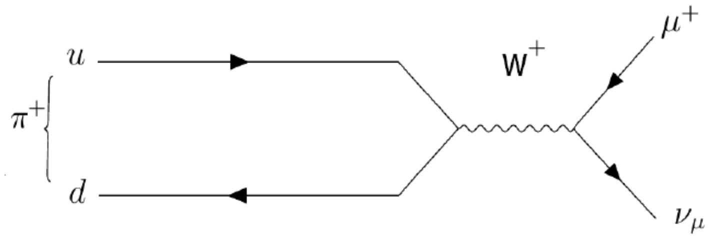
Classification of the processes in the SM

Leptonic Decays

the prototype of these decays is given by

$$\pi^+ \longrightarrow \mu^+ + \nu_\mu \tag{35}$$

which at the fundamental level is given by



$$\Gamma \propto |V_{ij}|^2 f_P^2$$

from lattice calculations

Other possible leptonic decays are given by

$$K^+ \longrightarrow \mu^+ + \nu_\mu$$
$$D^+ \longrightarrow \mu^+ + \nu_\mu$$
$$B^+ \longrightarrow \tau^+ + \nu_\tau$$
$$\pi^+ \longrightarrow e^+ + \nu_e$$

the latter process is suppressed by chirality

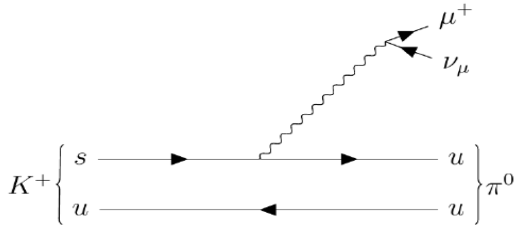
Semi-leptonic Decays

these are the better sources to measure the absolute values of the CKM matrix elements |V ij|

$$n \longrightarrow p + e^- + \overline{\nu_e}$$

other possible semi-leptonic decays are the following

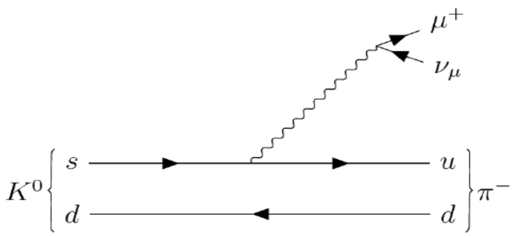
$$\pi^+ \longrightarrow \pi^0 + e^+ + \nu_e$$
$$K^+ \longrightarrow \pi^0 + \mu^+ + \nu_\mu$$



$$\Gamma \propto |V_{ij}|^2 |f(q^2)|$$

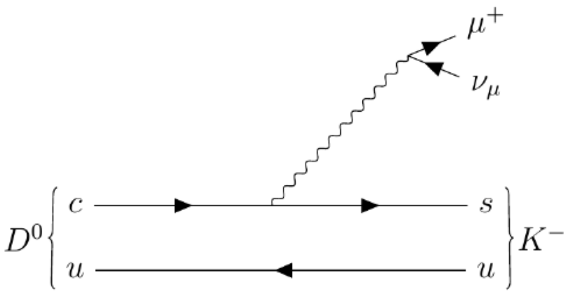
$$\frac{1}{\Gamma}\frac{d\Gamma}{dq^2} \propto |f(q^2)|$$

$$K^0 \longrightarrow \pi^- + \mu^+ + \nu_\mu$$



from lattice calculations

$$D^0 \longrightarrow K^+ + \mu^+ + \nu_\mu$$

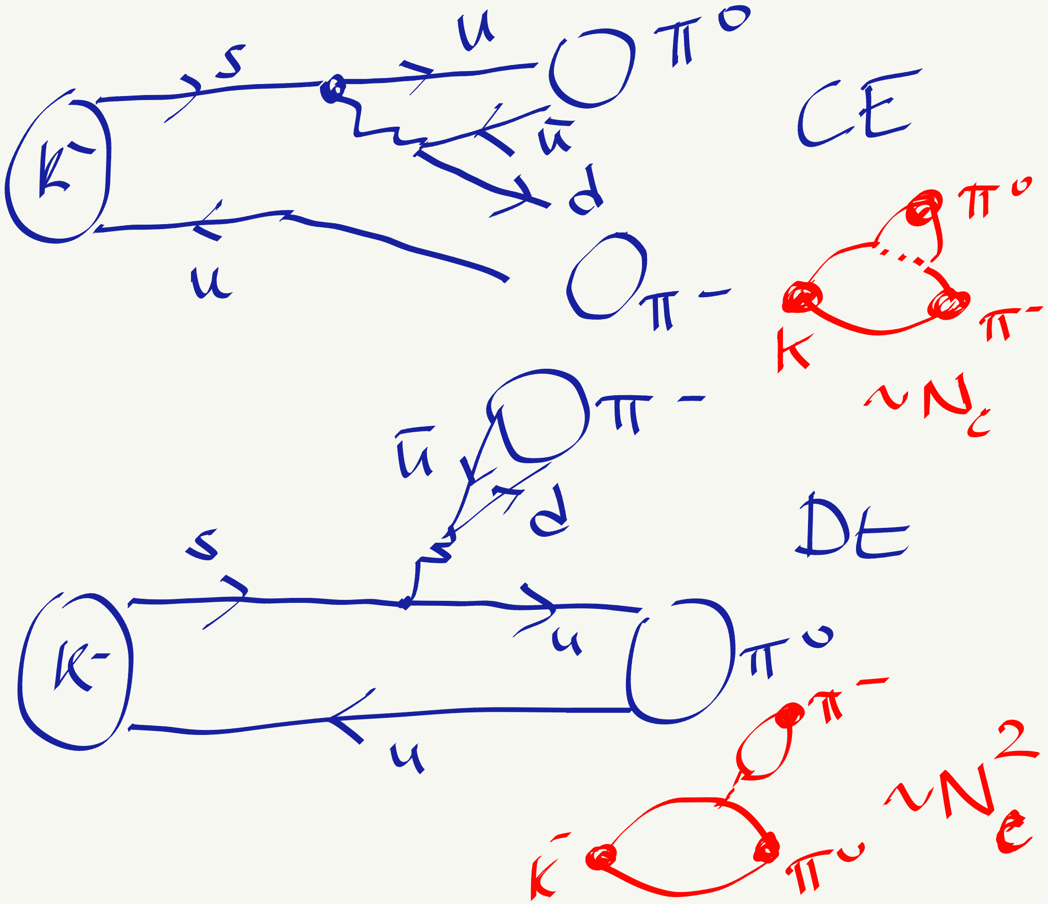


Non-leptonic Decays

Penguins contractions and all that

$$K^- \to \pi^- \pi^0$$

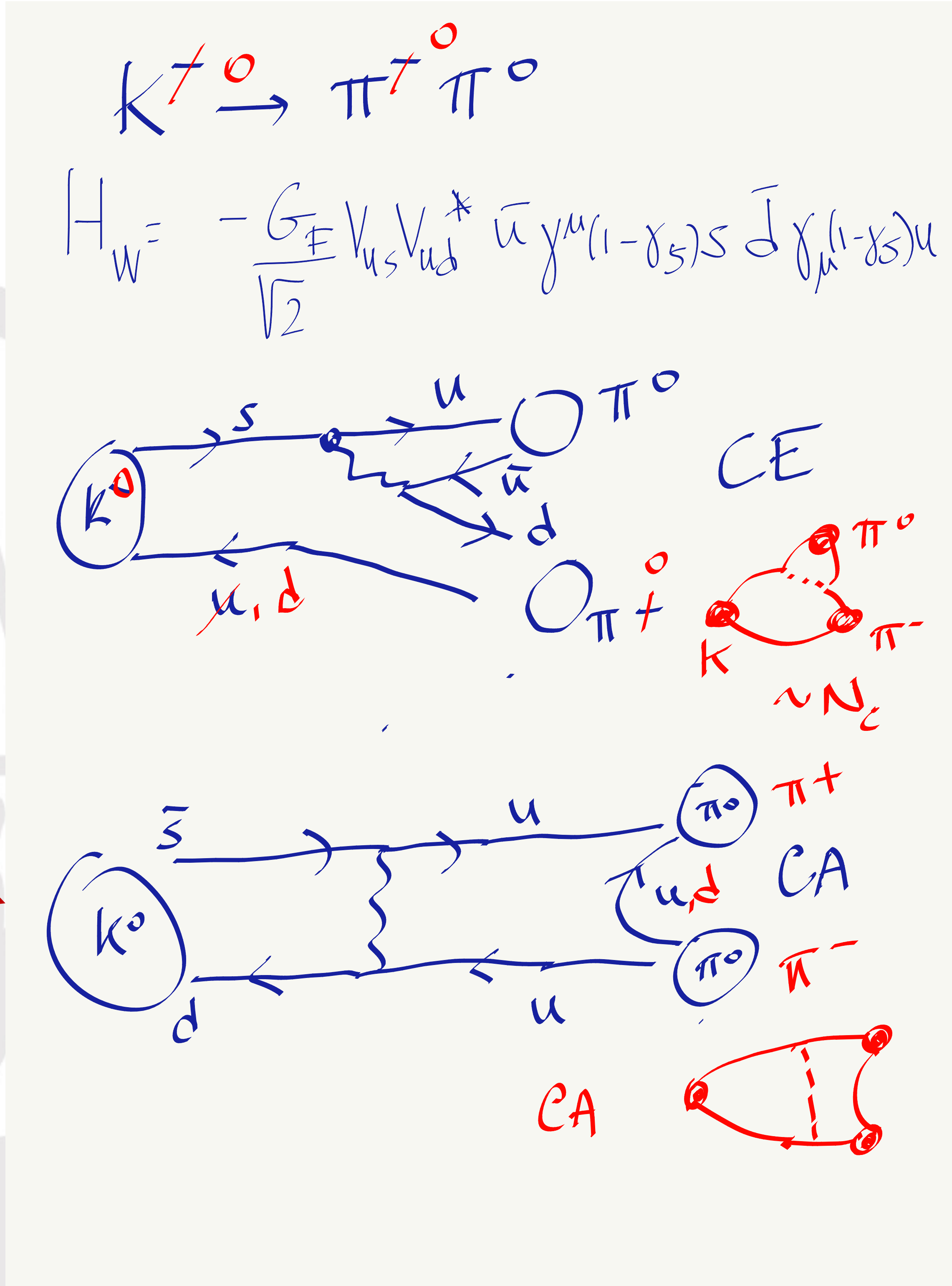
$$H_W = -\frac{G_F}{\sqrt{2}} V_{us} V_{ud}^* \, \bar{u} \gamma^\mu (1-\gamma_5) s \, \bar{d} \gamma_\mu (1-\gamma_5) u$$



Non-leptonic Decays

Penguins contractions and all that

annihilations



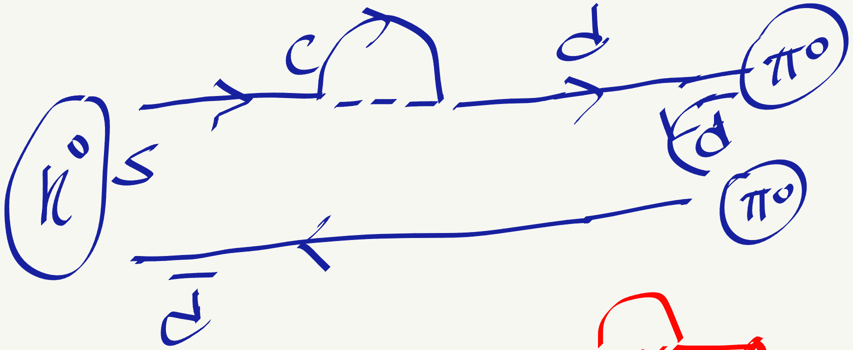
Non-leptonic Decays

Penguins contractions and all that

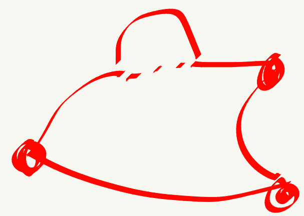
penguin contractions

Penguins diagrams

$$H = -\frac{G_F}{\sqrt{2}} V_{cs} V_{cd}^{*} \; \bar{c}\gamma_\mu(1-\gamma_5)s \; \bar{d}\gamma^\mu(1-\gamma_5)c$$



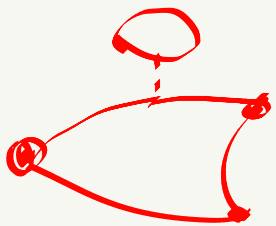
CP



CP

other ops

$$\sim V_{cs} V_{cd}^{*} \quad \bar{d}\gamma_\mu(1-\gamma_5)s \; \bar{c}\gamma^\mu(1-\gamma_5)c$$



DP

All Topologies

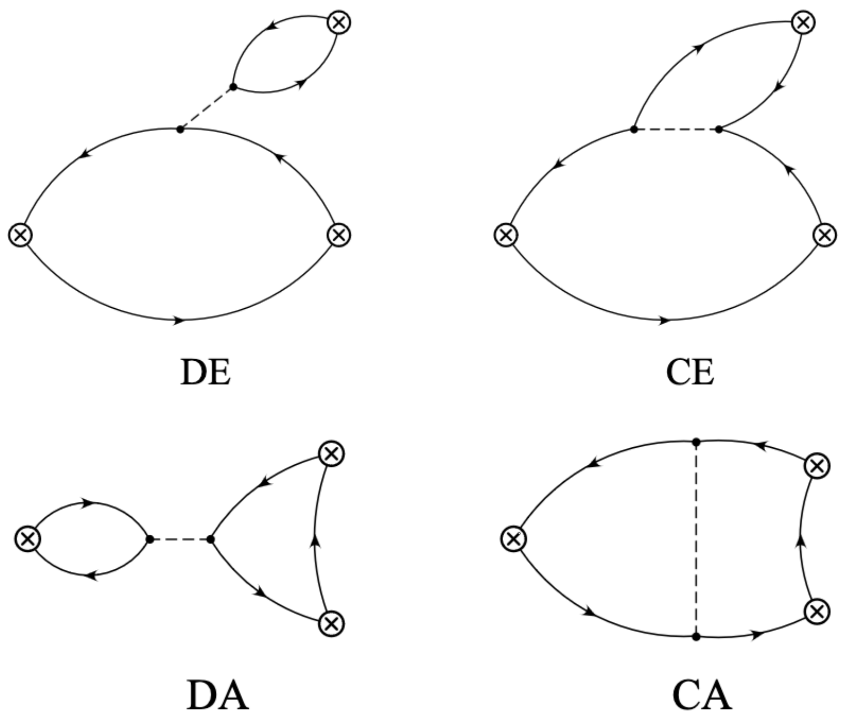
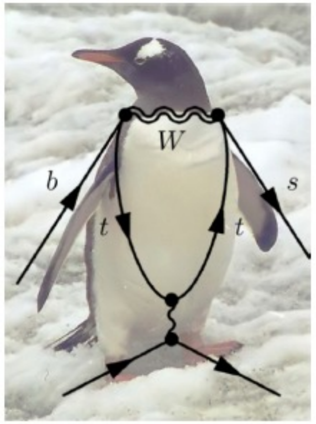


Figure 1: *Non-penguin diagrams. The dashed line represents the four-fermion operator.*

All Topologies



D. Kovalskyi's seminar @ CERN (26/7/22)

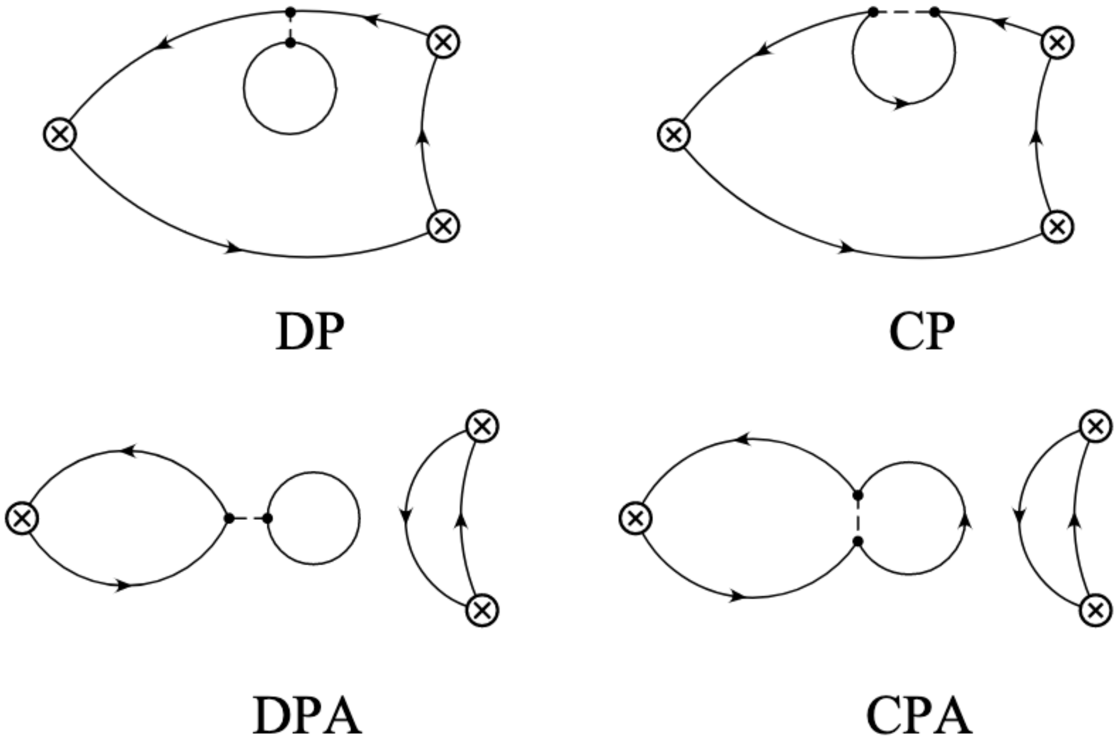
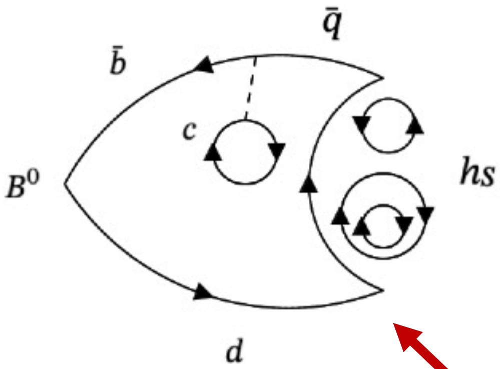


Figure 2: *Penguin diagrams.*



for heavy mesons many particles in the final state

Rare Penguin Radiative Decays

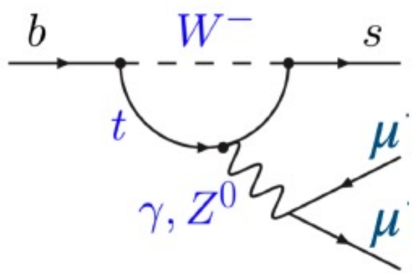
The main issue in this Group Meeting will be represented by b → s quark transitions: the best example is offered by the neutral-current semileptonic B → K(*) l+ l- transitions!

Many interesting properties:

1. **Loop-level processes (FCNCs are forbidden at tree level in the Standard Model)**
2. **CKM-suppressed decays, where**

$$J^{\mu}_{\text{charged}} = \bar{u}^i_L V^{ij}_{CKM} \gamma^\mu d^j_L + \bar{\nu}^i_L \gamma^\mu \ell^i_L$$

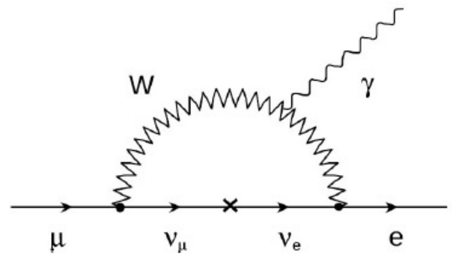
➡ **Rare transitions!**



L. Vittorio (LAPTh & CNRS, Annecy)

$$B^+ \to K^{(*)+}\gamma \qquad B^+ \to K^{(*)+}\mu^+\mu^-$$

since different neutrinos have a mass and they can mix, $\mu \to e\gamma$ is a possible decay which satisfies all the symmetry constraints



$$\mathcal{B}(\mu \to e\gamma) \sim \alpha \frac{m_\nu^4}{m_W^4} \sim 10^{-52}$$

note that the photon is emitted by the W boson, analogy radiative B decays

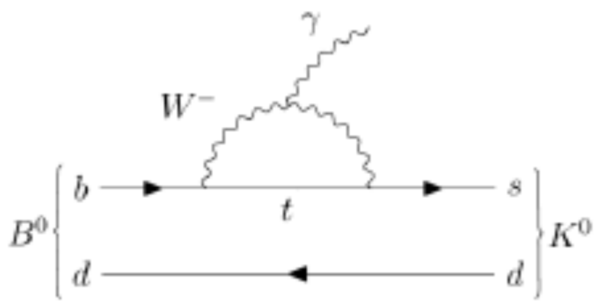


Figura 4: quark process

Radiative Penguins

PENGUINS AND BOXES

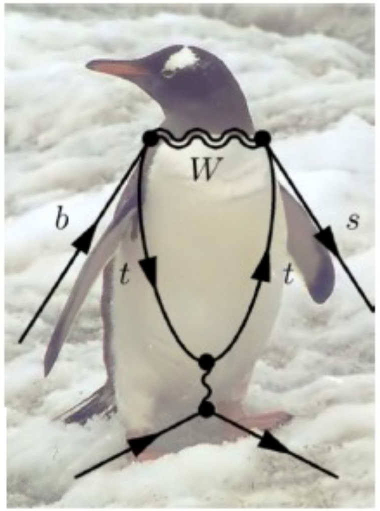
Pure leptonic Bs decays

$$Br(B_s \to l^+l^-) = \tau(B_s)\frac{G_F^2}{\pi}\left(\frac{\alpha}{4\pi\sin^2\Theta_W}\right)^2 F_{B_s}^2 m_l^2 m_{B_s}\sqrt{1-4\frac{m_l^2}{m_{B_s}^2}}|V_{tb}^*V_{ts}|^2Y^2(x_t)$$

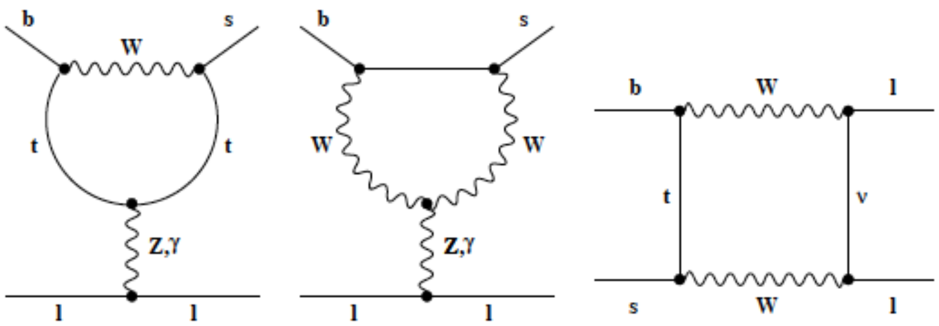
G. Buchalla and A.J. Buras, Nucl. Phys. B 400 (1993) 225.

Many interesting properties:

1. **Helicity suppressed**
2. **Non-perturbative hadronic contributions enter via Bs decay constant**



valskyi's seminar @ CERN (26/7/22)



(a) (b) (c)

Lowest order diagrams
QCD corrections at NLO or NNLO

BOXES

Mixing of Neutral Mesons

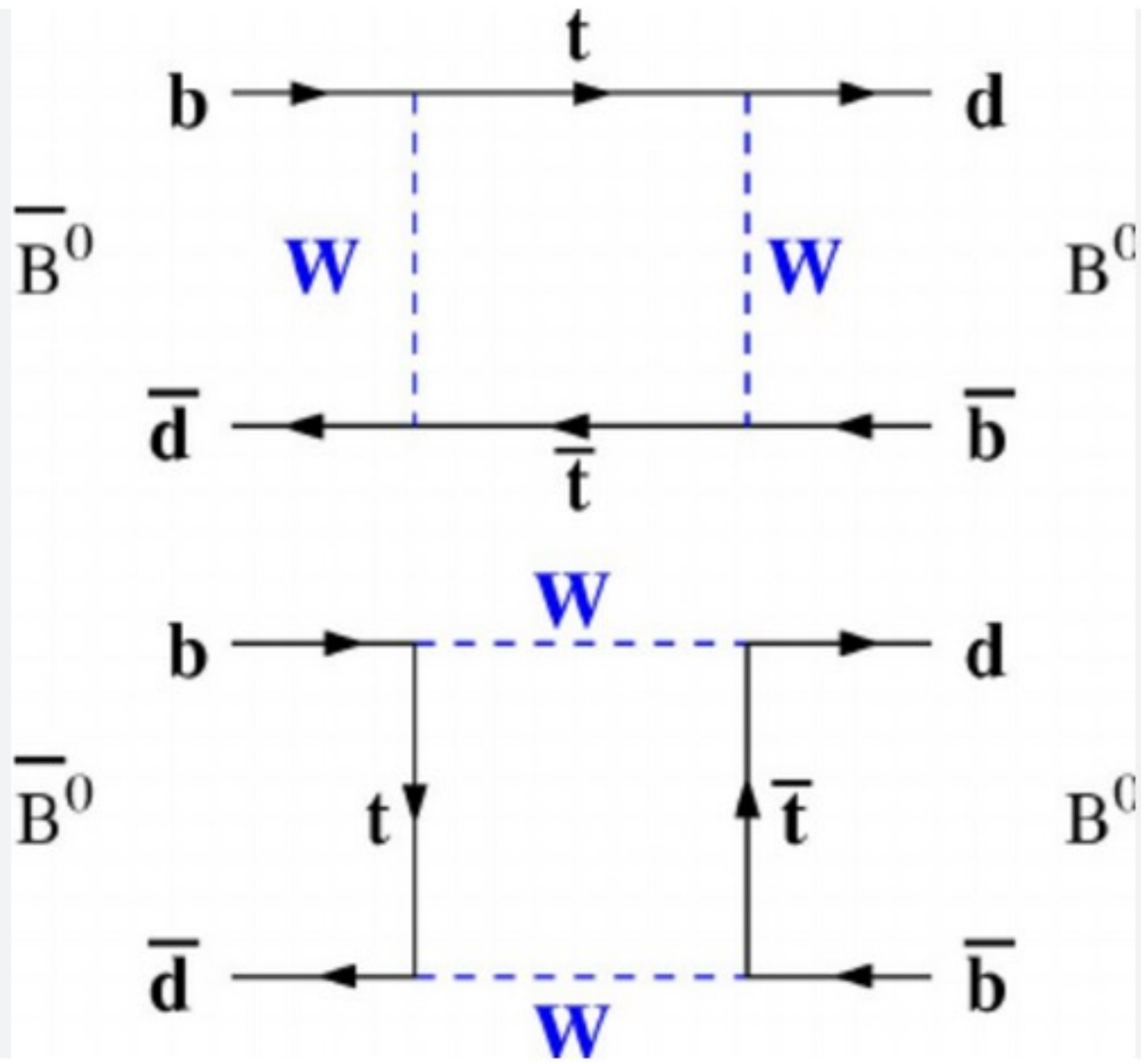
$$K^0 \leftrightarrow \bar{K}^0$$

$$D^0 \leftrightarrow \bar{D}^0$$

$$B^0 \leftrightarrow \bar{B}^0$$

in the case of kaons
also charm and up quarks contribute

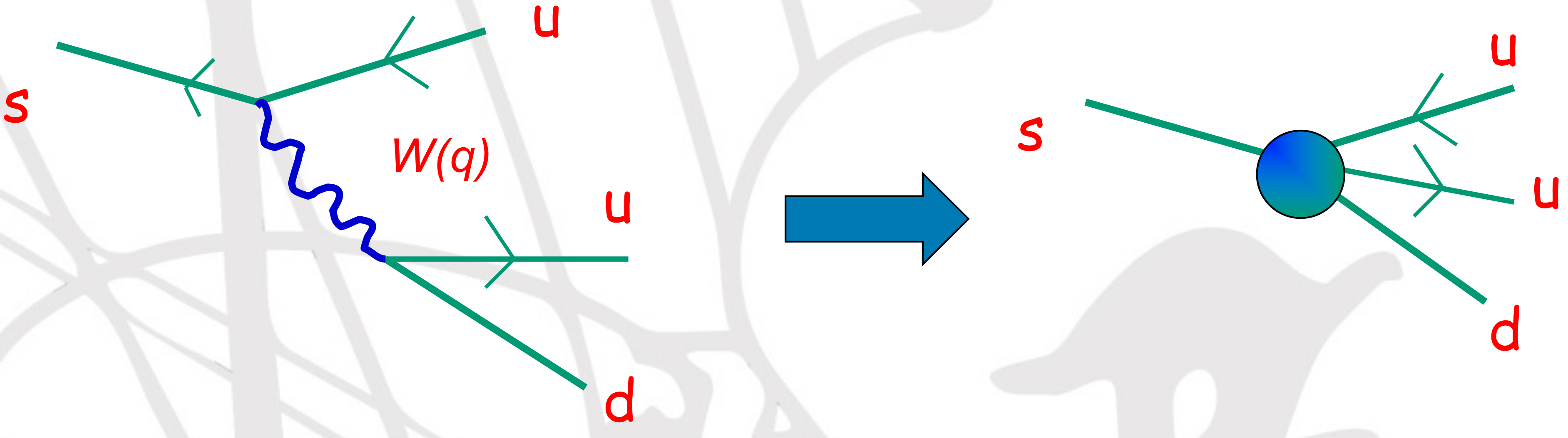
for D and K meson mixing there are important long distance contributions



$$\mathcal{H}_{\rm eff}^{\Delta B=2} = \frac{G_{\rm F}^2}{16\pi^2} M_W^2 \left(V_{tb}^* V_{tq}\right)^2 \eta_B S_0(x_t) \times \\ \times \left[\alpha_s^{(5)}(\mu_b)\right]^{-6/23} \left[1 + \frac{\alpha_s^{(5)}(\mu_b)}{4\pi} J_5\right] Q^q(\Delta B = 2) + h.c.$$

QCD corrections

The Effective Hamiltonian, Wilson OPE and QCD Corrections



$$q \sim m_K \ll M_W$$

$$\mathcal{H}_{eff} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \, (\bar{s}\gamma_\mu (1-\gamma_5) u) \, (\bar{u}\gamma^\mu (1-\gamma_5) d)$$

GENERAL FRAMEWORK: THE OPE

$$A_{\mathbf{FI}}\,(2\pi^4)\,\delta^4\,(p_F - p_I) = \int d^4x\, d^4y\, D_{\mu\nu}(x, M_W)$$

$$\langle F\,|T[\,J_\mu(y+x/2)\,J^\dagger_\nu(y-x/2)]\,|\,I\,\rangle$$

$$\langle F\,|\,H^{\Delta S=1}\,|\,I\,\rangle =$$

$$G_F/\sqrt{2}\,V_{ud}\,V_{us}^* \quad \Sigma_i\,C_i(\mu)\,\frac{\langle F\,|\,Q_i(\mu)\,|\,I\,\rangle}{(M_W)^{di-6}}$$

di= dimension of the operator $Q_i(\mu)$

$C_i(\mu)$ Wilson coefficient: it depends on M_W/μ and $\alpha_W(\mu)$

$Q_i(\mu)$ local operator renormalized at the scale μ

GENERAL FRAMEWORK

$$H^{\Delta S=1} = G_F/\sqrt{2}\, V_{ud}\, V_{us}^{*} \Big[\, (1-\tau)\, \Sigma_{i=1,2}\, z_i\, (Q_i - Q^c_i) + \tau\, \Sigma_{i=1,10} (\, z_i + y_i\,)\, Q_i \;\Big]$$

Where y_i and z_i are short distance coefficients, which are known in perturbation theory at the NLO (Buras et al. + Ciuchini et al.)

$$\tau = -V_{ts}^{*} V_{td} / V_{us}^{*} V_{ud}$$

We have to compute $A^{I=0,2}_{i} = \langle\, (\pi\, \pi)_{I=0,2}\, |\, Q_{\,i}\, |\, K\, \rangle$ with a non perturbative technique (lattice, QCD sum rules, 1/N expansion etc.)

$$A_0 = \sum_i C_i(\mu) \langle (\pi\pi) | Q_i(\mu) | K \rangle_{I=0} (1 - \Omega_{IB})$$

μ = renormalization scale
μ-dependence cancels if operator matrix elements are consistently computed

ISOSPIN BREAKING

$$\mathcal{A}_2 = \sum_i C_i(\mu) \langle (\pi\pi) | Q_i(\mu) | K \rangle_{I=2}$$

Ω_{IB} = 0.25± 0.08 (Munich from Buras & Gerard)
0.25± 0.15 (Rome Group) 0.16± 0.03 (Ecker et al.)
0.10± 0.20 Gardner & Valencia, Maltman & Wolf, Cirigliano & al.

$$A^{I=0,2}_i(\mu) = \langle (\pi\pi)_{I=0,2} | Q_i(\mu) | K \rangle$$
$$= Z_{ik}(\mu a) \langle (\pi\pi)_{I=0,2} | Q_k(a) | K \rangle$$

Where $Q_i(a)$ is the bare lattice operator
And a the lattice spacing.

The effective Hamiltonian can then be read as:

$$\langle F | H^{\Delta S=1} | I \rangle = G_F/\sqrt{2} V_{ud} V_{us}^* \Sigma_i C_i(1/a) \langle F | Q_i(a) | I \rangle$$

In practice the renormalization scale (or 1/a) are the scales which separate short and long distance dynamics

GENERAL FRAMEWORK

$$\langle H^{\Delta S=1} \rangle = G_F/\sqrt{2}\, V_{ud}\, V_{us}^{*} \ldots \Sigma_i\, C_i\,(\mathbf{a})\, \langle Q_i(\mathbf{a}) \rangle$$

$M_W = 100$ GeV

perturbative regime

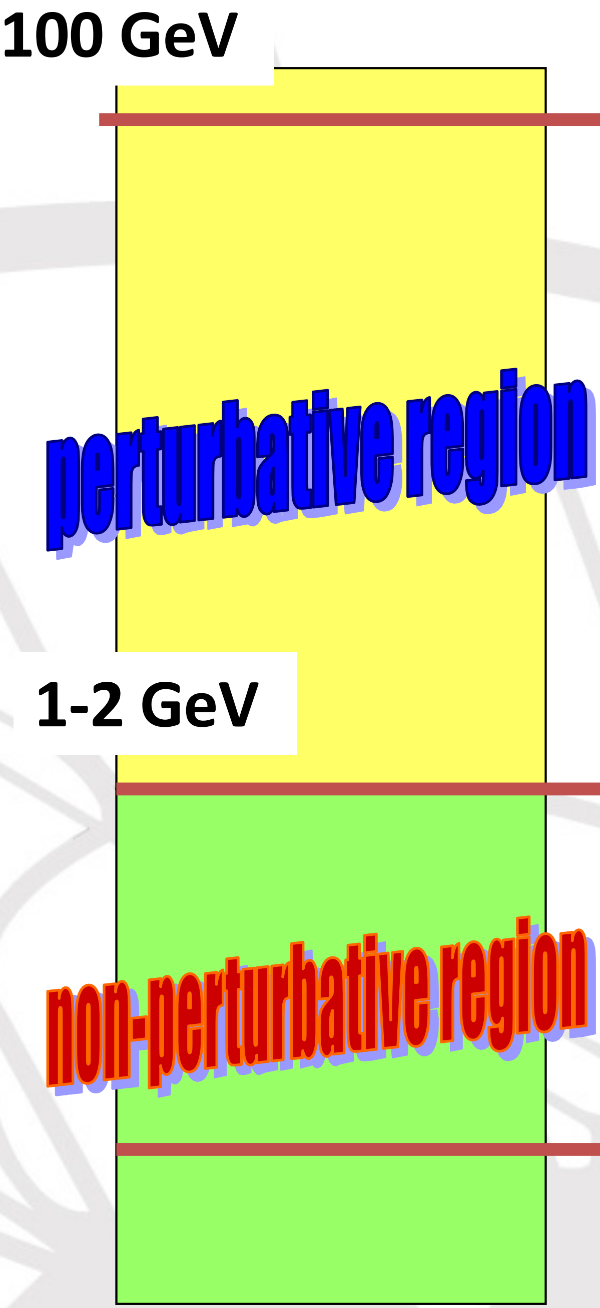
Effective Theory - quark & gluons

$a^{-1} = 2\text{-}5$ GeV

Hadronic non-perturbative region

Chiral regime

Λ_{QCD}, M_K = 0.2-0.5 GeV



Large mass scale: heavy degrees of freedom (m_t , M_W, M_s) are removed and their effect included in the Wilson coefficients

renormalizazion scale μ (inverse lattice spacing 1/a); this is the scale where the quark theory is matched to the effective hadronic theory

Scale of the low energy process $\Lambda \sim M_W$

THE SCALE PROBLEM: Effective theories prefer low scales, Perturbation Theory prefers large scales

if the scale μ is too low
problems from higher dimensional operators

(Cirigliano, Donoghue, Golowich)

- it is illusory to think that the problem is solved by using dimensional regularization

on the lattice this problem is called

DISCRETIZATION ERRORS

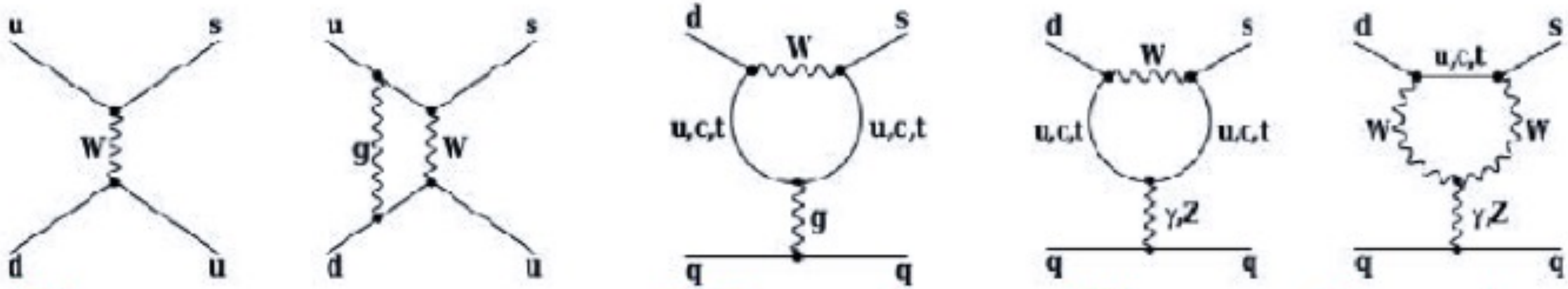
(reduced by using improved actions and/or scales $\mu > 2\text{-}4$ GeV

Weak Hamiltonian for $K \to \pi\pi$

Weak Hamiltonian is given by local four-quark operator *Courtesy by Xu Feng*

$$\mathcal{H}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left\{ \sum_{i=1}^{10} [z_i(\mu) + \tau y_i(\mu)]\, Q_i \right\}, \quad \tau = -\frac{V_{td} V_{ts}^*}{V_{ud} V_{us}^*}$$

- $\tau = -\frac{V_{td} V_{ts}^*}{V_{ud} V_{us}^*} = 1.543 + 0.635i$
- $z_i(\mu)$ and $y_i(\mu)$ are perturbative Wilson coefficients
- Q_i are local four-quark operator



Current-current operator Q_1, Q_2 dominate $\mathrm{Re}[A_0]$, $\mathrm{Re}[A_2]$

QCD penguin $Q_3 - Q_6$; Q_6 dominate $\mathrm{Im}[A_0]$

Electro-weak penguin $Q_7 - Q_{10}$; Q_7, Q_8 dominate $\mathrm{Im}[A_2]$

New local four-fermion operators are generated

$$Q_1 = (\bar{s}_L^A \gamma_\mu u_L^B)\,(\bar{u}_L^B \gamma_\mu d_L^A) \qquad \text{Current-Current}$$
$$Q_2 = (\bar{s}_L^A \gamma_\mu u_L^A)\,(\bar{u}_L^B \gamma_\mu d_L^B)$$

$$Q_{3,5} = (\bar{s}_R^A \gamma_\mu d_L^A)\sum_q (\bar{q}_{L,R}^B \gamma_\mu q_{L,R}^B) \qquad \text{Gluon}$$
$$Q_{4,6} = (\bar{s}_R^A \gamma_\mu d_L^B)\sum_q (\bar{q}_{L,R}^B \gamma_\mu q_{L,R}^A) \qquad \text{Penguins}$$

$$Q_{7,9} = 3/2(\bar{s}_R^A \gamma_\mu d_L^A)\sum_q e_q\,(\bar{q}_{R,L}^B \gamma_\mu q_{R,L}^B) \quad \text{Electroweak}$$
$$Q_{8,10} = 3/2(\bar{s}_R^A \gamma_\mu d_L^B)\sum_q e_q\,(\bar{q}_{R,L}^B \gamma_\mu q_{R,L}^A) \quad \text{Penguins}$$

+ Chromomagnetic end electromagnetic operators

$$\mathcal{A}(K \to \pi\pi) = \sum_i C_W^i(\mu)\langle \pi\pi | O_i(\mu) | K \rangle$$

$\not{C}\not{P}$ Violation in the Neutral Kaon System

Expanding in several "small" quantities

$$\eta^{00} = \frac{\langle \pi^0\pi^0 | \mathcal{H}_W | K_L \rangle}{\langle \pi^0\pi^0 | \mathcal{H}_W | K_S \rangle} \sim \varepsilon - 2\,\varepsilon'$$

$$\eta^{+-} = \frac{\langle \pi^+\pi^- | \mathcal{H}_W | K_L \rangle}{\langle \pi^+\pi^- | \mathcal{H}_W | K_S \rangle} \sim \varepsilon + \varepsilon'$$

Conventionally:

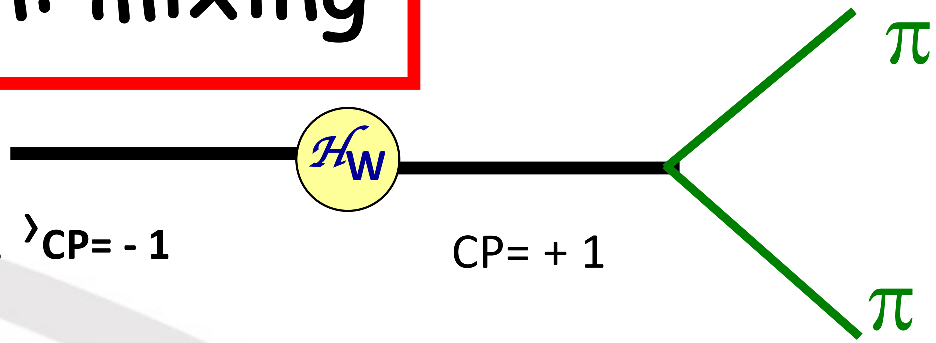
$$| K_S \rangle = | K_1 \rangle_{CP=+1} + \varepsilon\, | K_2 \rangle_{CP=-1}$$

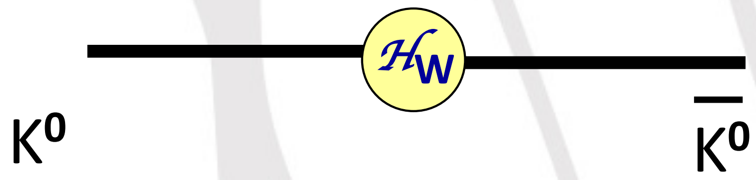
$$| K_L \rangle = | K_2 \rangle_{CP=-1} + \varepsilon\, | K_1 \rangle_{CP=+1}$$

Indirect CP violation: mixing

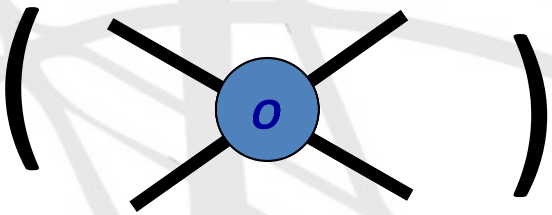
ε_K

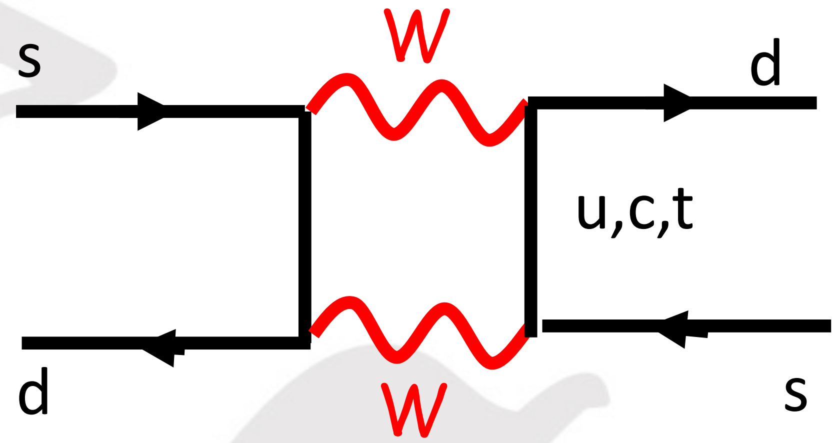
$|K_L\rangle = |K_2\rangle_{CP=-1}$





ΔS=2





Box diagrams:
They are also responsible for B^0 - $\overline{B}^0$ mixing

$\Delta m_{d,s}$

Complex ΔS=2 effective coupling

$$| \varepsilon_K | \sim C_\varepsilon A^2 \lambda^6 \sigma \sin \delta$$

$$\{F(x_c, x_t) + F(x_t)[A^2 \lambda^4 (1 - \sigma \cos \delta)] - F(x_c)\} B_K$$

Inami-Lin Functions + QCD Corrections (NLO)

$\eta = \sigma \sin \delta \quad \rho = \sigma \cos \delta$

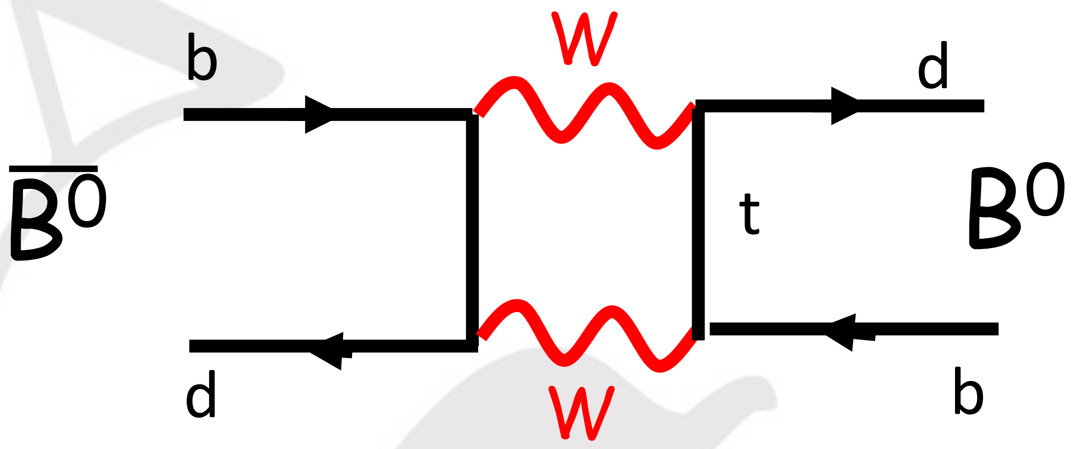
$$C_\varepsilon = \frac{G_F^2 M_W^2 M_K f_K^2}{6 \sqrt{2} \pi^2 \Delta \mathcal{M}_K}$$

$$\langle \overline{K^0} | (\bar{s} \gamma_\mu (1 - \gamma_5) d)^2 | K^0 \rangle = 8/3 f_K^2 M_K^2 B_K$$

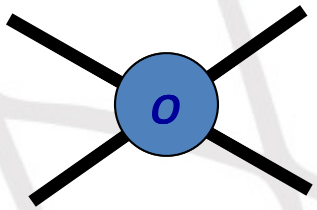
B^0 - B^0 mixing

$$H = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix}$$

ΔB=2 Transitions



$\mathcal{H}_{eff}^{\Delta B=2}$ =



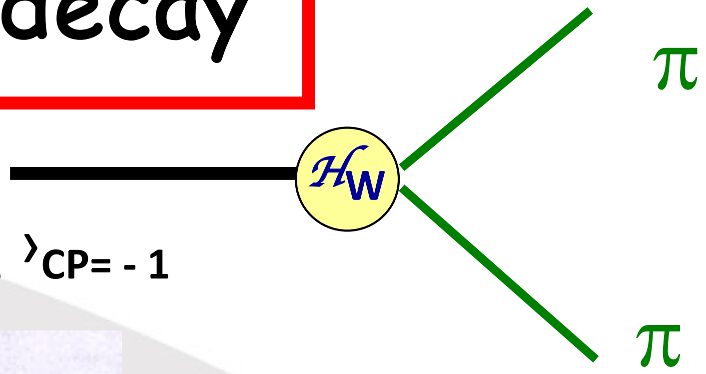
$$\propto \left(\bar{d}\, \gamma_\mu (1 - \gamma_5)\, b \right)^2$$

CKM

Hadronic matrix element

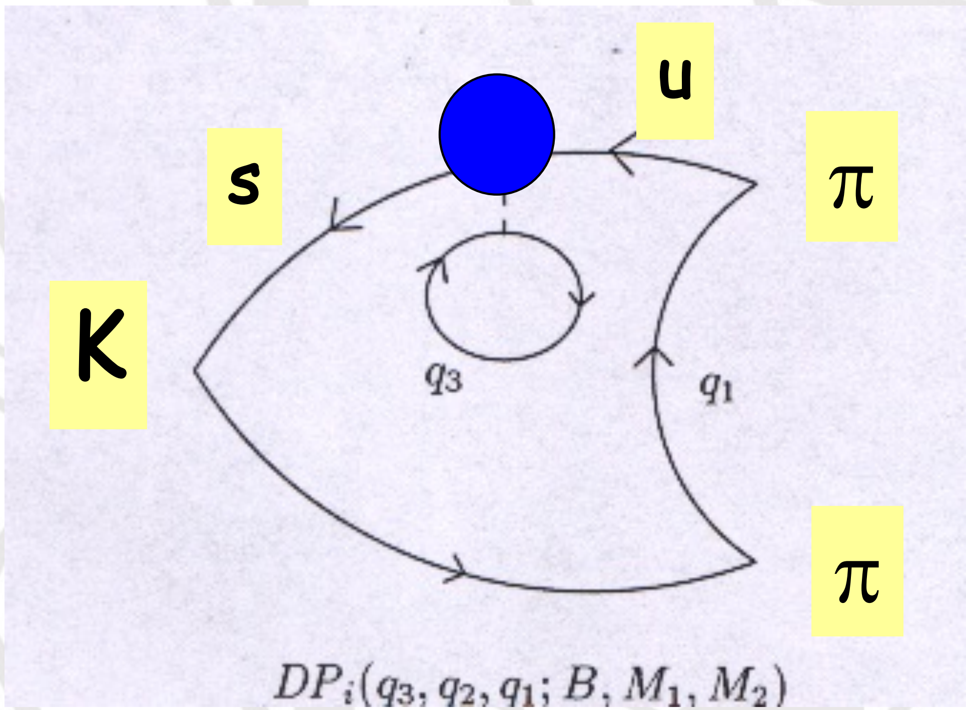
$$\Delta m_{d,s} = \frac{G^2_F\, M^2_W}{16\, \pi^2}\, A^2\, \lambda^6\, F_{tt}\left(\frac{m^2_t}{M^2_W}\right) \langle\, O\, \rangle$$

Direct CP violation: decay



$| K_L \rangle = | K_2 \rangle_{CP=-1}$

CP= + 1



S

Complex ΔS=1 effective coupling

$$\mathcal{L}^{\not{CP}} = \mathcal{L}^{\Delta F=0} + \mathcal{L}^{\Delta F=1} + \mathcal{L}^{\Delta F=2}$$

ΔF=0 $d_e < 1.5\ 10^{-27}$ e cm $d_{\mathcal{N}} < 6.3\ 10^{-26}$ e cm

ΔF=1 ε'/ε

+ B decays (see later)

ΔF=2 ε and $B \longrightarrow J/\psi\ K_s$

The extraordinary progress of the experimental measurements requires accurate theoretical predictions

Precision flavor physics requires the control of hadronic effects for which lattice QCD simulations are essential

$$Q^{EXP} = V_{CKM}\langle F|\hat{O}|I\rangle$$

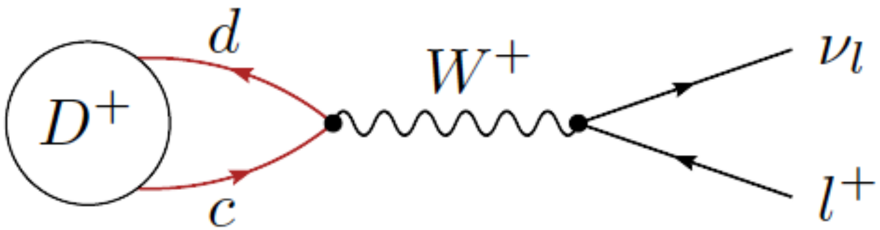
SM

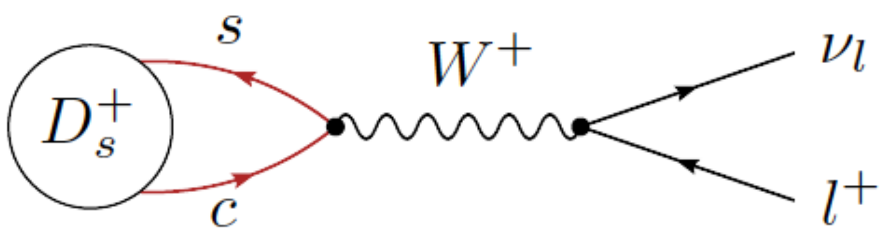
$$Q^{EXP} = \sum_i C^i_{SM}(M_W, m_t, \alpha_s)\langle F|\hat{O}_i|I\rangle + \sum_{i'} C^{i'}_{Beyond}(\tilde{m}_\beta, \alpha_s)\langle F|\hat{O}_{i'}|I\rangle$$

BSM

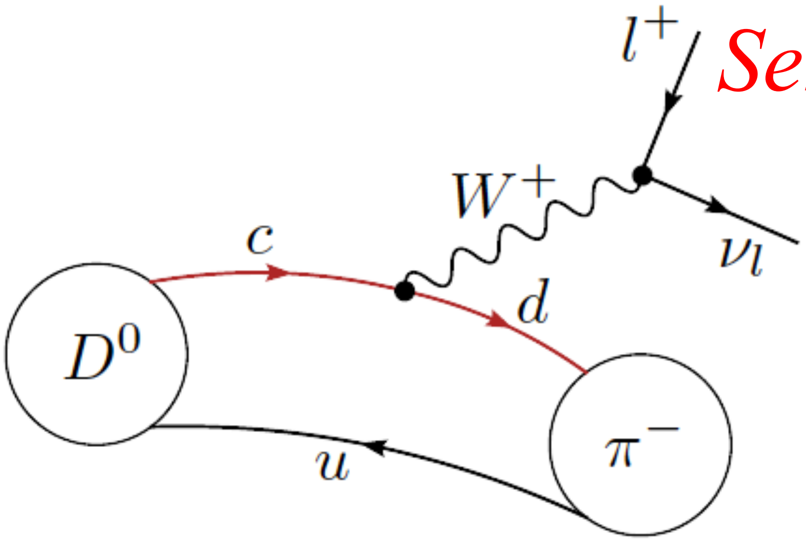
What can be computed and what cannot be computed

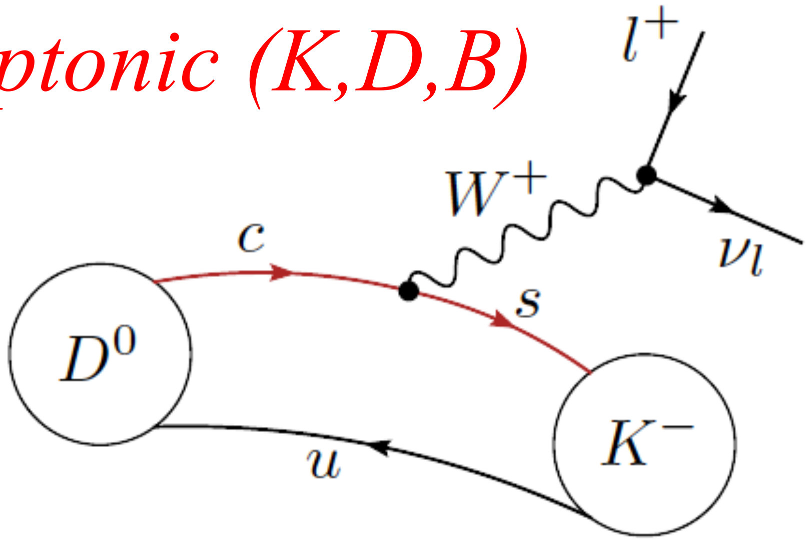
Leptonic (π,K,D,B)





Semileptonic (K,D,B)

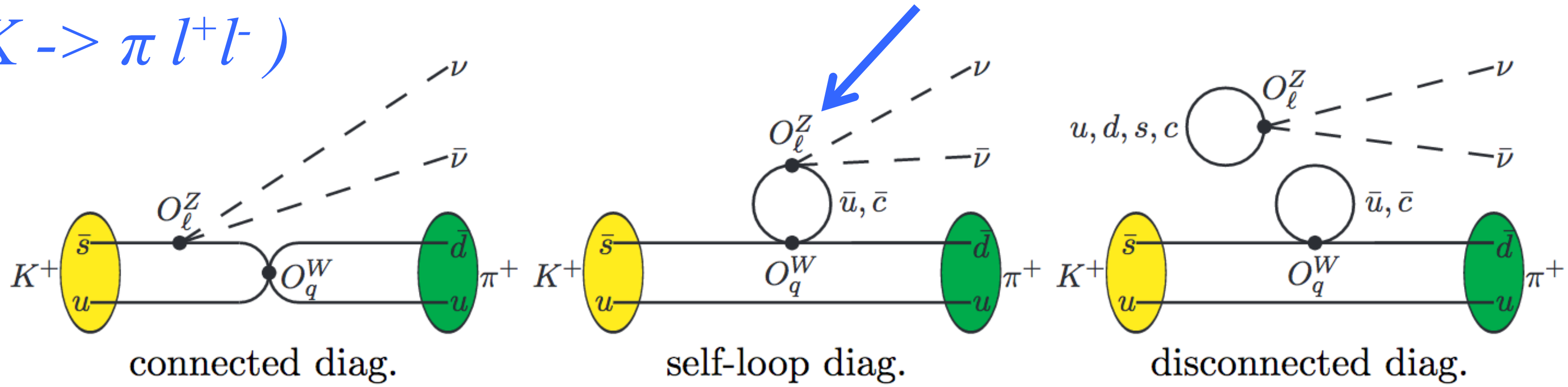




(some) Radiative and Rare

long distance effects

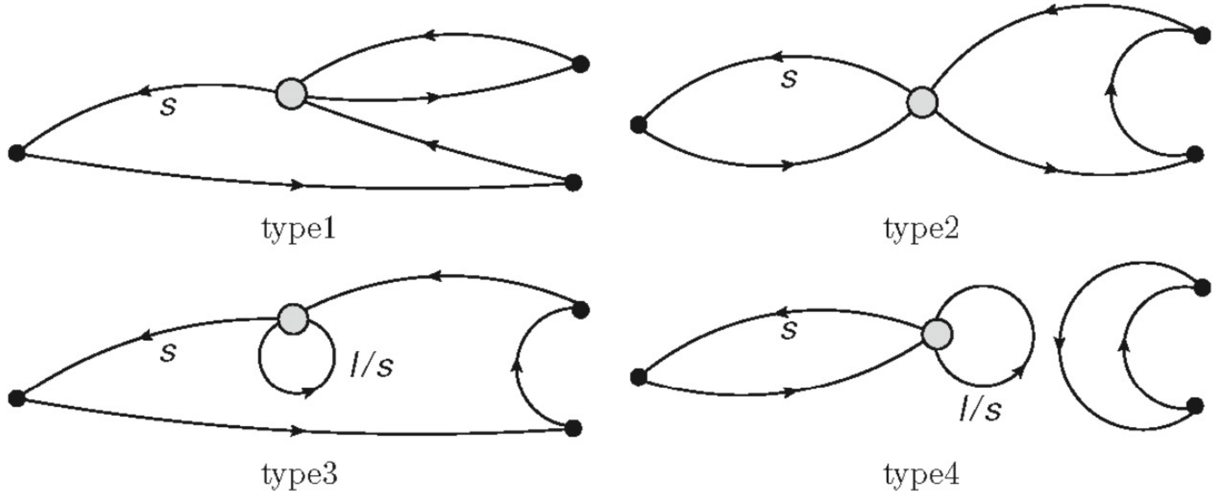
(also $K \to \pi\, l^+ l^-$)



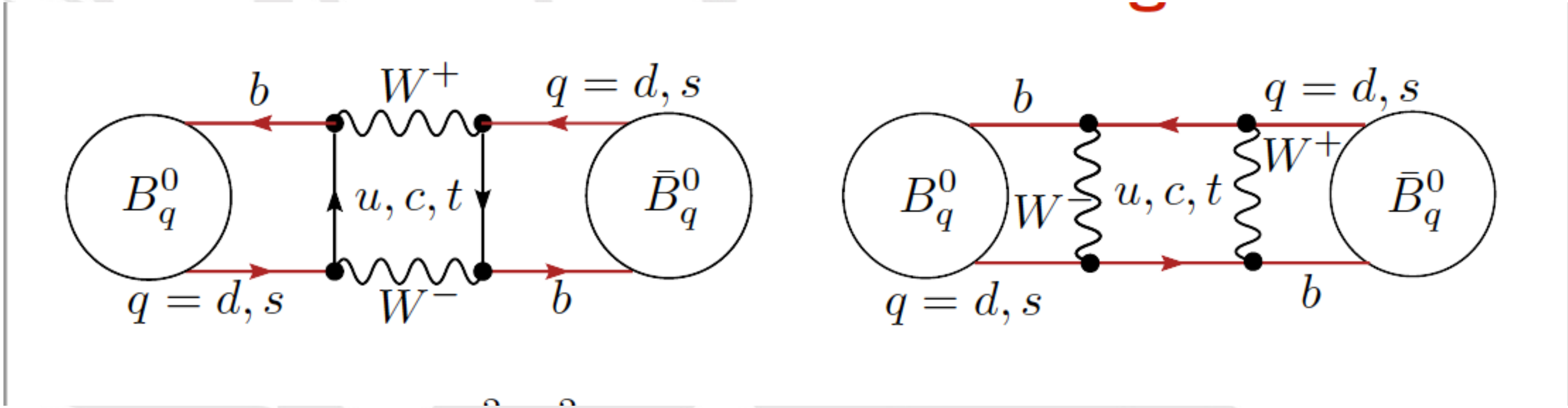
Non-leptonic

but only below the inelastic threshold (may be also 3 body decays)

B -> ππ,Kπ, etc. No !



Neutral meson mixing (local)



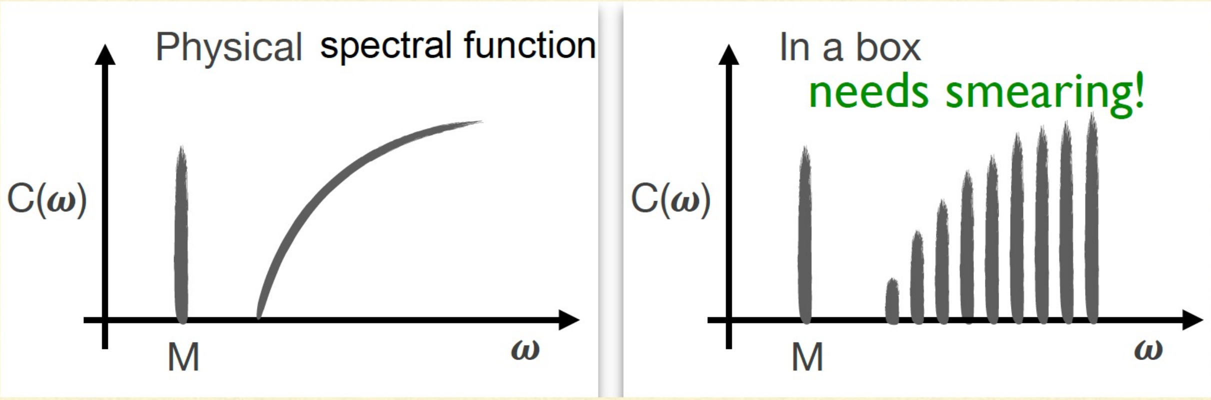
+ some long distance contributions to K and D neutral meson mixing + short distance contributions to B-> $K^{(*)}$ l^+l^-

INCLUSIVE DECAYS ON THE LATTICE

Inclusive processes impractical to treat directly on the lattice. Vacuum current correlators computed in euclidean space-time are related to $e^+e^- \to$ hadrons or τ decay via analyticity. In our case the correlators have to be computed in the B meson, but analytic continuation more complicated: two cuts, decay occurs only on a portion of the physical cut.

While the lattice calculation of the spectral density of hadronic correlators is an ***ill-posed problem***, the spectral density is accessible after smearing

Hansen, Meyer, Robaina, Hansen, Lupo, Tantalo, Bailas, Hashimoto, Ishikawa

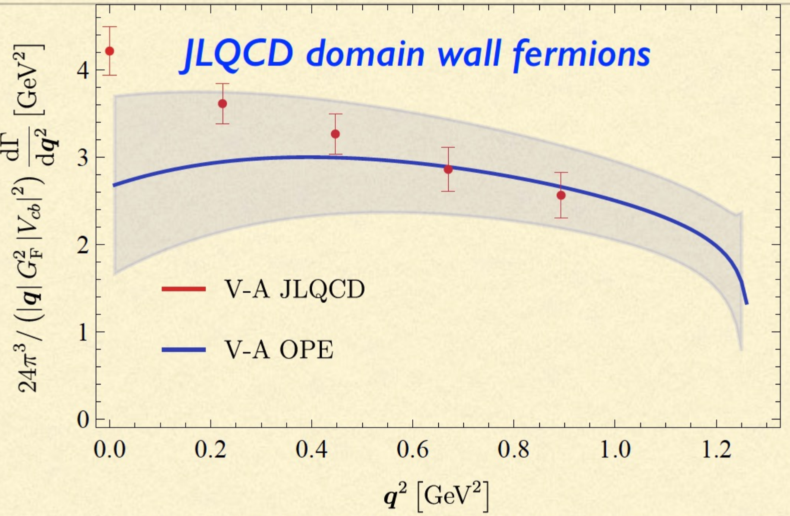


W. Jay @Snowmass workshop

courtesy of P. Gambino

PG, Hashimoto, Maechler, Panero, Sanfilippo, Simula, Smecca, Tantalo, 2203.11762

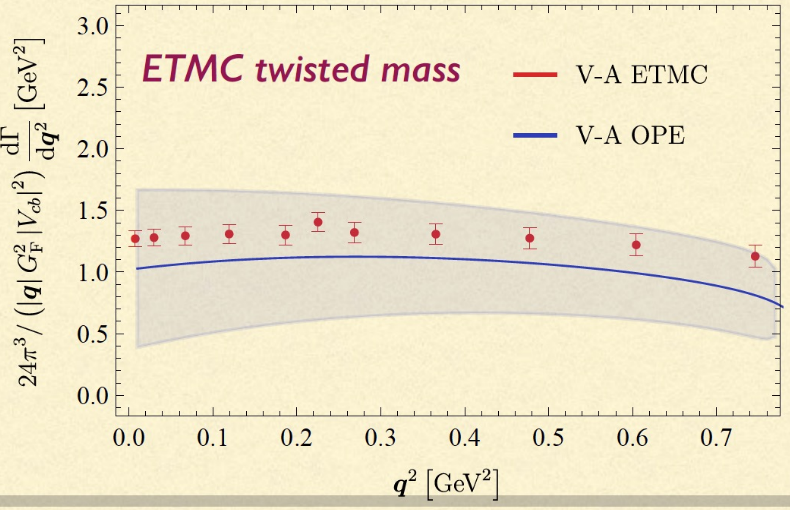
LATTICE vs OPE



m_b^{kin} (JLQCD)	2.70 ± 0.04
$\overline{m}_c$(2 GeV) (JLQCD)	1.10 ± 0.02
m_b^{kin} (ETMC)	2.39 ± 0.08
$\overline{m}_c$(2 GeV) (ETMC)	1.19 ± 0.04
μ_π^2	0.57 ± 0.15
ρ_D^3	0.22 ± 0.06
$\mu_G^2(m_b)$	0.37 ± 0.10
ρ_{LS}^3	-0.13 ± 0.10
$\alpha_s^{(4)}$(2 GeV)	0.301 ± 0.006

OPE inputs from fits to exp data (physical m_b), HQE of meson masses on lattice

1704.06105, J.Phys.Conf.Ser. 1137 (2019) 1, 012005



We include $O(1/m_b^3)$ and $O(\alpha_s)$ terms

Hard scale $\sqrt{m_c^2 + \mathbf{q}^2} \sim 1 - 1.5\,\mathrm{GeV}$

We do not expect OPE to work at high $|\mathbf{q}|$

Twisted boundary conditions allow for any value of $\vec{q}^2$

Smaller statistical uncertainties

courtesy of P. Gambino

Evaluating hadronic amplitudes on the lattice through the spectral representation

Giuseppe Gagliardi, INFN Sezione di Roma Tre

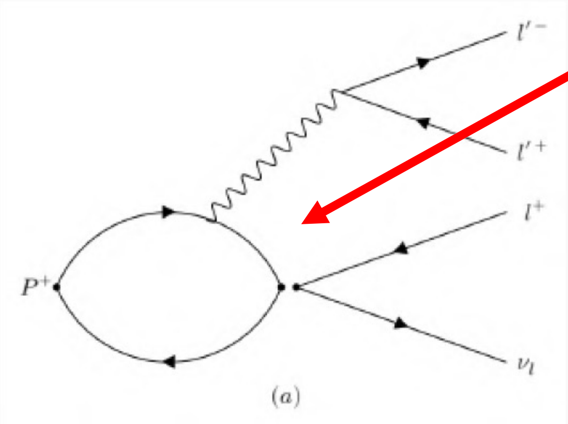
R. Frezzotti, V. Lubicz, G. Martinelli, F. Mazzetti, C.T. Sachrajda,
F. Sanfilippo, S. Simula, N. Tantalo

ETMC meeting, 8-10 February 2023, Bern.

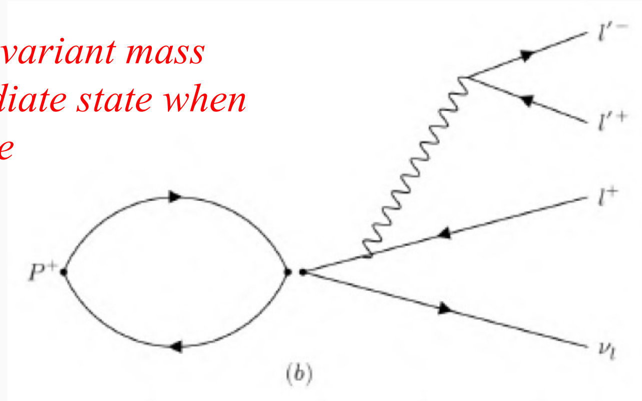
Radiative decays

$D_s^{\pm} \to l'^{+} l'^{-} l^{\pm} \nu_l$ **decays**

The $P^{+} \equiv D\gamma^5 U \to l'^{+} l'^{-} l^{+} \nu_l$ decays



Large invariant mass intermediate state when q^2 large



- Diagram (b) is perturbative, only QCD input is decay constant f_P.
- Diagram (a) is non-perturbative. Virtual photon γ^* emitted from either a $U-$type or a $D-$type quark line. For $P^{+} = D_s^{+}$: $U = c$, $D = s$.

Non-perturbative QCD contribution encoded in the hadronic tensor

$$H_W^{\mu\nu}(k, \boldsymbol{p}) = \int d^4x\, e^{ik\cdot x} \langle 0 | T[J_{\rm em}^{\mu}(x) J_W^{\nu}(0)] | P(\boldsymbol{p}) \rangle, \quad W = V, A$$

- $k = (E_\gamma, \boldsymbol{k})$ is photon 4-momentum, $\boldsymbol{p}$ is P-meson 3-momentum.
- We neglect $\mathrm{SU}(3)$-vanishing quark-line disconnected diagrams.

sin 2β is measured directly from $B \to J/\psi\, K_s$ decays at Babar & Belle

$$\mathcal{A}_{J/\psi\, K_S} = \frac{\Gamma(B_d^0 \to J/\psi\, K_s, t) - \Gamma(\overline{B}_d^0 \to J/\psi\, K_s, t)}{\Gamma(B_d^0 \to J/\psi\, K_s, t) + \Gamma(\overline{B}_d^0 \to J/\psi\, K_s, t)}$$

$$\boxed{\mathcal{A}_{J/\psi\, K_S} = \sin 2\beta \;\; \sin(\Delta m_d t)}$$

DIFFERENT LEVELS OF THEORETICAL UNCERTAINTIES (STRONG INTERACTIONS)

1) First class quantities, with reduced or negligible uncertainties

$$A_{CP}(B \to J/\psi K_s) \quad \gamma \; from \; B \to DK$$
$$K^0 \to \pi^0 \nu \bar{\nu}$$

2) Second class quantities, with theoretical errors of O(10%) or less that can be reliably estimated

$$\varepsilon_K \quad \Delta M_{d,s}$$
$$\Gamma(B \to c, u), \quad K^+ \to \pi^+ \nu \bar{\nu}$$

3) Third class quantities, for which theoretical predictions are model dependent (BBNS, charming, etc.)

In case of discrepacies we cannot tell whether is *new physics or we must blame the model*

$$B \to K\pi \quad B \to \pi^0 \pi^0$$
$$B \to \phi K_s$$

Flavour Physics

1963: Cabibbo Angle
1964: CP violation in K decays *
1970 GIM Mechanism
1973: CP Violation needs at least three quark families (CKM) *
1975: discovery of the tau lepton – 3rd lepton family *
1977: discovery of the b quark - 3rd quark family *
2003/4: CP violation in B meson decays

* Nobel Prize

Discoveries from Flavor Physics

CP Violation !!

- the tiny branching ratio of the decay $K_L \to \mu^+\mu^-$ led to the prediction of the charm quark to suppress FCNCs
  (Glashow, Iliopoulos, Maiani 1970)

- the measurement of the frequency of kaon anti-kaon oscillations allowed a successful prediction of the charm quark mass
  (Gaillard, Lee 1974)

(direct discovery of the charm quark in 1974 at SLAC and BNL)

- the observation of CP violation in kaon anti-kaon oscillations led to the prediction of the 3rd generation of quarks
  (Kobayashi, Maskawa 1973)

- the measurement of the frequency of B - $\bar{B}$ oscillations allowed to predict the large top quark mass
  (various authors in the late 80's)

(direct discovery of the bottom quark in 1977 at Fermilab)
(direct discovery of the top quark in 1995 at Fermilab)

Recent developments in Flavor physics, the Unitarity Fit, Anomalies (Much ado about nothing) and all that

Guido Martinelli
INFN Sezione di Roma
Università La Sapienza

DIPARTIMENTO DI FISICA

SAPIENZA
UNIVERSITÀ DI ROMA

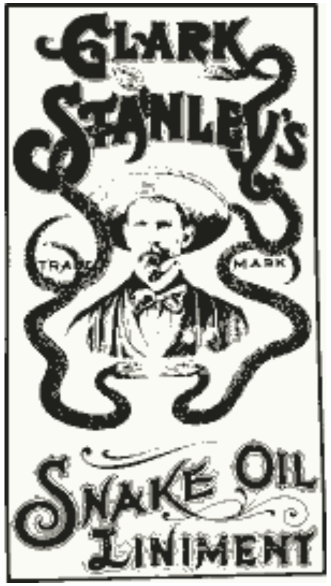
Monopoli 20 September 2023

UTfit

Extended Twisted Mass Collaboration

PRACE

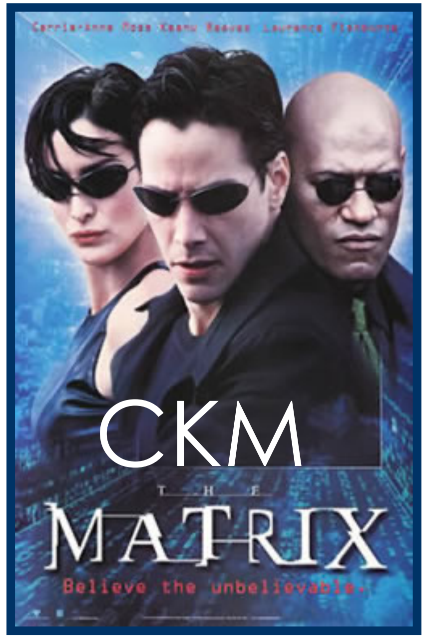
INFN
Istituto Nazionale di Fisica Nucleare



UTfit

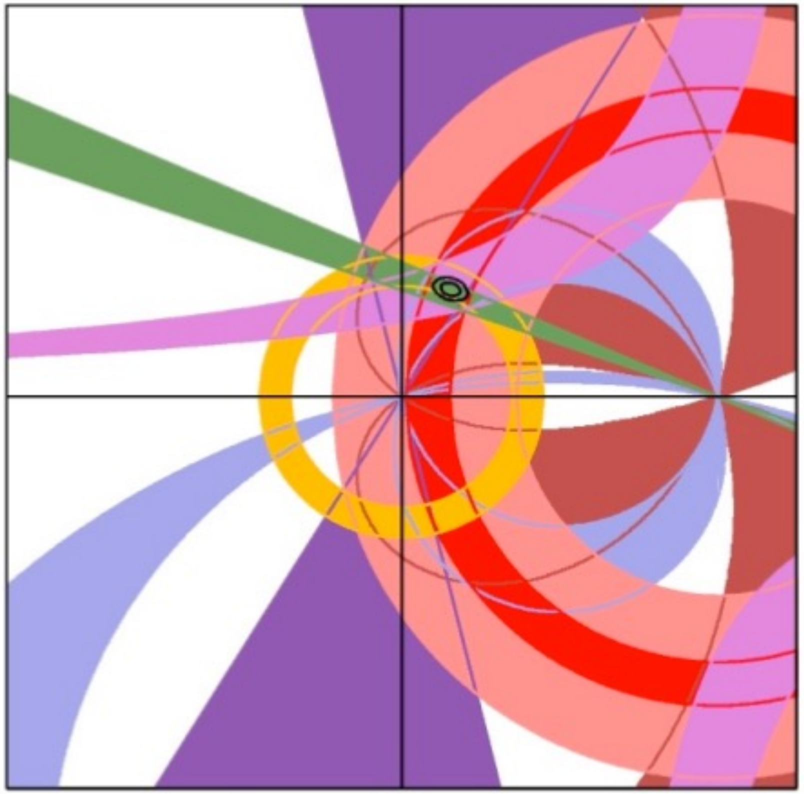
www.utfit.org

RELOADED



M.Bona, M. Ciuchini, D. Derkach, F. Ferrari, E. Franco, V. Lubicz, G. Martinelli, D. Morgante, M. Pierini, L. Silvestrini, S. Simula, C. Tarantino, V. Vagnoni, M. Valli, and L.Vittorio

- *General introduction to the Unitary Triangle Fit*
- *SM Analysis*
- *Tensions and unknown*
- *Future directions, new/old ideas*
- *Conclusion*



New UTfit Analysis of the Unitarity Triangle in the Cabibbo-Kobayashi-Maskawa scheme

Rend.Lincei Sci.Fis.Nat. 34 (2023) 37-57
arXiv:2212.03894

With respect to the published paper several theoretical and experimental new unputs and updated results

Thanks to
M. Bona, A. Di Domenico, C. Kelly, V. Lubicz, C. Sachrajda, L. Silvestrini, S. Simula, L. Vittorio

Flavor physics in the Standard Model

In the SM, the quark mass matrix, from which the CKM matrix and CP violation originate, is determined by the coupling of the Higgs boson to fermions.

$$\mathcal{L}^{\mathbf{quarks}} = \mathcal{L}^{\mathbf{kinetic}} + \mathcal{L}^{\mathbf{gauge}} + \mathcal{L}^{Yukawa}$$

CP invariant

CP and symmetry breaking are striclty correlated

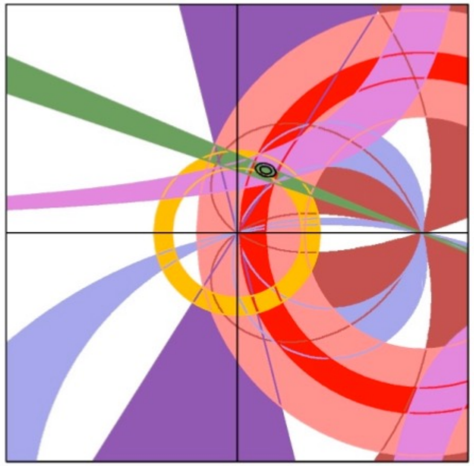
$$\mathcal{L}(\Lambda_{Fermi}) = \mathcal{L}(\Lambda, H, H^\dagger) + \mathcal{L}^{kin} + \mathcal{L}_{SM}^{gauge} + \mathcal{L}_{SM}^{Yukawa} + \frac{\mathcal{L}_5}{\Lambda} + \frac{\mathcal{L}_6}{\Lambda^2} + \ldots$$

EWSB

has many accidental simmetries

may violate accidental simmetries

STANDARD MODEL UNITARITY TRIANGLE ANALYSIS (Flavor Physics)

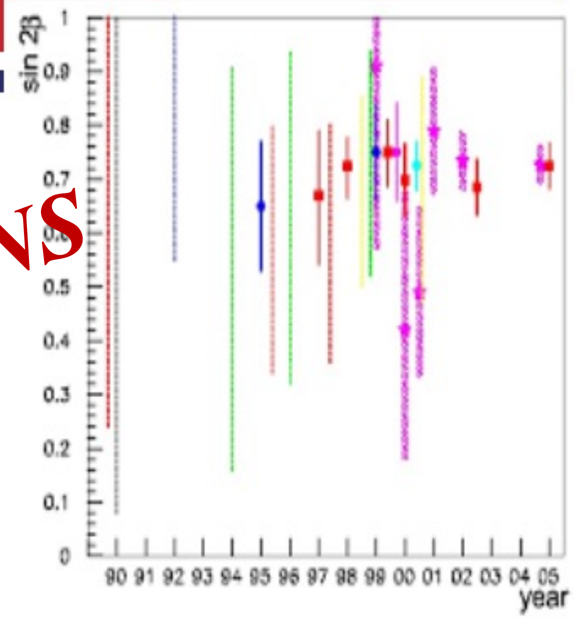


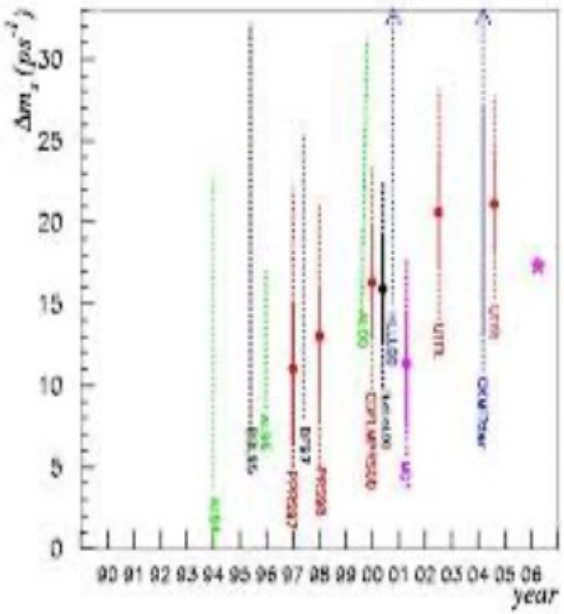
- *Provides the best determination of the CKM parameters;*
- *Tests the consistency of the SM (``direct" vs ``indirect" determinations) @ the quantum level;*
- *Provides predictions for SM observables (in the past for example sin 2β and Δm_s)*
- *It could lead to new discoveries (CP violation, Charm, !?)*
- *The discovery potential of precision flavor physics should not be underestimated*

30 years of UT fit

- Since early '90s, the UT framework has been established to probe CP violation in the flavor sector
  - sin2b (CPV in $B_d\bar{B}_d$ mixing) the reference quantity
  - very loose predictions once its value
  - jump in accuracy ~ '95, when the first full statistical analysis was attempted, strongly benefiting of the first determination of the top mass. The UT analysis was born, predicting a few still unknown quantities
    - $\sin 2\beta = 0.65 \pm 0.12$
  - In 2000, Rome and Orsay/Genova groups (running similar fits) joined forces. This was the beginning of the UTfit collaboration

PREDICTIONS





2000 CKM-TRIANGLE ANALYSIS
A Critical Review with Updated Experimental Inputs and Theoretical Parameters

M. Ciuchini[a], G. D'Agostini[b], E. Franco[b], V. Lubicz[a], G. Martinelli[b], F. Parodi[c], P. Roudeau[d] and A. Stocchi[d]

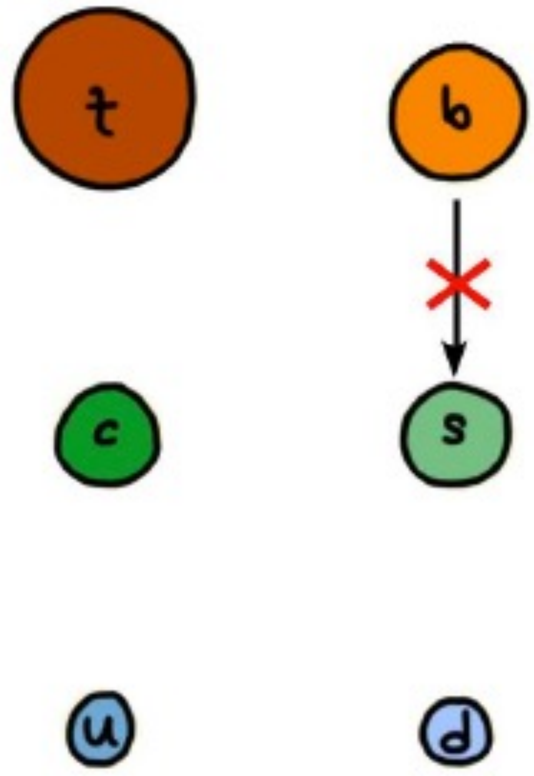
Courtesy by M. Pierini

Absence of FCNC at tree level (& GIM suppression of FCNC @loop level)

Almost no CP violation at tree level

Flavour Physics is extremely sensitive to New Physics (NP)

In competition with Electroweak Precision Measurements



WHY RARE DECAYS ?

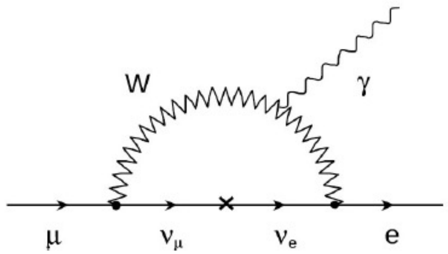
Rare decays are a manifestation of broken (accidental) symmetries e.g. of physics beyond the Standard Model

Proton decay — **baryon and lepton number conservation**

$\mu \rightarrow e + \gamma$

lepton flavor number

$\nu_i \rightarrow \nu_k$ **found !**



$$\mathcal{B}(\mu \to e\gamma) \sim \alpha \frac{m_\nu^4}{m_W^4} \sim 10^{-52}$$

RARE DECAYS WHICH ARE ALLOWED IN THE STANDARD MODEL

FCNC:

$q_i \; -> \; q_k + \nu \; \bar{\nu}$

$q_i \; -> \; q_k + l^+ \; l^-$

$q_i \; -> \; q_k + \gamma$

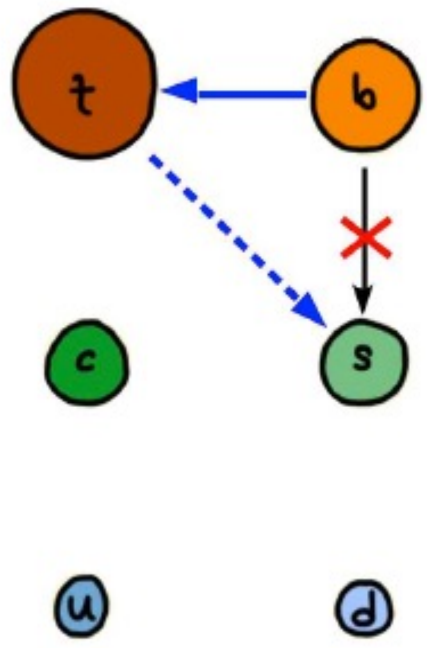
these decays occur only via loops because of GIM and are suppressed by CKM

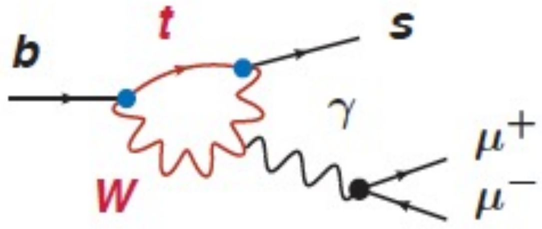
THUS THEY ARE SENSITIVE TO NEW PHYSICS

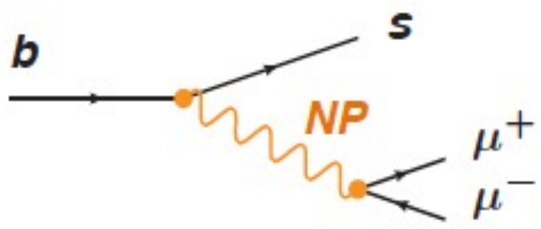
Flavor Changing Neutral Currents in the SM

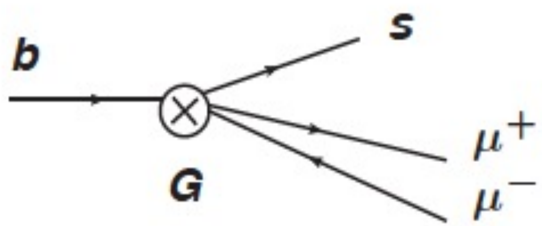
In the SM, flavor changing neutral currents (FCNCs)
are absent at the tree level

FCNCs can arise at the loop level
they are suppressed by loop factors
and small CKM elements









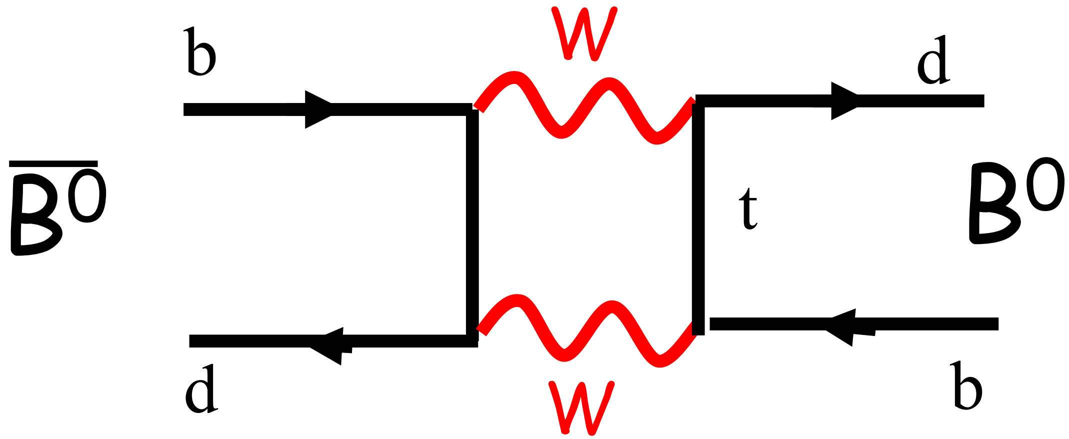
$$G \sim \frac{1}{16\pi^2} \frac{g^4}{m_W^2} \frac{m_t^2}{m_W^2} V_{tb} V_{ts}^* + \frac{C_{NP}}{\Lambda_{NP}^2}$$

→ measuring low energy flavor observables gives information
on new physics flavor couplings and the new physics mass scale

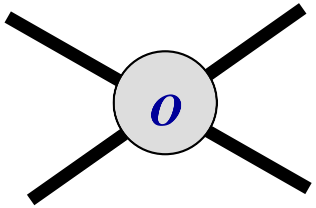
B^0 - $\overline{B^0}$ mixing

$$H = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix}$$

ΔB=2 Transitions



$$\mathcal{H}_{eff}^{\Delta B=2} =$$



$$\propto \left(\bar{d}\, \gamma_\mu (1 - \gamma_5)\, b \right)^2$$

Hadronic matrix element

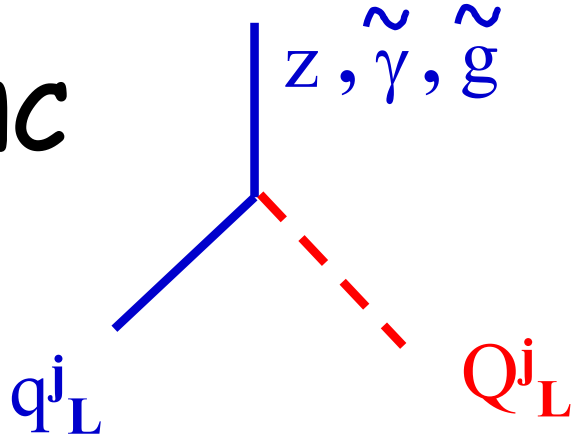
CKM

$$\Delta m_{d,s} = \frac{G_F^2 M_W^2}{16 \pi^2} A^2 \lambda^6 F_{tt}\left(\frac{m_t^2}{M_W^2}\right) \langle O \rangle$$

In general the mixing mass matrix of the SQuarks (SMM) is not diagonal in flavour space analogously to the quark case We may either

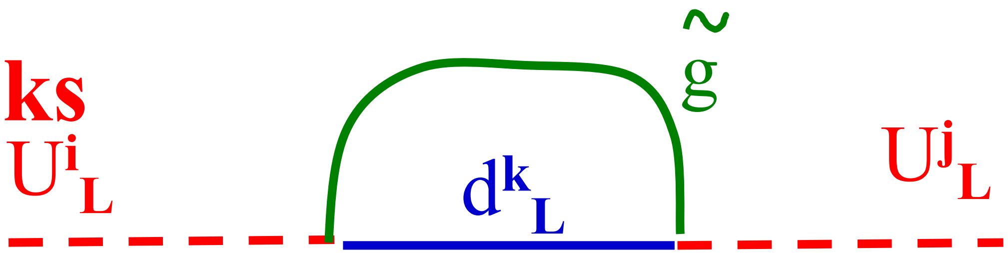
Diagonalize the SMM

FCNC

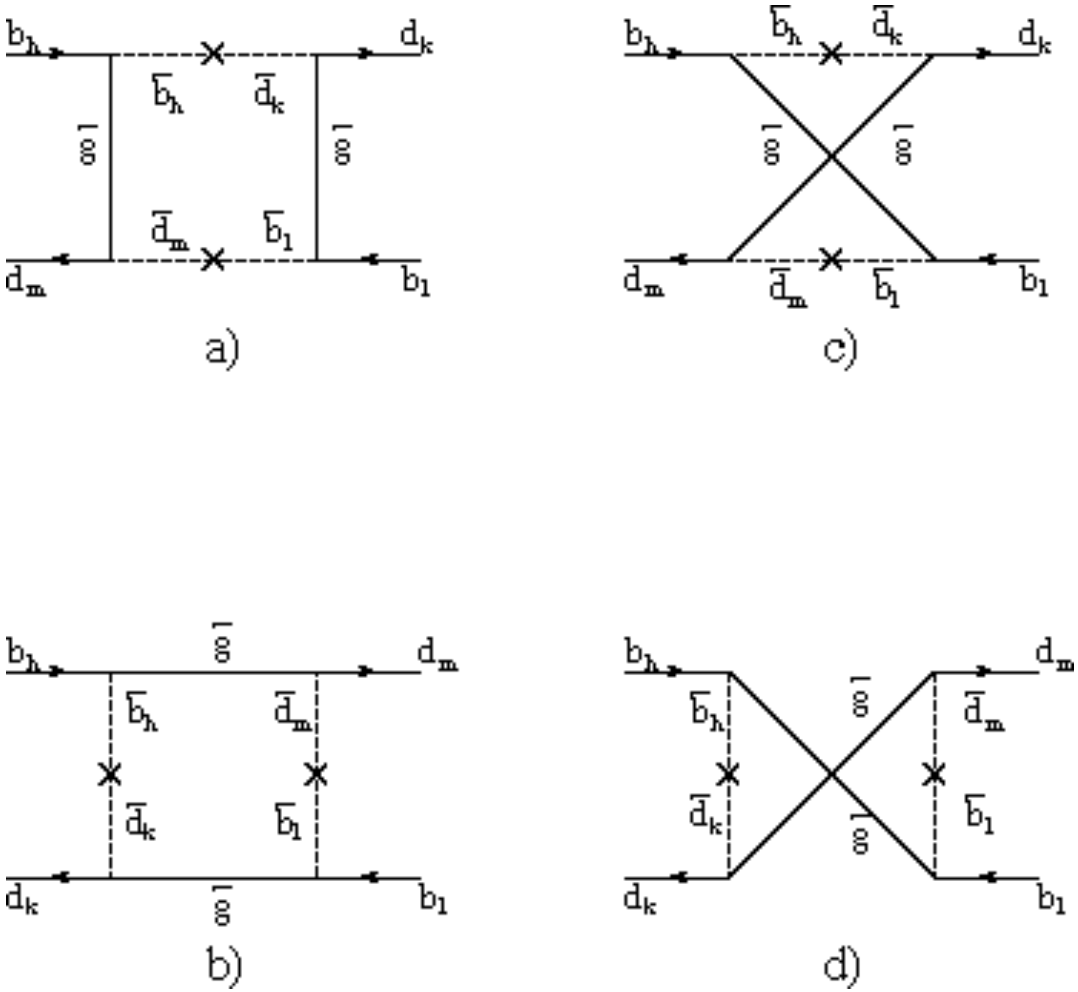


or **Rotate by the same matrices the SUSY partners of the u- and d- like quarks**

$(Q^j_L)' = U^{ij}_L \, Q^j_L$

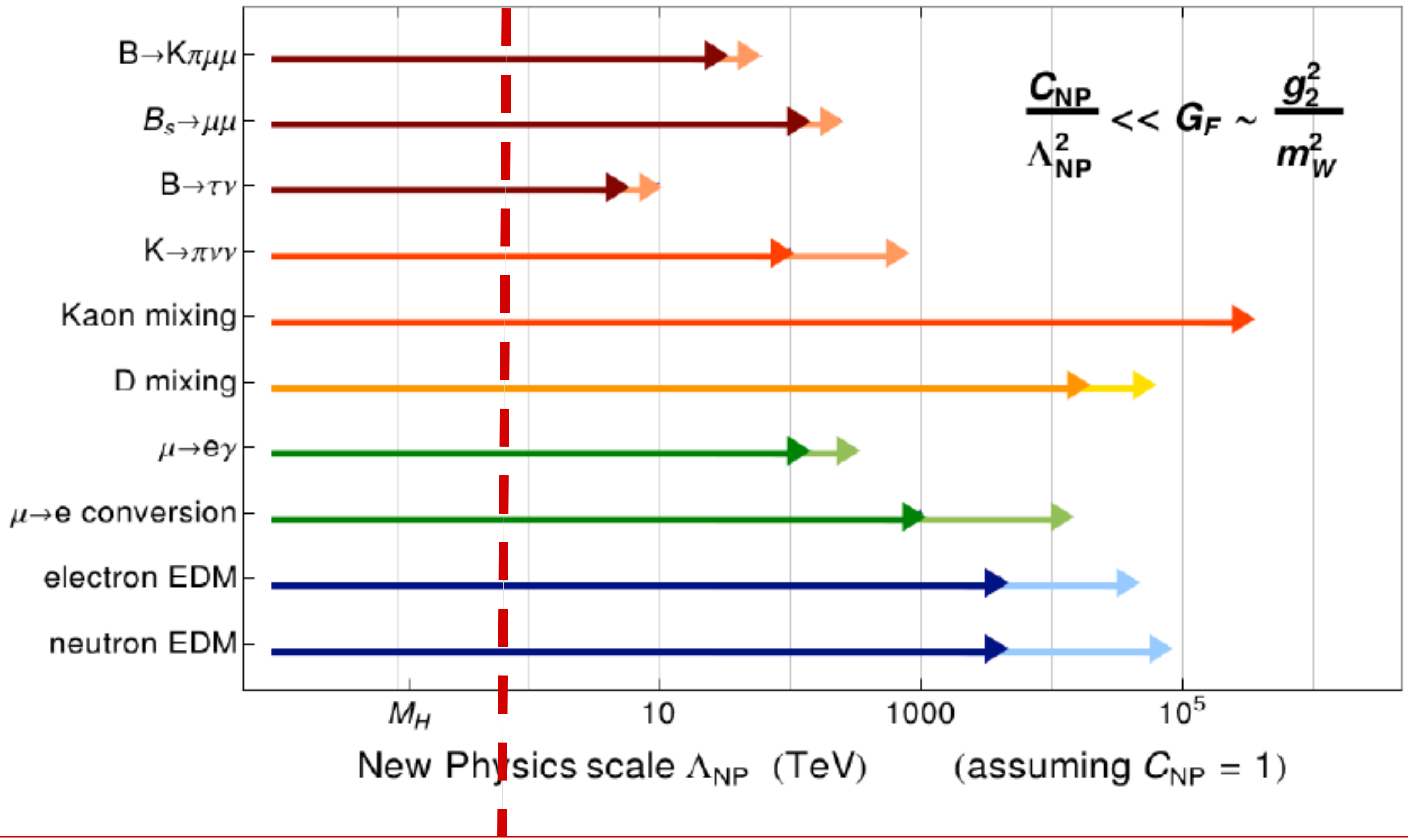


In the latter case the Squark Mass Matrix is not diagonal



$$(m^2{}_Q)_{ij} = m^2{}_{average}\, 1_{ij} + \Delta m_{ij}{}^2 \qquad \delta_{ij} = \Delta m_{ij}{}^2 / m^2{}_{average}$$

Sensitivity to New Physics from Flavor



Approximate LHC direct reach

N(N-1)/2 angles and (N-1)(N-2) /2 phases

N=3 3 angles + 1 phase KM

the phase generates complex couplings i.e. CP violation

6 masses +3 angles +1 phase = 10 parameters

The Unitarity Triangle Analysis

- Flavor-changing processes and CP violation in the SM ruled by 4 parameters in the 3x3 CKM (unitary) matrix

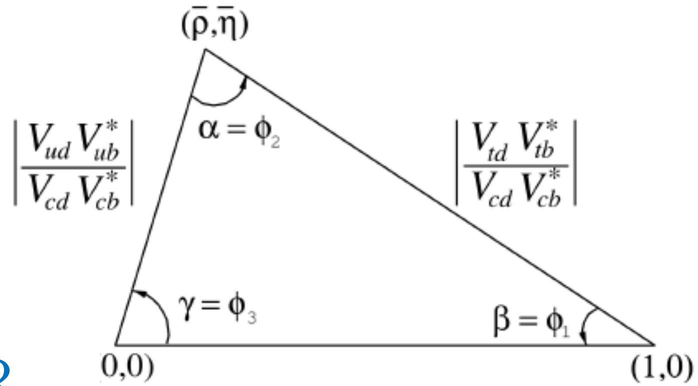
$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

- $A, \lambda, \bar\rho$ and $\bar\eta$

$$\bar\rho = \rho(1-\lambda^2/2+\ldots) \quad \bar\eta = \eta(1-\lambda^2/2+\ldots)$$

- Small value sin of Cabibbo angle (λ) makes the CKM matrix close to diagonal

- Unitarity implies relations between elements, that can be represented as a triangle in a plane



$\sin\theta_{12} = \lambda$

$\sin\theta_{23} = A\,\lambda^2$

$\sin\theta_{13} = A\,\lambda^3(\rho - i\,\eta)$

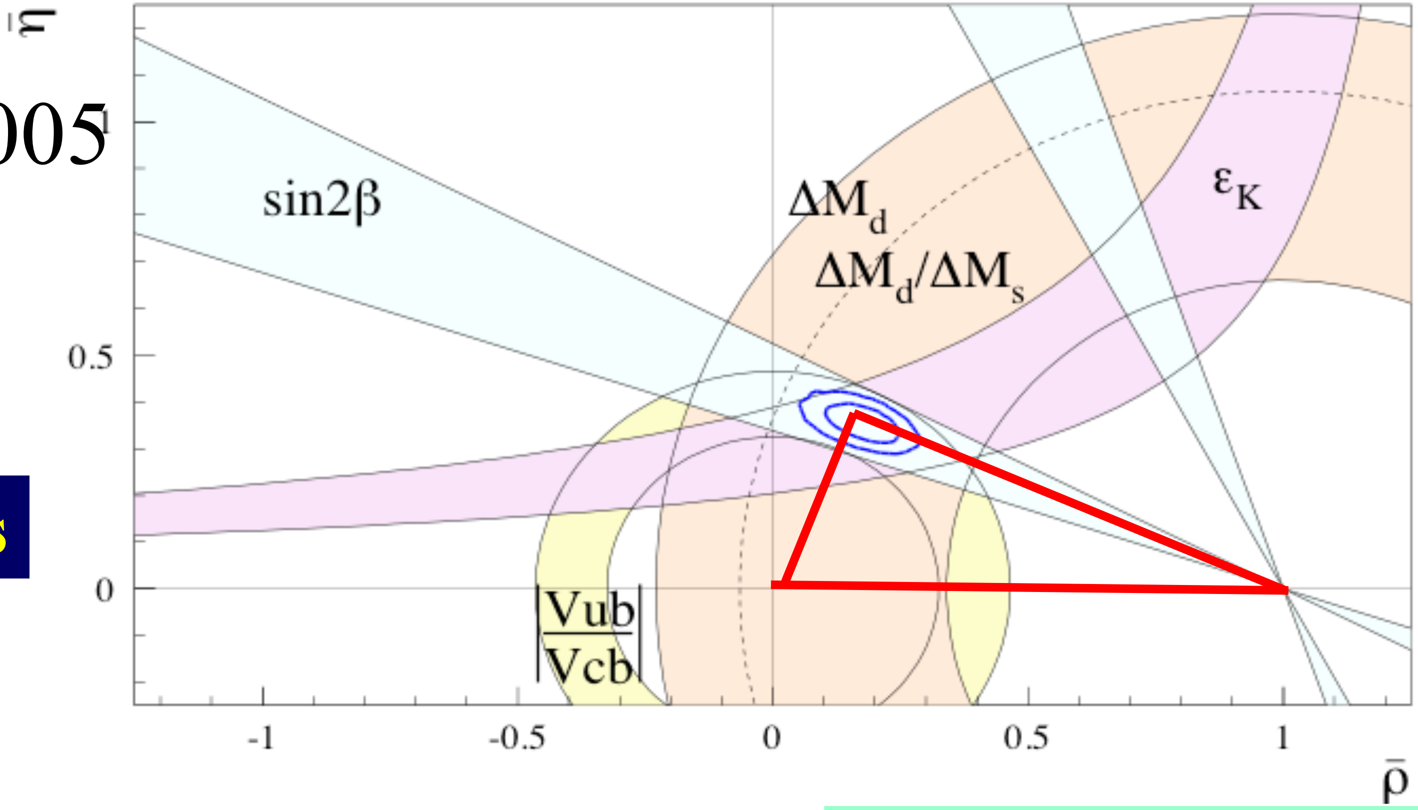
$\lambda \sim 0.2 \quad A \sim 0.8$

$\eta \sim 0.2 \quad \rho \sim 0.3$

UTfit

Unitary Triangle SM

2005



semileptonic decays

Experimental coı

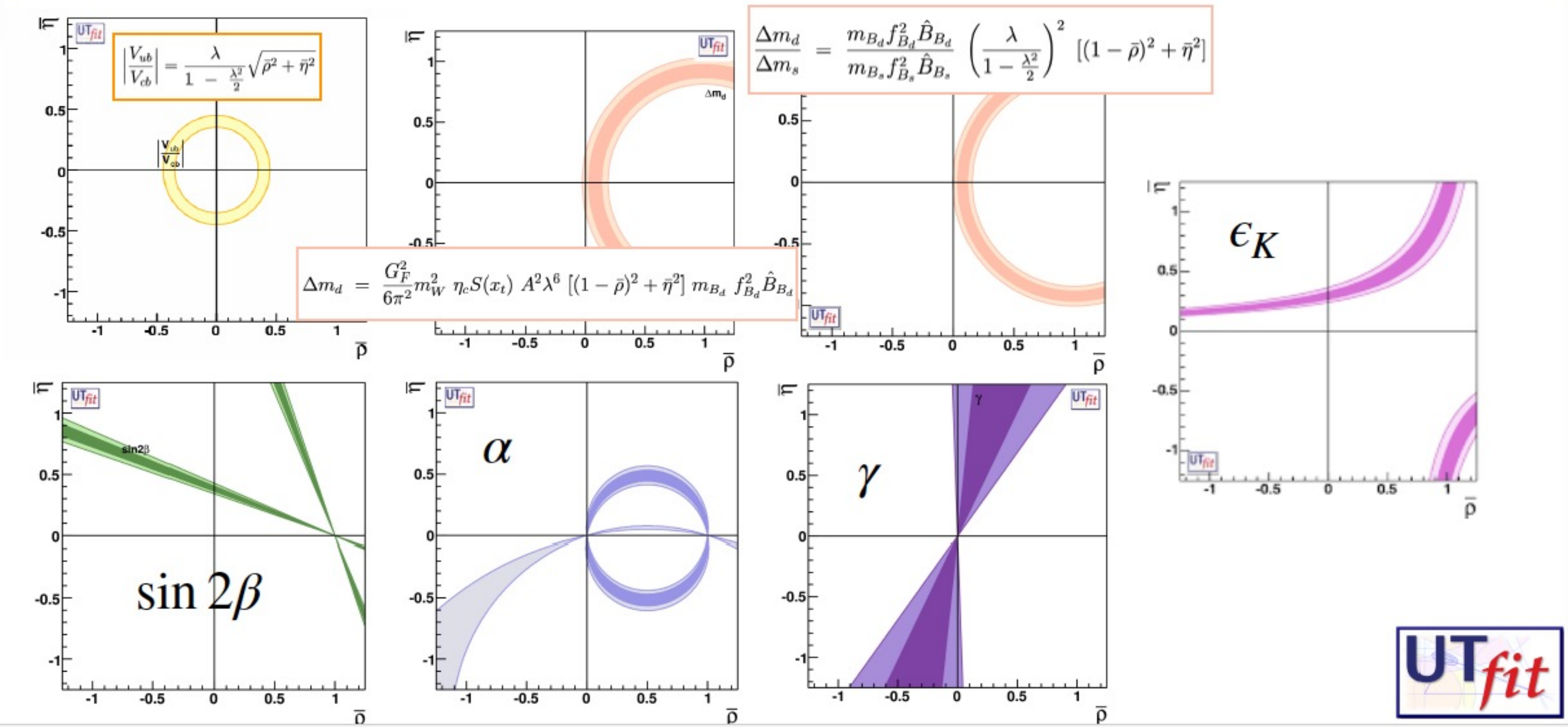
Meas.	$V_{CKM}\times$other	$(\bar{\rho},\bar{\eta})$
$\frac{b\to u}{b\to c}$	$\lvert V_{ub}/V_{cb}\rvert^2$	$\bar{\rho}^2+\bar{\eta}^2$
Δm_d	$\lvert V_{td}\rvert^2 f_{B_d}^2 B_{B_d}$	$(1-\bar{\rho})^2+\bar{\eta}^2$
$\frac{\Delta m_d}{\Delta m_s}$	$\left\lvert\frac{V_{td}}{V_{ts}}\right\rvert^2 \xi^2$	$(1-\bar{\rho})^2+\bar{\eta}^2$
ϵ_K	$f(A,\bar{\eta},\bar{\rho},B_K)$	$\propto \bar{\eta}(1-\bar{\rho})$
$A(J/\psi K^0)$	$sin2\beta$	$\frac{2\bar{\eta}(1-\bar{\rho})}{\sqrt{\bar{\eta}^2+(1-\bar{\rho})^2}}$

K^0 - $\overline{K}^0$ mixing

$B^0_{d,s}$ - $\overline{B}^0_{d,s}$ mixing

B_d

UT constraints



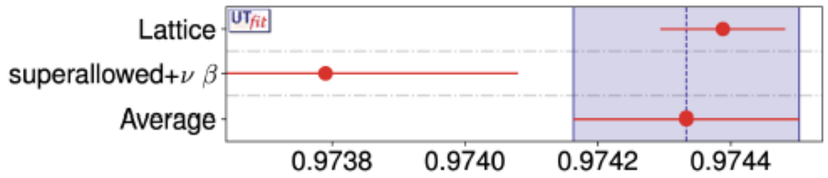
redundancy is the big strength of the UT analysis

one can remove a subset of inputs and still determine the CKM

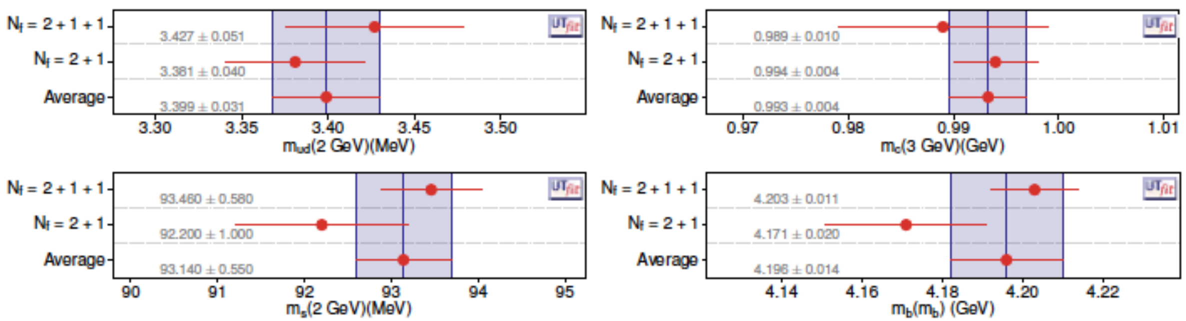
one can exclude η=0 using only CP conserving processes

What's new for EPS23

- Theory updates:
  - New V_{ud} extraction from neutron decays, following V. Cirigliano et al. arXiv:2306.03138
  - New lattice values for masses
  - New lattice form factors for exclusive $b \to q\ell\nu$
- Experiment updates:
  - New sin2β by LHCb
  - New γ by LHCb
  - New α
  - …

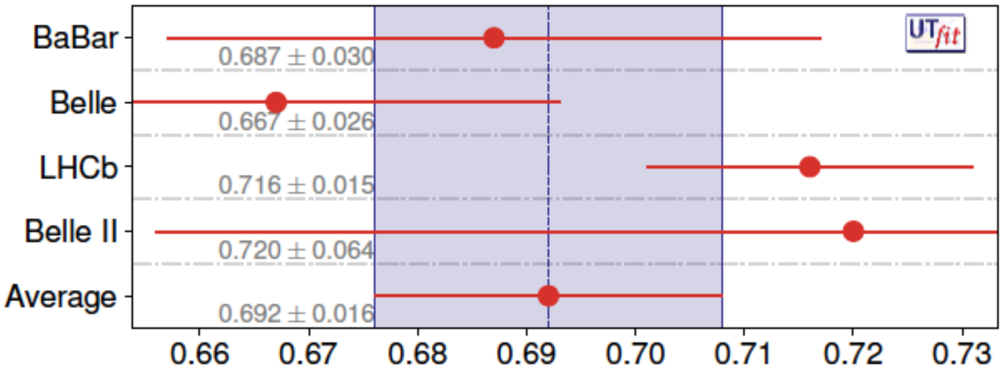


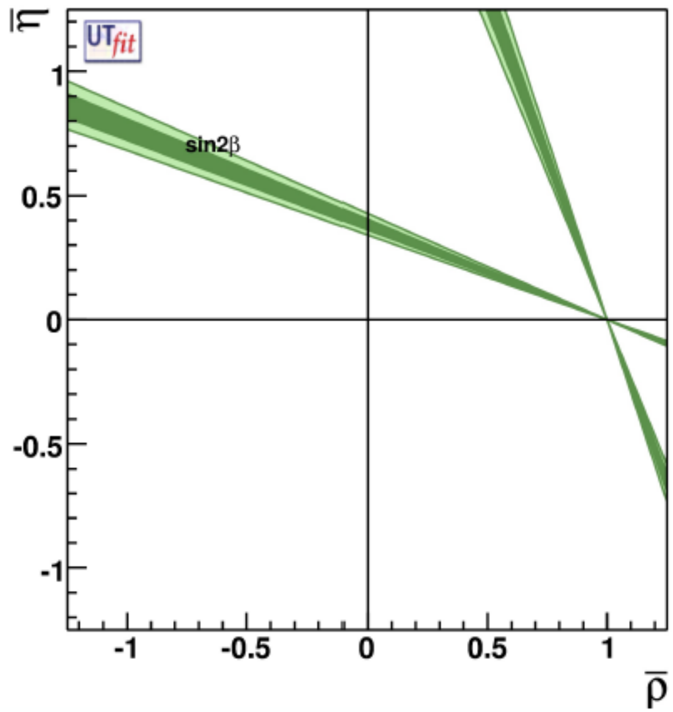
All masses computed in $\overline{\mathrm{MS}}$ and averaged with PDG scale factors



What's new for EPS23: $sin(2\beta)$

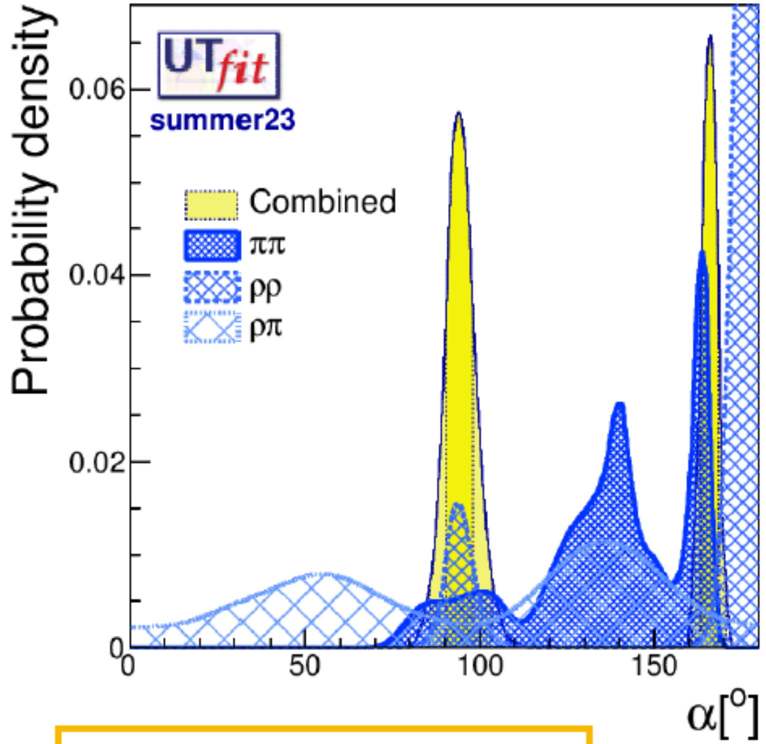
- Averaged charmonium values
- New sin2β from LHCb
- Average including correction due to Cabibbo-suppressed penguin contribution:
  - Most recent estimate $\Delta(\sin 2\beta) = -0.1 \pm 0.1$
  - Theoretical uncertainty comparable to experimental error





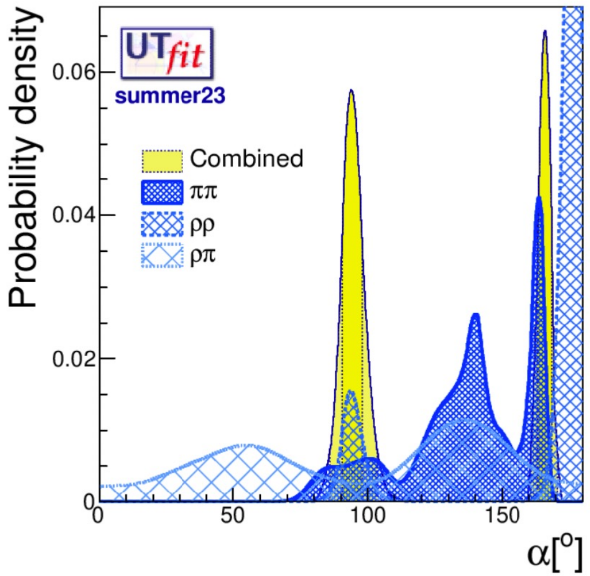
What's new for EPS23

- Updated the bound on α with
  - Bounds from ππ and ρρ derived from PDG averages (including PDG rescaling of the error)
  - Bound from ρπ derived from same inputs used by HFLAV
- As usual, main difference wrt other combinations is in the treatment of the multiple solutions
  - Profiling vs marginalization: in our case, multiple overlapping solutions counts more than a single solution when integrating out the other quantities (T, P, and strong phases)

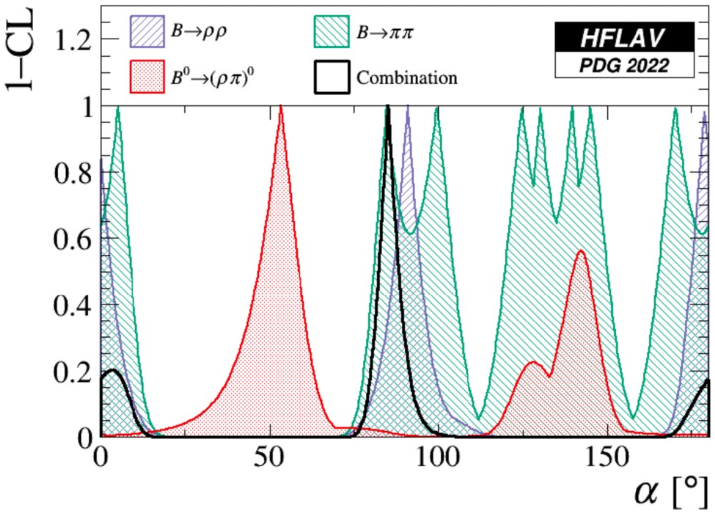


$$\alpha = (93.8 \pm 4.5)^o$$

More on α



$\alpha^{exp} = 93.8° \pm 4.5°$



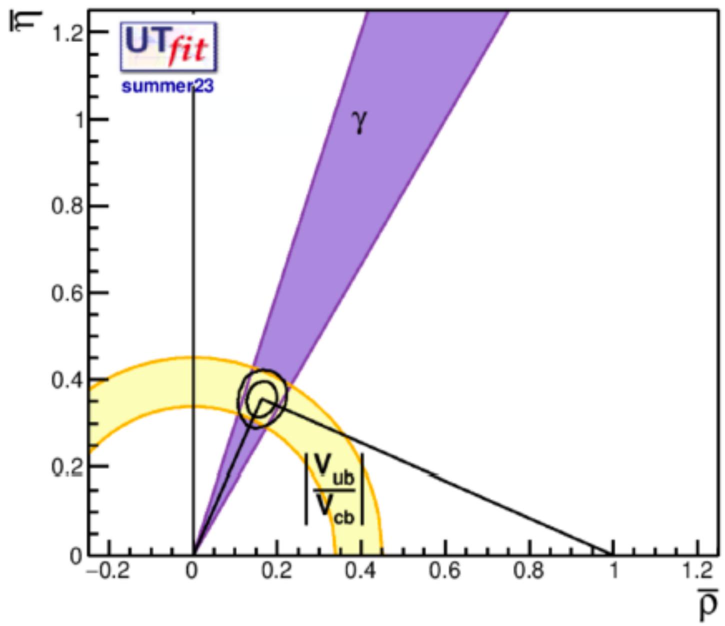
$\alpha_{HFLAV} = 85.5 \pm 4.6$

Inputs are slighly different from what HFLAV because for the BR averages we use the PDG (with the error inflation if there is a tension), while HFLAV would use their averages without error inflation.
So the pipi BR inputs are slightly different. We also use the updated rhopi.

HFLAV
It seems that the reason why the combination falls on the pipi solution on the left of the rhorho peak (while the right solution would be just as probable and even not distinguishable) is due to the small bump from the rhopi distribution which instead goes to zero for the pipi solution on the right.

What's new for EPS23

- Determination combining all $D^{(*)}K^{(*)}$ modes
  - Simultaneous extraction of γ and $D\bar{D}$ mixing parameters (which enter the BSM analysis)
  - ~~Details are given in dedicated talk by R. Di Palma on Friday~~
- Tree-level determination
  - Baseline determination of CP violation in the SM, assuming BSM effects enter only at loop
  - With $|V_{ub}/V_{cb}|$, allows for a robust fit of the CKM parameters in the SM, even in presence of new physics



$$\bar{\rho} = \pm 0.163 \pm 0.024$$
$$\bar{\eta} = \pm 0.356 \pm 0.027$$

UTfit

CERN

Di Palma and Silvestrini in preparation

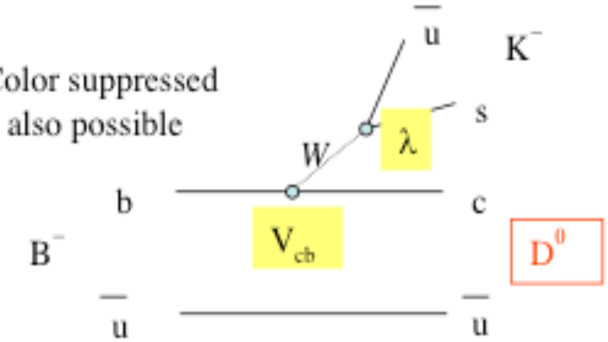
Very old slides

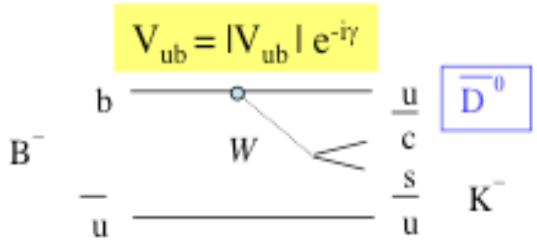
γ from $B \to D^{(*)} K^{(*)}$

Direct CP violation occurs because there are two different ways of reaching the same final state

In this particular case sensitive to γ
D^0 and $\overline{D}^0$ are involved

Color suppressed also possible





$$A(B^- \to D^0 K^-) = A_B$$

$$A(B^+ \to \overline{D}^0 K^+) = A_B$$

$$A(B^- \to \overline{D}^0 K^-) = A_B r_B e^{i(\delta_B - \gamma)}$$

$$A(B^+ \to D^0 K^+) = A_B r_B e^{i(\delta_B + \gamma)}$$

A_B strong amplitude (the same for V_{ub} and V_{cb} mediated transitions

$\delta_B = \delta_1 - \delta_2$ strong phase difference between V_{ub} and V_{cb} mediated transitions

$$r_B = \left| \frac{A(B^- \to \overline{D}^0 K^-)}{A(B^- \to D^0 K^-)} \right|$$

GLW (Gronau,London,Wyler) Method

$$|D^0_{CP\pm}\rangle = \frac{1}{\sqrt{2}}\left(|D^0\rangle \pm |\overline{D}^0\rangle \right)$$

Look at D^0(CP) states

$$\sqrt{2}A(B^+ \to D^0_{CP+}K^+) = A(B^+ \to D^0 K^+) + A(B^+ \to \overline{D}^0 K^+)$$

$$\sqrt{2}A(B^- \to D^0_{CP+}K^-) = A(B^- \to D^0 K^-) + A(B^- \to \overline{D}^0 K^-)$$

$$\sqrt{2}A(B^+ \to D^0_{CP-}K^+) = A(B^+ \to D^0 K^+) - A(B^+ \to \overline{D}^0 K^+)$$

$$\sqrt{2}A(B^- \to D^0_{CP-}K^-) = A(B^- \to D^0 K^-) - A(B^- \to \overline{D}^0 K^-)$$

ADS (Atwood, Dunietz, Soni) Method

D^0 and $\overline{D}^0 \to f$

D^0 and $\overline{D}^0$ give the same final

GLW (Gronau,London,Wyler) Method

$$A_{CP\pm} = \frac{\Gamma(B^+ \to D^0_{CP\pm}K^+) - \Gamma(B^- \to D^0_{CP\pm}K^-)}{\Gamma(B^+ \to D^0_{CP\pm}K^+) + \Gamma(B^- \to D^0_{CP\pm}K^-)} = \frac{\pm 2r_B \sin\gamma \sin\delta_B}{1 + r_B{}^2 \pm 2r_B \cos\gamma\cos\delta_B}$$

$$R_{CP\pm} = \frac{\Gamma(B^+ \to D^0_{CP\pm}K^+) + \Gamma(B^- \to D^0_{CP\pm}K^-)}{\Gamma(B^+ \to \overline{D}^0 K^+) + \Gamma(B^- \to D^0 K^-)} = 1 + r_B{}^2 \pm 2r_B \cos\gamma\cos\delta_B$$

ADS (Atwood, Dunietz, Soni) Method (only Babar)

$$R_{ADS} = \frac{\Gamma(B^- \to [K^+\pi^-]_D K^-) + \Gamma(B^+ \to [K^-\pi^+]_D K^+)}{\Gamma(B^- \to [K^-\pi^+]_D K^-) + \Gamma(B^+ \to [K^+\pi^-]_D K^+)}$$
$$= r^2_{DCS} + r^2_B + 2r_B r_{DCS}\cos\gamma\cos(\delta_B + \delta_D)$$

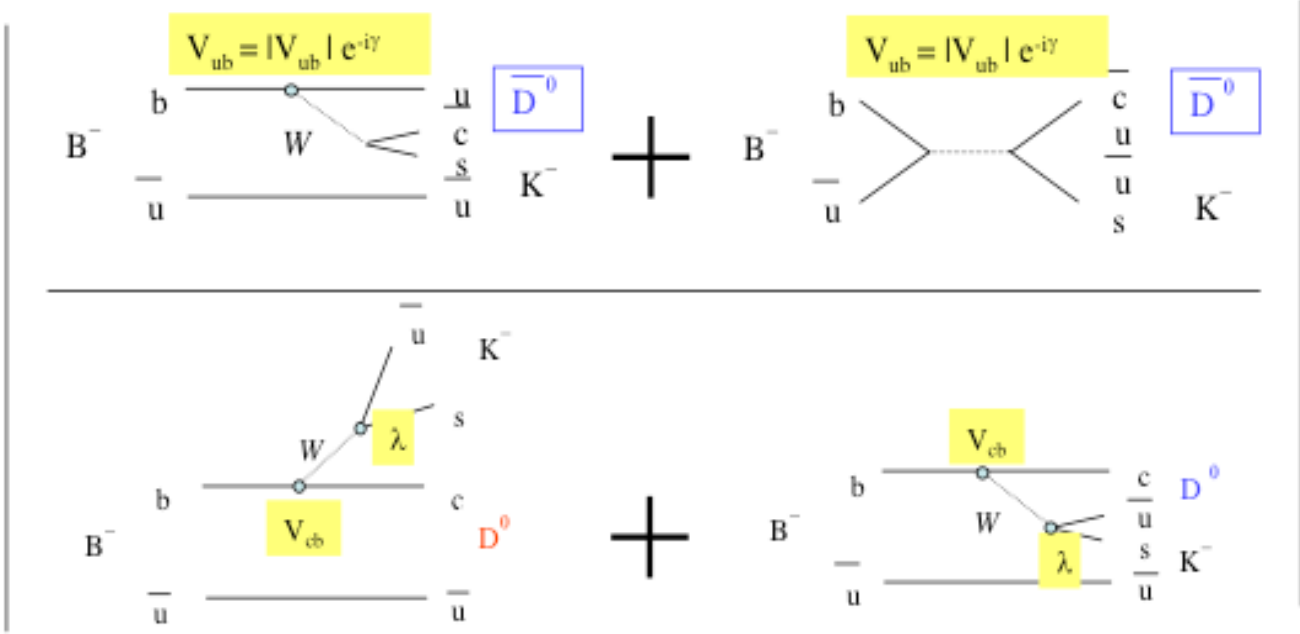
$$A_{ADS} = \frac{\Gamma(B^- \to [K^+\pi^-]_D K^-) - \Gamma(B^+ \to [K^-\pi^+]_D K^+)}{\Gamma(B^- \to [K^+\pi^-]_D K^-) + \Gamma(B^+ \to [K^-\pi^+]_D K^+)}$$
$$= 2r_B r_{DCS}\sin\gamma\sin(\delta_B + \delta_D)/R_{ADS}$$

$$r_{DCS} = \left|\frac{A(D^0 \to K^+\pi^-)}{A(D^0 \to K^-\pi^+)}\right|$$

$(3.62 \pm 0.29)10^{-3}$

r_B is a crucial parameter. It drives the sensitivity on γ

What about r_B ? $r_B = \left| \frac{A(B^- \to \overline{D}^0 K^-)}{A(B^- \to D^0 K^-)} \right|$



$= \left| \frac{C + A}{T + \overline{C}} \right|$

$r_B = | RB \times RCT | = \left| \frac{V_{ub} V_{cs}^*}{V_{cb} V_{us}^*} \right| \left| \frac{C + A}{T + \overline{C}} \right| = \sqrt{\overline{\eta}^2 + \overline{\rho}^2} \left| \frac{C + A}{T + \overline{C}} \right|$ $RB = 0.36 \pm 0.04$

Evaluation can be done if Annihilation diagram is neglected $RCT \approx \sqrt{\frac{Br(\overline{B}^0 \to D^0 \overline{K}^0)}{Br(B^- \to D^0 K^-)}} = 0.34 \pm 0.10$ $r_B = 0.12 \pm 0.04$

Beyond this approx. If |A/C|~0.3 (max?) (+- 30% according to the interference between A and C)

$r_B = 0.12 \pm 0.04(stat) \pm 0.04(theo.)$

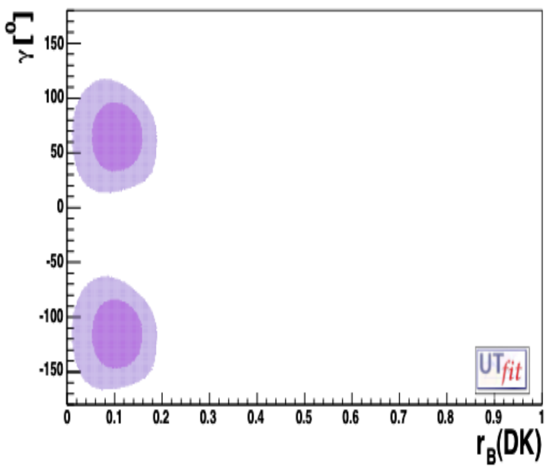
Conclusions : should be measured on data

Repeat with several f_{CP} final states

Observable	DK	D^*K	DK^*
A_{CP^+}(GLW)	0.22 ± 0.11	-0.14 ± 0.18	-0.07 ± 0.18
A_{CP^-}(GLW)	0.02 ± 0.12	0.26 ± 0.26	-0.16 ± 0.29
R_{CP^+}(GLW)	0.91 ± 0.12	1.25 ± 0.20	1.77 ± 0.39
R_{CP^-}(GLW)	1.02 ± 0.12	0.94 ± 0.29	$0.76^{+0.30}_{-0.33}$
R_{ADS}	0.017 ± 0.009	$< 0.16 @ 90\%$ C.L.	-
A_{ADS}	$0.49^{+0.53}_{-0.46}$	-	-
r_B(Dalitz)-Belle	$0.21 \pm 0.08 \pm 0.03 \pm 0.04$	$0.12^{+0.16}_{-0.11} \pm 0.02 \pm 0.04$	-
$\gamma[^\circ]$(Dalitz)-Belle	$68 \pm 15 \pm 13 \pm 11$	$75 \pm 57 \pm 11 \pm 11$	-
	... 90% $C.L.$	$0.155^{+0.070}$ ...	
	70 ± 2 ...		

Tree level diagrams, not influenced by new physics

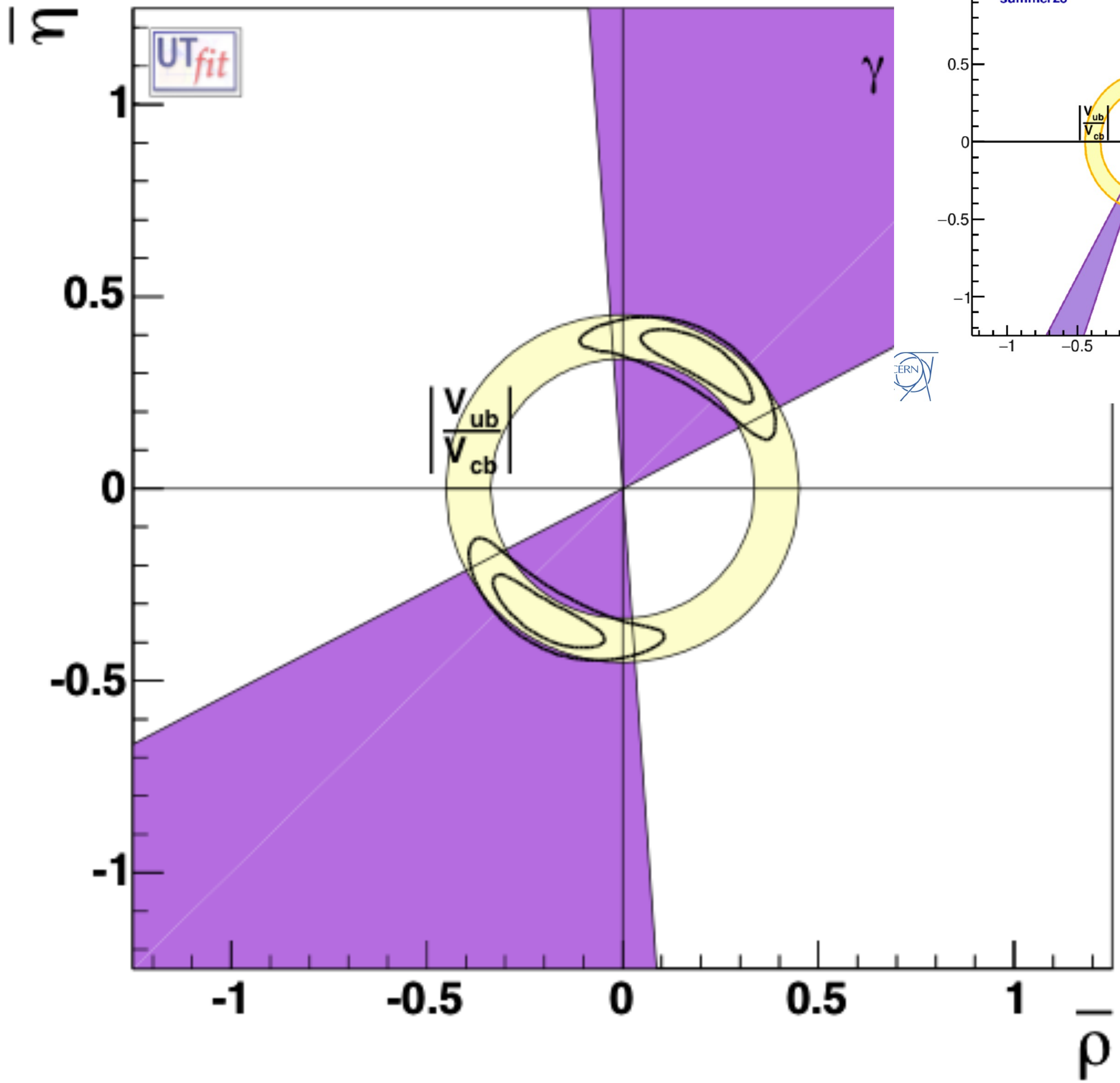
Using also the Dalitz Plot Method

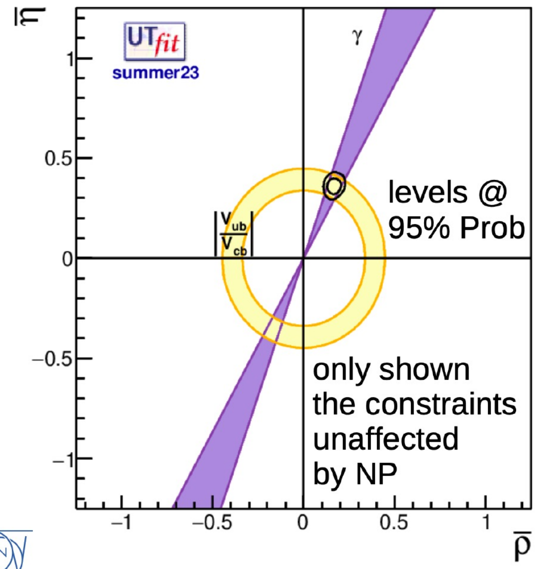


$$\gamma[^\circ] = 60.3 \pm 6.8 \; ([47.0, 74.2] \; at \; 95\%) \; indirect - UT fit$$

$$\gamma[^\circ] = \begin{cases} 59.1 \pm 16.7 \cup -120.3 \pm 17.2 \\ ([24.7, 97.9] \cup [-155.4, -82.7] \; at \; 95\%) \end{cases} \; direct - D^{(*)}K^{(*)}$$

Only tree level processes





$\bar{\rho} = \pm 0.163 \pm 0.024$
$\bar{\eta} = \pm 0.356 \pm 0.027$

UT Fit - using only $\left\lvert\frac{V_{ub}}{V_{cb}}\right\rvert$ and γ		
	SM Solution	2^{nd} Solution
$\bar{\rho}$	0.18 ± 0.11	-0.18 ± 0.11
$\bar{\eta}$	0.41 ± 0.05	-0.41 ± 0.05
$\sin 2\beta$	0.784 ± 0.062	-0.639 ± 0.079
γ	$(66 \pm 17)^o$	$(-114 \pm 17)^o$
α	$(86 \pm 14)^o$	$(-46 \pm 14)^o$
$2\beta + \gamma$	$(122 \pm 12)^o$	$(-153 \pm 12)^o$

Table 2: *Results for several UT parameters, obtained using the constraints from* $\left\lvert\frac{V_{ub}}{V_{cb}}\right\rvert$ *and* γ.

$\alpha = (92.4 \pm 1.4)^0$

$\sin 2\beta = 0.703 \pm 0.014$

$\beta = (22.46 \pm 0.68)^0$

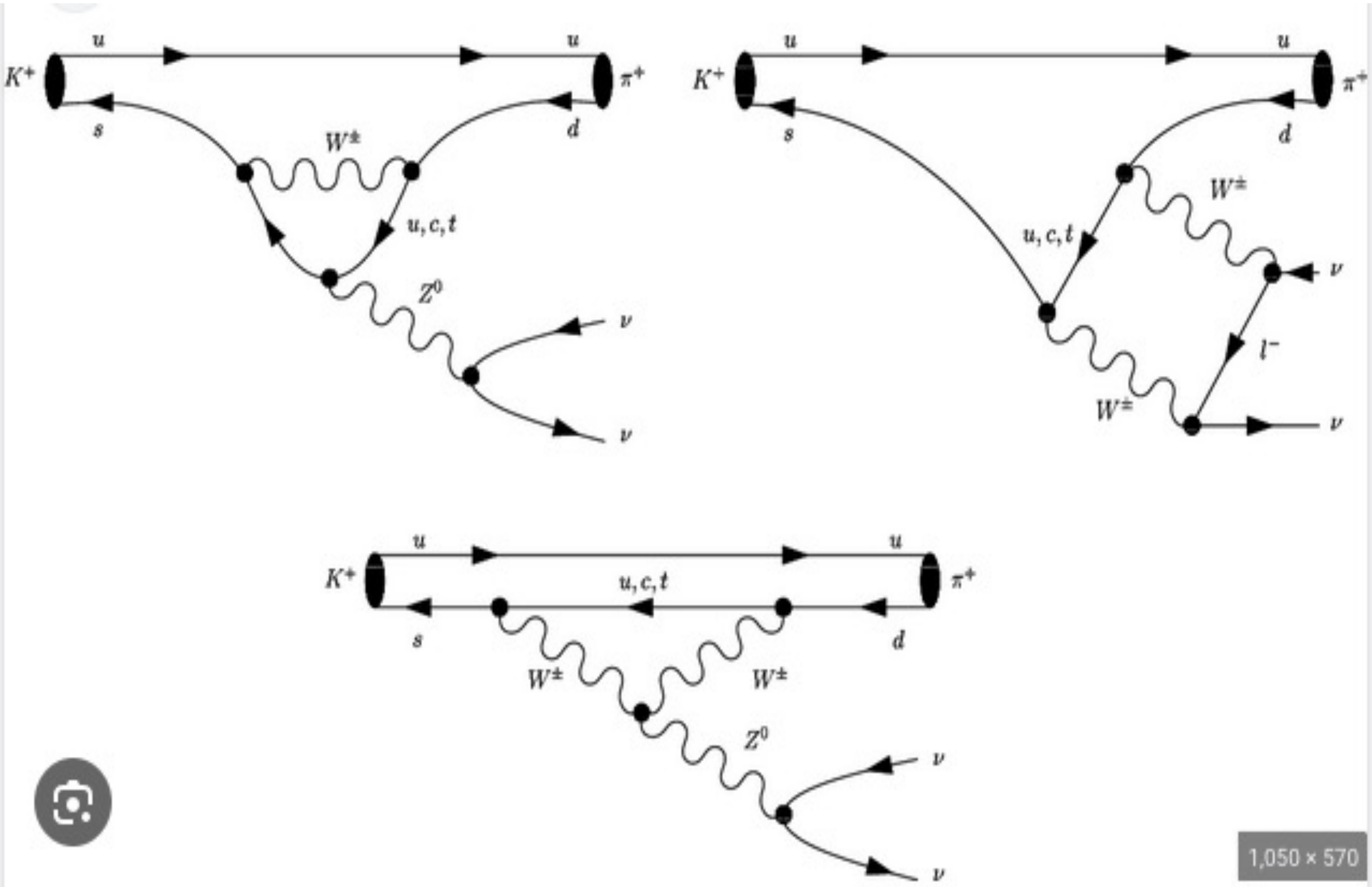
$\gamma = (65.1 \pm 1.3)^0$

$A = 0.828 \pm 0.011$

$\lambda = 0.22519 \pm 0.00083$

2022

$$K \to \pi\nu\bar{\nu}$$

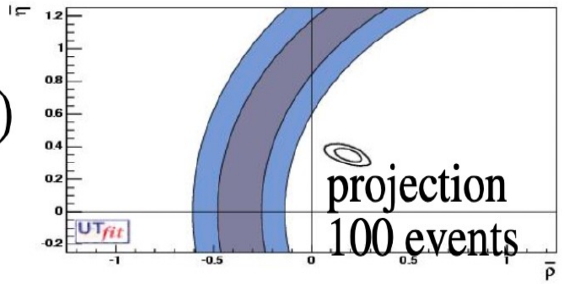


some old plots coming back to fashion:

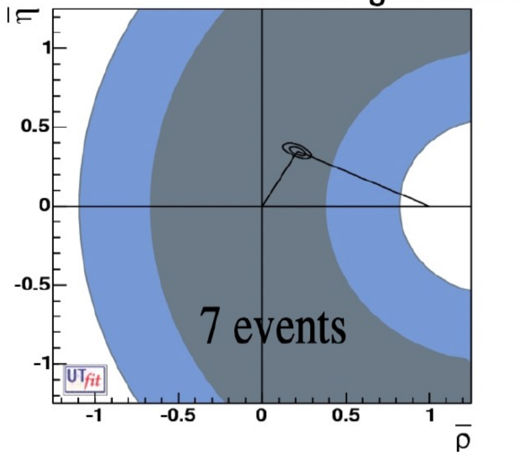
As NA62 and KOTO are analysing data:

$BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})$

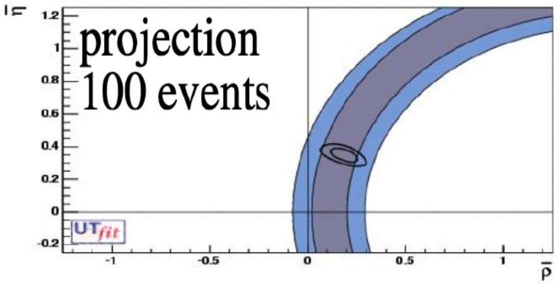
E949 central value

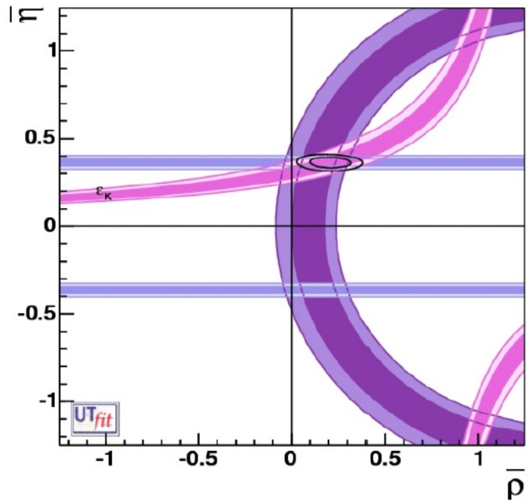


2007 global fit a

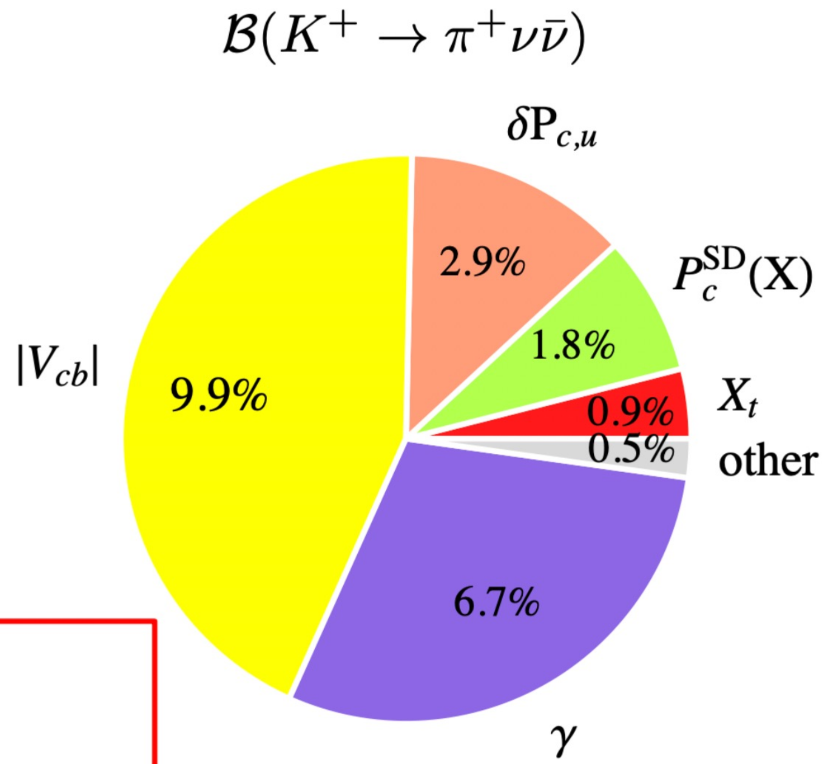


SM central value





including
$BR(K^0 \rightarrow \pi^0 \nu \bar{\nu}$
SM central va

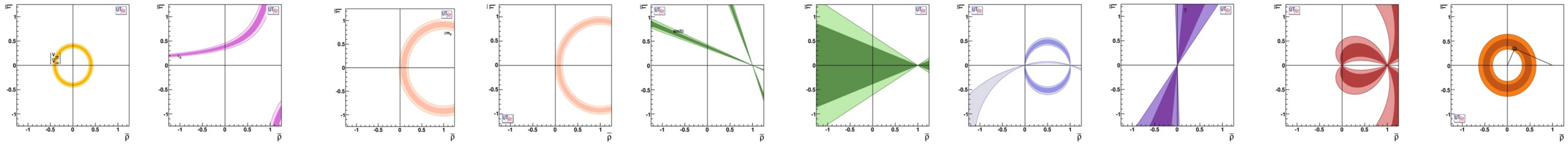


SM branching ratios:
[arXiv:2109.11032]

$K^+ \to \pi^+ \nu\nu(\gamma) = (8.62 \pm 0.42) \times 10^{-11}$

$K_L \to \pi^0 \nu\nu = (2.94 \pm 0.15) \times 10^{-11}$

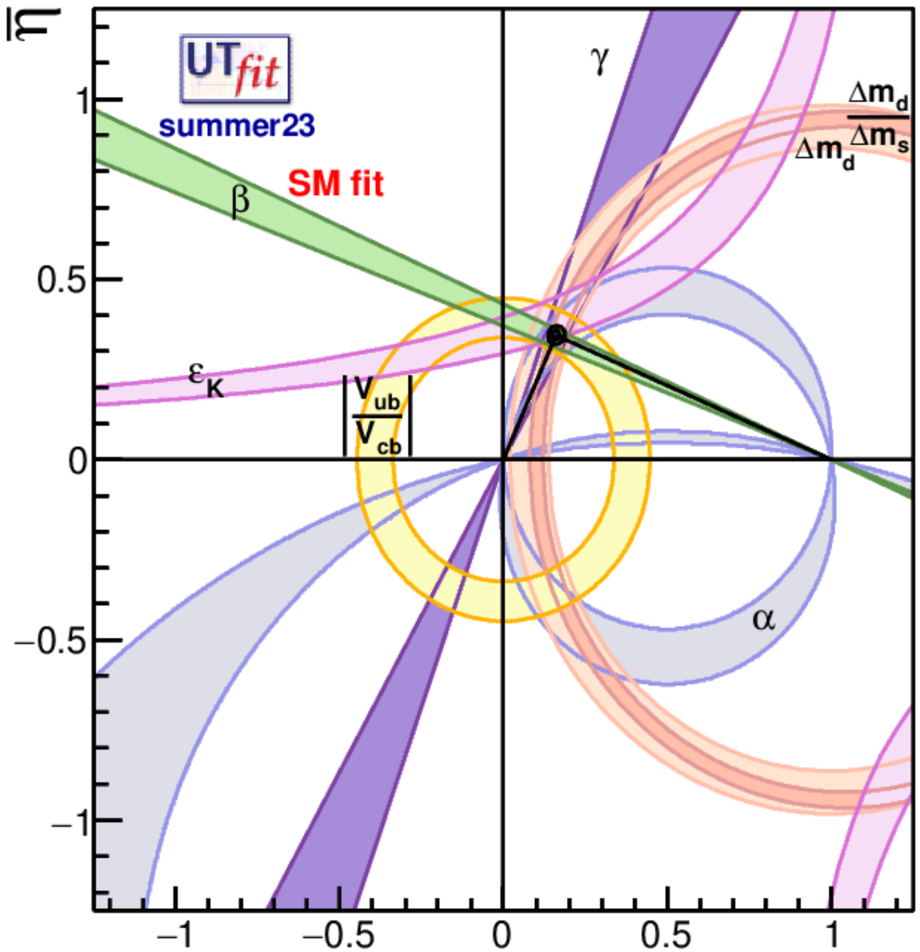
Courtesy by G. D'Ambrosio



2023 results

$\bar{\rho} = 0.160 \pm 0.009 \quad \bar{\eta} = 0.345 \pm 0.011$

In the hadronic sector, the SM CKM pattern represents the principal part of the flavor structure and of CP violation



$\alpha = (92.4 \pm 1.4)^0$

$\sin 2\beta = 0.703 \pm 0.014$

$\beta = (22.46 \pm 0.68)^0$

$\gamma = (65.1 \pm 1.3)^0$

$A = 0.828 \pm 0.011$

$\lambda = 0.22519 \pm 0.00083$

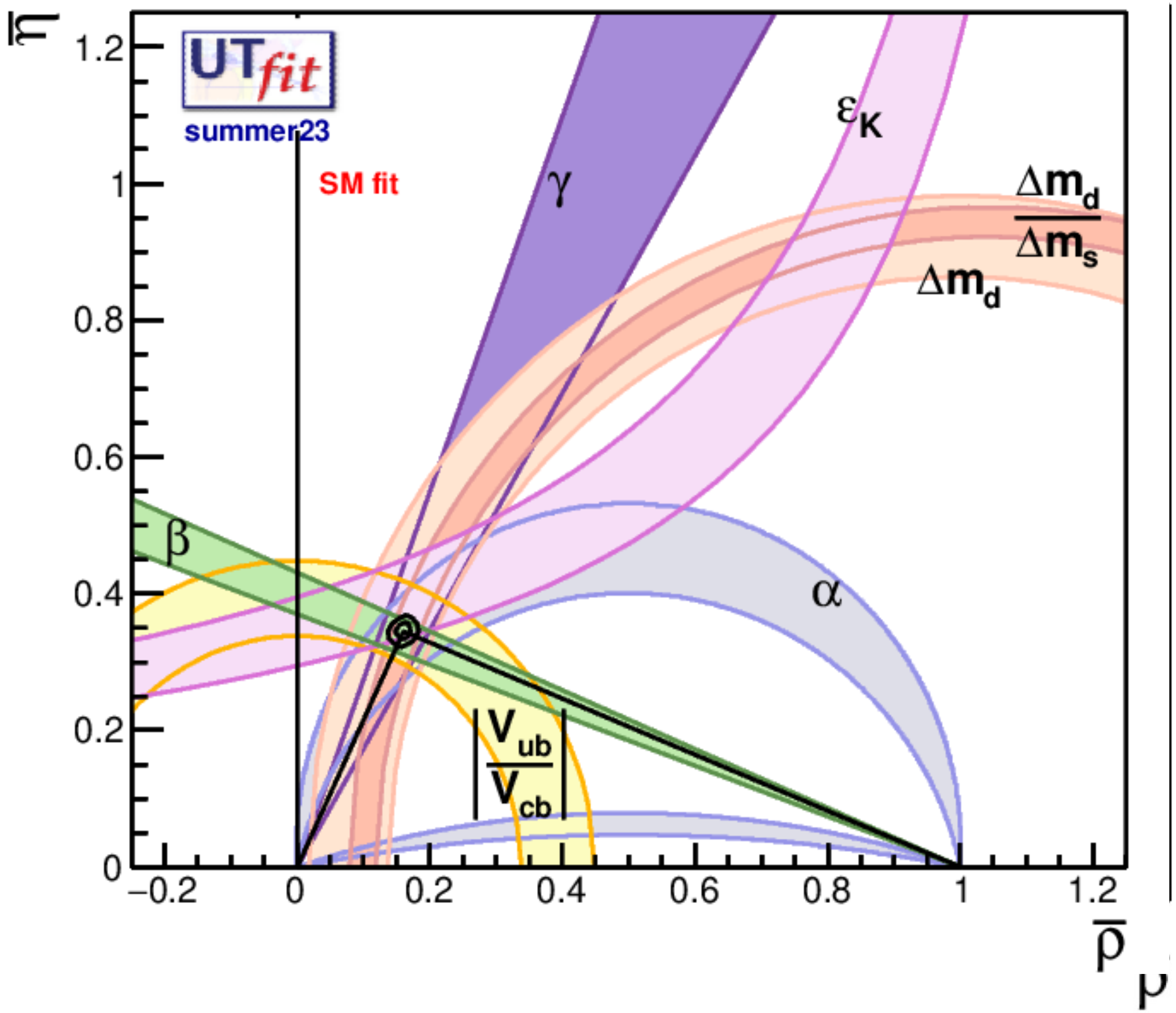
2022

Consistence on an over constrained fit of the CKM parameters

CKM matrix is the dominant source of flavour mixing and CP violation

Unitarity Triangle analysis in the SM:

zoomed in..



levels @ 95% Prob

~6%

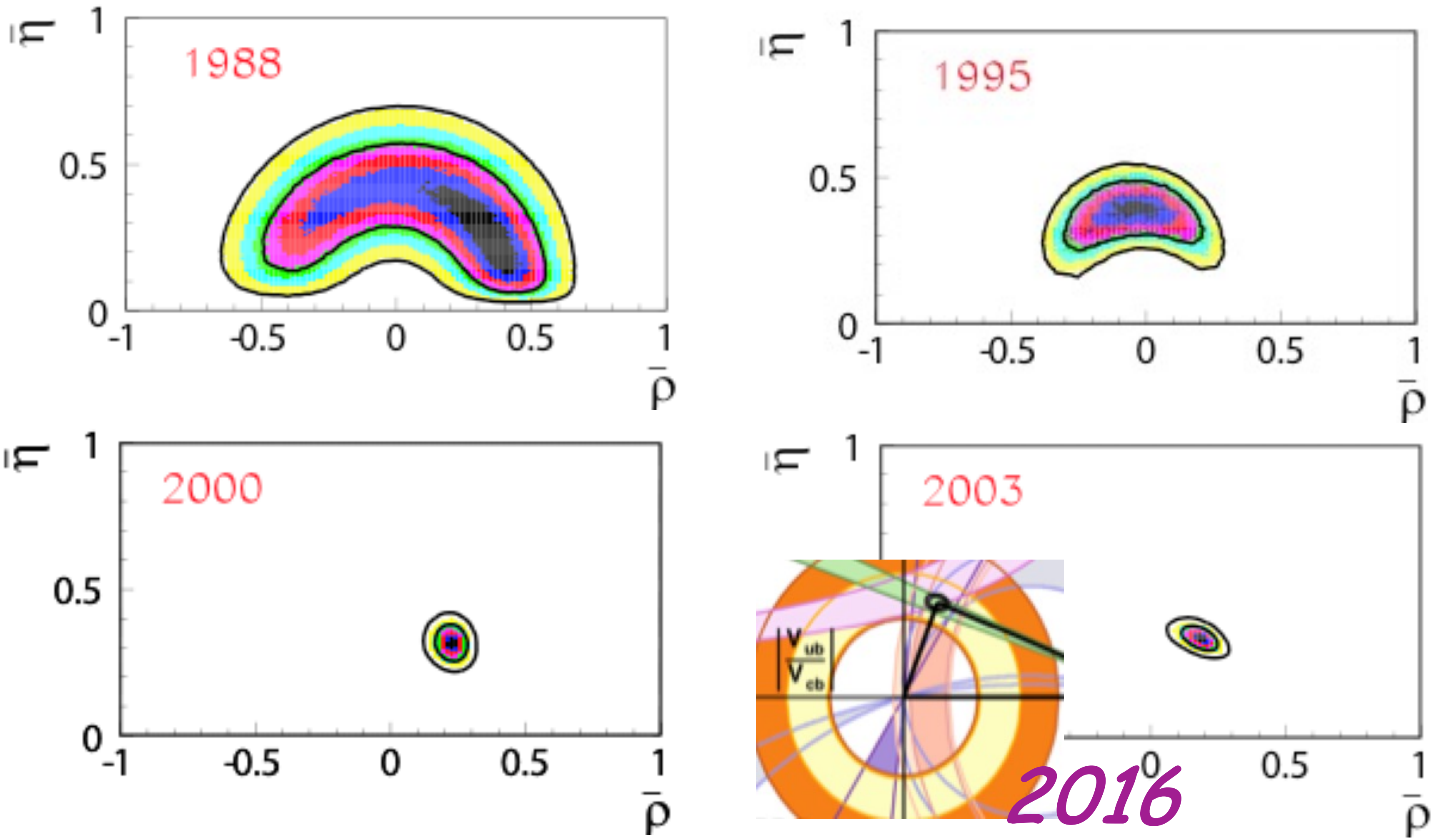
$\rho = 0.160 \pm 0.009$

$\eta = 0.345 \pm 0.011$

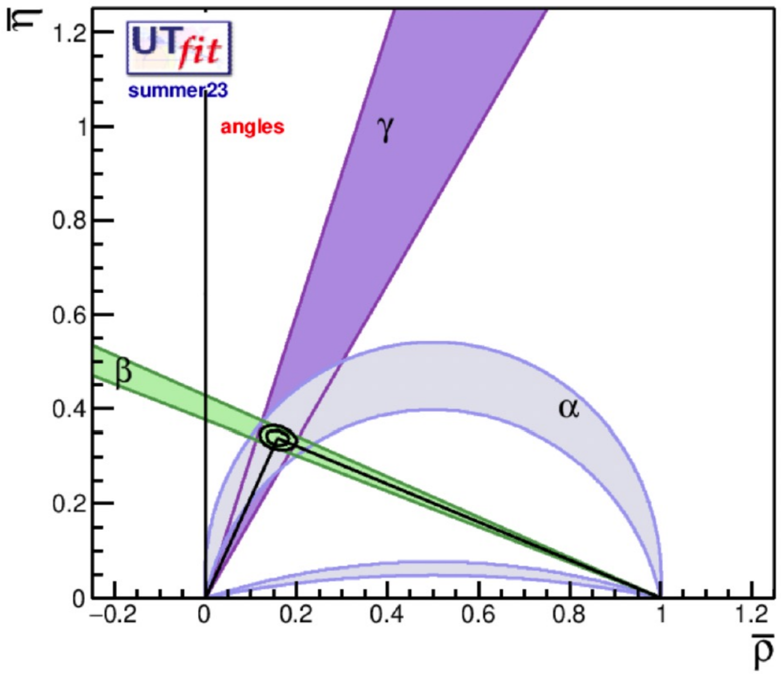
~3%

PROGRESS SINCE 1988

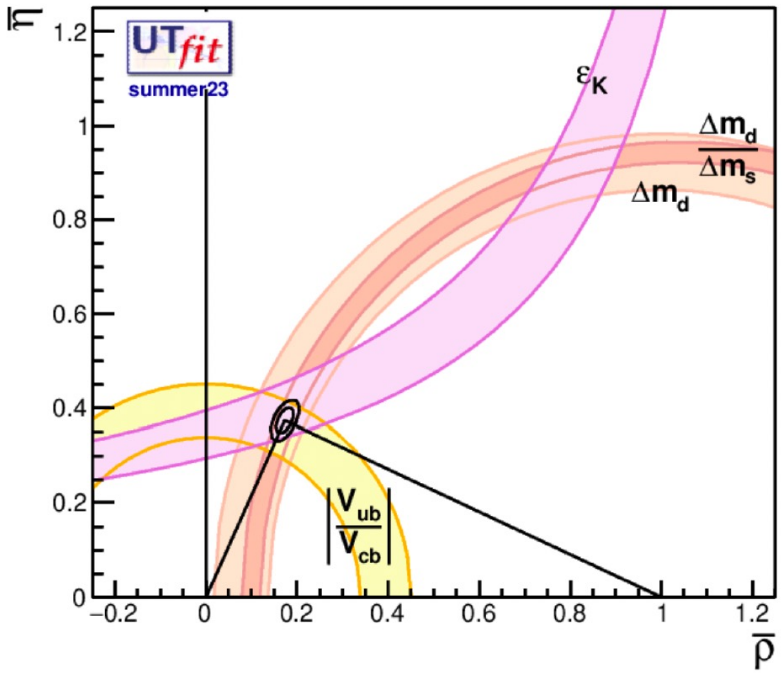
Experimental progress so impressive that we can fit the hadronic matrix elements (in the SM)



Standard Model Fit result



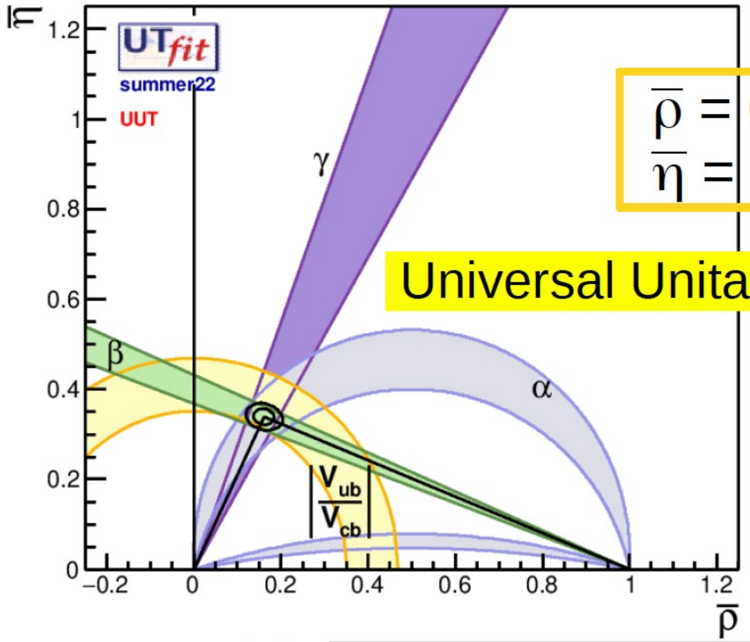
$$\bar{\rho} = 0.159 \pm 0.016$$
$$\bar{\eta} = 0.339 \pm 0.010$$



$$\bar{\rho} = 0.173 \pm 0.012$$
$$\bar{\eta} = 0.374 \pm 0.019$$

December 2022

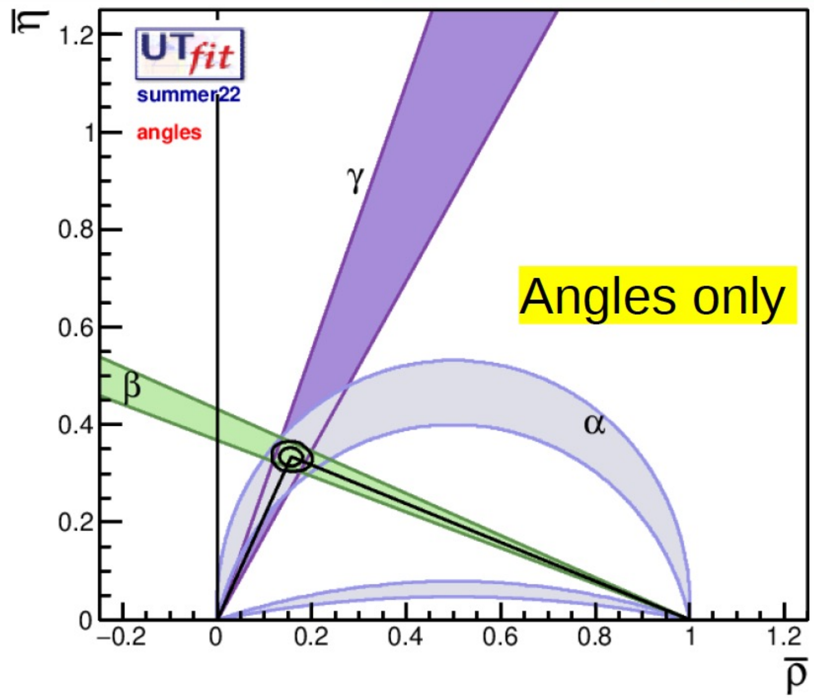
Some interesting configurations



Universal Unitary Triangle

$\bar{\rho} = 0.161 \pm 0.017$ ~10%

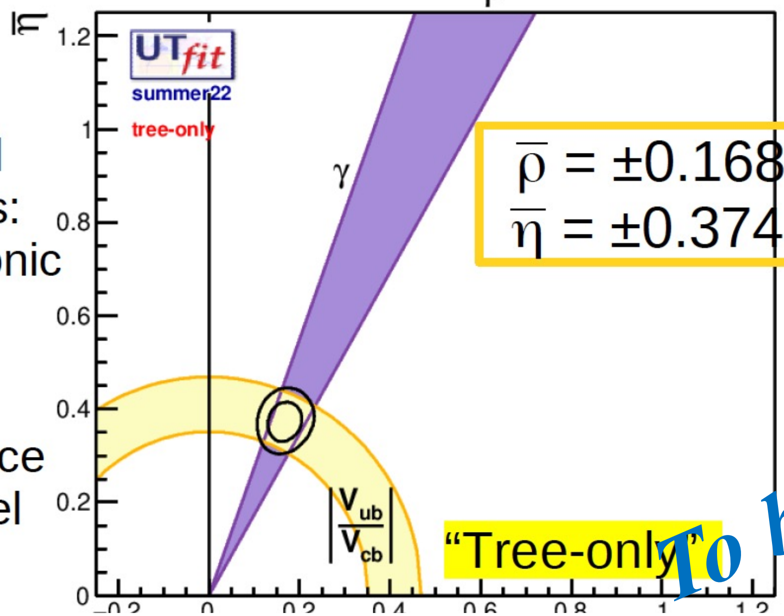
$\bar{\eta} = 0.339 \pm 0.011$ ~3%



Angles only

Tree-level processes: Semileptonic and DK B decays

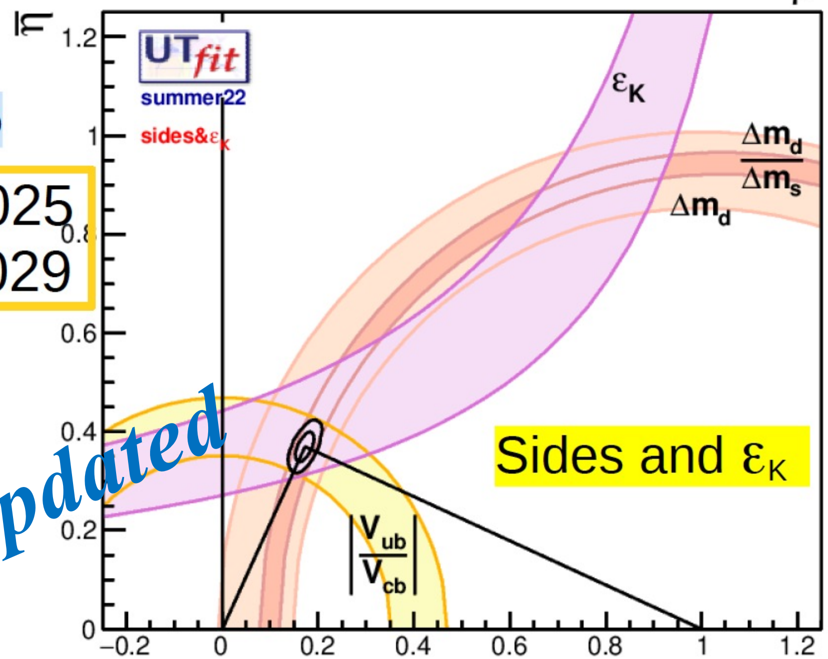
→ reference for model building



"Tree-only"

$\bar{\rho} = \pm 0.168 \pm 0.025$ ~15%

$\bar{\eta} = \pm 0.374 \pm 0.029$ ~8%



Sides and ε_K

To be updated

compatibility plots

A way to “measure” the agreement of a single measurement with the indirect determination from the fit using all the other inputs: test for the SM description of the flavour physics

2022

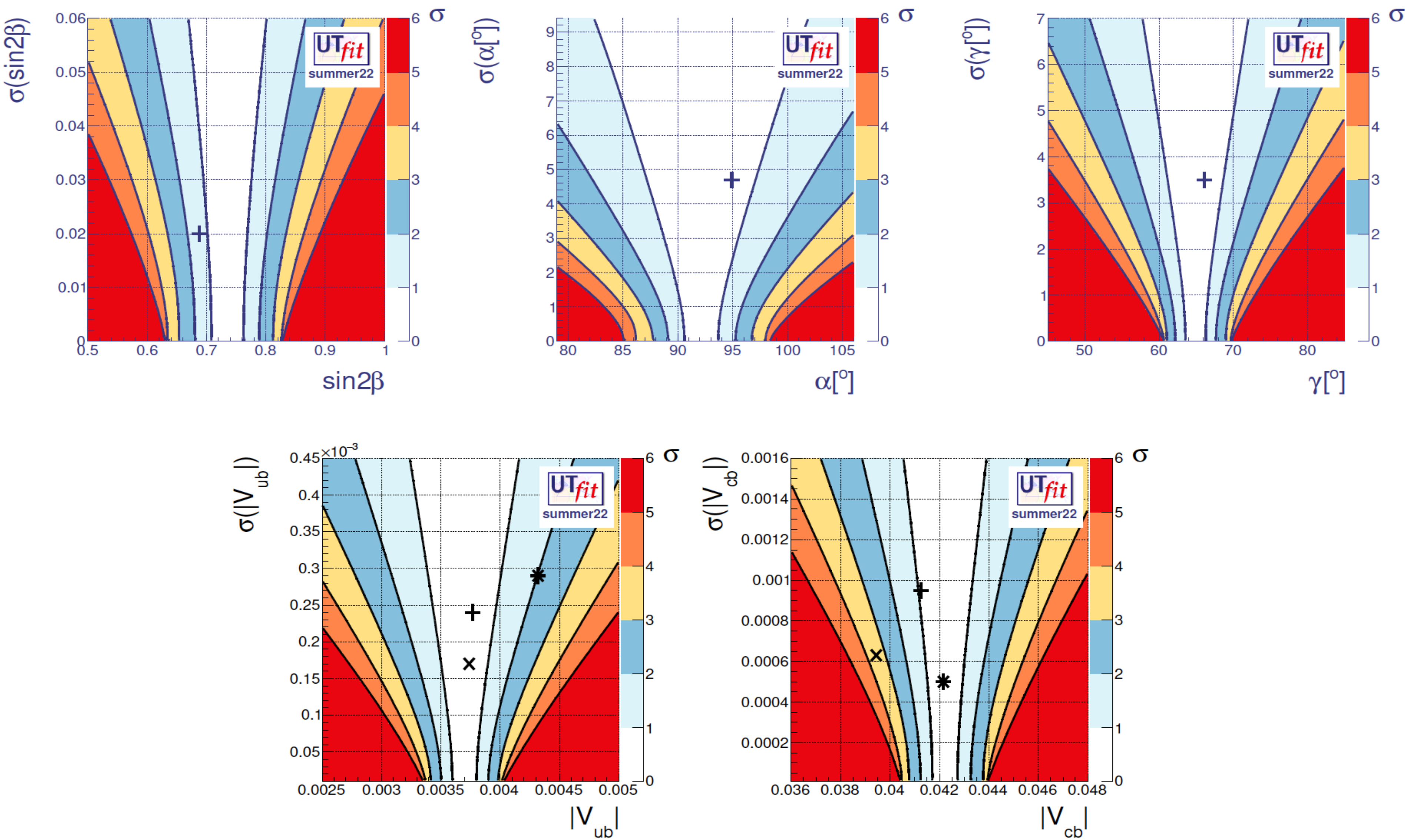
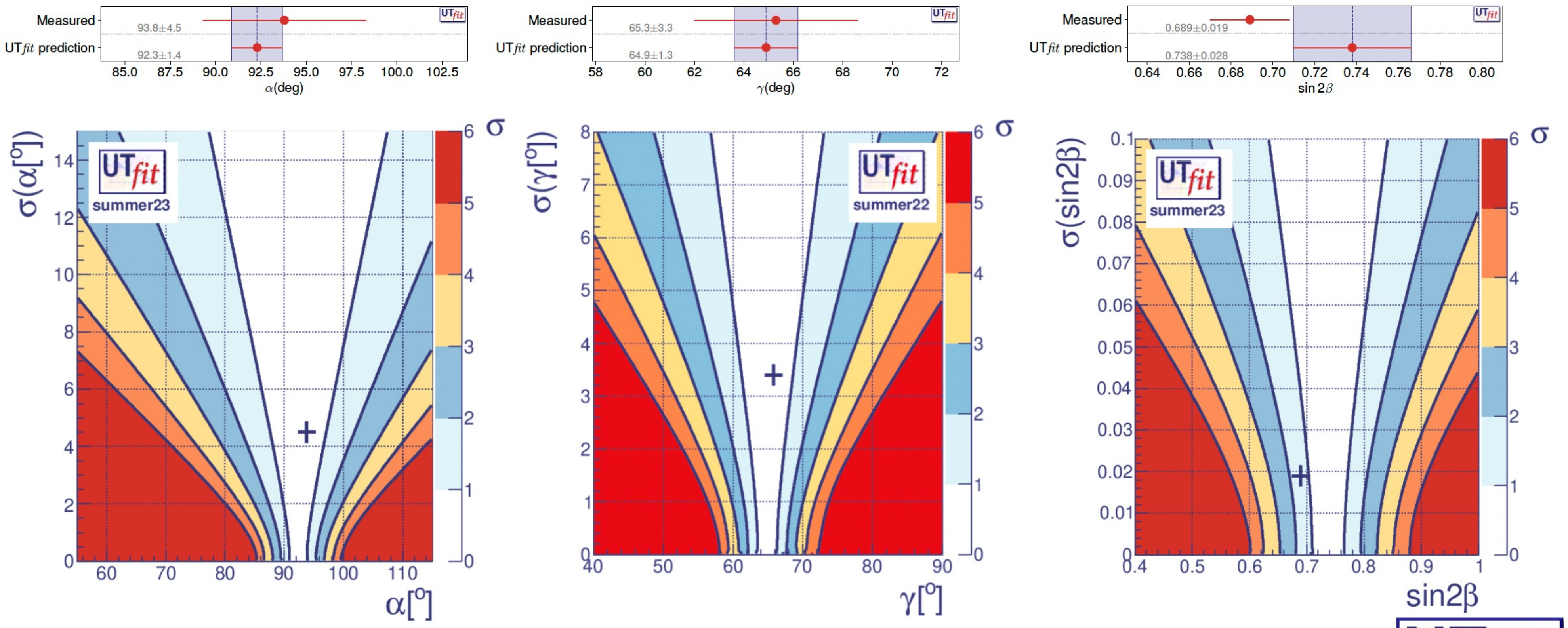


FIG. 5. *Pull plots (see text) for* $\sin 2\beta$ *(top-left),* α *(top-centre),* γ *(top-right),* $|V_{ub}|$ *(bottom-left) and* $|V_{cb}|$ *(bottom-right) inputs. The crosses represent the input values reported in Table I. In the case of* $|V_{ub}|$ *and* $|V_{cb}|$ *the* x *and the* * *represent the values extracted from exclusive and inclusive semileptonic decays respectively.*

2023

Standard Model Fit compatibility



V_{cb} and V_{ub}

from FLAG 2021

$|V_{cb}|$ *(excl)* = $(39.44 \pm 0.63)\ 10^{-3}$

NEW $(40.55 \pm 0.46)\ 10^{-3}$

$|V_{cb}|$ *(incl)* = $(42.16 \pm 0.50)\ 10^{-3}$

~ 3.2σ discrepancy

from Bordone et al.
arXiv:2107.00604

$V_{cb}|$ *(incl)* = $(41.69 \pm 0.63)\ 10^{-3}$

from ***Bernlochner*** et al.
arXiv:2205.10274

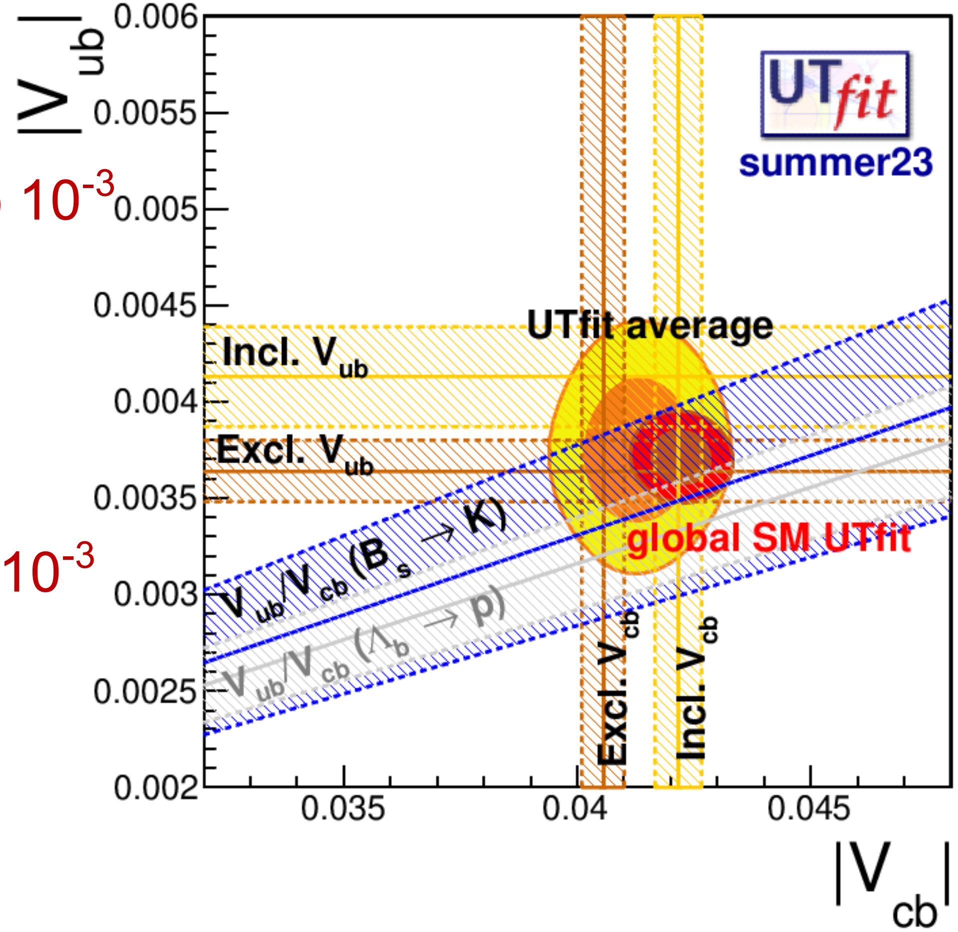
NEW $(3.64 \pm 0.16)\ 10^{-3}$

$|V_{ub}|$ *(excl)* = $(3.74 \pm 0.17)\ 10^{-3}$

$|V_{ub}|$ *(incl)* = $(4.32 \pm 0.29)\ 10^{-3}$

from GGOU HFLAV 2021
adding a flat uncertainty
covering the spread
of central values

~1.6σ discrepancy



From B_s to K at high q^2

$|V_{ub} / V_{cb}|$ *(LHCb)* = $(9.46 \pm 0.79)\ 10^{-2}$

NEW $(8.27 \pm 1.17)\ 10^{-2}$

$|V_{ub} / V_{cb}|$ *(LHCb)* = $(7.9 \pm 0.6)\ 10^{-2}$

From Λ_b, excluded following FLAG guidelines

From global SM fit $|V_{cb}| = (42.00 \pm 0.47)\ 10^{-3}$ $|V_{ub}| = (3.715 \pm 0.093)\ 10^{-3}$

Utfit Prediction Vcb= $(42.22 \pm 0.51)\ 10^{-3}$ *Vub*= $(3.70 \pm 0.11)\ 10^{-3}$

WORK IN PROGRESS *(G.M., S.Simula, L.Vittorio)*

NEW Vcb= $(40.55 \pm 0.46)\ 10^{-3}$
EXCLUSIVE from B-> D* INCLUSIVE $(42.16 \pm 0.50)\ 10^{-3}$ $(41.69 \pm 0.63)\ 10^{-3}$

NEW Vub/Vcb = $(8.27 \pm 1.17)\ 10^{-2}$
FLAG UNDERESTIMATES OF THE UNCERTAINTY
The larger error reduces the correlation between Vub nd Vcb

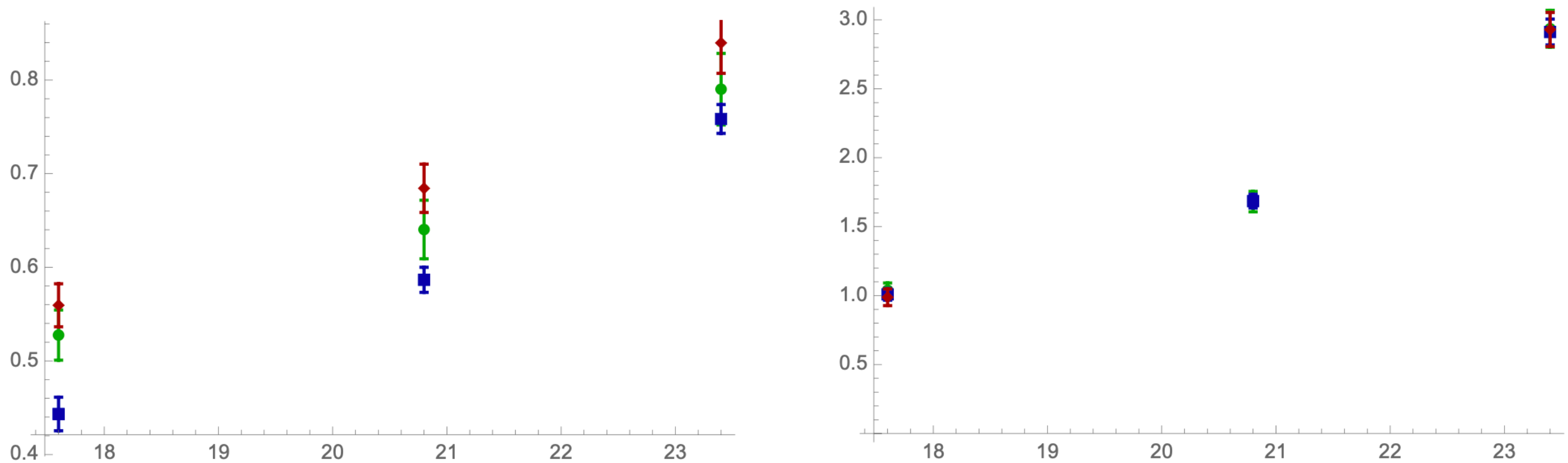
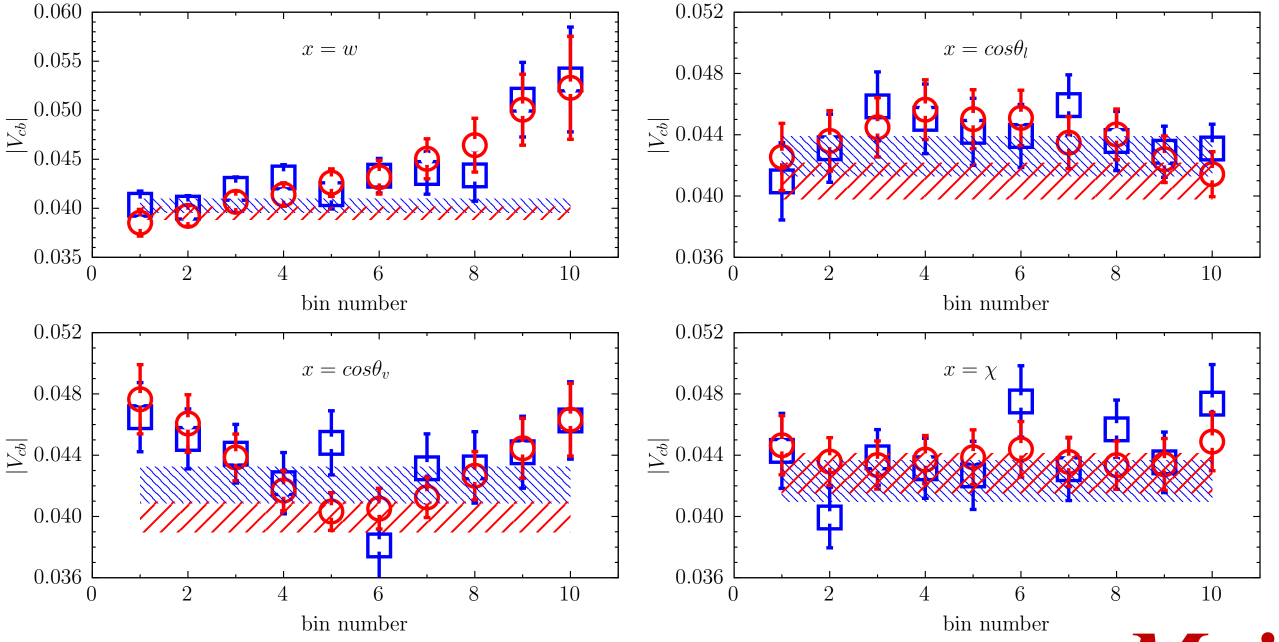


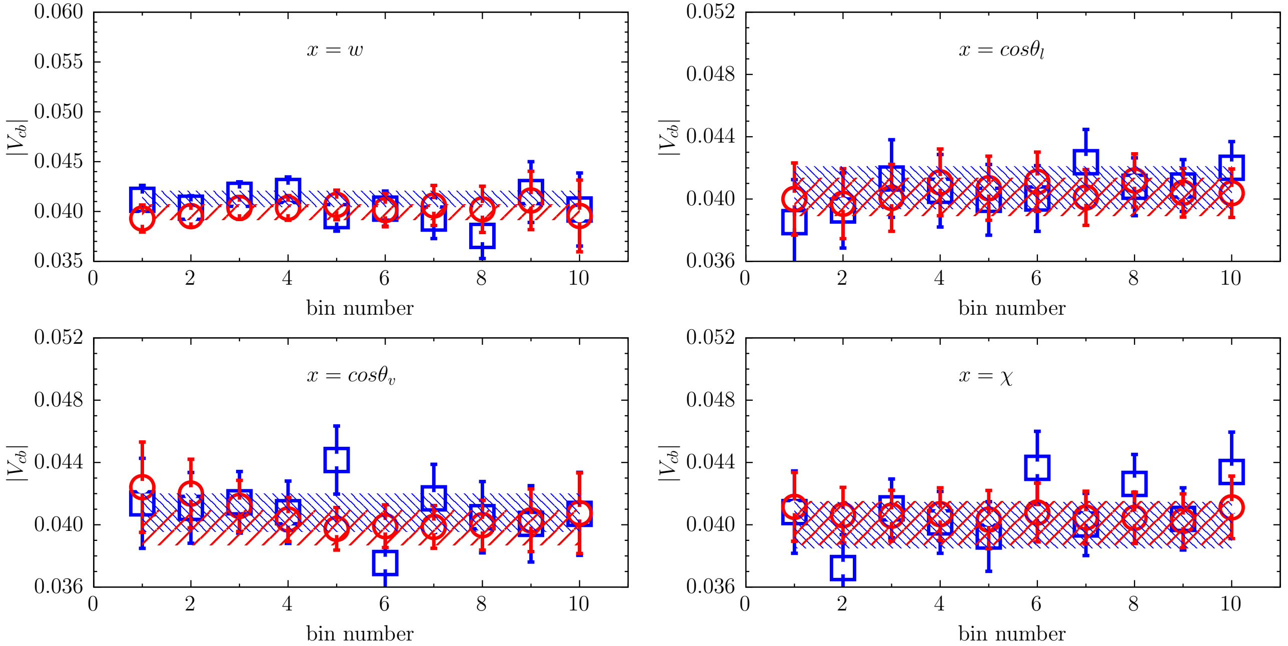
Fig. 8. *Available lattice results for the FFs $f_0(q^2)$ (left panel) and $f_+(q^2)$ (right panel) relevant for $B_s \to K\ell\nu_\ell$ decays. The RBC/UKQCD [6] (diamond), FNAL/MILC [31] (squares) and HPQCD [32, 33] (circles).*

Utfit Prediction Vcb= $(42.21 \pm 0.51)\ 10^{-3}$
Vub= $(3.70 \pm 0.09)\ 10^{-3}$



FNAL/MILC

Mainly due to $F_1(w)$



JLQCD

GM,S. Simula,L.Vittorio

The importance of $|V_{cb}|$

An important CKM unitarity test is the Unitarity Triangle (UT) formed by

$$1 + \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0$$

V_{cb} plays an important role in UT

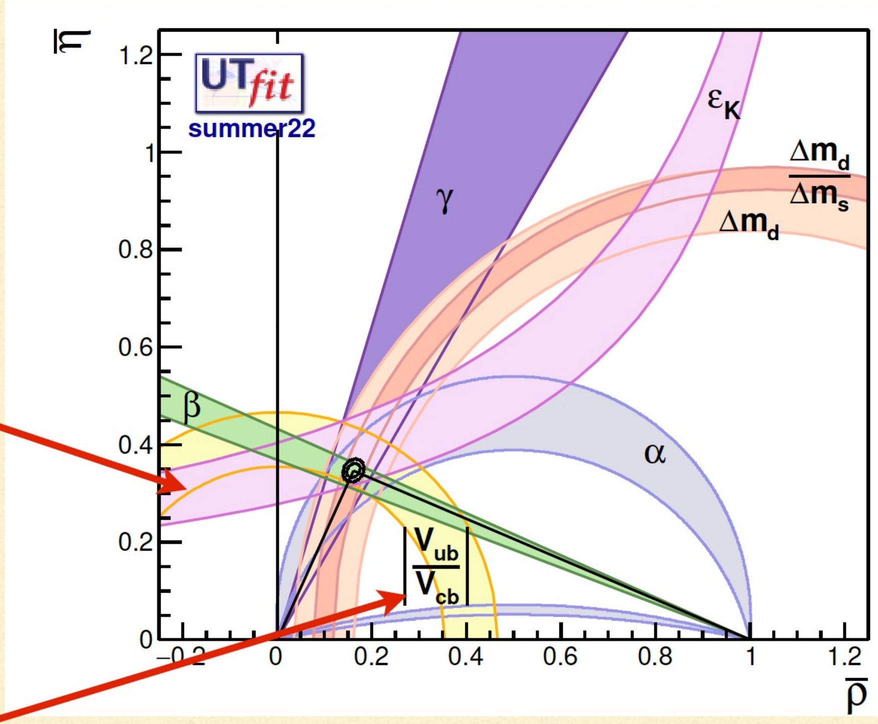
$$\varepsilon_K \approx x|V_{cb}|^4 + \ldots$$

and in the prediction of FCNC:

$$\propto |V_{tb}V_{ts}|^2 \simeq |V_{cb}|^2\left[1 + O(\lambda^2)\right]$$

where *it often dominates the theoretical uncertainty.*

V_{ub}/V_{cb} constrains directly the UT

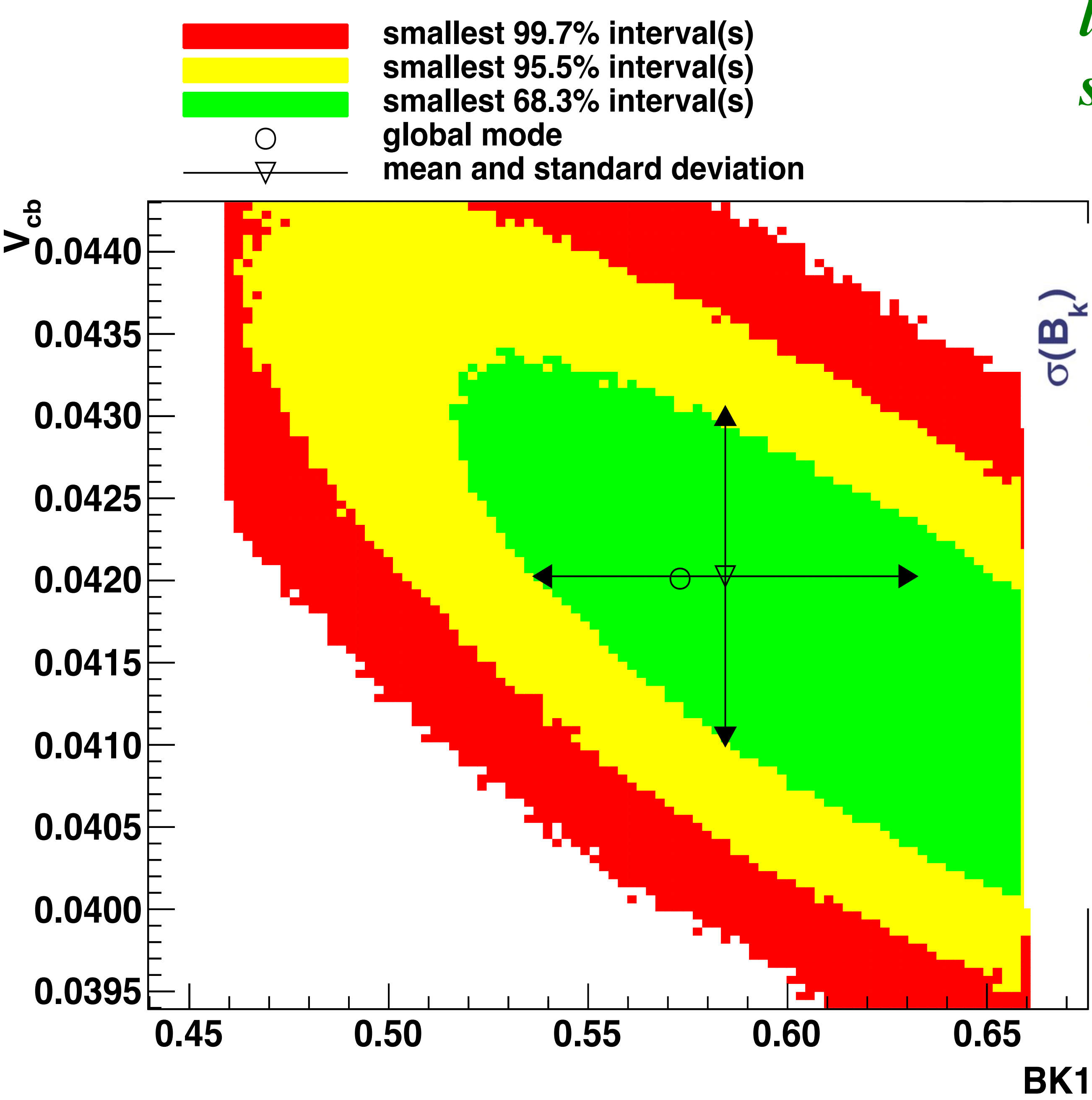


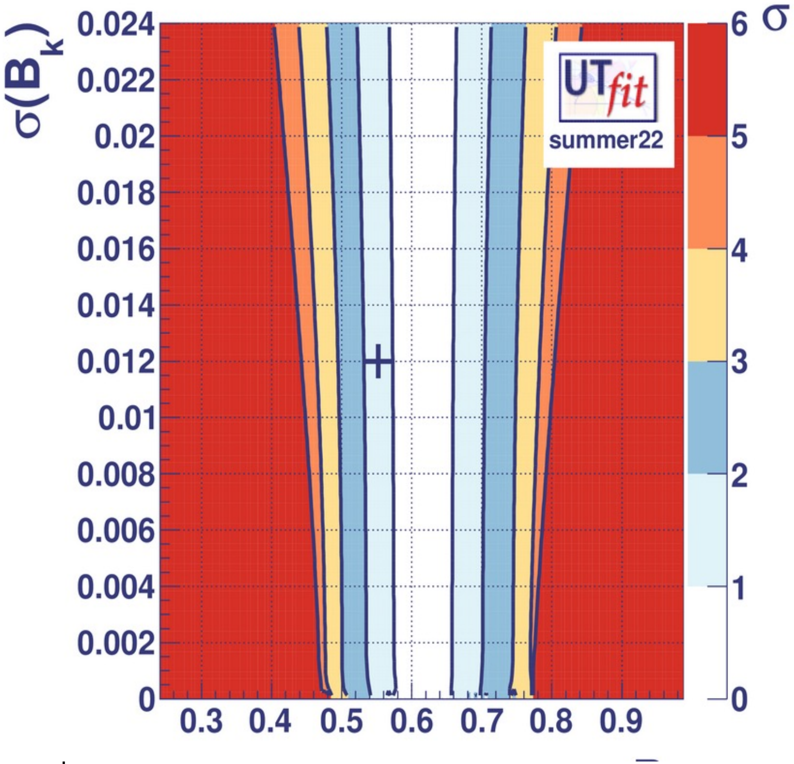
Our ability to determine precisely V_{cb} is crucial for indirect NP searches

Courtesy by Gambino

UT-fit Preliminary

- ε_K large Vcb
-B mixing with large lattice matrix elements smaller Vcb





2022

Power corrections to the CP-violation parameter ε_K

M. Ciuchini[(a)], E. Franco[(b)], V. Lubicz[(c,a)],
G. Martinelli[(d,b)], L. Silvestrini[(b)], C. Tarantino[(c,a)]

$$\varepsilon_K^{exp} = 2.228 \pm 0.011) \cdot 10^{-3}$$

2021: an estimate from the 1/mc expansion of the effective Hamiltonian + UTfit

$$\varepsilon_K = 2.00\,(15) \times 10^{-3}$$

Computing the long-distance contributions to ε_K

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Norman Christ* †
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E-mail: nhc@phys.columbia.edu

RBC and UKQCD Collaborations

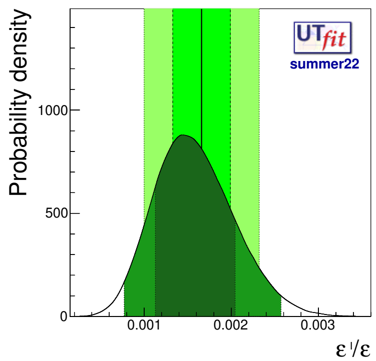
2015: a real exploratory calculation
no physical masses, no extrapolation to the continuum

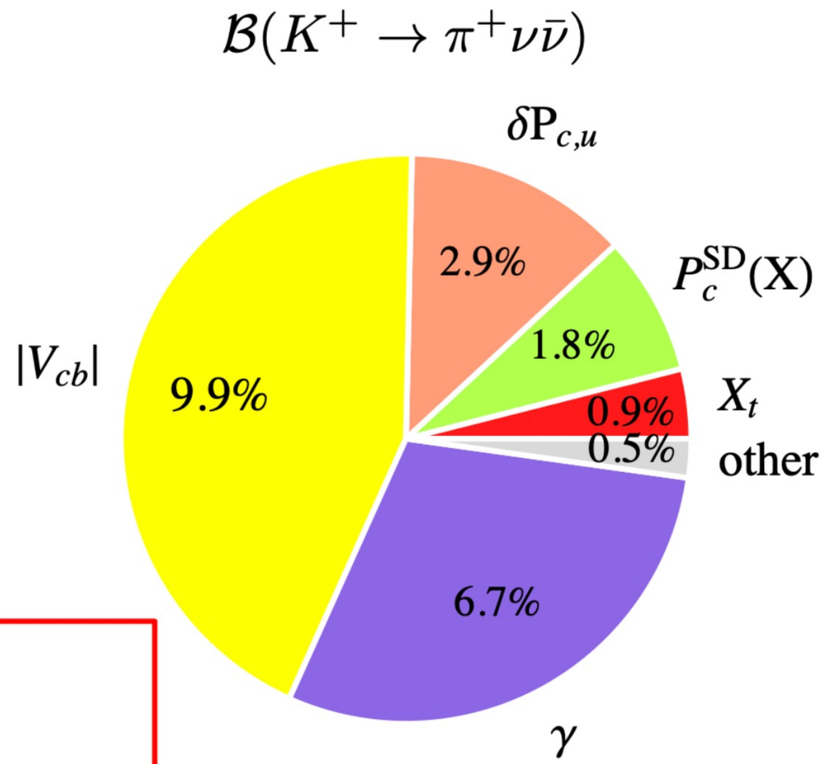
$$|\varepsilon| = (1.806(41) + 0.891(11) + 0.209(6) + 0.112(13)) \times 10^{-3} = 3.019(45) \times 10^{-3}$$

tt $\quad ut_{SD}$ $\quad ut_{LD}$ $\quad \mathrm{Im}(A_0)$,

e'/e from RBC now in Utfit:

$$e'/e = 15.2(4.7)\ \mathrm{x}10^{-4}$$





SM branching ratios:
[arXiv:2109.11032]

$K^+ \to \pi^+ \nu\nu(\gamma) = (8.62 \pm 0.42) \times 10^{-11}$

$K_L \to \pi^0 \nu\nu = (2.94 \pm 0.15) \times 10^{-11}$

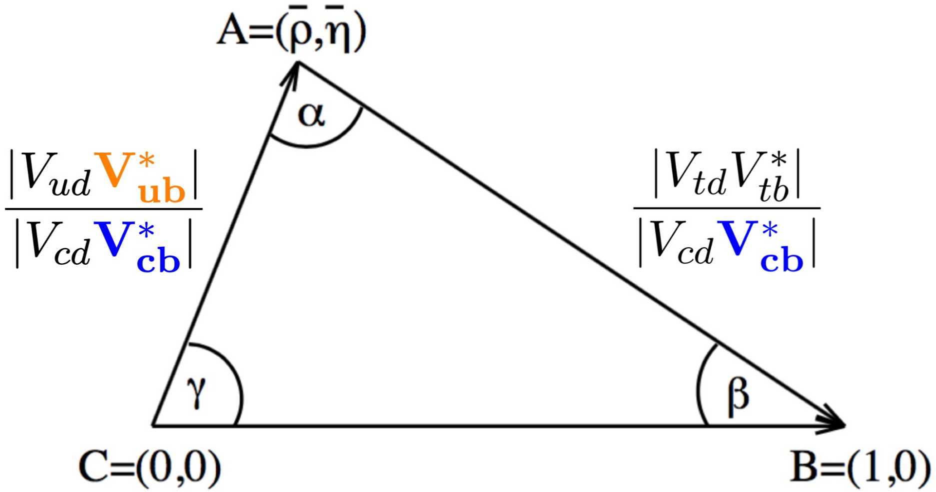
Courtesy by G. D'Ambrosio

Exclusive semileptonic B → {D(*),π} decays through unitarity (and other developments)

Work in collaboration with M. Naviglio. S. Simula and L. Vittorio

(PRD ’21 (2105.02497), PRD ’21 (2105.07851), PRD ’22 (2105.08674), 2109.15248, 2204.05925, 2202.10285)

See talk by A. Vaquero



Mr. Nosferatu
from Transylvania

The central role of the Form Factors (FFs) in excl. semil. *B* decays

- Production of a pseudoscalar meson (*i.e. D,π*):

$$\frac{d\Gamma}{dw} = \frac{G_F^2 |V_{cb}|^2 \eta_{EW}^2}{48\pi^3} \frac{4r\, m_D^3\, (m_B + m_D)^2\, (w^2-1)^{3/2}}{(1+r)^2} |f_+(w)|^2$$

- Production of a vector meson (*i.e. D**):

$$\frac{d\Gamma(B \to D^*(\to D\pi)\ell\nu)}{dw\, d\cos\theta_\ell\, d\cos\theta_v\, d\chi} = \frac{G_F^2 |V_{cb}|^2 \eta_{EW}^2}{4(4\pi)^4} 3 m_B m_{D^*}^2 \sqrt{w^2-1}$$

$$\times B(D^* \to D\pi)\{(1-\cos\theta_\ell)^2 \sin^2\theta_v |H_+|^2$$
$$+ (1+\cos\theta_\ell)^2 \sin^2\theta_v |H_-|^2 + 4\sin^2\theta_\ell \cos^2\theta_v |H_0|^2$$
$$- 2\sin^2\theta_\ell \sin^2\theta_v \cos 2\chi H_+ H_-$$
$$- 4\sin\theta_\ell (1-\cos\theta_\ell) \sin\theta_v \cos\theta_v \cos\chi H_+ H_0$$
$$+ 4\sin\theta_\ell (1+\cos\theta_\ell) \sin\theta_v \cos\theta_v \cos\chi H_- H_0\},$$

$$H_\pm(w) = f(w) \mp m_B m_{D^*} \sqrt{w^2-1}\, g(w)$$

$$H_0(w) = \frac{\mathcal{F}_1(w)}{\sqrt{m_B^2 + m_D^2 - 2 m_B m_D w}}$$

relation between the momentum transfer and the recoil

$$q^2 = m_B^2 + m_P^2 - 2 m_B m_P\, w$$

If the lepton is **NOT** massless? Two other FFs!

$$f_0(w)\ \text{(pseudoscalar)},\quad P_1(w)\ \text{(vector)}$$

small q^2 large discretization systematics

Semileptonic heavy-to-light meson decay on the lattice

L^{-1} finite box	$\ll$ physics of interest $\ll$	a^{-1} lattice spacing

Kinematics: example $B \longrightarrow \pi l \nu$

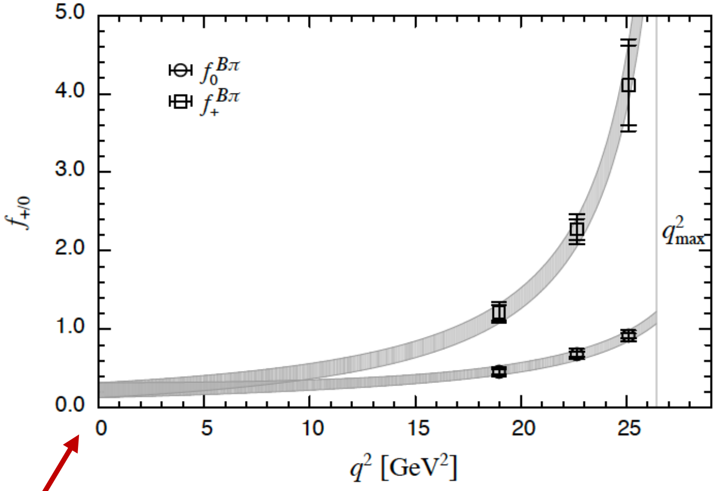
$$q^2_{\rm max} = (m_B - m_\pi)^2 \approx 26.4\,\mathrm{GeV}^2$$

Lowest Fourier modes on $L = 4$ fm lattice

$\lvert\vec{n}^2\rvert$	0	1	2	3	4
E_π / GeV	0.139	0.338	0.457	0.551	0.631
q^2 / GeV^2	26.4	24.3	23.1	22.1	21.2

Limited coverage of q^2 range

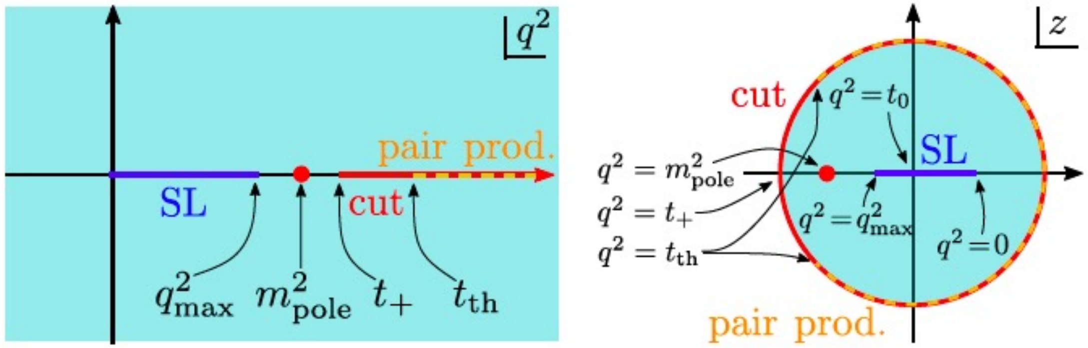
RBC-UKQCD PRD91 074510 2015[1]



Kinematical constraint $f_0(0) = f_+(0)$

courtesy of J. Flynn

Analytic structure of the Form Factors



BGL approach

(Boyd, Grinstein and Lebed '95-'97)

* the hadronic form factors corresponding to definite spin-parity can be represented as an expansion, originating from unitarity, analyticity and crossing symmetry, in terms of the conformal variable z ($|z| \leq 1$)

$$f_+(q^2) = \frac{1}{\sqrt{\chi_{1^-}(q_0^2)}} \frac{1}{\phi_+(z(q^2), q_0^2)\ P_+(z(q^2))} \sum_{n=0}^{\infty} a_n\ z^n(q^2)$$

$$z(t) \equiv \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$

$t_0 \to t_-$

$t_\pm \equiv (m_B \pm m_D)^2$

$\phi_+(z(q^2), q_0^2)$ = kinematical function (q_0^2 = auxiliary quantity)

$P_+(z(q^2))$ = Blaschke factor including resonances below the pair-production threshold t_+

$$\chi_{1^-}(q_0^2) = \text{transverse vector susceptibility} \equiv \frac{1}{2} \frac{\partial^2}{\partial (q_0^2)^2} \left[q_0^2\, \Pi_{1^-}(q_0^2) \right] = \frac{1}{\pi} \int_0^\infty ds \frac{s\, \mathrm{Im}\Pi_{1^-}(s)}{(s - q_0^2)^3}$$

calculable nonperturbatively from appropriate 2-point lattice correlators (see arXiv:2105.07851)

unitarity constraint: $\sum_{n=0}^{\infty} a_n^2 \leq 1$

BGL used by almost all the FF studies in the past (CLN)

BGL: frequentist fit

Input (eg lattice ff)

$$\mathbf{f}^T = (\mathbf{f}_+, \mathbf{f}_0)^T$$
$$= \big(f_+(q_0^2), f_+(q_1^2), \ldots, f_+(q_{N_+-1}^2), f_0(q_0^2), f_0(q_1^2), \ldots, f_0(q_{N_0-1}^2)\big)$$

Output (BGL params)

$$\mathbf{a}^T = (\mathbf{a}_+, \mathbf{a}_0)^T = (a_{+,0}, a_{+,1}, \ldots, a_{+,K_+-1}, a_{0,1}, a_{0,2}, \ldots, a_{0,K_0-1})$$

Frequentist fit

$$\chi^2(\mathbf{a}, \mathbf{f}) = (\mathbf{f} - Z\mathbf{a})^T C_{\mathbf{f}}^{-1}(\mathbf{f} - Z\mathbf{a})$$

Frequentist result

$$\mathbf{a} = (Z^T C_{\mathbf{f}}^{-1} Z)^{-1} Z C_{\mathbf{f}}^{-1} \mathbf{f}, \qquad C_{\mathbf{a}} = (Z^T C_{\mathbf{f}}^{-1} Z)^{-1}$$

- Z contains BGL ansatz and kinematic constraint
- Written here using constraint to eliminate $a_{0,0}$:

$$a_{0,0} = \frac{B_0(0)\phi_0(0)}{B_+(0)\phi_+(0)} \sum_{k=0}^{K_+-1} a_{+,k} z(0)^k - \sum_{k=1}^{K_0-1} a_{0,k} z(0)^k$$

courtesy of J. Flynn

Dispersion Matrix (DM) approach

PRD '21 (2105.02497)

* reappraisal and improvement of the method originally proposed by Bourrely et al. NPB '81 and Lellouch in NPB '96

$$\mathcal{M} = \begin{pmatrix} <\phi f|\phi f> & <\phi f|g_t> & <\phi f|g_{t_1}> & \cdots & <\phi f|g_{t_N}> \\ <g_t|\phi f> & <g_t|g_t> & <g_t|g_{t_1}> & \cdots & <g_t|g_{t_N}> \\ <g_{t_1}|\phi f> & <g_{t_1}|g_t> & <g_{t_1}|g_{t_1}> & \cdots & <g_{t_1}|g_{t_N}> \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ <g_{t_N}|\phi f> & <g_{t_N}|g_t> & <g_{t_N}|g_{t_1}> & \cdots & <g_{t_N}|g_{t_N}> \end{pmatrix}$$

inner product: $<g|h> \equiv \frac{1}{2\pi i}\int_{|z|=1} \frac{dz}{z}\,\overline{g}(z)\,h(z)$

$$g_t(z) \equiv \frac{1}{1-\overline{z}(t)\,z}$$

$$<g_t|\phi f> \equiv \phi(z,q_0^2)\,f(z) \qquad <g_t|g_{t_m}> = \frac{1}{1-\overline{z}(t_m)\,z(t)}$$

t_1, t_2, …, t_N are the N values of the squared 4-momentum transfer where the form factor *f* has been computed and t is its value where we want to compute *f*(t)

$$\text{unitarity bound:} \quad <\phi f|\phi f> \equiv \frac{1}{2\pi i}\int_{|z|=1} \frac{dz}{z}\,|\phi(z,q_0^2)\,f(z)|^2 \leq \chi(q_0^2)$$

in the case of interest $z_i \equiv z(t_i)$ and $\phi_i f_i \equiv \phi(z_i, q_0^2) f(t_i)$ are real numbers and the positivity of the inner product implies:

$$\det[\overline{\mathcal{M}}] = \begin{vmatrix} \chi(q_0^2) & \phi f & \phi_1 f_1 & \cdots & \phi_N f_N \\ \phi f & \frac{1}{1-z^2} & \frac{1}{1-z z_1} & \cdots & \frac{1}{1-z z_N} \\ \phi_1 f_1 & \frac{1}{1-z_1 z} & \frac{1}{1-z_1^2} & \cdots & \frac{1}{1-z_1 z_N} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \phi_N f_N & \frac{1}{1-z_N z} & \frac{1}{1-z_N z_1} & \cdots & \frac{1}{1-z_N^2} \end{vmatrix} \geq 0$$

* the explicit solution is a band of values: $\beta - \sqrt{\gamma} \leq f(z) \leq \beta + \sqrt{\gamma}$

$$\beta = \frac{1}{d(z)\,\phi(z)} \sum_{j=1}^{N} f_j\, \phi_j d_j \frac{1 - z_j^2}{z - z_j} \qquad \gamma = \frac{1}{d^2(z)\,\phi^2(z)} \frac{1}{1 - z^2} \left[\chi - \sum_{i,j=1}^{N} f_i f_j\, \phi_i \phi_j d_i d_j \frac{(1 - z_i^2)(1 - z_j^2)}{1 - z_i z_j} \right]$$

χ, f_i : nonperturbative input quantities, $\phi(z), d(z), \phi_i, d_i$: kinematical coefficients depending on z_i

* unitarity is satisfied when $\gamma \geq 0$, which implies: $\chi \geq \sum_{i,j=1}^{N} f_i f_j\, \phi_i \phi_j d_i d_j \frac{(1 - z_i^2)(1 - z_j^2)}{1 - z_i z_j}$

*** this is the parameterization-independent unitarity test of the set of input data $\{f_j\}$ ***

* important feature: when $z \to z_j$ one has $\beta \to f_j$ and $\gamma \to 0$, i.e. the DM band collapses to f_j for $z = z_j$

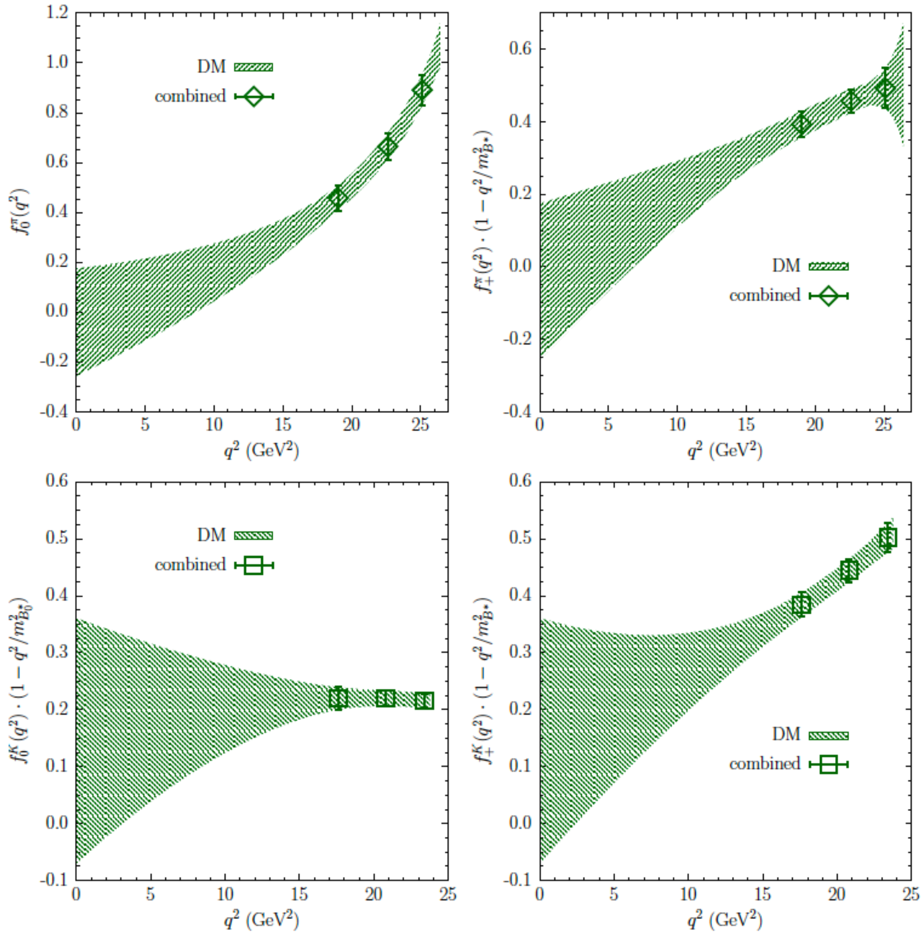
↓

for any given set of input data the DM approach reproduces exactly the known data and it allows to extrapolate the form factor in the whole kinematical range in a parameterization-independent way providing a band of values representing the results of all possible BGL fits satisfying unitarity and passing through the known points (important for estimating uncertainties)

* the DM band represents a uniform distribution which is combined with the multivariate distribution of the input data $\{f_j\}$ to generate the final band for $f(z)$

* kinematical constraint(s) can be easily and rigorously implemented in the DM approach (see for details arXiv:2105.02497)

Dispersive matrix method results



- plots from JHEP 08 022 2022[16]
  top: $B \longrightarrow \pi$ RBC-UKQCD 15[1] FNAL-MILC 15[14]
  bottom: $Bs \longrightarrow K$ HPQCD 14[17], RBC-UKQCD 15[1], FNAL-MILC 19[15]
- χ's from lattice-computed current-current correlators
- indirect implementation of kinematic constraint
- use input data from different sources by combining form-factors at common q^2 points
- ~~lacks frequentist interpretation~~

Di Carlo et al PRD104 054502 2021[18]; Martinelli et al PRD104 094512 2021[19], PRD105 034503 2022[20], JHEP 08 022 2022[16], PRD106 093002 2022[21]

Bayesian BGL form factor fit

- Frequentist fit
  - $N_{\text{dof}} = N_{\text{data}} - N_{\text{params}} \geq 1$ means in practice truncation of z expansion at low order
  - induced systematic
- Bayesian fit [RBC-UKQCD 2303.11280[22]; JF, Jüttner, Tsang 2303.11285[13]]
  - aim to fit full z expansion (no truncation)
  - need regulator to control higher-order coefficients — use unitarity constraint
  - compute (functions of) z-expansion coefficients as expectation values

$$\langle g(\mathbf{a})\rangle = N \int d\mathbf{a}\, g(\mathbf{a})\, \pi(\mathbf{a}|\mathbf{f}, C_{\mathbf{f}})\, \pi_{\mathbf{a}}$$

with probability for parameters given model and data

$$\pi(\mathbf{a}|\mathbf{f}, C_{\mathbf{f}}) \propto \exp\left(-\frac{1}{2}\chi^2(\mathbf{a},\mathbf{f})\right) \qquad \text{where} \qquad \chi^2(\mathbf{a},\mathbf{f}) = (\mathbf{f} - Z\mathbf{a})^T C_{\mathbf{f}}^{-1}(\mathbf{f} - Z\mathbf{a})$$

and prior knowledge from unitarity constraint

$$\pi_{\mathbf{a}} \propto \theta(1 - |\mathbf{a}_+|_\alpha^2)\,\theta(1 - |\mathbf{a}_0|_\alpha^2)$$

- use MC integration: sample $\mathbf{a}$ from multivariate normal and drop samples incompatible with unitarity
- in practice, low probability to satisfy unitarity when K_+ and K_0 large
- modify

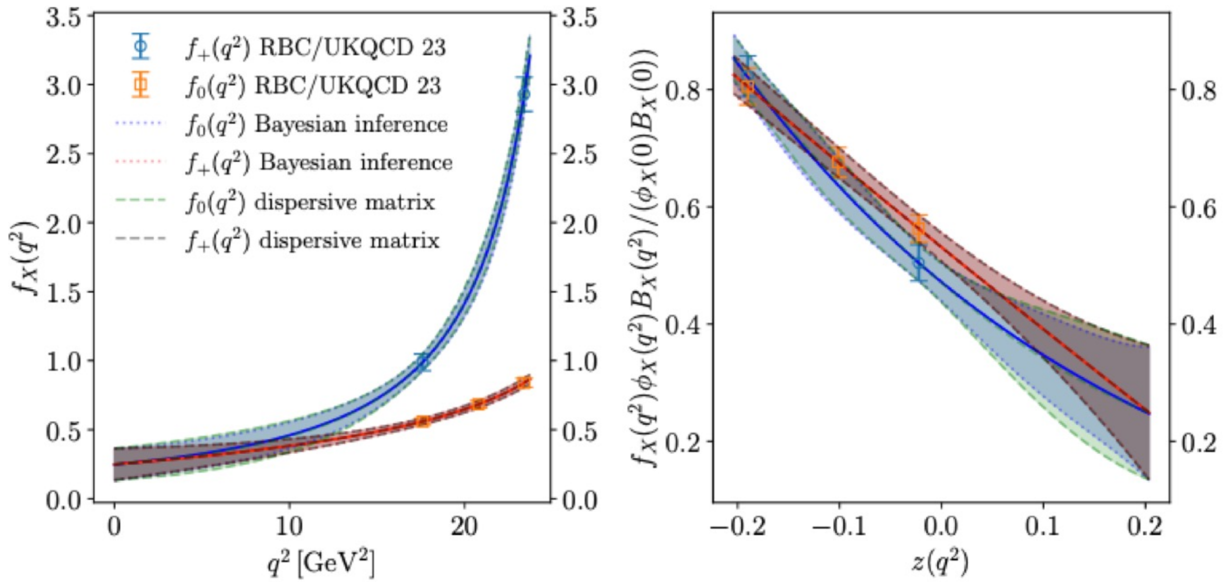
$$\pi(\mathbf{a}|\mathbf{f}_p, C_{\mathbf{f}_p})\,\pi_{\mathbf{a}}(\mathbf{a}_p|M) \propto \theta(\mathbf{a})\,\exp\left(-\frac{1}{2}(\mathbf{f}_p - Z\mathbf{a})^T C_{\mathbf{f}_p}^{-1}(\mathbf{f}_p - Z\mathbf{a}) - \frac{1}{2}\mathbf{a}^T\frac{M}{\sigma^2}\mathbf{a}\right)$$

  - choose M such that $\mathbf{a}^T M\mathbf{a} \leq 2$ in presence of kinematic constraint
  - draw random number
  - correct with accept-reject with probability

$$p \leq \frac{\exp(-1/\sigma^2)}{\exp(-\mathbf{a}^T\frac{M}{2\sigma^2}\mathbf{a})}$$

A recent counter-check of the DM method

Results III: Bayesian Inference vs Dispersive Matrix Method



Application to $B_s \to K$: **identical results!**

- BI and DM produce equivalent results
- BI simpler to implement + easily to combine with multiple data sets
- BI provides numerical value of the coefficients

problems with lattice calculations vs experimental data

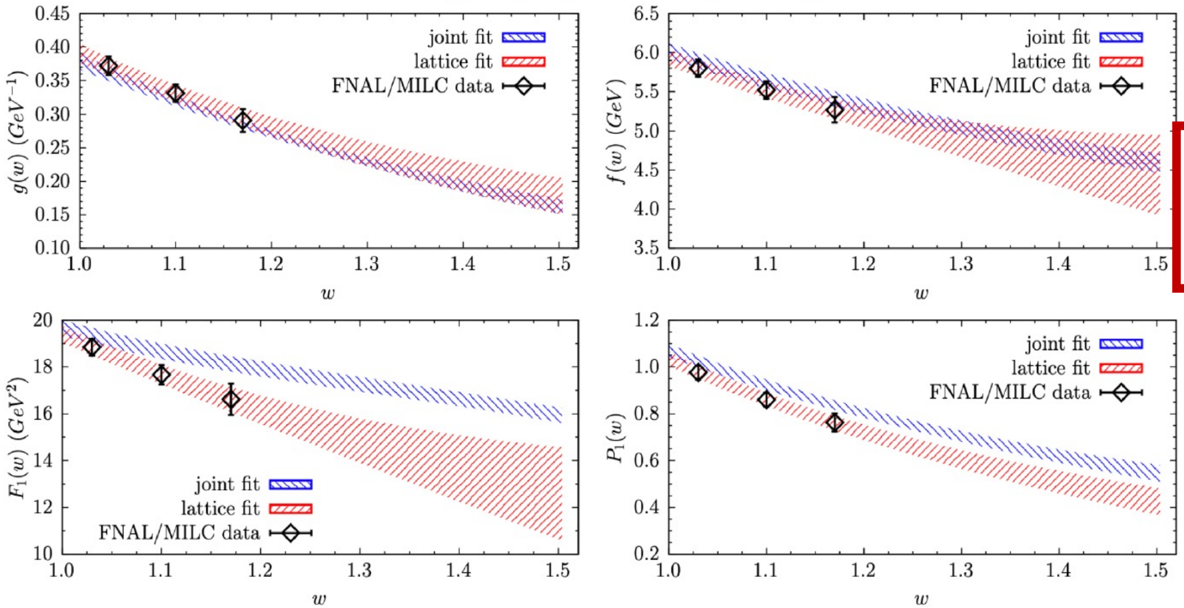
Why not doing a global fit of lattice and exp. data

Note that one can use **also** experimental data (in addition to the LQCD ones) to constrain the shape of the FFs…

$$d\Gamma/dx,\ \ x = w, \cos\theta_l, \cos\theta_v, \chi$$

Belle Coll.: arXiv:1702.01521, PRD '19 [arXiv:1809.03290]

Let us see this in detail: *let us consider the BGL fits performed by FNAL/MILC Collaborations in EPJC '22 arXiv:2105.14019*



joint fit:
BGL fit of LQCD points + Belle + BaBar exp. data

$|V_{cb}| \cdot 10^3 = 38.40 \pm 0.74$

$R(D^*) = 0.2483 \pm 0.0013$

lattice fit:
quadratic BGL fit of LQCD points only

$|V_{cb}| > |V_{cb}|^{\text{joint fit}}$?

$R(D^*) = 0.265 \pm 0.013$

simultaneous fit of the lattice points and experimental data to determine the shape of the FFs and to extract $|V_{cb}|$

*** slope differences between exp's and theory $\rightarrow$ bias on $|V_{cb}|^{\text{joint fit}}$? ***

State-of-the-art of the semileptonic B → {D(*),π} decays

Two critical issues

- V_{cb}
- $R_{D^{(*)}}$

- exclusive/inclusive $|V_{cb}|$ puzzle:

exclusive (FLAG '21): $|V_{cb}|(BGL) \cdot 10^3 = 39.36\ (68)$

inclusive (HFLAV '21): $|V_{cb}| \cdot 10^3 = 42.19\ (78)$

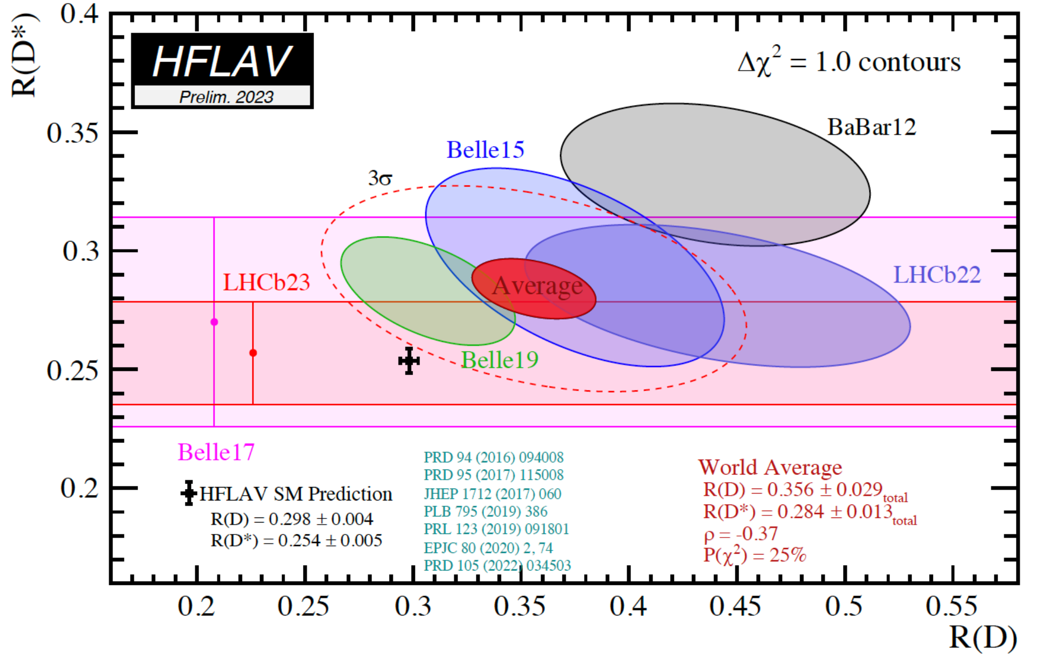
difference of ~ 2.7 σ

$|V_{cb}| \cdot 10^3 = 42.16\ (50)$

(Bordone et al. 2107.00604)

$$\mathcal{R}(D) = \frac{\mathcal{B}(B \to D\tau\nu_\tau)}{\mathcal{B}(B \to D\ell\nu_\ell)},$$

$$\mathcal{R}(D^*) = \frac{\mathcal{B}(B \to D^*\tau\nu_\tau)}{\mathcal{B}(B \to D^*\ell\nu_\ell)}$$

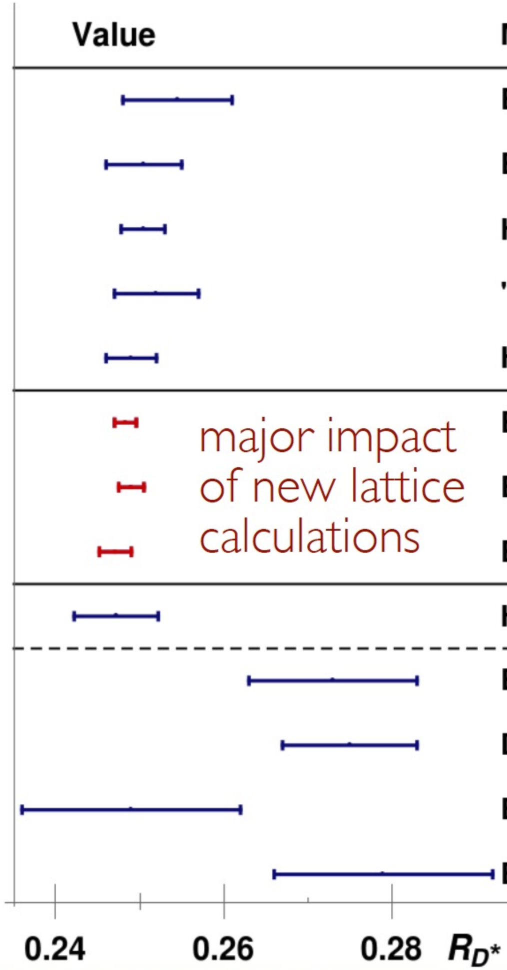


HFLAV Collaboration, PRD '23 [arXiv:2206.07501] (updated plot)

2022

3.2σ tension

Overview over predictions for $R(D^*)$



Method	Input Theo	Input Exp	Reference
BGL	Lattice, HQET	Belle'17'18	Gambino et al.'19
BGL	Lattice, HQET	Belle'18	Jaiswal et al.'20
HQET@$1/m_c^2,\alpha_s$	Lattice, LCSR, QCDSR	Belle'17'18	Bordone et al.'20
"Average"			HFLAV'21
HQET$_{RC}$@$1/m^2,\alpha_s^{(2)}$	Belle'17'18	Lattice	Bernlochner et al.'22
BGL	Lattice	Belle'18, Babar'19	Vaquero et al.'21v2
BGL	Lattice	Belle'18	JLQCD prel. (MJ)
BGL	Lattice	Belle'18	Davies, Harrison'23
HQET@$1/m_c^2,\alpha_s$	Lattice, LCSR, QCDSR	---	Bordone et al.'20
BGL	Lattice	---	Vaquero et al.'21v2
DM	Lattice	---	Martinelli et al. FNAL/MILC
BGL	Lattice	---	JLQCD prel. (MJ)
BGL	Lattice	---	Davies, Harrison'23

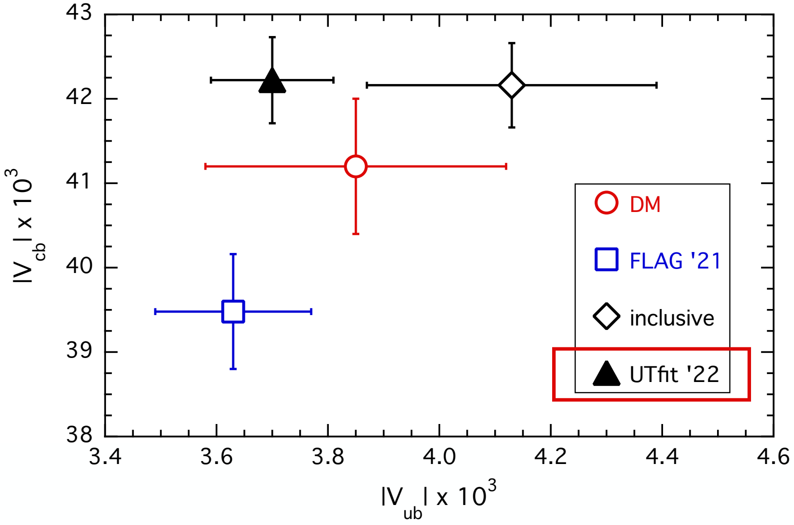
FNAL 0.275 \pm 0.008
JLQCD 0.248 \pm 0.008
HPQCD 0.276 \pm 0.009

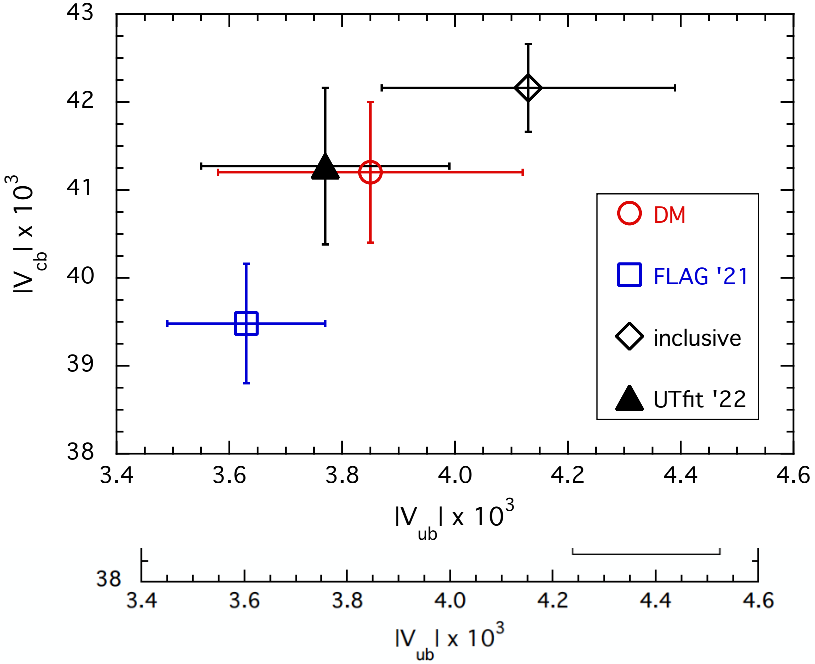
Predictions based only on Fermilab & HPQCD lead to larg[e] ... agreement with exp, mostly because of the suppression at high w of the denominator.

I see no reason not to use experimental data for a SM test, especially in presence of tensions in lattice data.

Courtesy by Gambino ***EXP 0.284 \pm 0.013***

May 2023

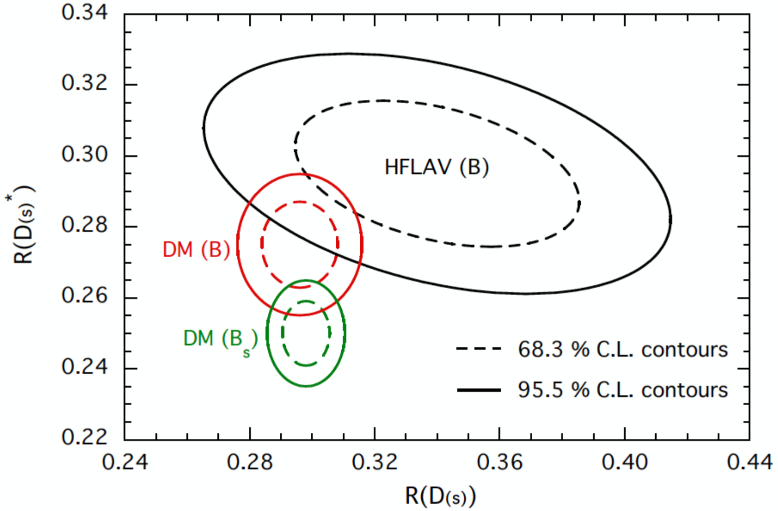




	decays	DM	FLAG '21	inclusive
$\vert V_{cb}\vert \cdot 10^3$	$B_{(s)} \to D_{(s)}^{(*)}$	41.4 (8)	39.48 (68)	42.16 (50)
$\vert V_{ub}\vert \cdot 10^3$	$B_{(s)} \to \pi, K$	3.85 (27)	3.63 (14)	4.13 (26)

SU(3) breaking effects need further investigation

see Ludovico's slides in discussion session



	DM	HFLAV '19
R(D)	0.296 (8)	0.340 (27) (13)
R(D*)	0.275 (8)	0.295 (11) (8)
R(Ds)	0.298 (5)	
R(Ds*)	0.250 (6)	

reduced tensions in both $|V_{cb}|$, $|V_{ub}|$ and $R(D^{(*)})$
when theory and experiments are not fitted simultaneously

RADIATIVE CORRECTIONS

The accuracy of lattice calculations of the hadron spectrum (and hence of the quark masses) and of the decay constants and form factors is such that isospin breaking and em effects cannot be neglected anymore:

f_π = 130.2(0.8) MeV ε =0.6% f_K = 155.7(0.3) MeV ε =0.2%

f_K/f_π = 1.1932(19) ε =0.16% $F^{K\pi}(0)$ =0.9698(17) ε =0.18%

A remark on useful and useless precision of lattice calculations:

1) ε_K and long distance charm contributions

2) isospin breaking and electromagnetic corrections to f_K and f_π

Radiative corrections to neutron decay, the Sacred Graal

leptonic decays of PS mesons ➡ extraction of CKM matrix elemen

$$\Gamma(PS^+ \to \ell^+\nu_\ell) = \frac{G_F^2}{8\pi} |V_{q_1q_2}|^2 m_\ell^2 \left(1 - \frac{m_\ell^2}{m_{PS^+}^2}\right) M_{PS^+} f_{PS}^2 S_{ew} \left(1 + \delta R_{IB}^{PS} + \delta R_{QED}^{PS}\right)$$

S_{ew}: universal electroweak correction (

f_{PS} : leptonic decay constant in isoQCD ($m_u = m_d$, $e_f = 0$)

δR_{IB}^{PS} : strong isospin breaking correction $\propto O[(m_d - m_u)/\Lambda_{QCD}] \simeq O(1\%)$

δR_{QED}^{PS} : QED correction $\propto O(\alpha_{em}) \simeq O(1\%)$

* lattice determinations of f_{PS} have reached an accuracy below the percent level

$\frac{f_K}{f_\pi}$: relative error of $\simeq$ (

FLAG-4 [EPJC '20]

⬇

need of determining δR_{IB}^{PS} and δR_{QED}^{PS} on the lattice

* **the infrared (IR) problem**: only $\Gamma(\Delta E_\gamma) = \Gamma_0 + \Gamma_1(\Delta E_\gamma)$ is IR finite [Block&Nordsiek '37]

Γ_n : n photons in the fi

RM123+Soton strategy: $\Gamma(\Delta E_\gamma) = \lim_{V\to\infty}\left[\Gamma_0 - \Gamma_0^{pt}\right] + \lim_{V\to\infty}\left[\Gamma_0^{pt} + \Gamma_1(\Delta E_\gamma)\right]$ pt = point-like

IR finite IR finite

PRD '15 arXiv:1502.00257 (master formula)
PRD '17 arXiv:1611.08497 (FVEs)
PRL '18 arXiv:1711.06537 (π and K)
PRD '19 arXiv:1904.08731 (π and K)

$\lim_{V\to\infty}\left[\Gamma_0 - \Gamma_0^{pt}\right]$ on the lattice

$\lim_{m_\gamma\to 0}\left[\Gamma_0^{pt} + \Gamma_1^{pt}(\Delta E_\gamma)\right]$ within the pt appro

From f_K/f_π and $f^{K\pi}(0)$ we can extract V_{us}/V_{ud} and V_{us}

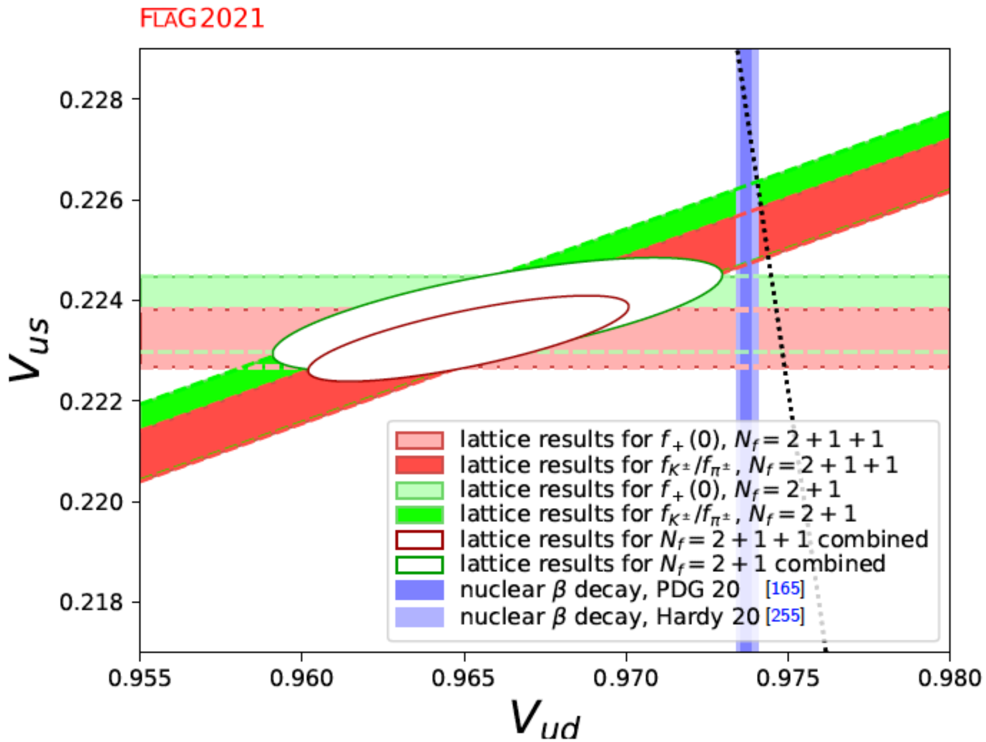


Figure 10: The plot compares the information for $|V_{ud}|$, $|V_{us}|$ obtained on the lattice for $N_f = 2+1$ and $N_f = 2+1+1$ with $|V_{ud}|$ extracted from nuclear β transitions Eqs. (71) and (72). The dotted line indicates the correlation between $|V_{ud}|$ and $|V_{us}|$ that follows if the CKM-matrix is unitary. For the $N_f = 2$ results see the 2016 edition [3].

Including radiative corrections

RM123+SOTON, PRL 120 (2018)

- with this method, our result for

$$\Gamma_P(E) = \Gamma_P^0 \{1 + \delta R_P(E)\} \;,$$

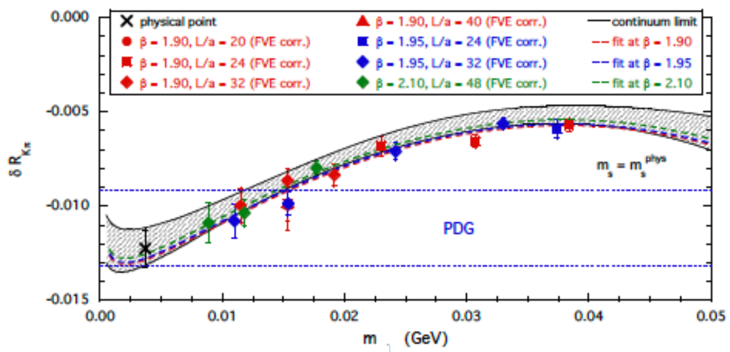
$$\delta R_{K\pi} = \delta R_K(E_K^{max}) - \delta R_\pi(E_\pi^{max})$$

is the following

$$\delta \mathbf{R}_{\mathbf{K}\pi}$$
$$= -0.0122(10)^{st}(2)^{tun}(8)^{\chi}(5)^{L}(4)^{a}(\cdots)^{qQED}$$

$$\frac{|V_{us}|}{|V_{ud}|} = 0.23134\,(24)_{\rm exp}\,(30)_{\rm th} = 0.23134\,(38)$$

$$= \mathbf{-0.0122(16)}$$



ETMC gauge configurations

$n_f = 1+1+1+1$

$a \geq 0.0619(18)$ fm

- this can (remember the caveat concerning the definition of QCD) be compared with the result currently quoted by the PDG and obtained in **v.cirigliano and h.neufeld, PLB 700 (2011)**

$$\delta R_{K\pi} = -0.0112(21)$$

NEW: Phys. Rev. D100 (2019) no. 3, 034514

$$\delta R_{K\pi}^{\rm phys} = \delta R_K^{\rm phys} - \delta R_\pi^{\rm phys} = -0.0126\,(14)$$

$$\frac{|V_{us}|}{|V_{ud}|} = 0.23134\,(24)_{\rm exp}\,(30)_{\rm th} = 0.23134\,(38)$$

$$|V_{us}| = 0.22538\,(24)_{\rm exp}\,(30)_{\rm th} = 0.22538\,(38)$$

Real photon emission

KLOE experiment $K \to e\nu_e\gamma$

[EPJC '09]

$$\Delta R^{exp,i} = \int_{E_\gamma^i}^{E_\gamma^{i+1}} dE_\gamma \frac{1}{\Gamma_{K\mu 2[\gamma]}} \left[\frac{d\Gamma(K_{e2\gamma})}{dE_\gamma}\right]_{p_e > 200\text{ MeV}} \to \Delta R^{pt,i} + \Delta R^{SD,i} + \Delta R^{INT,i}$$

(kinematical cut due to K_{e3} decays)

five bins : $E_\gamma^i = \{10,\ 50,\ 100,\ 150,\ 200,\ 250\}$ MeV

$E_\gamma^{max} \simeq 250$ MeV

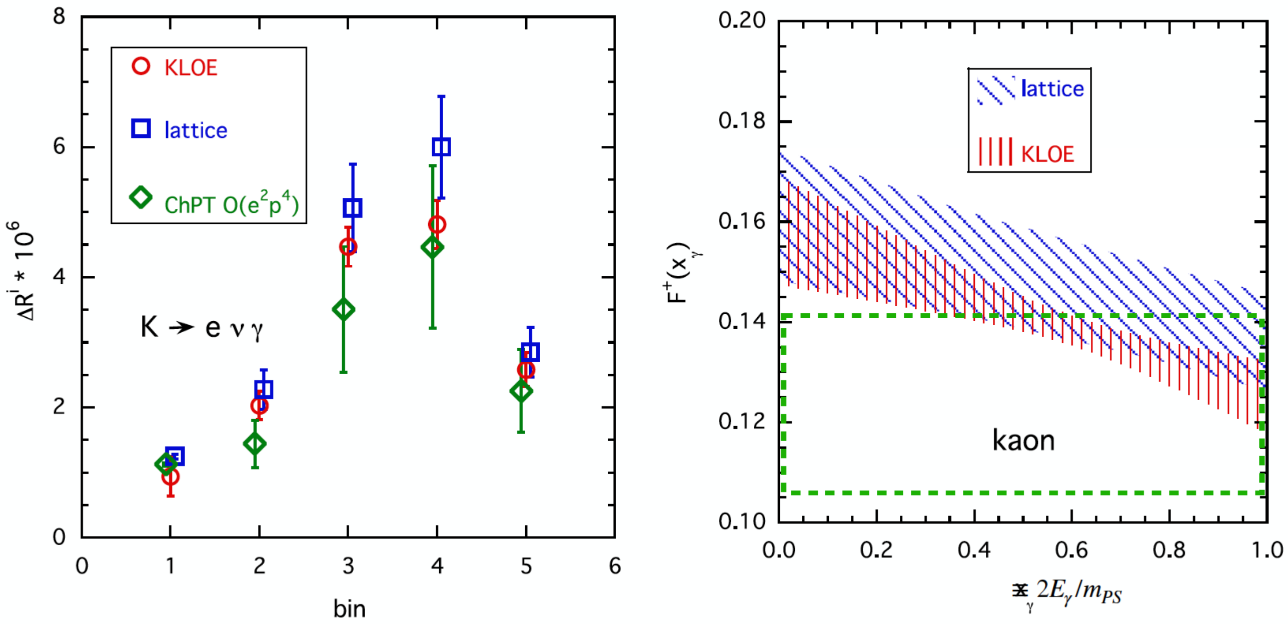


FIG. 1. Left panel: comparison of the KLOE experimental data $\Delta R^{\exp,i}$ [9] (red circles) with the theoretical predictions $\Delta R^{\text{th},i}$, (blue squares) evaluated with the vector and axial form factors of Ref. [8] given in Eqs. (13)–(17), for the 5 bins (see Table IV). The green diamonds correspond to the prediction of ChPT at order $\mathcal{O}(e^2 p^4)$, based on the vector and axial form factors given in Eq. (53). Right panel: comparison of the form-factor $F^+(x_\gamma)$ extracted by the KLOE collaboration in Ref. [9] and the theoretical prediction from Eqs. (13)–(17). The shaded areas represent uncertainties at the level of 1 standard deviation.

$\Delta R^{pt,i}$: relevant in the first bin only

$\Delta R^{INT,i}$: negligible

$$\Delta R^{SD,i} \propto \left[F_V(x_\gamma) + F_A(x_\gamma)\right]^2$$

ChPT $O(e^2p^4)$: $F_V(x_\gamma) = \dfrac{m_{PS}}{4\pi^2 f_{PS}}$

$$F_A(x_\gamma) = \frac{8 m_{PS}}{f_{PS}}(L_9^r + L_{10}^r)$$

$$F^+(x_\gamma) = F_V(x_\gamma) + F_A(x_\gamma) \simeq 0.123 \pm 0.018$$

***** **good consistency** *****

B meson real photon emissions

Factorization at leading power in an expansion of the decay amplitude in Λ_{QCD}/E_γ and Λ_{QCD}/mb has been established to all orders in the strong coupling α_s. In this approximation, the branching fraction depends only on the leading-twist B-meson light-cone distribution amplitude (LCDA) $\phi_+(\omega,\mu)$

More precisely, it is proportional to $1/\lambda_B$, the most important LCDA parameter in exclusive decays, is uncertain by a large factor ranging from 200 MeV favoured by non-leptonic decays to 460 MeV from QCD sum rules.

The radiative leptonic decay has therefore been suggested as a measurement of λ_B

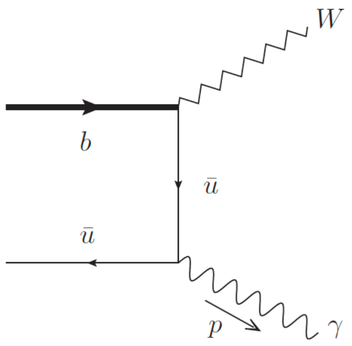


Figure 1. Leading contribution to $B \to \gamma \ell \nu_\ell$.

For large photon energies the form factors can be written as [9]

$$F_V(E_\gamma) = \frac{e_u f_B m_B}{2E_\gamma \lambda_B(\mu)} R(E_\gamma,\mu) + \xi(E_\gamma) + \Delta\xi(E_\gamma)\,,$$
$$F_A(E_\gamma) = \frac{e_u f_B m_B}{2E_\gamma \lambda_B(\mu)} R(E_\gamma,\mu) + \xi(E_\gamma) - \Delta\xi(E_\gamma)\,. \quad (2.7)$$

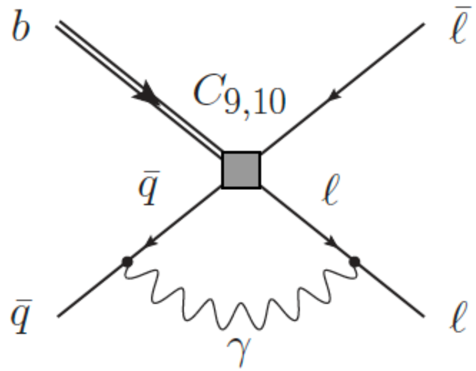
The first term is equal in both expressions and represents the leading-power contribution in the heavy-quark expansion (HQE). It originates only from photon emission from the light spectator quark in B meson (Fig. 1). In the above, f_B is the decay constant of B meson, and the quantity λ_B is the first inverse moment of the B-meson LCDA,

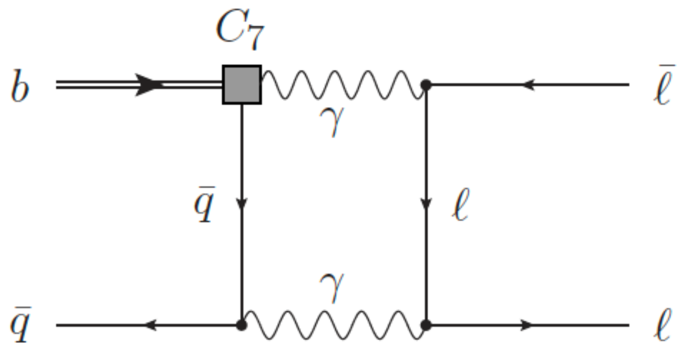
$$\frac{1}{\lambda_B(\mu)} = \int_0^\infty \frac{d\omega}{\omega}\, \phi_+(\omega,\mu)\,. \quad (2.8)$$

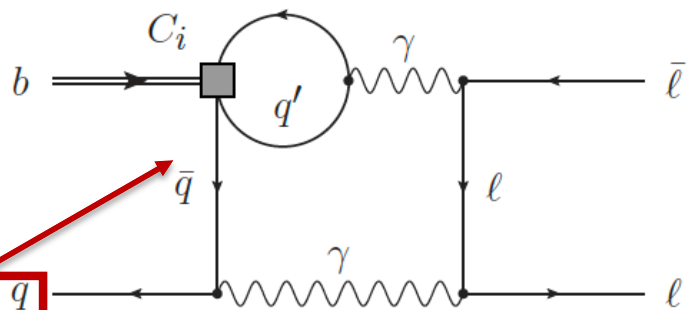
Further applications in decays of heavy neutral B mesons: Virtual corrections (some questions still open)

Enhanced electromagnetic correction to the rare B-meson decay $B_{s,d} \to \mu^+\mu^-$

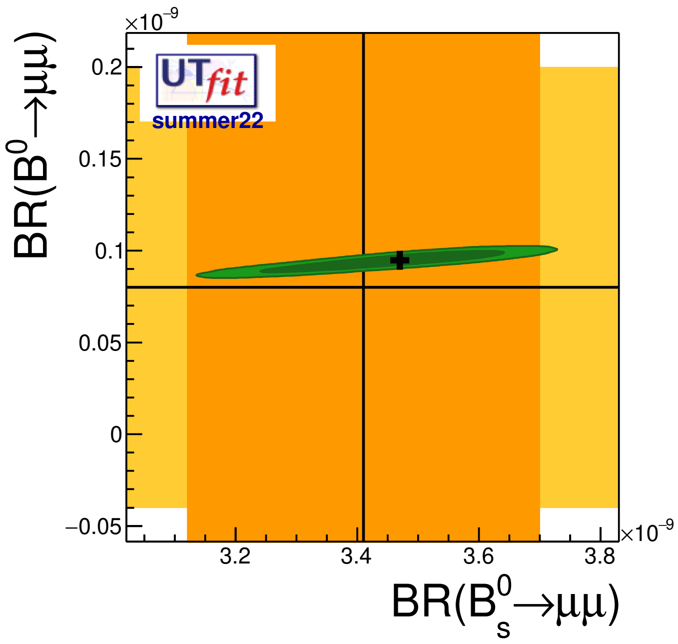
Martin Beneke,[1] Christoph Bobeth,[1,2] and Robert Szafron[1]







is this really reabsorbed in the coefficient of O_9?



Further applications in decays of heavy neutral B mesons:
real corrections (some questions still open)
see the talk by L. Vittorio

$B_s^0 \to \mu^+\mu^-\gamma$ from $B_s^0 \to \mu^+\mu^-$

Francesco Dettori[a], Diego Guadagnoli[b] and Méril Reboud[b,c]

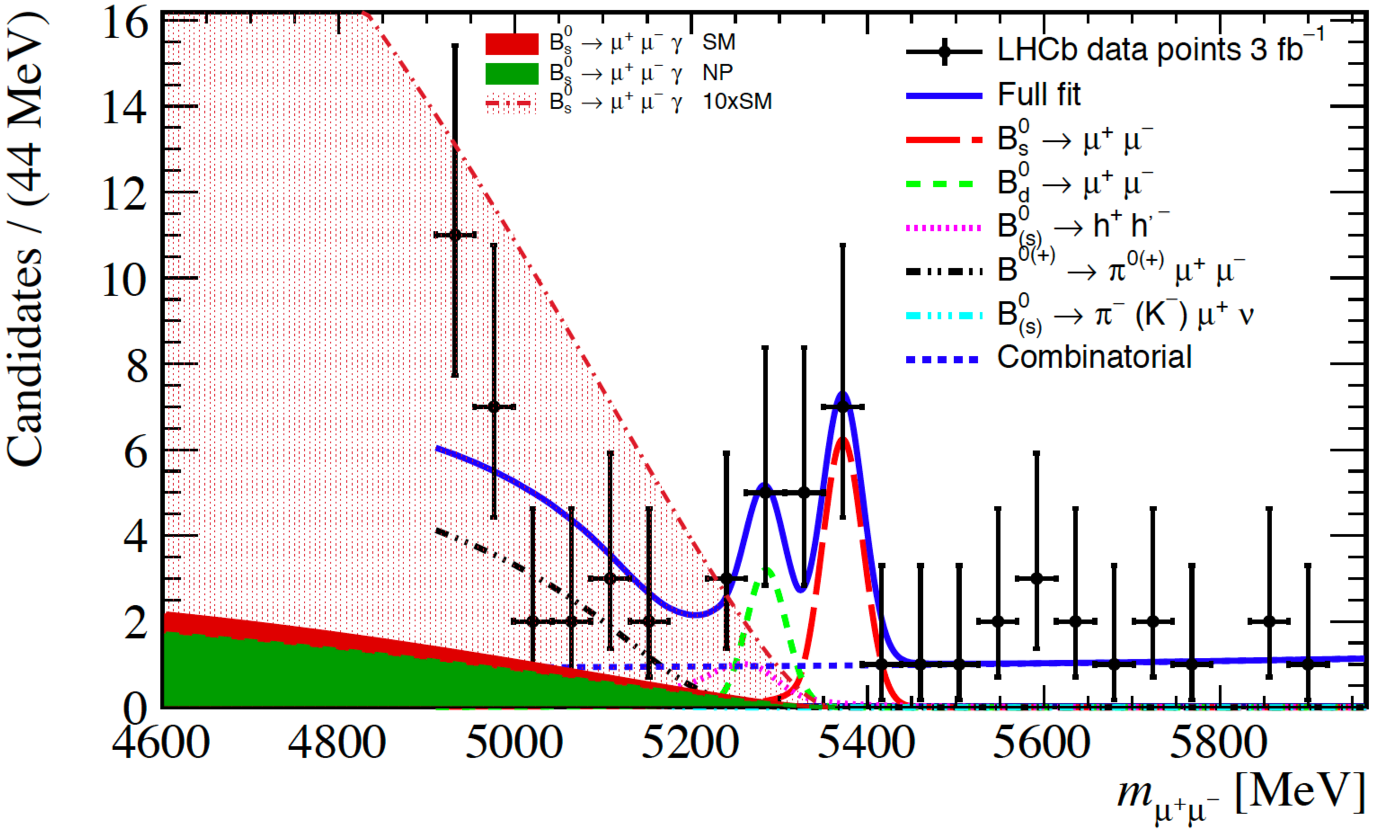
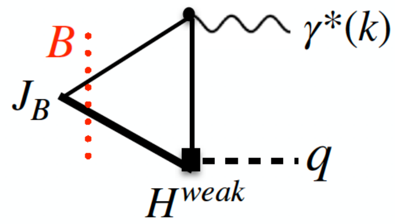
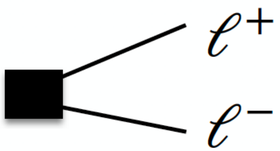


Figure 3: Dimuon invariant mass distribution from LHCb's measurement of $\mathcal{B}(B_s^0 \to \mu^+\mu^-)$ [52] overlayed with the contribution expected from $B_s^0 \to \mu^+\mu^-\gamma$ decays (ISR only). Assumes flat efficiency versus $m_{\mu^+\mu^-}$. The line denoted as '$B_s^0 \to \mu^+\mu^-\gamma$ NP' refers to the $V-A$ case with $\delta C_9 = -12\% \, C_9^{\rm SM}$ (see also Fig. 2). The two filled curves are not stacked onto each other.

Particle(s) from weak vertex with momenta q



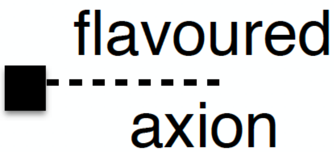
- **FCNC** Qb= Qq (need long distance in addition) : $F(q^2, k^2)$



H^{weak} ~ O$_{9,10}$: $B_{d,s} \to \ell^+ \ell^- \gamma$ $\quad F(q^2) = F(q^2, 0)$

Bobeth's talk

H^{weak} ~ O$_7$: $B_{d,s} \to \ell^+ \ell^- \gamma$ $\quad F^*(k^2) = F(0, k^2)$



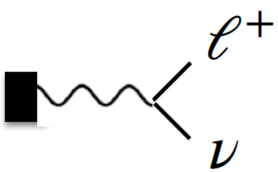
H^{weak} ~ $\bar{q} \gamma_\mu b_L \partial^\mu a$: $B_{d,s} \to \ell^+ \ell^- a$ $\quad F(m_a^2, k^2) \to F^*(k^2)$

Ziegler's talk

or dark photon, scalar DM, …

Xin-Yu Tuo et al. arXiv:2103.11331
G. Gagliardi et al. arXiv:2202.03833 [hep-lat]

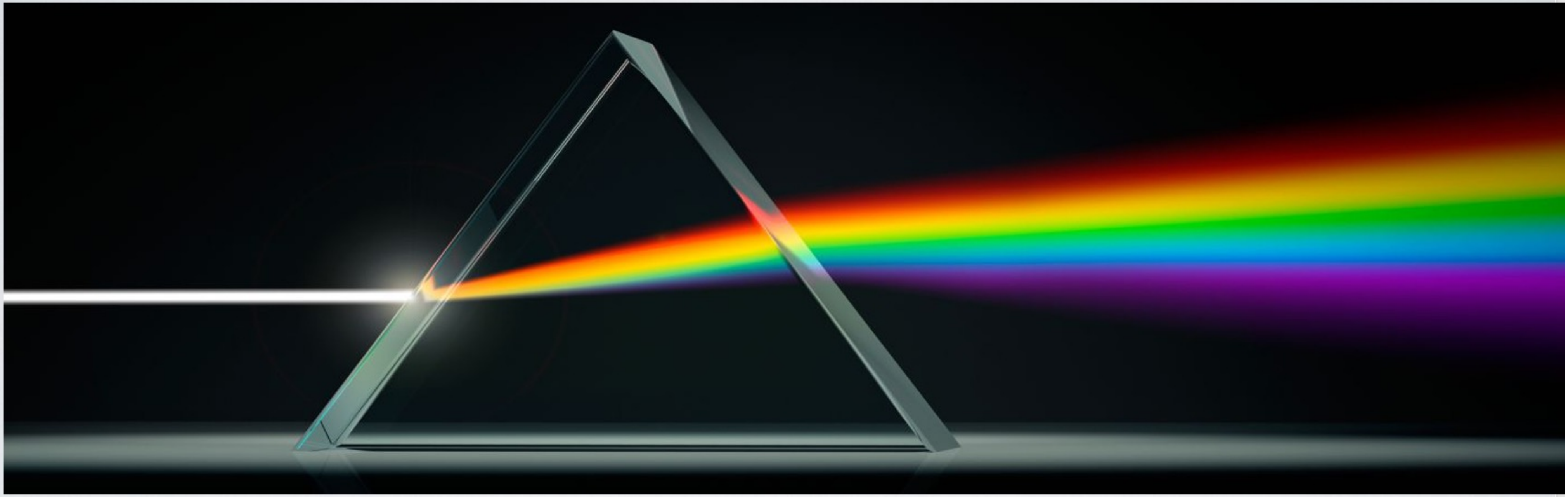
- **FCCC** Qb ≠ Qq :



H^{weak} ~ $V_{ub} \bar{u} \gamma_\mu b_L \ell \gamma^\mu \nu_L$: $B_u \to \ell^+ \nu \gamma$

- Physics: helicity suppression of $B \to f_i \bar{f}_j$ relieved in radiative decay!

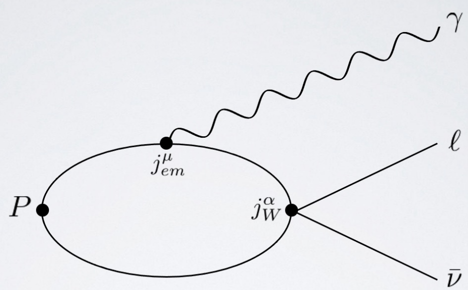
Radiative leptonic decay rates of pseudoscalar mesons



RM123 Collaboration

Antonio Desidero, Giulia de Divitiis, Marco Garofalo, **Martin Hansen**, Roberto Frezzotti, Nazario Tantalo, ***Massimo di Carlo, Davide Giusti, Vittorio Lubicz, Guido Martinelli, Chris Sachrajda, Francesco Sanfilippo, Silvano Simula, Cecilia Tarantino***

- Diagrammatically



$P \in \{\pi, K, D, D_s\}$

$$H_W^{r\nu}(k,\boldsymbol{p}) = \epsilon_\mu^r(k)\, H_W^{\mu\nu}(k,\boldsymbol{p}) = \epsilon_\mu^r(k) \int d^4y\, e^{ik\cdot y}\ \langle 0|\, \mathrm{T}[\, j_W^\nu(0) j_{\mathrm{em}}^\mu(y)]\, \big|D_s^+(\boldsymbol{p})\rangle$$

Why QED Corrections?

1) Pushing precision under percent level

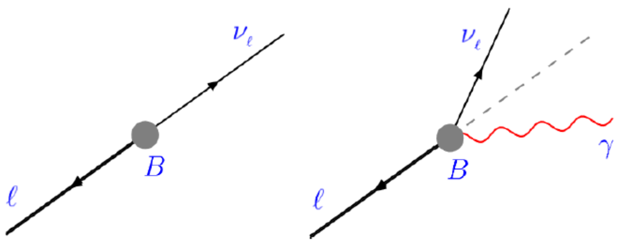
CKM matrix element extraction from leptonic decays

[Di Carlo et arXiv:1904.08731 (2019)] and

[Boyle et Al arXiv:2211.12865 (2022)] for K and π

real photon emission has <u>**always**</u> to be included when considering O(α_{em}) corrections **[Block&Nordsiek mechanism]**

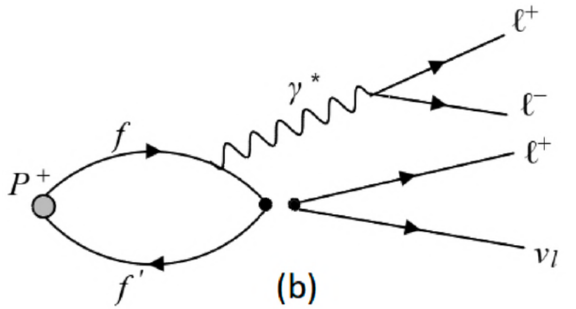
2) Removal of helicity suppression through photon emission



despite α_{em} there is an enhancement of $(m_P/m_l)^2$

3) Indirect search of New Physics

Operators that parametrize new physics are involved in processes where also QED has to be included.



$\propto \alpha_{em}^2$

NP is more likely to be detected

Courtesy of F. Mazzetti

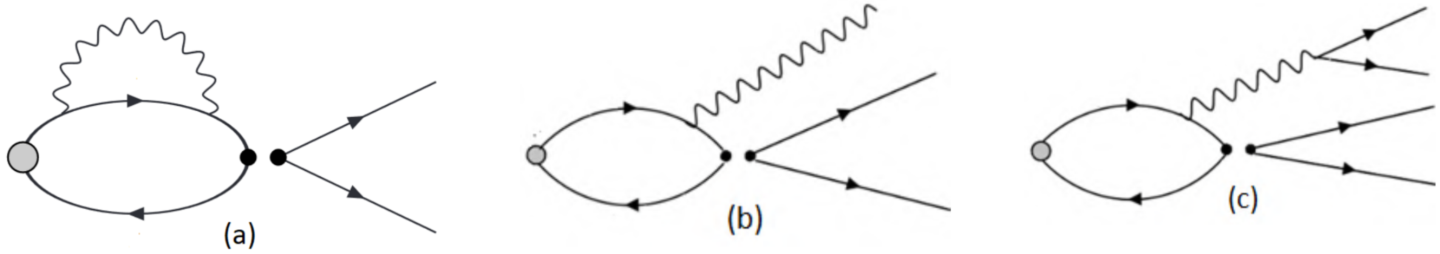
$O(\alpha_{em})$ corrections

NP constraints

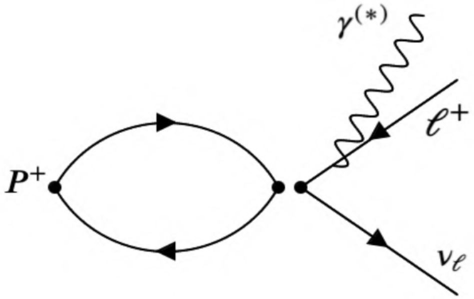
- Virtual corrections (a)
- Real photon emission (b)
- Virtual photon emission (c)

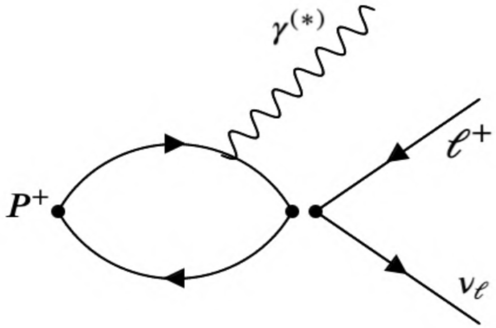
dynamical enhancement despite α_{em} suppression

$\propto \alpha_{em}^2$



$P^+ \to \ell^+ \nu_\ell \gamma^{(*)}$ decays



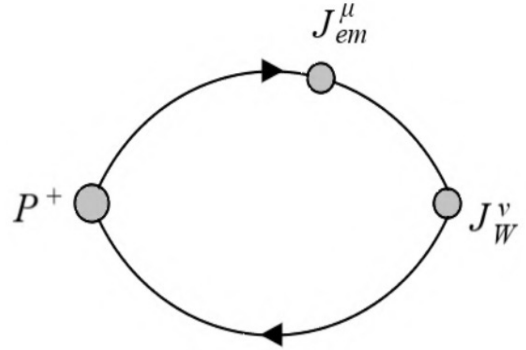


- Can be computed in perturbation theory, by simply knowing f_P

- (Virtual) photon interacts with the internal hadronic structure of P
- Non perturbative strong dynamics encoded in the hadronic tensor

Hadronic Tensor and Form Factors

$$H^{\mu\nu}(k,p) = \int d^4x e^{ik\cdot x} \langle 0|T[J^\mu_{em}(x) J^\nu_W(0)]|P(p)\rangle$$



$$H^{\mu\nu} = H^{\mu\nu}_{pt} + H^{\mu\nu}_{SD},$$

Point-like, IR contribution

$$H^{\mu\nu}_{pt} = f_P \left[g^{\mu\nu} - \frac{(2p-k)^\mu (p-k)^\nu}{(p-k)^2 - m_P^2} \right],$$

SD form factors

$$H^{\mu\nu}_{SD} = \frac{H_1}{m_P}(k^2 g^{\mu\nu} - k^\mu k^\nu) + \frac{H_2}{m_P}\frac{[(k\cdot p - k^2)k^\mu - k^2(p-k)^\mu]}{(p-k)^2 - m_P^2}(p-k)^\nu$$
$$+ \frac{F_A}{m_P}[(k\cdot p - k^2) g^{\mu\nu} - (p-k)^\mu k^\nu] - i\frac{F_V}{m_P}\epsilon^{\mu\nu\alpha\beta} k_\alpha p_\beta .$$

Non perturbative functions of k^2 and $(p-k)^2$

For real photon only F_A and F_V contribute!

1 pointlike f_P + 2 Real Photon Form Factors
+ 2 Virtual Photon Form Factors

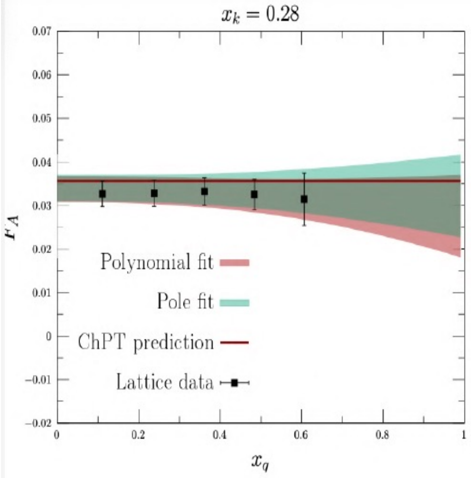
A look at the past: $K^+ \to \ell^+ \nu_\ell \ell'^+ \ell'^-$

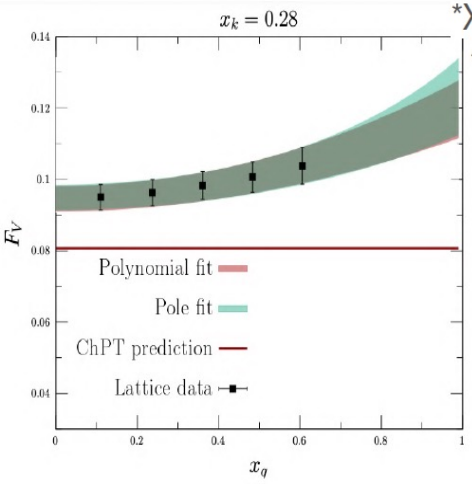
Channels	our Lattice	Tuo et al.*	ChPT**	experiments
$Br[K \to \mu\nu_\mu e^+ e^-]$	$8.26(13) \times 10^{-8}$	$10.59(33) \times 10^{-8}$	$9.8 - 8.2 \times 10^{-8}$	$7.93(33) \times 10^{-8}$***
$Br[K \to e\nu_e \mu^+ \mu^-]$	$0.762(49) \times 10^{-8}$	$0.72(5) \times 10^{-8}$	$1.1 - 0.6 \times 10^{-8}$	$1.72(45) \times 10^{-8}$****
$Br[K \to e\nu_e e^+ e^-]$	$1.95(11) \times 10^{-8}$	$1.77(16) \times 10^{-8}$	$3.4 - 1.7 \times 10^{-8}$	$2.91(23) \times 10^{-8}$***
$Br[K \to \mu\nu_\mu \mu^+ \mu^-]$	$1.178(35) \times 10^{-8}$	$1.45(6) \times 10^{-8}$	$1.5 - 1.1 \times 10^{-8}$	——

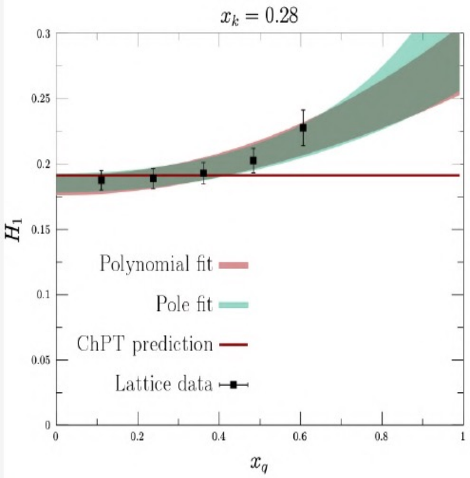
*X.Y. Tuo et Al arXiv:2103.11331 (2022).
**J. Bijnens et Al arXiv:9411311 (1993)
*** A. A. Poblaguev et Al arXiv: 0204006 (2002)
**** H. Ma et Al arXiv:0505011(2006)

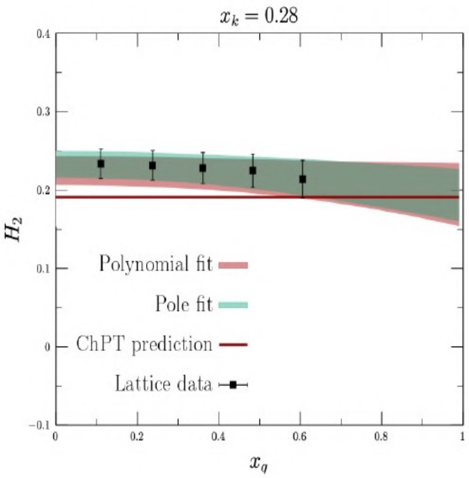
(based on one gauge ensemble only)

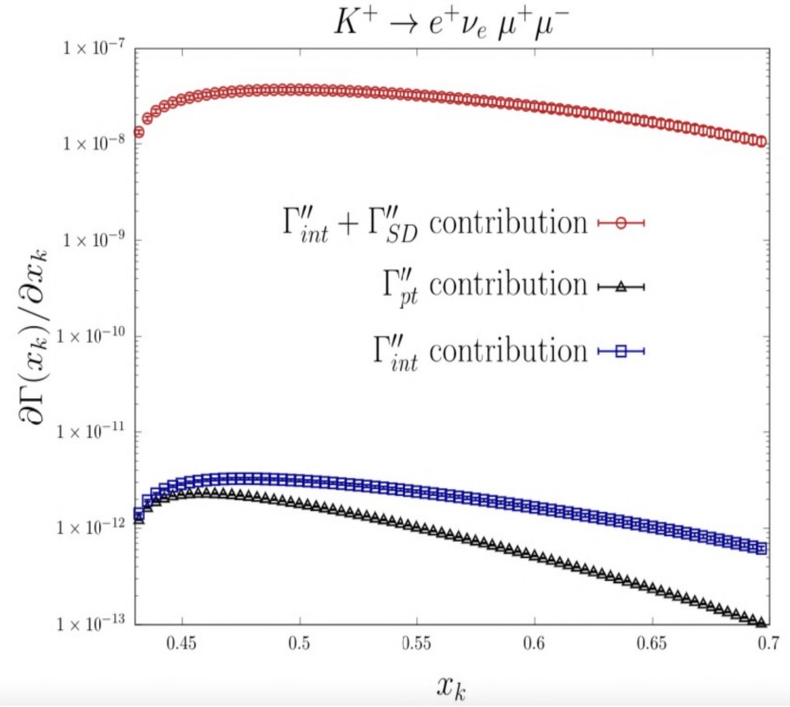
[G.Gagliardi et Al arXiv:2202.03833 (2022)]



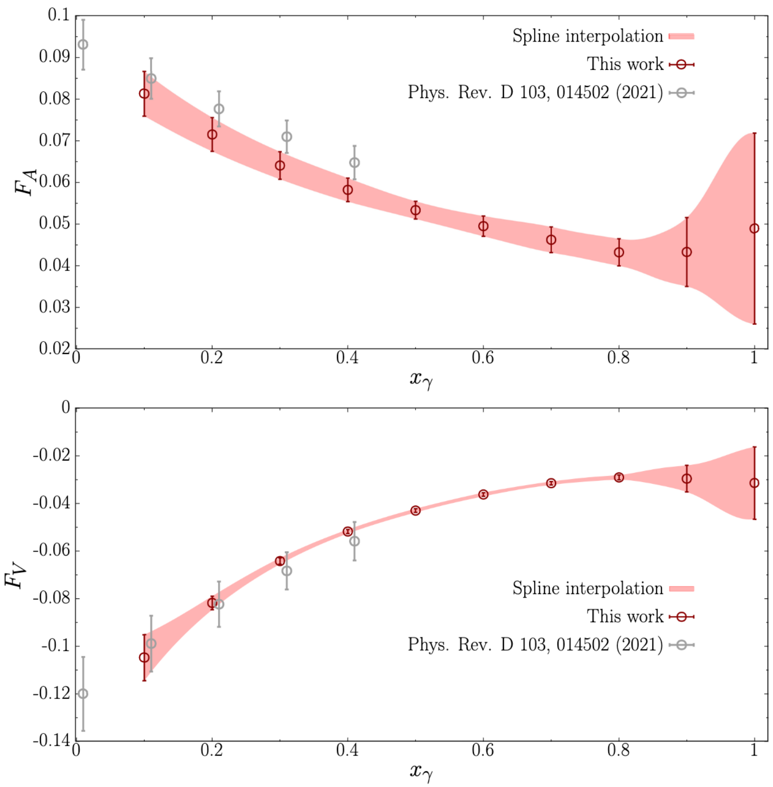






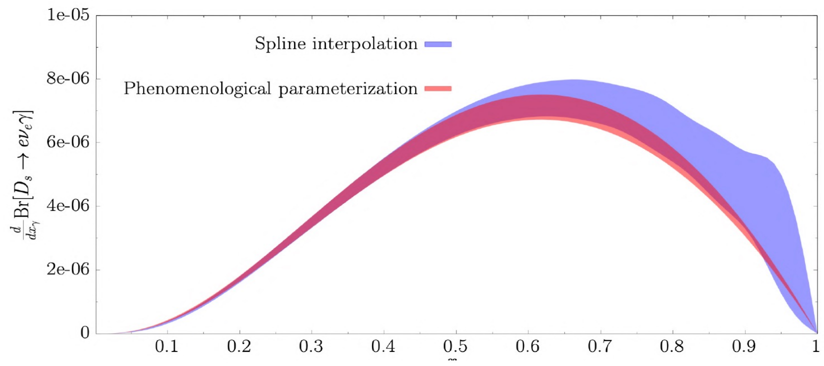


Reasonable agreement with other theoretical calculations
Less with experimental measurements



$D_s^+ \to e^+ \nu_e \gamma$ decay rate

$$\frac{d\Gamma(D_s \to \ell\nu\gamma)}{dx_\gamma} = \frac{\alpha_{em}}{4\pi}\Gamma^{(0)}\left\{\frac{dR^{pt}}{dx_\gamma} + \frac{dR^{int}}{dx_\gamma} + \frac{dR^{SD}}{dx_\gamma}\right\}$$



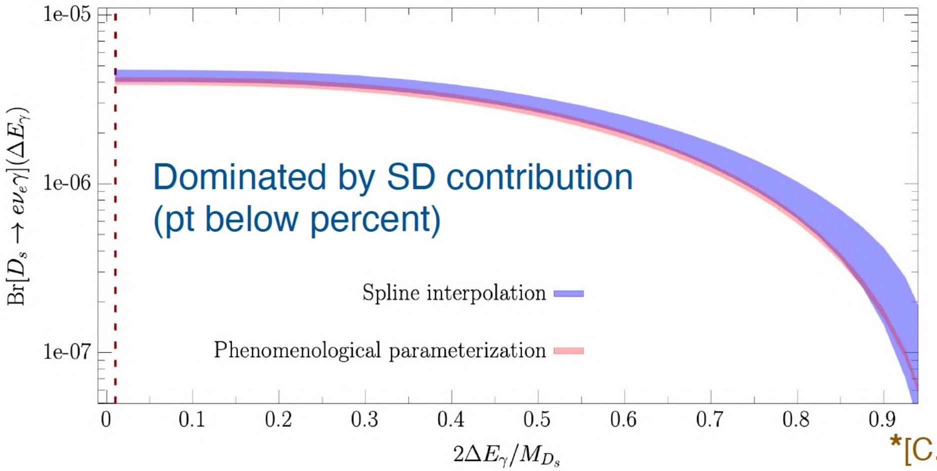
For final electron it dominates the rate

Large contributions from high values of x_γ

$$F_W(x_\gamma) = \frac{C_W}{\sqrt{R_W^2 + \frac{x_\gamma^2}{4}}\left(\sqrt{R_W^2 + \frac{x_\gamma^2}{4}} + \frac{x_\gamma}{2} - 1\right)} + B_W\,, \quad W = \{A, V\}$$

$D_s^+ \to e^+ \nu_e \gamma$ branching fraction as a function of the cutoff on the photon energy

$\mathrm{Br}(\Delta E_\gamma = 10\ \mathrm{MeV}) = 4.4(3) \times 10^{-6} \ll 1.3 \times 10^{-4}$ BESIII exp upper bound



Quark Model Predictions
$10^{-5} - 10^{-4}$ *

pQCD+HQEFT predictions
10^{-3} **

Test of QCD and of BSM contributions

*[C.Q. Geng et Al ArXiv:0012066 (2000)]
and [C.D. Lu et Al ArXiv:0212363 (2003)]

Relating $F_V \to g_{D_s D_s^* \gamma}$

$$= -\frac{M_{D_s^*} f_{D_s^*} g_{D_s^* D_s \gamma}}{2 M_{D_s}}$$

$$F_V(x_\gamma) = \frac{C_V}{\sqrt{R_{D_s^*}^2 + \frac{x_\gamma^2}{4}}\left(\sqrt{R_{D_s^*}^2 + \frac{x_\gamma^2}{4}} + \frac{x_\gamma}{2} - 1\right)}$$

Properly reproduced by lattice data

$\frac{R_V - R_{D_s^*}}{R_{D_s^*}} < 3\,\%$ 1.5σ compatibility

- Striking agreement with direct HPQCD calculation
- disagreement with LCSR calculation, traced down to $g^{(s)}$

	LCSR *	HPQCD **	This work
$g_{D_s^* D_s \gamma}$ [GeV^{-1}]	0.60(19)	0.10(2)	0.118(13)
$g^{(s)}_{D_s^* D_s \gamma}$ [GeV^{-1}]	1.0	0.50(3)	0.532(15)
$g^{(c)}_{D_s^* D_s \gamma}$ [GeV^{-1}]	-0.4	$-0.40(2)$	$-0.415(16)$
$\frac{g^{(s)}}{g^{(c)}}$	-2.5	$-1.25(10)$	$-1.282(61)$

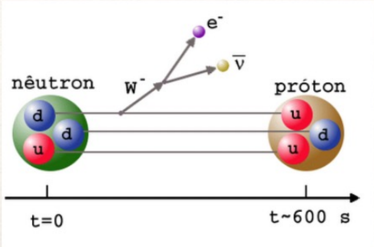
*[B. Pullin et Al ArXiv:2106.13617 (2021)]

**[G. C. Donald ArXiv:1312.5264 (2014)]

Status of Lattice Calculations of g_A

André Walker-Loud
BERKELEY LAB

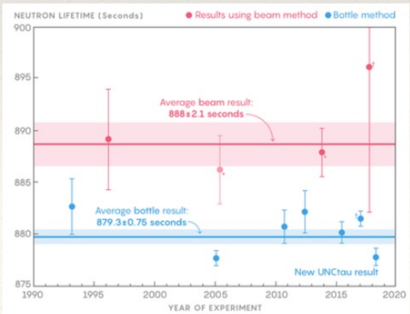
Neutron lifetime and the axial coupling

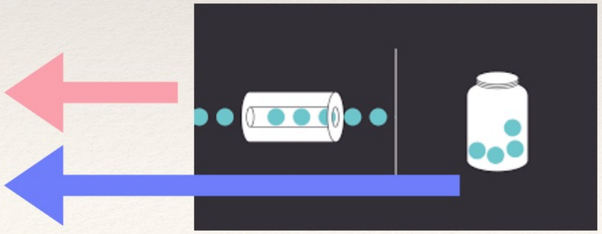


$$\frac{1}{\tau_n} = \frac{G_\mu^2 |V_{ud}|^2}{2\pi^3} m_e^5 (1 + 3g_A^2)(1 + RC) f_{V,A}$$

Radiative corrections are the Holy Grail

- The neutron lifetime and g_A (neutron decay) are used to probe the limits of the Standard Model
- We should have a (meaningful) Standard Model prediction for g_A - LQCD (lattice QCD)
- To gain confidence in the application of LQCD to nuclear physics, we must benchmark (calibrate) our calculations against well known quantities of interest, such as g_A
- In order for the theoretical uncertainty on g_A to match the larger uncertainty in the neutron lifetime measurements, we must determine g_A with < 0.2% uncertainty - **is this crazy?**

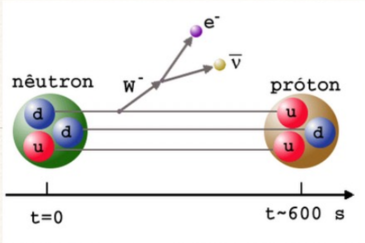


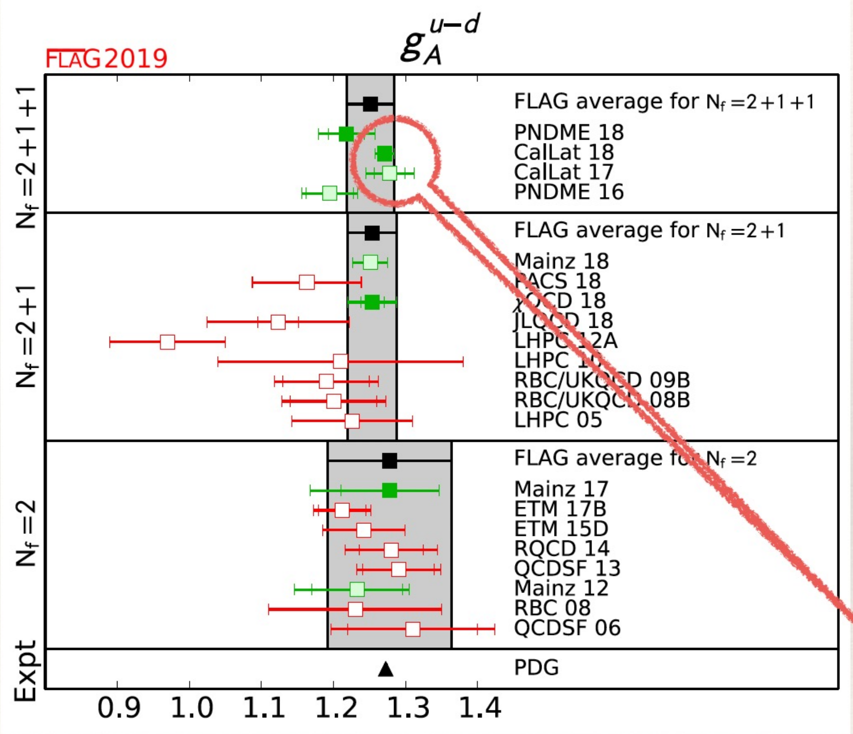


$\tau_n^{\text{beam}} = 888.0(2.0)s$

$\tau_n^{\text{bottle}} = 879.4(0.6)s$

nucleon axial coupling from LQCD





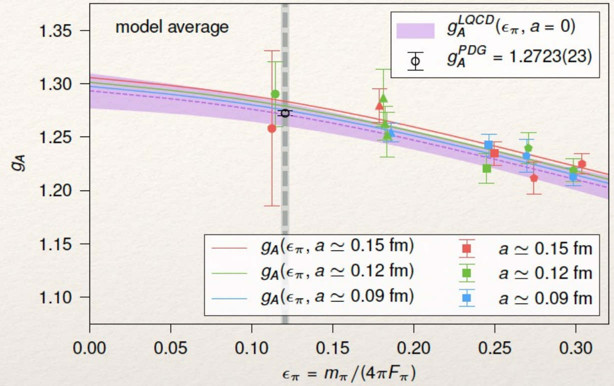
- To gain confidence in the application of Lattice QCD to nuclear physics, we must benchmark (calibrate) our calculations against well known quantities of interest
- g_A was supposed to be a good benchmark calculation for single nucleon structure - but it proved to have significant systematic challenges, preventing results with the precision anticipated
- FLAG 2019 has included single nucleon quantities in their averaging for the first time
- Notice one result is significantly more precise than the others

Nature 558 (2018) no.7708, 91-94
Chang et al.
[arXiv:1805.12130]

improving the determination of g_A

Final result

statistical	0.81%
chiral extrapolation	0.31%
$a \to 0$	0.12%
$L \to \infty$	0.15%
isospin	0.03%
model selection	0.43%
total	0.99%



$$g_A^{\rm QCD} = 1.2711(103)^s(39)^\chi(15)^a(19)^V(04)^I(55)^M$$

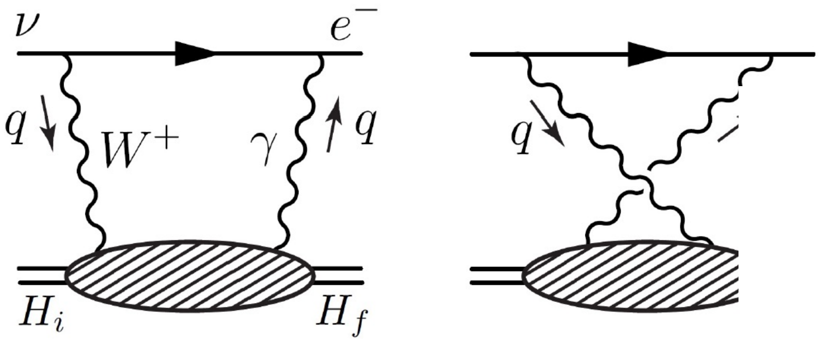
- More precise results at the physical pion mass will improve the three largest uncertainties:
  - statistical (s), extrapolation (χ) and model selection (M) **NOTE**, **a12m130** has 2.3% uncertainty
- Following our existing strategy, we anticipate getting to 0.5% by the end of this year
- Getting below (or maybe to 0.5%) will require a 4th lattice spacing as well (~0.06fm)
- Adding a FV study on additional pion mass points will improve the FV uncertainty
- The isospin uncertainty seems unnecessary…

$|V_{ud}|$ is extracted from neutron and superallowed Beta decays

The precision of the experiments is such that isospin breaking effects and radiative corrections are very important

Recent efforts to reduce systematic uncertainties in the calculation of radiative corrections and in particular the contribution of the box diagrams

Peng-Xiang Ma et al. arXiv:2308.16755v



ure 1. The γW-box diagrams for the semileptonic
cess $H_i \to H_f e \bar{\nu}_e$.

$$\square_{\gamma W}^{VA} = 3.65(8)_{\text{lat}}(1)_{\text{PT}} \times 10^{-3}.$$

$$\Delta_R^V = \frac{\alpha}{2\pi}\left[3\ln\frac{M_Z}{m_p} + \ln\frac{M_Z}{M_W} + \tilde{a}_g\right] + \delta_{\text{HO}}^{\text{QED}} + 2\square_{\gamma W}^{VA} \quad (20)$$

C. Seng et al. arXiv:1812.03352v3

$$\left(\square_{\gamma W}^{VA}\right)^{MS} = 2.81(16)\frac{\alpha}{2\pi} = 3.26(19) \times 10^{-3}$$

$$\left(\square_{\gamma W}^{VA}\right)^{\text{new}} = 3.26(9)\frac{\alpha}{2\pi} = 3.79(10) \times 10^{-3}$$

$$\Delta_R^{V,\text{old}} = 0.02361(38) \to \Delta_R^{V,\text{new}} = 0.02467(22).$$

$$|V_{ud}|^2 = \frac{0.97154(22)_{\text{exp}}(54)_{\text{NS}}}{(1+\Delta_R^V)}, \quad \text{superallowe}$$

$$|V_{ud}|^2 = \frac{0.9728(6)_{\tau_n}(16)_{g_A}}{(1+\Delta_R^V)}, \quad \text{free neutron.}$$

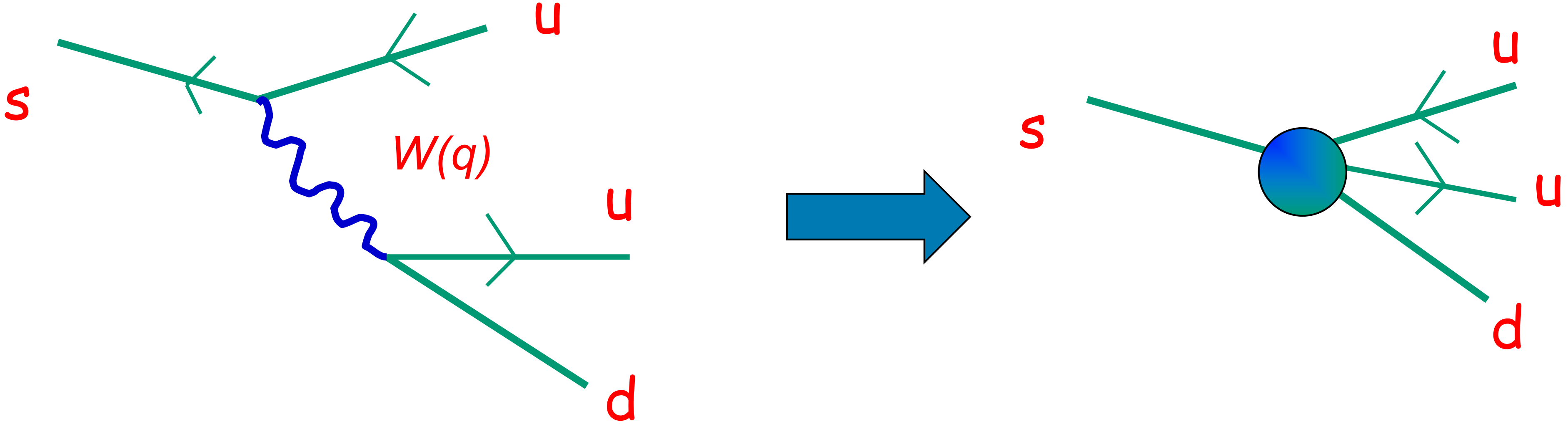
$$\Delta_R^V = 0.02439(19),$$

$$|V_{ud}| = 0.97386(11)_{\text{exp.}}(9)_{\text{RC}}(27)_{\text{NS}}$$

Czarnecki et al. arXiv:1907.06737v1

$$\Delta_R^V = 0.02421(32) \text{ AdS BjSR Approach.}$$

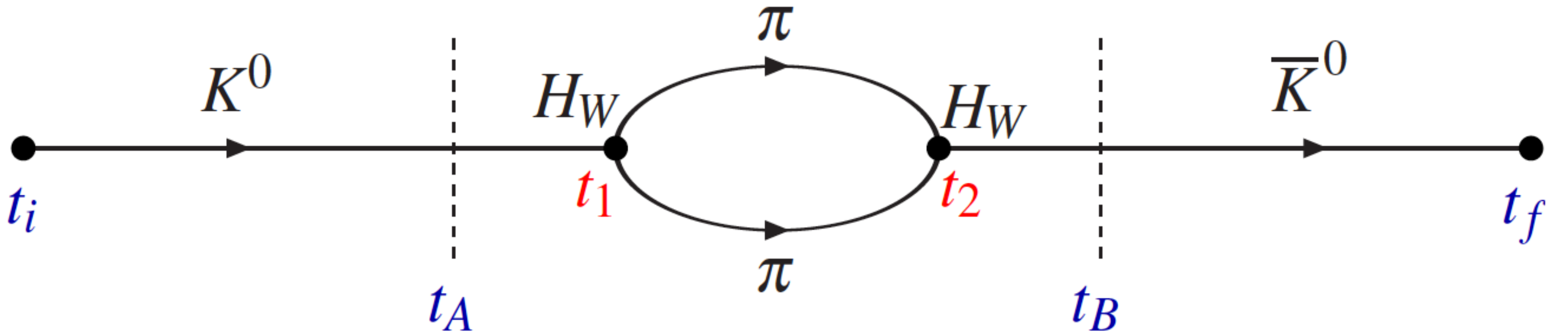
The Effective Hamiltonian



$$q \sim m_K \ll M_W$$

$$\mathcal{H}_{eff} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \; (\bar{s}\gamma_\mu (1-\gamma_5) u)\,(\bar{u}\gamma^\mu (1-\gamma_5) d)$$

Non-leptonic Decays



- Δm_K is given by

$$\Delta m_K \equiv m_{K_L} - m_{K_S} = 2\mathscr{P}\sum_\alpha \frac{\langle \bar{K}^0 \,|\, \mathscr{H}_W \,|\, \alpha\rangle \, \langle \alpha \,|\, \mathscr{H}_W \,|\, K^0\rangle}{m_K - E_\alpha} = 3.483(6)\times 10^{-12}\,\mathrm{MeV}.$$

- The above correlation function gives ($T = t_B - t_A + 1$)

$$C_4(t_A,t_B;t_i,t_f) \;=\; |Z_K|^2 e^{-m_K(t_f-t_i)} \sum_n \frac{\langle \bar{K}^0 \,|\, \mathscr{H}_W \,|\, n\rangle \, \langle n \,|\, \mathscr{H}_W \,|\, K^0\rangle}{(m_K - E_n)^2} \times \left\{ e^{(M_K-E_n)T} - (m_K - E_n)T - 1 \right\}.$$

dangerous terms

physical terms

- From the coefficient of T we can therefore obtain

$$\Delta m_K^{\rm FV} \equiv 2\sum_n \frac{\langle \bar{K}^0 \,|\, \mathscr{H}_W \,|\, n\rangle \, \langle n \,|\, \mathscr{H}_W \,|\, K^0\rangle}{(m_K - E_n)} \,.$$

RBC-UK QCD

NEW PHYSICS IN KAON DECAYS?

$$\varepsilon'/\varepsilon = (1.4 \pm 7.0)\cdot 10^{-4}$$

$$\left(\frac{\mathrm{Re}\,A_0}{\mathrm{Re}\,A_2}\right) = 31.0 \pm 6.6$$

$$(\varepsilon'/\varepsilon)_{exp} = (16.6 \pm 2.3)\cdot 10^{-4}$$

$$\left(\frac{\mathrm{Re}\,A_0}{\mathrm{Re}\,A_2}\right)_{exp} = 22.4$$

Courtesy by A. Buras 2015

Results for $\mathrm{Re}[A_0]$, $\mathrm{Im}[A_0]$ and $\mathrm{Re}[\epsilon'/\epsilon]$

Xu Feng Lattice 2017

[RBC-UKQCD, PRL115 (2015) 212001]

- Determine the $K \to \pi\pi(I = 0)$ amplitude A_0
  - Lattice results

$$\mathrm{Re}[A_0] = 4.66(1.00)_{\mathrm{stat}}(1.26)_{\mathrm{syst}} \times 10^{-7}\ \mathrm{GeV}$$
$$\mathrm{Im}[A_0] = -1.90(1.23)_{\mathrm{stat}}(1.08)_{\mathrm{syst}} \times 10^{-11}\ \mathrm{GeV}$$

  - Experimental measurement

$$\mathrm{Re}[A_0] = 3.3201(18) \times 10^{-7}\ \mathrm{GeV}$$
$$\mathrm{Im}[A_0]\ \text{is unknown}$$

- Determine the direct CP violation $\mathrm{Re}[\epsilon'/\epsilon]$

$$\mathrm{Re}[\epsilon'/\epsilon] = 0.14(52)_{\mathrm{stat}}(46)_{\mathrm{syst}} \times 10^{-3} \quad \text{Lattice}$$
$$\mathrm{Re}[\epsilon'/\epsilon] = 1.66(23) \times 10^{-3} \quad \text{Experiment}$$

Phase of final state interaction smaller than the experimental value

$2.1\ \sigma$ deviation $\Rightarrow$ require more accurate lattice results

Four dominant contributions to ε′/ε in the SM

AJB, Jamin, Lautenbacher (1993); AJB, Gorbahn, Jäger, Jamin (2015)

From $\mathrm{Re}A_0$ (→ −3.7, 21.2) From $\mathrm{Re}A_2$ (→ 1.1, −9.6)

$$\mathrm{Re}(\varepsilon'/\varepsilon) = \left[\frac{\mathrm{Im}\left(V_{td}V_{ts}^*\right)}{1.4\cdot 10^{-4}}\right] 10^{-4}\left[-3.7 + 21.2\cdot B_6^{(1/2)} + 1.1 - 9.6\cdot B_8^{(3/2)}\right]$$

(Q_4) (V-A) ⊗ (V-A) QCD Penguins	(V-A) ⊗ (V+A) QCD Penguins	(V-A) ⊗ (V-A) EW Penguins	(V-A) ⊗ (V+A) EW Penguins

Assumes that $\mathrm{Re}A_0$ and $\mathrm{Re}A_2$ ($\Delta I=1/2$ Rule) fully described by SM (includes isospin breaking corrections)

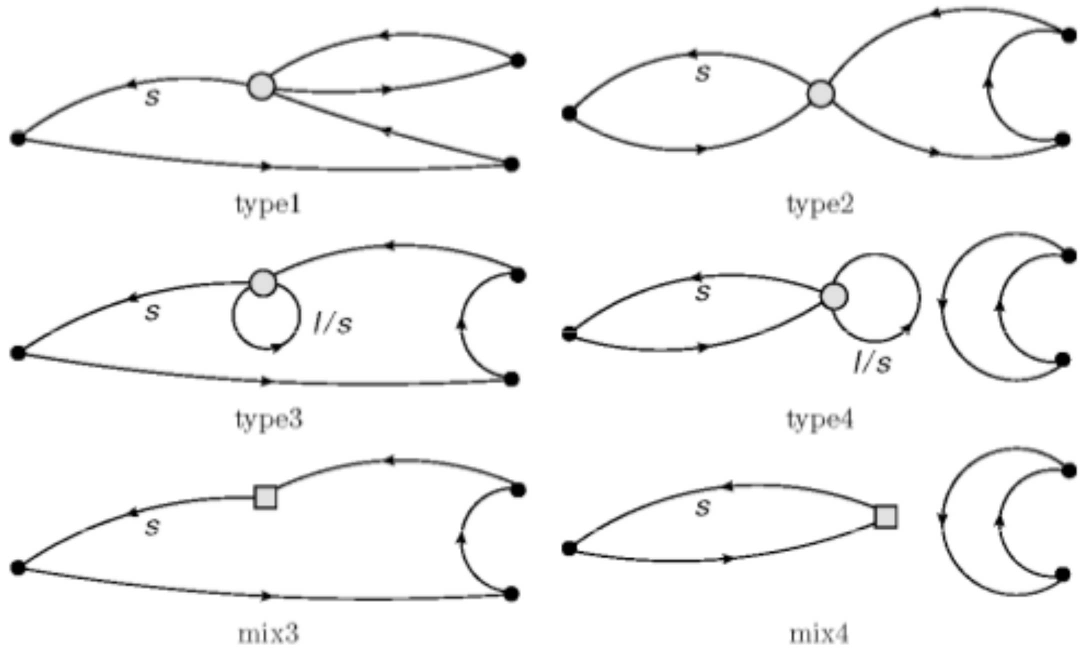
ε′/ε from RBC-UKQCD

Calculate all contributions directly (no isospin breaking corrections)

$$\left[-(6.5\pm3.2) + 25.3\cdot B_6^{(1/2)} + (1.2\pm0.8) - 10.2\cdot B_8^{(3/2)}\right]$$

$\Delta I = 1/2\ K \rightarrow \pi\pi$
(Qi Liu)

- Code 50 different contractions
- For each of 400 configurations invert with source at each of 32 times.
- Use Ran Zhou's deflation code



Anatomy of ε'/ε – A new flavour anomaly?

AJB, Gorbahn, Jäger, Jamin,, 1507.xxxx

RBC-UKQCD

$$\varepsilon'/\varepsilon = (1.4 \pm 7.0) \cdot 10^{-4}$$

RBC-QCD values

(3.2σ) $\varepsilon'/\varepsilon = (2.2 \pm 3.8) \cdot 10^{-4}$

$B_6^{(1/2)} = 0.57 \pm 0.15$

$B_8^{(3/2)} = 0.76 \pm 0.05$

$\varepsilon'/\varepsilon = (6.3 \pm 2.5) \cdot 10^{-4}$

large N bounds (AJB, Gérard)

$B_6^{(1/2)} = B_8^{(3/2)} = 0.76$

$\varepsilon'/\varepsilon = (9.1 \pm 3.3) \cdot 10^{-4}$

large N bounds (AJB, Gérard)

$B_6^{(1/2)} = B_8^{(3/2)} = 1.0$

exp: $\varepsilon'/\varepsilon = (16.6 \pm 3.3) \cdot 10^{-4}$

Systematic error budget

Christopher Kelly
(RBC & UKQCD collaborations)

- Primary systematic errors of 2015 work:
  - Finite lattice spacing: 12%
  - Wilson coefficients: 12%
  - Renormalization (mostly PT matching): 15%
  - Excited-state: $\leq$ 5% but now known to be significantly underestimated
  - Lellouch-Luscher factor (derivative of $\pi\pi$ phase shift wrt. energy): 11%
- In our new work we have used step-scaling to raise the renormalization scale from 1.53 → 4.00 GeV: 15% → 5%
- 3 operators have dramatically improved understanding of $\pi\pi$ system: Lellouch-Luscher factor 11% → 1.5%
- Detailed analysis shows no evidence of remaining excited-state contamination: Excited state error now negligible!
- Still single lattice spacing: Discretization error unchanged.
- Evidence that Wilson coefficient systematics are driven by using PT for 3-4f matching, not improved by higher μ: Wilson coeff error unchanged.

Final result for ε'

- Combining our new result for Im(A_0) and our 2015 result for Im(A_2), and again using expt. for the real parts, we find

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \mathrm{Re}\left\{\frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\varepsilon}\left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right]\right\}$$

$$= 0.00217(26)(62)(50)$$

(26) ← stat; (62) ← sys; (50) ← IB + EM

Consistent with experimental result:

$$\mathrm{Re}(\epsilon'/\epsilon)_{\mathrm{expt}} = 0.00166(23)$$

RBC/UKQCD: *e'/ e= 16.7 x10^{-4}*

Utfit: *e'/ e= 15.2(4.7) x10^{-4}*

A second group should do this calculation!!


…. beyond
the Standard Model

$$Q^{EXP} = V_{CKM}\langle F|\hat{O}|I\rangle$$

$$Q^{EXP} = \sum_i C^i_{SM}(M_W, m_t, \alpha_s)\langle F|\hat{O}_i|I\rangle + \sum_{i'} C^{i'}_{Beyond}(\tilde{m}_\beta, \alpha_s)\langle F|\hat{O}_{i'}|I\rangle$$

UT generalization Beyond the Standard Model

- fit simultaneously for the CKM and the NP parameters (generalized UT analysis)
  - parameterize BSM effects in $\Delta F = 2$ Hamiltonian in model-independent
  - use all available experimental information
- find out NP contributions to ΔF=2 transitions

$$A_q = C_{B_q} e^{2i\phi_{B_q}} A_q^{SM} e^{2i\phi_q^{SM}} = \left(1 + \frac{A_q^{NP}}{A_q^{SM}} e^{2i(\phi_q^{NP} - \phi_q^{SM})}\right) A_q^{SM} e^{2i\phi_q^{SM}}$$

$$\Delta m_{q/K} = C_{B_q/\Delta m_K} (\Delta m_{q/K})^{SM}$$
$$A_{CP}^{B_d \to J/\psi K_s} = \sin 2(\beta + \phi_{B_d})$$
$$A_{SL}^q = \mathrm{Im}\left(\Gamma_{12}^q / A_q\right)$$
$$\varepsilon_K = C_\varepsilon \varepsilon_K^{SM}$$
$$A_{CP}^{B_s \to J/\psi\phi} \sim \sin 2(-\beta_s + \phi_{B_s})$$
$$\Delta\Gamma^q / \Delta m_q = \mathrm{Re}\left(\Gamma_{12}^q / A_q\right)$$

CERN

UTfit

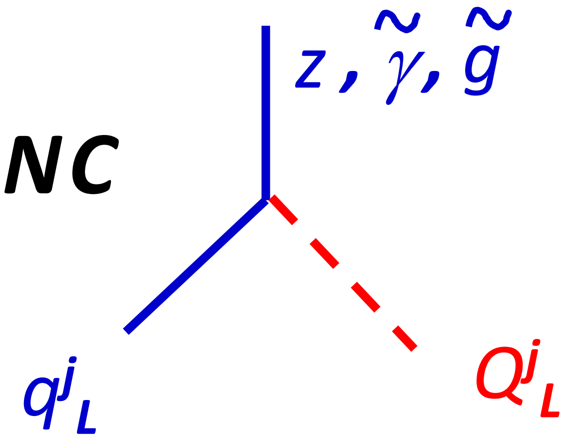
$\not{C}\not{P}$ beyond the SM (Supersymmetry)

Standard Model			Supersymmetric partners		
Spin 1/2	Quarks	q_L, u_R, d_R	Spin 0	SQuarks	Q_L, U_R, D_R
	Leptons	l_L, e_R		SLeptons	L_L, E_R
Spin 1	Gauge bosons	W, Z, γ, g	Spin 1/2	Gauginos	$\tilde{w}$, $\tilde{z}$, $\tilde{\gamma}$, $\tilde{g}$
Spin 0	Higgs bosons	H_1, H_2	Spin 1/2	Higgsinos	$\tilde{H}_1$, $\tilde{H}_2$

In general the mixing mass matrix of the SQuarks (SMM) is not diagonal in flavour space analogously to the quark case

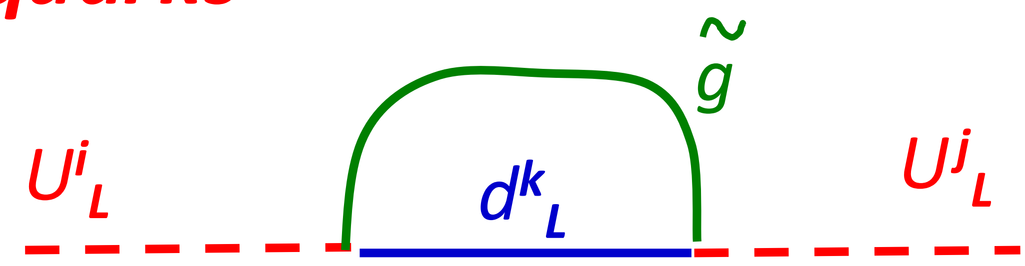
We may either ***Diagonalize the SMM***

FCNC



or ***Rotate by the same matrices the SUSY partners of the u- and d- like quarks***

$(Q^j_L)' = \boldsymbol{U}^{ij}_L \, Q^j_L$



New local four-fermion operators are generated

$Q_1 = (\bar{b}_L^A \gamma_\mu d_L^A)(\bar{b}_L^B \gamma_\mu d_L^B)$ SM

$Q_2 = (\bar{b}_R^A d_L^A)(\bar{b}_R^B d_L^B)$

$Q_3 = (\bar{b}_R^A d_L^B)(\bar{b}_R^B d_L^A)$

$Q_4 = (\bar{b}_R^A d_L^A)(\bar{b}_L^B d_R^B)$

$Q_5 = (\bar{b}_R^A d_L^B)(\bar{b}_L^B d_R^A)$

+ those obtained by $L \leftrightarrow R$

Similarly for the s quark e.g. $(\bar{s}_R^A d_L^A)(s_R^B d_L^B)$

$$\langle \bar{K}^0 | O_1(\mu) | K^0 \rangle = \frac{8}{3} M_K^2 f_K^2 B_1(\mu) ,$$

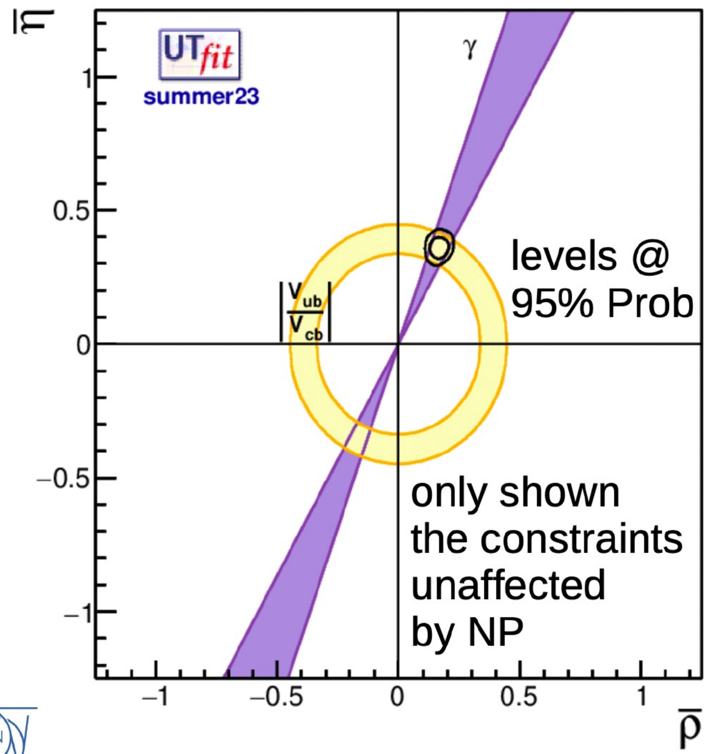
$$\langle \bar{K}^0 | O_2(\mu) | K^0 \rangle = -\frac{5}{3} \left(\frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 M_K^2 f_K^2 B_2(\mu) ,$$

$$\langle \bar{K}^0 | O_3(\mu) | K^0 \rangle = \frac{1}{3} \left(\frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 M_K^2 f_K^2 B_3(\mu) ,$$

$$\langle \bar{K}^0 | O_4(\mu) | K^0 \rangle = 2 \left(\frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 M_K^2 f_K^2 B_4(\mu) ,$$

$$\langle \bar{K}^0 | O_5(\mu) | K^0 \rangle = \frac{2}{3} \left(\frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 M_K^2 f_K^2 B_5(\mu) ,$$

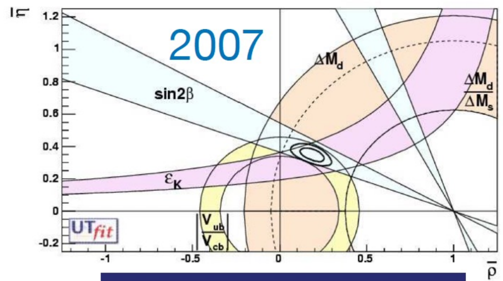
Results of BSM analysis: CKM parameters



CKM parameters from BSM analysis

$$\bar{\rho} = 0.167 \pm 0.025$$
$$\bar{\eta} = 0.361 \pm 0.027$$

CKM parameters known (even in presence of NP effects) with similar precision of pre-LHC SM analysis
2004



$$\bar{\rho} = 0.164 \pm 0.028$$
$$\bar{\eta} = 0.340 \pm 0.016$$

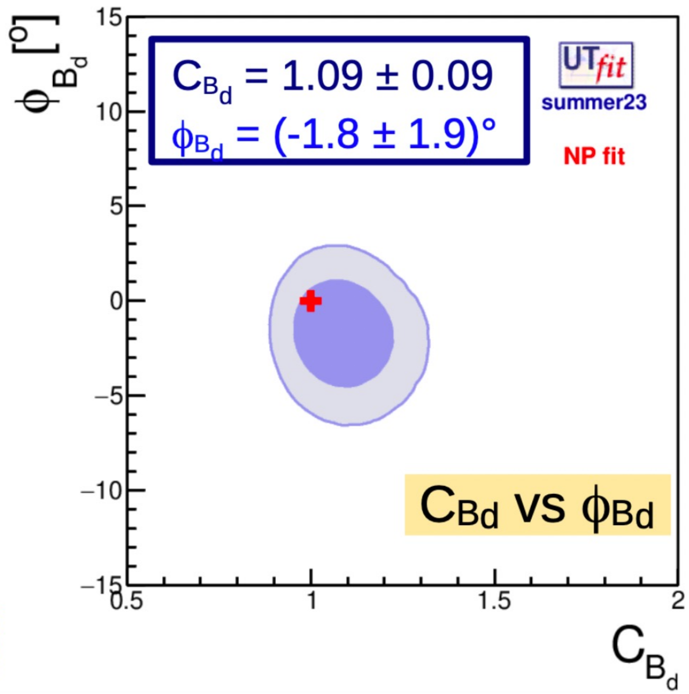
Results of BSM analysis: New Physics parameters

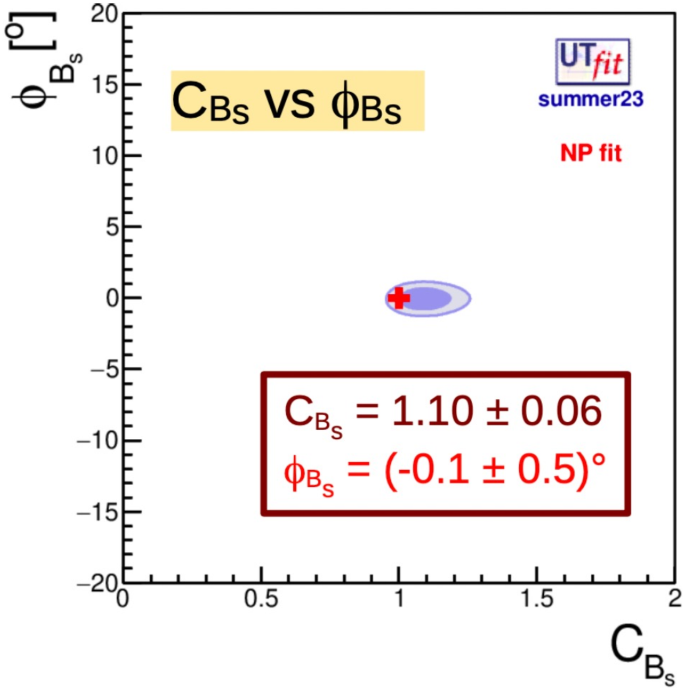
$A_q = C_{B_q} e^{2i\phi_{B_q}} A_q^{SM} e^{2i\phi_q^{SM}}$

K system

$C_{\epsilon_K} = 1.09 \pm 0.10$

dark: 68%
light: 95%
SM: red cross





Results of BSM analysis: New Physics parameters

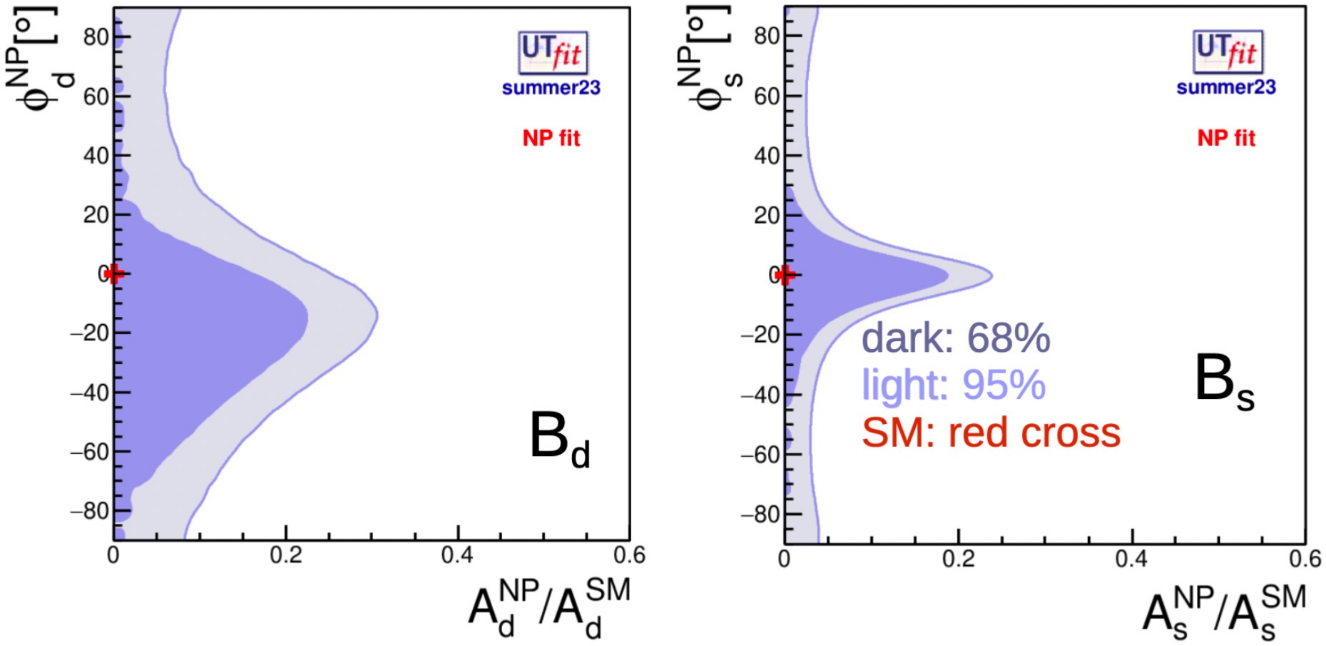
$$A_q = \left(1 + \frac{A_q^{NP}}{A_q^{SM}} e^{2i(\phi_q^{NP} - \phi_q^{SM})}\right) A_q^{SM} e^{2i\phi_q^{SM}}$$

The ratio of NP/SM amplitudes is:

< 25% @68% prob. (35% @95%) in B_d mixing

< 25% @68% prob. (30% @95%) in B_s mixing

dark: 68%
light: 95%
SM: red cross

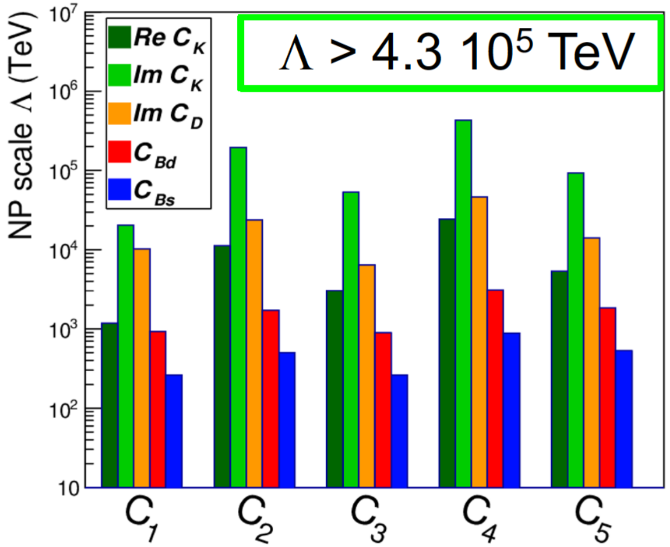


Beyond the SM

Wilson Coefficients results

*for lower bound for loop-mediated contributions, simply multiply by α_s (~ 0.1) or by α_W (~ 0.03).

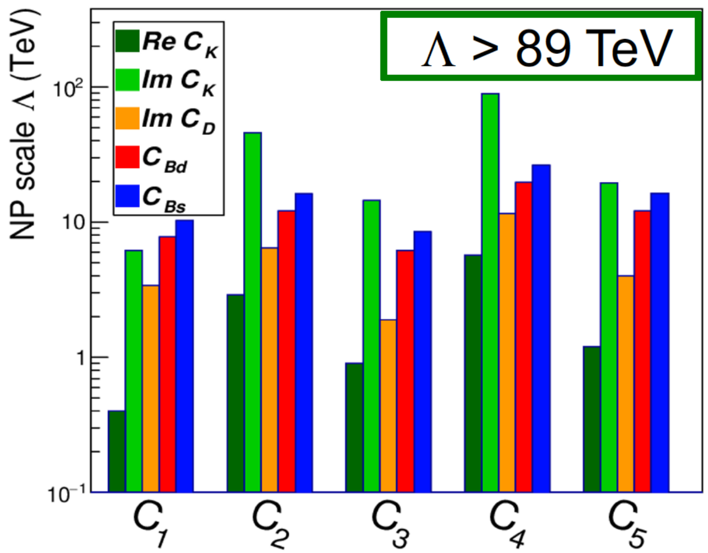
Generic: $C(\Lambda) = \alpha/\Lambda^2$, F_i~1, arbitrary phase, α ~ 1 for strongly coupled NP



- α ~ α_W in case of loop coupling
- through weak interactions*

$\Lambda > 1.3\ 10^4$ TeV

NMFV: $C(\Lambda) = \alpha \times |F_{SM}|/\Lambda^2$, F_i~$|F_{SM}|$, arbitrary phase



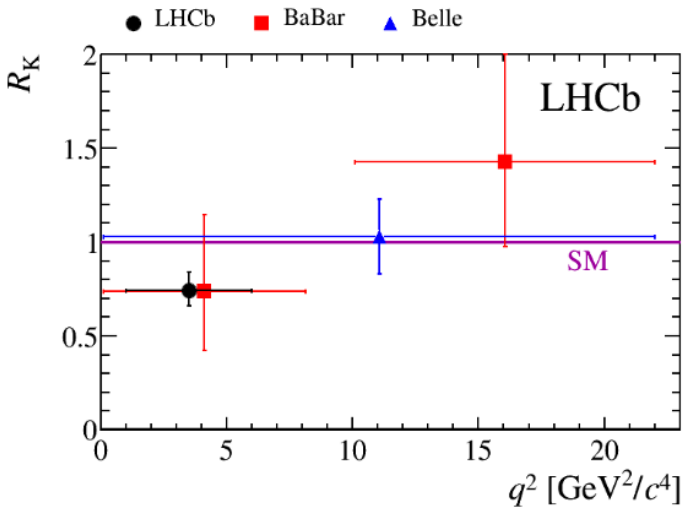
- α ~ α_W in case of loop coupling
- through weak interactions*

$\Lambda > 2.7$ TeV

2022

Reminder: $R_K = B(B^+ \to K^+\mu^+\mu^-)/B(B^+ \to K^+e^+e^-)$

- Test of lepton universality : $R_K \sim 1$ in SM, with negligible theoretical uncertainties



LHCb, PRL 113 151601
Belle, PRL 103 171801
BaBar, PRD 86 032012

$$R_K(1 < q^2 < 6\ \mathrm{GeV}^2) = 0.745^{+0.090}_{-0.074}(\mathrm{stat}) \pm 0.036(\mathrm{syst})$$

Old slide

- Compatible with SM at 2.6σ
- Experimentally challenging
  - lower trigger efficiency for electrons, resolution deteriorated by bremsstrahlung
- Other modes suitable for same test: $B^0 \to K^{*0} l^+ l^-$, $B_s \to \phi\, l^+ l^-$, $\Lambda_B \to \Lambda\, l^+ l^-$

Excitement

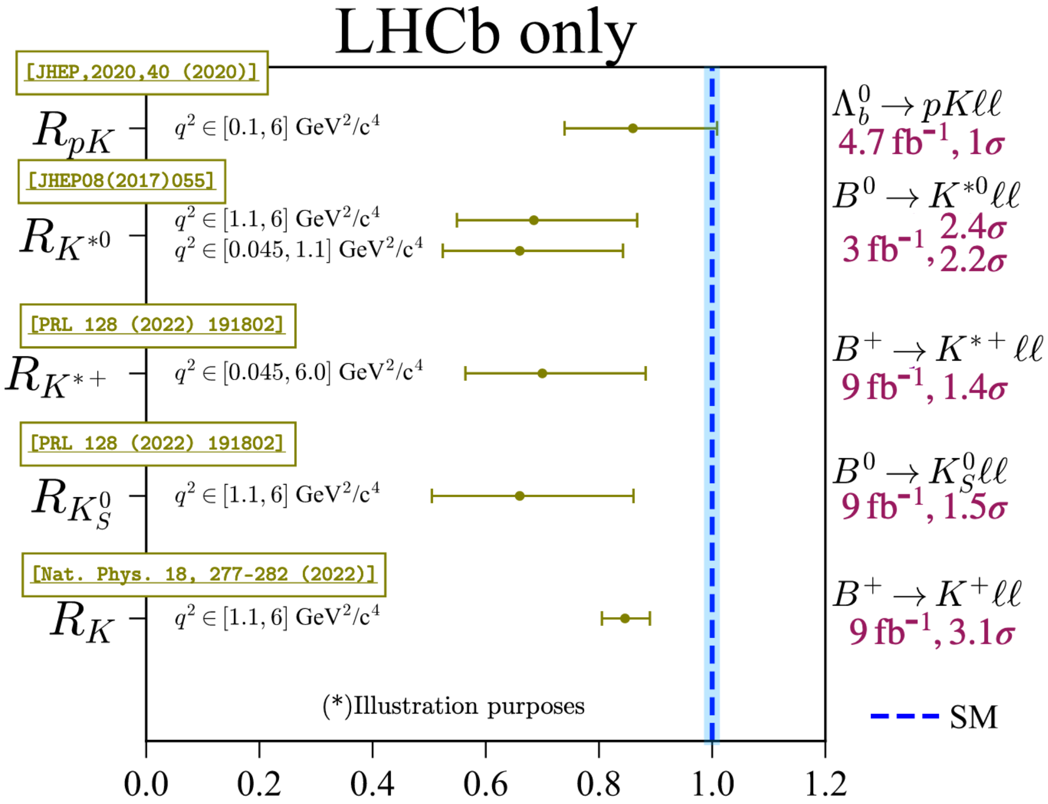
Analysis

Lepton Flavour Universality (LFU) tests in $b \to s\ell^+\ell^-$

- Coherent pattern of tension to SM in LFU test with $b \to s\ell^+\ell^-$ transition:

- R_X ratio extremely well predicted in SM
  - Cancellation of hadronic uncertainties at 10^{-4}
  - $\mathcal{O}(1\%)$ QED correction [Eur.Phys.J.C 76 (2016) 8]
  - Statistically limited

- **Any departure from unity is a clear sign of New Physics**



$$R_X = \frac{\mathcal{B}(b \to s\mu^+\mu^-)}{\mathcal{B}(b \to se^+e^-)}$$

(*) Measurements from Belle not shown (larger statistical uncertainties)

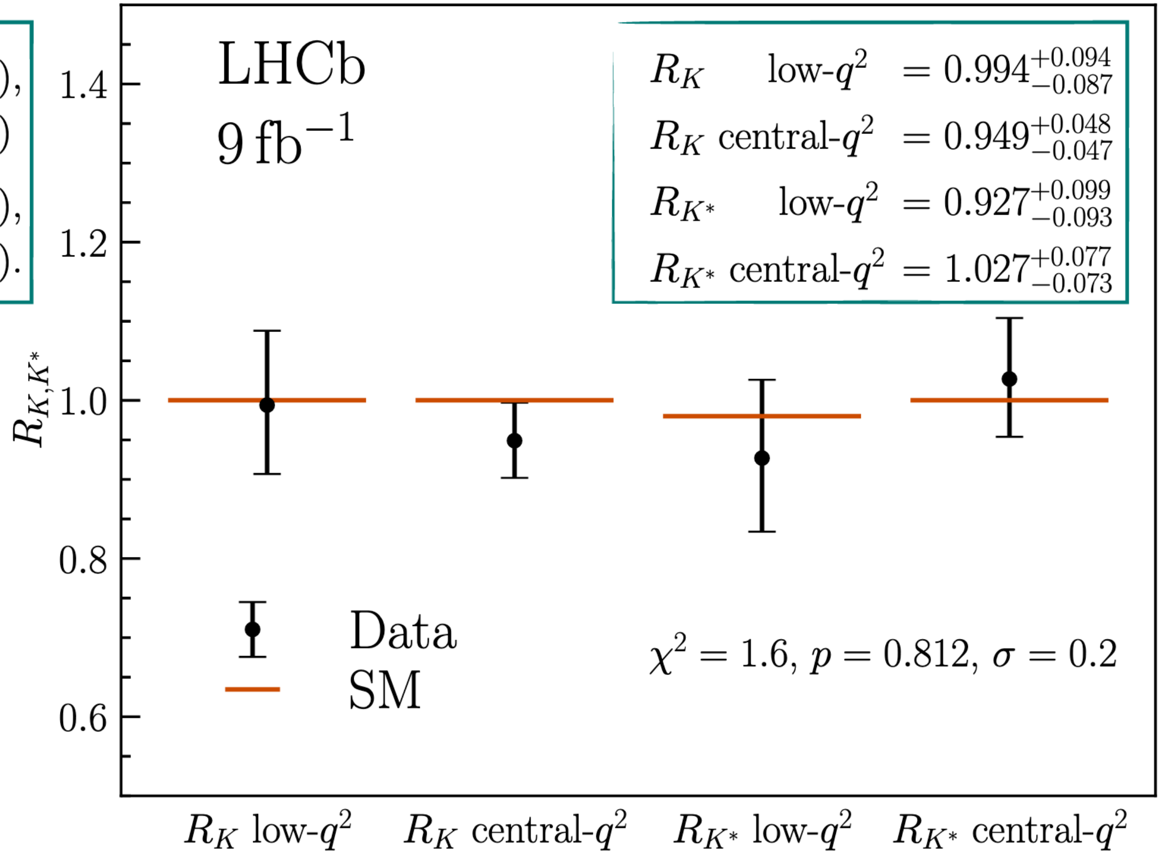
Harakiri!

Results

$$\text{low-}q^2 \begin{cases} R_K &= 0.994\ ^{+0.090}_{-0.082}\ (\text{stat})\ ^{+0.027}_{-0.029}\ (\text{syst}), \\ R_{K^*} &= 0.927\ ^{+0.093}_{-0.087}\ (\text{stat})\ ^{+0.034}_{-0.033}\ (\text{syst}) \end{cases}$$

$$\text{central-}q^2 \begin{cases} R_K &= 0.949\ ^{+0.042}_{-0.041}\ (\text{stat})\ ^{+0.023}_{-0.023}\ (\text{syst}), \\ R_{K^*} &= 1.027\ ^{+0.072}_{-0.068}\ (\text{stat})\ ^{+0.027}_{-0.027}\ (\text{syst}). \end{cases}$$

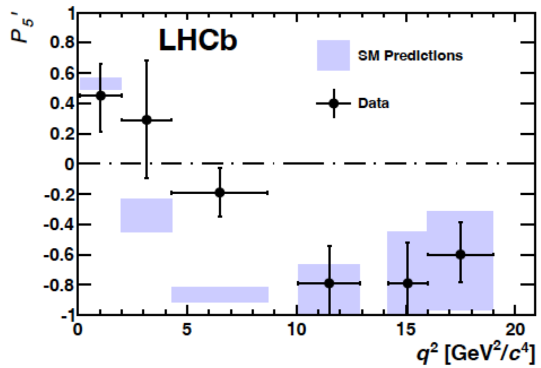
- Most precise and accurate LFU test in $b \to s\ell\ell$ transition
- Compatible with SM with a simple χ^2 test on 4 measurement at 0.2 σ



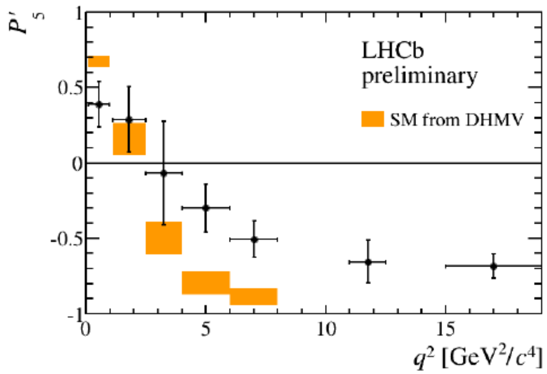
Tension in the angular observables

$B^0 \to K^{*0}\mu^+\mu^-$ angular observables, in particular P'_5 / S_5

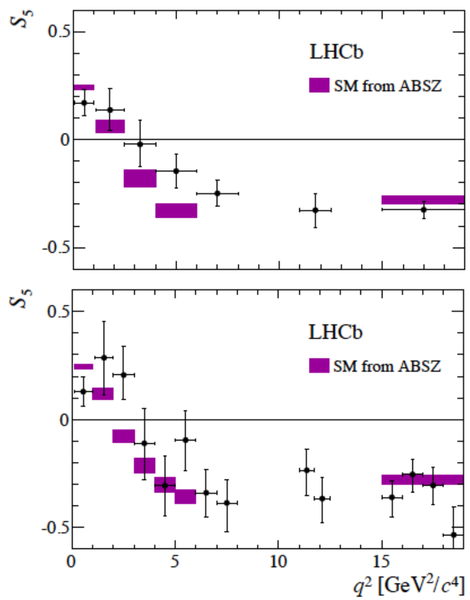
- 2013 (1 fb^{-1}): disagreement with the SM for P_2 and P'_5 **(PRL 111, 191801 (2013))**
- March 2015 (3 fb^{-1}): confirmation of the deviations **(LHCb-CONF-2015-002)**
- Dec. 2015: 2 analysis methods, both show the deviations **(JHEP 1602, 104 (2016))**



3.7σ deviation in the 3rd bin

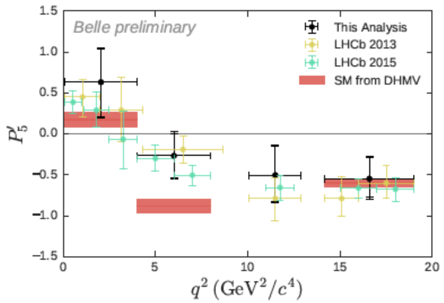


2.9σ in the 4th and 5th bins
(3.7σ combined)



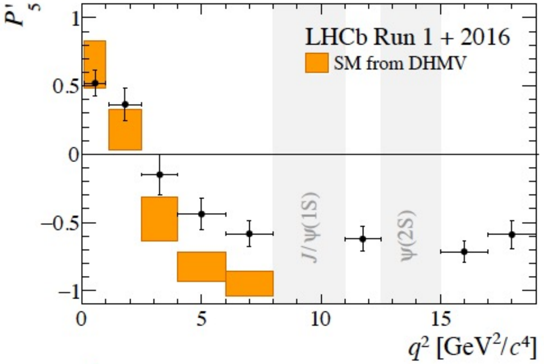
3.4σ combined fit (likelihood)

Belle supports LHCb
(arXiv:1604.04042)
tension at 2.1σ

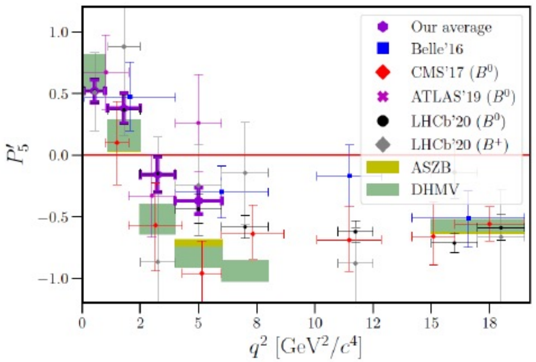


Tension in the angular observables - 2020 updates

$P_5'(B^0 \to K^{*0}\mu^+\mu^-)$: 2020 LHCb update with 4.7 fb^{-1}: $\sim 2.9\sigma$ local tension



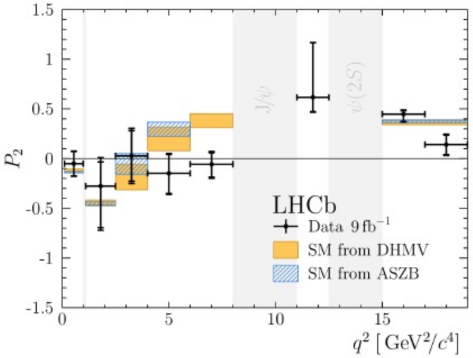
Phys. Rev. Lett. 125, 011802 (2020)

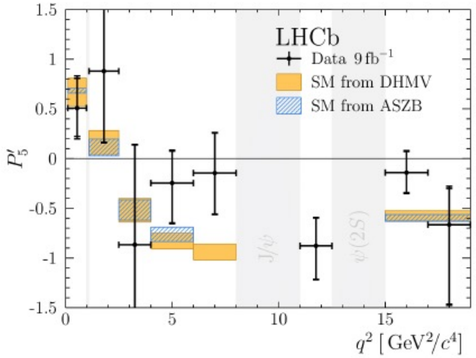


ATLAS-CONF-2017-023; CMS-PAS-BPH-15-008

First measurement of $B^+ \to K^{*+}\mu^+\mu^-$ angular observables

using the full Run 1 and Run 2 dataset (9 fb^{-1}):





Phys. Rev. Lett. 126, 161802 (2021)

The results confirm the global tension with respect to the SM!

with respect to the, model dependent, SM inspired theoretical calculations

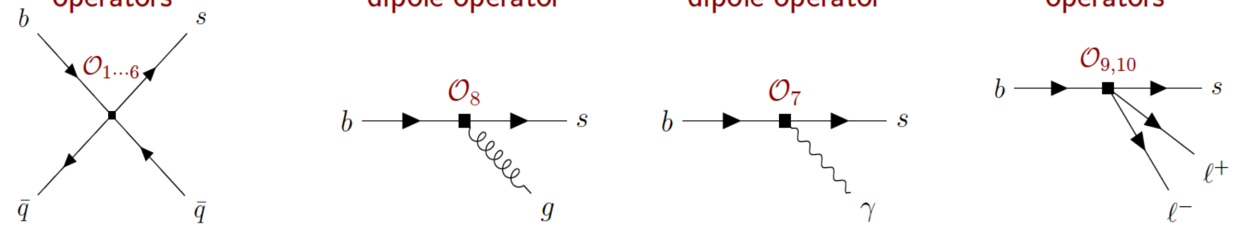
Theoretical framework

Effective field theory

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \Big(\sum_{i=1\cdots10,S,P} (C_i(\mu)\mathcal{O}_i(\mu) + C_i'(\mu)\mathcal{O}_i'(\mu)) \Big)$$

Operator set for $b \to s$ transitions:

4-quark operators | chromomagnetic dipole operator | electromagnetic dipole operator | semileptonic operators



$\mathcal{O}_{1,2} \propto (\bar{s}\Gamma_\mu c)(\bar{c}\Gamma^\mu b)$ $\quad$ $\mathcal{O}_8 \propto (\bar{s}\sigma^{\mu\nu} T^a P_R) G^a_{\mu\nu}$ $\quad$ $\mathcal{O}_7 \propto (\bar{s}\sigma^{\mu\nu} P_R) F^a_{\mu\nu}$ $\quad$ $\mathcal{O}_9^\ell \propto (\bar{s}\gamma^\mu b_L)(\bar{\ell}\gamma_\mu \ell)$

$\mathcal{O}_{3,4} \propto (\bar{s}\Gamma_\mu b)\sum_q (\bar{q}\Gamma^\mu q)$ $\quad$ $\mathcal{O}_{10}^\ell \propto (\bar{s}\gamma^\mu b_L)(\bar{\ell}\gamma_\mu \gamma_5 \ell)$

+ the chirality flipped counter-parts of the above operators, $\mathcal{O}_i'$

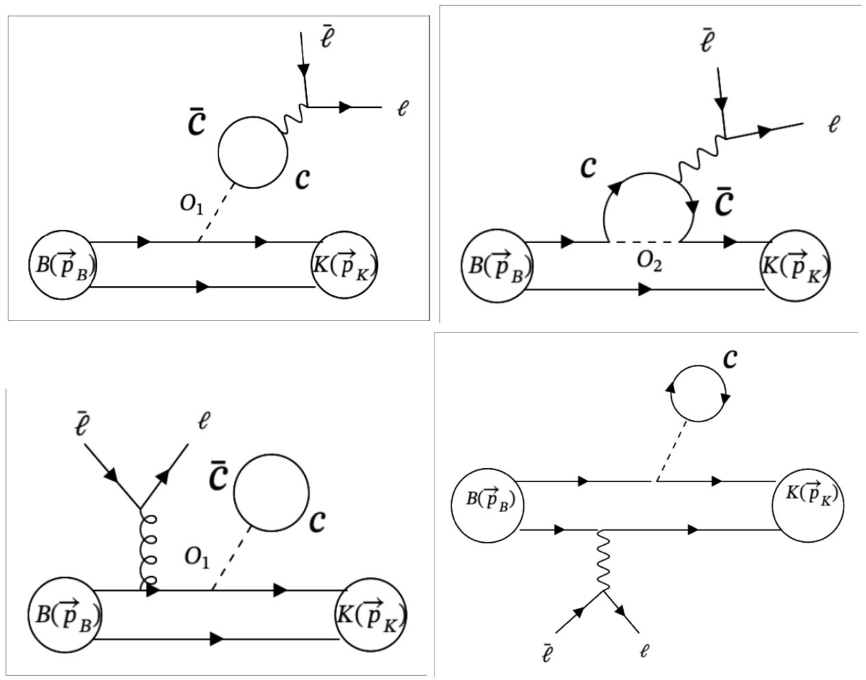
Wilson coefficients:

The Wilson coefficients are calculated perturbatively and are process independ
SM contributions known to NNLL **(Bobeth, Misiak, Urban '99; Misiak, Steinhauser '04, Gorl Gorbahn, Haisch, Misiak '05; Czakon, Haisch, Misiak '06,...)**

$C_7 \sim -0.3 \qquad C_9 \sim 4.2 \qquad C_{10} \sim -4.2$

Local Contributions

Penguin Non-local Contributions



Global fits

Global fits of the observables obtained by minimisation of

$$\chi^2 = \left(\vec{O}^{\rm th} - \vec{O}^{\rm exp}\right) \cdot (\Sigma_{\rm th} + \Sigma_{\rm exp})^{-1} \cdot \left(\vec{O}^{\rm th} - \vec{O}^{\rm exp}\right)$$

$(\Sigma_{\rm th} + \Sigma_{\rm exp})^{-1}$ is the inverse covariance matrix.

198 observables relevant for leptonic and semileptonic decays:

- BR($B \to X_s\gamma$)
- BR($B \to X_d\gamma$)
- BR($B \to K^*\gamma$)
- $\Delta_0(B \to K^*\gamma)$
- BR$^{\rm low}(B \to X_s\mu^+\mu^-)$
- BR$^{\rm high}(B \to X_s\mu^+\mu^-)$
- BR$^{\rm low}(B \to X_s e^+e^-)$
- BR$^{\rm high}(B \to X_s e^+e^-)$
- BR($B_s \to \mu^+\mu^-$)
- BR($B_s \to e^+e^-$)
- BR($B_d \to \mu^+\mu^-$)
- R_K in the low q^2 bin
- R_{K^*} in 2 low q^2 bins
- BR($B \to K^0\mu^+\mu^-$)
- $B \to K^+\mu^+\mu^-$: BR, F_H
- $B \to K^* e^+e^-$: BR, F_L, A_T^2, A_T^{Re}
- $B \to K^{*0}\mu^+\mu^-$: BR, F_L, A_{FB}, S_3, S_4, S_5, S_7, S_8, S_9
  in 8 low q^2 and 4 high q^2 bins
- $B^+ \to K^{*+}\mu^+\mu^-$: BR, F_L, A_{FB}, S_3, S_4, S_5, S_7, S_8, S_9
  in 5 low q^2 and 2 high q^2 bins
- $B_s \to \phi\mu^+\mu^-$: BR, F_L, S_3, S_4, S_7
  in 3 low q^2 and 2 high q^2 bins
- $\Lambda_b \to \Lambda\mu^+\mu^-$: BR, A_{FB}^{ℓ}, A_{FB}^{h}, $A_{FB}^{\ell h}$, F_L in the high q^2 bin

Computations performed using **SuperIso** public program

Single operator fits

Comparison of one-operator NP fits:

All observables 2022 ($\chi^2_{\rm SM} = 253.3$)			
	b.f. value	$\chi^2_{\rm min}$	$\mathrm{Pull}_{\rm SM}$
δC_9	-0.95 ± 0.13	215.8	6.1σ
δC_9^e	0.82 ± 0.19	232.4	4.6σ
δC_9^μ	-0.92 ± 0.11	195.2	7.6σ
δC_{10}	0.08 ± 0.16	253.2	0.5σ
δC_{10}^e	-0.77 ± 0.18	230.6	4.8σ
δC_{10}^μ	0.43 ± 0.12	238.9	3.8σ
$\delta C_{\rm LL}^e$	0.42 ± 0.10	231.4	4.7σ
$\delta C_{\rm LL}^\mu$	-0.43 ± 0.07	213.6	6.3σ

All observables 2023 ($\chi^2_{\rm SM} = 231.3$)			
	b.f. value	$\chi^2_{\rm min}$	$\mathrm{Pull}_{\rm SM}$
δC_9	-0.96 ± 0.13	230.7	6.3σ
δC_9^e	0.21 ± 0.16	269.2	1.3σ
δC_9^μ	-0.69 ± 0.12	240.4	5.5σ
δC_{10}	0.15 ± 0.15	270.0	1.0σ
δC_{10}^e	-0.18 ± 0.14	269.3	1.3σ
δC_{10}^μ	0.16 ± 0.10	268.3	1.6σ
$\delta C_{\rm LL}$	-0.54 ± 0.12	249.1	4.7σ
$\delta C_{\rm LL}^e$	0.10 ± 0.08	269.2	1.3σ
$\delta C_{\rm LL}^\mu$	-0.23 ± 0.06	257.4	3.7σ

$\delta C_{\rm LL}^\ell$ basis corresponds to $\delta C_9^\ell = -\delta C_{10}^\ell$.

absence says more than presence

FRANK HERBERT
(Dune)

THANKS FOR YOUR ATTENTION