Modular Invariance in Flavor Physics

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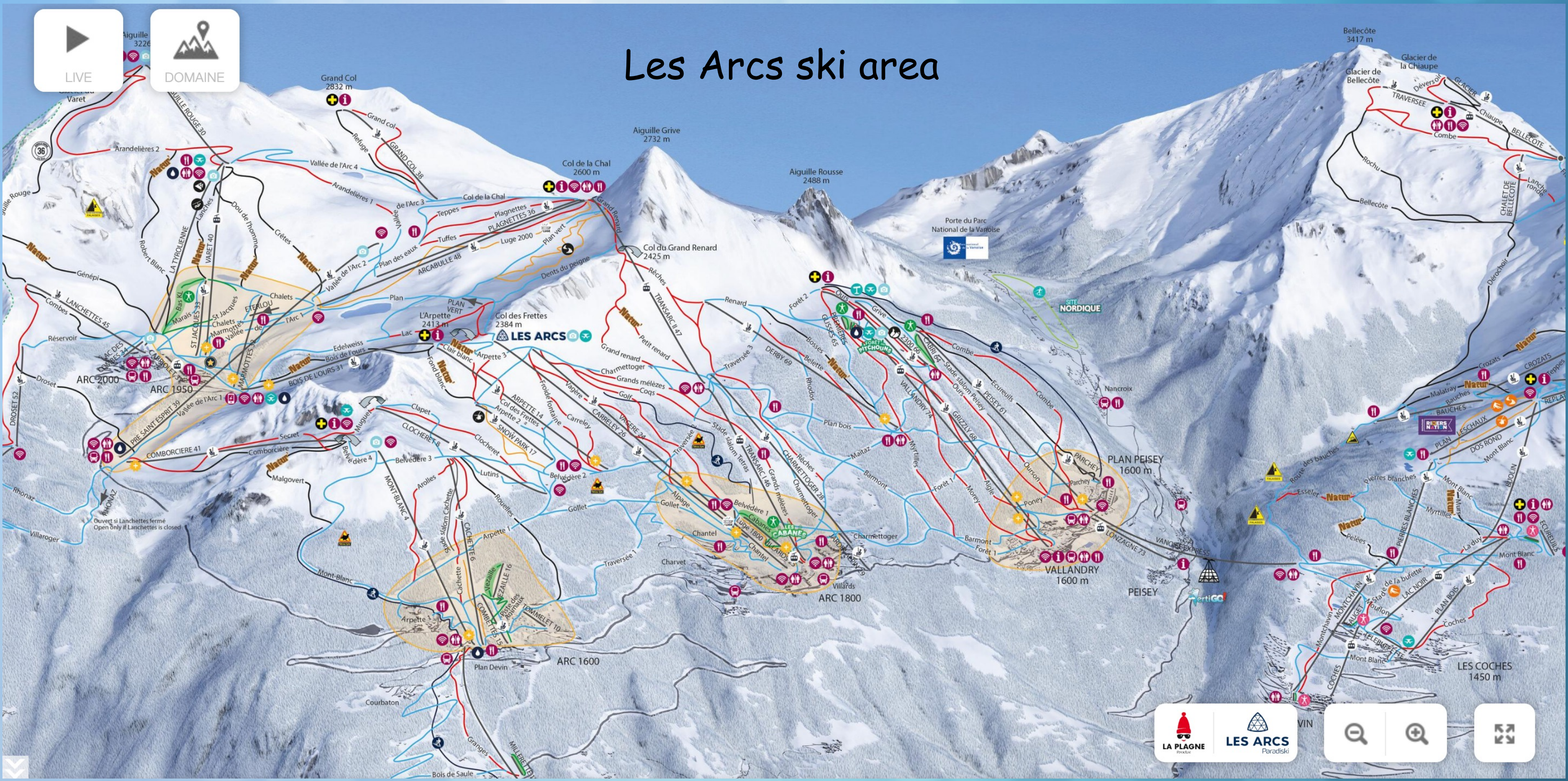
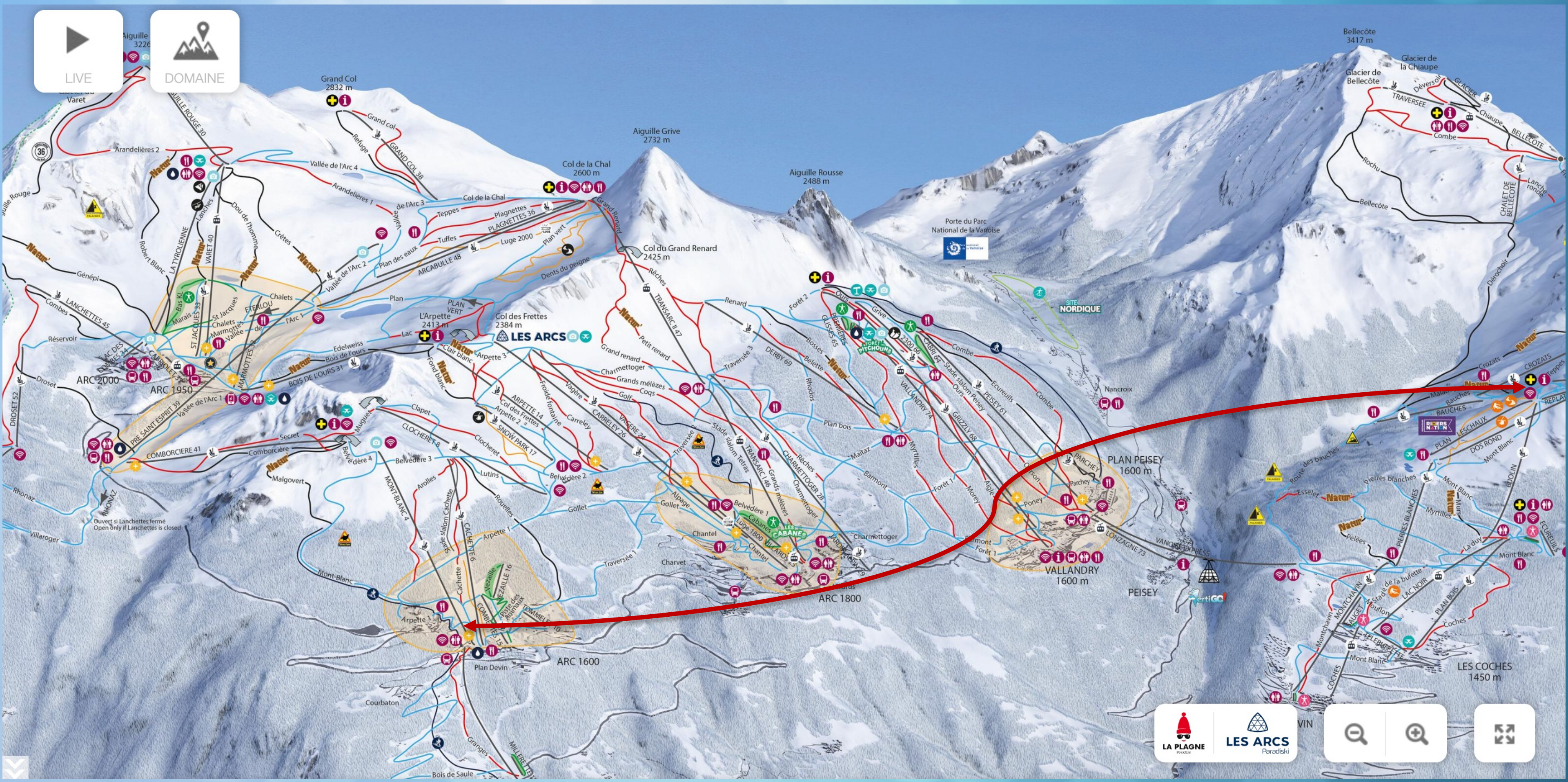
The first adventure



Les Arcs ski area

Circa 1987: 7th Moriond Workshop: Searches for New and Exotic Phenomena



the flavour puzzle from a symmetry perspective

- fermion masses and mixing angles require (up to) 22 parameters
  6+6 masses, 3+3 mixing angles, 1+3 phases

$$\mathcal{L}_Y = -\,\Psi^c \mathcal{Y}\,\Phi\,\Psi - \frac{1}{\Lambda}(\Phi\,\Psi)\mathcal{W}\,(\Phi\,\Psi)$$

- assume a symmetry group G acts in generation space

symmetric limit

$$m_{ij}(\tau) = m^0_{ij}$$

the flavour puzzle from a symmetry perspective

- τ_α — symmetry breaking sector: set of dimensionless, gauge invariant scalar fields, charged under G_{fl}

[τ_α stands for $\langle\tau_\alpha\rangle/\Lambda_F$ where the scale Λ_F has been set to 1]

symmetric limit

higher dimensional operators

<- many free parameters

$$m_{ij}(\tau) = m_{ij}^{0} + m_{ij}^{1\alpha}\,\tau_\alpha + m_{ij}^{1\bar{\alpha}}\,\bar{\tau}_{\bar{\alpha}} + m_{ij}^{2\alpha\beta}\,\tau_\alpha\tau_\beta + \ldots$$

vacuum alignment in SB sector

SUSY breaking effects
RGE corrections
(Λ_{UV}, m_{SUSY} , $\tan\beta$)

- huge number of models: G_{fl} continuous/discrete, global/local,……
no baseline model in bottom-up approach

in a SUSY model

$$m(\tau) = Z_A^{-\frac{1}{2}}(\tau,\bar{\tau})\ \mathcal{Y}(\tau)\, Z_B^{-\frac{1}{2}}(\tau,\bar{\tau})$$

the holomorphic part $\mathcal{Y}(\tau)$ can be tightly constrained by modular invariance

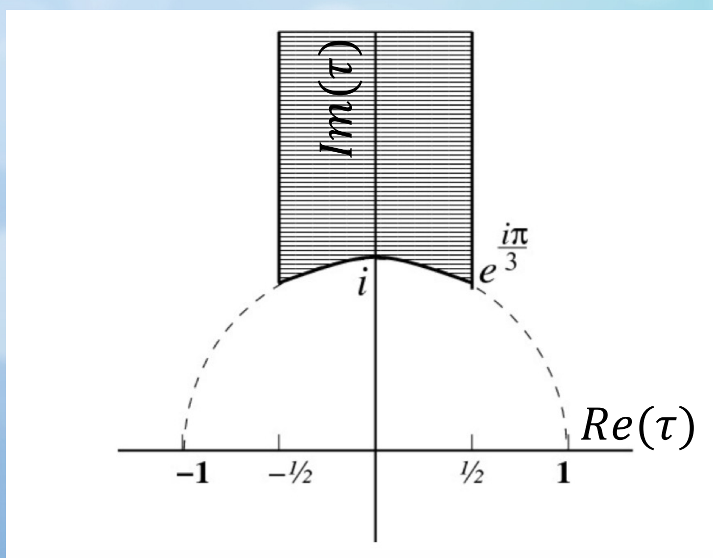
$$\tau \to \frac{a\tau + b}{c\tau + d}$$

$$\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,Z)$$

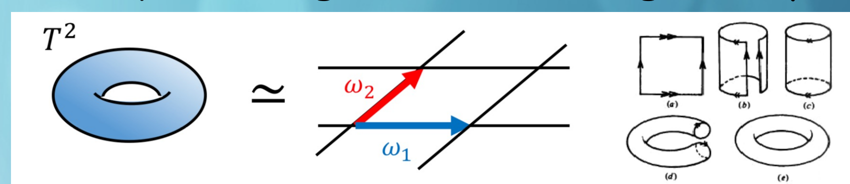
$$\varphi \ \to (c\,\tau + d)^{-k_\varphi}\ \rho_\varphi(\gamma)\ \varphi$$

unitary representation of the finite modular group $SL(2, Z_N)$

$N = 1,2,3, \ldots$



an ubiquitous ingredient in string theory



$\tau = \frac{\omega_2}{\omega_1} \quad Im(\tau) > 0$

Example

$SL(2, Z_3) \approx A_4{}'$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \to (c\tau + d)^{-1} \rho(\gamma) \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

$k_\nu = +1$

~ 3 of $SL(2, Z_3)$

$$w(\tau, \nu) = m_0\, \nu\, \mathcal{Y}(\tau) \nu + h.c.$$

modular form of level 3
$k = +2$ and $\rho \subset 3 + 1 + 1' + 1''$

$$d(\mathcal{M}_2(\Gamma_3)) = 3$$
$$\rho = 3$$

$$\mathcal{Y}(\tau) = \mathcal{Y}_0 \begin{pmatrix} 2Y_1(\tau) & -Y_3(\tau) & -Y_2(\tau) \\ -Y_3(\tau) & 2Y_2(\tau) & -Y_1(\tau) \\ -Y_2(\tau) & -Y_1(\tau) & 2Y_3(\tau) \end{pmatrix}$$

[F.F. 1706.08749]

Yukawas completely determined in terms of τ up to an overall constant

no corrections from higher order operators in the exact SUSY limit

Generalised CP symmetry in modular-invariant models of flavour

P.P. Novichkov, J.T. Penedo, S.T. Petcov & A.V. Titov

Journal of High Energy Physics **2019**, Article number: 165 (2019) | Cite this article

The formalism of combined finite modular and generalised CP (gCP) sym-metries for theories of flavour is developed. The corresponding consistency conditions for the two symmetry transformations acting on the modulus τ and on the matter fields are derived. The implications of gCP symmetry in theories of flavour based on modular invariance described by finite modular groups are illustrated with the example of a modular S_4 model of lepton flavour. Due to the addition of the gCP symmetry, viable modular models turn out to be more constrained, with the modulus τ being the only source of CP violation.

- a unique CP law consistent with the modular group [Im(τ) > 0]

$$\tau \rightarrow -\tau^*$$

[up to modular transformations]

in a modular and CP invariant theory

- CP conserved $\leftrightarrow$ τ imaginary or at the border of the fundamental domain

otherwise CP spontaneously broken by $\langle\tau\rangle$

Fermion mass hierarchies, large lepton mixing and residual modular symmetries

P. P. Novichkov, J. T. Penedo & S. T. Petcov

Journal of High Energy Physics **2021**, Article number: 206 (2021) | Cite this article

with discrete nonlinear transformations

$$\tau \to \frac{a\tau + b}{c\tau + d}$$

there is no notion of "small"

$$\tau \to \tau + n \qquad \gamma = \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix} \in SL(2, Z)$$

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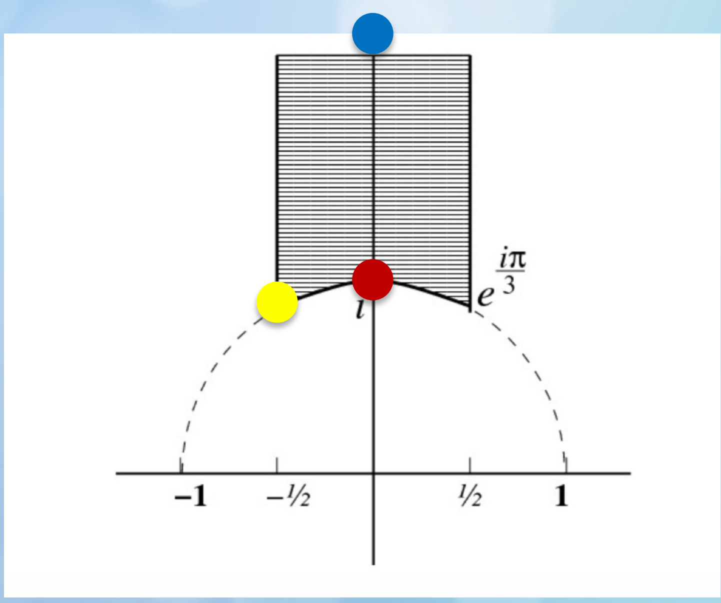
[F., Gherardi, Romanino and Titov, arXiv:2101.08718]



fixed point

$\tau = i$	$\tau \xrightarrow{S} -\frac{1}{\tau}$	$\mathbb{Z}_4^S$
$\tau = e^{i\,2\pi/3}$	$\tau \xrightarrow{ST} -\frac{1}{\tau+1}$	$\mathbb{Z}_2^{ST} \times \mathbb{Z}_2^{S^2}$
$\tau = i\,\infty$	$\tau \xrightarrow{T} \tau + 1$	$\mathbb{Z}^T \times \mathbb{Z}_2^{S^2}$

residual symmetry

- assume $\tau \approx i$

make use of new coordinates: $u = \dfrac{\tau - i}{\tau + i}$

S
$u \to -u$

- u is small

$$m_{ij}(u) = m_{ij}^{0} + m_{ij}^{1} u + m_{ij}^{\bar{1}}\, \bar{u} + m_{ij}^{2} u^{2} + \ldots$$

- near $\tau = i$, this looks like a $\mathbb{Z}_4^S$ theory spontaneously broken by the SB parameter u

mass hierarchies can be generated this way

models of lepton masses

- modular invariance successful in describing the lepton sector

  reproduce neutrino masses (3), mixing angles (3) and CP phases (3) in terms of τ and 2 or 3 additional parameters.

- 1. Large freedom leads to > 100 viable models

$$\varphi \;\; \rightarrow (c\,\tau + d)^{-k_\varphi}\,\rho_\varphi(\gamma)\,\varphi$$

unitary representation of the finite modular group $SL(2, Z_N)$

- 2. non-holomorphic contribution assumed flavor universal

$$m(\tau) = \; Z_A^{-\frac{1}{2}}(\tau,\bar{\tau})\;\; \mathcal{Y}(\tau)\, Z_B^{-\frac{1}{2}}(\tau,\bar{\tau}) = z_0 \mathcal{Y}(\tau)$$

- what can we learn?

distribution of models in τ space

all CP invariant

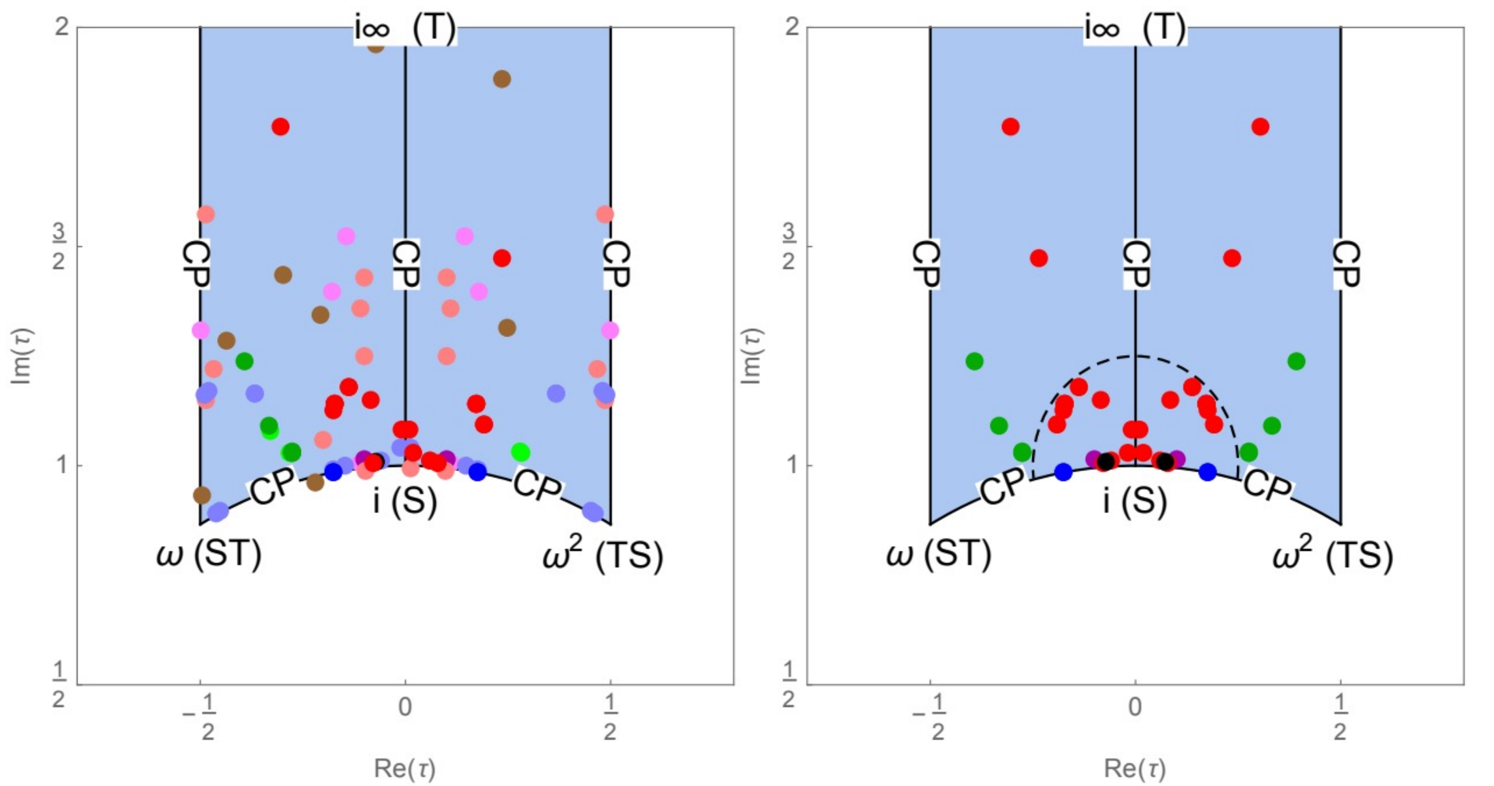


Figure 4: Fundamental domain $\mathcal{F}$ (light blue region) and fixed points (see text). Dots are the best-fit values of τ in models of ref. [66,68] (Γ_3 - light red), [69,70] ($\Gamma_3\&CP$ - red), [71] (Γ_4 - light magenta), [67] ($\Gamma_4\&CP$ - magenta), [72] (Γ'_4 - light blue), [72] ($\Gamma'_4\&CP$ - blue), [73] ($\Gamma'_5\&CP$ - black), [74] (Γ'_6 - light green), [74] ($\Gamma'_6\&CP$ - green), [75] (Γ_7 - brown). We use the notation $\Gamma'_N = SL(2,\mathbb{Z}_N)$ and $\Gamma_N = SL(2,\mathbb{Z}_N)/\{\pm\mathbb{1}\}$. In the left panel all models are displayed. The right panel includes only CP invariant models, for which the full pair of points τ and $-\bar{\tau}$ is shown. The dashed line represents the contour $|\tau - i| = 0.25$.

[F. 2211.00659,2302.11580]

lepton doublets are in an irrep of $SL(2, Z_N)$ [F. 2211.00659,2302.11580]

for any choice of level N, weights k_φ and kinetic terms $Z_\varphi^{-\frac{1}{2}}(\tau,\bar{\tau})$

models fall into 3 classes, and the successful one predicts

$$m_\nu^{-1} = m_{0\nu}^{-1} \begin{pmatrix} x_{11}\, x & x_{12}^0 & x_{13}^0 \\ \cdot & x_{22}\, x & x_{23}\, x \\ \cdot & \cdot & x_{33}\, x \end{pmatrix} + \mathcal{O}(x^2)$$

$$x = |u| = \left|\frac{\tau - i}{\tau + i}\right|$$

x_{ij} real

$$sin^2\vartheta_{12} = \frac{1}{2}(1 + c_{12}\, x) + \cdots$$

$$sin^2\vartheta_{23} = c_{23} + \cdots$$

$$sin^2\vartheta_{13} = c_{13}\, x^2 + \cdots$$

$$m_3 = m_0 \frac{c_3}{x} + \cdots$$

$$m_{1,2} = m_0(1 \pm c_m x) + \cdots$$

$$\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}} = c_{s/a}\, x^3 + \cdots$$

$$\delta_{CP} = c_\delta + \cdots$$

$$\alpha_{21} = \pi + \cdots$$

$$\alpha_{31} = c_\alpha + \cdots$$

data reproduced by

$m_0 \approx 10$ meV $\quad$ $x = |u| \approx 0.1$ $\quad$ $c_i = O(1)$

Modular flavour symmetries and modulus stabilisation

P. P. Novichkov, J. T. Penedo & S. T. Petcov

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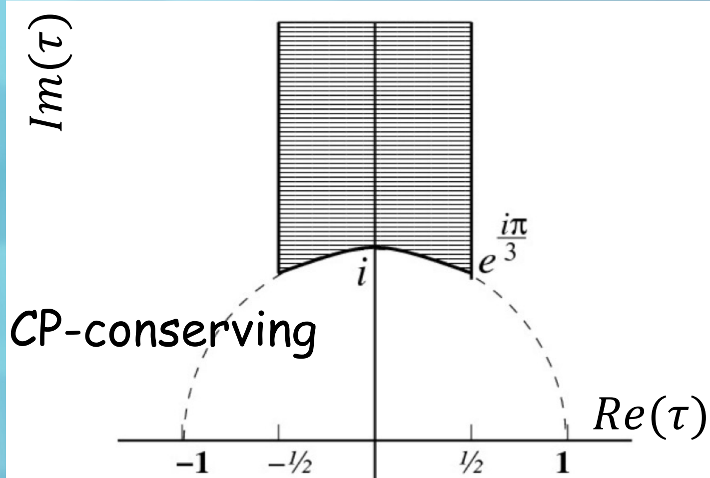
We study the problem of modulus stabilisation in the framework of the modular symmetry approach to the flavour problem. By analysing simple UV-motivated CP-invariant potentials for the modulus τ we find that a class of these potentials has (non-fine-tuned) CP-breaking minima in the vicinity of the point of $\mathbb{Z}_3^{ST}$ residual symmetry, $\tau \simeq e^{2\pi i/3}$. Stabilising the modulus at these novel minima breaks spontaneously the CP symmetry and can naturally explain the mass hierarchies of charged leptons and possibly of quarks.

what determines the value of τ ?

- anthropic selection
- cosmological evolution
- extrema of $V(\tau)$

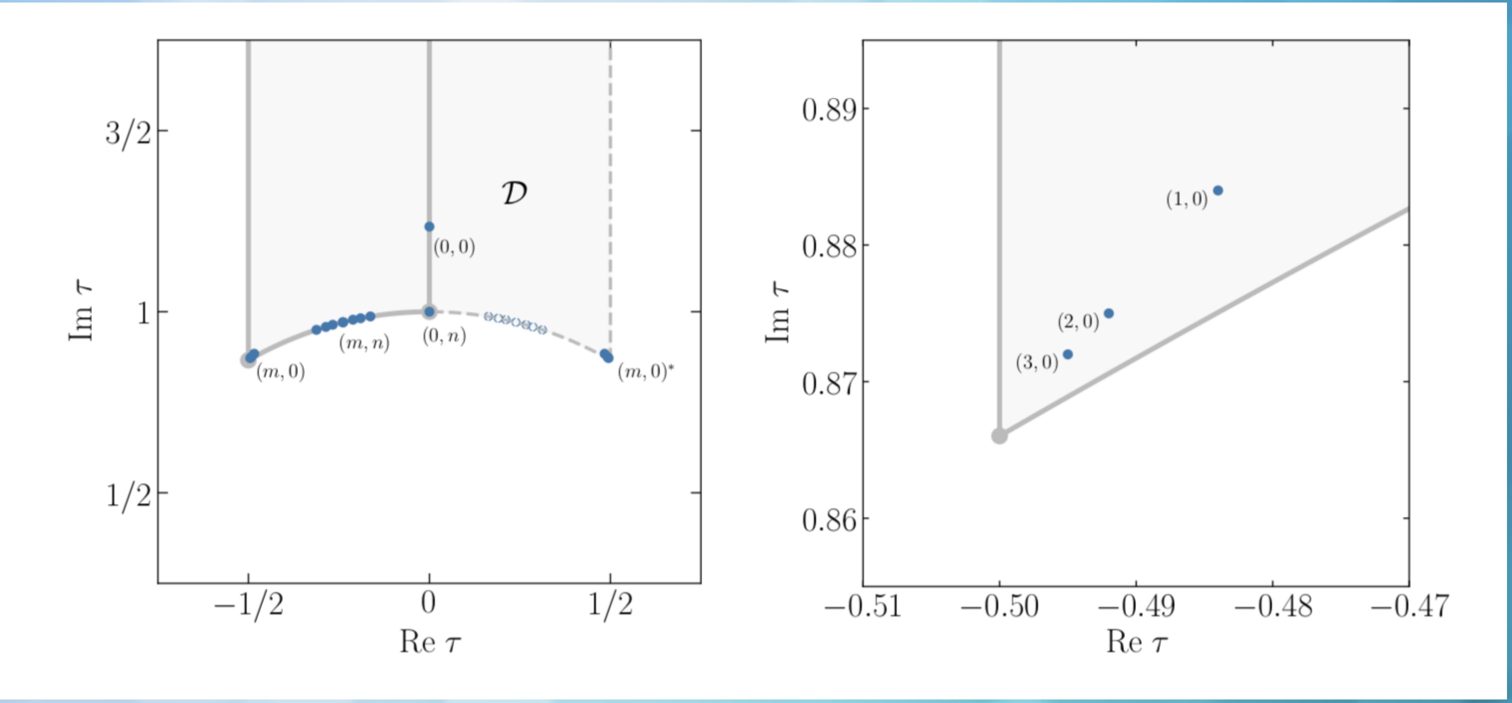
extrema of $V(\tau)$ at the border of the fundamental region and along the $Im(\tau)$ axis ?



[Cvetic, Font, Ibanez, Lust and Quevedo, Nucl.Phys.B 361 (1991) 194]

CP-violating minima from local SUSY

[Novichkov, Penedo and Petcov, 2201.02020]



Congratulations Serguey!

there are still many adventures on our way!

back-up slides

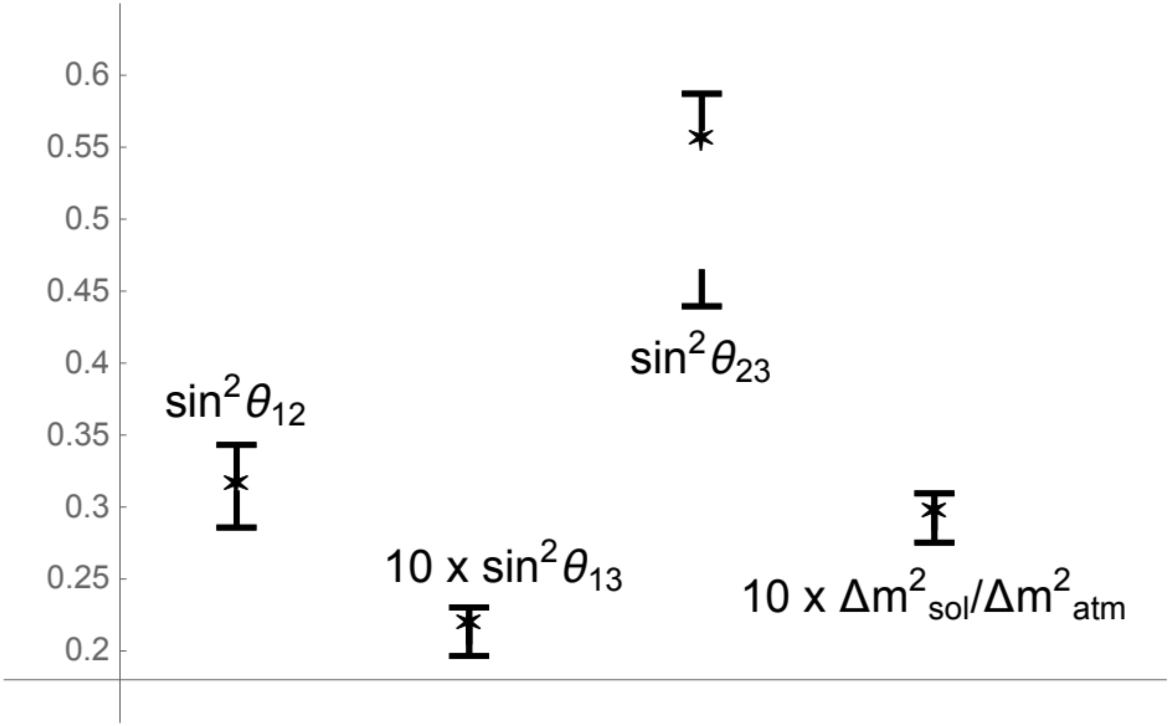


Figure 5: Mixing angles and $\Delta m^2_{sol}/\Delta m^2_{atm}$ of 14 pairs of CP and modular invariant models featuring normal ordering and $|\tau - i| < 0.25$ (see text). Shown in the plot are the intervals covered by the model predictions. A star indicates the average over the 14 models.

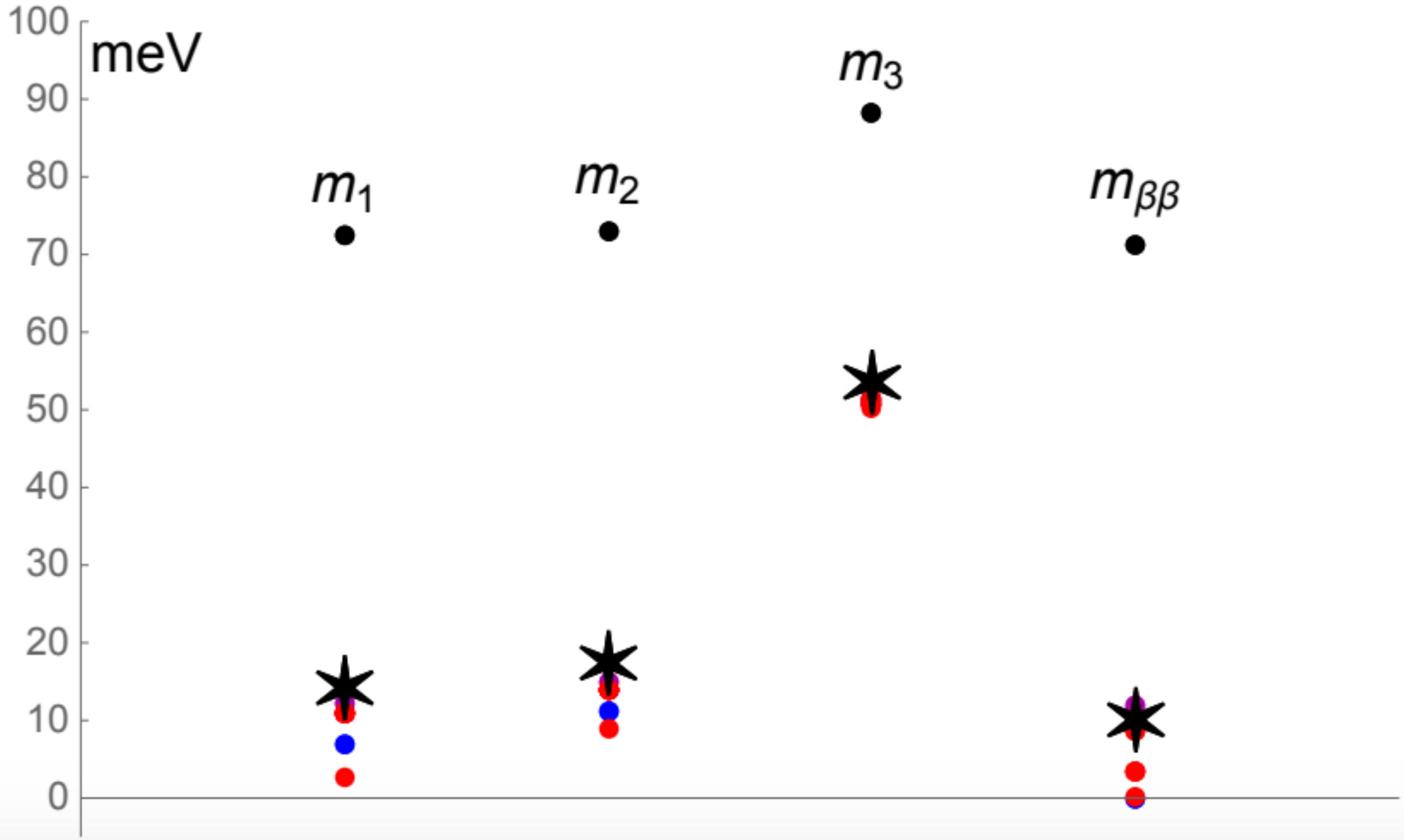


Figure 6: Mass parameters and phases of 14 pairs of CP and modular invariant models featuring normal ordering and $|\tau - i| < 0.25$ (see text). The full distributions of predictions are displayed. The color code is identical to the one in fig. 4. A star shows the average over the 14 models. CP violating phases refer to models where $\mathtt{Re}\tau > 0$.

$\mathcal{N}$=1 SUSY modular invariant theories

Yukawa interactions in $\mathcal{N}$=1 global SUSY [extension to $\mathcal{N}$=1 SUGRA straightforward]

$$S = \int d^4x d^2\theta \, w(\tau,\varphi) + h.c + kinetic\ terms$$

invariance satisfied by "minimal" Kahler potential

$$w(\tau,\varphi) = \sum_n Y_{I_1 \dots I_n}(\tau)\varphi^{(I_1)} \dots \varphi^{(I_n)}$$

field-dependent Yukawa couplings

invariance of $w(\Phi)$ guaranteed by an holomorphic $Y_{I_1 \dots I_n}(\tau)$ such that

$$Y_{I_1 \dots I_n}(\gamma\tau) = (c\tau + d)^{k_Y(n)} \rho(\gamma) \, Y_{I_1 \dots I_n}(\tau)$$

1. $k_Y(n) - k_{I_1} - \dots - k_{I_n} = 0$
2. $\rho \times \rho^{I_1} \times \ \dots \times \rho^{I_n} \supset 1$

modular forms of level N and weight k_Y → form a linear space $\mathcal{M}_k(\Gamma_N)$) of finite dimension