Scattering amplitudes and Feynman integrals

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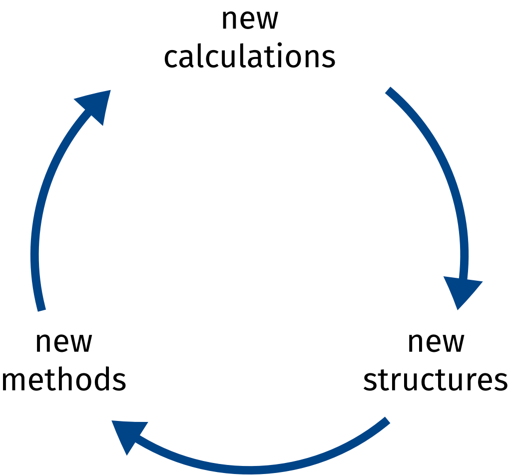
Motivation



Philosophy

Problem: Lagrangians and Feynman diagrams are highly redundant

- field redefinitions:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi\left(1 + \lambda_1\phi + \frac{1}{2!}\lambda_2\phi^2 + \ldots\right) \quad \sim \quad \mathcal{L} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi$$

$$\mathcal{A}_6 = \mathcal{O}\left(10^2\right) \text{ Feynman diagrams} \quad = \quad 0$$

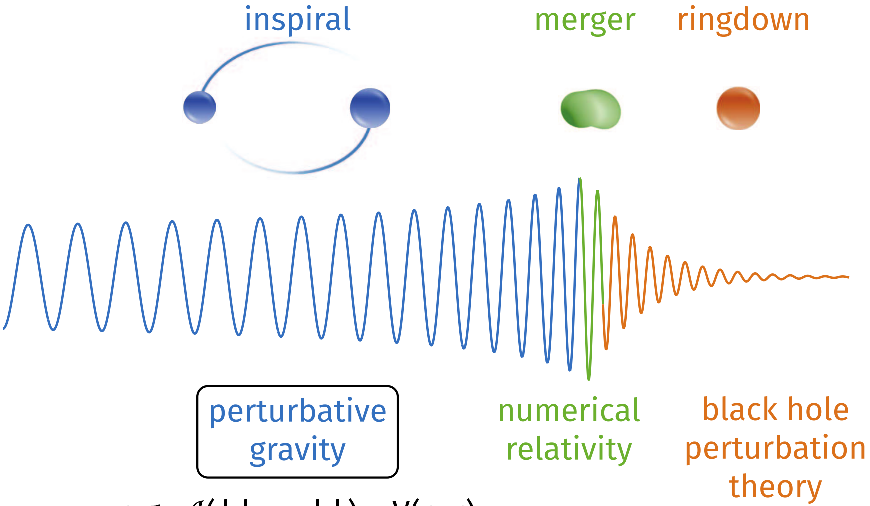
- gauge:

$$\mathcal{A}(\gamma^\pm, \ldots) = \varepsilon^\pm_\mu \ldots A^{\mu\ldots}$$

Idea: Focus on observables → scattering amplitudes

Amplitudes meet gravitational waves

Waveform templates



e.g. $\mathcal{A}(\phi\phi \to \phi\phi) \sim V(p, r)$

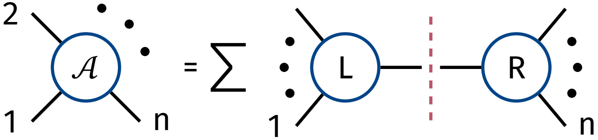
[Adapted from 1610.03567]

Tree-level recursion relations

$$\mathcal{A}^{\text{gauge}}_{\text{tree},3} \to \mathcal{A}^{\text{gauge}}_{\text{tree},n} \to \mathcal{A}^{\text{gravity}}_{\text{tree},n} \to \mathcal{J}^{\text{gravity}}_{\text{loop},4} \to \mathcal{A}^{\text{gravity}}_{\text{loop},4}$$

Parke–Taylor formula: $\mathcal{A}_n(1^-2^-3^+\dots n^+) = \dfrac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\dots\langle n1\rangle}$

BCFW recursion:



Double copy

$$\mathcal{A}^{\text{gauge}}_{\text{tree},3} \to \mathcal{A}^{\text{gauge}}_{\text{tree},n} \to \mathcal{A}^{\text{gravity}}_{\text{tree},n} \to \mathcal{I}^{\text{gravity}}_{\text{loop},4} \to \mathcal{A}^{\text{gravity}}_{\text{loop},4}$$

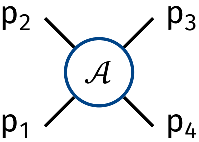
$$\mathcal{A}^{\text{gauge}}_{\text{L}} = \sum \frac{\text{Color}_{\text{L}} \cdot \text{Kin}_{\text{L}}}{\prod \text{Propagators}}$$

$$\mathcal{A}^{\text{gauge}}_{\text{R}} = \sum \frac{\text{Color}_{\text{R}} \cdot \text{Kin}_{\text{R}}}{\prod \text{Propagators}}$$

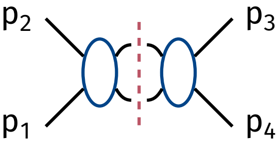
$$\mathcal{A}^{\text{gravity}}_{\text{L}\otimes\text{R}} = \sum \frac{\text{Kin}_{\text{L}} \cdot \text{Kin}_{\text{R}}}{\prod \text{Propagators}}$$

Generalized unitarity

$$\mathcal{A}^{\text{gauge}}_{\text{tree},3} \to \mathcal{A}^{\text{gauge}}_{\text{tree},n} \to \mathcal{A}^{\text{gravity}}_{\text{tree},n} \to \mathcal{I}^{\text{gravity}}_{\text{loop},4} \to \mathcal{A}^{\text{gravity}}_{\text{loop},4}$$



$$= \sum \int d\ell \, \text{Num}(\ell, k; \text{coefs}) \cdot \frac{1}{q_1^2 - m_1^2} \cdot \frac{1}{q_2^2 - m_2^2} \cdots$$



$$= \sum \int d\ell \, \text{Num}(\ell, k; \text{coefs}) \cdot \frac{1}{q_1^2 - m_1^2} \cdot \delta\left(q_2^2 - m_2^2\right) \cdots$$

Feynman integrals

$$\mathcal{A}^{\text{gauge}}_{\text{tree},3} \to \mathcal{A}^{\text{gauge}}_{\text{tree},n} \to \mathcal{A}^{\text{gravity}}_{\text{tree},n} \to \mathcal{I}^{\text{gravity}}_{\text{loop},4} \to \mathcal{A}^{\text{gravity}}_{\text{loop},4}$$

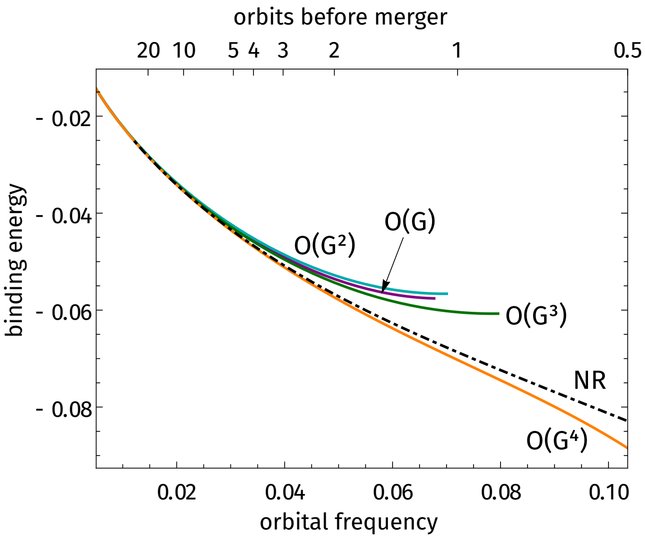
1. Integration-by-part (IBP) reduction:

$$\int d\ell \frac{\text{Num}(\ell; x)}{(q_1^2 - m_1^2)\dots} \stackrel{\text{IBP}}{=} \sum_n c_n(x) F_n(x) \quad \text{master integrals}$$

2. Differential equations for masters:

$$\frac{\partial F_m(x)}{\partial x} = \sum_n A_{mn}(x) F_n(x)$$

Was it worth it?



[Adapted from 2204.05194]

Summary

1. Amplitudes push the state of the art for high-precision calculations in collider and gravitational-wave physics

2. Amplitudes further our fundamental understanding of QFT

3. Many important open questions (integration, classical limit, spinning objects, unbound vs. bound, etc.)

Some references

1. C. Cheung. "TASI Lectures on Scattering Amplitudes". (2018). Ed. by R. Essig and I. Low, pp. 571–623. 1708.03872 [hep-ph]

2. Z. Bern and J. Trnka. "Snowmass TF04 Report: Scattering Amplitudes and their Applications". (2022). 2210.03146 [hep-th]

3. A. Buonanno et al. "Snowmass White Paper: Gravitational Waves and Scattering Amplitudes". (2022). 2204.05194 [hep-th]