Tracking and Vertexing

and the impact on detector design
part 1

XXXIV International School
“Francesco Romano”

September 22, 2023

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Outline

- **Design criteria**
  for particle physics experiments
  - momentum/energy reconstruction
  - tracking
  - particle identification
  - occupancy and rate
- **Tracking**
  - concepts and techniques
  - implications on detector design
- **Vertexing**
  - concepts and techniques
  - implications on detector design
- **Particle identification**
  - available methods
  - integration in experiments
- **Detector design and layout**
  - strategies
  - LHC upgrades as examples

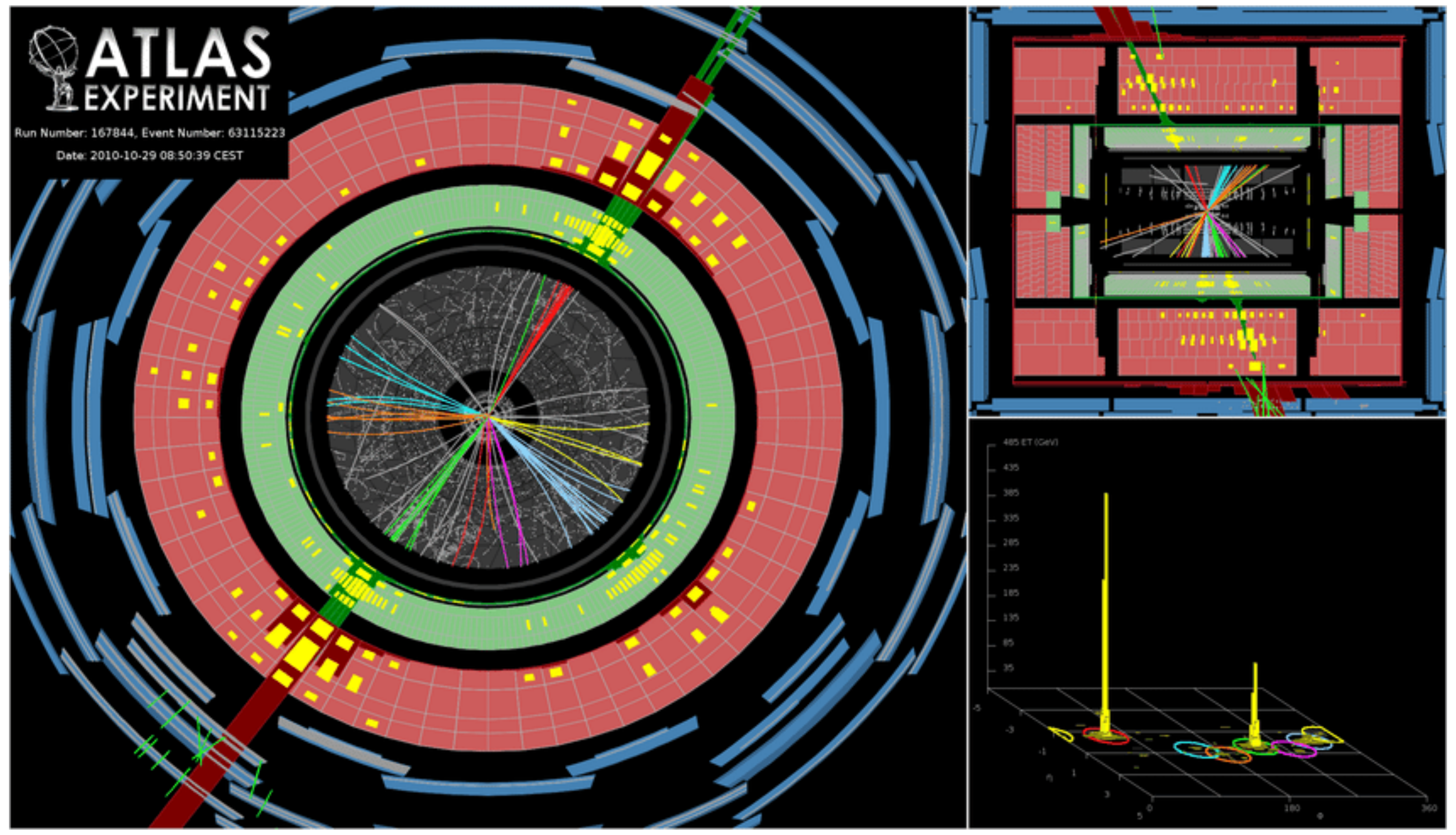
Experimental needs

- Experiments rely on sufficiently precise measurements of **properties of some or almost all particles in a reaction**, typically
  - momentum
  - energy
  - pointing towards primary vertex
  - mass → species
- **Priorities depend on physics objectives**
  → optimisation of detector various for different experiments

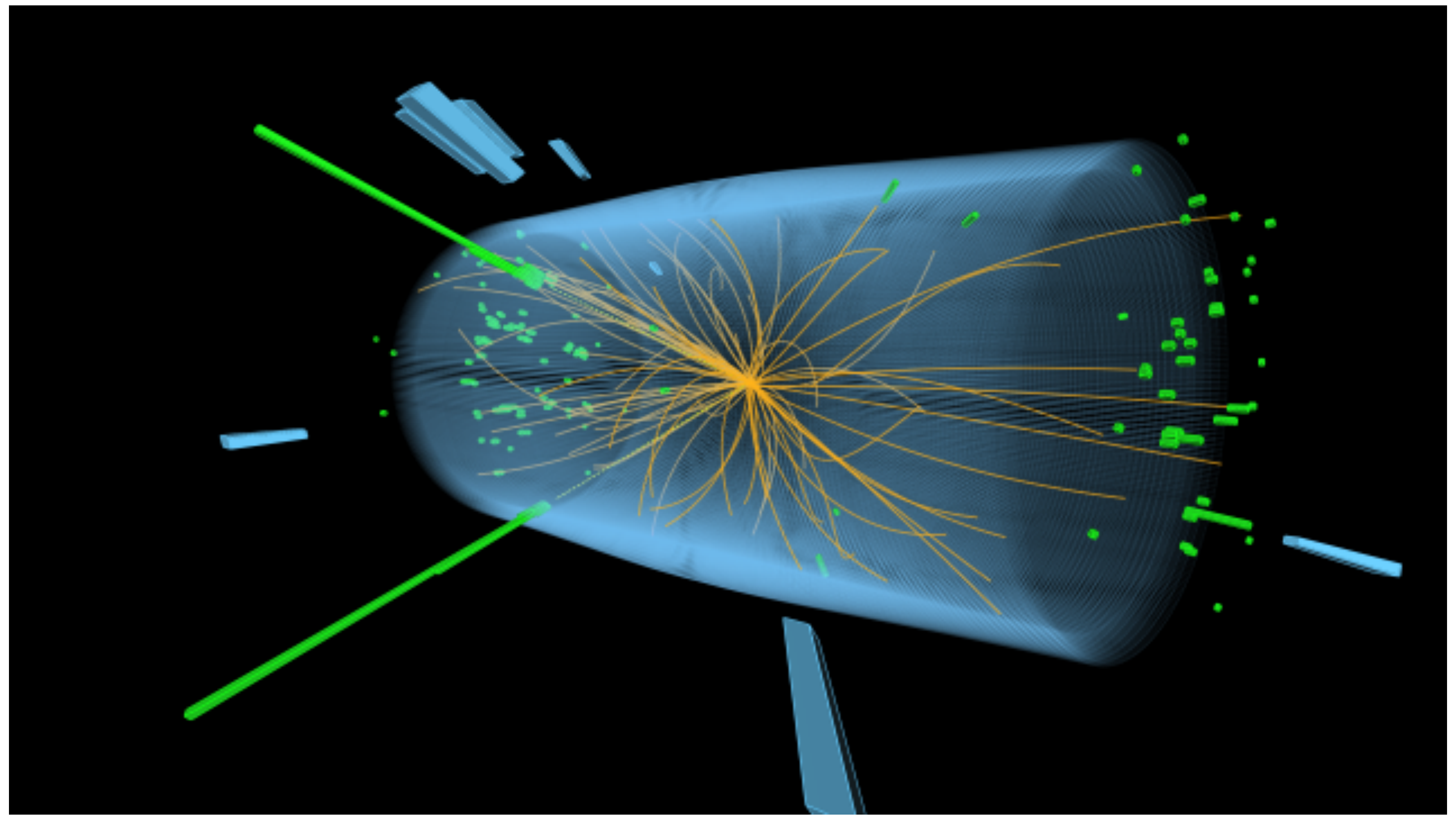
LHC experiments as examples

ATLAS/CMS

- **Physics priorities**
  - Higgs measurements
  - BSM searches
- **Analysis needs**
  - precise reconstruction of high-p_T probes and photons
  - sample large luminosities
  - identification of relevant signatures
- **Detector focus**
  - tracking and calorimetry
  - rate capability



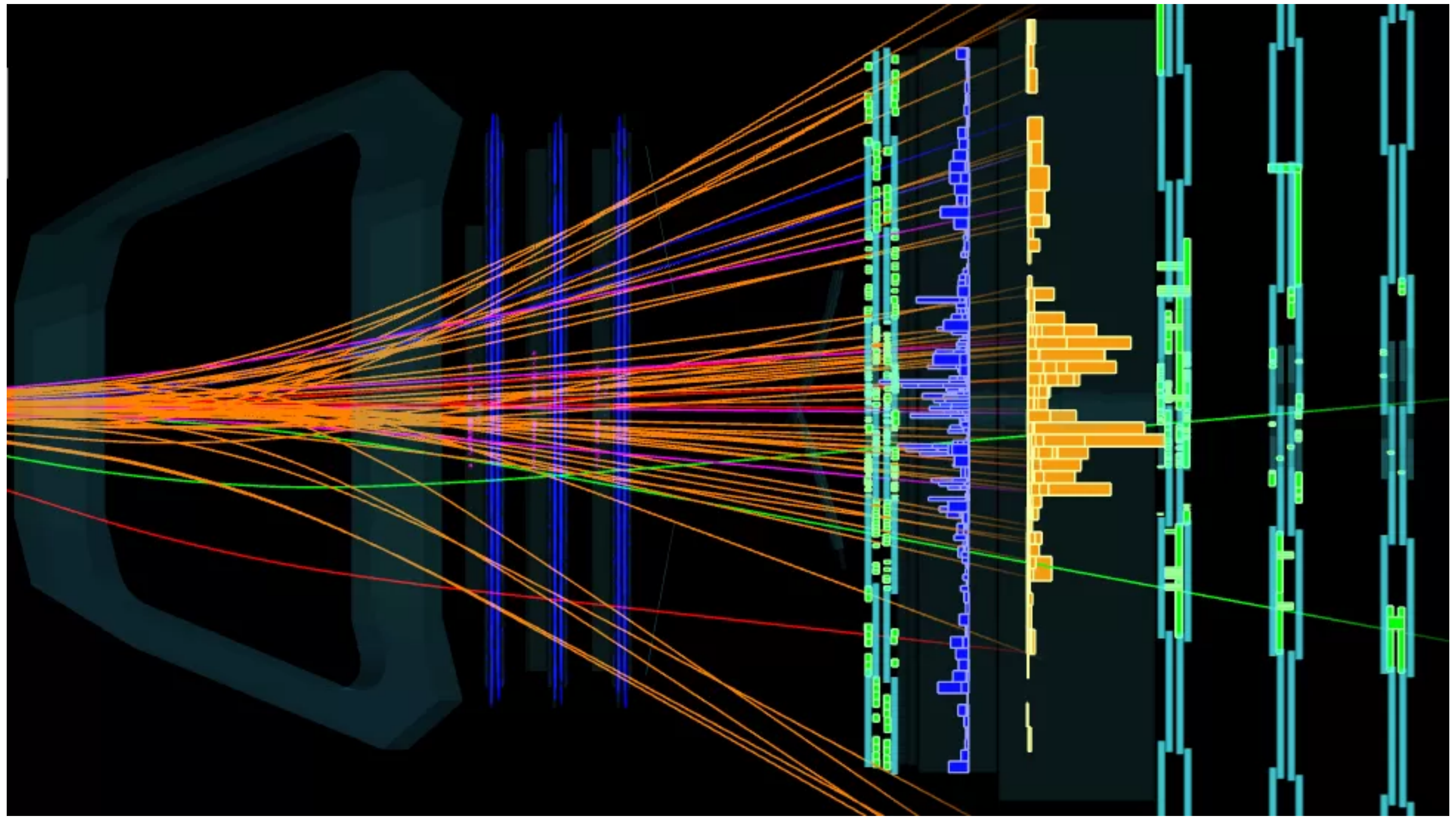
ATLAS



CMS

LHCb

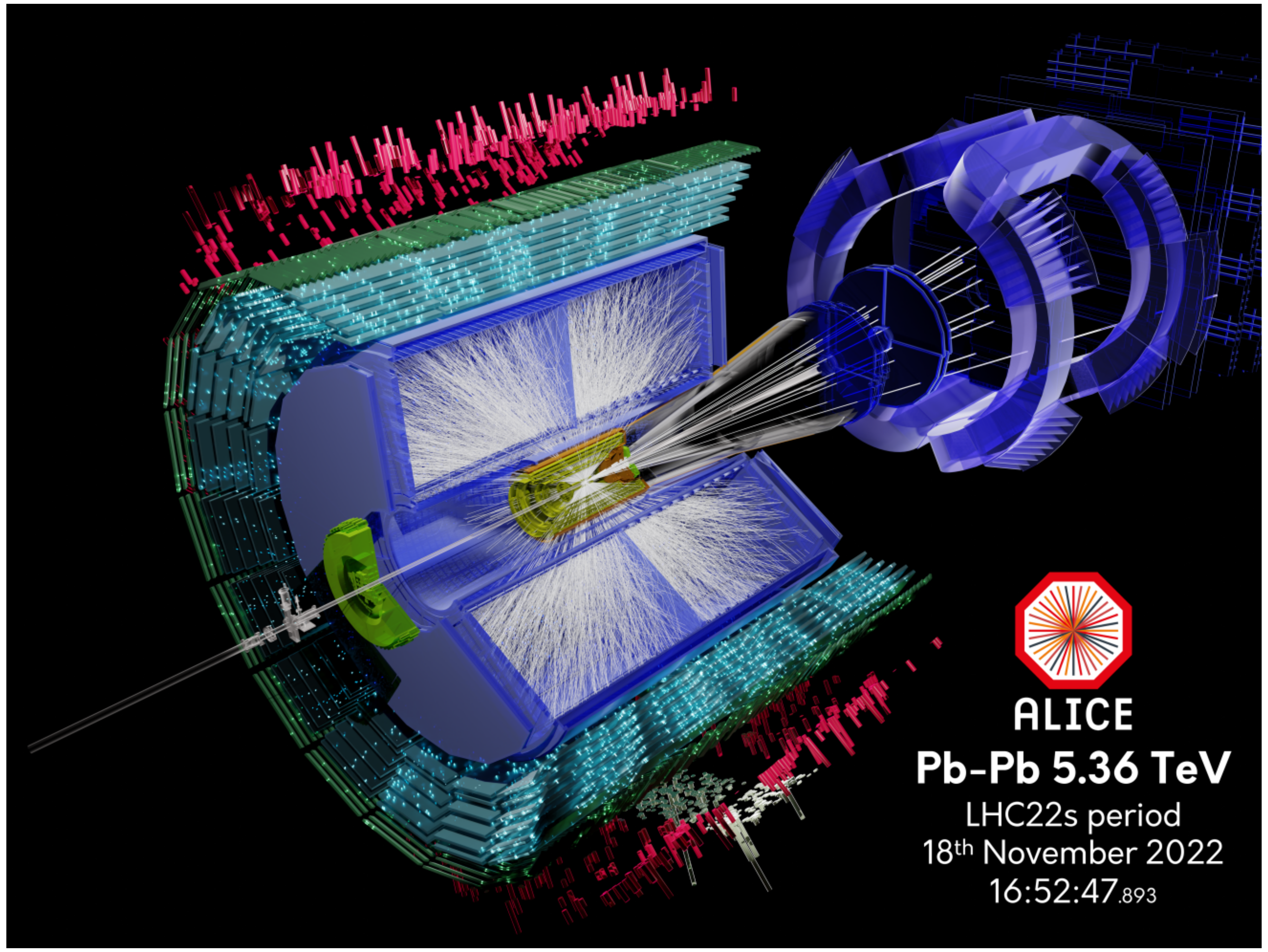
- **Physics priorities**
  - CP violation
  - rare decays in b physics
- **Analysis needs**
  - precise tracking
  - reconstruction of secondary vertices
  - particle identification
- **Detector focus**
  - vertexing
  - particle identification
  - forward coverage



LHCb

ALICE

- **Physics priorities**
  - physics of the quark-gluon plasma
  - study of strong interaction
- **Analysis needs**
  - measurement and identification of all particles (incl. yields)
  - complete reconstruction of events
- **Detector focus**
  - tracking down to low transverse momenta
  - vertexing
  - particle identification



ALICE

Relevant detector concepts

- **Tracking and vertexing** ← focus here
  from hits along particle trajectories (in magnetic field)
- **Particle identification** ← impacts design
  from a variety of general-purpose or specific techniques
  - specific energy loss
  - time-of-flight
  - calorimetry
  - muon systems
- **Energy measurement** (destructive) ← outside of trackers
  from electromagnetic and hadronic calorimetry

Tracking

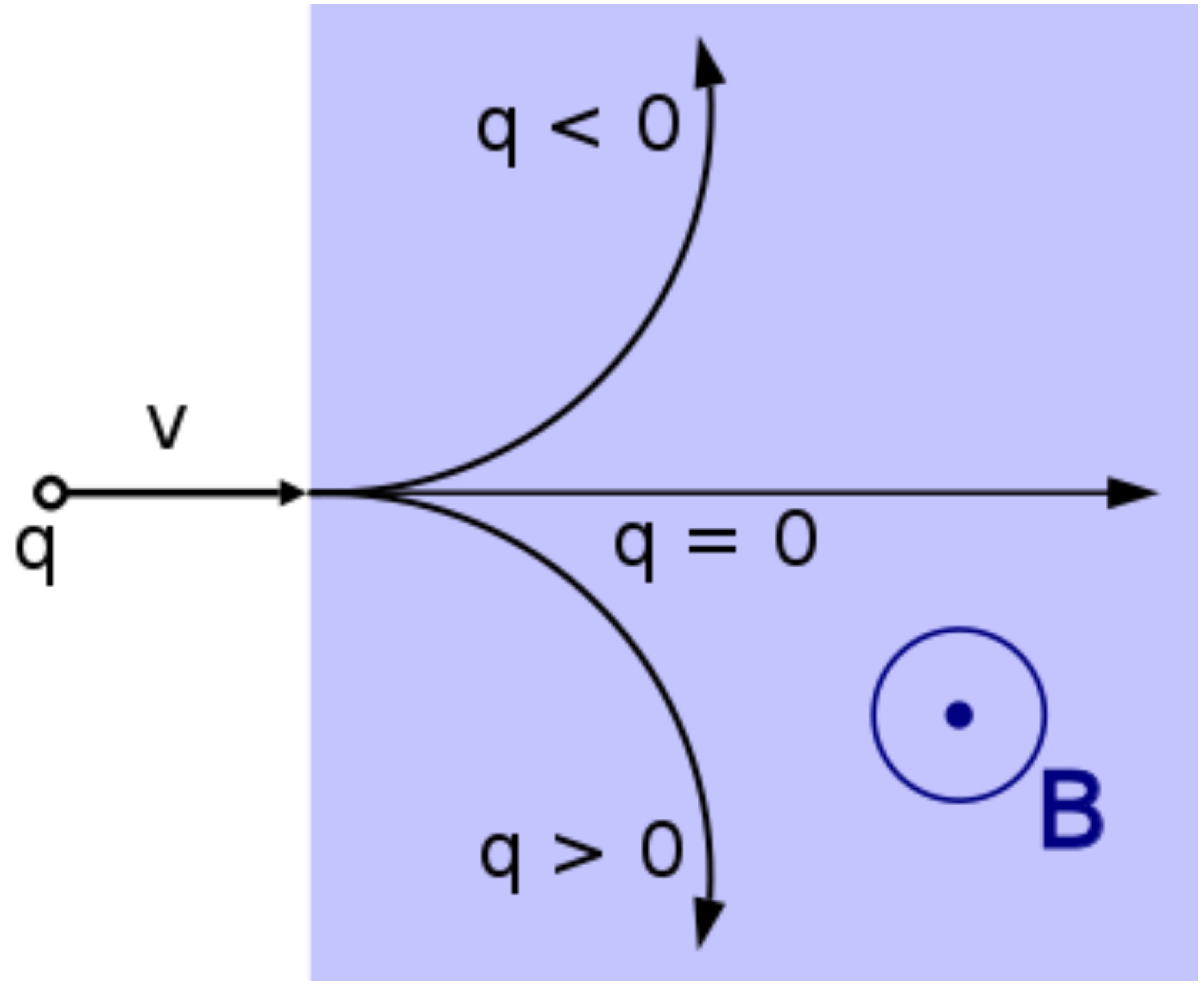
reconstruction of particle motion,
typically in magnetic fields

Motion of charged particles

- Motion of charged particle in a magnetic field determined by **Lorentz force**

$$\vec{F} = q\left(\vec{v} \times \vec{B}\right) \Rightarrow \dot{\vec{v}} = \frac{q}{\gamma m}\left(\vec{v} \times \vec{B}\right)$$

  - force perpendicular to direction of motion
    ➠ in magnetic field **only change of direction**
    ➠ no change of energy ➠ γ constant
  - **valid even relativistically**
    (despite non-covariant formulation)



Helix motion in homogeneous field

- Charged particle in homogeneous magnetic field
  → **helix motion** coaxial with magnetic field (in *z* direction)

$$x = \frac{v_\mathrm{T}}{\eta\omega_B}\sin(\eta\omega_B t + \psi_0) + x_0 \qquad y = \frac{v_\mathrm{T}}{\eta\omega_B}\cos(\eta\omega_B t + \psi_0) + y_0 \qquad z = v_3 t + z_0$$

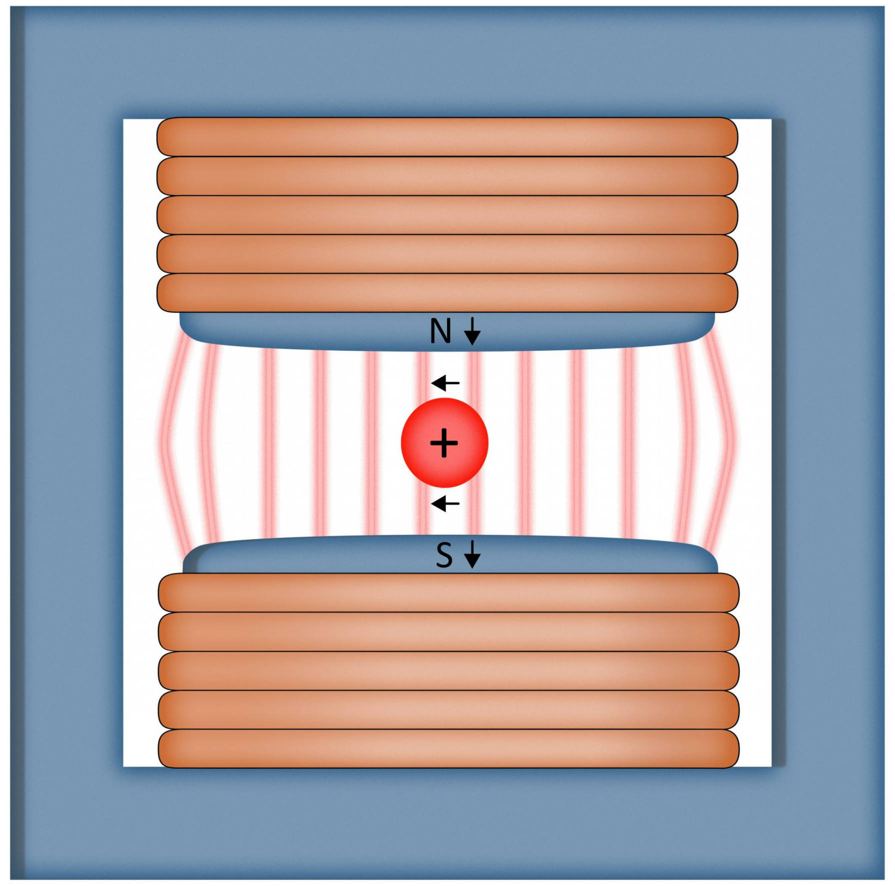
with cyclotron frequency $\omega_B = \dfrac{|q|B}{\gamma m}$ and $\eta = q/|q|$

- **radius of curvature** given by $p_\perp = |q|\,B\,R$,
  when using standard units (GeV/*c*, T, m) ➟ $p_\perp = 0.3\,B\,R$

1 GeV/c particle in 1 T field
→ ~3 m radius

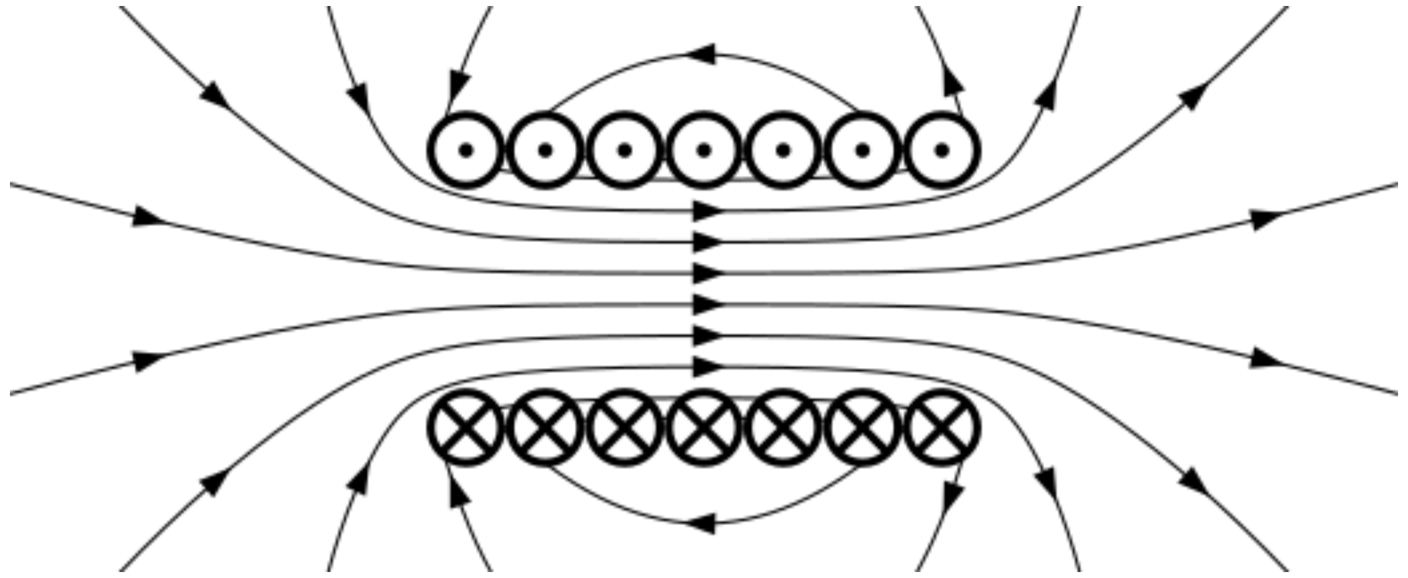
curvature in magnetic field
directly linked to (transverse) momentum

Magnetic field configurations

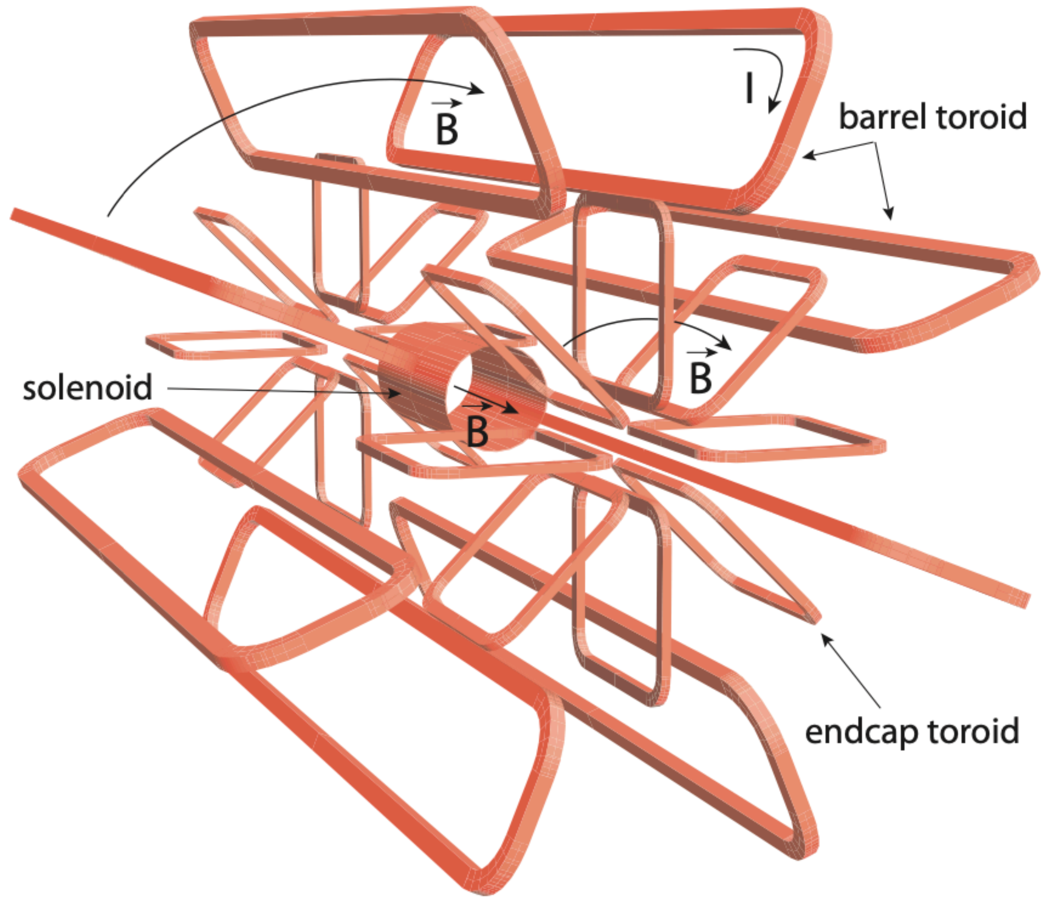


- **Dipole** fields
  - often used in fixed-target experiments or forward part of collider experiments

- **Solenoidal** fields
  - often used in collider experiments (rotationally symmetric)



- **Toroidal** fields
  - often used as extension of solenoidal fields (perpendicular, rotationally symmetric)

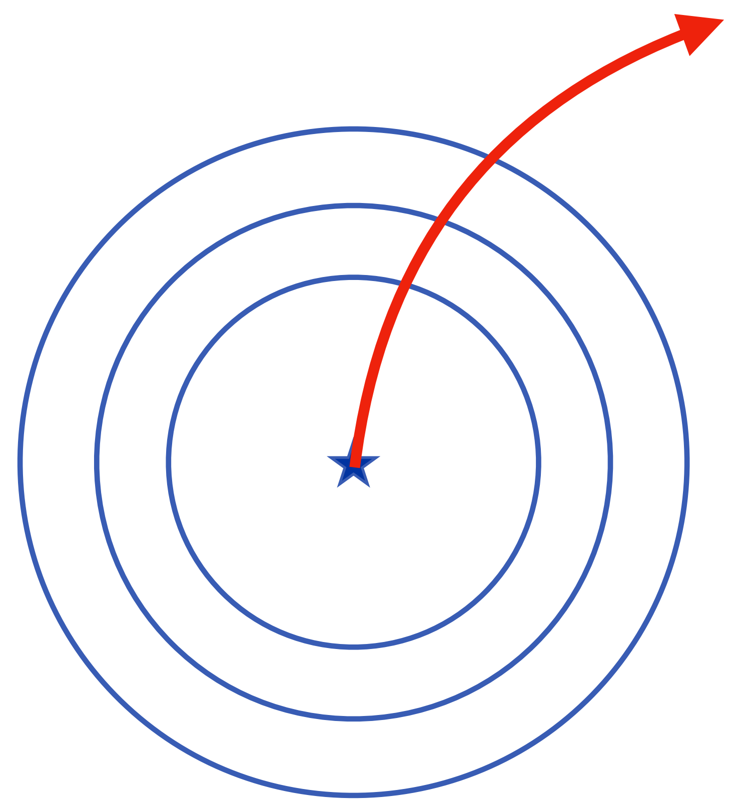


Deflection depends on motion perpendicular to magnetic fields

Tracking

- **Reconstruction of trajectory** of a particle through the detector with the goal of determining properties of the particle, in particular momentum
  - connection of hits in one or more detectors (originating from the same particle)
    ➟ **track finding**
  - fitting of track parameters from hits (taking into account interaction with material and magnetic fields)
    ➟ **track fitting**

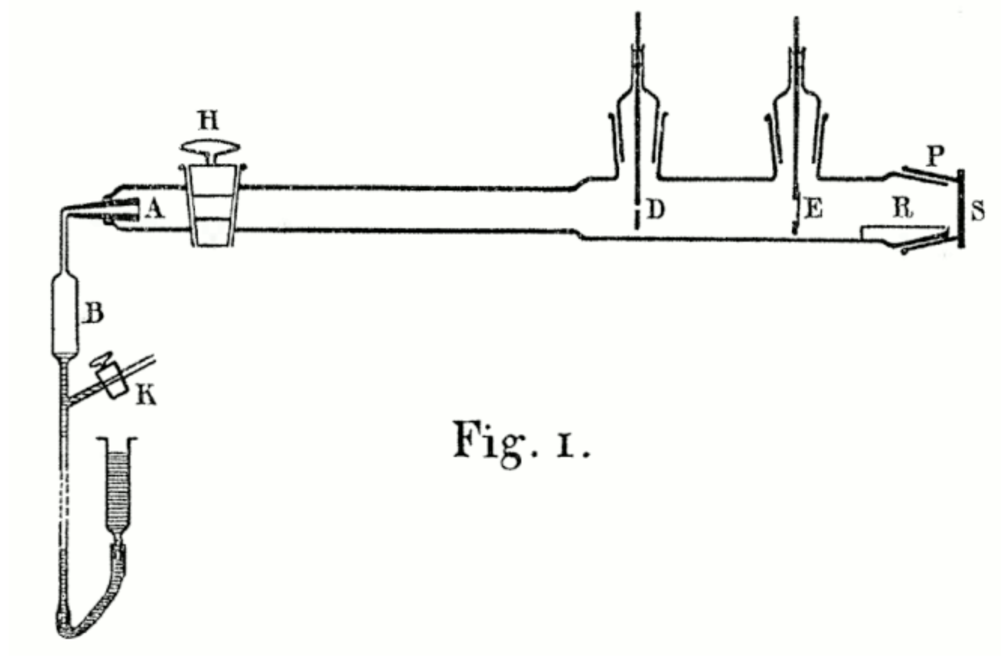
- **Requirements for tracking detectors**
  - 3 space points fully describe trajectory (under ideal conditions)
  - additional measurements can reduce uncertainties
  - need for redundancy and suppression of fake tracks (occupancy)



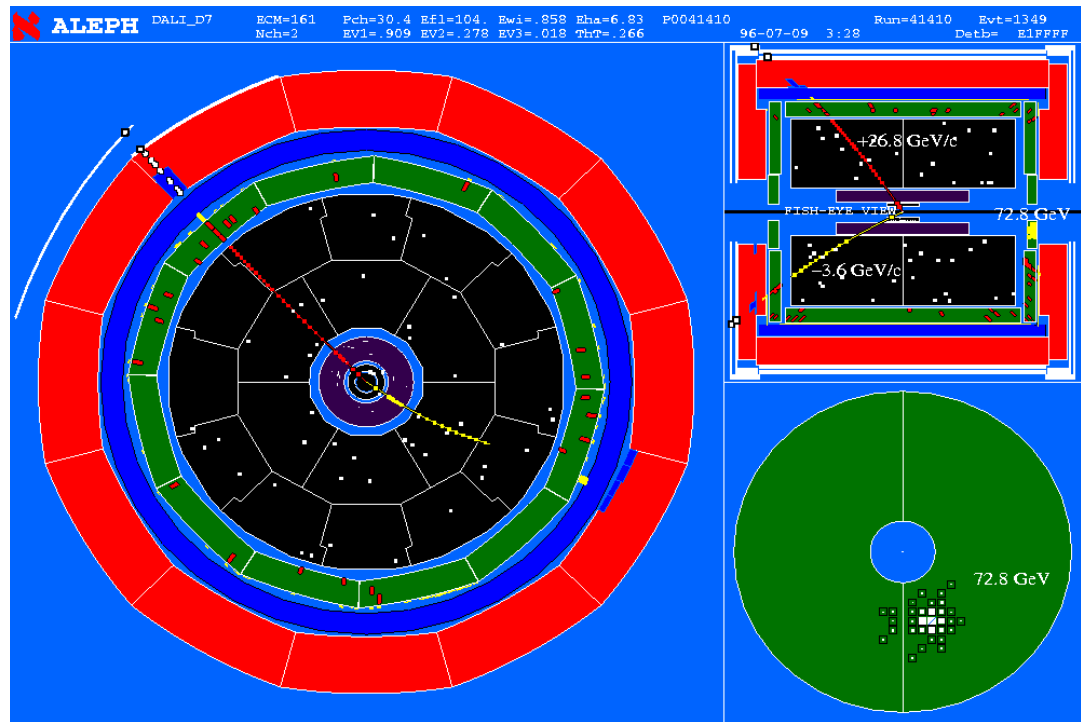
Trajectory through cylindrical layers

Evolution of experiments

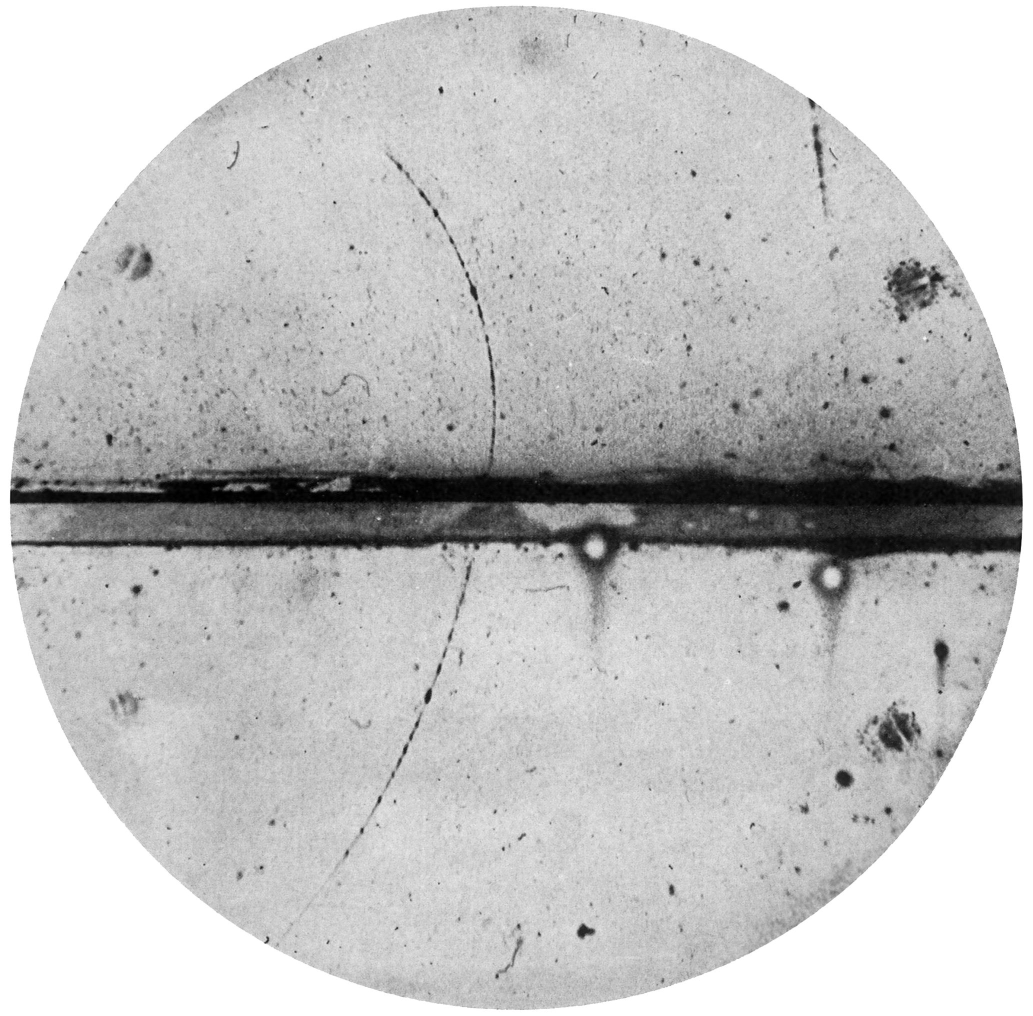
- **Scintillating screens**
  → counting experiments
- **Emulsions** and **cloud/bubble chambers**
  → imaging experiments
- **Modern experiments**
  → electronic readout and computer-based reconstruction of trajectories



Rutherford experiment (1909)



ALEPH @ LEP (1982 - 2000)



Discovery of positron
in cloud chamber (1932)

Criteria for tracking performance

- **Ideal reconstruction**
  - **assignment of all hits to correct track**
    → all tracks are reconstructed, no impact from wrongly assigned hits
  - **precise estimation of track parameters**
    → unbiased, minimal uncertainties
- **Figures of merit**
  - acceptance → fraction of area covered by detector
  - efficiency → fraction of reconstructed tracks
  - fake hit probability → probability of assigning hits from other particles
  - extrapolation uncertainties → propagation beyond volume covered by tracker
  - momentum resolution

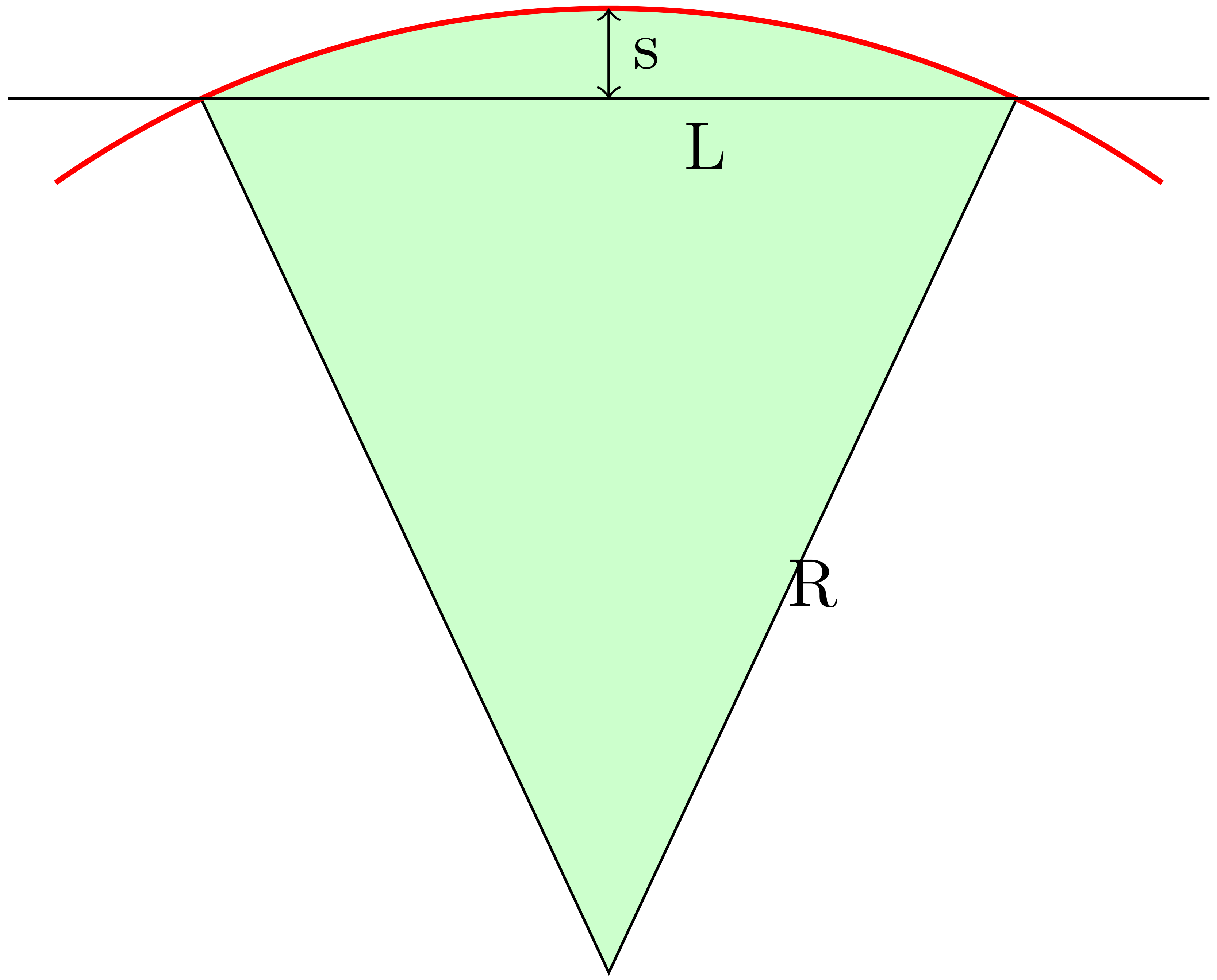
Sagitta

- Momentum measurements boils down to **measurement of curvature** in magnetic field
  → extraction from bubble chamber images

- **Sagitta** → maximum deviation
  from line between endpoints of track segment

$$s = R - R\cos\frac{\vartheta}{2} = 2R\sin^2\frac{\vartheta}{4} \approx \frac{R\vartheta^2}{8} \approx \frac{L^2}{8R} = \frac{qBL^2}{8p_\perp}$$

  - **quadratic dependence on lever arm**
  - relative uncertainty of momentum proportional to relative uncertainty of sagitta



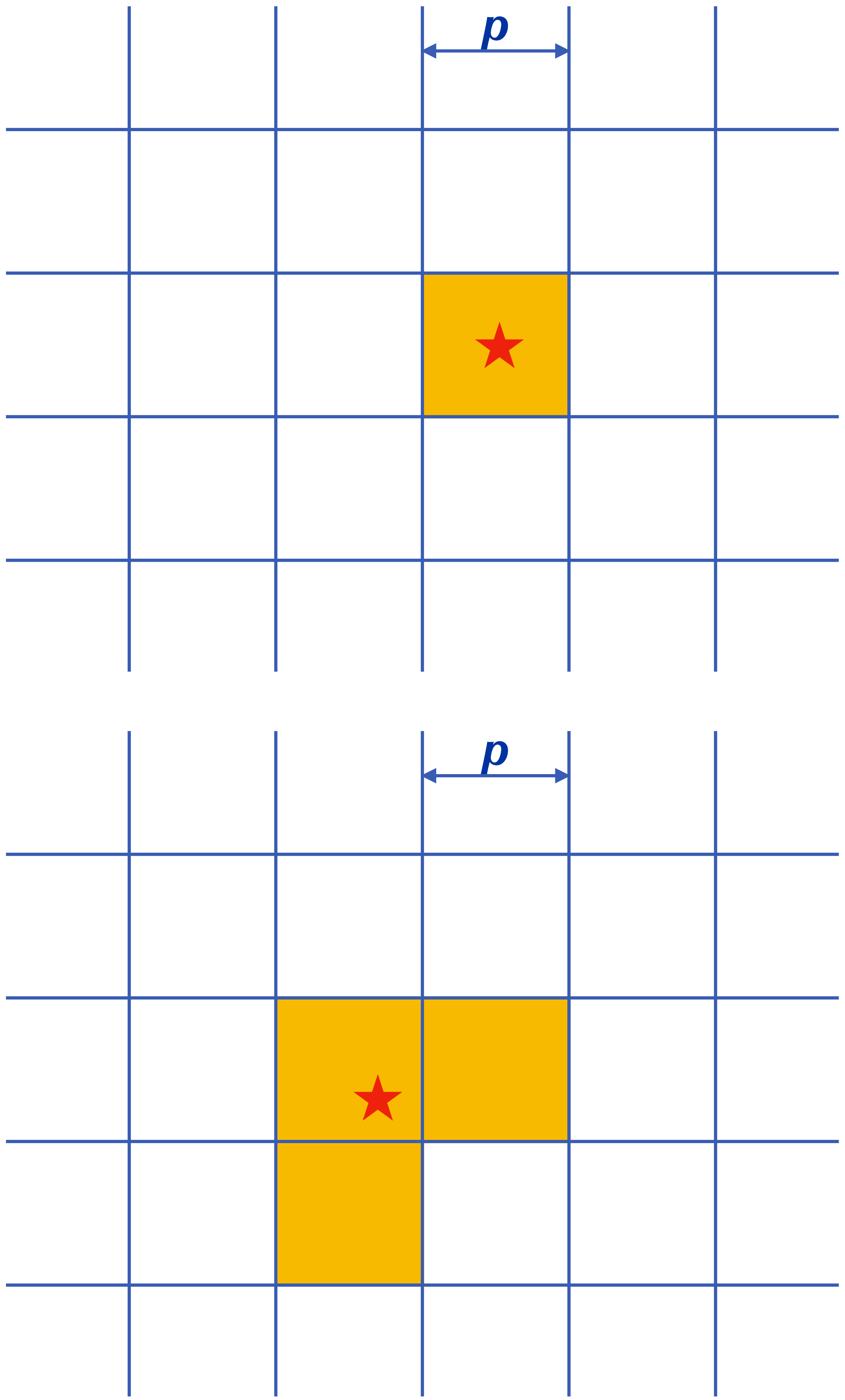
geometric approach, next consider measurement of points along trajectory

Position resolution

- Measurement of hit position
  limited by detector resolution
  - uncertainty for rectangular structure with **binary readout pitch p**

    $$\sigma_{\text{pos}}^2 = \int_{-p/2}^{p/2} \mathrm{d}x\, x^2 \frac{1}{p} = \frac{p^2}{12} \Rightarrow \sigma_{\text{pos}} = \frac{p}{\sqrt{12}}$$

  - uncertainty reduced when signal spreads across multiple bins (**charge sharing**), possibly with information on charge per bin
  - gas detectors typically $\mathcal{O}(100\ \mu\text{m})$, semiconductor detectors typically $\mathcal{O}(10\ \mu\text{m})$



Multiple scattering

- Material (incl. sensitive volume!) leads to multiple scattering
  → **random change of direction** on top of curvature in magnetic field
  - width of distribution

$$\sigma_\alpha = \frac{0.0136\,\mathrm{GeV}/c}{\beta p}\sqrt{\frac{d}{X_0}}$$

    - **inversely proportional to momentum**
    - **scales with square root of material thickness**

- Multiple scattering poses **fundamental limit on measurement precision**
  → cannot be mitigated by improved position resolution
  - angular effect, i.e. $\sigma_x \propto \sigma_\alpha \cdot \Delta x$

[R.L. Workman et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2022, 083C01 (2022)]

Track fit

- **Global χ^2 minimisation** of parameterised function f(x) for measured points (x_i, y_i)
  - assume linear dependency on parameters, e.g.
    - straight line
      → $y_i = f(x_i) = a_0 + a_1 x_i$
    - parabola (as approximation to circular shape)
      → $y_i = f(x_i) = a_0 + a_1 x_i + a_2 x_i^2/2$ $(a_2 = 1/R)$
  - **find parameters *a* to minimise**
    $\chi^2 = (y - Ga)^T W (y - Ga)$ with $G := x_i^j$ and weights W
    (W is identity matrix for equal weights)
  - minimum achieved for **solution of normal equation**
    $G^T W G a = G^T W y$
    solvable by matrix inversion or as system of linear equations (numerically more stable)

Tracking uncertainties

- Uncertainties of measurements described by **covariance matrix**
  $C_y = E((y - Ey)(y - Ey)^T)$

- **Propagation of uncertainties** to parameters a
  $C_a = BC_yB^T$ with $B = (G^TWG)^{-1}G^TW$ (from previous slide)

- **Optimal choice of W**
  to achieve unbiased parameter estimation with minimal uncertainties
  $W = C_y^{-1}$

- **Uncertainties for ideal weights** are
  $C_a = (G^TC_y^{-1}G)^{-1}$

Momentum resolution

- Transverse momentum extracted from **curvature in magnetic field**: $p_\perp = |q|\,B\,R$
  - approximated circle by parabola $f(x_i) = a_0 + a_1\,x_i + a_2\,x_i^2/2$ with $a_2 = 1/R$
  - momentum resolution proportional to uncertainty of curvature:
    ➟ $\dfrac{\Delta p_T}{p_T} = \dfrac{\Delta R}{R} = \dfrac{\Delta a_2}{a_2} = \dfrac{p_T}{|q|B}\Delta a_2$

- **Position resolution** of N+1 equidistant layers leads to uncertainty

$$\frac{\Delta p_T}{p_T} = \frac{\sigma\, p_T}{|q|BL^2}\sqrt{\frac{720N^3}{(N-1)(N+1)(N+2)(N+3)}} \approx \frac{\sigma\, p_T}{|q|BL^2}\sqrt{\frac{720}{N+4}}$$

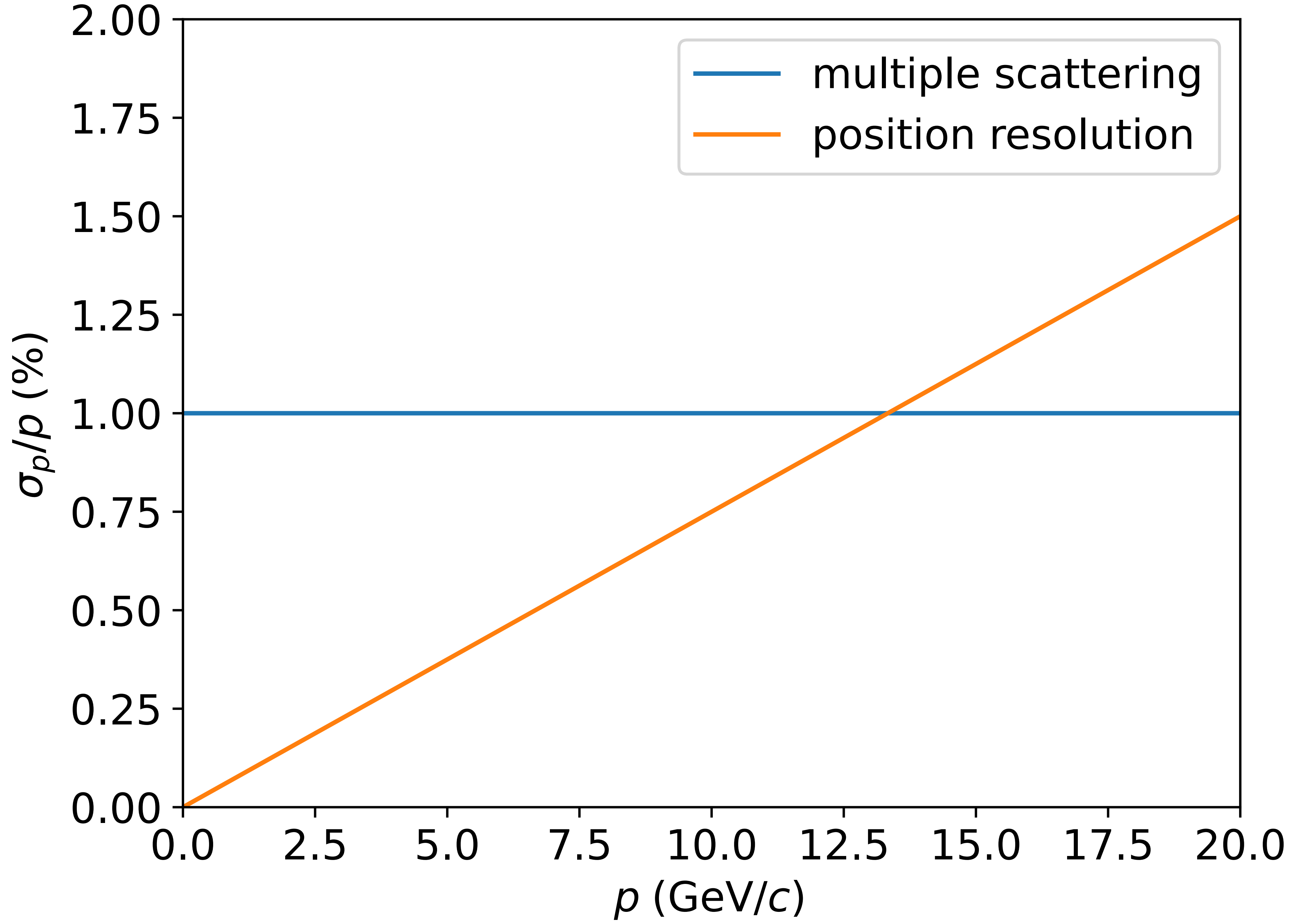
(Glückstern formula, NIM 24 (1963) 381)

- **Multiple scattering** in N+1 equidistant layers leads to uncertainty

$$\frac{\Delta p_T}{p_T} = \frac{N}{\sqrt{(N+1)(N-1)}}\frac{p\sigma_\alpha\sqrt{N+1}}{BL} = \frac{N}{\sqrt{(N+1)(N-1)}}\frac{p\,0.0136\,\mathrm{GeV}/c\,\sqrt{d_{tot}/X_0}}{\beta BL}$$

Considerations for detector layout

- **Multiple scattering** contributes
  $$\propto \frac{\sqrt{d_{\mathrm{tot}}/X_0}\,\cosh\eta}{\beta B L}$$
  - independent of momentum (for $\beta \approx 1$)
  - linear dependence on lever arm
  - scales with square root of material
- **Position resolution** contributes
  $$\propto \frac{\sigma p_{\mathrm{T}}}{B L^2}$$
  - linear rise with momentum
  - quadratic dependence on lever arm



m.s. sets irreducible limit, p.r. takes over at large momenta

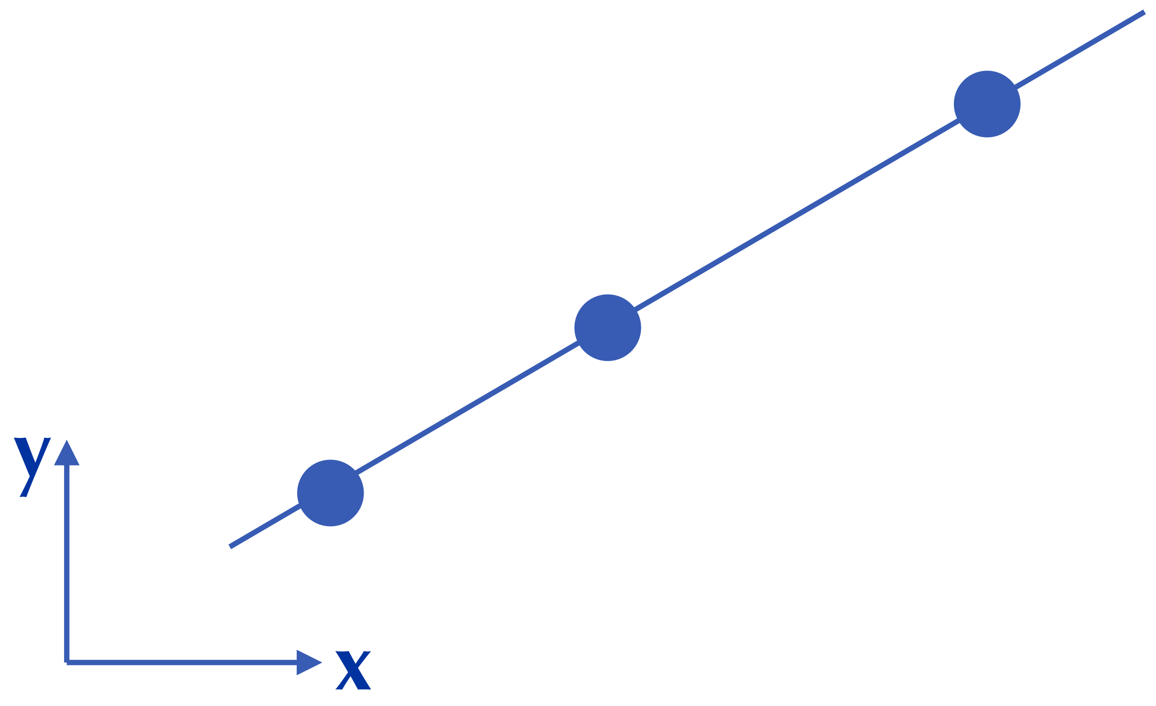
Track finding

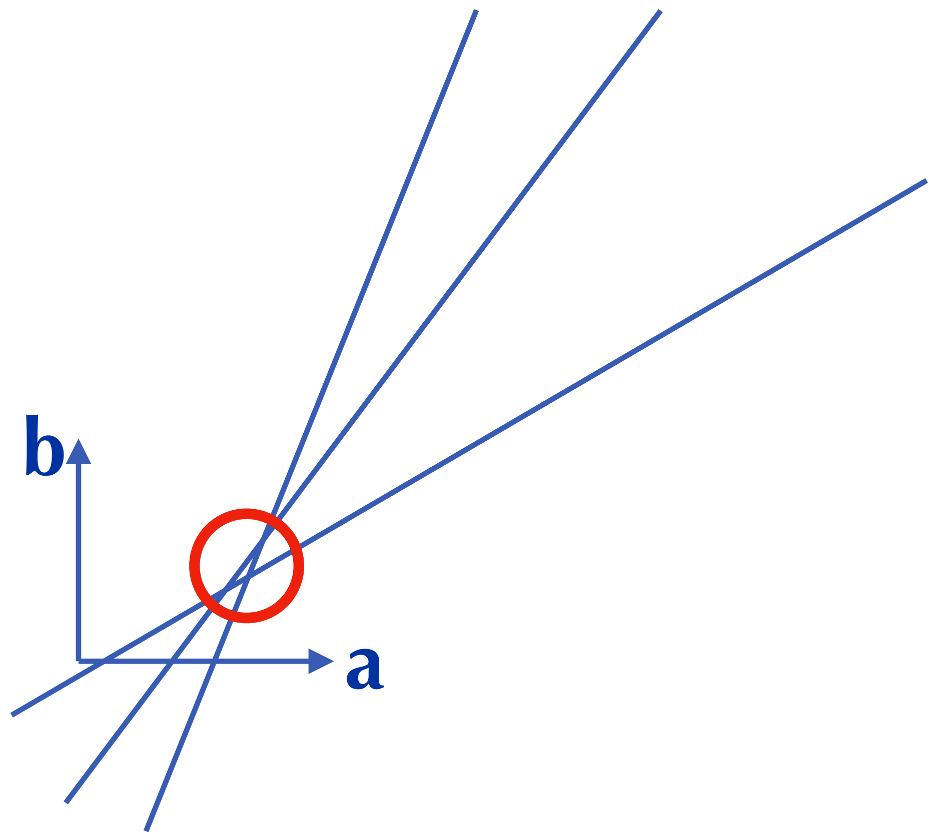
- So far track fit assuming knowledge of contributing hits
  → in reality need to **identify hit-to-track association from data**
  → task of **pattern recognition**

- **Variety of methods available** (and used),
  usability often determined by computational effort
  - fit all combinations (and reject based on χ^2)
  - Hough transformation
  - cellular automaton
  - Kalman filter
  - machine learning

Hough transformation

- Idea: **transformation from space coordinates to parameter coordinates**,
  e.g. describing a straight line y = ax + b
  - every hit (x, y) transforms into a **line in the parameter space**
  - **parameter lines intersect** in a single point (ideal case) or cluster around a value (with uncertainties)
  - **accumulation point** in the parameter space transforms into a line in real space
- Can be generalised to more complex parameterisations





Cellular automaton

- **Connect hits from adjacent layers** within a search window
  → tree of connections
  - combine track segments
  - longest paths are candidates
  - select candidates based on track fit
- **Advantages**
  - local matching of hits
    → contain computational effort

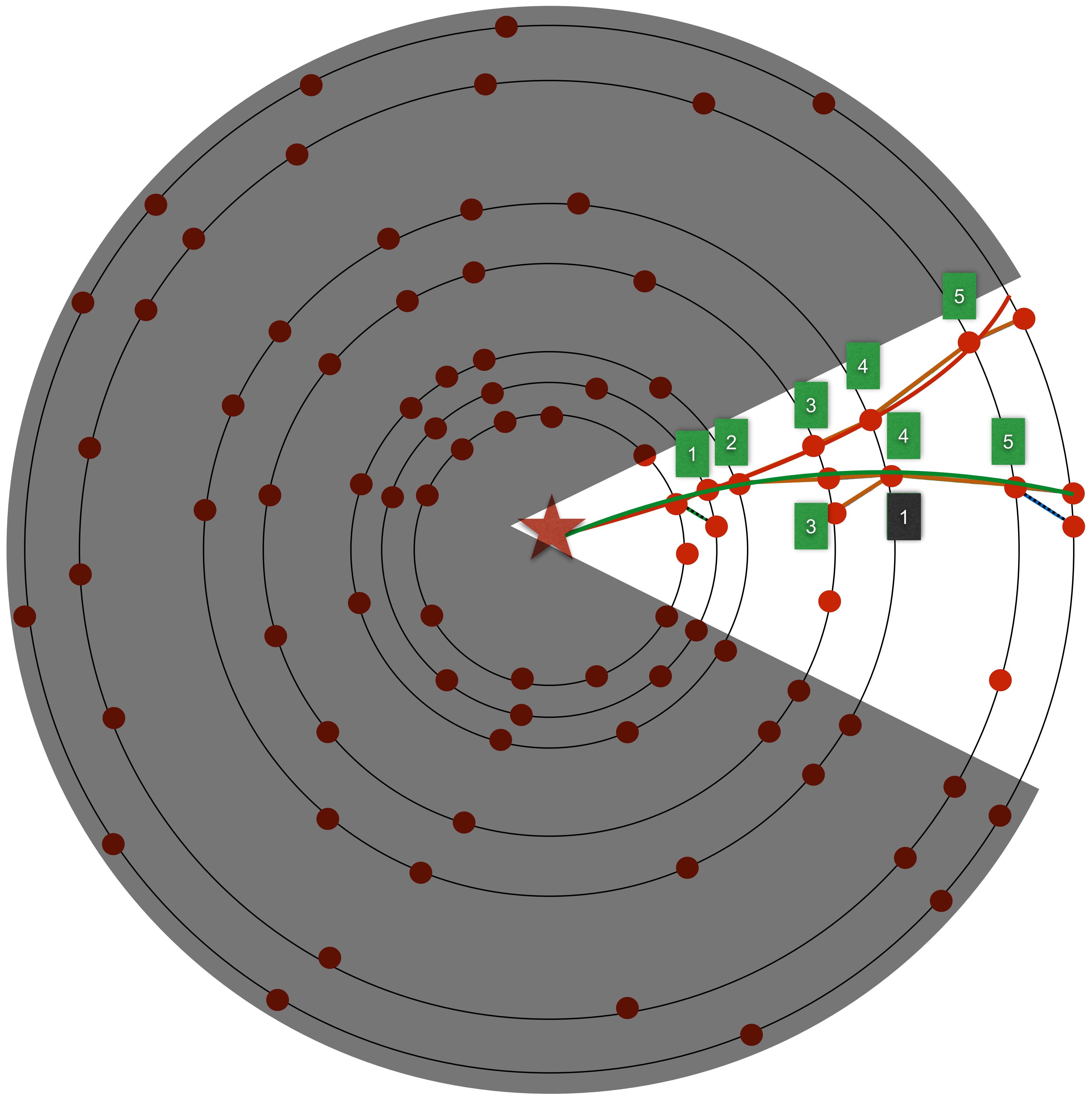
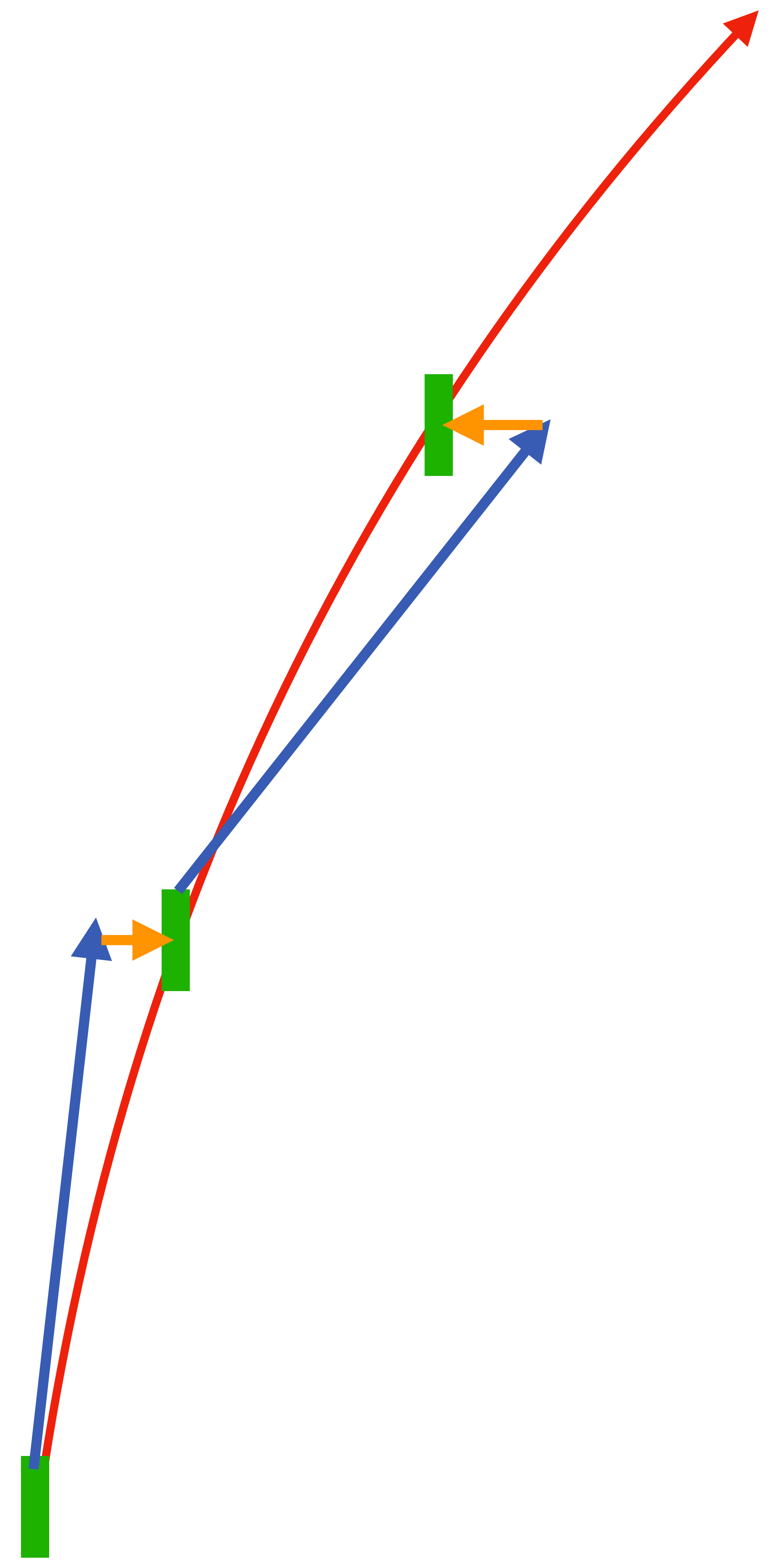


Figure by M. Puccio

Kalman filter

- Idea: propagate track through detector and update parameters
  → combination of track finding and fitting steps
  - Define parametrisation of track at given point together with propagator to next layer
    - propagator can take into account effects such as energy loss in material
  - Find a seed, i.e. rough track parametrisation from a single hit or track segment
  - Iterate over layers
    - propagate track to next layer
    - update parameterisation based on hit in this layer
- Algorithm can be repeated in opposite direction for refined fit and smoothing



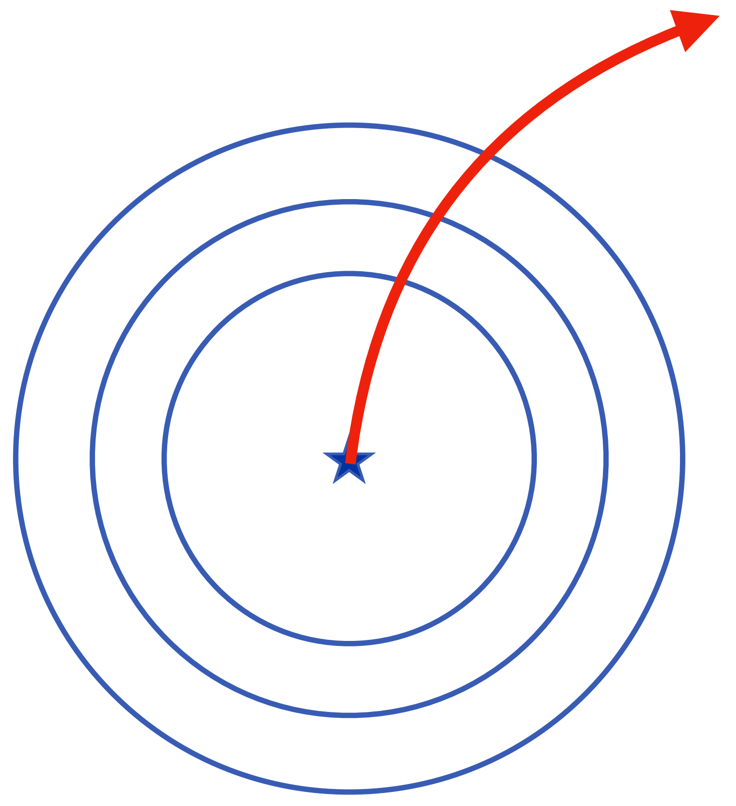
Detector optimisation

- Impact of **number of layers** on
  - track finding
  - track fitting
- Impact of **placement of layers** on
  - track finding
  - track fitting
- Impact of **lever arm and magnetic field** on
  - momentum resolution
- Impact of **dead zone and inefficiencies** on
  - corrections for analysis

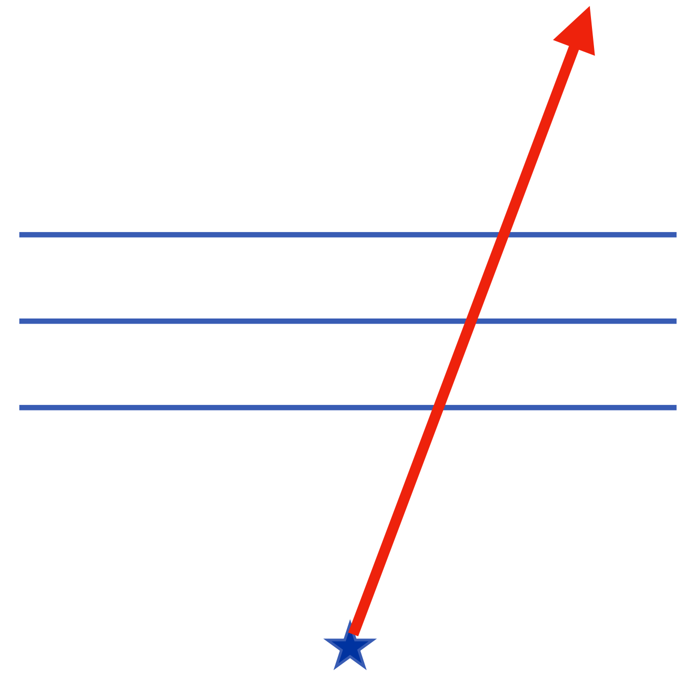
Momentum resolution

- Consider **momentum resolution in solenoidal field**
  - $\propto \frac{\sqrt{d_{\text{tot}}/X_0 \cosh \eta}}{\beta B L}$ for multiple scattering
  - $\propto \frac{\sigma p_{\text{T}}}{B L^2}$ for position resolution

- **Objectives**
  - choose **lever arm L** required for momentum resolution:
    area (and cost) scales quadratically with L (for fixed η coverage)
  - choose **magnetic field**:
    higher field improves mom. resolution, limits acceptance,
    increases magnet cost
  - optimise **number of layers**:
    more layers add material, help with track finding, increase cost
  - minimise **material per layer**:
    challenge on power consumption, cooling, mechanics



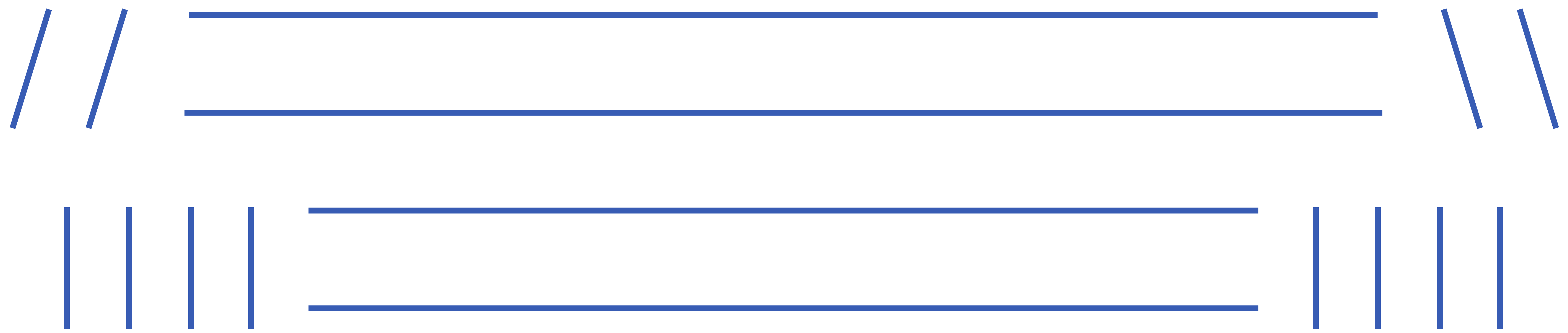
Trajectory through cylindrical layers



forward coverage with barrel layers
- **large areas → expensive**
- **deterioration of performance**

Layer arrangement

- **Objectives**
  - reduce instrumented area
  - minimise path length in material (avoid shallow angles)
- **Barrel layers** well suited to cover central region (up to $\eta \approx 1.5$)
- **Disks** well suited to cover forward region (beyond $\eta \approx 1.5$)
- **Inclined layers** can be used for transition region



Layer placement

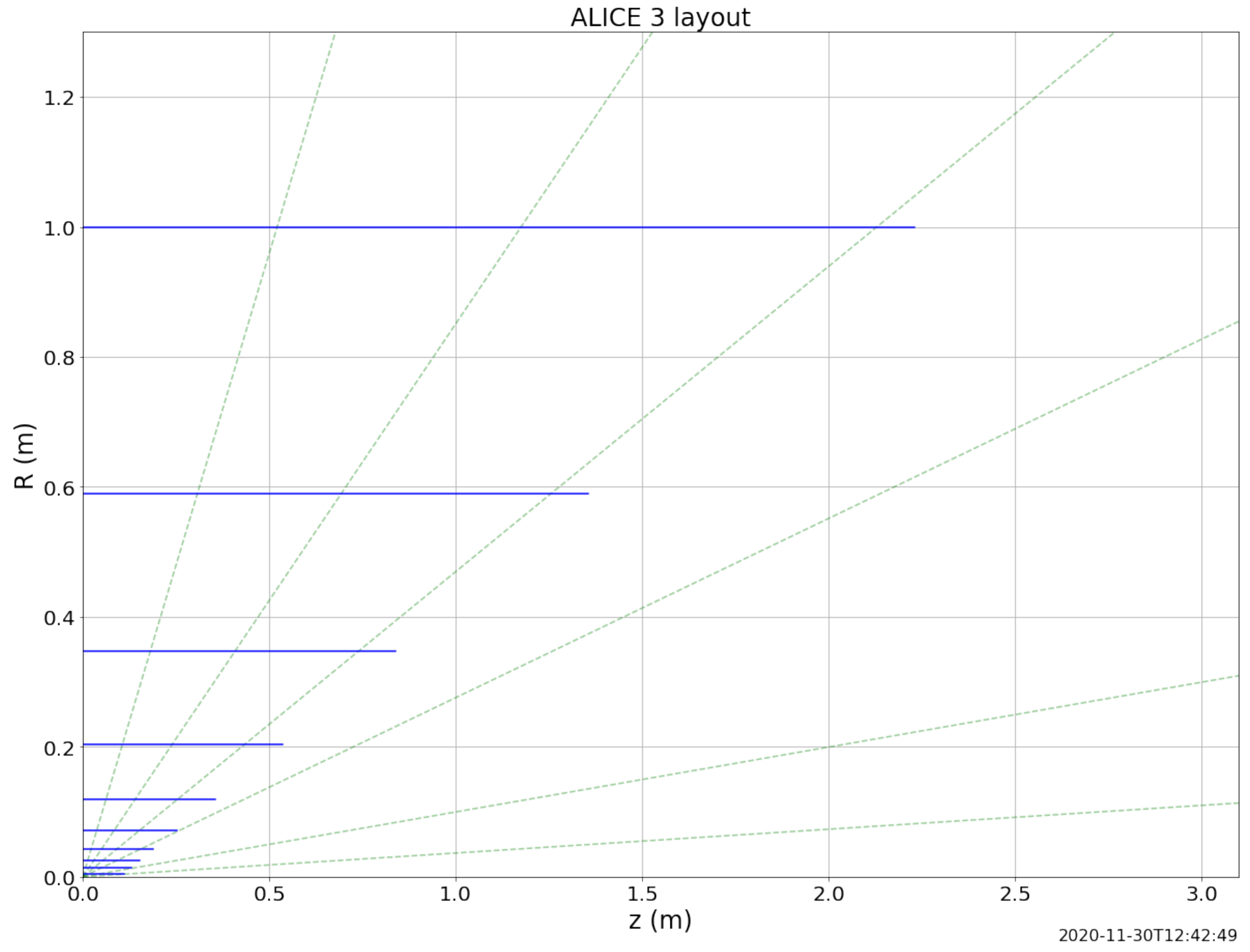
- One objective among others:
  optimise momentum resolution

- Consider tracker with following properties fixed:
  lever arm, magnetic field, material thickness per layer, position resolution
  - **number of layers** has small impact on momentum resolution
  - **ideal layer positions** depend on dominating effects
    - multiple scattering
    - position resolution
    - track finding (pattern recognition)

**Optimisation for momentum resolution
gives only weak indication on ideal layer placement**

Mismatch probability

- **Objective**
  - minimise assignment of hits not originating from a given track
- **A conceptual approach**
  - probability of wrong assignment
    scales with number of hits in vicinity
    $$P \propto \frac{\Delta R^2}{R^2} \cdot x/X_0$$
    here: assuming quadratic decay of particle flux
  - distance from preceding layer to achieve constant probability
    $$\Delta R \propto \sqrt{\frac{P}{x/X_0}} \cdot R$$
- **Inefficiencies and dead areas**
  can lead to significant deterioration of matching
  - layers often arranged in pairs to avoid lack of position information



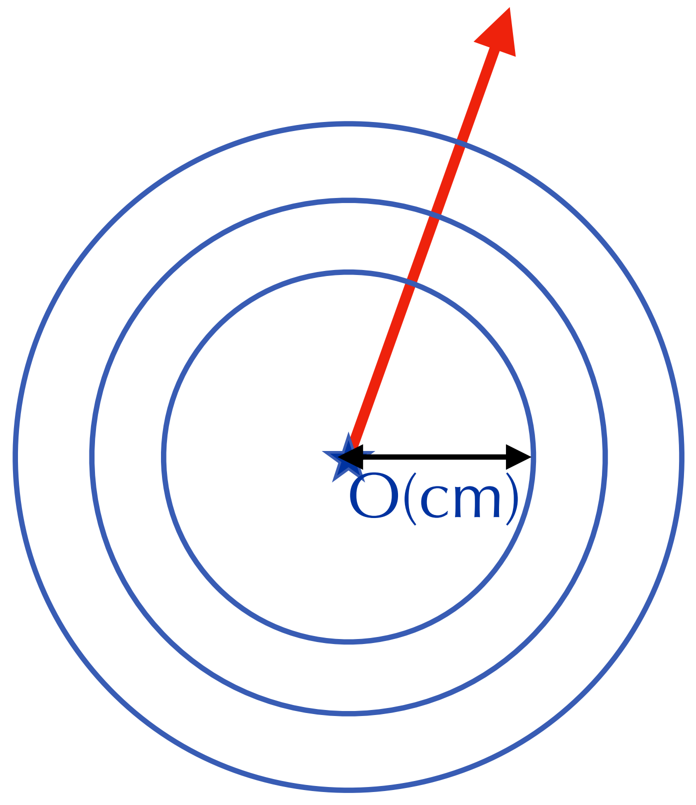
Additional considerations

- **Momentum range**
  - limited by particles not reaching outer layers
  - limited by momentum resolution
- **Propagation to other detectors**
  - limited by pointing resolution towards other (outer) detectors
- **Track length determination**
  - limited by precision of propagation inside tracker
- **Reconstruction of secondary particles**
  - limited by number of hits
    for particles produced at a distance from the primary vertex

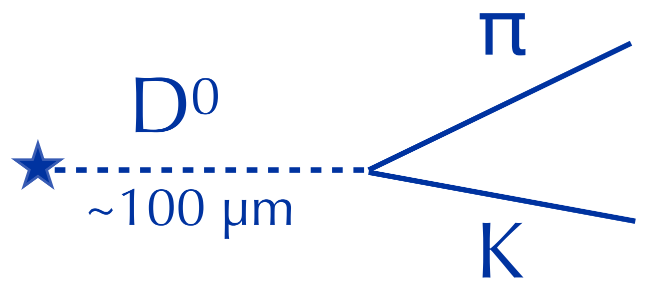
Vertexing

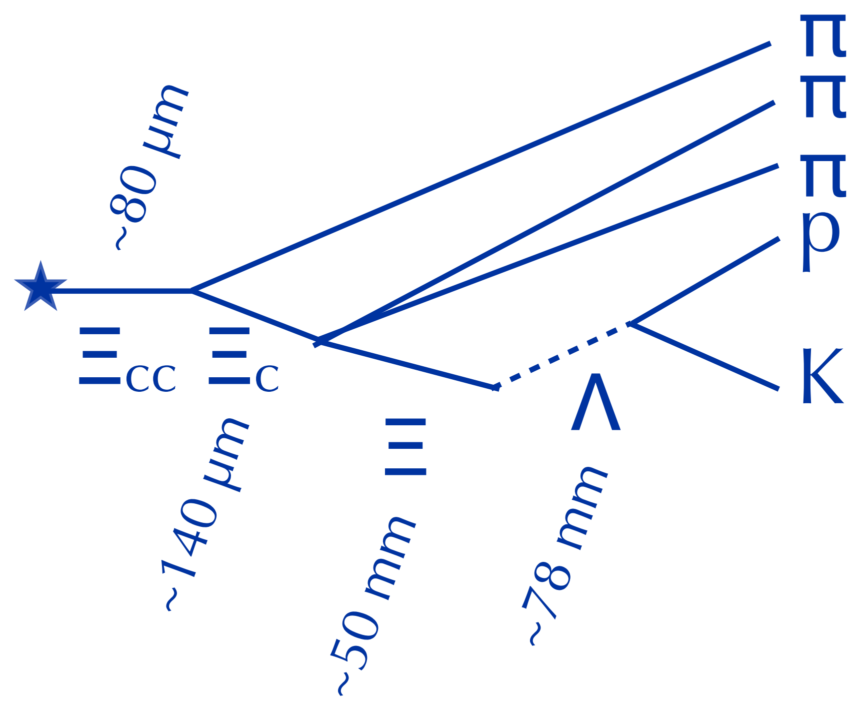
Vertices

- Reconstruction of **primary vertex**, i.e. point of underlying interaction, from emerging tracks
  - need to separate primary vertices, e.g. ~200 in a single HL-LHC bunch crossing
  - assignment of tracks to interactions
- Reconstruction of **decay vertices**
  - distinction of prompt and non-prompt particles
  - reconstruction of decay vertices and decay chains
- Need for pointing resolutions on the order of 100 μm



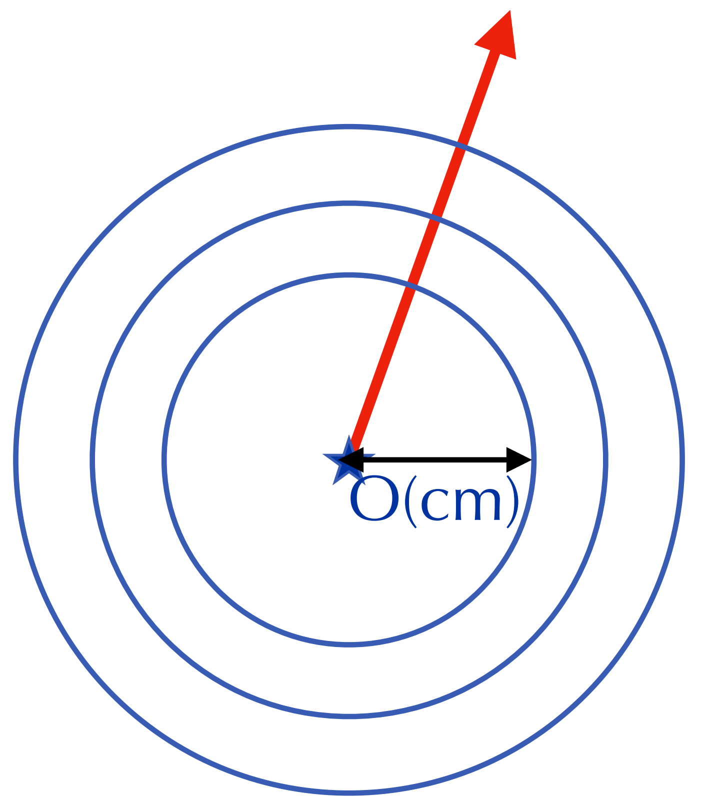
decays with cτ < 100 μm



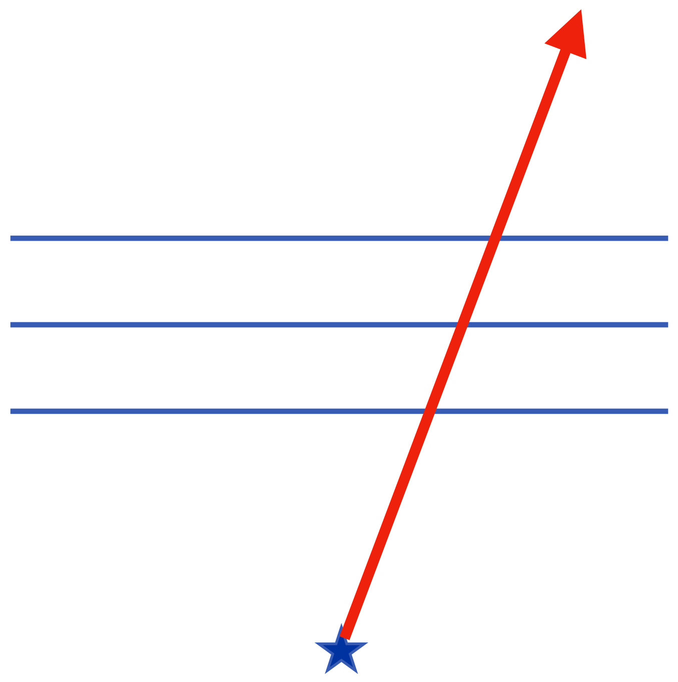


Track propagation

- **Track finding** like for tracking
- Track and propagation defined by **straight line through 2 hits**
  - magnetic field can often be neglected because of short distances
  - typically vertex detectors feature at least three layers for track finding
- **Extrapolation limited** by
  - position resolution
  - multiple scattering

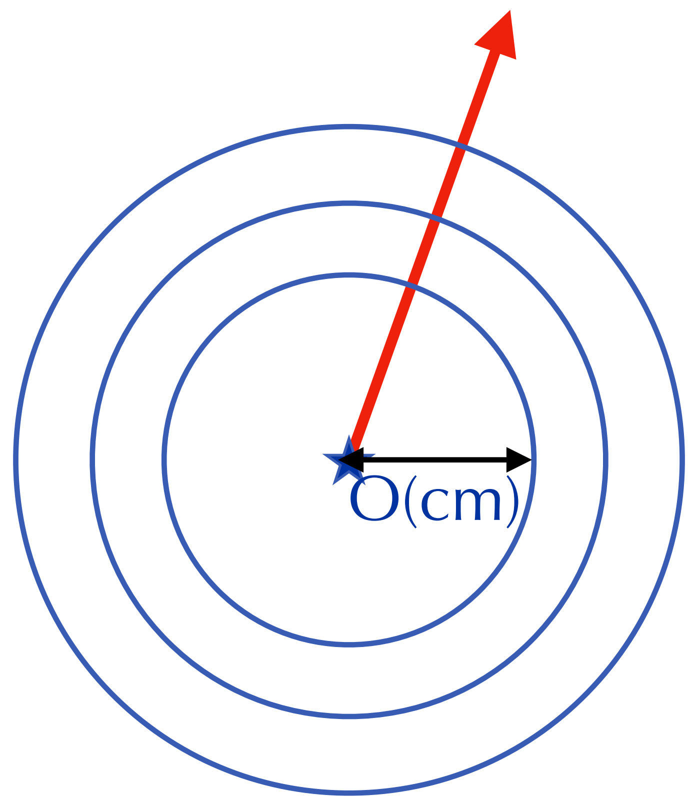


decays with $c\tau < 100\ \mu m$



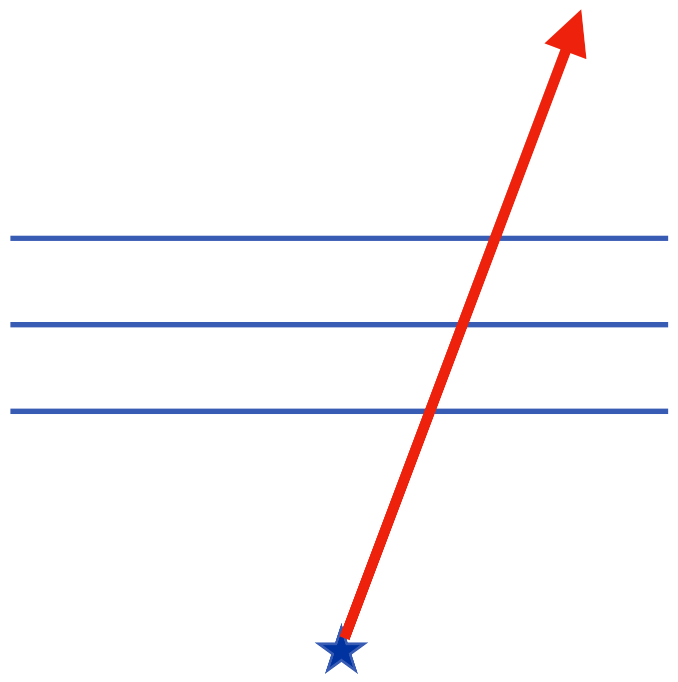
Position resolution

- Impact parameter obtained from track extrapolation
  - approximation by straight line
    $f(x_i) = a_0 + a_1\, x_i$
  - with first layer at radius r_0, uncertainty given by
    $\sigma_{xy} = \sigma_{a_1} \cdot r_0$
    $\sigma_z = \sigma_{a_1} \cdot r_0 \cdot \cosh \eta$



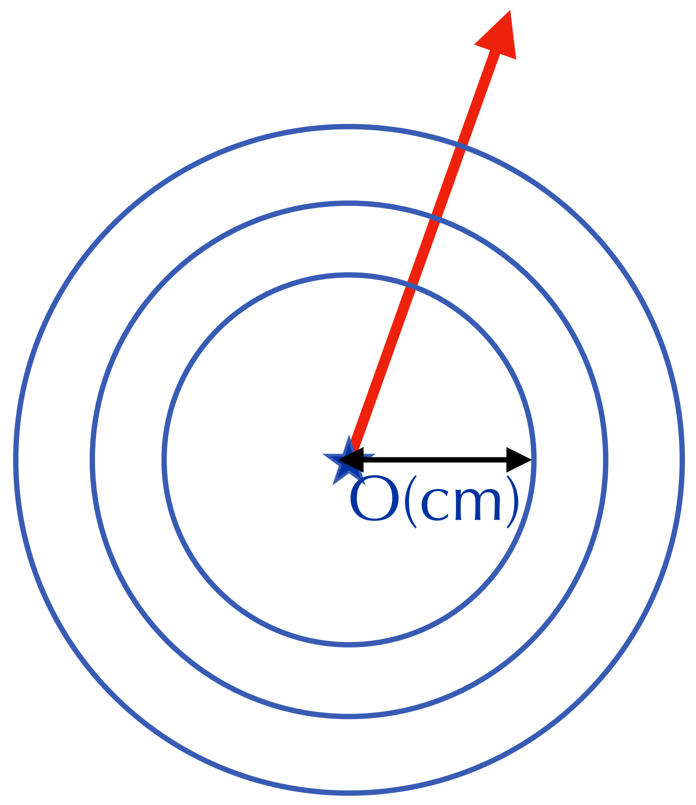
decays with cτ < 100 µm

- Position resolution of N+1 equidistant layers leads to
  - $\sigma_{d_{xy}} \propto \sigma_{r\varphi}$ (transverse impact parameter)
  - $\sigma_{d_z} \propto \sigma_z$ (longitudinal impact parameter)



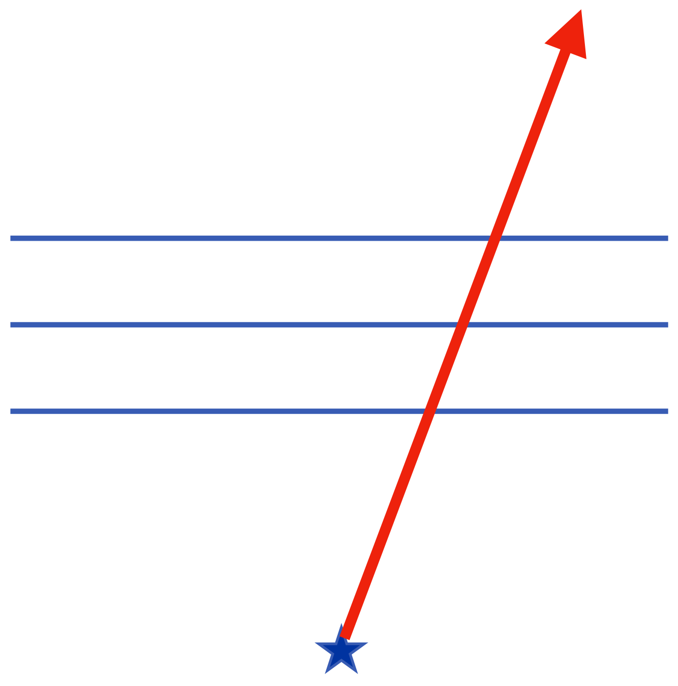
Multiple scattering

- **Multiple scattering in N+1 equidistant layers** leads to
  - $\sigma_\varphi \propto \frac{\sqrt{d/X_0 \cosh\eta}}{\beta p_\mathrm{T}}$
  - $\sigma_\vartheta \propto \frac{\sqrt{d/X_0 \cosh\eta}}{\beta p} \propto \frac{\sqrt{d/X_0}}{\beta p_\mathrm{T}\sqrt{\cosh\eta}}$



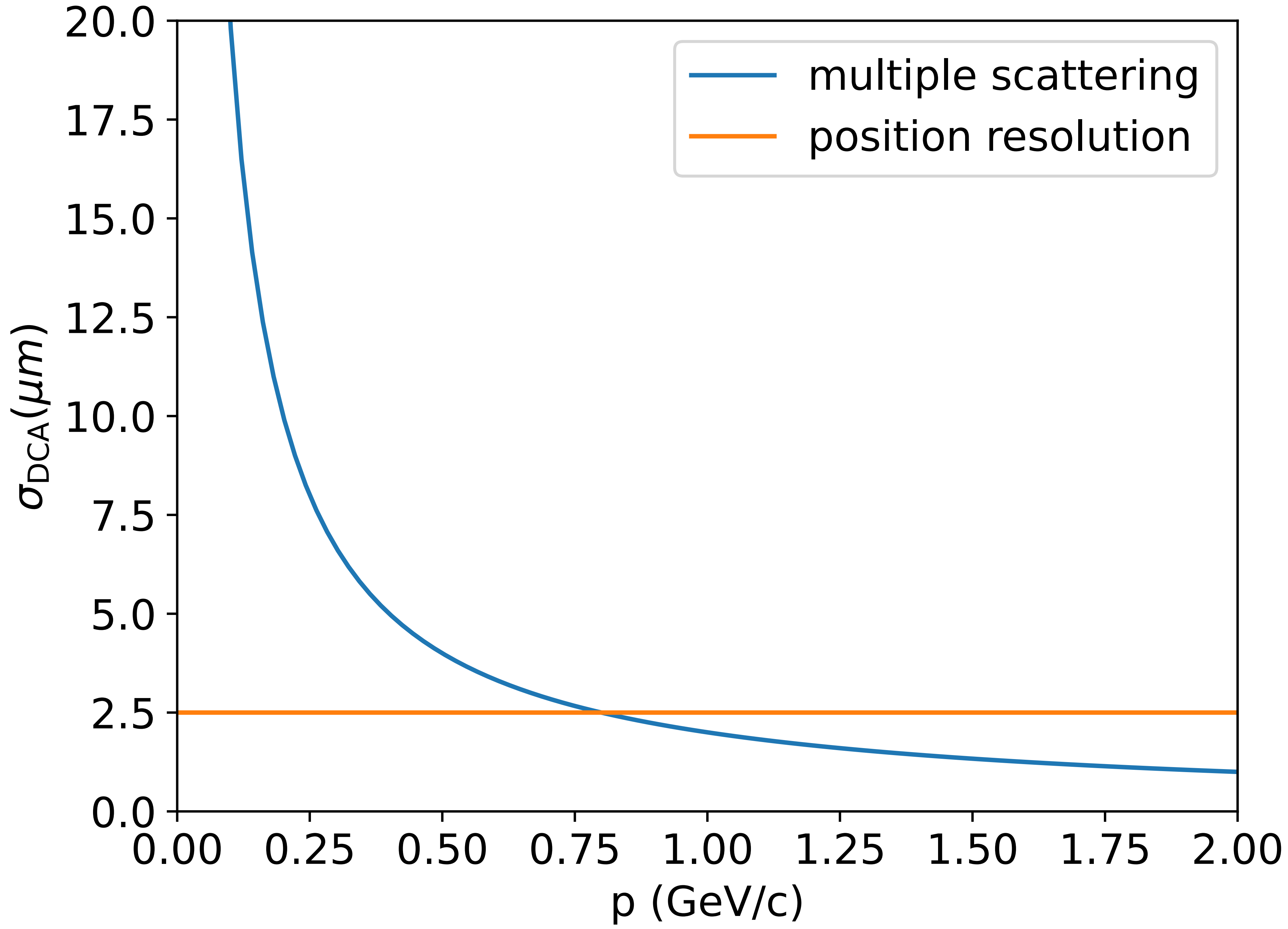
decays with cτ < 100 μm

- **Propagation towards interaction point** leads to
  - $\sigma_{d_{xy}} \propto \sigma_\varphi \cdot r_0 \propto \frac{\sqrt{d/X_0 \cosh\eta}}{\beta p_\mathrm{T}} r_0$ (transverse)
  - $\sigma_{d_z} \propto \sigma_\vartheta \cdot r_0 \cdot \cosh^2\eta \propto \frac{\sqrt{d/X_0}}{\beta p_\mathrm{T}} \cdot r_0 \cdot \sqrt{\cosh\eta}^3$ (longitudinal)



Detector design

- **Pointing resolution scales**
  - with distance of closest layer
    → priority to be as close as possible
  - with square root of material (m.s. ∝ 1/p)
    → minimise material
  - with position resolution
    → keep sub-dominant (within range)
- **More layers do not improve precision**
  but can be needed for track finding



Wrap-up: resolution scaling

(for cylindrical layers)

	position resolution	**multiple scattering**
momentum	$\propto \frac{\sigma p_\mathrm{T}}{BL^2}$	$\propto \sqrt{d_\mathrm{tot}/X_0}\frac{\sqrt{\cosh\eta}}{\beta BL}$
transverse DCA	$\propto \sigma_{xy}$	$\propto r_0\sqrt{d/X_0}\frac{\sqrt{\cosh\eta}}{\beta p_\mathrm{T}}$
longitudinal DCA	$\propto \sigma_z$	$\propto r_0\sqrt{d/X_0}\frac{\sqrt{\cosh\eta}^3}{\beta p_\mathrm{T}}$

Summary part 1

- Tracking and vertexing based on **reconstruction from hits along particle trajectory**, often in magnetic field to extract curvature and, thus, momentum
- Performance determined by
  - multiple scattering → material
  - position resolution → detector technique and performance
  - magnetic field
  - lever arm
- Scaling with momentum and pseudo-rapidity

Time for discussion 😊