Optimizing the Higgs self-coupling measurement at ILC and C³

Second ECFA Workshop on e+e- Higgs/EW/Top Factories | 2023/10/12 | Paestum

Bryan Bliewert[1,2], Jenny List[1], Julie Munch Torndal[1,3],
Dimitris Ntounis[4], Caterina Vernieri [4], Junping Tian[5]

[1] DESY Hamburg
[2] Technische Universität München
[3] Universität Hamburg
[4] SLAC
[5] University of Tokio

HELMHOLTZ

Higgs self-coupling λ in the Standard Model (SM)

- Higgs-sector in SM after SSB: only one free parameter

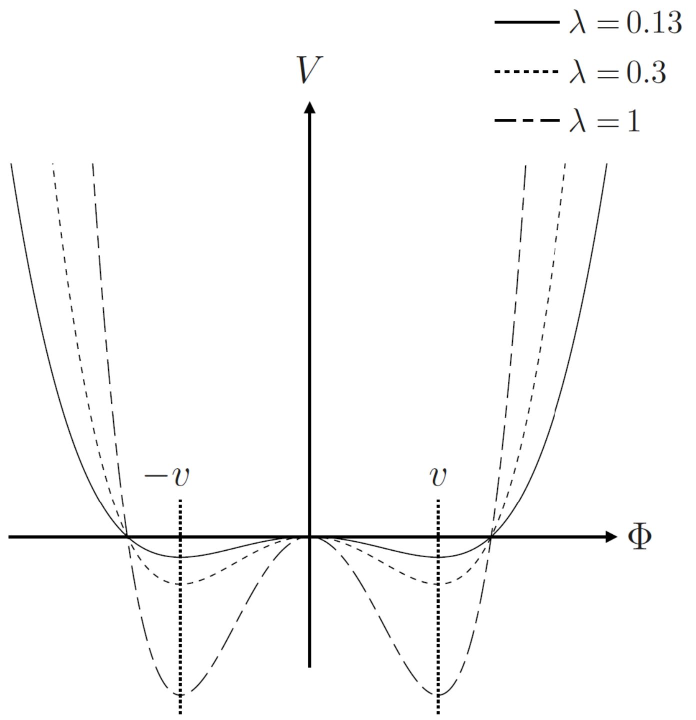
$$V(h) = \frac{1}{2} m_H^2 h^2 + \lambda_3 v h^3 + \frac{1}{4} \lambda_4 h^4$$

$$\frac{m_H^2}{2v^2} = \lambda_3^{SM} = \lambda_4^{SM}$$

- self-coupling λ defines shape of Higgs potential

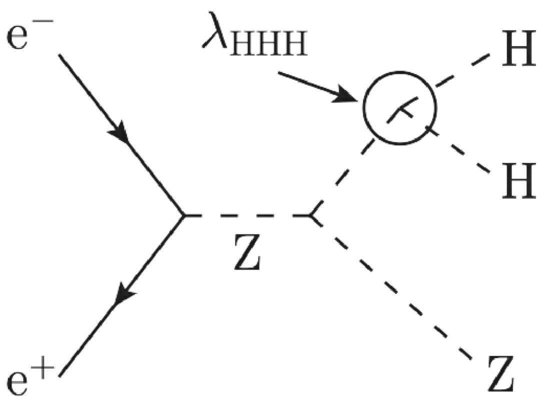
$$\lambda + \delta\lambda = \frac{m_H^2}{2v^2} \pm \frac{\delta m_H}{v^2} m_H \approx 0.13 \pm 10^{-3}$$

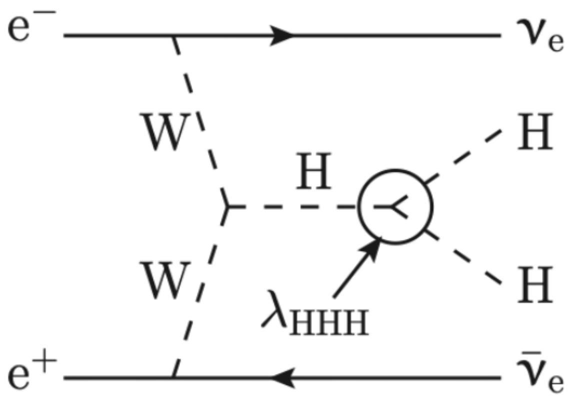
- sensitive to BSM physics by loop corrections

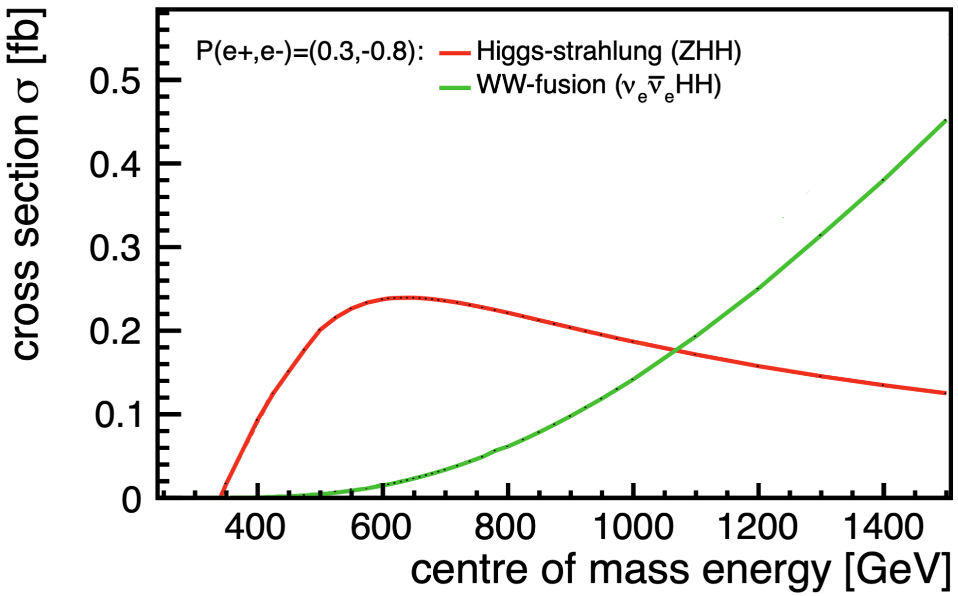


Measuring the Higgs self-coupling at e+e- colliders

- *direct access* to λ possible through double-Higgs production
  - Di-Higgs strahlung (dominant < 1 TeV)
  - vector boson fusion (dominant > 1 TeV)





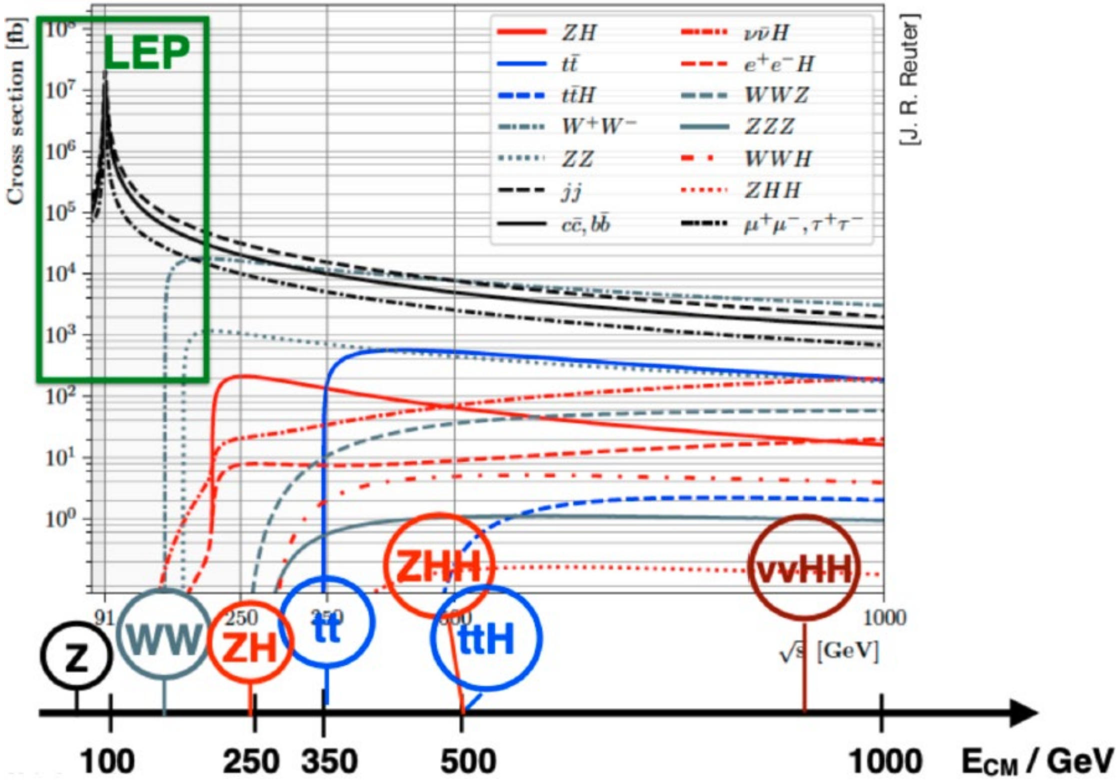


Ph.D. Thesis Dürig, DESY [2016]

Overview of future e^+e^- colliders

- different center-of-mass (COM) energies $\sqrt{s}/\mathrm{GeV}$ for physics programs

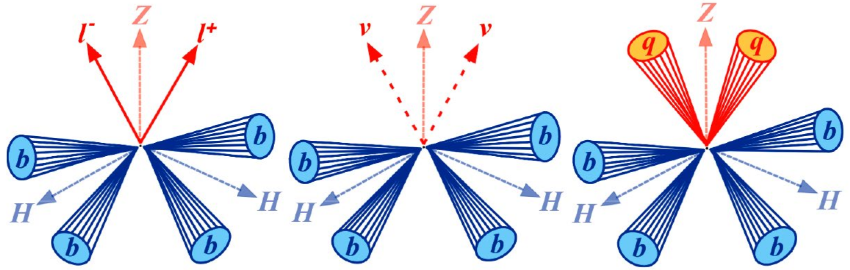
	Z	WW	ZH	$t\bar{t}$	ZHH	$\nu\nu HH$
ILC	–	–	250	350	500	1000
C³	–	–	250	–	550	–
CLIC	–	–	380	380	–	3000
FCC-ee	91	160	240	365	–	–
CEPC	91	160	240	360	–	–



- ILC/C³ designed to operate at peak of di-Higgs production by Higgs strahlung
  - direct measurement of λ

The ZHH analysis

- extensive projections at ILC with proposed $\sqrt{s} = 500\ \mathrm{GeV}$ (DESY-Thesis-2016-027)



- based on ILD detector concept (DBD2013, IDR2020)
- precision reach after running $4\mathrm{ab}^{-1}$ at $500\ \mathrm{GeV}$ $(\mathrm{HH} \rightarrow \mathrm{b\bar{b}b\bar{b}} + \mathrm{HH} \rightarrow \mathrm{b\bar{b}W^{\pm}W^{\mp}})$

  $^{\Delta\sigma_{\mathrm{ZHH}}}/_{\sigma_{\mathrm{ZHH}}} = 16.8\%$

  $^{\Delta\lambda_{\mathrm{SM}}}/_{\lambda_{\mathrm{SM}}} = 26.6\%$

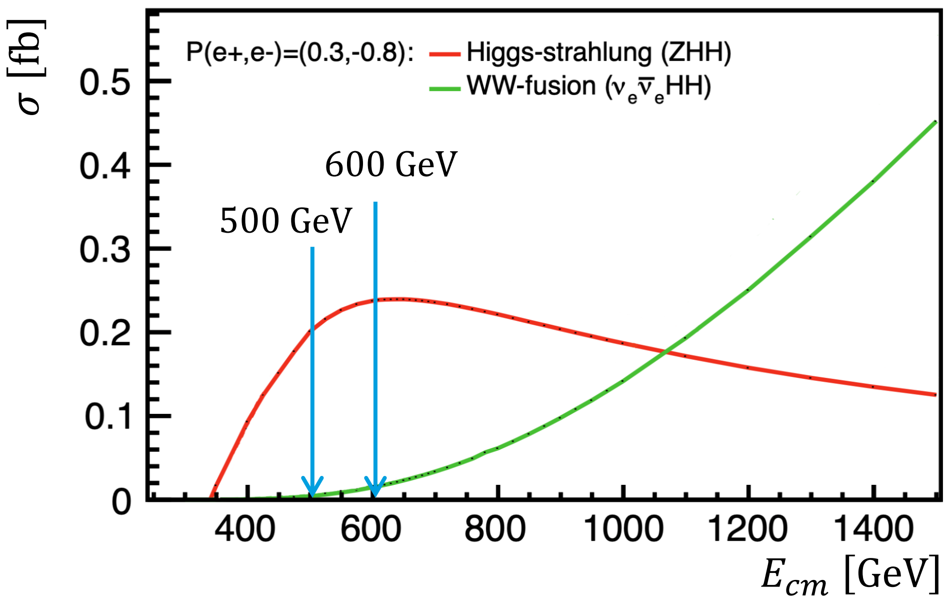
  $^{\Delta\lambda_{\mathrm{SM}}}/_{\lambda_{\mathrm{SM}}} = 10\%$ with additional upgrade to $1\ \mathrm{TeV}$

- **better than 20% sensitivity** expected with state-of-the-art reconstruction tools
- open questions:
  - How do better reconstruction tools improve the sensitivity to the Higgs self-coupling?
  - What’s the quantitative effect of increasing the COM-energy?

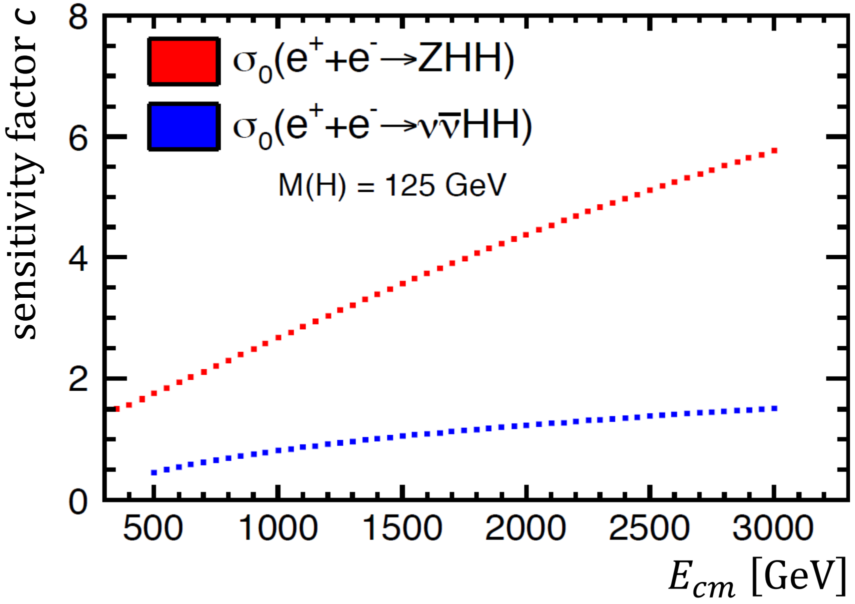
Choice of COM energy

- higher $\sqrt{s}$ → higher ZHH cross section **but** increasing uncertainty on self-coupling

Ph.D. Thesis Dürig, DESY [2016]



Ph.D. Thesis Dürig, DESY [2016]



Advantages of higher COM energies

- more boosted jets
  - better clustering, better jet-pairing?
  - improved b-tagging efficiencies?
  - better separation between signal and background?

Disadvantages of higher COM energies

- sensitivity factor c on self-coupling λ increases with E_{CM}

$$\frac{\Delta\lambda}{\lambda} = c \cdot \frac{\Delta\sigma}{\sigma}$$

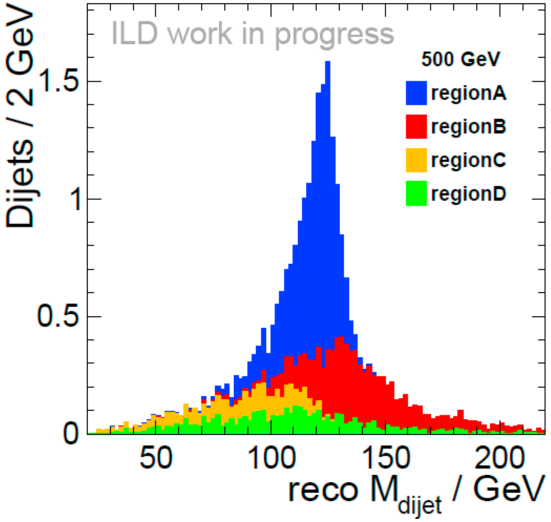
  - less sensitivity to λ?

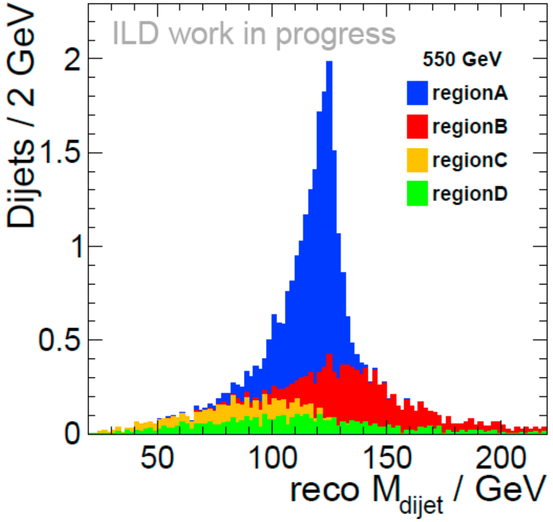
Jet clustering

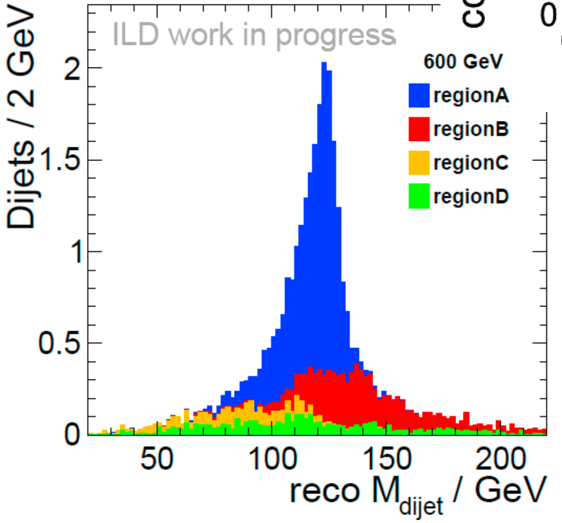
- ambigious clustering of jets and jet-matching
  → misclustering
- quantify with misclustering categories:
  - overlap fraction between true and reco energy

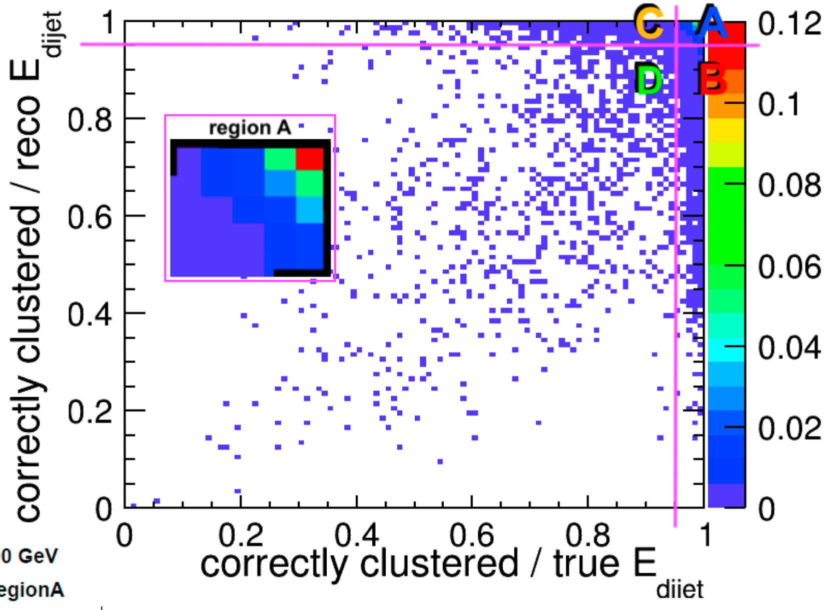
J. Torndal, J. List [2023]

Increasing energy









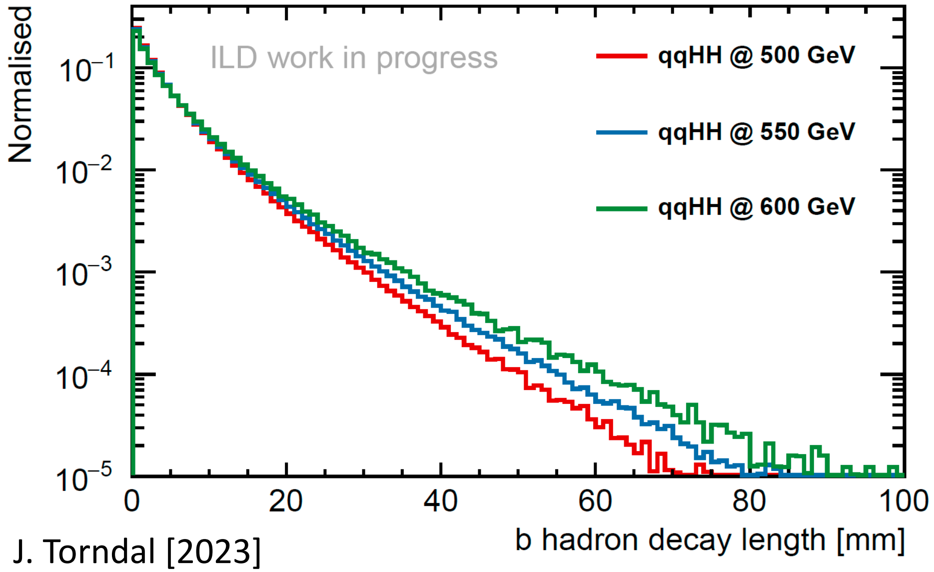
fraction of dijets in category A:

$\sqrt{s}$ [GeV]	A [%]
500	45.5
550	50.5
600	53.7

- WIP: investigating Graph-Neural-Networks (GNN) as promising alternative to Durham alg.

Flavor tagging

- improved b-tagging efficiency since state-of-the-art projections from 2016
  - 5% relative improvement in ϵ_{b-tag}
  - 11% expected improvement in $\Delta\sigma_{ZHH}/\sigma_{ZHH}$
- b-hadron decay length increases with COM energy



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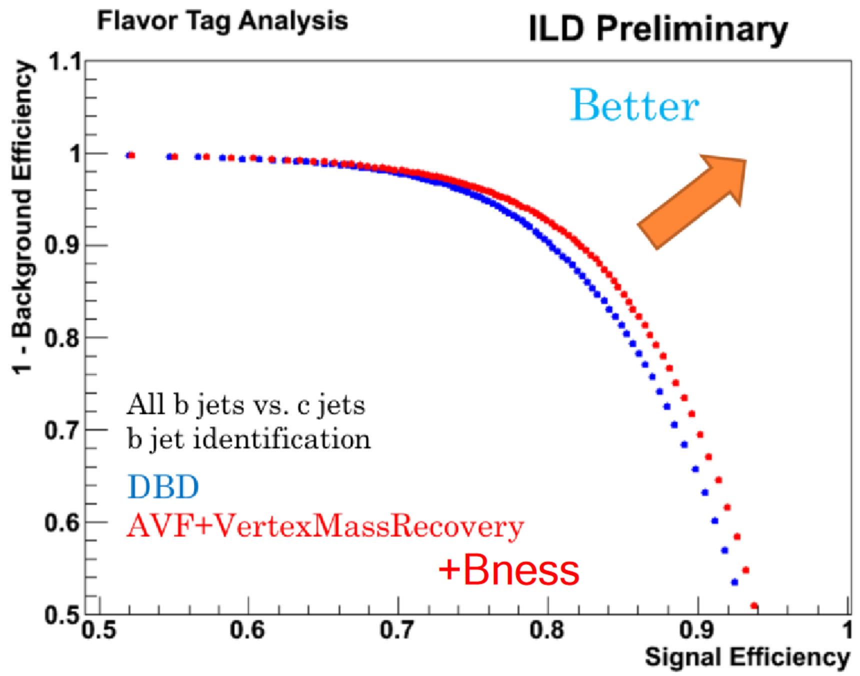


T. Suehara [2017]

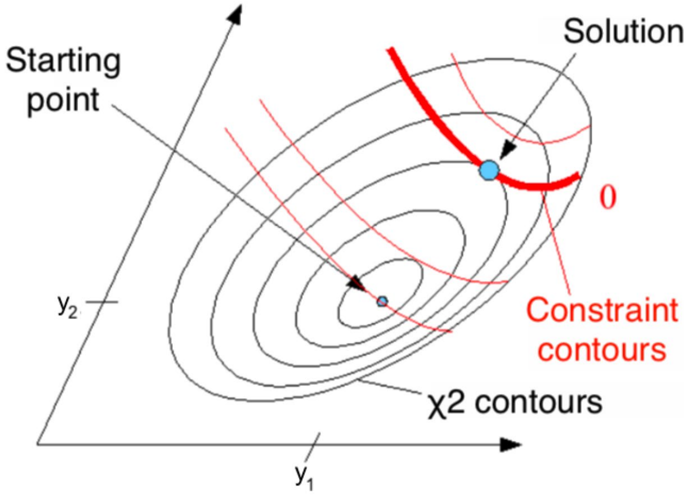
See also:
Talk by Taikan Suehara

Kinematic fitting

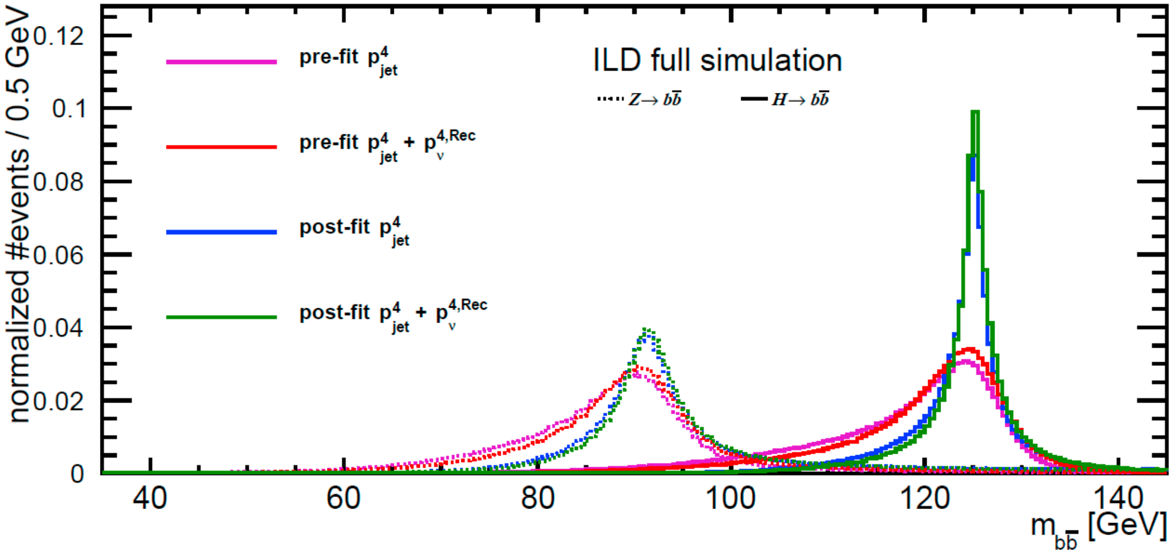
- exploit known initial state in e^+e^- colliders for
  - improving kinematics
  - hypothesis testing
  - jet-pairing
- based on method of Lagrange multipliers
- additionally: **ErrorFlow**
  - parametrizes sources of uncertainties for **individual** jets [Yasser Radkhorrami, 2023]

$$\sigma_{E_{jet}} = \sigma_{Det} \oplus \sigma_{Conf} \oplus \sigma_{\nu} \oplus \sigma_{Clus} \oplus \sigma_{Had} \oplus \sigma_{\gamma\gamma}$$

    - σ_{Det} detector resolution
    - σ_{Conf} particle confusion in Particle Flow algorithm
    - σ_{ν} neutrino correction



See also:
Talk by Leonhard Reichenbach
Poster by Jenny List



Jet pairing from kinematic fits

ZHH hypothesis

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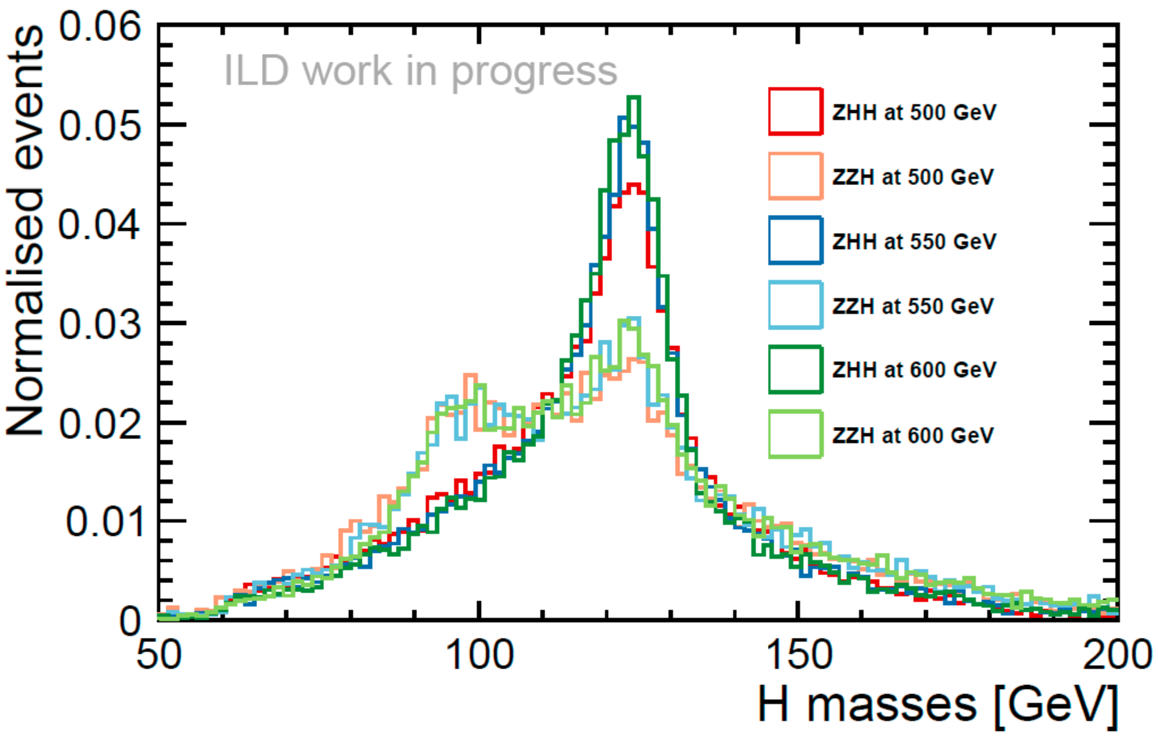

- assuming signal hypothesis: sharper dijet mass distributions for higher energies
  - however, also for ZZH events

ZZH hypothesis

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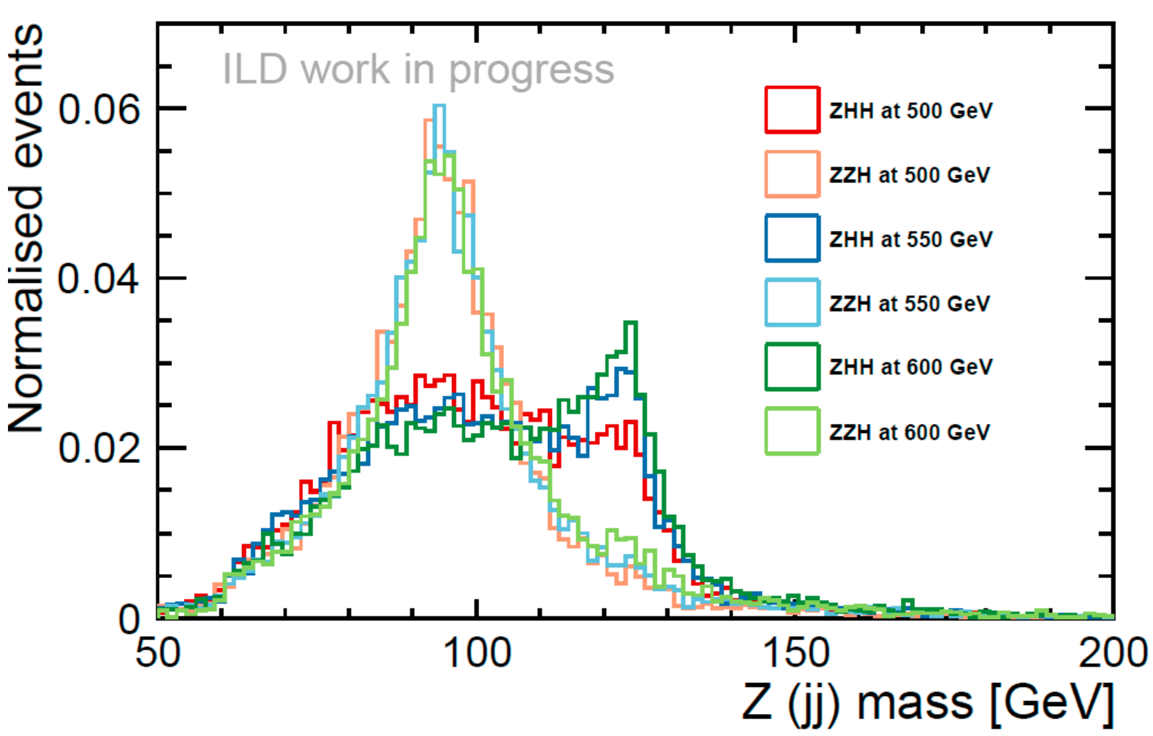
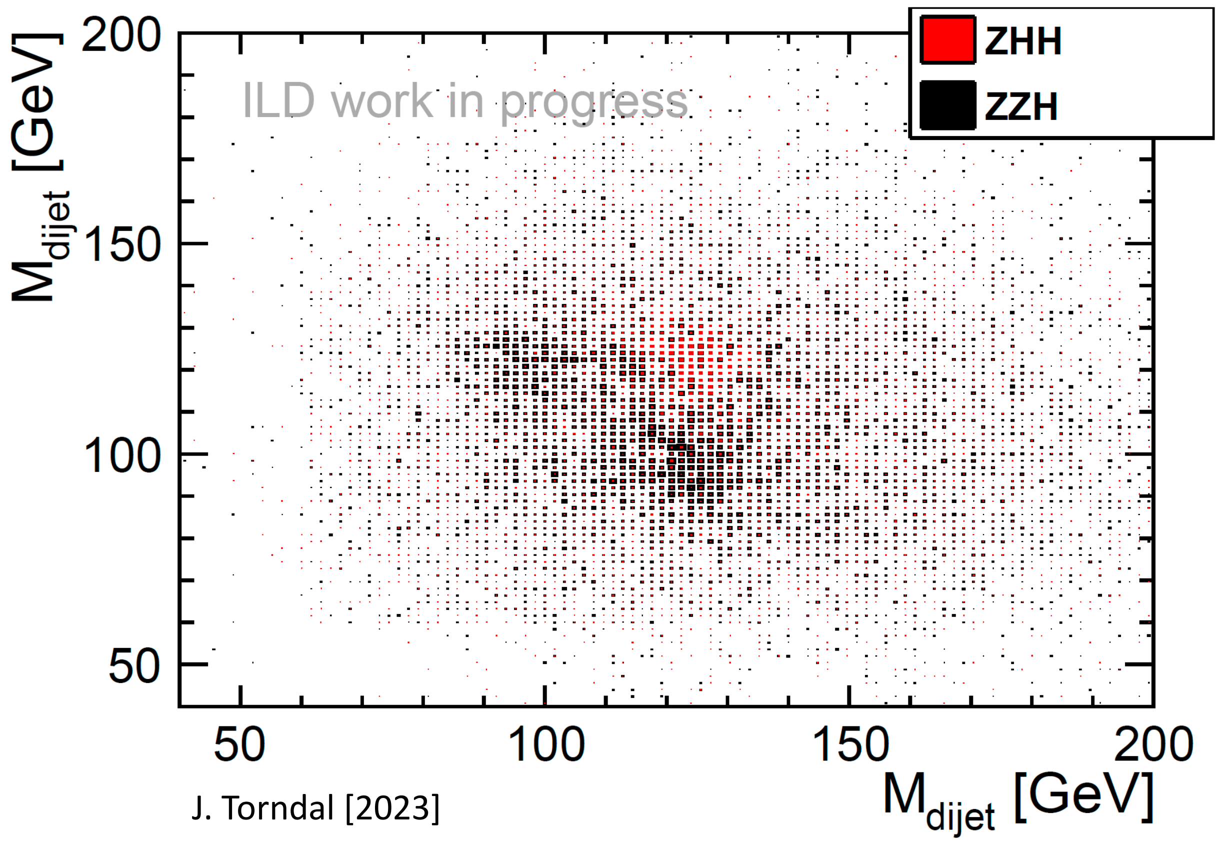


- applying background hypothesis, more ZHH events remain at m_H for higher energies

Hypothesis testing with kinematic fitting



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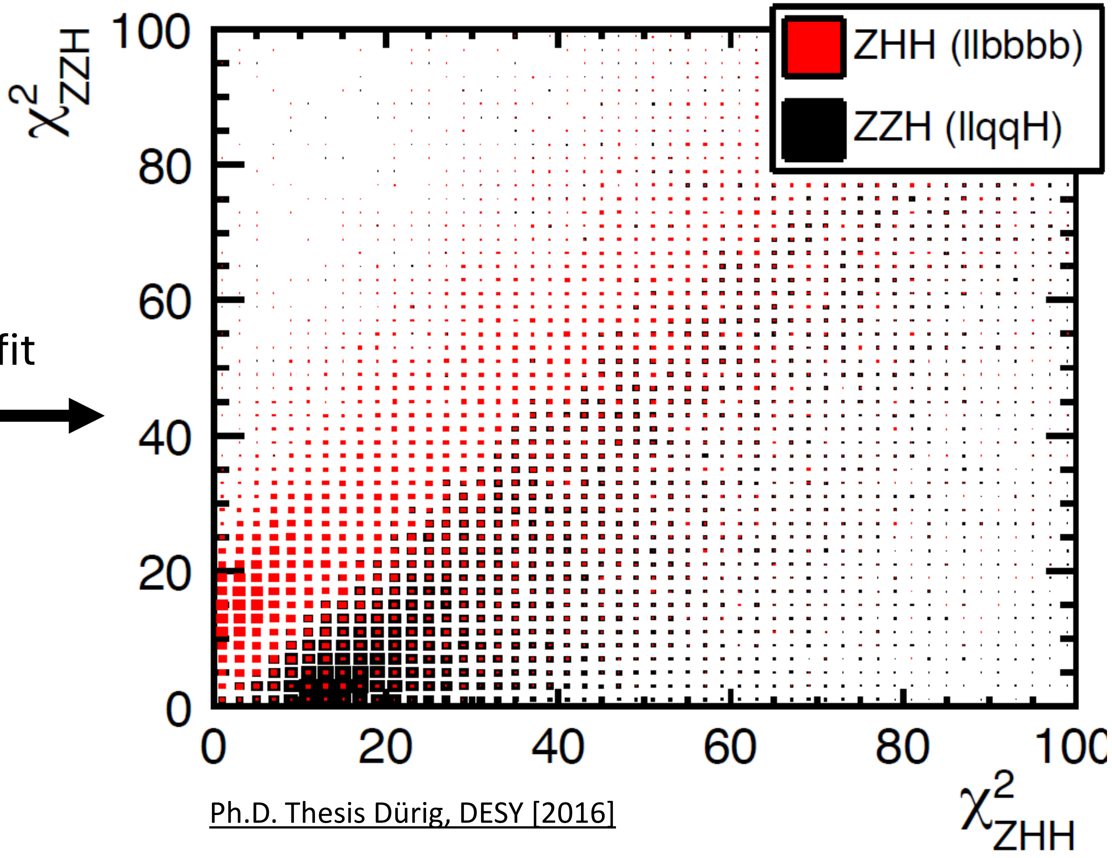
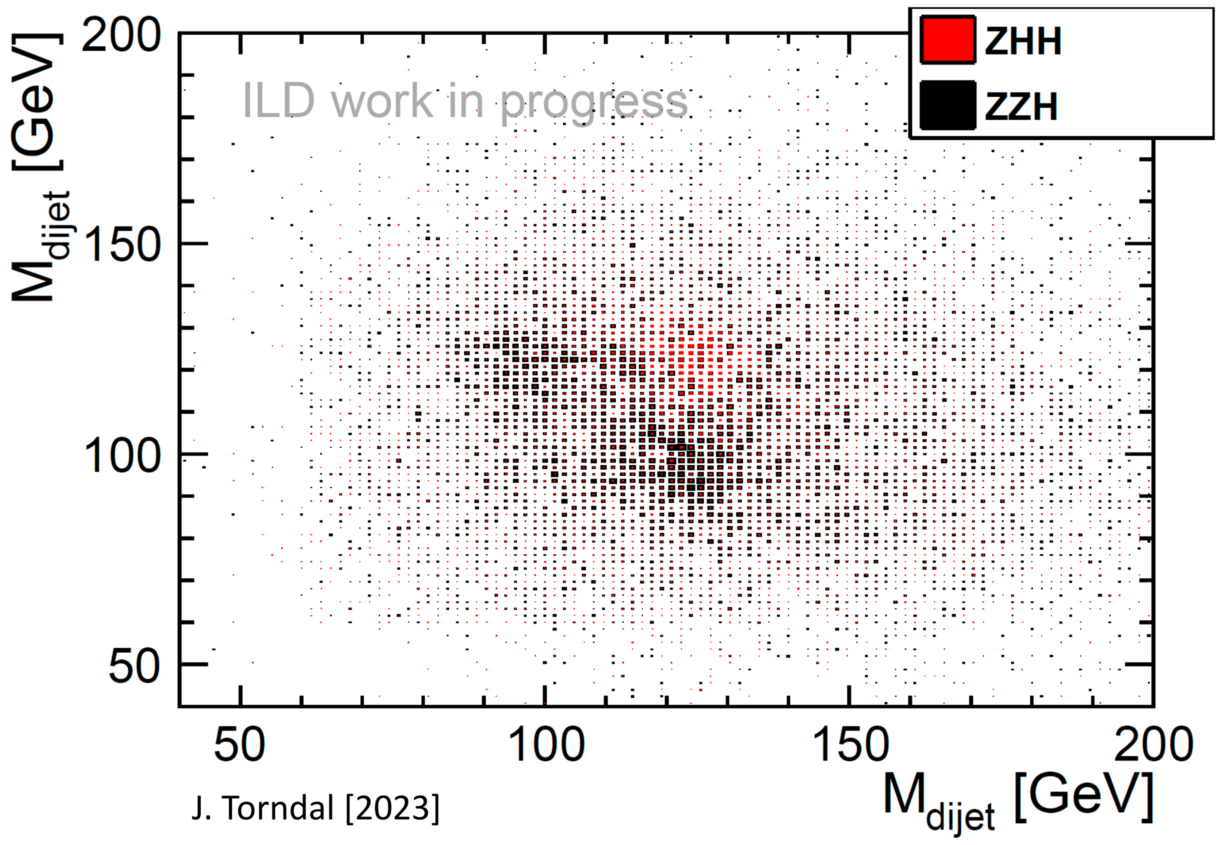
Kinfit

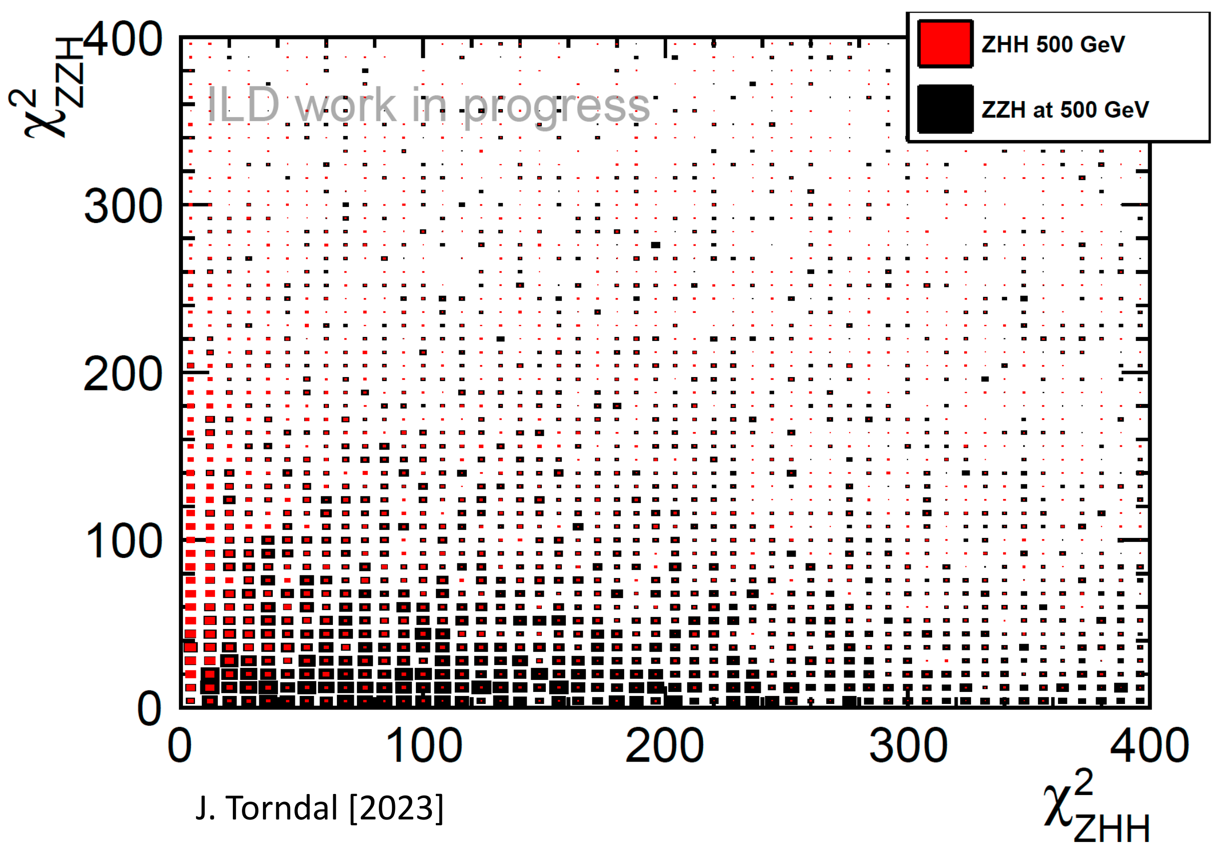


Ph.D. Thesis Dürig, DESY [2016]

- pre-fitted dijet-masses: large overlap between signal (ZHH) and background (ZZH)
- good separation for low χ^2-values of signal and background in previous analysis

Hypothesis testing with kinematic fitting





- pre-fitted dijet-masses: large overlap between signal (ZHH) and background (ZZH)
- with ErrorFlow: different separation of signal/background (WIP)

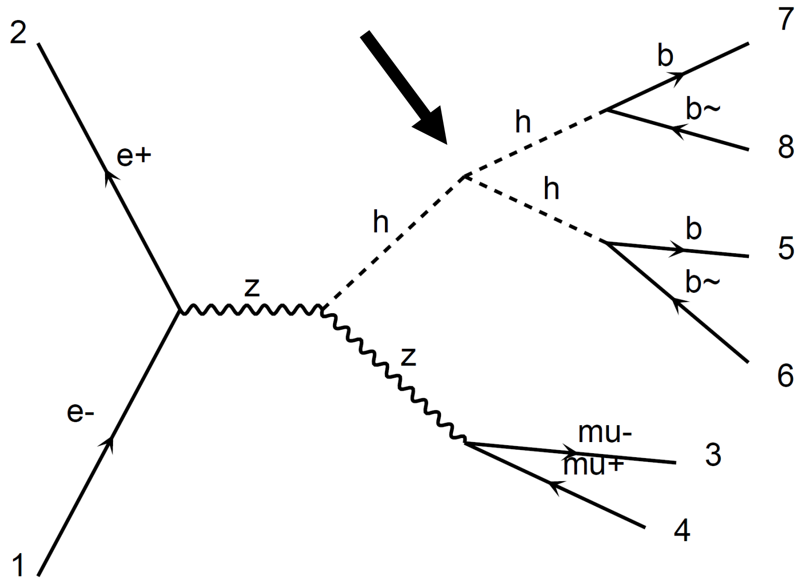
The Matrix Element Method (MEM)

- method for calculating event-likelihoods; use cases:
  - hypothesis testing (Neyman-Pearsson lemma)
  - parameter estimation
- example here: separate signal vs main irreducible background ZHH vs. ZZH $\rightarrow \mu^-\mu^+ b\bar{b}b\bar{b}$
- for each event $\boldsymbol{y}$ and process i (ZHH, ZZH), solve

$$P_i\,(\boldsymbol{y}\mid\boldsymbol{a}) = \frac{1}{\sigma_i(\boldsymbol{a})\cdot A_i(\boldsymbol{a})}\int |M_i(\boldsymbol{x},\boldsymbol{a})|^2 W_i(\boldsymbol{y}\mid\boldsymbol{x})\epsilon_i(\boldsymbol{x})d\boldsymbol{\Phi}_n(\boldsymbol{x})$$

  - $M_i(\boldsymbol{x},\boldsymbol{a})$ LO matrix element (**ME**): HELAS-based Physsim (J. Tian)
  - $W_i(\boldsymbol{y}|\boldsymbol{x})$ detector transfer functions (**DTFs**): probability density for measuring $\boldsymbol{y}$ given $\boldsymbol{x}$; fitted from ILD full-simulation
- discriminator: $D_{bkg}(\boldsymbol{y}) = \left(1 + \frac{P_{ZZH}(\boldsymbol{y})}{P_{ZHH}(\boldsymbol{y})}\right)$

$\boldsymbol{\lambda_{HHH}}$



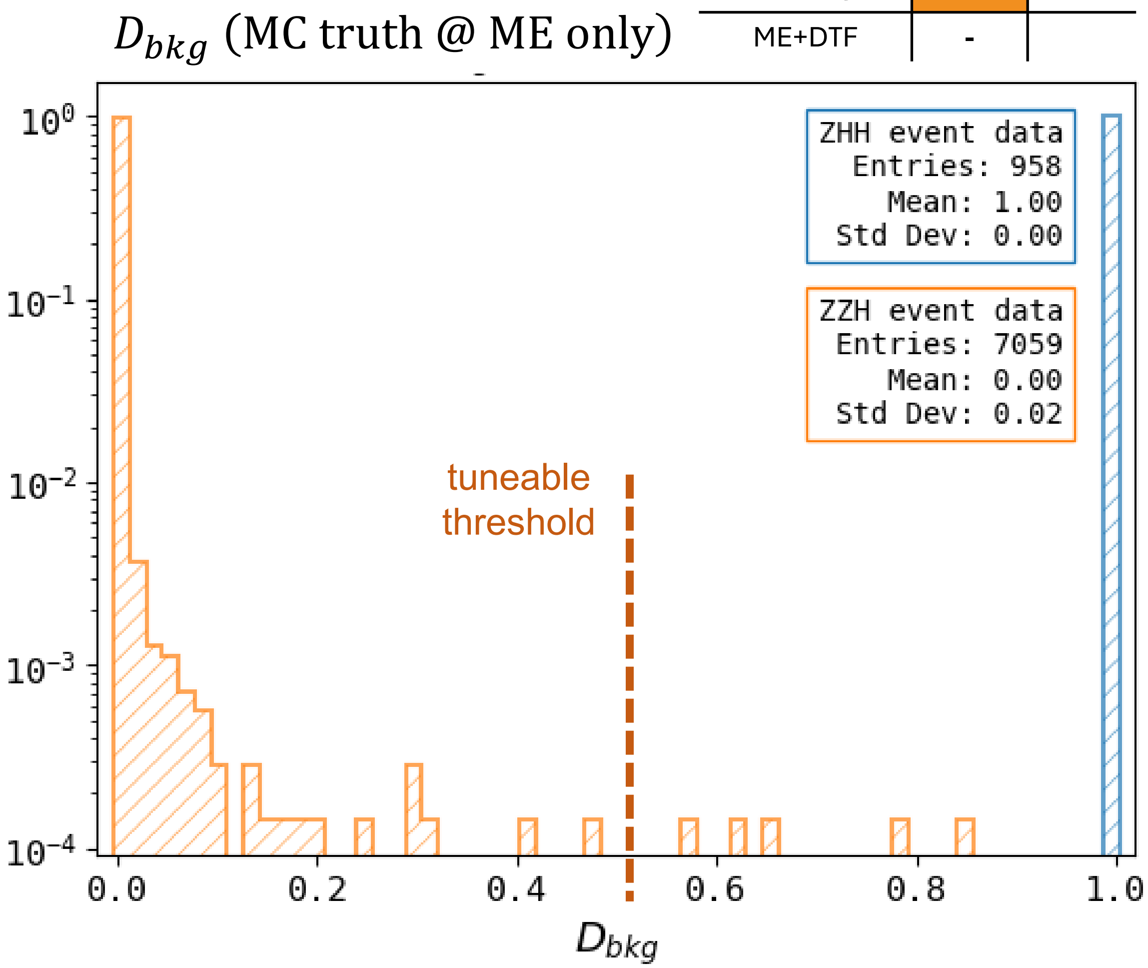
$\boldsymbol{a}$: theory parameters; e.g. λ_{HHH}
$A_i(\boldsymbol{a})$: signal acceptance
$\epsilon_i(\boldsymbol{x})$: detector efficiency

Hypothesis testing with the MEM

MC truth + Matrix Elements (ME) only

4-vectors / MEM type	MC truth	Reco
ME only	■	
ME+DTF	-	

- use case: generator-level check
  - calculate discriminator just from $M_i(y_{truth})$ and σ_i
  - no transfer function
- perfect separation, as expected

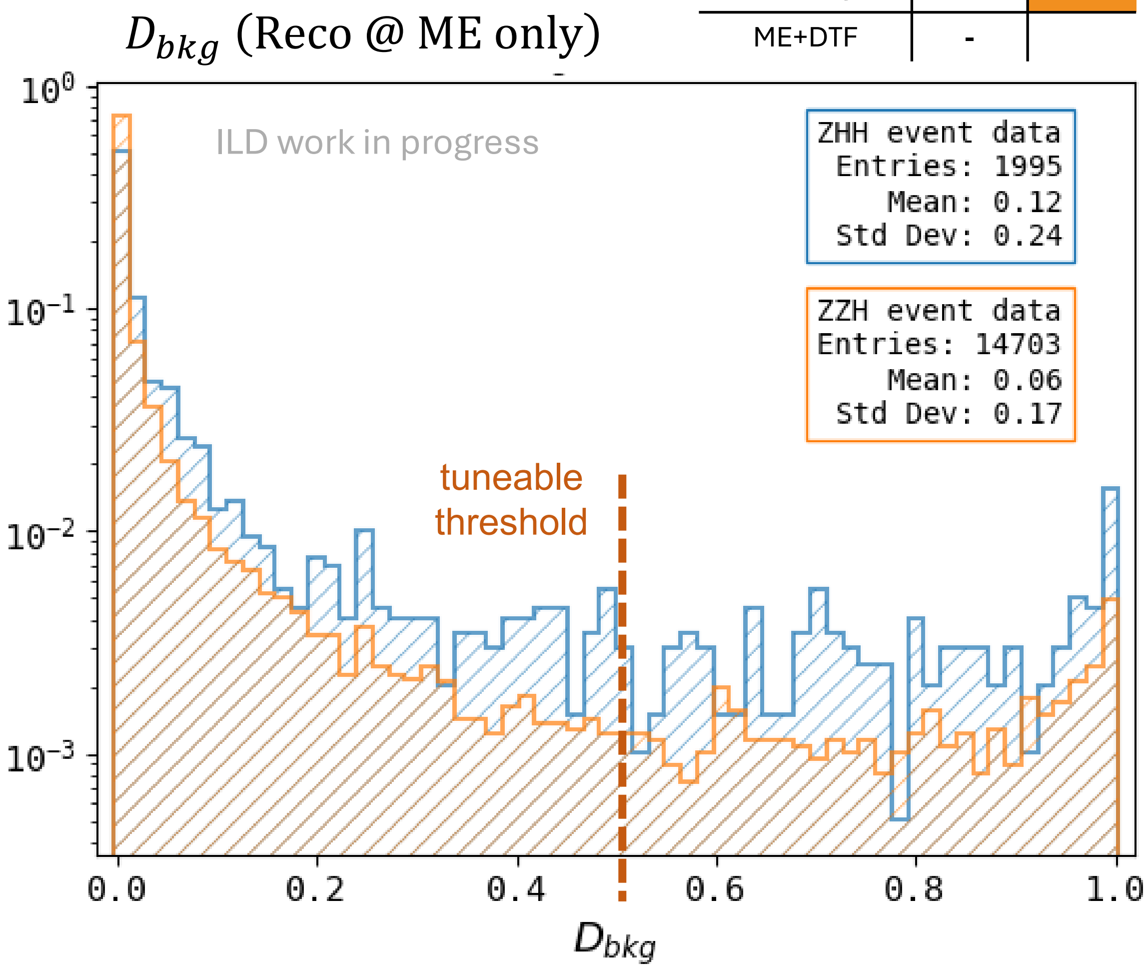


Hypothesis testing with the MEM

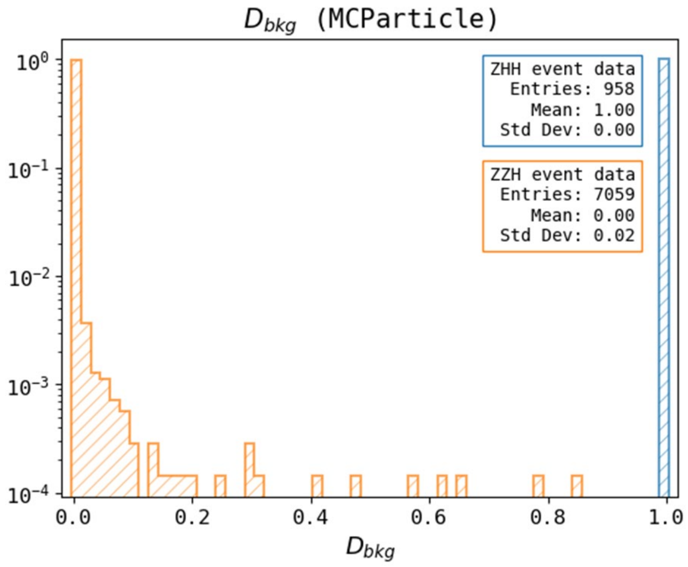
4-vectors / MEM type	MC truth	Reco
ME only		
ME+DTF	-	

Reconstructed data + ME only

- smearing and loss of separation power due to
  - parton showering, hadronization
  - detector effects
  - misclustering etc.



For comparison:
ME only with MC data

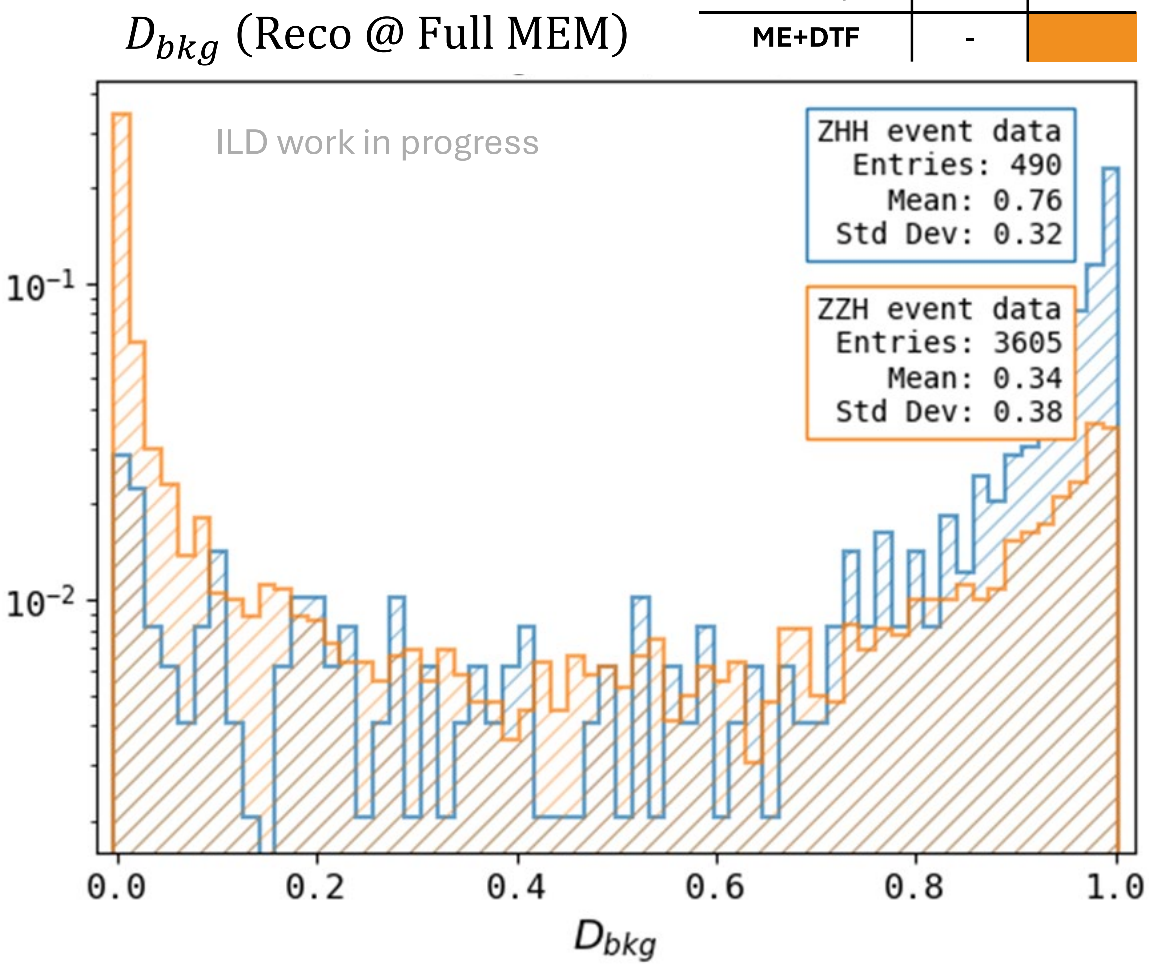


Hypothesis testing with the MEM: Result

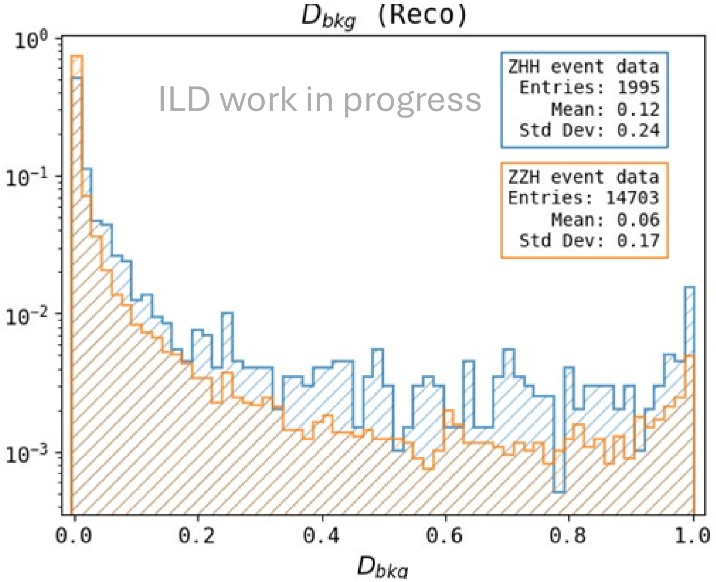
4-vectors / MEM type	MC truth	Reco
ME only		
ME+DTF	-	

Reconstructed data + Full-MEM

- gained separation power by including detector effects
- possibly: MEM output as input to other MVA
- computationally demanding → WIP: investigate approach with invertible neural networks (INNs)



For comparison:
ME only with reco data



Hypothesis testing with the MEM: Result

Reconstructed data + Full-MEM

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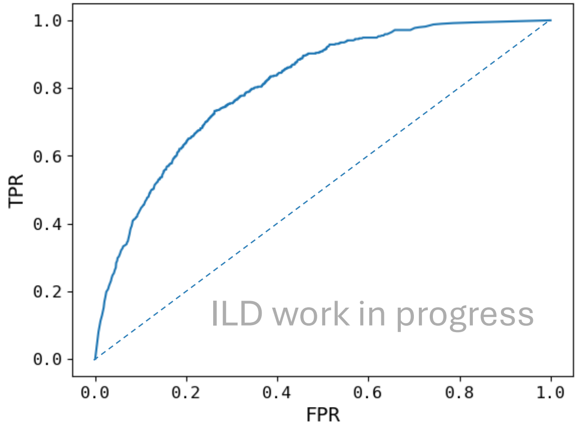
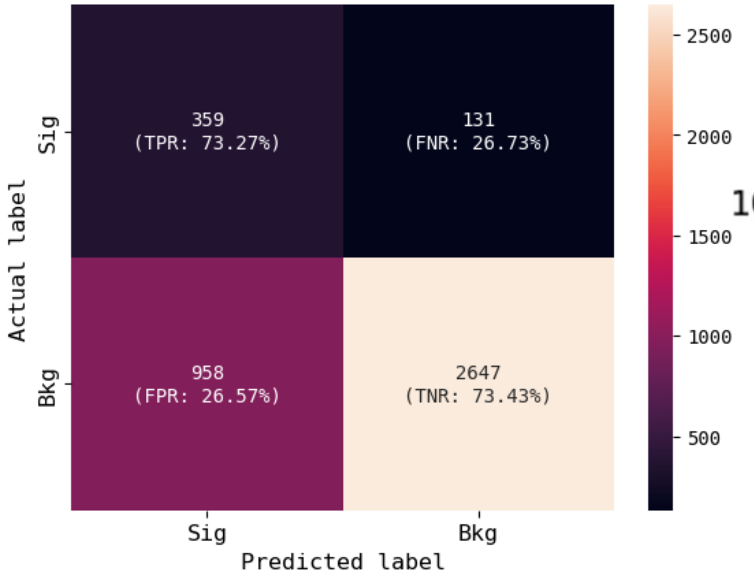
MEM type \ 4-vectors	MC truth	Reco
ME only		
ME+DTF	-	

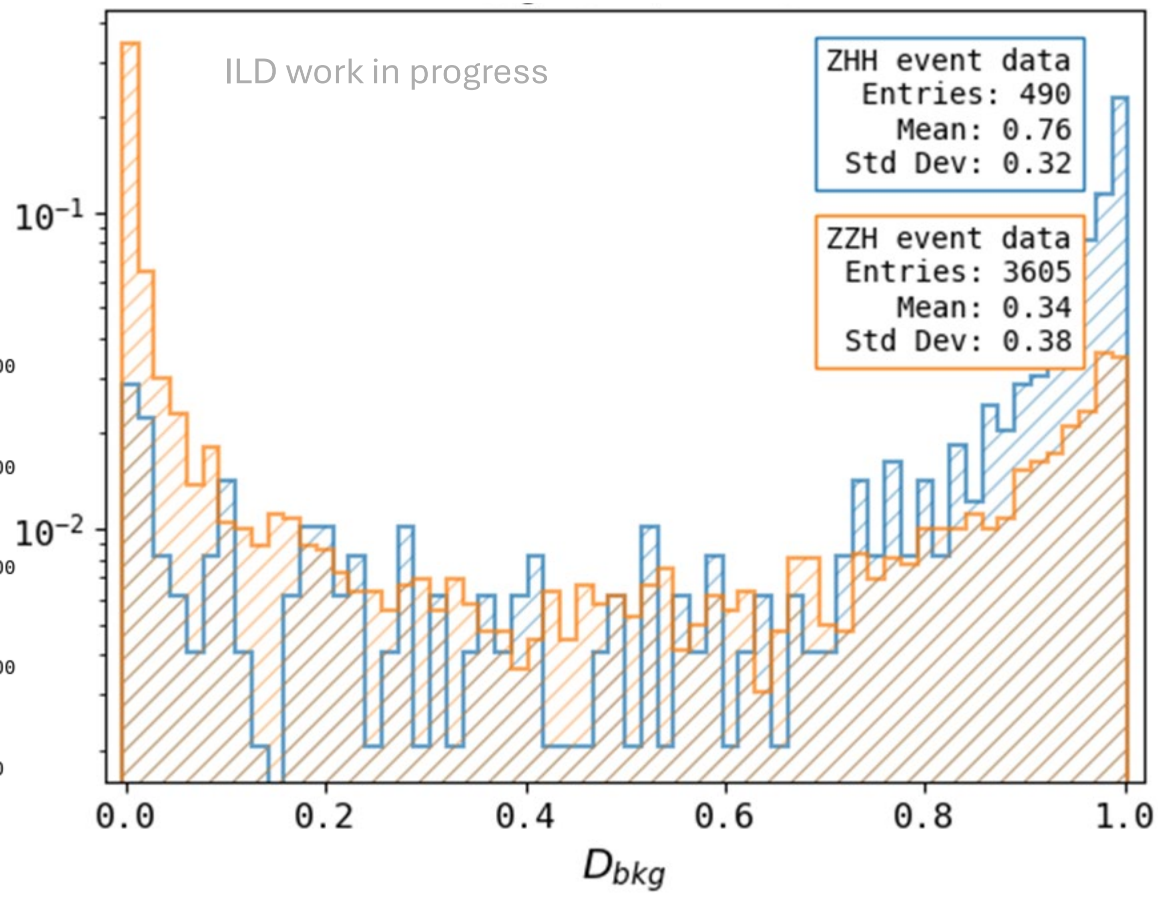
ROC (Reco @ Full – MEM) AUC = 0.758





Efficiency: Rows sum to 1

D_{bkg} (Reco @ Full MEM)



Choice of COM energy

Advantages of higher COM energies

- more boosted jets
  - better clustering, better jet-pairing?
  - improved b-tagging efficiencies?
  - better separation between signal and background?

Disadvantages of higher COM energies

- sensitivity factor c on self-coupling λ increases with E_{CM}

$$\frac{\Delta\lambda}{\lambda} = c \cdot \frac{\Delta\sigma}{\sigma}$$

  - less sensitivity to λ?

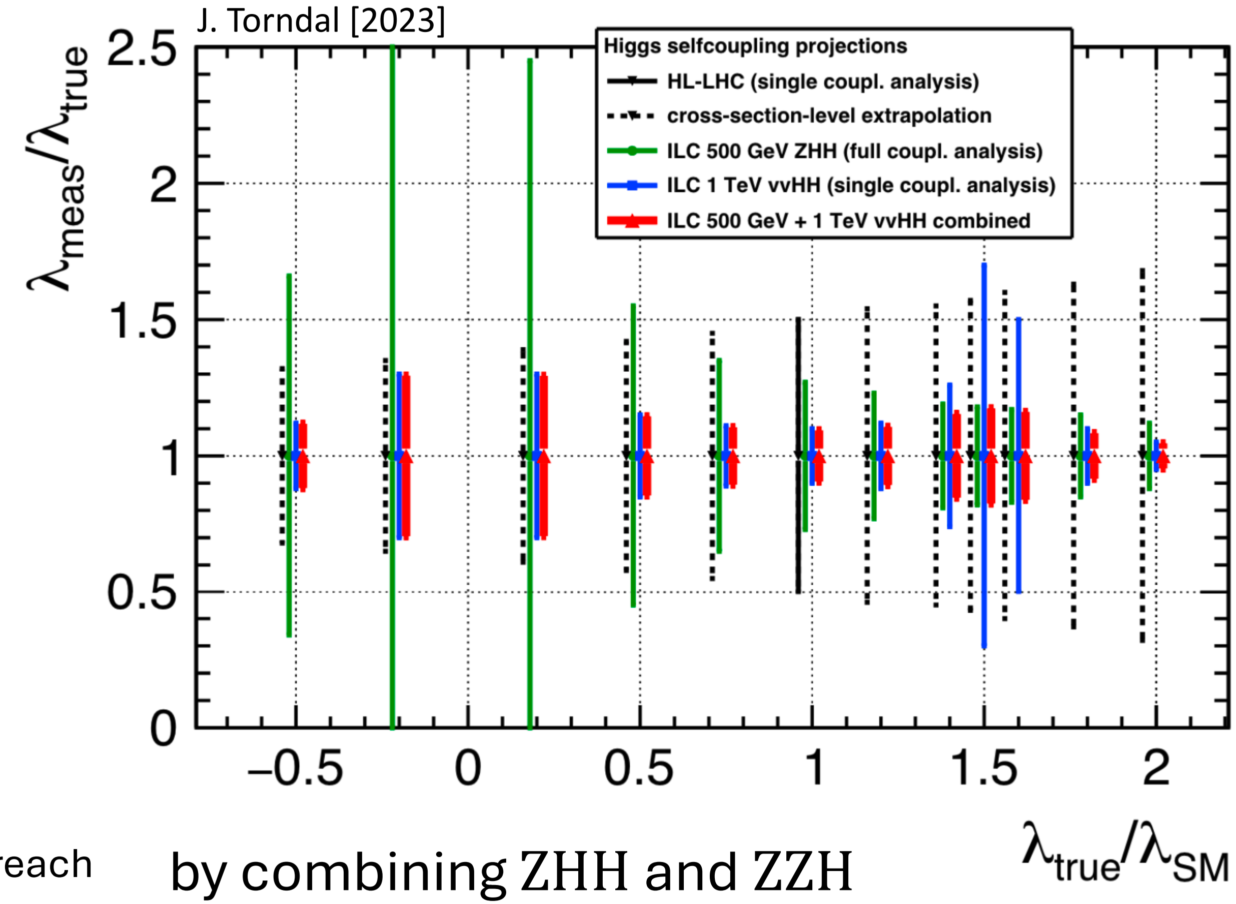
➔ so far: small effects in all expected places that could improve event reconstruction

➔ need to also consider reduced dependence of λ on the cross-section

Precision on Higgs self-coupling at future colliders

collider	indirect h	Direct hh
HL-LHC	$100 - 200\%$	50%
ILC250	–	–
ILC500	58%	20%*
ILC500+1000	52%	10%*
CLIC380	–	–
CLIC1500	–	36%
CLIC1500+3000	–	9%
FCC-ee 240+365	44%	–
FCC-ee (4 IPs)	27%	–
FCC-hh	–	$3.4 - 7.8\%$

*improvements in reconstruction tools propagated to precision reach



- only valid for $\lambda = \lambda_{SM}$
  - precision depends on λ itself

by combining ZHH and ZZH

- **10% precision on λ for $\lambda_{true} = \lambda_{SM}$**
- at least **30% for any value of λ_{true}**

Conclusion

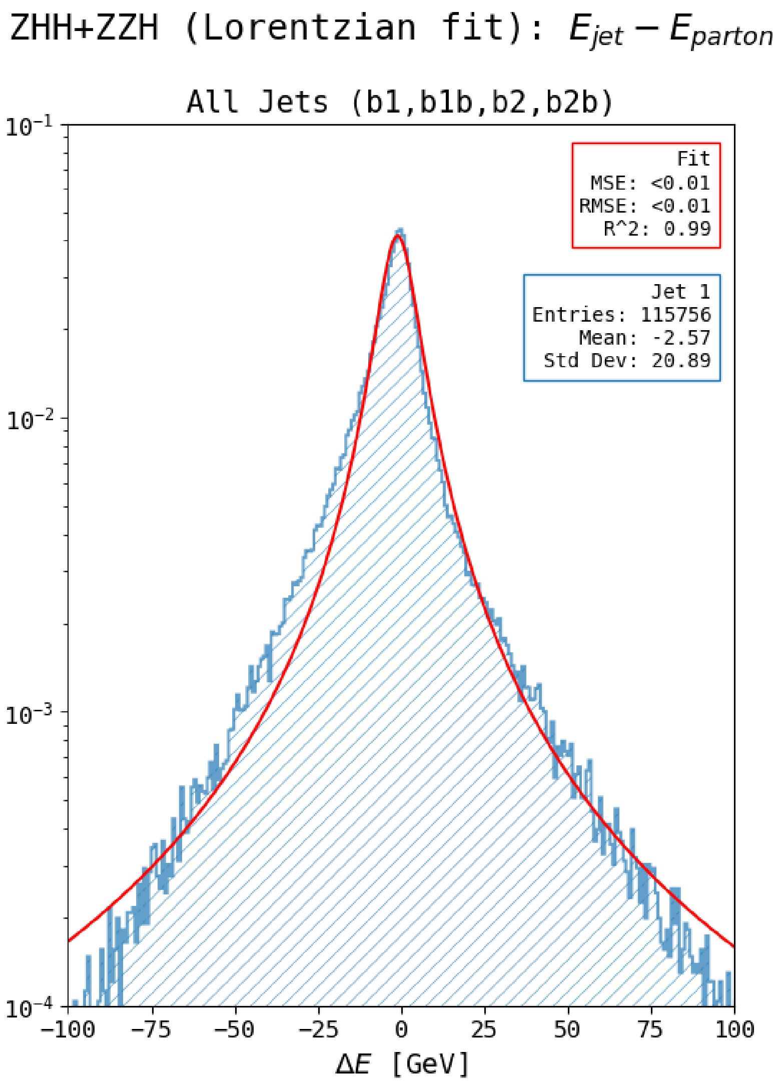
- **discovery potential of Higgs self-coupling at ILC clearly demonstrated** in the past
- sensitivity improvement to **better than 20% at ILC500** expected due to improvements in reconstruction tools
  - update to state-of-the-art projections for ILC underway at 500, 550 and 600 GeV COM energies
- by combining ZHH and $\nu\nu$HH measurements at **ILC500+ILC1000**
  - **10% precision on λ** for $\lambda_{true} = \lambda_{SM}$
  - at least **30% precision on any value of λ**
- improving signal/background discrimination using MEM
  - investigate ML-based approaches, especially conditional INNs

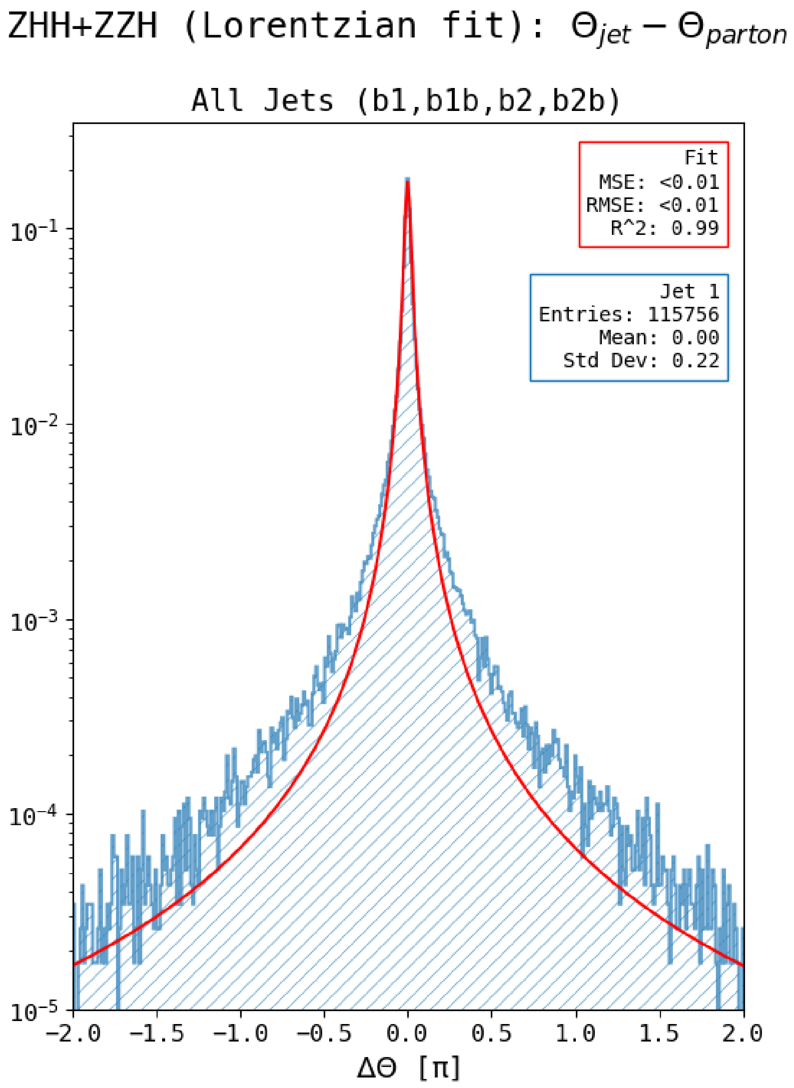
Thank you!

Backup

MEM detector transfer functions

- PDF for energies/angles between reconstructed and parton-level particles
- „conventional approach“: fitting transfer functions explicitly
- separate transfer functions possible for signal/background hypothesis





MEM Monte-Carlo integration

$$P_i\,(\boldsymbol{y}\,|\,\boldsymbol{a}) = \frac{1}{\sigma_i(\boldsymbol{a}) \cdot A_i(\boldsymbol{a})} \int W_i(\boldsymbol{y}\,|\,\boldsymbol{x},\boldsymbol{a}) |M_i(\boldsymbol{x},\boldsymbol{a})|^2 T_i(\boldsymbol{x},\boldsymbol{a}) d\boldsymbol{\Phi}_n$$

$$d\boldsymbol{\Phi}_n = \prod_i^{\mu^-,\mu^+,b_1,\overline{b_1},b_2,\overline{b_2}} \frac{d^3\boldsymbol{p_i}}{(2\pi)^3 2E_i}$$

- leptons well measured ➔ no integration for μ^-, μ^+
- conservation of four momentum and narrow-width-approximation ➔ reduction of integration to 7 dimensions
- integration variables: $\Theta_{b1}, \phi_{b1}, \rho_{b1}, \theta_{b1b}, \phi_{b1b}, \rho_{b2}, \Theta_{b2}$
- with VEGAS+ and integrand in C++, computation time 1-3 minutes per process (including setup of integration grid)
- „accept-and-reject“ MC

itn	integral	wgt average	chi2/dof	Q
1	4.2(3.6)e-09	4.2(3.6)e-09	0.00	1.00
2	6.7(2.7)e-10	6.9(2.7)e-10	0.94	0.33
3	6.0(2.1)e-10	6.4(1.7)e-10	0.50	0.60
4	2.69(55)e-10	3.05(52)e-10	1.81	0.14
5	3.49(58)e-10	3.24(39)e-10	1.44	0.22
6	2.96(43)e-10	3.12(29)e-10	1.20	0.31
7	5.0(1.2)e-10	3.23(28)e-10	1.42	0.20
8	4.78(94)e-10	3.35(27)e-10	1.58	0.14
9	8.6(2.2)e-10	3.43(27)e-10	2.11	0.03
10	5.9(1.8)e-10	3.48(26)e-10	2.07	0.03

result = 3.48(26)e-10 Q = 0.03

itn	integral	wgt average	chi2/dof	Q
1	1.58(18)e-09	1.58(18)e-09	0.00	1.00
2	1.68(19)e-09	1.63(13)e-09	0.13	0.72
3	1.94(19)e-09	1.72(11)e-09	0.96	0.38
4	1.91(13)e-09	1.800(82)e-09	1.04	0.37
5	1.98(27)e-09	1.815(79)e-09	0.88	0.48
6	2.73(99)e-09	1.821(78)e-09	0.88	0.50
7	1.78(10)e-09	1.807(62)e-09	0.74	0.61
8	2.03(17)e-09	1.834(59)e-09	0.86	0.54
9	1.72(13)e-09	1.816(54)e-09	0.82	0.58
10	1.813(83)e-09	1.815(45)e-09	0.73	0.68

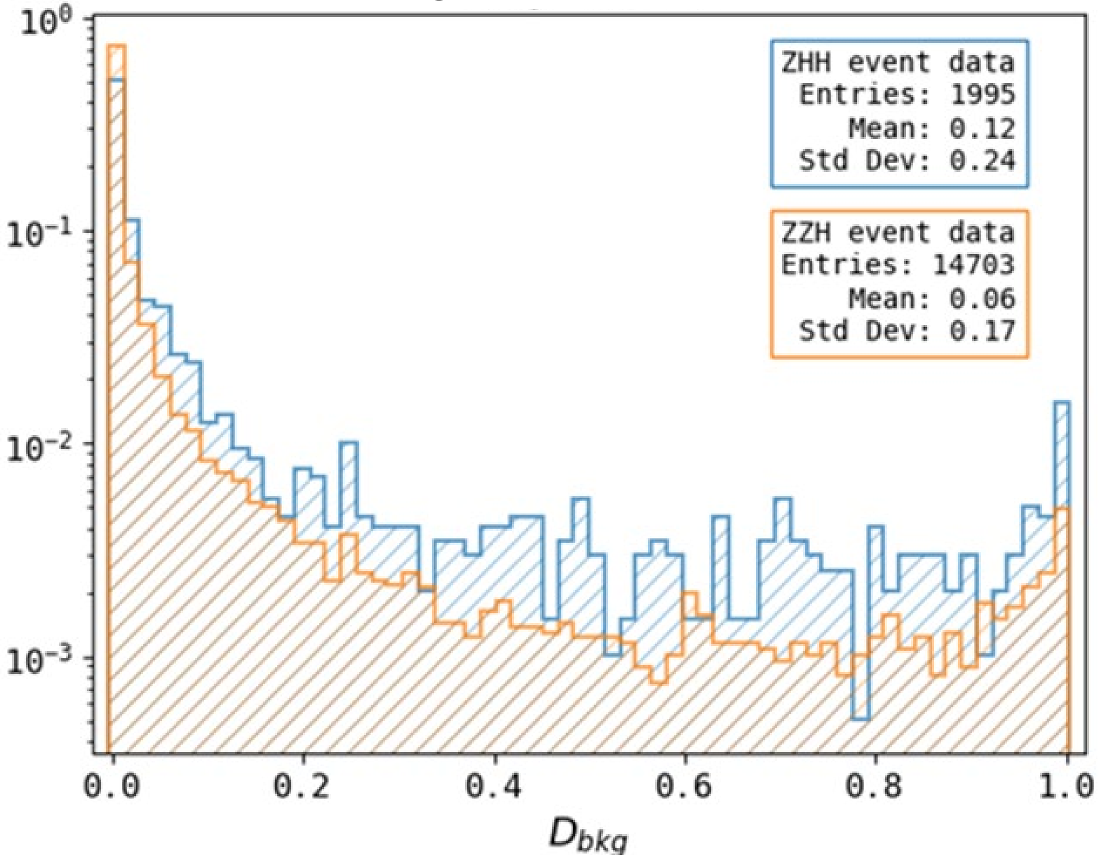
result = 1.815(45)e-09 Q = 0.68

MEM results for example ZHH (top) and ZZH (bottom) event

MEM with+without transfer functions

- distributions before (left) and after (right) including transfer functions

D_{bkg} (Reco @ ME only)



D_{bkg} (Reco @ Full MEM)