Precise predictions for the trilinear Higgs coupling in arbitrary models

Based on

arXiv:2305.03015 in collaboration with Henning Bahl, Martin Gabelmann and Georg Weiglein

Johannes Braathen

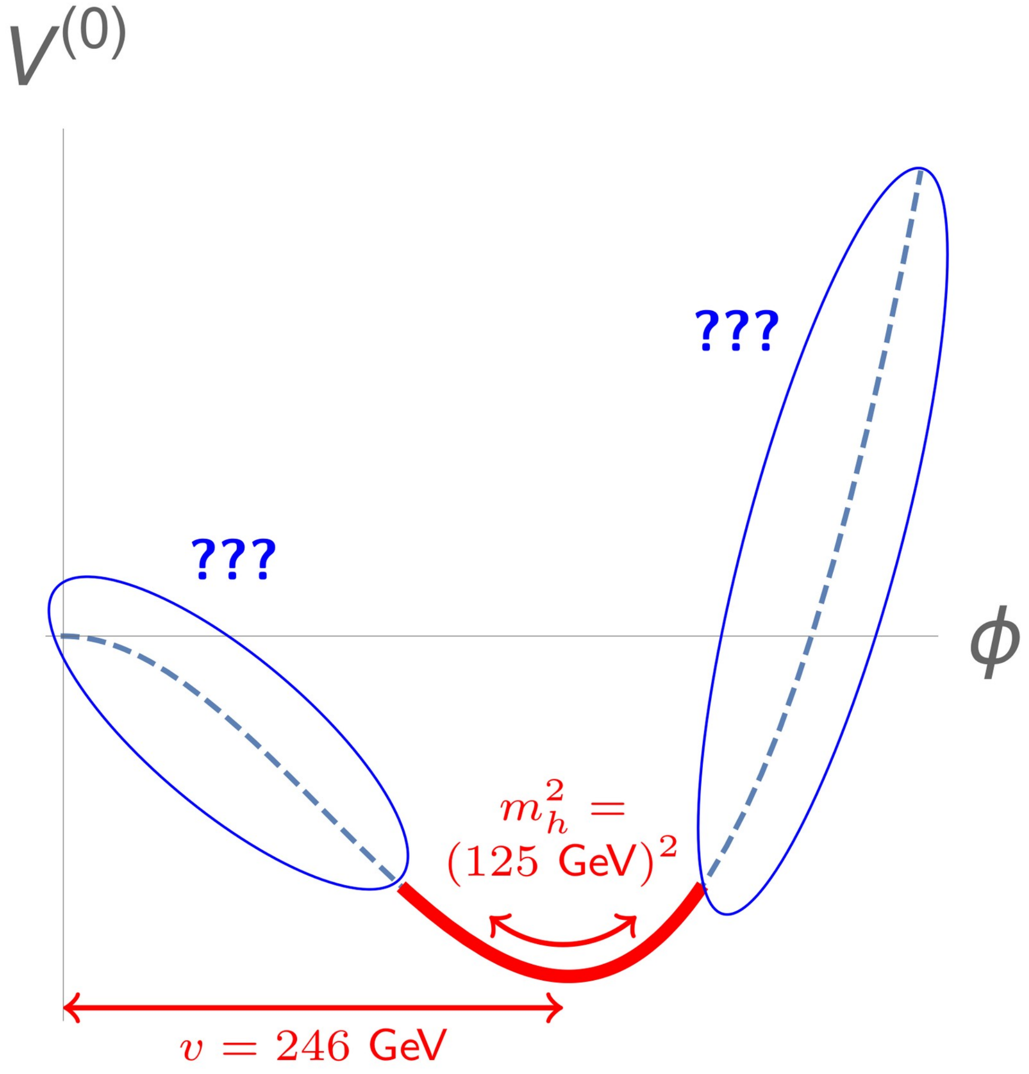
Second ECFA Workshop on e+e- Higgs/EW/Top factories

Paestum, Italy | 12 October 2023

Why study the trilinear Higgs coupling?

- **Probing the Higgs potential:**
  Since the Higgs discovery, the existence of the Higgs potential is confirmed, but at the moment we only know:
  → the location of the EW minimum:
  $v = 246$ GeV
  → the curvature of the potential around the EW minimum:
  $m_h = 125$ GeV
  However we still don't know the **shape** of the potential, away from EW minimum → **depends on λ_{hhh}**

- **λ_{hhh} determines the nature of the EWPT**!
  ⇒ deviation of λ_{hhh} from its SM prediction typically needed to have a strongly first-order EWPT → necessary for EWBG
  [Grojean, Servant, Wells '04], [Kanemura, Okada, Senaha '04]



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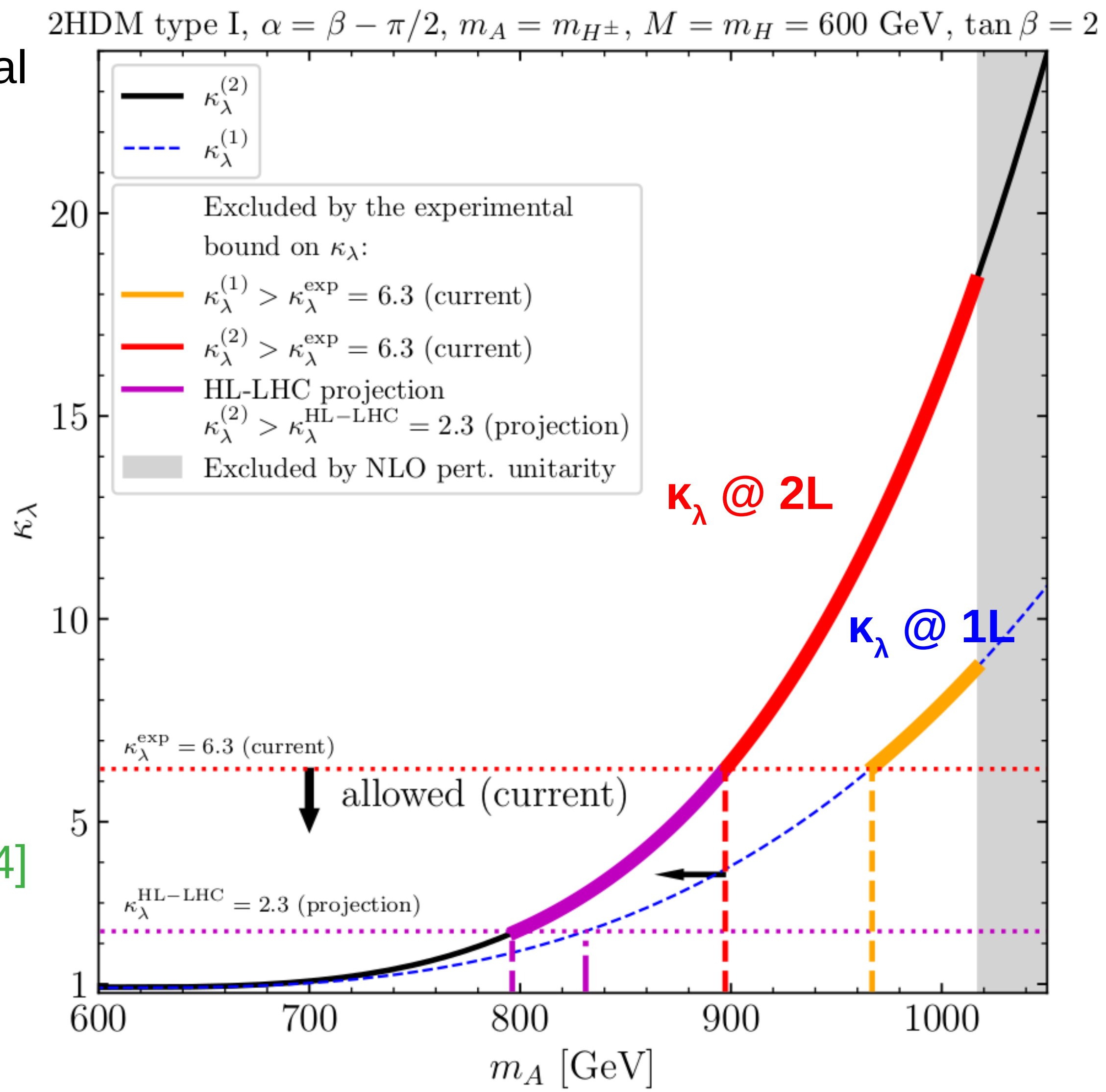
- Comparing latest exp. bounds **$-0.4 < \kappa_\lambda \equiv \lambda_{hhh}/(\lambda_{hhh}^{(0)})^{SM} < 6.3$** [ATLAS PLB '23] with **precise theory predictions** for λ_{hhh} provides a **powerful new tool to constrain BSM models**



[Bahl, JB, Weiglein *Phys.Rev.Lett.* '22]

Computing λ_{hhh} in BSM theories

- Calculations of λ_{hhh} are important, and receive increasing attention
  - More and more model specific results at 1L

    SM + singlet [Kanemura et al. ’16]; ***2HDMs*** [Kanemura et al. ’04], [Basler et al. ’17]; ***N2HDM (2HDM + singlet)*** [Basler et al. ’19]; ***triplet extensions*** [Aoki et al. ’12], [Chiang et al. ’18]; ***MSSM*** [Hollik, Penaranda ‘04]; ***NMSSM*** [Dao et al. ’13]; ***models with classical scale invariance*** [Hashino, Kanemura, Orikasa ‘16], etc.

    … and at 2L

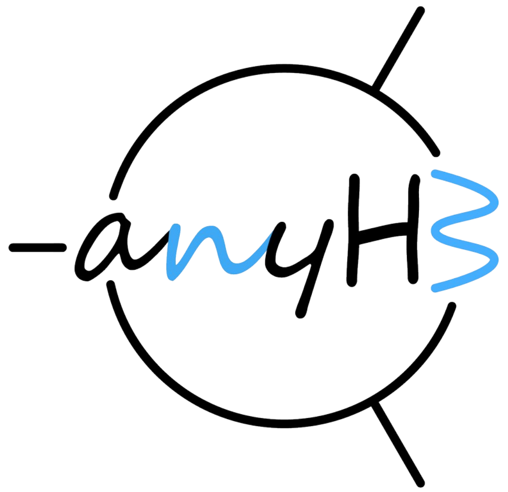
    SM + singlet [JB, Kanemura ’19]; ***2HDMs*** [Senaha ‘18], [JB, Kanemura ’19]; ***MSSM*** [Brucherseifer et al. ’13]; ***NMSSM*** [Dao et al. ‘15], [Borschensky et al ‘22] ; ***models with classical scale invariance*** [JB, Kanemura, Shimoda ‘20], etc.

    but many more models to investigate!

- For many (pseudo-)observables, automated tools exist

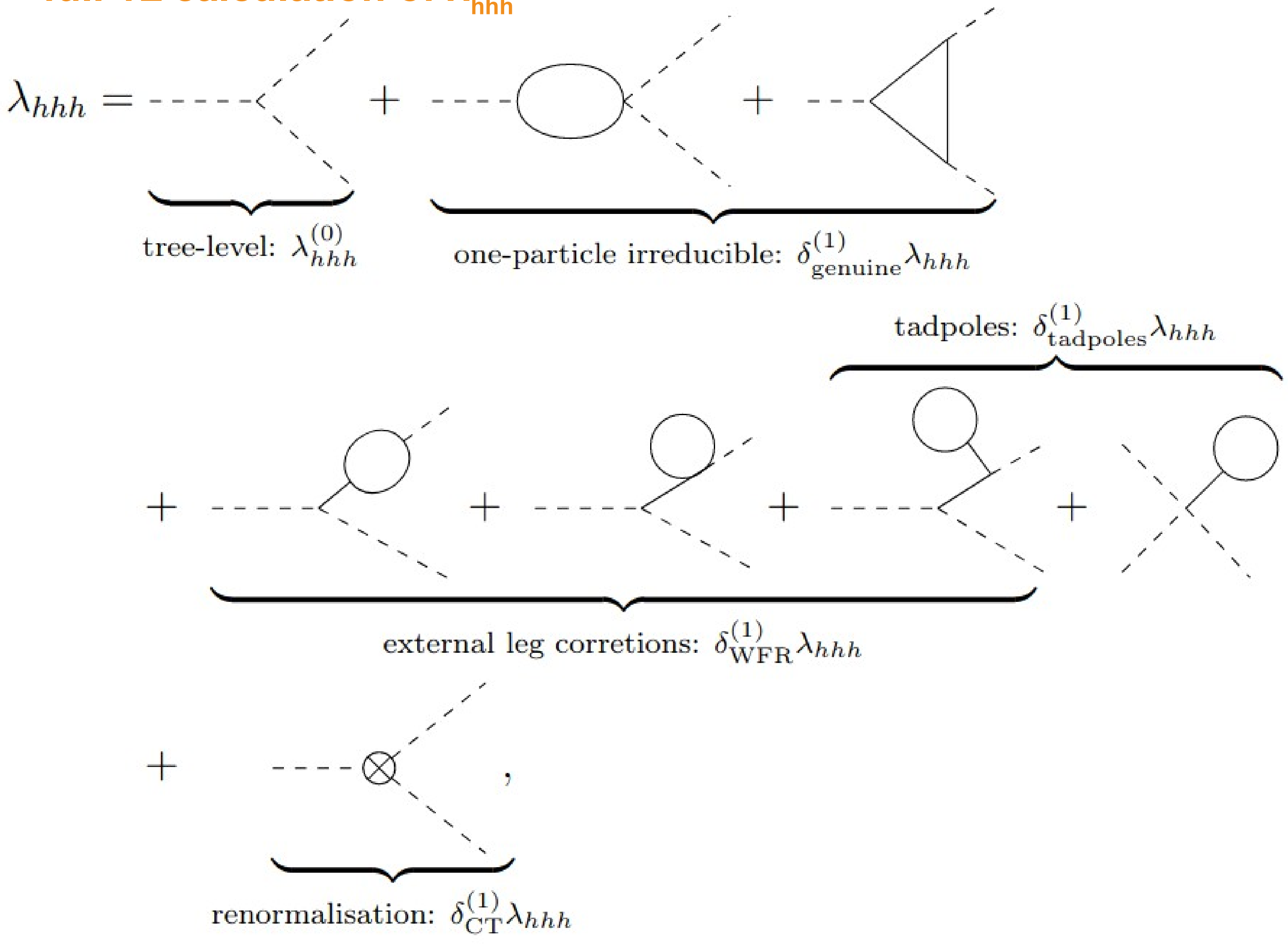
- What about for the trilinear Higgs coupling?
  → none so far
  → `anyH3` [Bahl, JB, Gabelmann, Weiglein `2305.03015`]

Generic predictions for λ_{hhh}



Computing λ_{hhh} in general renormalisable theories: ingredients

anyH3 → full 1L calculation of λ_{hhh}



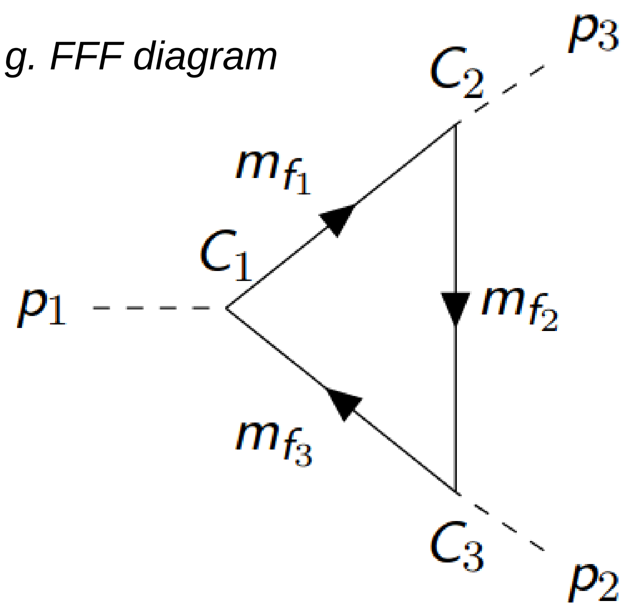
- Solid lines:
  - *scalars,*
  - *fermions,*
  - *gauge/vector bosons,*
  - *ghosts*

- Restrictions on particles and/or topologies possible

Computing λ_{hhh} in general renormalisable theories: method

Our method: we derive and implement analytic results for **generic diagrams**, i.e. assuming generic

e.g. FFF diagram



- Couplings $C_i = C_i^L P_L + C_i^R P_R, \text{ where } P_{L,R} \equiv \frac{1}{2}(1 \mp \gamma_5)$
- Masses on the internal lines m_{fi}, i=1,2,3
- External momenta p_i, i=1,2,3

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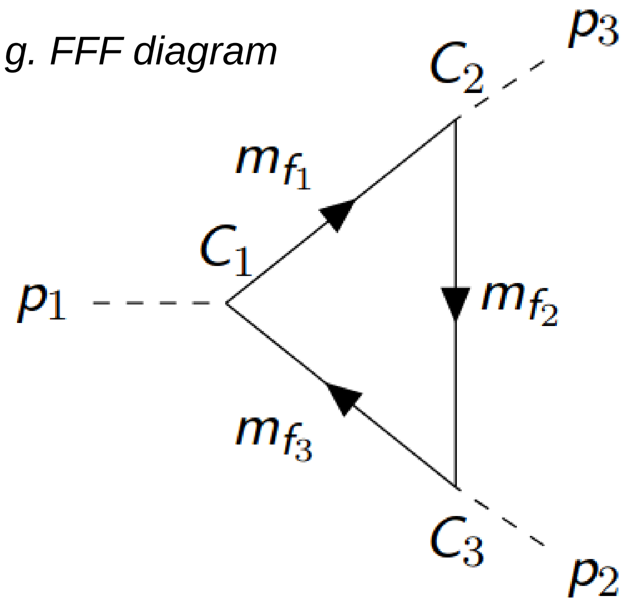

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$$
\begin{aligned}
= {} & 2\mathbf{B0}(p_3^2, m_2^2, m_3^2)\big(C_1^L(C_2^L C_3^R m_{f_1} + C_2^R C_3^R m_{f_2} + C_2^R C_3^L m_{f_3}) + C_1^R(C_2^R C_3^L m_{f_1} + \\
& C_2^L C_3^L m_{f_2} + C_2^L C_3^R m_{f_3})\big) + m_{f_1}\mathbf{C0}(p_2^2, p_3^2, p_1^2, m_1^2, m_3^2, m_2^2)\big((C_1^L C_2^L C_3^R + \\
& C_1^R C_2^R C_3^L)(p_1^2 + p_2^2 - p_3^2) + 2(C_1^L C_2^L C_3^L + C_1^R C_2^R C_3^R) m_{f_2} m_{f_3} + \\
& 2m_{f_1}(C_1^L(C_2^L C_3^R m_{f_1} + C_2^R C_3^R m_{f_2} + C_2^R C_3^L m_{f_3}) + C_1^R(C_2^R C_3^L m_{f_1} + C_2^L C_3^L m_{f_2} + \\
& C_2^L C_3^R m_{f_3}))\big) + \mathbf{C1}(p_2^2, p_3^2, p_1^2, m_1^2, m_3^2, m_2^2)\big(2p_2^2(C_1^L C_3^R(C_2^L m_{f_1} + C_2^R m_{f_2}) + \\
& C_1^R C_3^L(C_2^R m_{f_1} + C_2^L m_{f_2})) + (p_1^2 + p_2^2 - p_3^2)((C_1^L C_2^L C_3^R + C_1^R C_2^R C_3^L) m_{f_1} + \\
& (C_1^L C_2^R C_3^L + C_1^R C_2^L C_3^R) m_{f_3})\big) + \mathbf{C2}(p_2^2, p_3^2, p_1^2, m_1^2, m_3^2, m_2^2)\big((p_1^2 + p_2^2 - \\
& p_3^2)(C_1^L C_3^R(C_2^L m_{f_1} + C_2^R m_{f_2}) + C_1^R C_3^L(C_2^R m_{f_1} + C_2^L m_{f_2})) + 2p_1^2((C_1^L C_2^L C_3^R + \\
& C_1^R C_2^R C_3^L) m_{f_1} + (C_1^L C_2^R C_3^L + C_1^R C_2^L C_3^R) m_{f_3})\big)
\end{aligned}
$$

(**B0**, **C0**, **C1**, **C2**: loop functions)

Computing λ_{hhh} in general renormalisable theories: method

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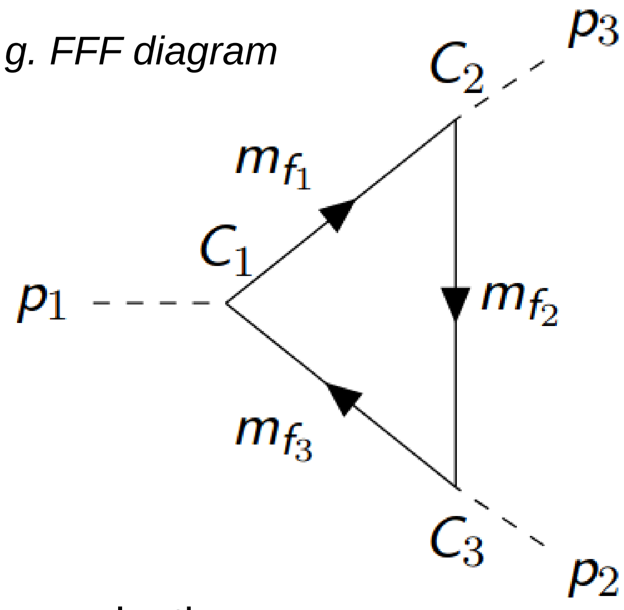

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& C_2^L C_3^L m_{f_2} + C_2^L C_3^R m_{f_3})) + m_{f_1}\mathbf{C0}(p_2^2, p_3^2, p_1^2, m_1^2, m_3^2, m_2^2)((C_1^L C_2^L C_3^R + \\
& C_1^R C_2^R C_3^L)(p_1^2 + p_2^2 - p_3^2) + 2(C_1^L C_2^L C_3^L + C_1^R C_2^R C_3^R) m_{f_2} m_{f_3} + \\
& 2m_{f_1}(C_1^L(C_2^L C_3^R m_{f_1} + C_2^R C_3^R m_{f_2} + C_2^R C_3^L m_{f_3}) + C_1^R(C_2^R C_3^L m_{f_1} + C_2^L C_3^L m_{f_2} + \\
& C_2^L C_3^R m_{f_3}))) + \mathbf{C1}(p_2^2, p_3^2, p_1^2, m_1^2, m_3^2, m_2^2)(2p_2^2(C_1^L C_3^R(C_2^L m_{f_1} + C_2^R m_{f_2}) + \\
& C_1^R C_3^L(C_2^R m_{f_1} + C_2^L m_{f_2})) + (p_1^2 + p_2^2 - p_3^2)((C_1^L C_2^L C_3^R + C_1^R C_2^R C_3^L) m_{f_1} + \\
& (C_1^L C_2^R C_3^L + C_1^R C_2^L C_3^R) m_{f_3})) + \mathbf{C2}(p_2^2, p_3^2, p_1^2, m_1^2, m_3^2, m_2^2)((p_1^2 + p_2^2 - \\
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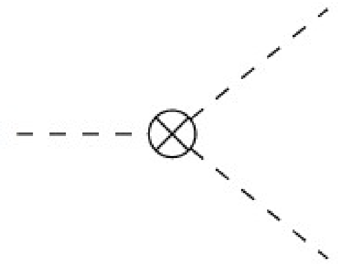
\end{aligned}
$$

For evaluation:

- Apply to concrete (B)SM model, using inputs in `UFO` format [Degrande et al., '11], [Darmé et al. '23]
- Evaluate loop functions via `COLLIER` [Denner et al '16] interface, `pyCollier`
- All included in public tool `anyH3` [Bahl, JB, Gabelmann, Weiglein '23]

(**B0**, **C0**, **C1**, **C2**: loop functions)

Flexible choice of renormalisation schemes

$\delta^{(1)}_{\mathrm{CT}}\lambda_{hhh} =$  $= \;?$

- **1L calculation** → renormalisation of all parameters entering λ_{hhh} at tree-level

- In general:

$$(\lambda^{(0)}_{hhh})^{\mathrm{BSM}} = (\lambda^{(0)}_{hhh})^{\mathrm{BSM}}(\underbrace{m_h \simeq 125\text{ GeV}, v \simeq 246\text{ GeV}}_{\text{SM sector}}, \underbrace{m_{\Phi_i}}_{\text{BSM masses}}, \underbrace{\alpha_i}_{\text{BSM mixing angles}}, \underbrace{v_i}_{\text{BSM VEVs}}, \underbrace{g_i}_{\text{indep. BSM coups.}})$$

- Most automated codes: $\overline{\text{MS}}/\overline{\text{DR}}$ only

- **anyH3**: much more flexibility, following **user choice**:
  - **SM sector** (m_h, v): fully OS or $\overline{\text{MS}}/\overline{\text{DR}}$
  - **BSM masses**: OS or $\overline{\text{MS}}/\overline{\text{DR}}$
  - **Additional couplings/vevs/mixings**: by default $\overline{\text{MS}}$, but **user-defined ren. conditions** also possible!

$$\delta^{(1)}_{\mathrm{CT}}\lambda_{hhh} = \sum_x \left(\frac{\partial}{\partial x}(\lambda^{(0)}_{hhh})^{\mathrm{BSM}}\right)\delta^{\mathrm{CT}}x\,, \qquad \text{with } x \in \{m_h, v, m_{\Phi_i}, v_i, \alpha_i, g_i, \text{etc.}\}$$

Renormalised in $\overline{\text{MS}}$, OS, in custom schemes, etc.

Example results from anyH3

A cross-check: the decoupling limit

- Consider the decoupling limit in several BSM models

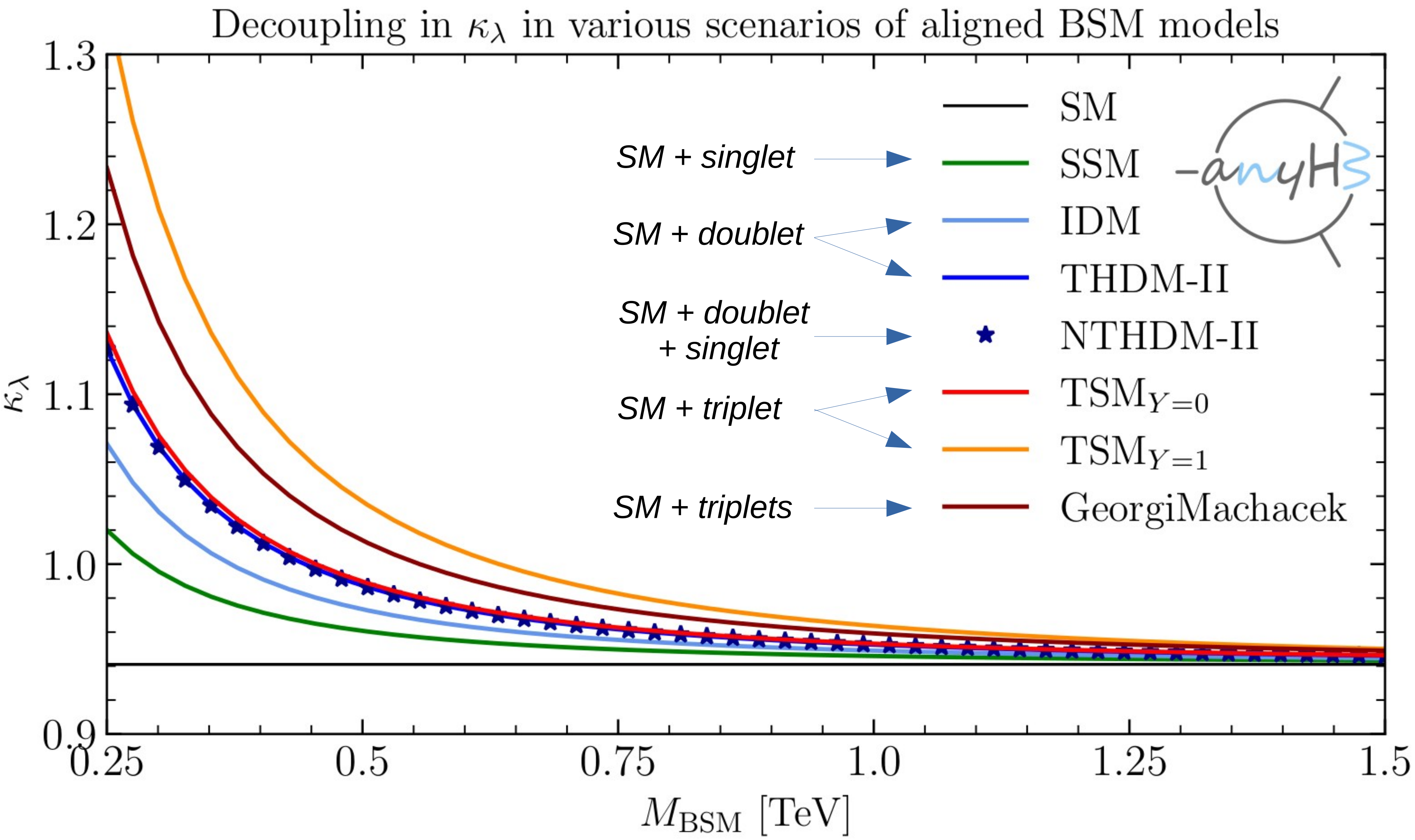
$$M_{\text{BSM}}^2 = \mathcal{M}^2 + \tilde{\lambda} v^2$$

$\mathcal{M}$: BSM mass scale
$\tilde{\lambda}$: Quartic couplings

- Increase BSM mass scale

$$\mathcal{M} \to \infty$$

- BSM corrections to should vanish (c.f. decoupling theorem [Appelquist, Carrazone '75])



New results I: non-decoupling effects in various BSM models

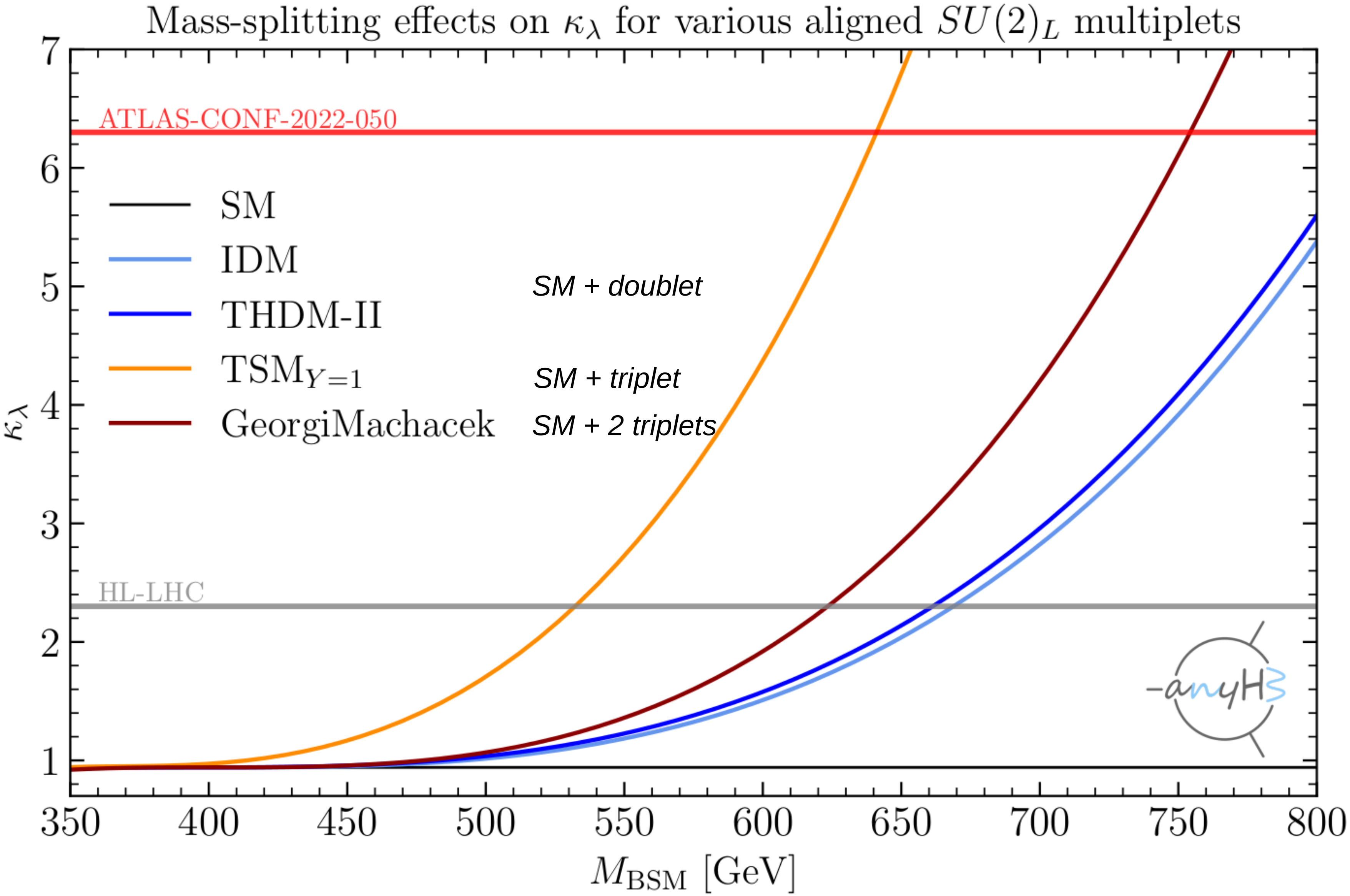
- Consider the non-decoupling limit in several BSM models

$$M^2_{\text{BSM}} = \mathcal{M}^2 + \tilde{\lambda}v^2$$

- Increase M_{BSM}, keeping $\mathcal{M}$ fixed
  → large mass splittings
  → **large BSM effects!**

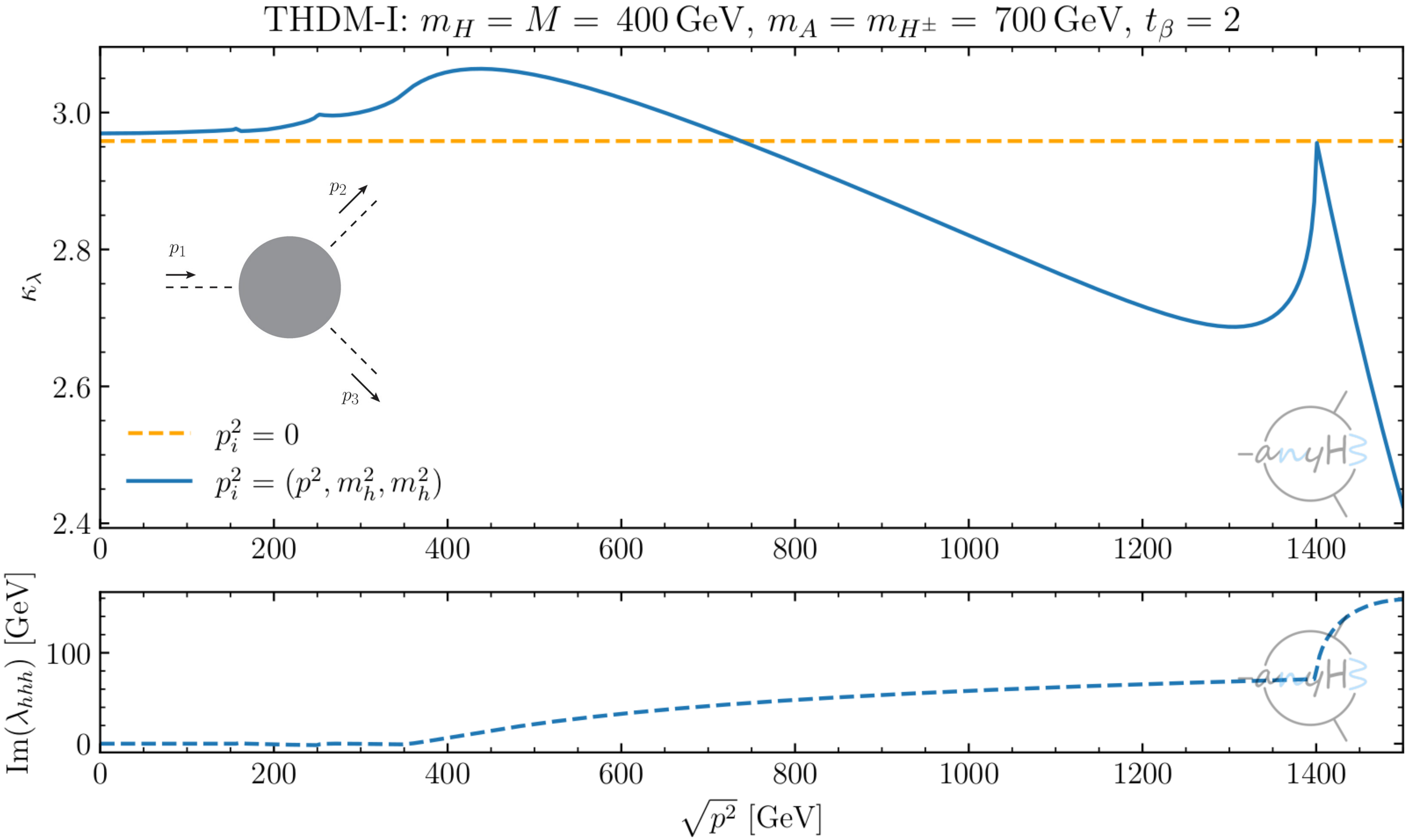
- Perturbative unitarity checked with `anyPerturbativeUnitarity`

- **Constraints on BSM parameter space!**

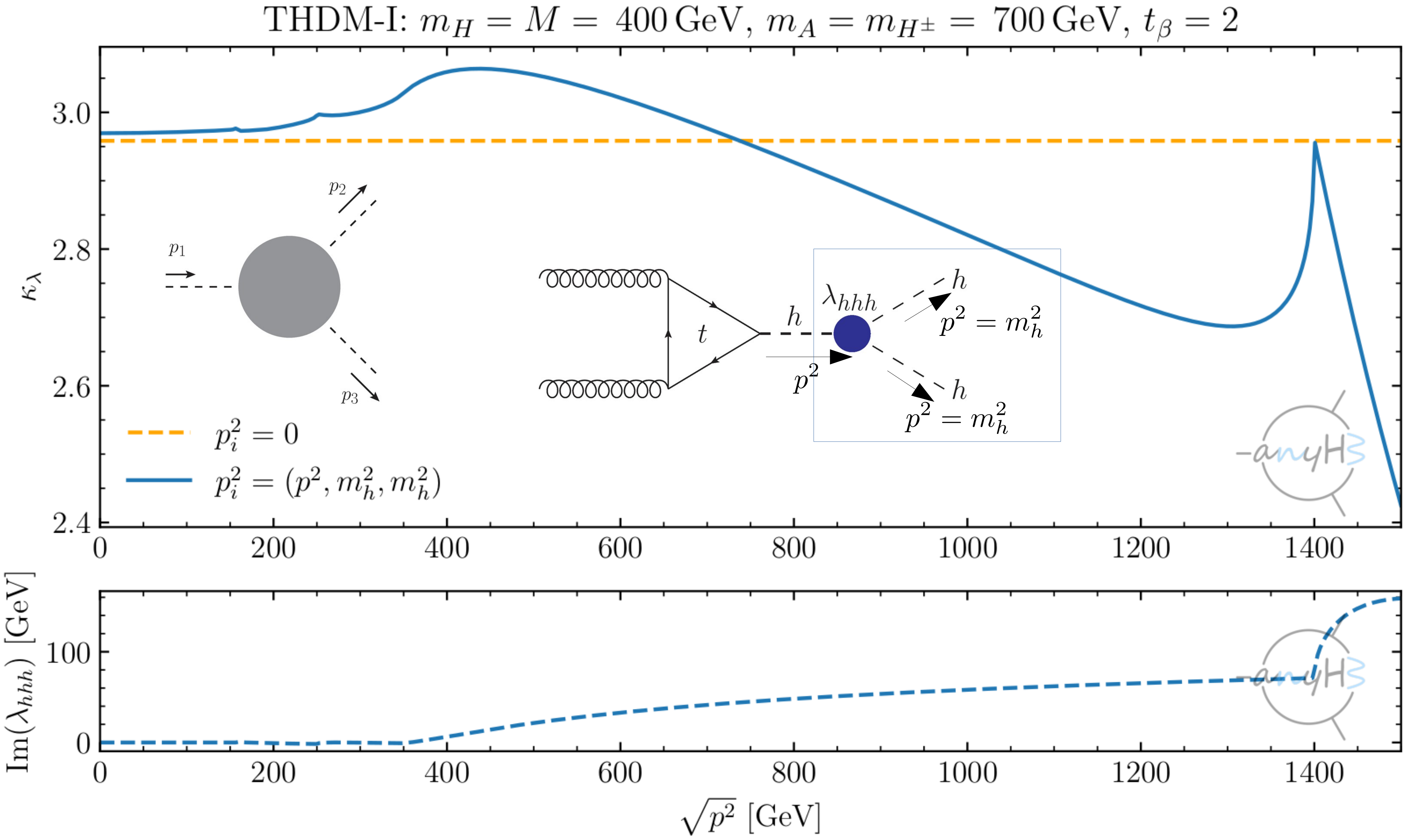


Here: scenarios with lightest BSM scalar mass + BSM mass param. at 400 GeV; other BSM scalar masses = M_{BSM}

New results II: momentum dependence in the 2HDM



New results II: momentum dependence in the 2HDM



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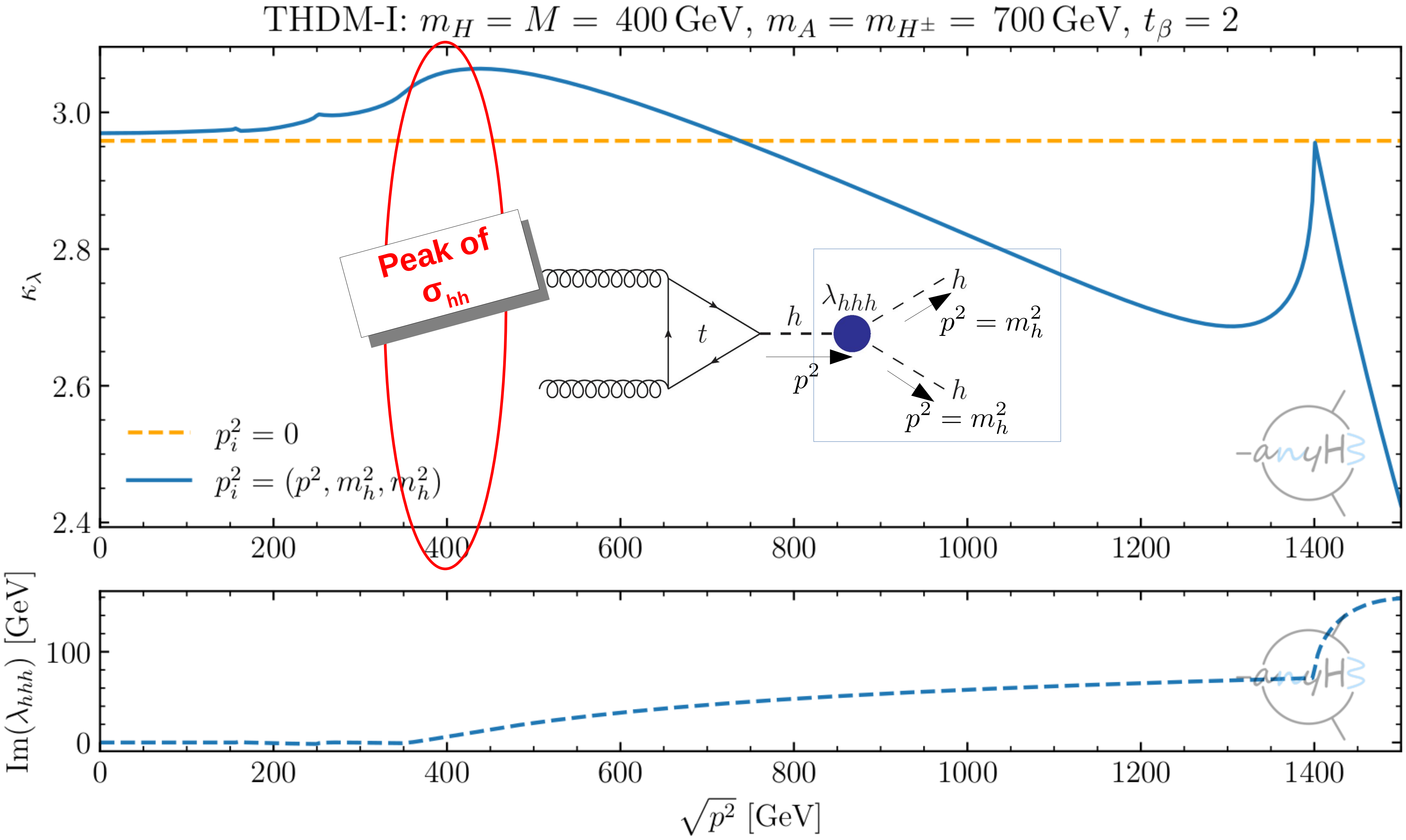

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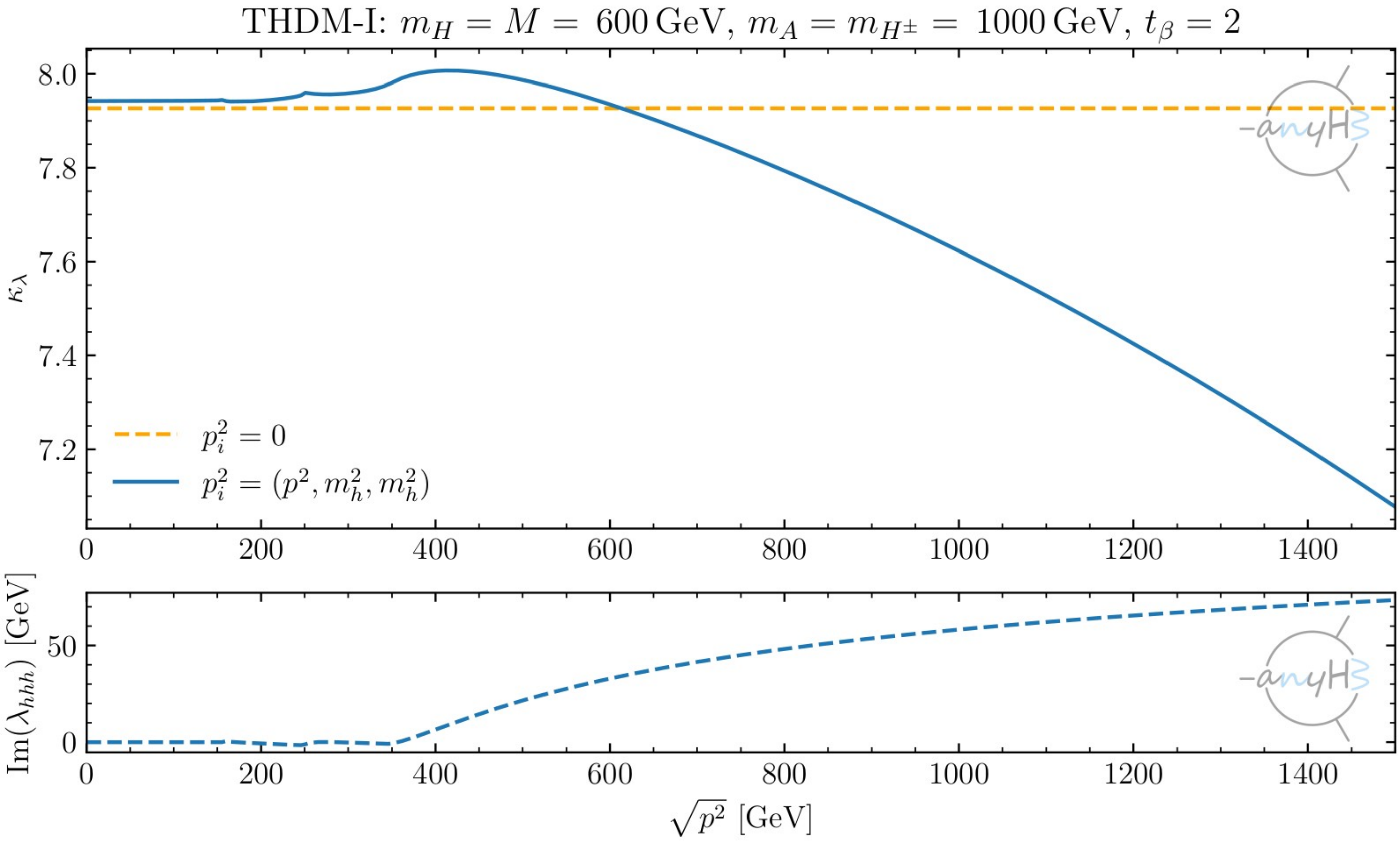
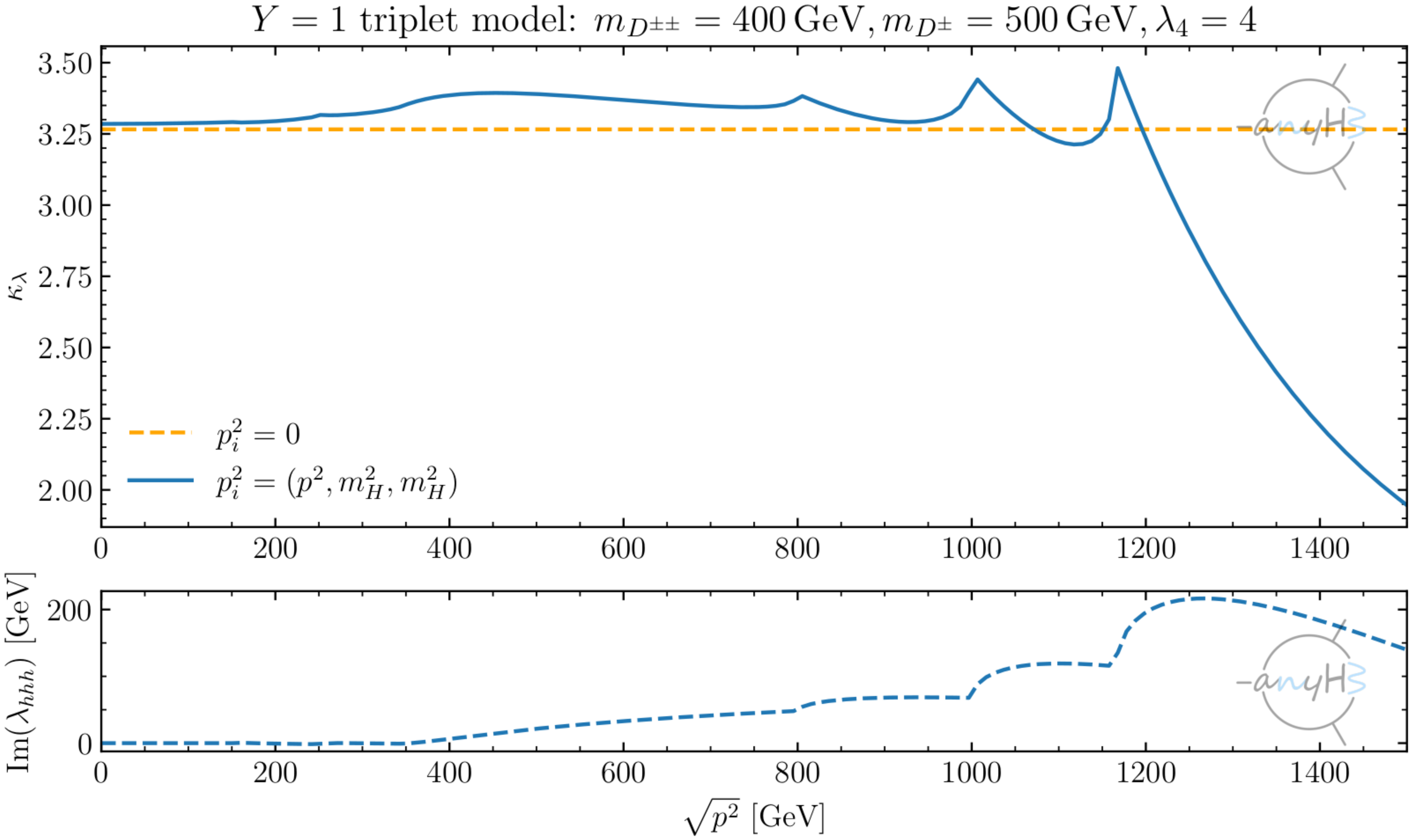


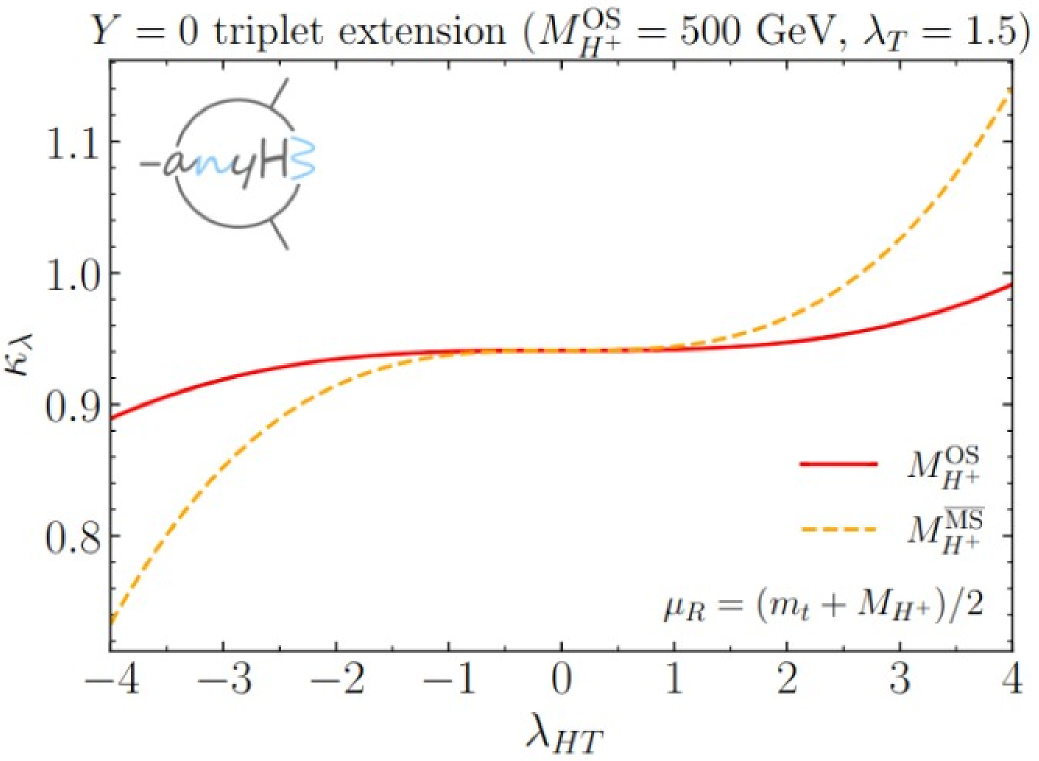
New results III: momentum dependence in a Y=1 triplet extension

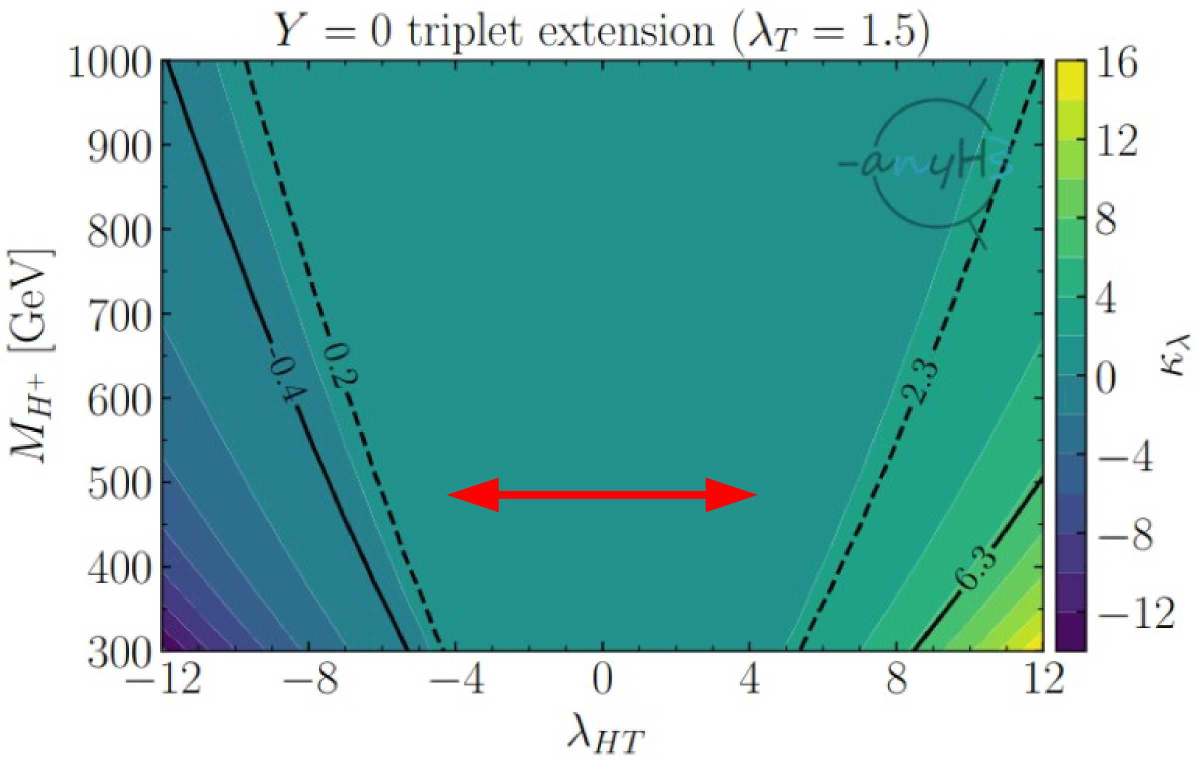


New results IV: renormalisation scheme comparisons

Real (VEV-less) triplet model:

$$V(\Phi, T) = \mu^2|\Phi|^2 + \frac{\lambda}{2}|\Phi|^4 + \frac{M_T^2}{2}|T|^2 + \frac{\lambda_T}{2}|T|^4 + \frac{\lambda_{HT}}{2}|T|^2|\Phi|^2, \;\; \langle T\rangle = 0, \; \langle \Phi \rangle = v_{SM}$$





- *Left*: scheme variation of charged triplet mass $M_{H\pm}$ (enters λ_{hhh} from 1L) → estimate of theoretical uncertainty from missing 2L corrections
- *Right*: κ_λ @ 1L in plane of $M_{H\pm}$ and λ_{HT} (portal coupling)

Future determination of λ_{hhh}

Achieved accuracy actually depends on the value of λ_{hhh}

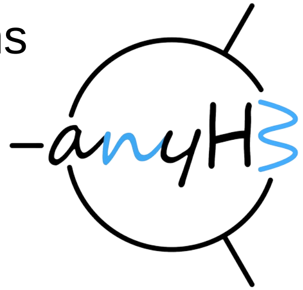
[J. List et al. '21]

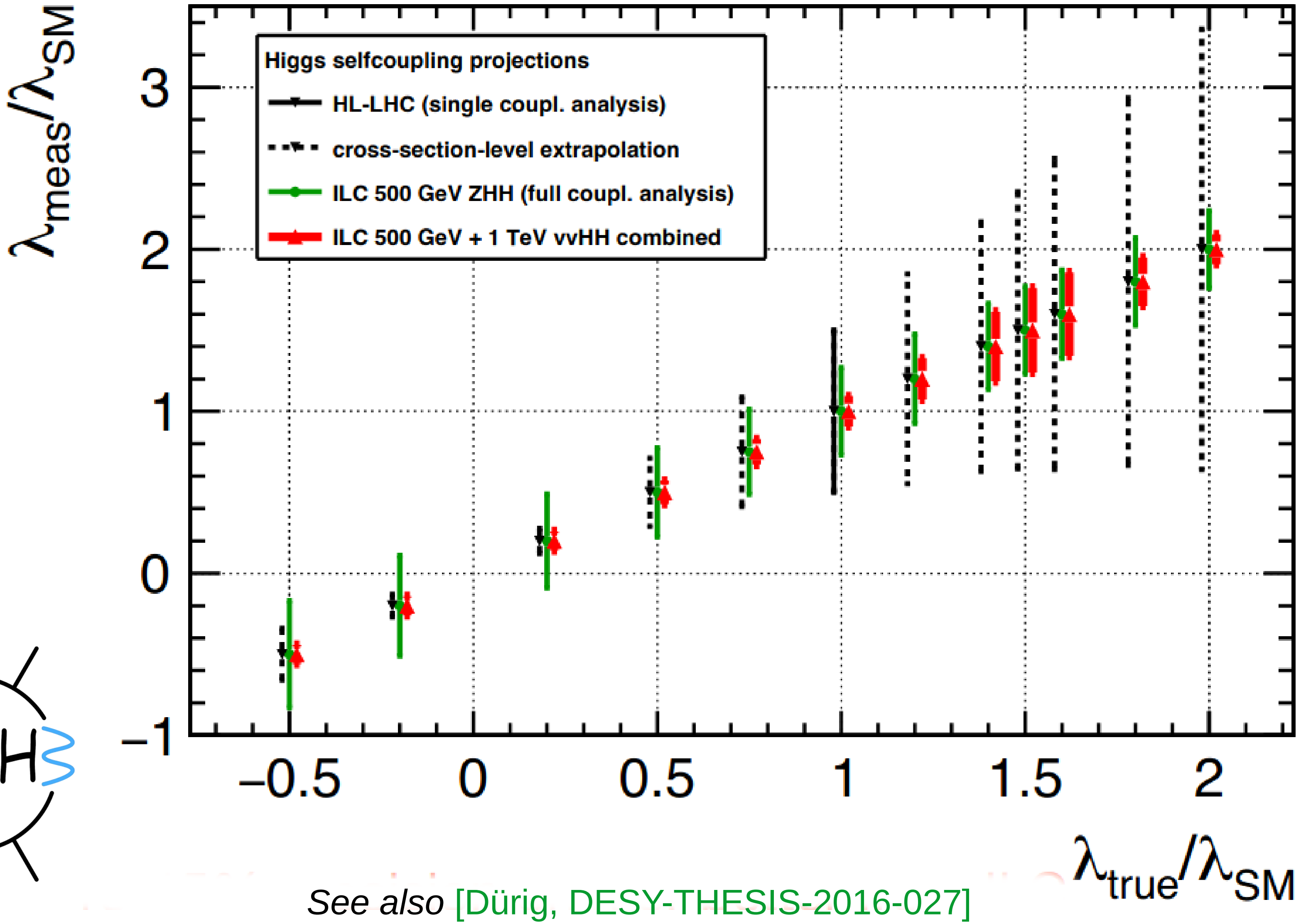
- Scenarios with large BSM deviation in κ_λ possible and motivated
  → EW phase transition
  → early-Universe Cosmology
  → collider signatures (like A → ZH)

- For κ_λ > 1, determination of λ_{hhh} at HL-LHC worse than expected

- **Lepton collider with $\sqrt{s} \geq 500$ GeV needed to access λ_{hhh} to high precision and probe EWPT scenarios!!**

- Necessary theory predictions
  → anyH3
  [Bahl, JB, Gabelmann, Weiglein '23]





See also [Dürig, DESY-THESIS-2016-027]

Summary

- λ_{hhh} plays a crucial role to understand the **shape of the Higgs potential**, and probe indirectly **signs of New Physics**
- **Python package `anyH3` allows calculation of λ_{hhh} for arbitrary renormalisable theories** with
  - Full 1L effects including p^2 dependence
  - Highly flexible choices of renormalisation schemes → predefined or by user
- Uses **UFO** model inputs (generated with SARAH, FeynRules or using custom ones)
- Analytical results (Python, Mathematica)
- Fast numerical results (with caching): SM → O(0.2s); MSSM → O(0.5s)
- Part of wider **anyBSM framework**, under development
- Currently 14 models included, easy inclusion of further models → **new ideas/requests welcome!**

Get started at https://anybsm.gitlab.io/
or directly in terminal with
`pip install anyBSM` **&** `anyBSM --help` **!**

Thank you very much for your attention!

Contact

DESY. Deutsches Elektronen-Synchrotron

www.desy.de

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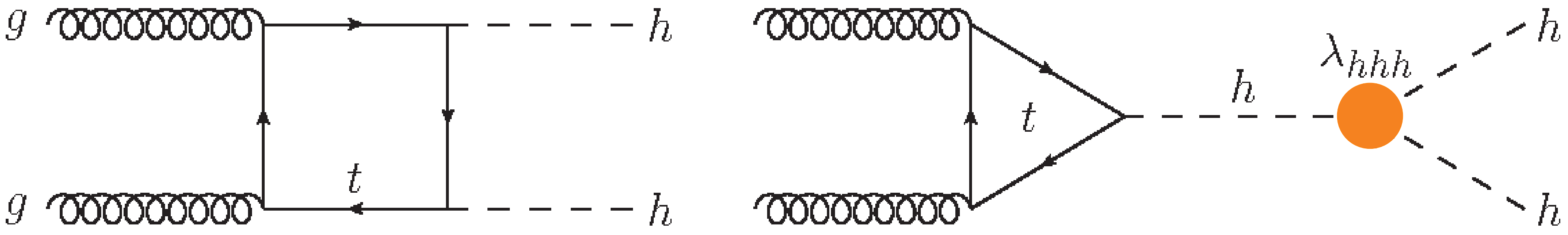
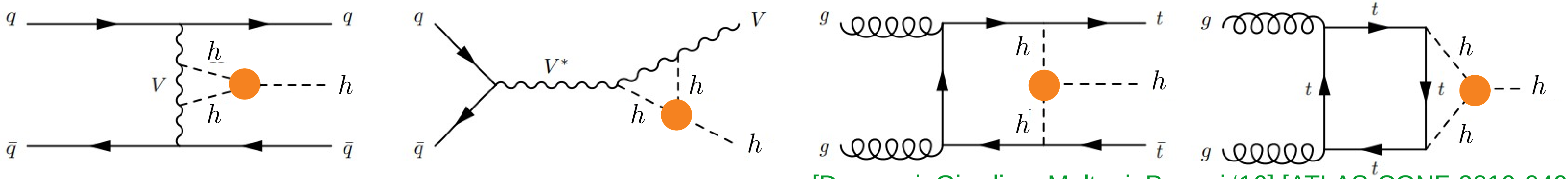
Backup

Experimental probes of λ_{hhh}

- **Double-Higgs production** → λ_{hhh} enters at leading order (LO) → **most direct probe!**

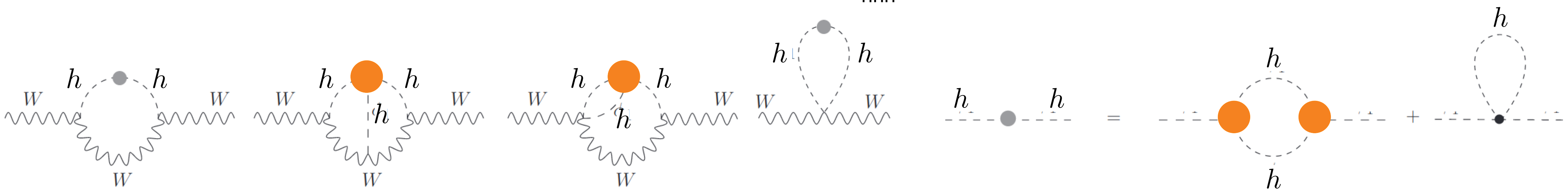


- **Single-Higgs production** → λ_{hhh} enters at NLO



[Degrassi, Giardino, Maltoni, Pagani '16] [ATLAS-CONF-2019-049]

- **Electroweak Precision Observables** (EWPOs) → λ_{hhh} enters at NNLO



[Degrassi, Fedele, Giardino '17]

Accessing λ_{hhh} via double-Higgs production

- **Double-Higgs production** → λ_{hhh} enters at LO → **most direct probe of λ_{hhh}**

Recent results from ATLAS hh-searches [ATLAS-CONF-2022-050] yield the limits:

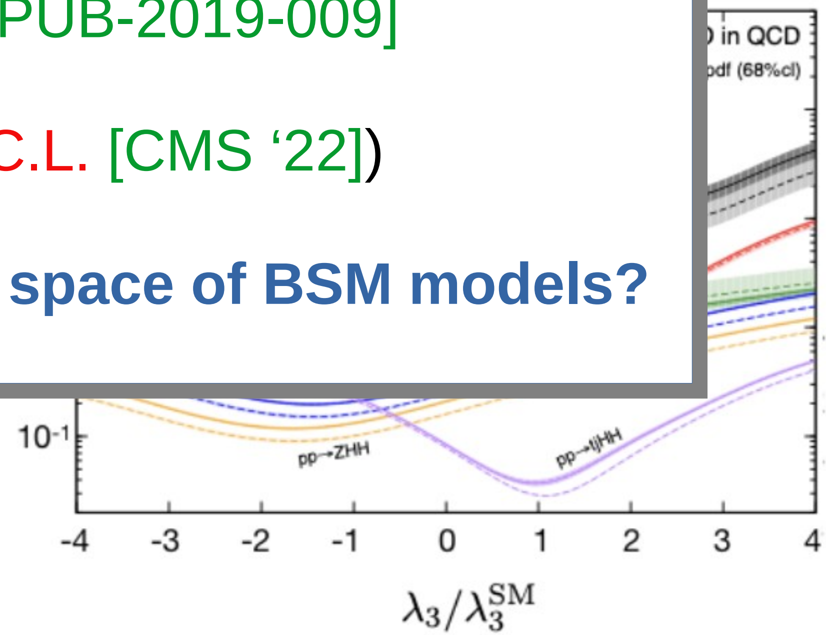
$-0.4 < \kappa_\lambda < 6.3$ at 95% C.L.

→ factor ~2 improvement compared to pre-2021 best ATLAS limits (from single-h prod.)
$-3.2 < \kappa_\lambda < 11.9$ at 95% C.L. [ATLAS-PHYS-PUB-2019-009]

(CMS recently gave **$-1.2 < \kappa_\lambda < 6.5$** at 95% C.L. [CMS '22])

→ **Can κ_λ now be used to constrain the parameter space of BSM models?**

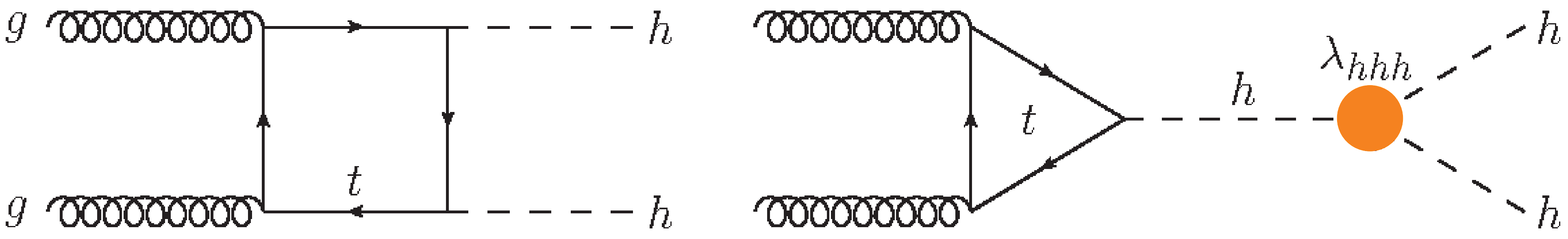
- Box …
  → sr…
  → B…
  hh-p…
- Uppe…
  $\kappa_\lambda \equiv \lambda$…
- κ_λ as an effective coupling → $\mathcal{L} \supset -\kappa_\lambda \times \frac{3m_h^2}{v^2} \cdot h^3 + \cdots$



[Frederix et al., '14]

Accessing λ_{hhh} experimentally

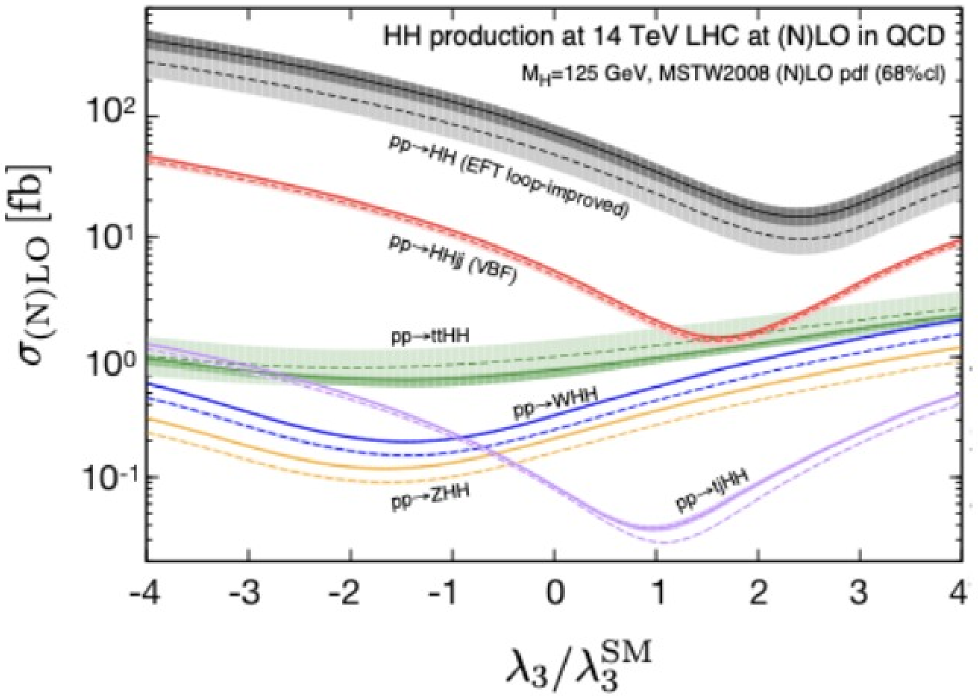
- **Double-Higgs production** → λ_{hhh} enters at LO → **most direct probe of λ_{hhh}**



- Box and triangle diagrams **interfere destructively**
→ small prediction in SM
→ BSM deviation in λ_{hhh} can **significantly alter double-Higgs production!**

- Upper limit on double-Higgs production cross-section
→ **limits on $\kappa_\lambda \equiv \lambda_{hhh}/(\lambda_{hhh}^{(0)})^{SM}$**

- κ_λ as an effective coupling → $\mathcal{L} \supset -\kappa_\lambda \times \frac{3m_h^2}{v^2} \cdot h^3 + \cdots$



[Frederix et al., '14]

Constraining BSM models with λ_{hhh} – details

- Latest experimental bounds

$-0.4 < \kappa_\lambda \equiv \lambda_{hhh}/(\lambda_{hhh}^{(0)})^{SM} < 6.3$

[ATLAS-CONF-2022-050]

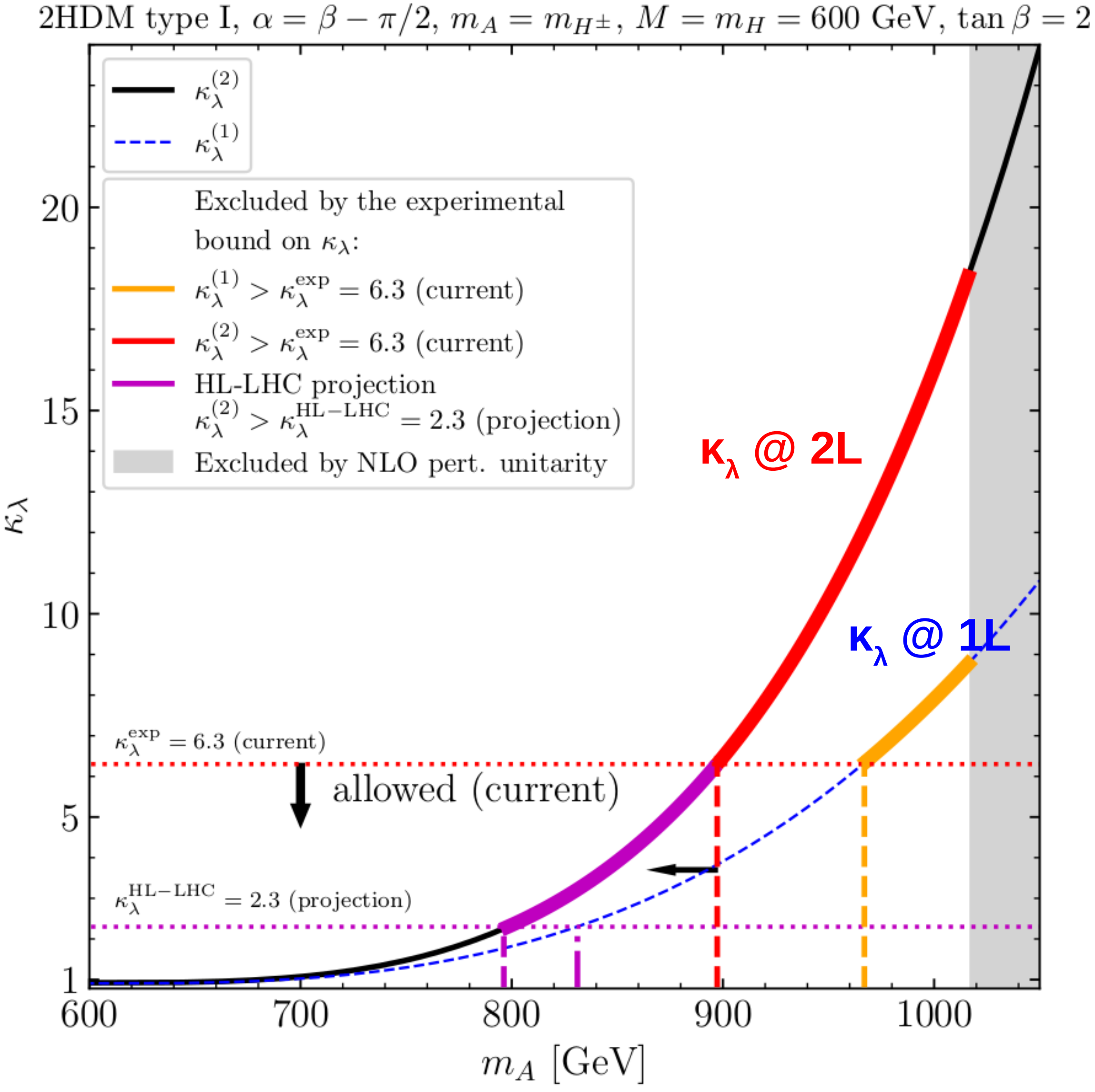
- Comparing these bounds with **precise theory predictions** for λ_{hhh} provides a **powerful new way of constraining BSM models**

- Assumptions for the extraction of bounds on κ_λ:
  - Other couplings of 125-GeV Higgs are SM-like
  - Deviation in di-Higgs production cross-section only due to deviation in κ_λ

→ **true for many BSM models, e.g. with alignment**
→ **couplings of 125-GeV Higgs SM-like at tree level**

- E.g. for an aligned 2HDM scenario
  [Bahl, JB, Weiglein Phys.Rev.Lett. '22]

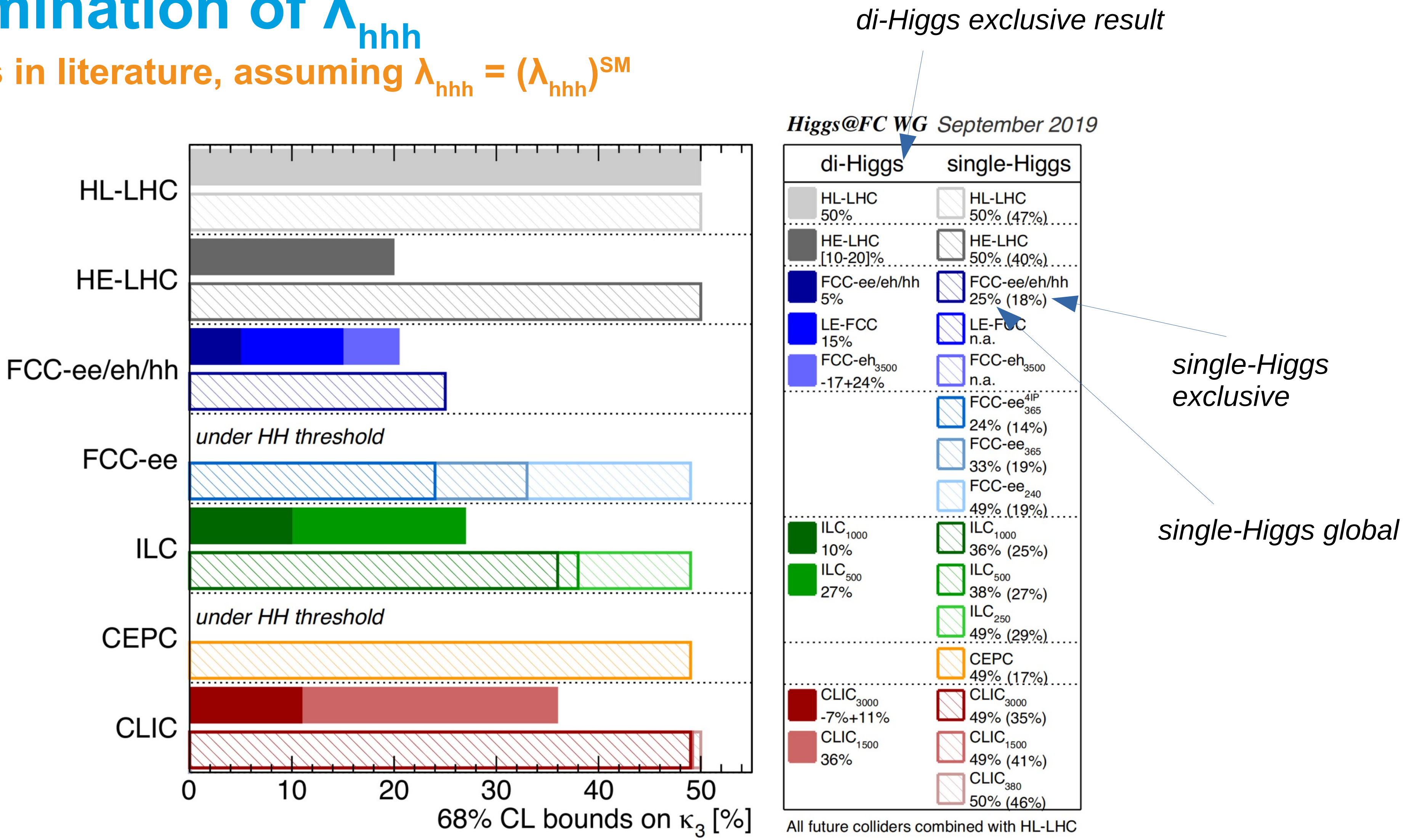
[Bahl, JB, Weiglein *Phys.Rev.Lett.* '22]



Future determination of λ_{hhh}

Expected sensitivities in literature, assuming $\lambda_{hhh} = (\lambda_{hhh})^{SM}$

Plot taken from
[de Blas et al., 1905.03764]



see also [Cepeda et al., 1902.00134], [Di Vita et al.1711.03978], [Fujii et al. 1506.05992, 1710.07621, 1908.11299], [Roloff et al., 1901.05897], [Chang et al. 1804.07130,1908.00753], *etc.*

Future determination of λ_{hhh}

Higgs production cross-sections (here double Higgs production) depend on λ_{hhh}

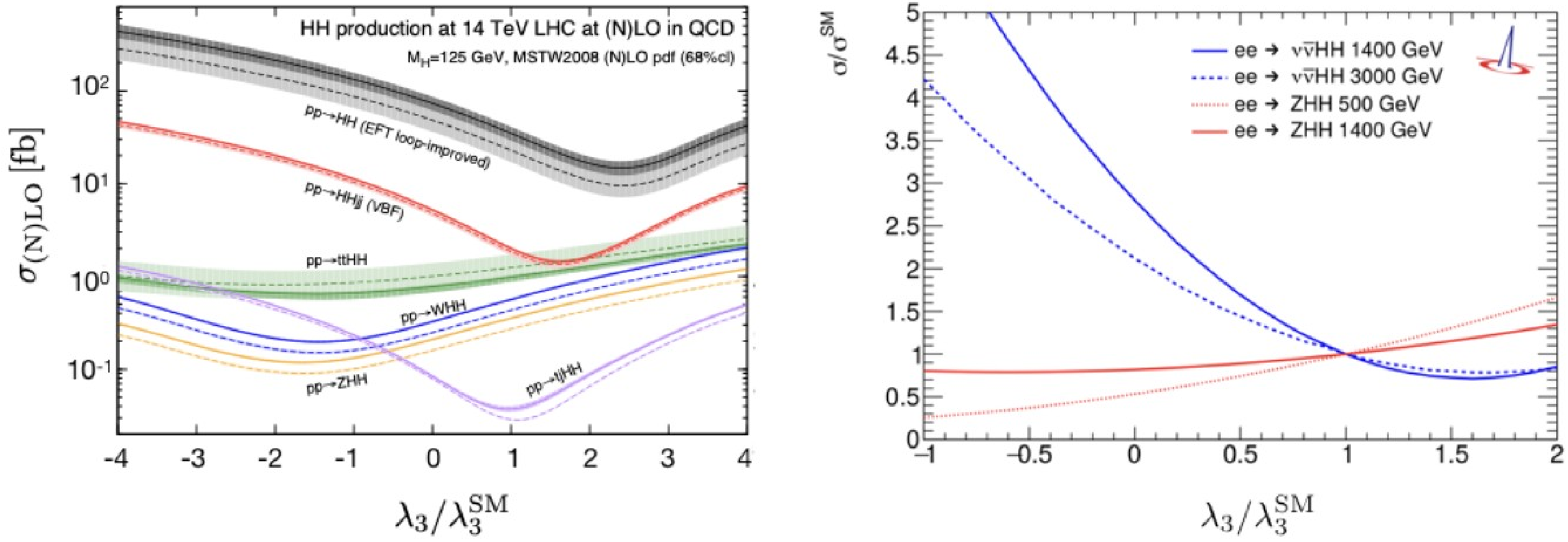
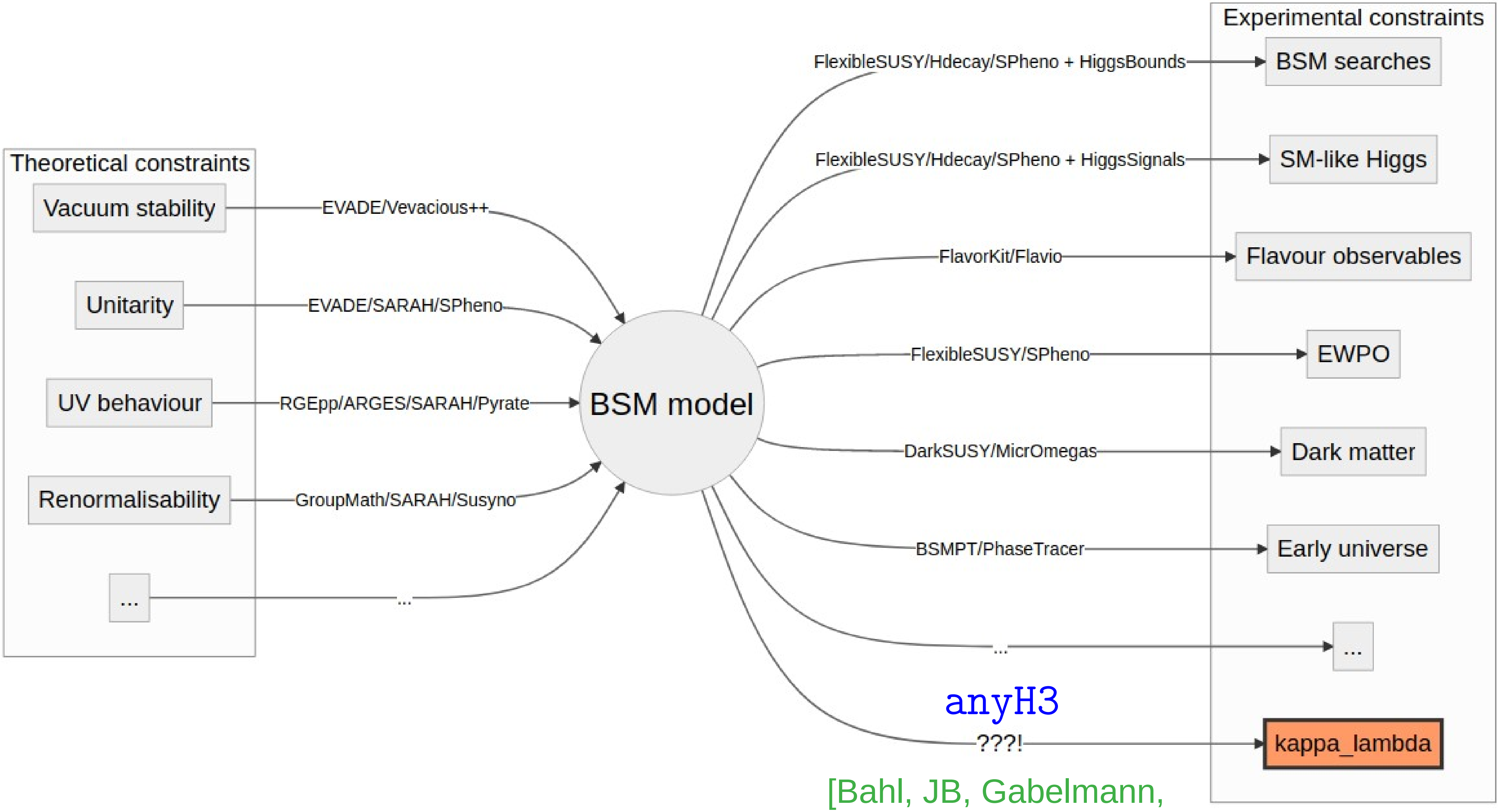


Figure 10. Double Higgs production at hadron (left) [65] and lepton (right) [66] colliders as a function of the modified Higgs cubic self-coupling. See Table 18 for the SM rates. At lepton colliders, the production cross sections do depend on the polarisation but this dependence drops out in the ratios to the SM rates (beam spectrum and QED ISR effects have been included).

[Frederix et al., 1401.7340]

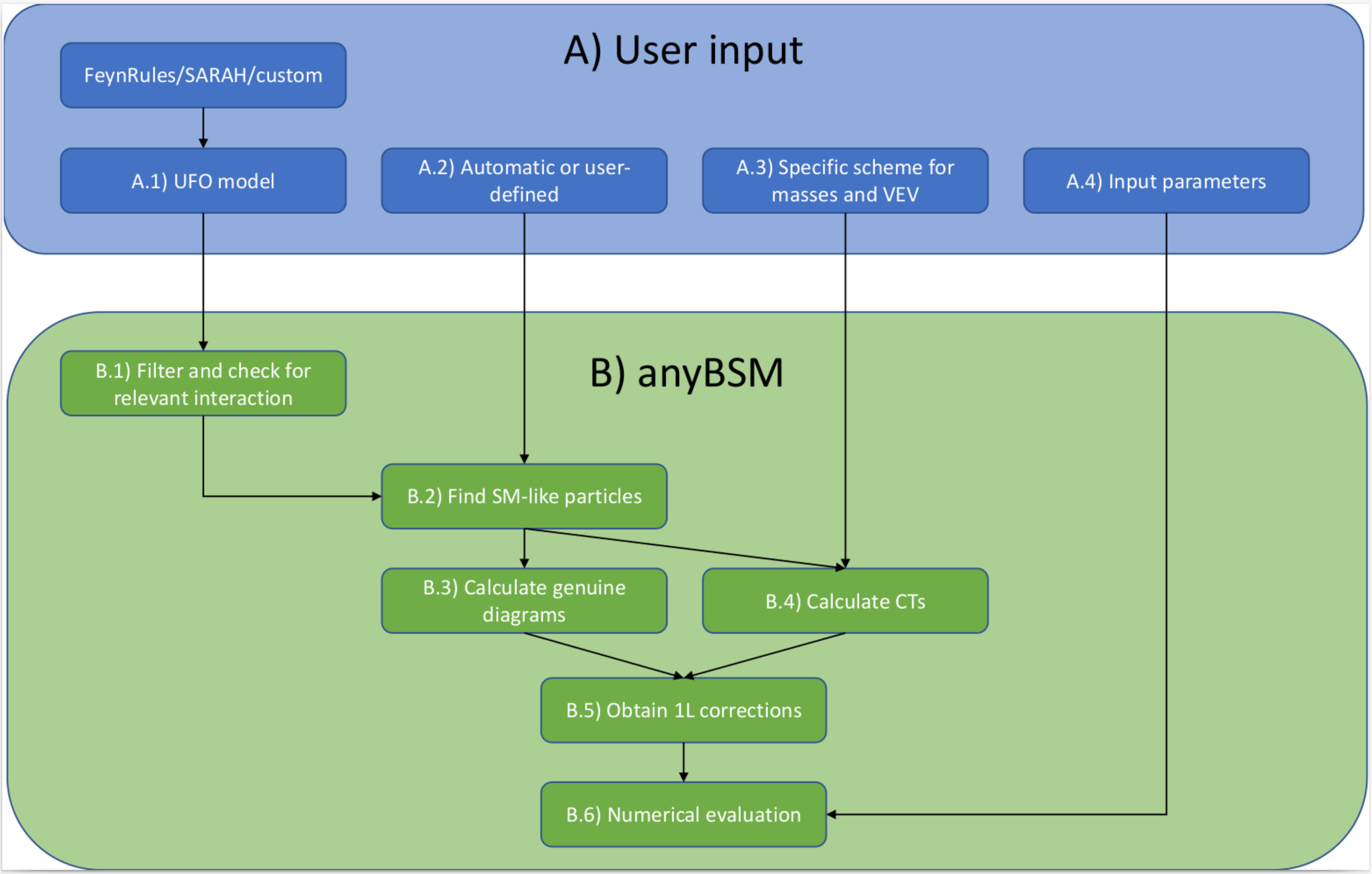
Plots taken from
[de Blas et al., 1905.03764]

λ_{hhh} within the landscape of automated tools



[Bahl, JB, Gabelmann, Weiglein 2305.03015]

Workflow of anyH3



(Default) Renormalization choice of $(v^{\text{SM}})^{\text{OS}}$ and $(m_i^2)^{\text{OS}}$

> $v^{\text{OS}} \equiv \frac{2M_W^{\text{OS}}}{e}\sqrt{1-\frac{M_W^{2\,\text{OS}}}{M_Z^{2\,\text{OS}}}}$ with

- $\delta^{(1)} M_V^{2\,\text{OS}} = \frac{\Pi_V^{(1),T}}{M_V^{2\,OS}}(p^2 = M_V^{2\,OS})\,,\, V = W, Z$
- $\delta^{(1)} e^{\text{OS}} = \frac{1}{2}\dot{\Pi}_\gamma(p^2=0) + \text{sign}(\sin\theta_W)\frac{\sin\theta_W}{M_Z^2\cos\theta_W}\Pi_{\gamma Z}(p^2=0)$

> attention *(i)*: $\rho^{\text{tree-level}} \neq 1 \rightarrow$ further CTs needed (depends on the model)

$\rightarrow$ ability to define *custom* renormalisation conditions

> scalar masses: $m_i^{\text{OS}} = m_i^{\text{pole}}$

- $\delta^{\text{OS}} m_i^2 = -\widetilde{\text{Re}}\Sigma_{h_i}^{(1)}|_{p^2=m_i^2}$
- $\delta^{\text{OS}} Z_i = \widetilde{\text{Re}}\frac{\partial}{\partial p^2}\Sigma_{h_i}^{(1)}|_{p^2=m_i^2}$

> attention *(ii)*: scalar mixing may also require further CTs/tree-level relations

All bosonic one- & two-point functions and their derivatives for general QFTs are required for flexible OS renormalisation.

Features of anyH3, so far

- Import/conversion of any UFO model
- Definition of renormalisation schemes

schemes.yml:

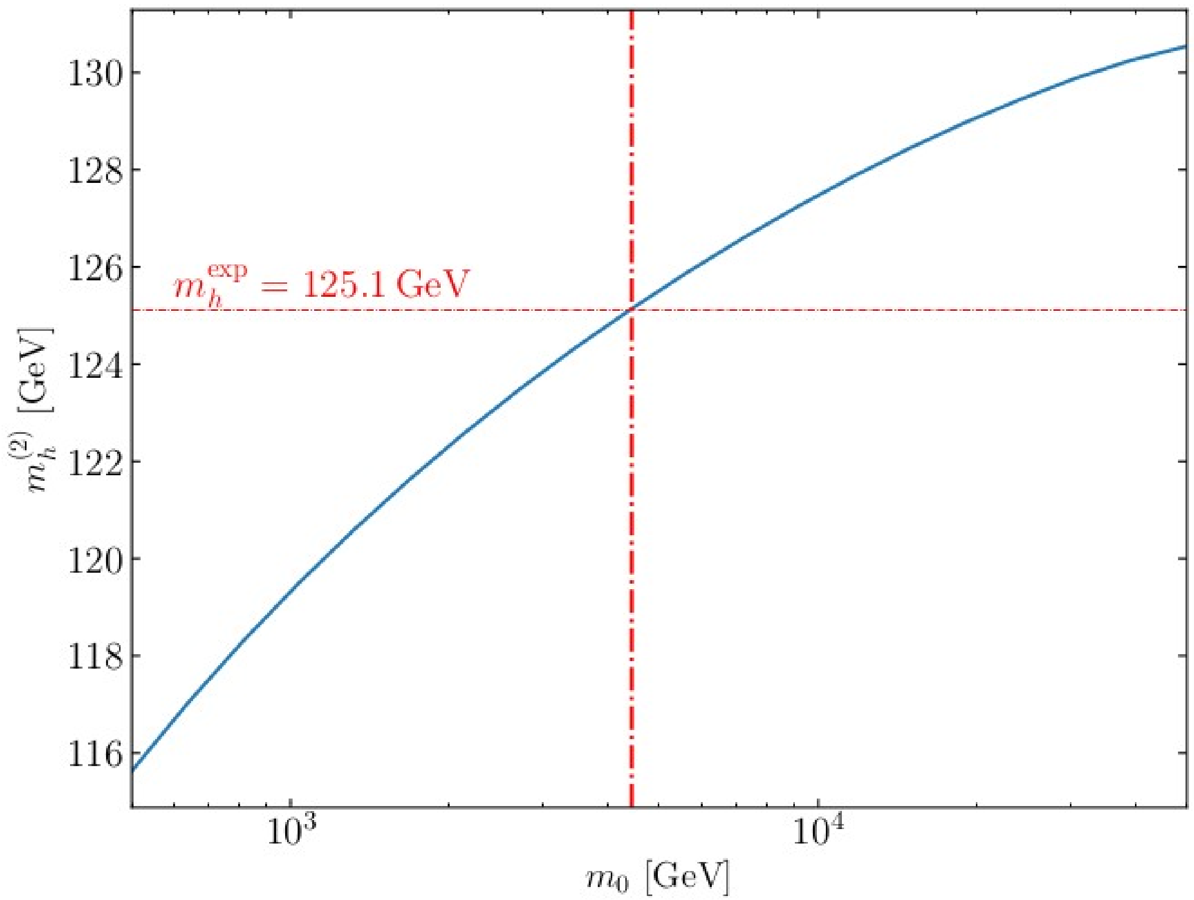
Example for 2HDM

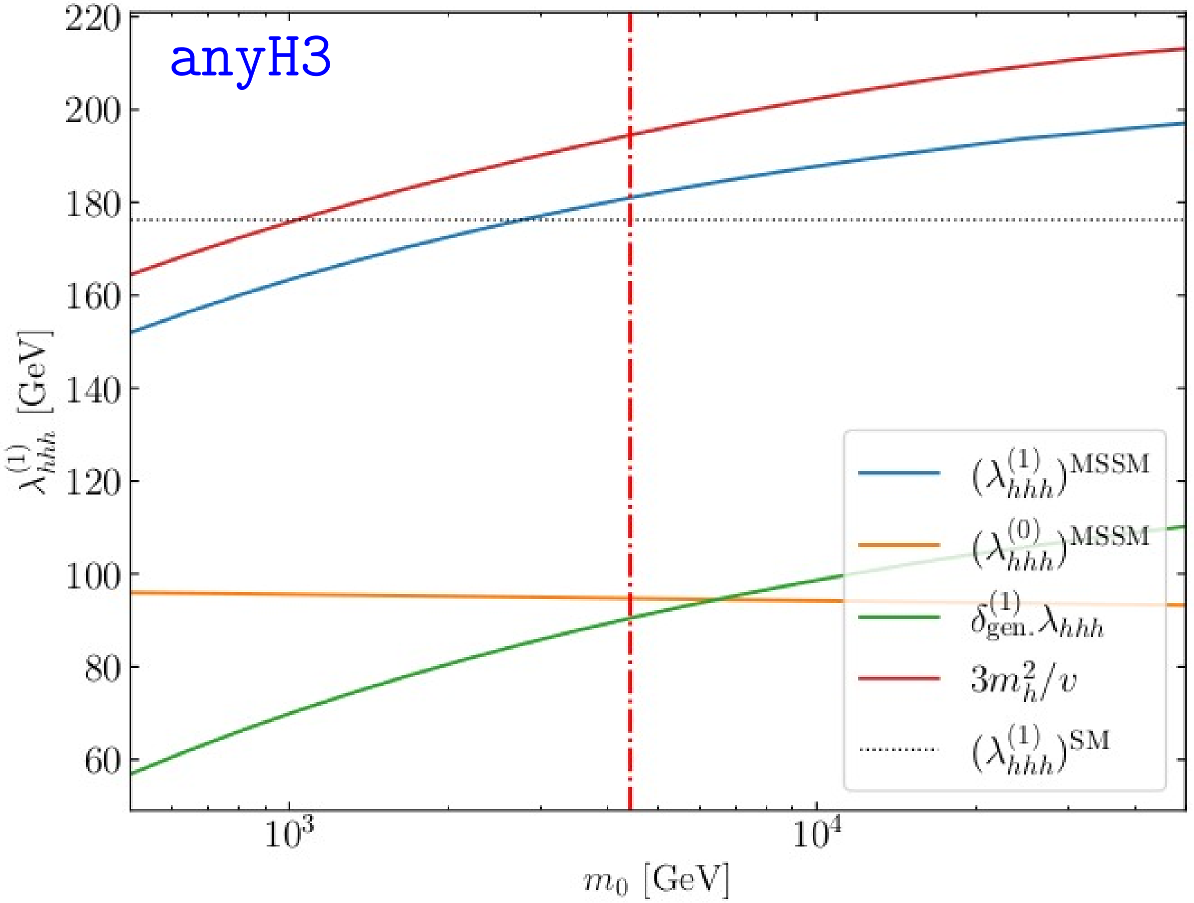
```yaml
default_scheme: OSalignment

renormalization_schemes:
  MS:
    description: all (B)SM parameters MS
    SM_names:
      Higgs-Boson: h1
    VEV_counterterm: MS
    mass_counterterms:
      h1: MS
      h2: MS

  OSalignment:
    description: $\overline{\mathrm{MS}}$ mixing angles and OS masses i.e. fully on-shell $\lambda_{hhh}$ for $\sin {\beta-\alpha}=1$
    SM_names:
      Higgs-Boson: h1
    VEV_counterterm: OS
    mass_counterterms:
      h1: OS
      h2: OS

  OS:
    description: OS conditions for scalar masses as well
```

- Analytical / numerical / LaTeX outputs
- Restrictions on topologies or on considered particles possible
- **3 user interfaces**:
  - Python library
  - Command line
  - Mathematica interface
- **Perturbative unitarity checks** available (at tree level and in high-energy limit for now)
- Can be used together with a spectrum generator and **handles SLHA format**
- Efficient **caching** available
- Etc.

New results V: full one-loop calculation of λ_{hhh} in the MSSM

CMSSM, $m_0 = m_{1/2} = -A_0$, $\tan\beta = 10$, $\mathrm{sgn}(\mu) = 1$, with m_h computed at 2L in SPheno





- Example for a very simple version of the constrained MSSM → BSM parameters m_0, $m_{1/2}$, A_0, sgn(μ), tanβ
- For each point, M_h computed at 2L with SPheno, and SLHA output of SPheno used as input of anyH3