{\text{Roche}} = 127\text{Hz}$

Parameter estimation - Results



Injection and recovery of a signal with the Einstein Telescope (SNR = 130):

- $\mathcal{M} = 10M_{\odot}$
- $q = 0.8$
- $M_B = 255M_{\odot}$
- $\chi_1 = 0.05$
- $\chi_2 = 0.35$
- $f_{\text{Roche}} = 50\text{Hz}$ ←

Conclusions and perspectives

We have seen how to develop a coherent waveform template for the inspiral of rotating self-interacting BSs taking finite size effects into account :

- We have characterized the tidal deformability and the quadrupole moment $M_2 \sim \chi^2$ of boson stars as smoking gun signatures from departures from the signal of a black hole binary
- With ET at SNR ~ 100 it is possible to constraint the fundamental couplings of the scalar theory with $\sim 1\%$ accuracy:

(m_1, m_2)	$\delta\mathcal{M}_{rel}$	$\delta\eta_{rel}$	δM_{Brel}	$\delta\chi_{1rel}$	$\delta\chi_{2rel}$
$(6.4, 5.2)M_\odot$	0.015%	0.8%	1.0%	48%	37%
$(12.8, 10.3)M_\odot$	0.05%	1.7%	1.5%	92%	43%

In agreement with F.M. analysis: *C. Pacilio, M. Vaglio, A. Maselli, P. Pani. Phys.Rev. D 102 (2020) 8, 083002*

Next steps and future works:

- Generalization to other BS's models: change $V(|\phi|^2)$, Vector bosonic stars, universal relations...
- Model selection between different potentials and boson star families

