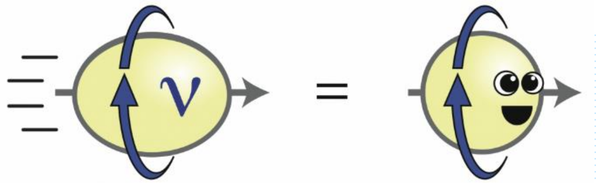
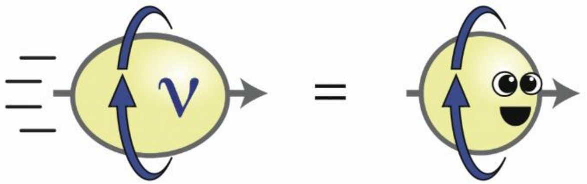
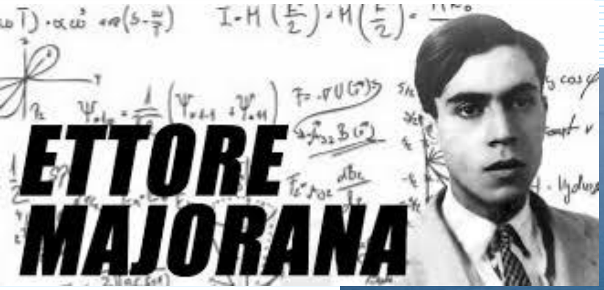
ICTP Trieste, April 24 (Mon), 2022



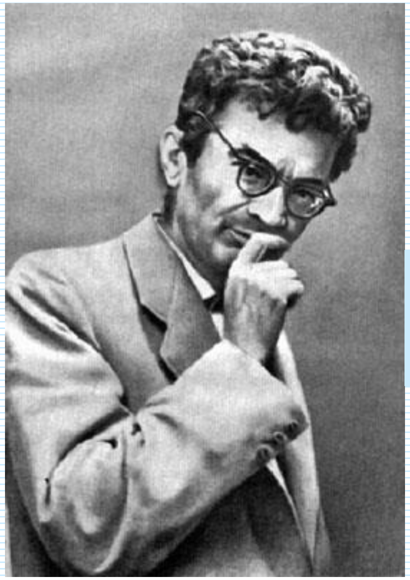
Neutrinoless Double-Beta Decay

Fedor Šimkovic





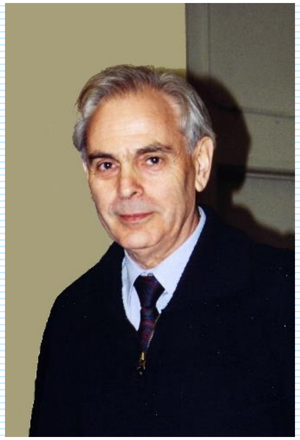
Family Tree



Isaak Yakovlevich Pomeranchuk

Samoil Mihelevich Bilenky
(S. Petcov,
E, Nedelcheva,
D. Bardin,
F. Š …)

Igor Yurevich Kobzarev
(M.I. Krivoruchenko
B.V. Martemyanov)



Lev Borisovich Okun

OUTLINE

I. ***Introduction***

II. The ***$0\nu\beta\beta$-decay experiments, status and perspectives***

(Gerda/Majorana/Legend, EXO,KamLAND-Zen,
NEXT, CUORE, CUPID, and Outlook)

III. ν-mass ***$0\nu\beta\beta$-decay mechanisms***

(QCSS scenario, neutrino mass spectrum predicted)

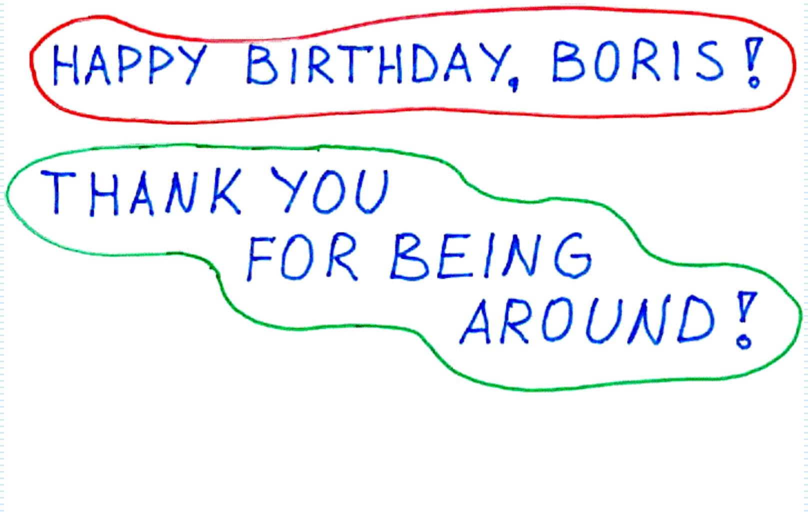
IV. Some notes about neutrino to antineutrino oscillations

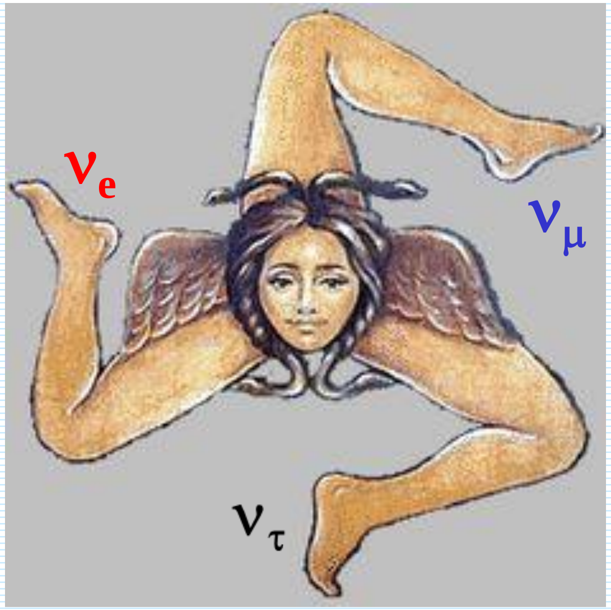
(QCSS scenario)

V. Neutrino oscillations in QFT revisited

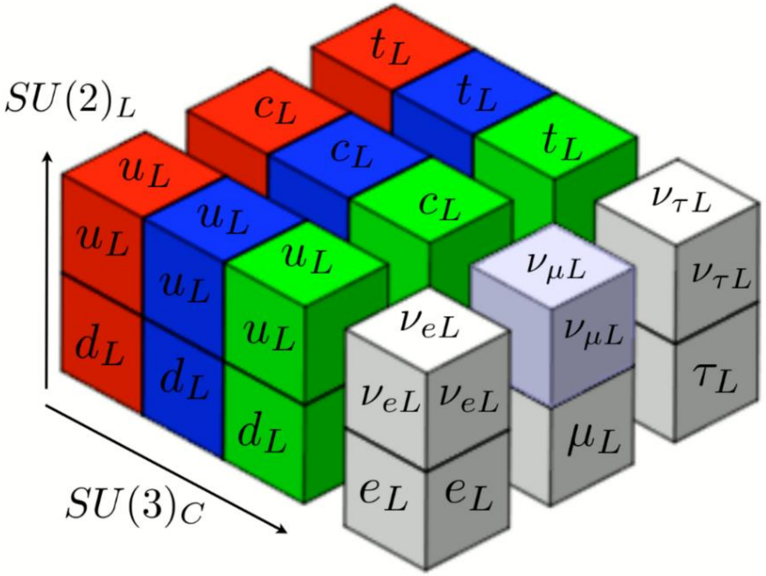
(without consideration of wave packets)

VI. ***Happy Birthday Dear Serguey***



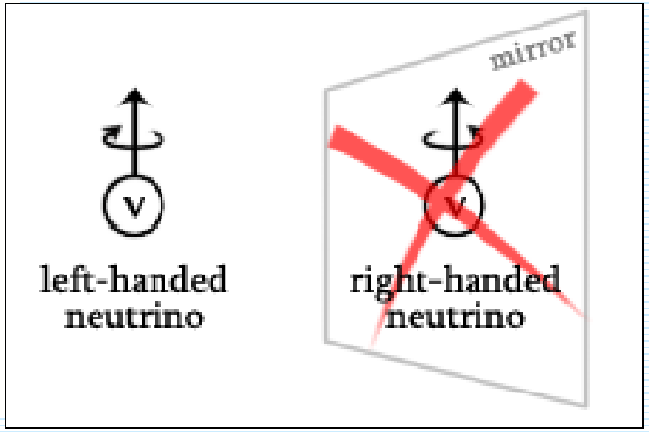


Standard Model
(an astonishing successful theory,
based on few principles)



ν is a special particle in SM:

- It is the only fermion that **does not carry electric charge** (like γ, g, H^0)
- There are only **left-handed ν's** (ν_{eL}, $\nu_{\mu L}$, $\nu_{\tau L}$)
- **ν-mass** can not be generated with any renormalizable coupling with the **Higgs fields** through **SSB**



ν's oscillations experiments
⇒ tiny neutrino masses (!)
⇒ Beyond SM physics (!)

SUPER K
SNO
KamLAND
Kamioka Liquid scintillator Anti-Neutrino Detector
DOUBLE CHOOZ
Daya Bay θ_{13}
RENO ν θ_{13}
T2K
NOνA

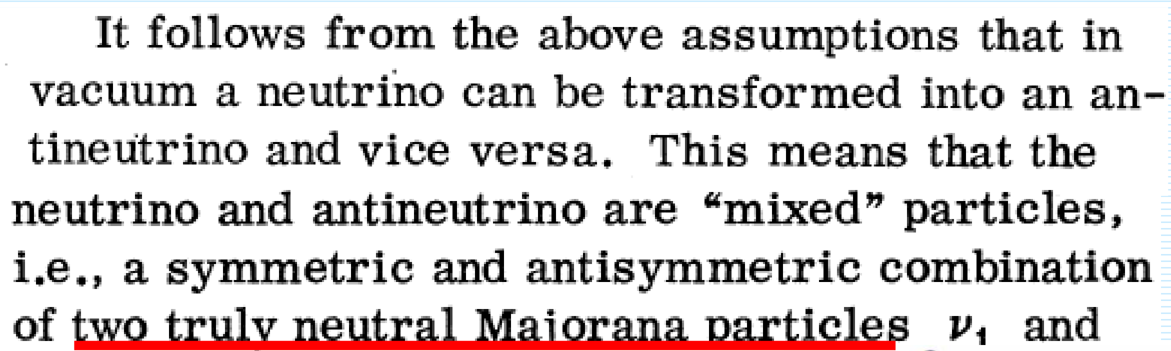
, etc

Majorana fermions

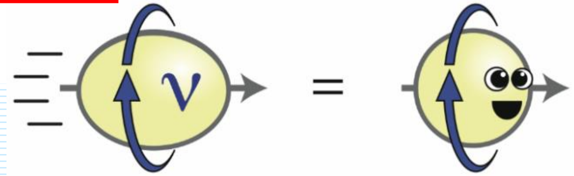
Ettore Majorana

Teoria simmetrica dell'elettrone e del positrone
(A symmetric theory of electrons and positrons).
Il Nuovo Cimento, 14: 171–184, 1937.) 171

ν is its own antiparticle



It follows from the above assumptions that in vacuum a neutrino can be transformed into an antineutrino and vice versa. This means that the neutrino and antineutrino are “mixed” particles, i.e., a symmetric and antisymmetric combination of two truly neutral Majorana particles ν_1 and ν_2 of different combined parity.[5]



ν ↔ anti-ν oscillation

Bruno Pontecorvo

Inverse beta processes and nonconservation of lepton charge
Zhur. Eksptl'. i Teoret. Fiz.
34, 247 (1958)

INTERNATIONAL JOURNAL OF HIGH-ENERGY PHYSICS
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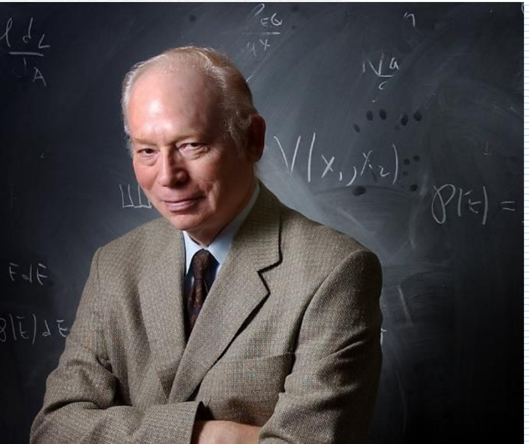
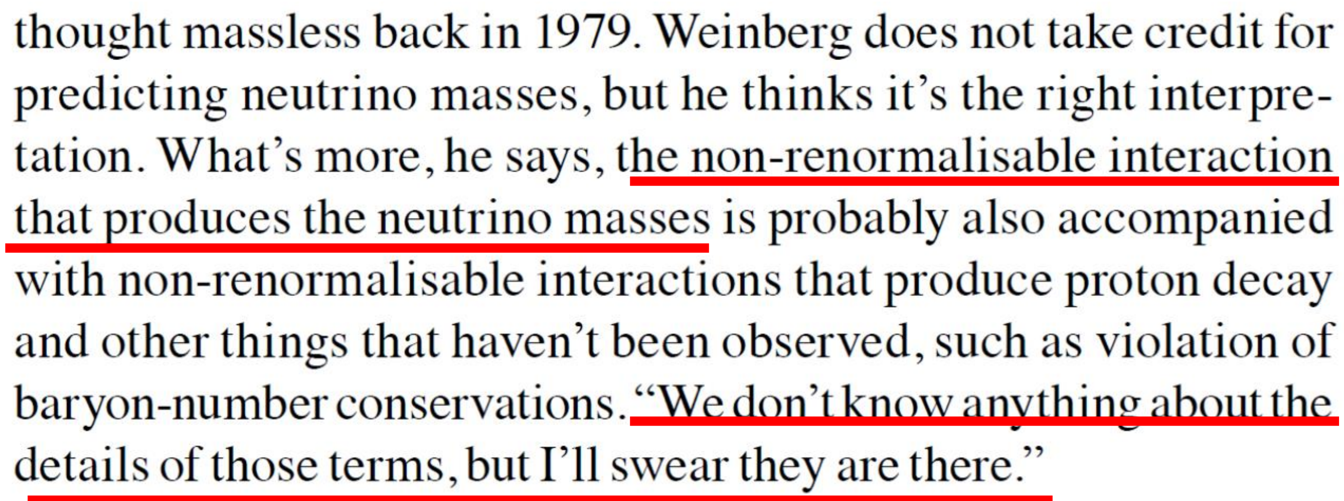
VOLUME 57 NUMBER 9 NOVEMBER 2017



Steve Weinberg

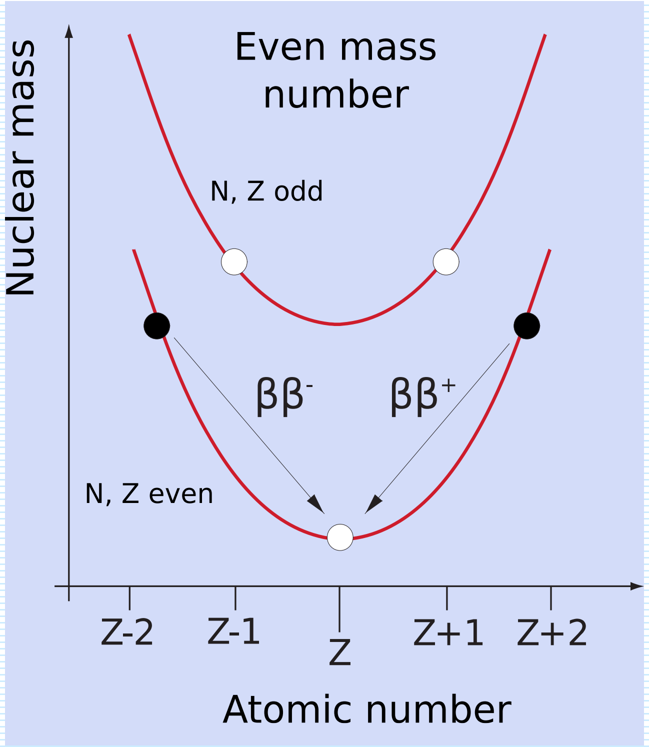
ν-mass generation
via d=5 eff. oper.
related to unknown
high energy scale (GUT?)

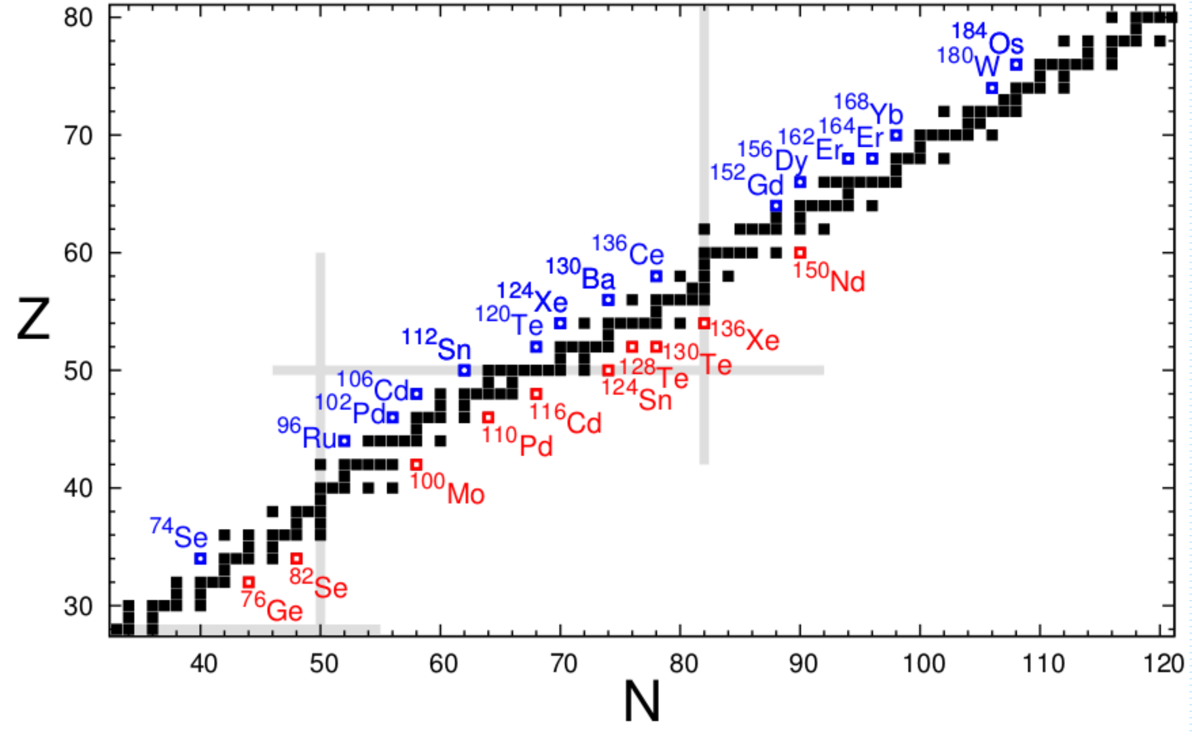


thought massless back in 1979. Weinberg does not take credit for predicting neutrino masses, but he thinks it’s the right interpretation. What’s more, he says, the non-renormalisable interaction that produces the neutrino masses is probably also accompanied with non-renormalisable interactions that produce proton decay and other things that haven’t been observed, such as violation of baryon-number conservations. “We don’t know anything about the details of those terms, but I’ll swear they are there.”

Nuclear double-β decay

(even-even nuclei, pairing int.)





Phys. Rev. 48, 512 (1935)

Two-neutrino double-β decay – LN conserved

$$(A,Z) \rightarrow (A,Z+2) + e^- + e^- + \nu_e + \nu_e$$

Goepert-Mayer – 1935. 1st observation in 1987

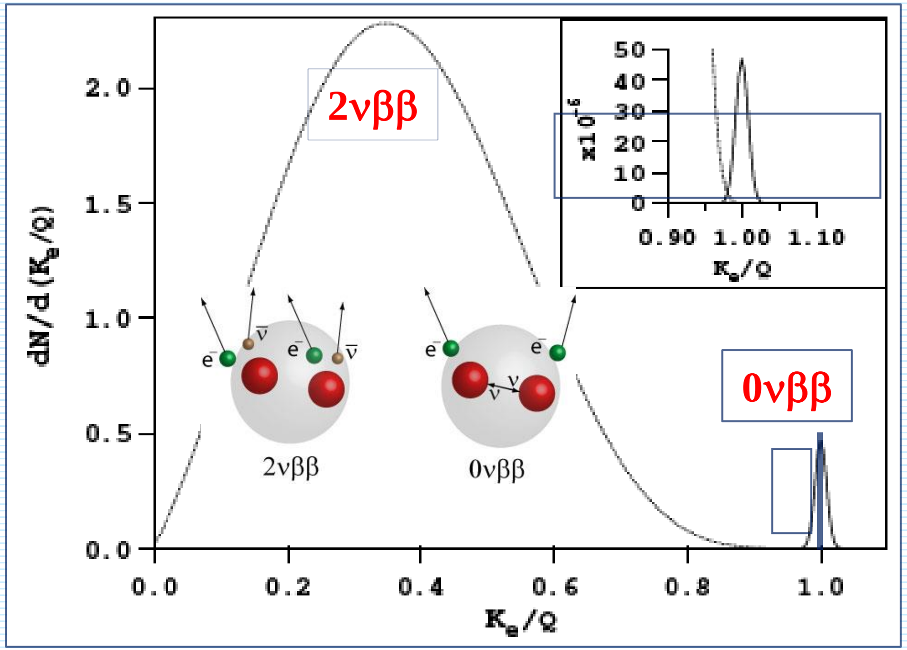
Nuovo Cim. 14, 322 (1937)

Phys. Rev. 56, 1184 (1939)

Neutrinoless double-β decay – LN violated

$$(A,Z) \rightarrow (A,Z+2) + e^- + e^- \quad \text{(Furry 1937)}$$

Not observed yet. Requires massive Majorana ν's



(A,Z) → (A,Z+2) + e⁻ + e⁻

0νββ-decay

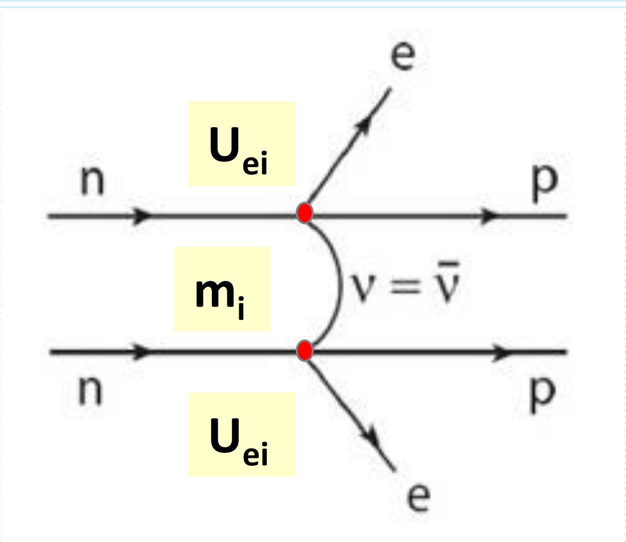
(LNV at ≈GUT scale, exchange of three light ν)

$$\left(T^{0\nu}_{1/2}\right)^{-1} = \left|\frac{m_{\beta\beta}}{m_e}\right|^2 \, g_A^4 \, \left|M^{0\nu}_{\nu}\right|^2 \, G^{0\nu}$$

Phase space factor well understood

?

***NME** must be evaluated using tools of nuclear theory*



Effective Majorana mass

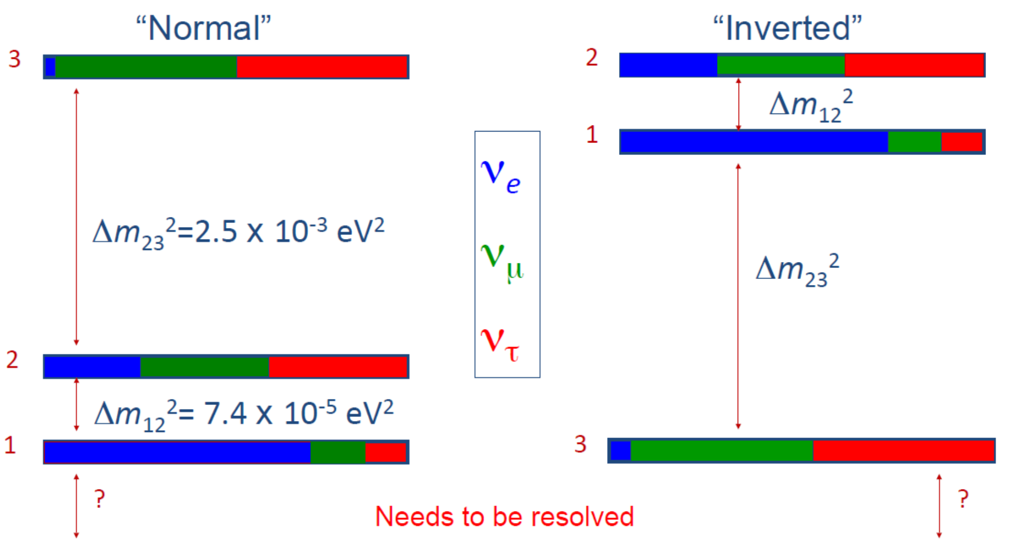
can be evaluated. It depends on

$m_1, m_2, m_3, \theta_{12}, \theta_{13}, \alpha_1, \alpha_2$

(3 unknown parameters: $m_1/m_3, \alpha_1, \alpha_2$

and ν-mass hierarchy)

$$m_{\beta\beta} = \left|c_{13}^2 c_{12}^2 e^{i\alpha_1} m_1 + c_{13}^2 s_{12}^2 e^{i\alpha_2} m_2 + s_{13}^2 m_3\right|$$



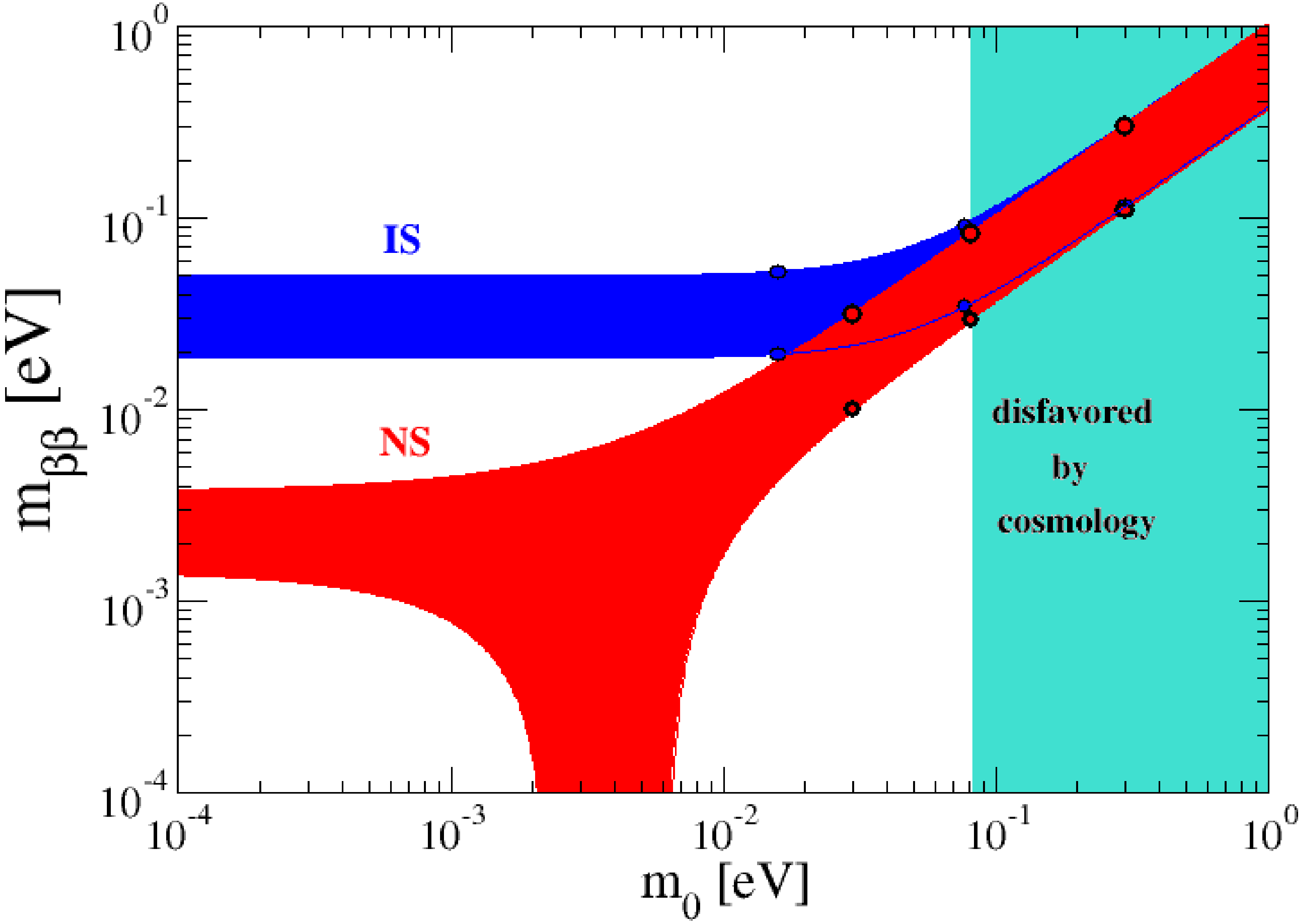
$$U^{PMNS} = \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & e^{-i\delta}s_{13} \\ -c_{23}s_{12} - e^{i\delta}c_{12}s_{13}s_{23} & c_{12}c_{23} - e^{i\delta}s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{12}s_{23} - e^{i\delta}c_{12}c_{23}s_{13} & -e^{i\delta}c_{23}s_{12}s_{13} - c_{12}s_{23} & c_{13}c_{23} \end{pmatrix} \begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Effective Majorana
ν-mass $m_{\beta\beta}$
(prediction
due ν-oscillations)

Constraint from cosmology

$\Sigma = m_1 + m_2 + m_3$
< 0.90 eV
< 0.26 eV (Planck coll.)
< 0.12 eV

Contrary, the constraint from 0νββ-decay (KLZ)
$m_{\beta\beta} < 0.036\text{-}0.156$ eV
implies
$\Sigma < 0.12$ eV



$0\nu\beta\beta$ decay isotopes and experiments

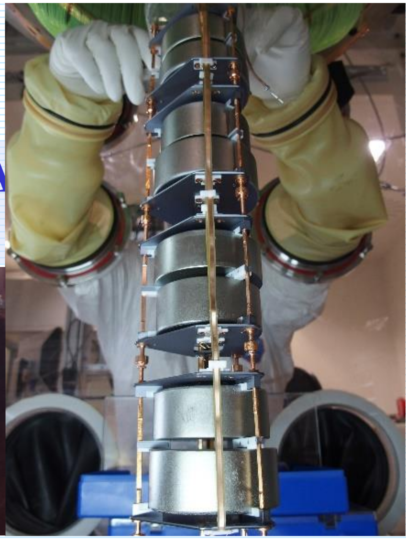
Candidates	$Q_{\beta\beta}$(MeV)	N.A. (%)
$^{48}Ca\rightarrow^{48}Ti$	**4.268**	**0.187**
$^{76}Ge\rightarrow^{76}Se$	**2.039**	**7.8**
$^{82}Se\rightarrow^{82}Kr$	**2.998**	**8.8**
$^{96}Zr\rightarrow^{96}Mo$	**3.356**	**2.8**
$^{100}Mo\rightarrow^{100}Ru$	**3.034**	**9.7**
$^{110}Pd\rightarrow^{110}Cd$	**2.017**	**11.7**
$^{116}Cd\rightarrow^{116}Sn$	**2.813**	**7.5**
$^{124}Sn\rightarrow^{124}Te$	**2.293**	**5.8**
$^{130}Te\rightarrow^{130}Xe$	**2.528**	**34.1**
$^{136}Xe\rightarrow^{136}Ba$	**2.458**	**8.9**
$^{150}Nd\rightarrow^{150}Sm$	**3.371**	**5.6**

【Current CANDLES detector】

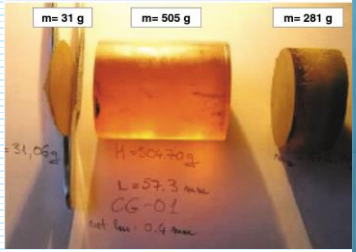
CANDLE
CaF scintillating crystal

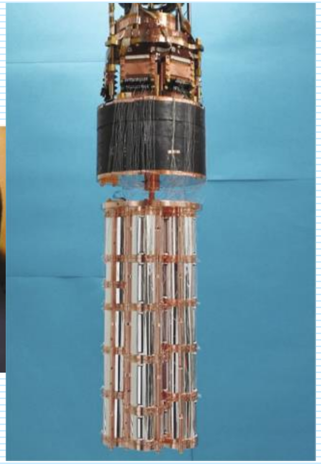
SuperNEMO
Se source foil

GERDA, MAJORANA
Ge crystal

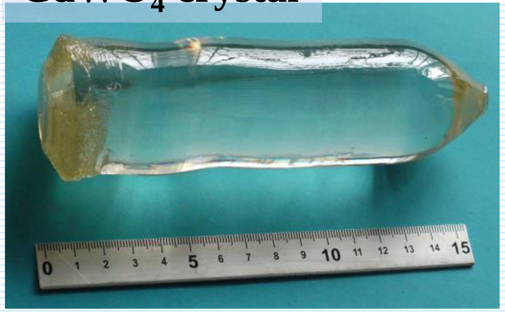


CUPID-0
ZnSe scintillating crystal

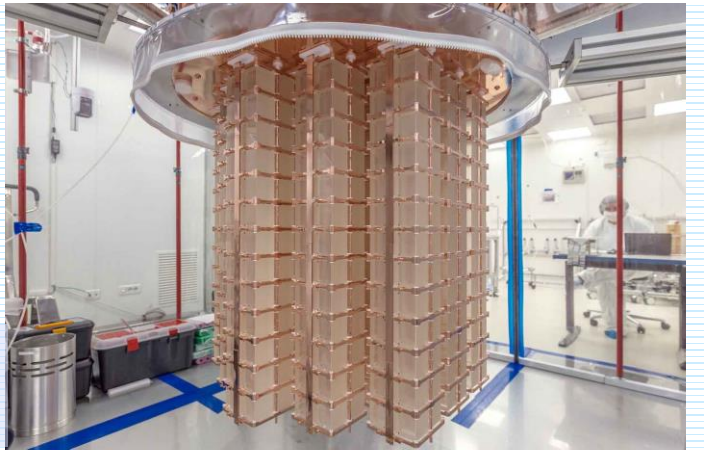




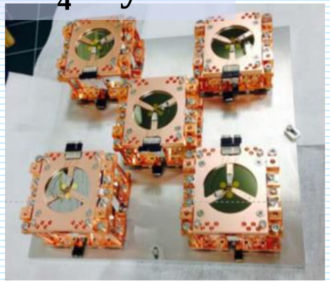
Aurora
CdWO$_4$ crystal



CUORE
TeO$_2$ crystal



Amore
CaMoO$_4$ crystal



EXO, KamLAND-Zen
Liquid Xe

Leading limits in each $0\nu\beta\beta$ isotope

A monoenergetic peak at the Q-value is searched for.
Need a large amount of decay isotope and low radioactive environment

Experiment	Isotope	Exposure [kg yr]	$T^{0\nu}_{1/2}[10^{25}$ yr]	$m_{\beta\beta}$ [meV]
Gerda	^{76}Ge	127.2	18	79-180
Majorana	^{76}Ge	26	2.7	200-433
CUPID-0	^{82}Se	5.29	0.47	276-570
NEMO3	^{100}Mo	34.3	0.15	620-1000
CUPID-Mo	^{100}Mo	2.71	0.18	280-490
Amore	^{100}Mo	111	0.095	1200-2100
CUORE	^{130}Te	1038.4	2.2	90-305
EXO-200	^{136}Xe	234.1	3.5	93-286
KamLAND-Zen	^{136}Xe	970	23	36-156

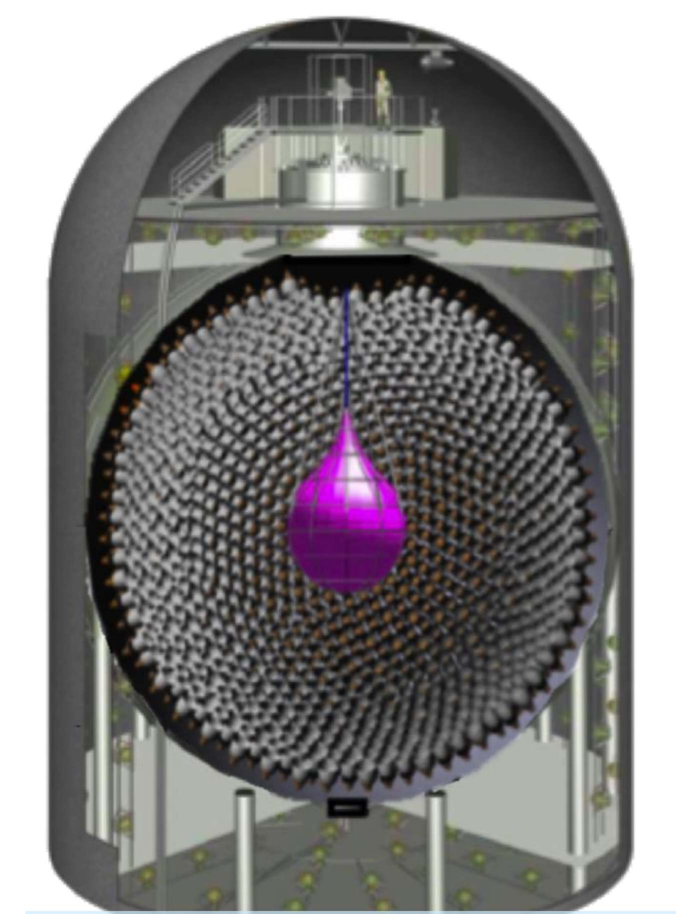
KamLAND-Zen
1 ton-class ^{136}Xe 0νββ experiment reaching IH region

KamLAND-Zen 400 and KamLAND-Zen 800 combined results

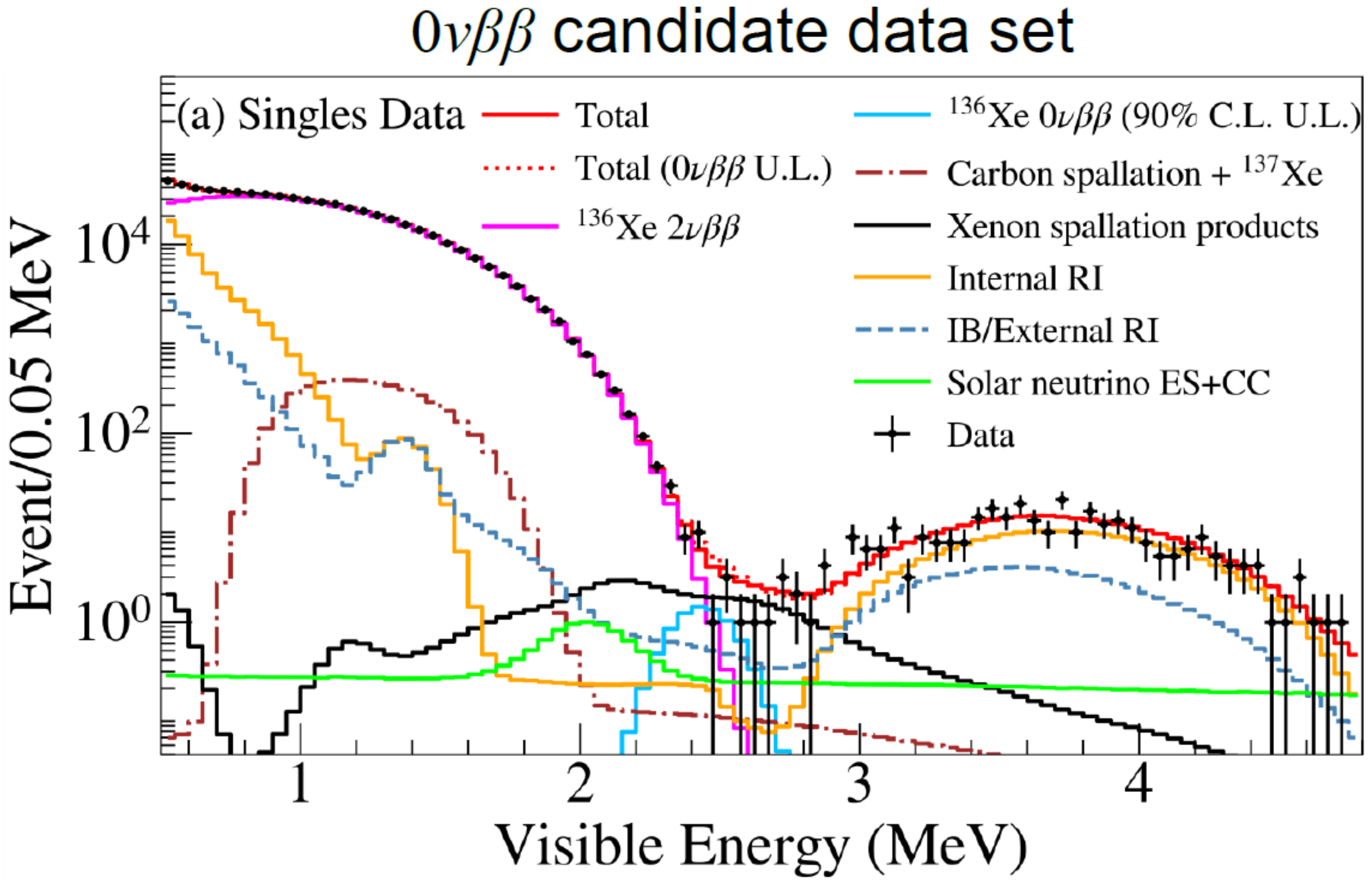
Limit: $T^{0\nu}_{1/2} > 2.3 \times 10^{26}$ year (90%C.L.), $m_{\beta\beta} < 36\text{–}156$ meV

(g_A=1.27, NME = 1.11-4.77 are assumed)

Currently, the most strict 0νββ limit



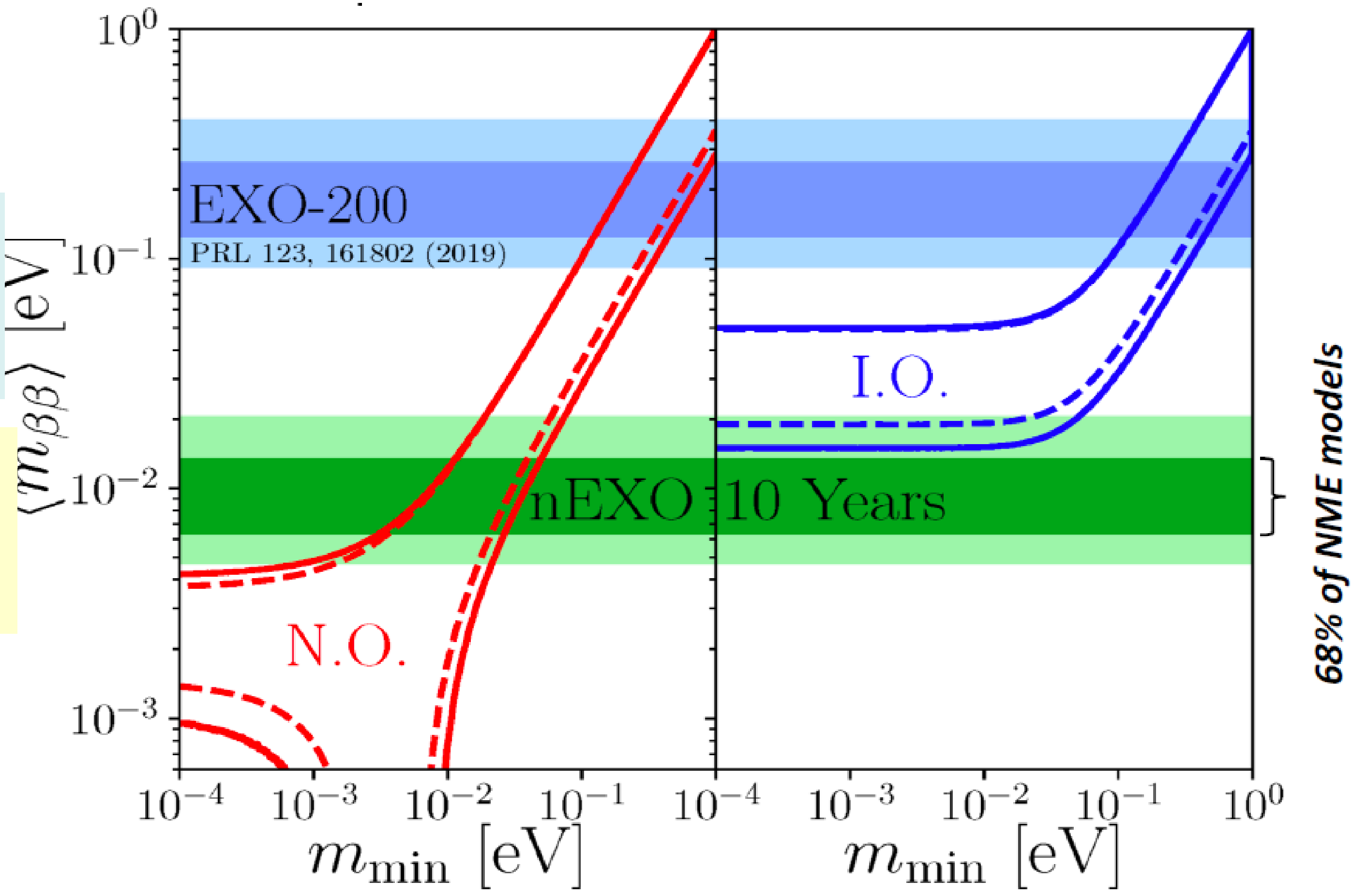
Large volume
liquid scintillator detector
LS: 30.5 m^3, ^{136}Xe: 677 kg

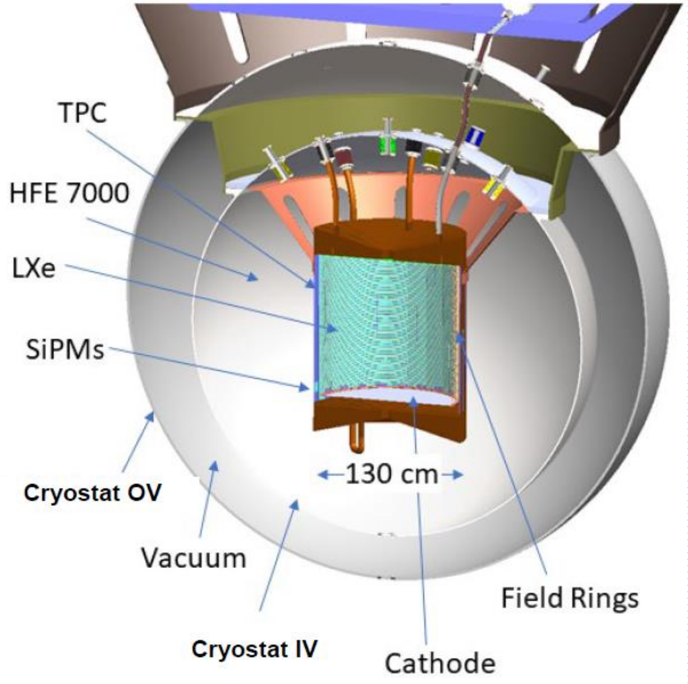

0νββ candidate data set

nEXO
5 ton-class ^{136}Xe 0νββ experiment

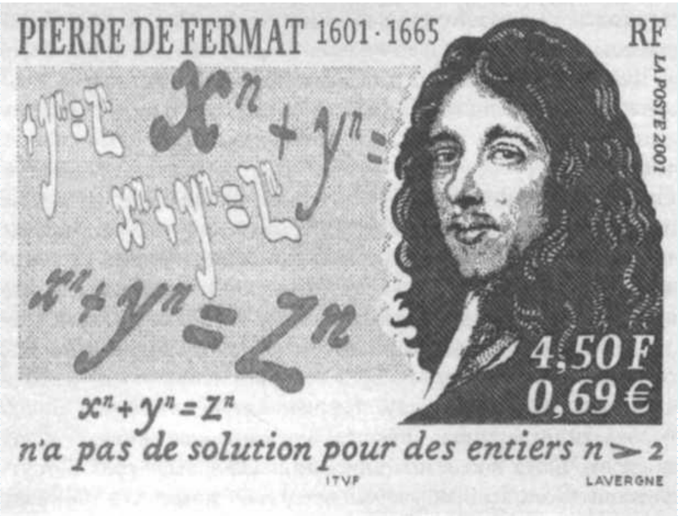
EXO-200, 1st 100 kg-class 0νββ-experiment, excellent background-essential for nEXO design, Sensitivity increased linearly with exposure.

nEXO, discovery 0νββ experiment, reaches sensitivity of **10^{28} yr** in **6.5 yr** data taking, probes $m_{\beta\beta}$ down to **15 meV**, scalable experiment.





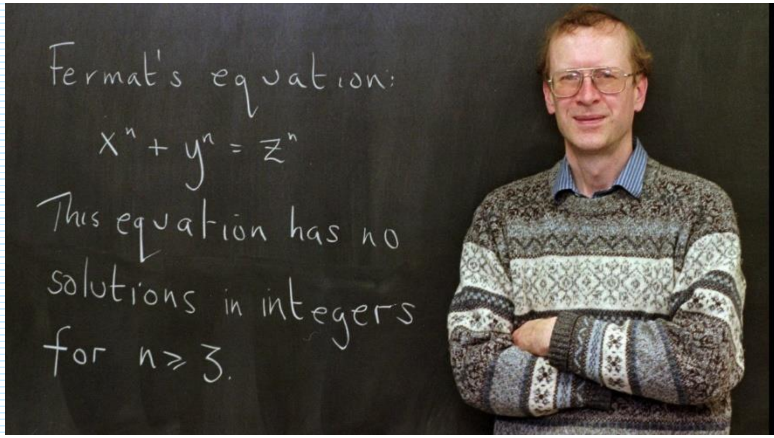
	isotope	$m_{\beta\beta}$ [meV] 90% excl. sensitivity	$m_{\beta\beta}$ [meV] 3 σ discovery potential
Legend	**^{76}Se**	**8.2**	**11.1**
CUPID	**^{100}Mo**	**11.1**	**12.0**
nEXO	**^{136}Xe**	**12.9**	**15.0**



Around 1637, Pierre de Fermat wrote
in the margin of a book that
the more general equation

$$a^n + b^n = c^n$$

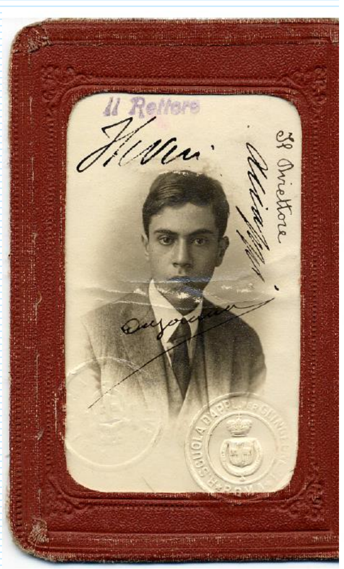
had no solutions in positive integers if n
is an integer greater than 2.



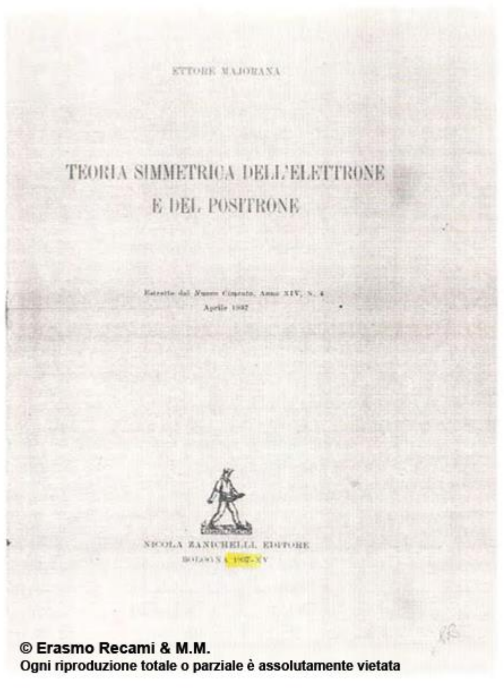
After **358** years

The proof was published by
Andrew Wiles in 1995.

Some long-standing tasks of humanity …



1937



ETTORE MAJORANA

TEORIA SIMMETRICA DELL'ELETTRONE
E DEL POSITRONE

Estratto dal Nuovo Cimento, Anno XIV, N. 4
Aprile 1937

NICOLA ZANICHELLI EDITORE
BOLOGNA 1937-XV

© Erasmo Recami & M.M.
Ogni riproduzione totale o parziale è assolutamente vietata

After
85
years

n-ton-class 0νββ exp.
with discovery potential
KamLAND-Zen 800
SNO+
LEGEND
nEXO
NEXT
CUPID
etc

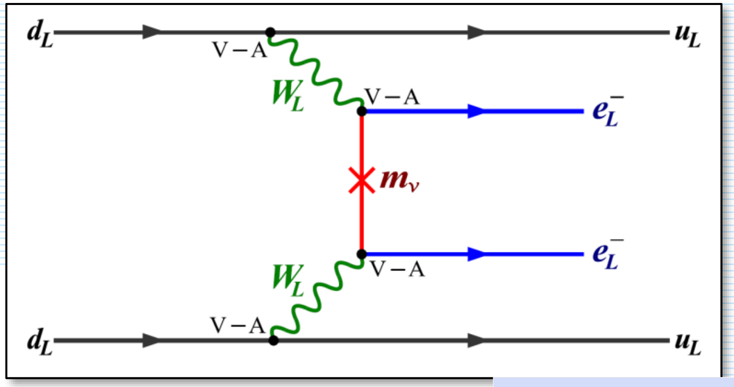
After
?
years

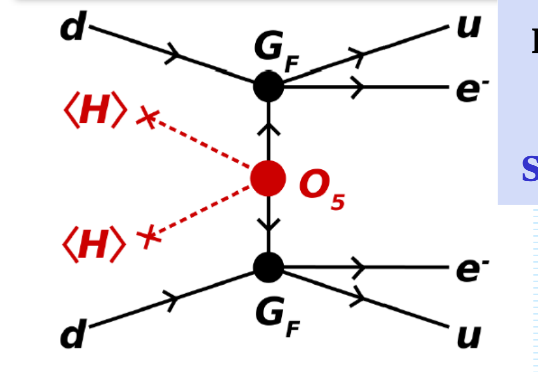
If $m_{\beta\beta} < 1$ meV,
what
technology is
needed for
observation
of 0νββ?

0νββ governed by exotic mechanisms

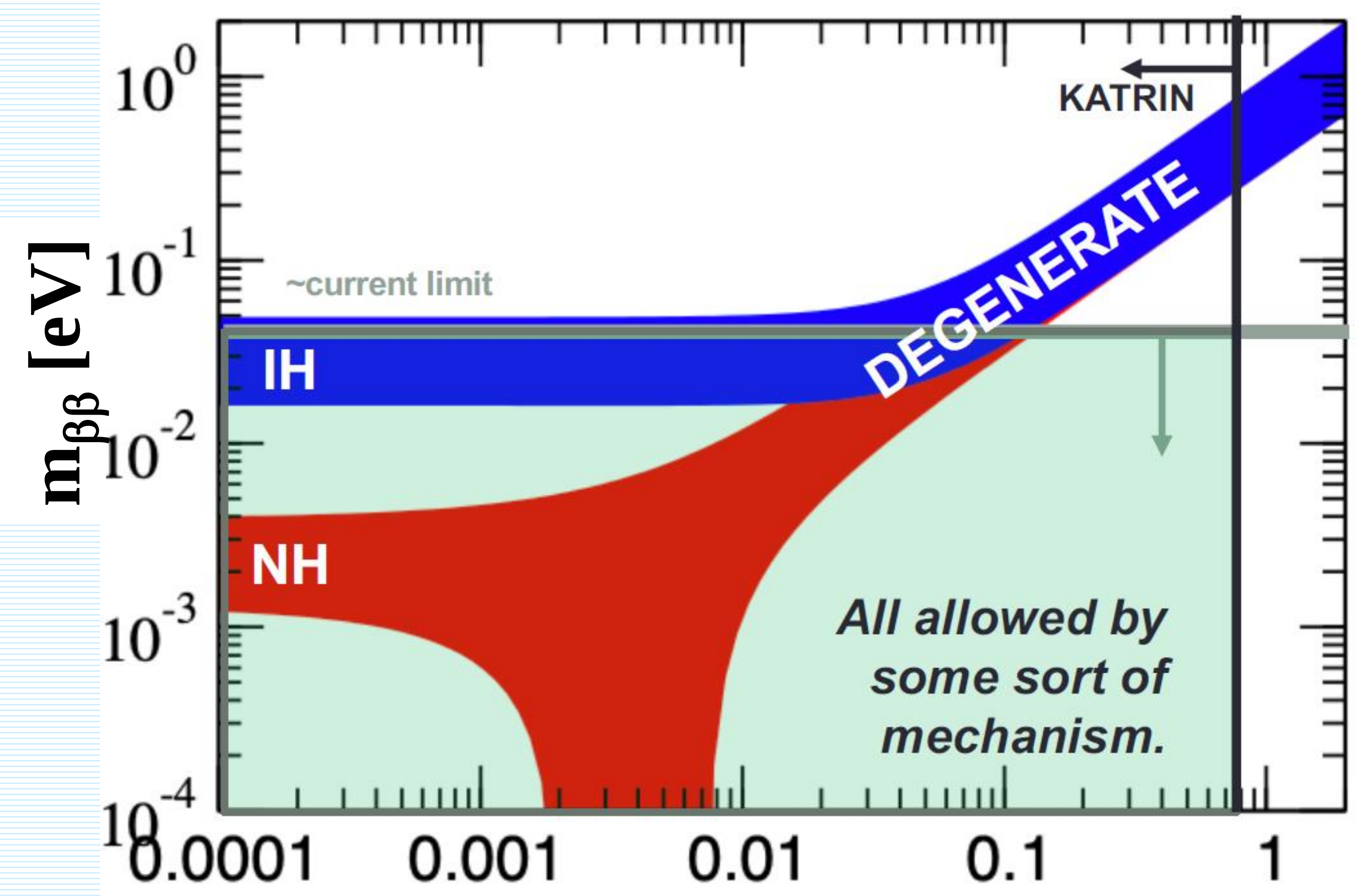
Light ν-mass mechanism can be strongly suppressed: **$m_{\beta\beta} < 1$ meV**

- It is not possible to discover 0νββ with **10-100 ton-class** experiment
- It should be a **subject of theory** to justify it
- There might be a dominance of other 0νββ mechanisms

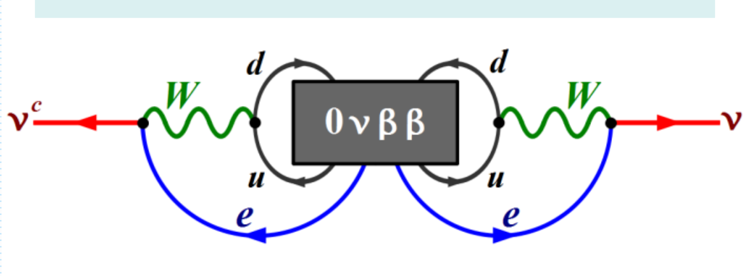




$m_{\beta\beta}$ mech. strongly suppressed



Any 0νββ mech. generates a small correction to ν-mass



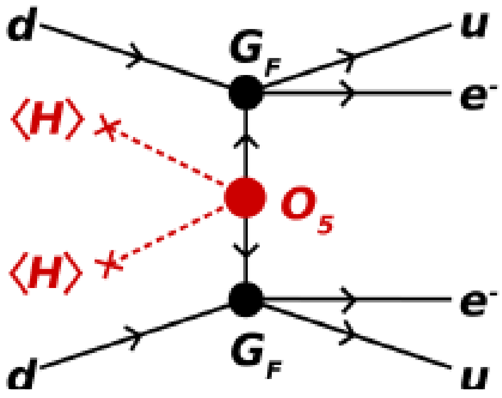
Schechter, Valle: PRD 1982

Beyond the SM physics

$$\mathcal{L} = \mathcal{L}_{SM}^{(4)} + \boxed{\frac{1}{\Lambda}\sum_i c_i^{(5)}\mathcal{O}_i^{(5)} + \frac{1}{\Lambda^2}\sum_i c_i^{(6)}\mathcal{O}_i^{(6)} + O(\frac{1}{\Lambda^3})}$$

Amplitude for
(A,Z)→(A,Z+2)+2e$^-$
can be divided into:

mass mechanism: d=5

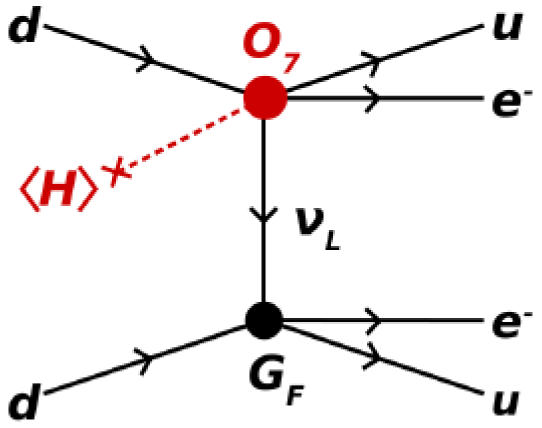


$$\mathcal{O}_W \propto \frac{c_{ij}}{\Lambda}(L_iH)(L_jH)$$

Weinberg, 1979

+

long range: d=7

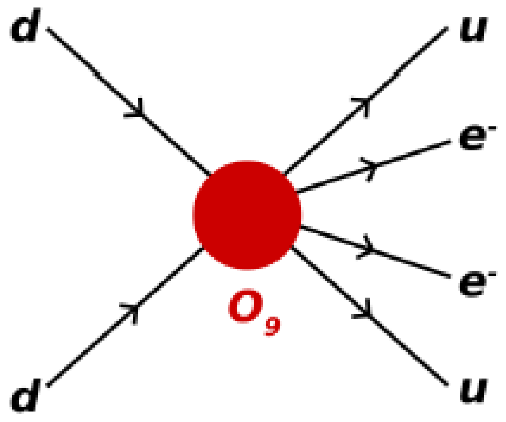


$$\mathcal{O}_2 \propto LLLe^cH$$
$$\mathcal{O}_3 \propto LLQd^cH$$
$$\mathcal{O}_4 \propto LL\bar{Q}\bar{u}^cH$$
$$\mathcal{O}_8 \propto L\bar{e}^c\bar{u}^cd^cH$$

Babu, Leung: 2001
de Gouvea, Jenkins: 2007

+

short range: d=9 (d=11)



$$\mathcal{O}_5 \propto LLQd^cHHH^\dagger$$
$$\mathcal{O}_6 \propto LL\bar{Q}\bar{u}^cHH^\dagger H$$
$$\mathcal{O}_7 \propto LQ\bar{e}^c\bar{Q}HHH^\dagger$$
$$\mathcal{O}_9 \propto LLLe^cLe^c$$
$$\mathcal{O}_{10} \propto LLLe^cQd^c$$
$$\mathcal{O}_{11} \propto LLQd^cQd^c$$

……

Valle

Quark Condensate Seesaw Mechanism for Neutrino Mass

PRD 103, 015007 (2021).

This operator contributes to the Majorana-neutrino mass matrix due to chiral symmetry breaking via the light-quark condensate.

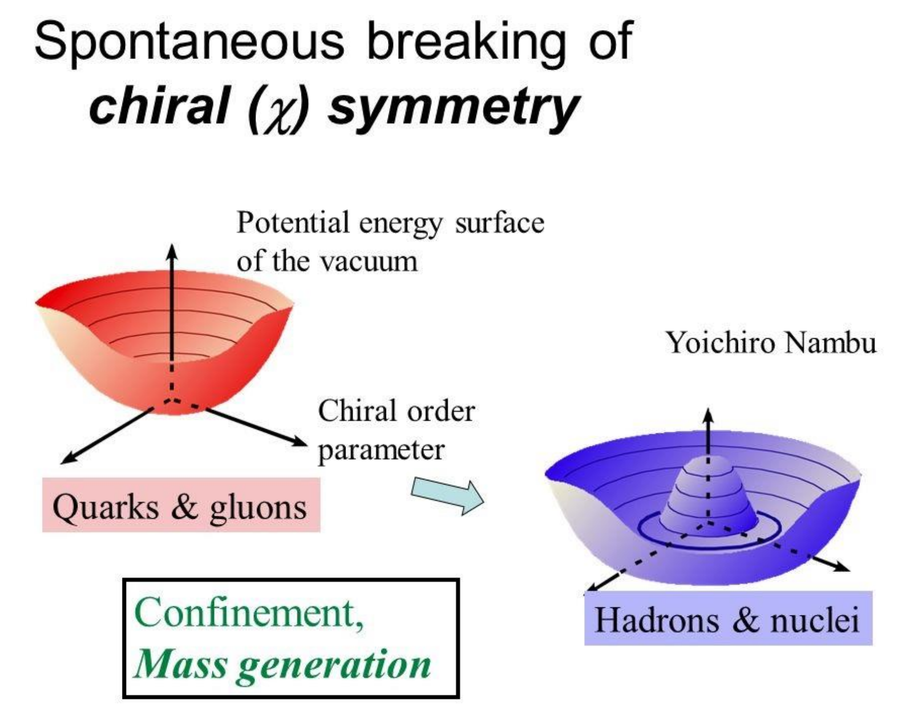
The SM gauge-invariant effective operators

$$\mathcal{O}_7^{u,d} = \frac{\tilde{g}^{u,d}_{\alpha\beta}}{\Lambda^3}\,\overline{L^C_\alpha}\,L_\beta\,H\,\left\{(\overline{Q}\,u_R),\ (\overline{d_R}\,Q)\right\}$$

After the EWSB and ChSB one arrives at the Majorana mass matrix of active neutrinos

$$\begin{aligned} {m_{\alpha\beta}}^{\nu} &= g_{\alpha\beta}\,v\,\frac{\langle\overline{q}q\rangle}{\Lambda^3} \\ &= g_{\alpha\beta}\,v\left(\frac{\omega}{\Lambda}\right)^3 \end{aligned}$$

$$\begin{aligned} g_{\alpha\beta} &= g^u_{\alpha\beta} + g^d_{\alpha\beta}, \quad v/\sqrt{2} = \langle H^0\rangle \\ \omega &= -\langle\overline{q}q\rangle^{1/3}, \quad \langle\overline{q}q\rangle^{1/3} \approx -283\,\mathrm{MeV} \end{aligned}$$



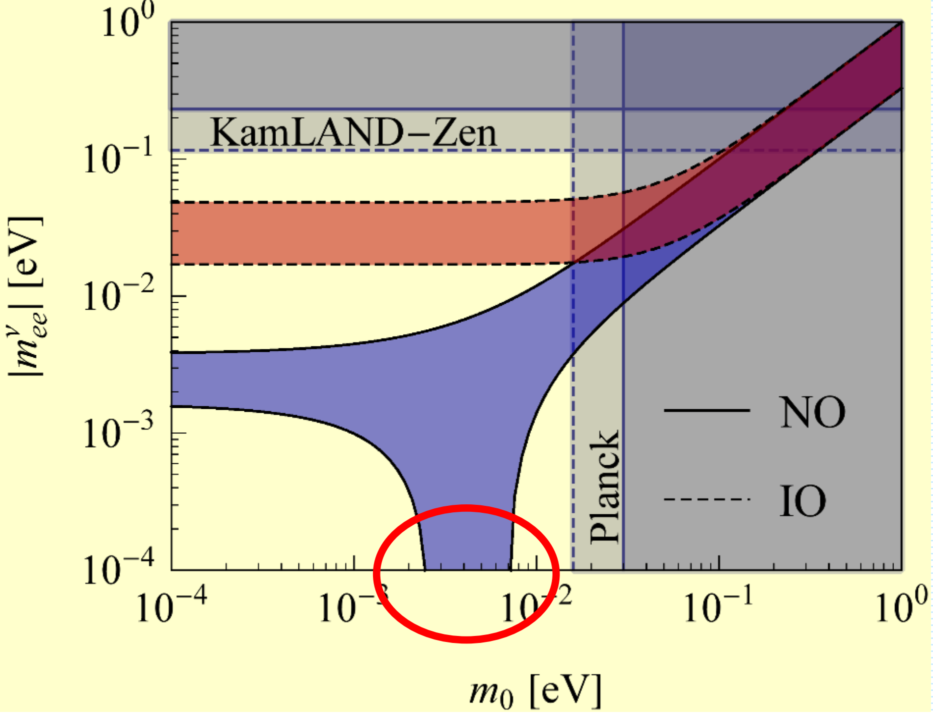
Λ ~ a few TeV
we get the neutrino mass in the sub-eV ballpark

The genuine QCSS scenario

(predicts NH and ν-mass spectrum)

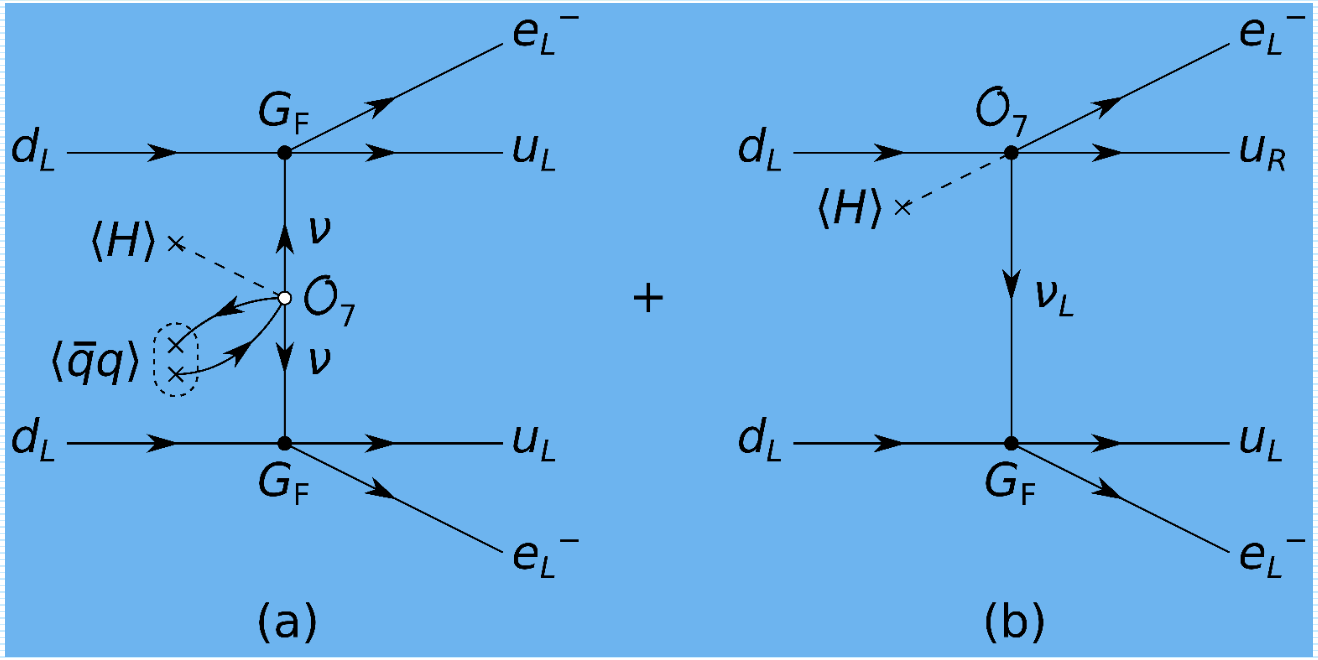
$$\mathcal{L}_7 = \frac{1}{\sqrt{2}}\sum_{\alpha\beta}\frac{v}{\Lambda^3}\overline{\nu^C_{\alpha L}}\nu_{\beta L}\left(g^u_{\alpha\beta}\overline{u_L}u_R + g^d_{\alpha\beta}\overline{d_R}d_L\right) + \text{H.c.}$$

$$m^\nu_{\alpha\beta} = -\frac{g_{\alpha\beta}}{\sqrt{2}}v\frac{\langle\bar{q}q\rangle}{\Lambda^3} = \frac{g_{\alpha\beta}}{\sqrt{2}}v\left(\frac{\omega}{\Lambda}\right)^3$$



(a) PRL 112, 142503 (2014).

(b) PLB 453, 194 (1999).



Neutrino spectrum (NH) !!!

2 meV < m_1 < 7 meV

9 meV < m_2 < 11 meV

50 meV < m_2 < 51 meV

Prediction for m_β

9 meV < m_b < 12 meV

Prediction for cosmology (Σ)

62 meV < $m_1 + m_2 + m_3$ < 69 meV

neutrino ↔ antineutrinos oscillations

governed by a related parameter

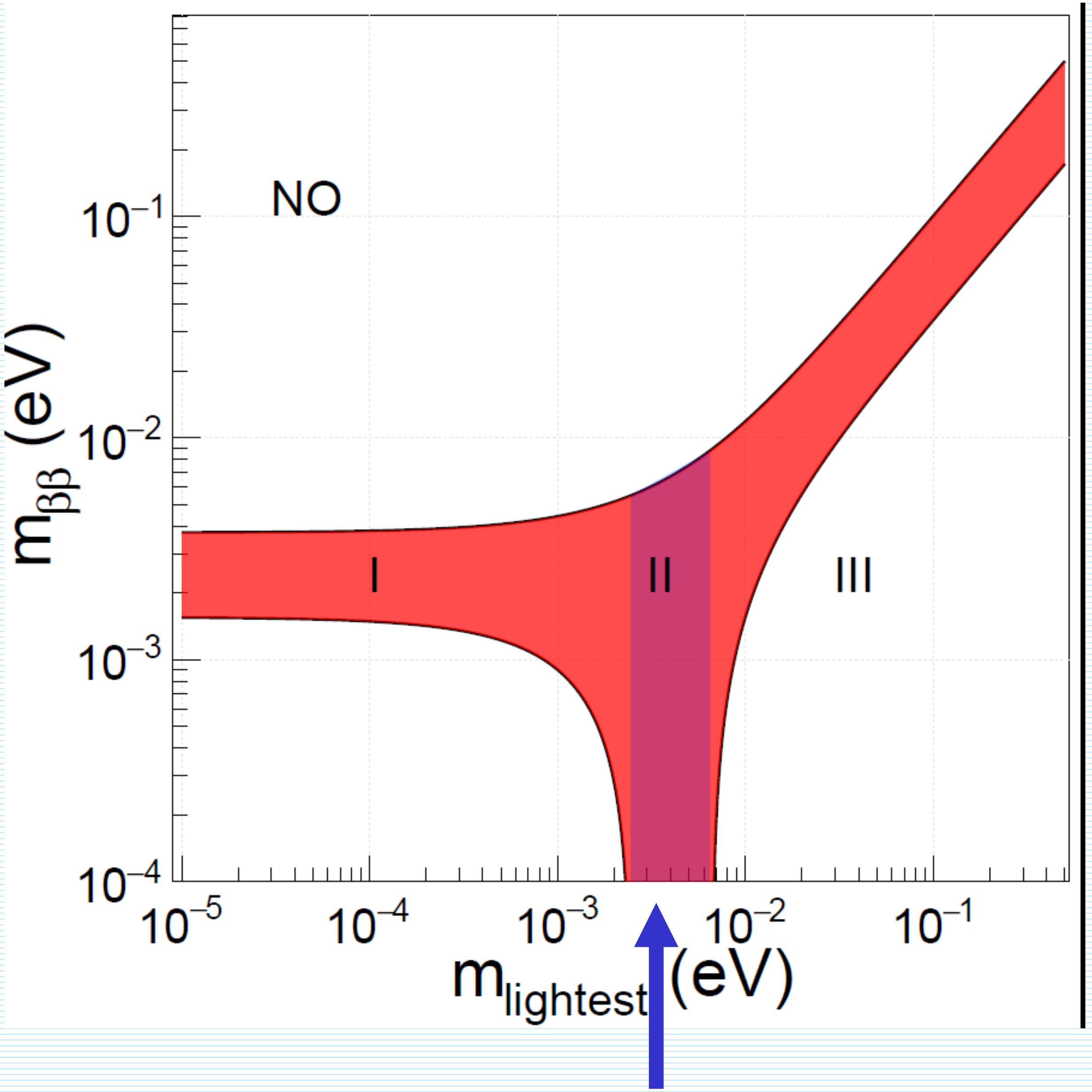
m^L_{ee} (L=0) = $m_{\beta\beta}$

$$A(\nu_\alpha \to \bar{\nu}_\gamma) = \sum_i [U^*{}_{\alpha i} U^*{}_{\gamma i}\ \frac{m_i}{E}\ exp(-i\frac{m_i^2}{2E}L)]\ \bar{K}$$

$$A(\bar{\nu}_\alpha \to \nu_\gamma) = \sum_i [U_{\alpha i} U_{\gamma i}\ \frac{m_i}{E}\ exp(-i\frac{m_i^2}{2E}L)]\ K$$

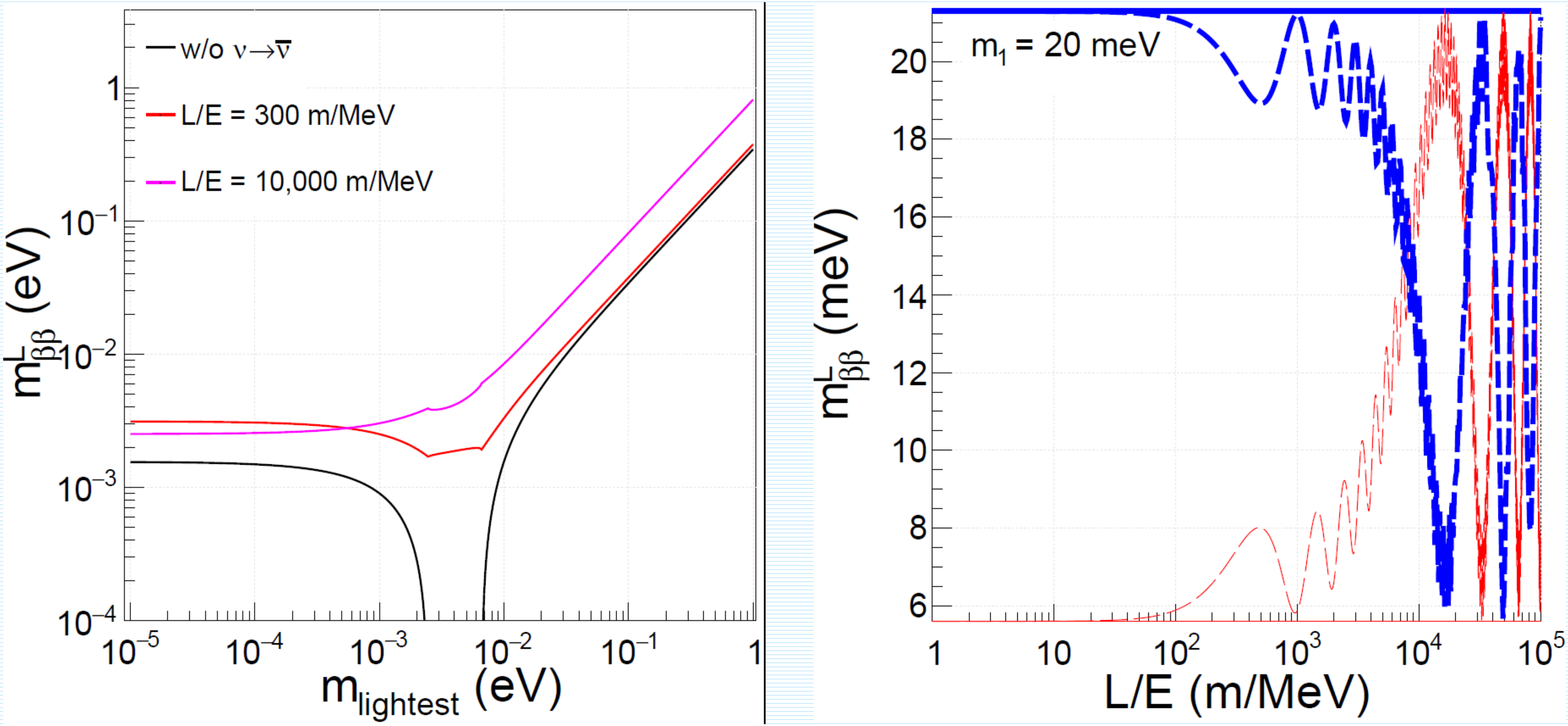
$$P(\bar{\nu}_\alpha \to \nu_\gamma) = \frac{|K|^2}{E^2}\ (m^L_{\alpha\gamma})^2$$

$$m^L_{\alpha\gamma} = \left|\sum_i U_{\alpha i} U_{\gamma i} m_i\ e^{-i\frac{m_i^2}{2E}L}\right|$$



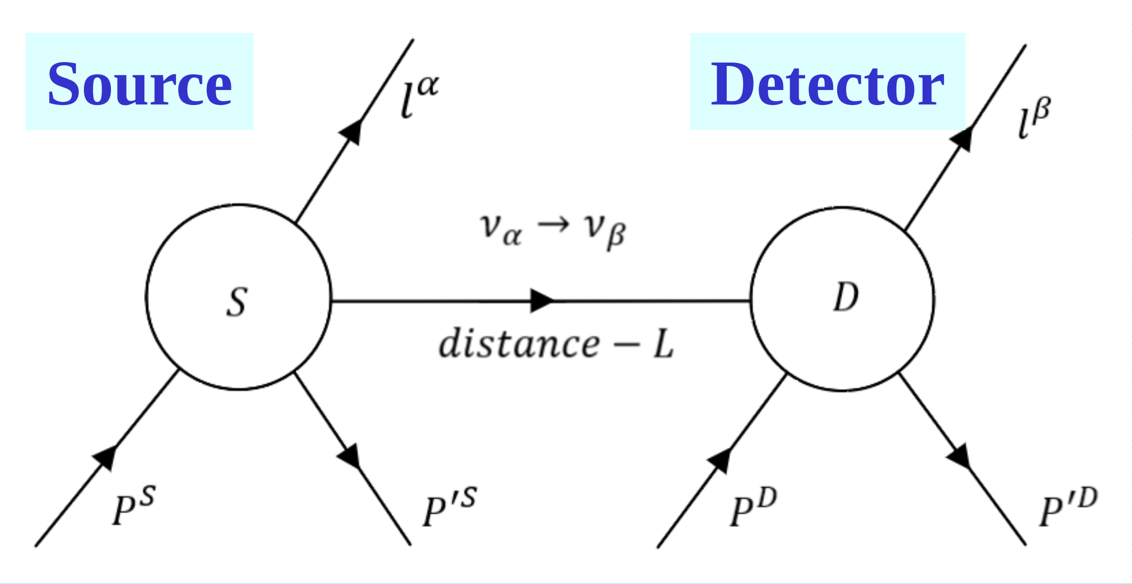
0νββ can be strongly suppressed (?!) …

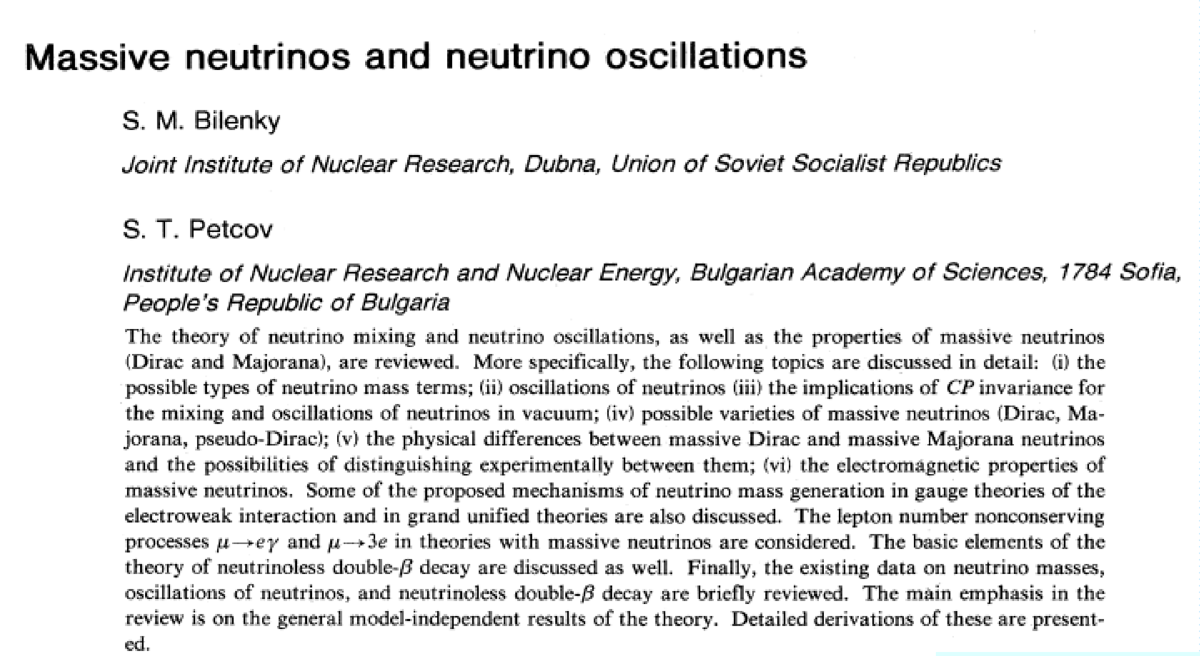
Dependence of m^L_{ee} on $m_{lightest}$ and L/E



Symmetry 14, 1383 (2022)

Neutrino oscillations
(Quantum Mechanics Approach)





Massive neutrinos and neutrino oscillations

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The theory of neutrino mixing and neutrino oscillations, as well as the properties of massive neutrinos (Dirac and Majorana), are reviewed. More specifically, the following topics are discussed in detail: (i) the possible types of neutrino mass terms; (ii) oscillations of neutrinos (iii) the implications of *CP* invariance for the mixing and oscillations of neutrinos in vacuum; (iv) possible varieties of massive neutrinos (Dirac, Majorana, pseudo-Dirac); (v) the physical differences between massive Dirac and massive Majorana neutrinos and the possibilities of distinguishing experimentally between them; (vi) the electromagnetic properties of massive neutrinos. Some of the proposed mechanisms of neutrino mass generation in gauge theories of the electroweak interaction and in grand unified theories are also discussed. The lepton number nonconserving processes $\mu \to e\gamma$ and $\mu \to 3e$ in theories with massive neutrinos are considered. The basic elements of the theory of neutrinoless double-β decay are discussed as well. Finally, the existing data on neutrino masses, oscillations of neutrinos, and neutrinoless double-β decay are briefly reviewed. The main emphasis in the review is on the general model-independent results of the theory. Detailed derivations of these are presented.

$$S \to S' + \ell^+_\alpha + \nu_\alpha$$

$$\nu_\alpha \to \nu_\beta$$

$$\nu_\beta + D \to D' + \ell^-_\beta$$

$$\Gamma_{osc} = \int \frac{d\Phi_\nu(E_\nu)}{dE_\nu} \, \frac{\mathcal{P}_{\alpha\beta}(E_\nu, L)}{4\pi L^2} \, \sigma(E_\nu) \, dE_\nu$$

Rev. Mod. Phys. 59, 671 (1987)
961 citations
(inspire hep)

Process is governed by the oscillation probability

$$S + D \to \ell^+_\alpha + \ell^-_\beta + S' + D'$$

$$\mathcal{P}_{\alpha\beta}(E_\nu, L) = \left| \sum_{j=1}^{3} U^*_{\alpha j} U_{\beta j} e^{-i \, m_j^2 \, L/(2E_\nu)} \right|^2$$

Neutrino oscillations

(within QFT, Walter Grimus approach revisited)

e-Print: *2212.13635* [hep-ph]

$$\langle f|S^{(2)}|i\rangle = -i\int d^4x_1 J^{\mu}_{S}(P'_S,P_S)e^{i(P_\alpha+P'_S-P_S)\cdot x_1}\times$$

$$\int d^4x_2 J^{\mu}_{D}(P'_D,P_D)e^{i(P_\beta+P'_D-P_D)\cdot x_2}\sum_{k=1}^{3}U^*_{\alpha k}U_{\beta k}\times$$

$$\overline{v}(P_\alpha;\lambda_\alpha)\gamma_\mu(1-\gamma_5)\ D(x_2-x_1,m_k)\ (1-\gamma_5)\gamma_\nu u(P_\beta;\lambda_\beta)$$

The neutrino emission and detection are identified with the charged-current vertices of a single second-order Feynman diagram for the underlying process, enclosing neutrino propagation between these two points.

$$D(x;m) = \theta(x_0)D^-(x;m) + \xcancel{\theta(-x_0)D^+(x;m)},$$

$$D^{\pm}(x;m) = \int \frac{d\mathbf{q}}{(2\pi)^3}\,\frac{\mp(-\mathbf{q}\cdot\vec{\gamma}+\omega\gamma^0)+m}{2\omega}e^{\pm i(-\mathbf{q}\cdot\mathbf{x}+\omega x_0)}$$

$$2\pi i\ \frac{\delta(E_\beta+E'_D-E_D+E_\alpha+E'_S-E_S)}{\omega+E_\alpha+E'_S-E_S+i\varepsilon}.$$

Neutrino propagation

$$\int \frac{d\mathbf{q}}{(2\pi)^3}\,\frac{\not{p}+m_k}{2\omega(\omega+E_\alpha+E'_S-E_S+i\varepsilon)}\,e^{i\mathbf{q}\cdot(\mathbf{x}_2-\mathbf{x}_1)}$$

$$\simeq \frac{1}{4\pi}\,\frac{e^{ip_k|\mathbf{x}_2-\mathbf{x}_1|}}{|\mathbf{x}_2-\mathbf{x}_1|}\,(\not{Q}_k+m_k)\simeq e^{i\mathbf{p}_k\cdot\mathbf{x}_D}\;e^{-i\mathbf{p}_k\cdot\mathbf{x}_S}\;\frac{e^{ip_kL}}{L}\,(\not{Q}_k+m_k)$$

$$Q_k\equiv(E_\nu,\mathbf{p}_k),\quad \mathbf{p}_k=p_k\,(\mathbf{x}_2-\mathbf{x}_1)\,/|\mathbf{x}_2-\mathbf{x}_1|,\quad p_k=\sqrt{E_\nu^2-m_k^2}$$

$$E_\nu=E_S-E'_S-E_\alpha\;(source)=E_\beta+E'_D-E_D\;(detector)$$

Energy conservation

Amplitude

Momentum conservation at source

Momentum conservation at detector

Energy conservation

$$\langle f|S^{(2)}|i\rangle = (2\pi)^7 \delta(E_f - E_i) \sum_k U_{\alpha k} U^*_{\beta k} \frac{e^{i p_k L}}{4\pi L} \times$$

$$T_k^{\alpha\beta}\ \delta^3_{V_S}(\mathbf{p}_k + \mathbf{p}_\alpha + \mathbf{p}'_S - \mathbf{p}_S)\ \delta^3_{V_D}(\mathbf{p}_\beta + \mathbf{p}'_D - \mathbf{p}_D - \mathbf{p}_k)$$

with

$$E_f - E_i = E_\beta + E'_D - E_D + E_\alpha + E'_S - E_S$$

$$T_k^{\alpha\beta} = J^\mu_S(P'_S, P_S) J^\nu_D(P'_D, P_D) \not{q}_k \gamma_\nu u(P_\beta; \lambda_\beta) \gamma_\nu u(P_\beta; \lambda_\beta)$$

Lepton current

Master formula

$$
\begin{aligned}
d\Gamma^{\alpha\beta}(L) = \sum_{km} U_{\alpha k}U^*_{\beta k}U_{\alpha m}U^*_{\beta m}\ \frac{e^{i(p_k-p_m)L}}{4\pi L^2} \times \mathcal{F}^{\alpha\beta}_{km} \\
\delta(\mathbf{p}_k + \mathbf{p}_\alpha + \mathbf{p}'_S - \mathbf{p}_S)\delta(\mathbf{p}_\beta + \mathbf{p}'_D - \mathbf{p}_D - \mathbf{p}_m) \\
\frac{(2\pi)^7}{4E_S E_D}\ \delta(E_\beta + E'_D - E_D + E_\alpha + E'_S - E_S)\times \\
\frac{1}{\hat{J}_S\hat{J}_D}\frac{d\mathbf{p}_\alpha}{2E_\alpha\ (2\pi)^3}\ \frac{d\mathbf{p}_\beta}{2E_\beta\ (2\pi)^3}\ \frac{d\mathbf{p}'_S}{2E'_S\ (2\pi)^3}\ \frac{d\mathbf{p}'_D}{2E'_D\ (2\pi)^3}
\end{aligned}
$$

with

$$
\mathcal{F}^{\alpha\beta}_{km} = 4\pi\sum_{\text{spin}}\frac{1}{2}\left(T^{\alpha\beta}_k\left(T^{\alpha\beta}_m\right)^* + T^{\alpha\beta}_m\left(T^{\alpha\beta}_k\right)^*\right)
$$

$$
\langle\Phi^{S,D}(\mathbf{P}_i)|\Phi^{S,D}(\mathbf{P}_k)\rangle = (2\pi)^3\ 2E_k\ \delta^3_{V_{S,D}}\ (\mathbf{P}_i - \mathbf{P}_k)
$$

Two normalization volumes:
i) source;
ii) Detector.

$$
\delta^3_V(\mathbf{Q}_n - \mathbf{P})\ \delta^3_V(\mathbf{Q}_m - \mathbf{P}) \simeq \frac{V}{(2\pi)^3}\ \frac{1}{2}\left(\delta^3_V(\mathbf{Q}_n - \mathbf{P}) + \delta^3_V(\mathbf{Q}_m - \mathbf{P})\right)
$$

An example:

e-Print: *2212.13635* [hep-ph]

$$\pi^+ + n \rightarrow \mu^+ + e^- + p$$

$$\pi^+ \rightarrow \mu^+ + \nu_\mu, \quad \nu_\mu \rightarrow \nu_e, \quad \nu_e + n \rightarrow p + e^-$$

Standard QM approach

$$\Gamma^{\pi^+ n}_{osc} = \int \frac{d\Phi_\nu(E'_\nu)}{dE'_\nu} \, \frac{\mathcal{P}_{\nu_\mu \nu_e}(E'_\nu)}{4\pi L^2} \, \sigma(E'_\nu) \, dE'_\nu$$

$$= \frac{1}{2\pi^2} \, G_\beta^2 \left(\frac{f_\pi}{\sqrt{2}}\right)^2 \frac{m_\mu^2}{m_\pi} \, E_\nu^2 \, \frac{P_{\nu_\mu \nu_e}(E_\nu)}{4\pi L^2} \, (g_V^2 + 3g_A^2) \, p_e E_e$$

with

$$\mathcal{P}_{\alpha\beta}(E_\nu, L) = \left| \sum_{j=1}^{3} U^*_{\alpha j} U_{\beta j} e^{-i m_j^2 L/(2E_\nu)} \right|^2$$

New QFT approach

$$\Gamma^{\pi^+ n}_{QFT} = \frac{1}{2\pi^2} \, G_\beta^2 \left(\frac{f_\pi}{\sqrt{2}}\right)^2 \frac{m_\mu^2}{m_\pi} \, E_\nu^2 \, \frac{\mathcal{P}^{QFT}_{\mu e}(E_\nu)}{4\pi L^2} \, (g_V^2 + 3g_A^2) \, p_e E_e$$

with

$$\mathcal{P}^{QFT}_{\alpha\beta}(E_\nu) = \frac{1}{2} \sum_{km} U_{\beta k} U^*_{\alpha k} \; U^*_{\beta k} U_{\alpha k} \; e^{i(p_m - p_k)L} \left(1 + \frac{p_k p_m}{E_\nu^2}\right)$$

Скъпи Сергей,
Честит Рожден ден 70 години!

I would like to wish you
strong health, happiness, and just to enjoy life and physics
to jubilee celebration!

You deserve much honor for all you have done in your 70 years.
Enjoy your special day.

Most cordially yours,
Fedor $+ \nu_e + \nu_\mu + \nu_\tau + (\nu_s)$