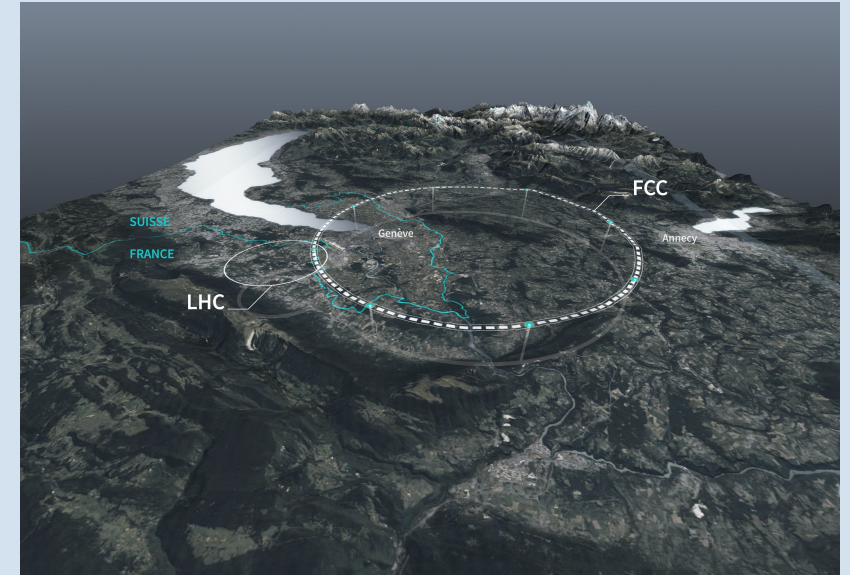


# Second ECFA Workshop on $e^+e^-$ Higgs/EW/Top Factories

## Paestum, October 11-13 2023



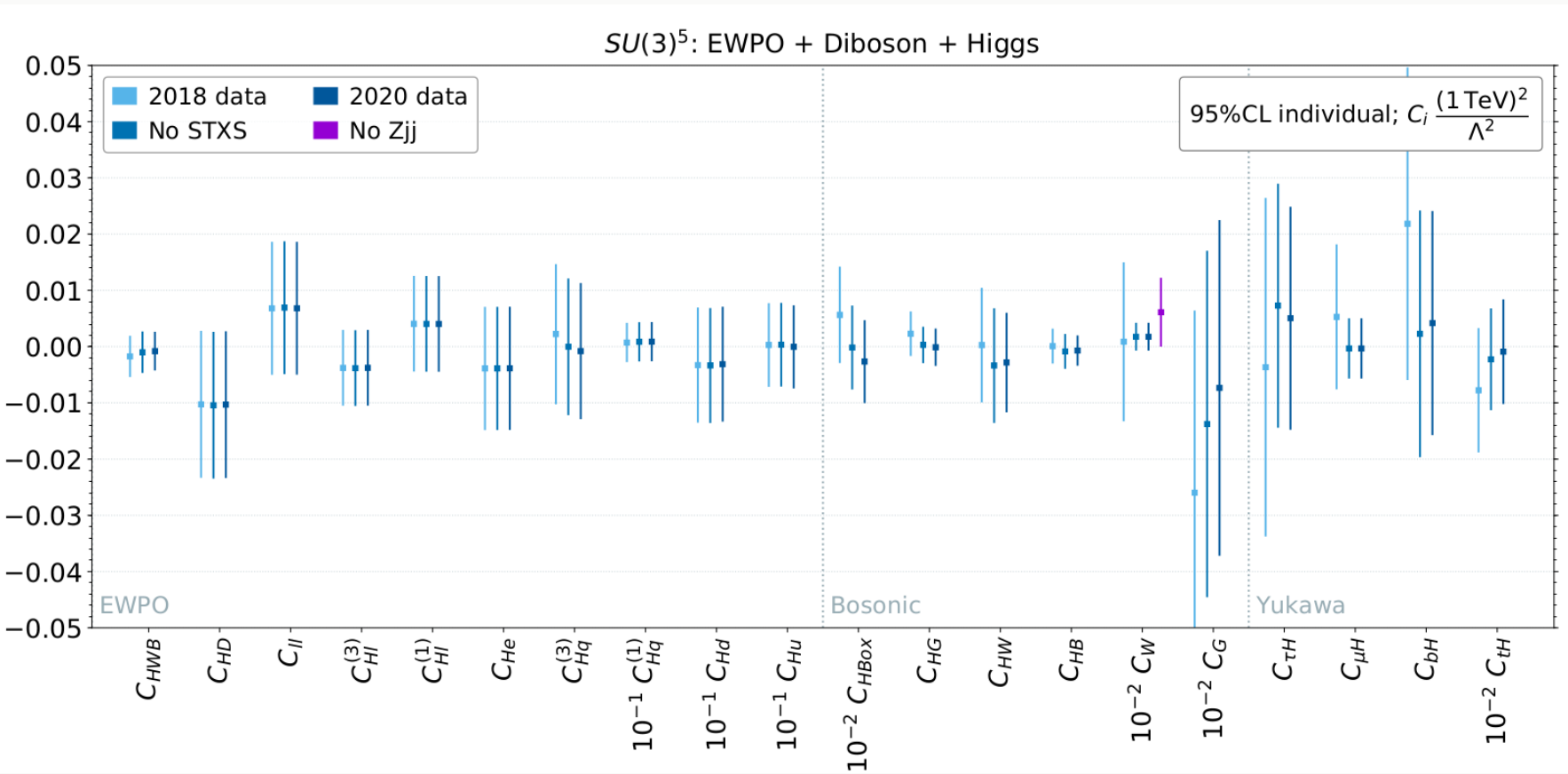
## Probing Radiative Neutrino Masses and Extra Fermions at the Future Circular Collider

Based on 2303.12232, CM and Aurora Melis

*Carlo Marzo,*  
Tallinn, Estonia



# Theory&Pheno in the Standard Model: $\sim 60$ years of harmony



Dim-6 operators live at TeV scale to appreciate per-cent level deviations

Heavy NP for  $\sim 0.01$  couplings

Theory&Pheno in the Standard Model:  
~ 60 years of harmony ...

Heavy NP for  
~ 0.01 couplings

but NP unavoidable

Neutrino Masses and Mixing

Strong CP

non-trivial CKM

Baryon Asymmetry

Inflation

Dark Matter

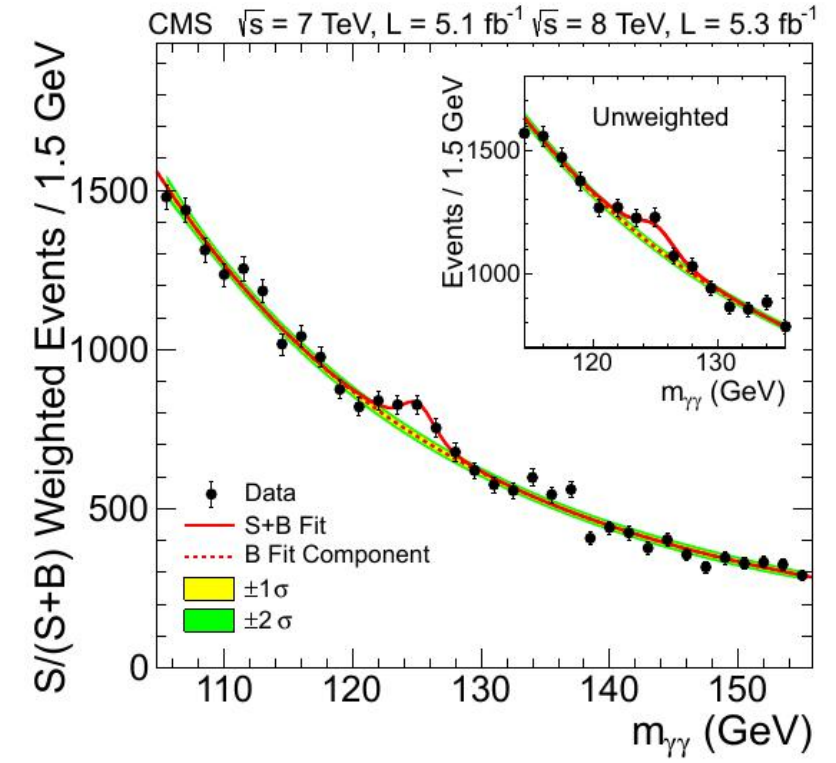
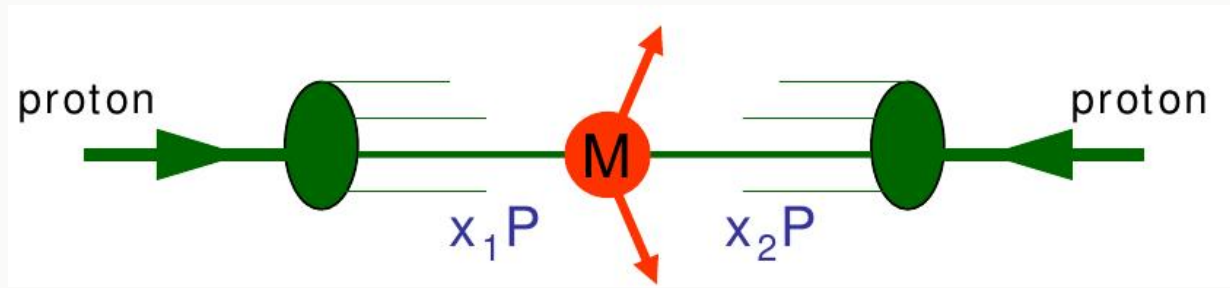
W-mass?

Dark Energy

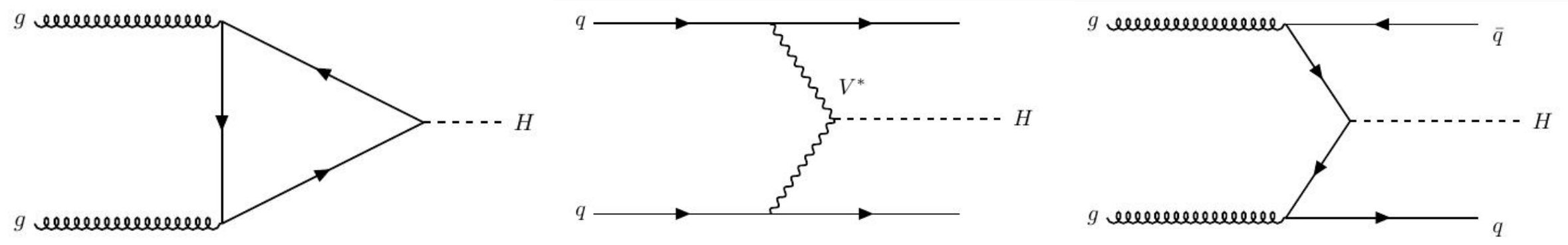
g-2?

# LHC: Discovery through production

## We found the Higgs!



8 TeV for  $\sim$  EW scale resonance



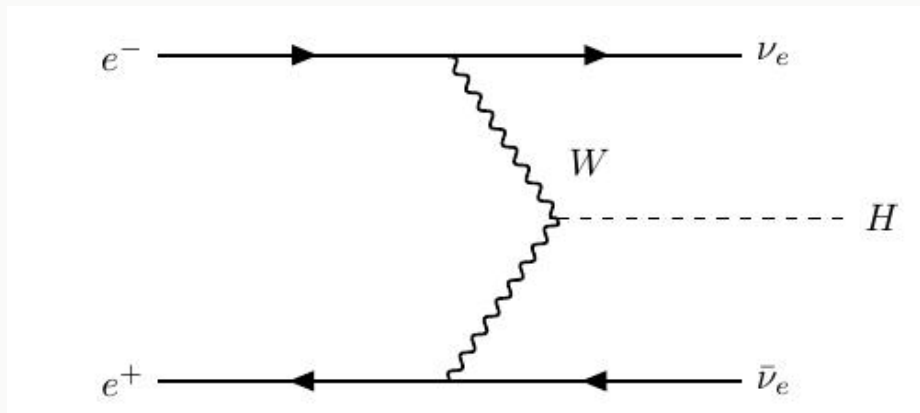
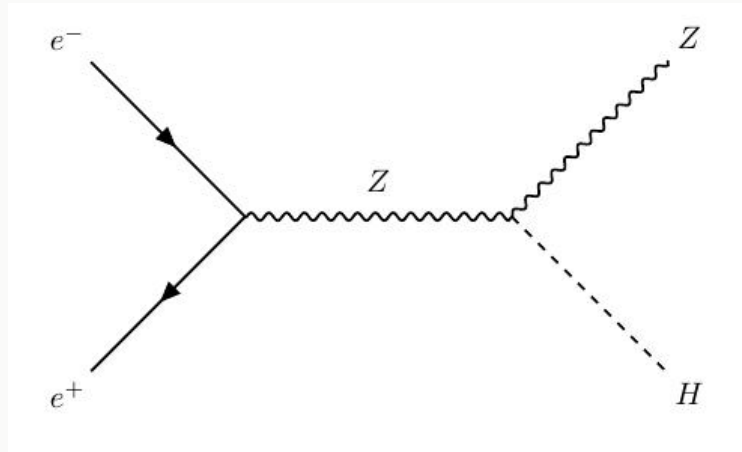
# FCC-ee: Discovery\* through precision

(\*Of NP)

(+ **HL-LHC**)

$e^+e^-$  at  $\sqrt{s} = 240, 365$  GeV

Higgs production:



”Baseline plan:

$\sim 10^6$  events in ZH

$\sim 10^5$  events in WWH ”

” $\sigma_{ZH}$  can be determined  
...with an ultimate  
statistical precision of 0.1%...”

**A special Higgs challenge**

Paolo Azzurri, Gregorio Bernardi, Sylvie Braibant, Davide d’Enterria et al.

**Opportunity for THEORY:**  
Heavy NP indirectly accessible

(like in LEP!)

# FCC-ee: Discovery through precision : Higgs-strahlung

**Opportunity for THEORY:**  
Heavy NP indirectly accessible

*helicity/polarization structure*

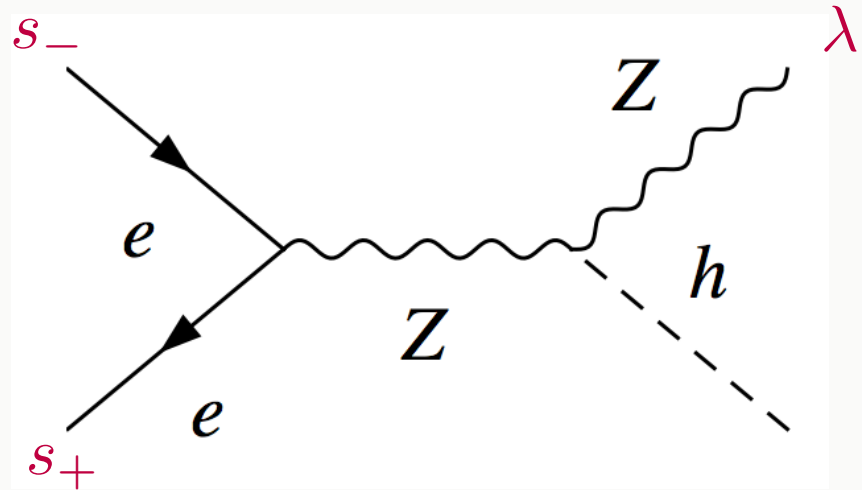
$$\mathcal{M}(\lambda; s_+, s_-) = \sum_j F_j \mathcal{M}_j(\lambda; s_+, s_-)$$

..if we have equally precise control over SM background

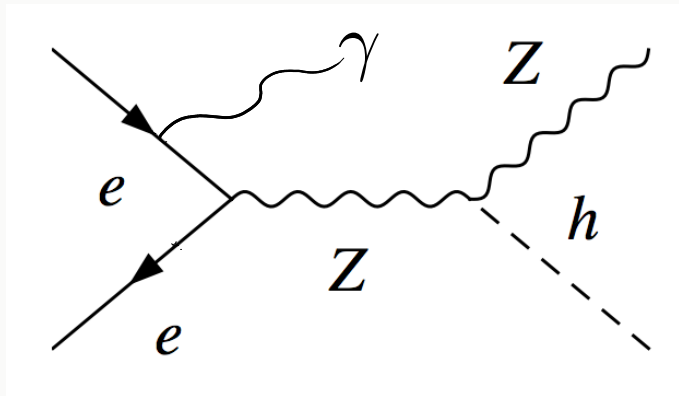
(+NLO)

$$|\mathcal{M}^{\text{NLO}}|^2 \simeq \sum_{ij} (F_i^{\text{LO}})^* (F_j^{\text{LO}} + 2 \delta F_j) \text{Re}[\mathcal{M}_i^* \mathcal{M}_j],$$

2 SM NLO contributions:



# FCC-ee: Discovery through precision : Higgs-strahlung

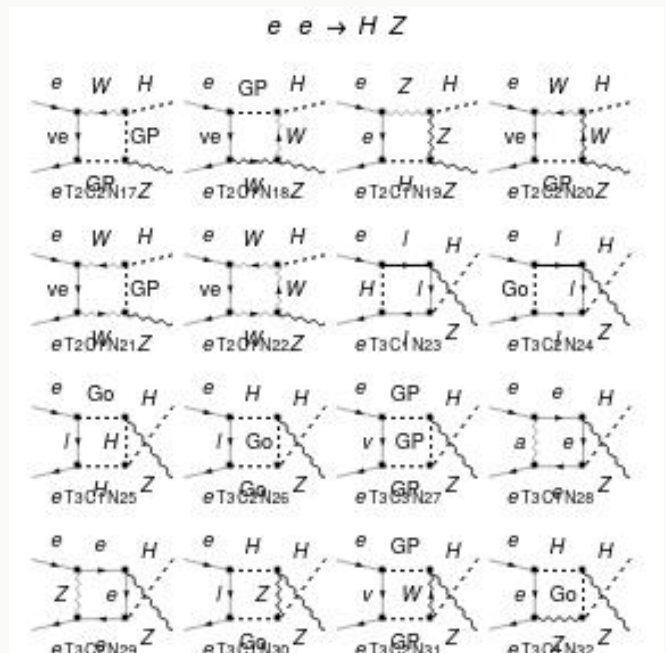
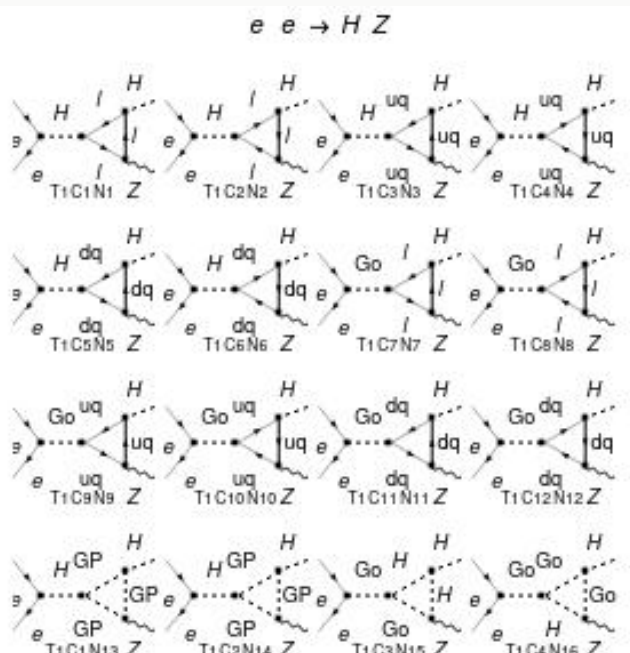
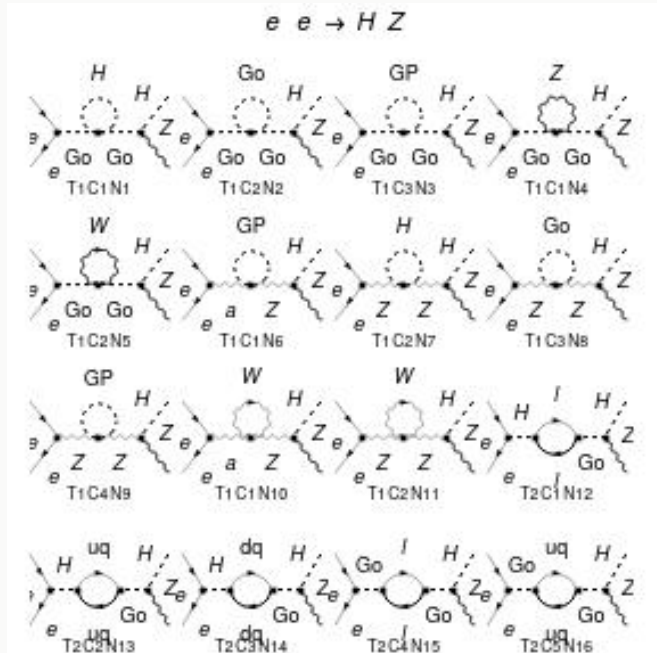


soft- $\gamma$  radiation (IR)

**Opportunity for THEORY:**  
Heavy NP indirectly accessible

..if we have equally precise control over SM background

+NLO



EW

# FCC-ee: Discovery through precision : Higgs-strahlung

..if we have equally precise control over SM background

**Getting finite results is desirable:**

Dimensional regularization

+

ON-Shell renormalization (Sirlin '80, see also Denner '91)

$$g_1, g_w, Y_u, Y_d, Y_e, v_h, \lambda, \mu_1^2$$



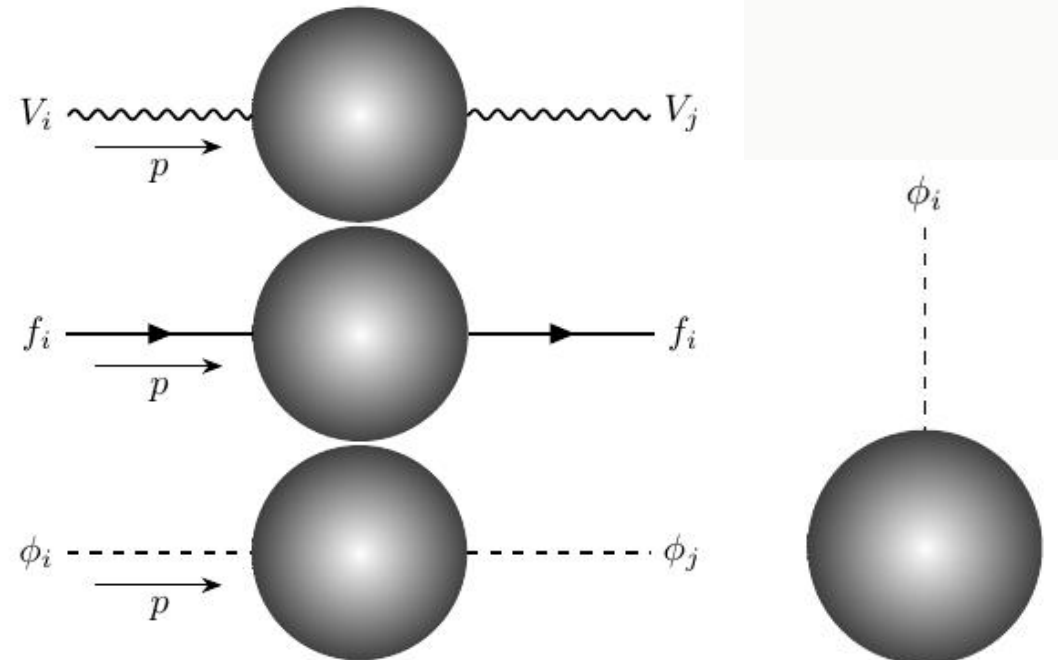
$$m_Z^2, m_W^2, e, m_{u_i}, m_{d_i}, m_{l=e,\mu,\tau}, m_h^2, t$$



$$\begin{aligned} m_Z^2 &\rightarrow m_Z^2 + \delta m_Z^2, & m_W^2 &\rightarrow m_W^2 + \delta m_W^2, \\ m_f &\rightarrow m_f + \delta m_f, & (f = u, d, l) \\ m_h^2 &\rightarrow m_h^2 + \delta m_h^2, & t &\rightarrow t + \delta t, \\ e &\rightarrow e(1 + \delta e) \end{aligned}$$



**Renormalization conditions:**





# FCC-ee: Discovery through precision : Higgs-strahlung

..if we have equally precise control over SM background

**Getting finite results is desirable:**

Dimensional regularization

+

ON-Shell renormalization (Sirlin '80, see also Denner '91)

=

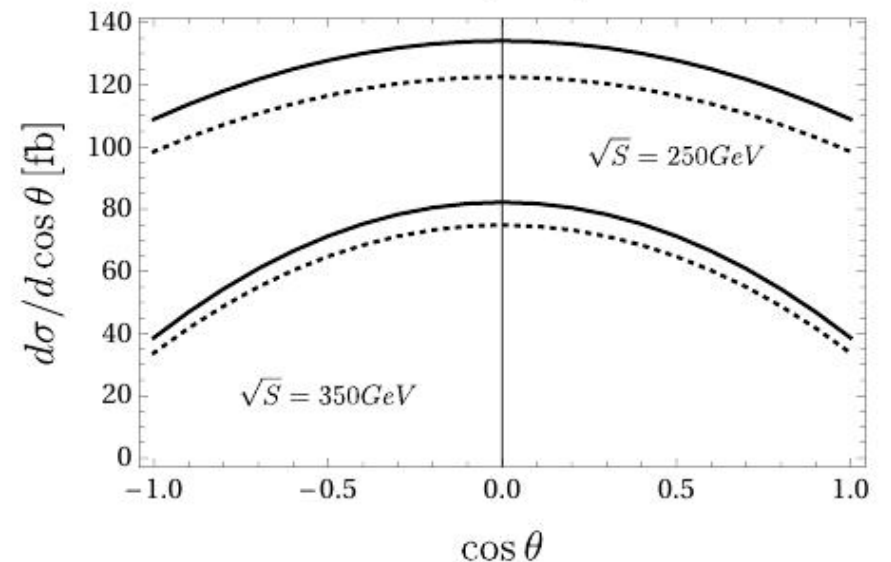
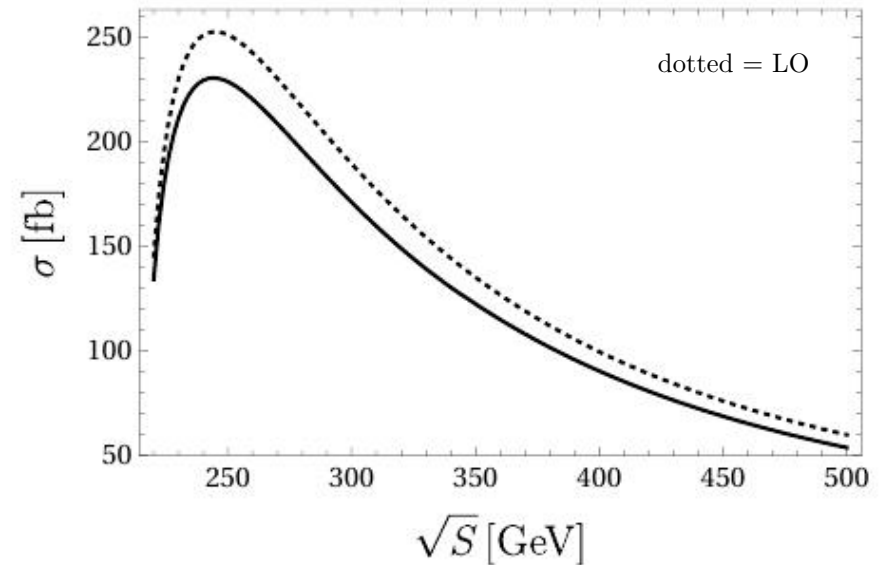
IR and UV finite result

$$|\mathcal{M}^{\text{NLO}}|^2 \simeq \sum_{ij} (F_i^{\text{LO}})^* (F_j^{\text{LO}} + 2 \delta F_j) \text{Re}[\mathcal{M}_i^* \mathcal{M}_j]$$

$$\sigma_{Zh}^{\text{SM}}(250\text{GeV}) = 228.748[\text{fb}],$$

$$\sigma_{Zh}^{\text{SM}}(350\text{GeV}) = 123.392[\text{fb}]$$

$$\underline{\text{unpol } \sigma_{Zh}^{\text{SM}} \sim 25[\text{fb}] \text{ LO-NLO}}$$



# FCC-ee: Discovery through precision : Higgs-strahlung

We are now ready to address BSM (affecting NLO):

$$\mathcal{M}(\lambda; s_+, s_-) = \sum_j F_j \mathcal{M}_j(\lambda; s_+, s_-)$$

$$|\mathcal{M}^{\text{NLO}}|^2 \simeq \sum_{ij} (F_i^{\text{LO}})^* (F_j^{\text{LO}} + 2\delta F_j) \text{Re}[\mathcal{M}_i^* \mathcal{M}_j]$$

(where  $F^{\text{LO}}$  = LO and  $\delta F_j$  = NLO form factors)

$$F_j^{\text{LO}} = F_j^{\text{Born}}$$
$$\delta F_j = \delta F_j^{\text{SM}} + \delta F_j^{\text{NP}}$$

$$|\mathcal{M}^{\text{NLO}}|^2 \simeq |\mathcal{M}^{\text{Born}}|^2 + 2\text{Re}[\mathcal{M}^{*\text{Born}} (\delta\mathcal{M}^{\text{SM}} + \delta\mathcal{M}^{\text{NP}})]$$

$$|\mathcal{M}^{\text{NLO}}|^2 \simeq |\mathcal{M}_{\text{SM}}^{\text{NLO}}|^2 + 2\text{Re}[\mathcal{M}^{*\text{Born}} \delta\mathcal{M}^{\text{NP}}]$$

$\sigma_{Zh}$  linear in NP!

# FCC-ee: Discovery through precision : Higgs-strahlung

A minimal/common extension: extra doublet.

## Inert Doublet Model

(2009.03250, Abouabid, Arhrib, Benbrik et Al.)

$$H_1 = \begin{pmatrix} G^+ \\ \frac{v_h + h + iG^0}{\sqrt{2}} \end{pmatrix}, \quad H_2 = \begin{pmatrix} H^+ \\ \frac{H^0 + iA^0}{\sqrt{2}} \end{pmatrix}$$

(EW vacuum, no vev for  $H_2$ )

$$\begin{aligned} \mathcal{L}^{\text{IDM}} = & \mathcal{L}^{\text{SM}} + |D_\mu H_2|^2 - \mu_2^2 |H_2|^2 - \lambda_2 |H_2|^4 \\ & - \lambda_3 |H_1|^2 |H_2|^2 - \lambda_4 |H_1^\dagger H_2|^2 \\ & - \frac{\lambda_5}{2} \left[ (H_1^\dagger H_2)^2 + \text{h.c.} \right] \end{aligned}$$

(no scalar mixing, no SM fermion coupling)

$$\begin{aligned} m_{H^\pm}^2 &= m_0^2 - \lambda_4 \frac{v_h^2}{2}, \\ m_{H^0}^2 &= m_0^2 + \lambda_5 \frac{v_h^2}{2}, \\ m_{A^0}^2 &= m_0^2 - \lambda_5 \frac{v_h^2}{2}. \end{aligned}$$

$$(m_0^2 \equiv \frac{1}{2}(m_{H^0}^2 + m_{A^0}^2) = \mu_2^2 + (\lambda_3 + \lambda_4) \frac{v_h^2}{2})$$

# Inert Doublet Model: a portal for BSM NP

(from 2009.03250, Abouabid, Arhrib, Benbrik et Al.)

$$\lambda_i < 8\pi \quad (\text{perturbativity})$$

$$\lambda_2 > 0, \quad \lambda_3, \lambda_3 + \lambda_4 - |\lambda_5| > -2\sqrt{\lambda_1\lambda_2} \quad (\lambda_1 \sim 0.132) \quad (\text{vacuum stability})$$

**Theo bounds:**

$$|e_i^\pm| < 8\pi \quad \begin{aligned} e_1^\pm &= \lambda_3 \pm \lambda_{4,5}, & e_2 &= \lambda_3 + 2\lambda_4 \pm 3\lambda_5, \\ e_4^\pm &= -(\lambda_1 + \lambda_2) \pm \sqrt{(\lambda_1 - \lambda_2)^2 + \lambda_{4,5}^2}, \\ e_5^\pm &= -3(\lambda_1 + \lambda_2) \pm \sqrt{9(\lambda_1 - \lambda_2)^2 + (2\lambda_3 + \lambda_4)^2} \end{aligned} \quad (\text{perturbative unitarity})$$

Precise measurements of  $Z$  and  $W$  widths at **LEP** forbids  
 $Z \rightarrow H^+H^-$ ,  $Z \rightarrow H^0A^0$  and  $W^\pm \rightarrow H^\pm H^0(A^0)$ :

$$2m_{H^\pm} > m_Z, \quad m_{H^0} + m_{A^0} > m_Z, \\ m_{H^\pm} + m_{H^0, A^0} > m_W$$

**Pheno bounds:**

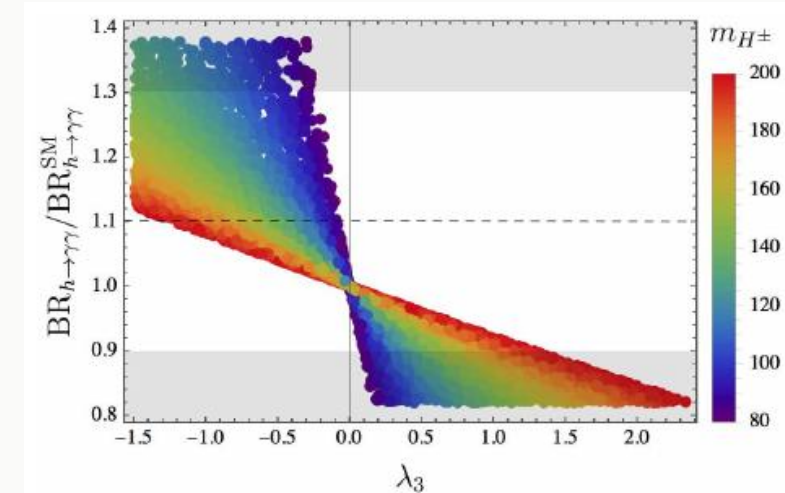
**LEP II** chargino searches:  $m_{H^\pm} > \{70 - 90\} \text{ GeV}$

**LEP II** neutralino searches  
do not apply!

Higgs data at **LHC**

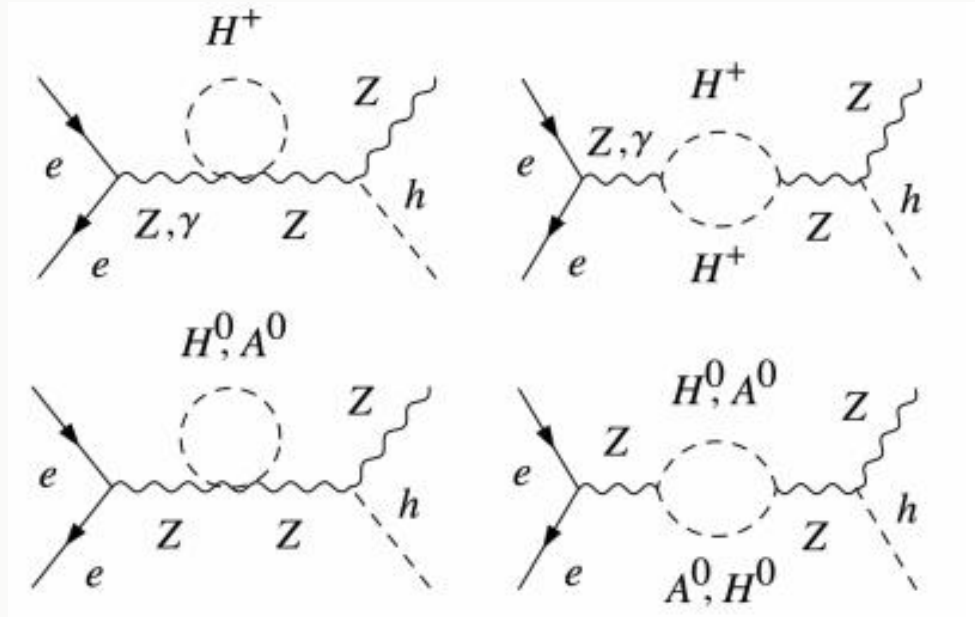
$h \rightarrow \text{invisible}$ :  $m_{H^0, A^0} > m_h/2$

Radiative contributions on  $h \rightarrow \gamma\gamma$  from  $H^\pm$ : constraint on  $\lambda_3$



~~Oblique Parameters?~~

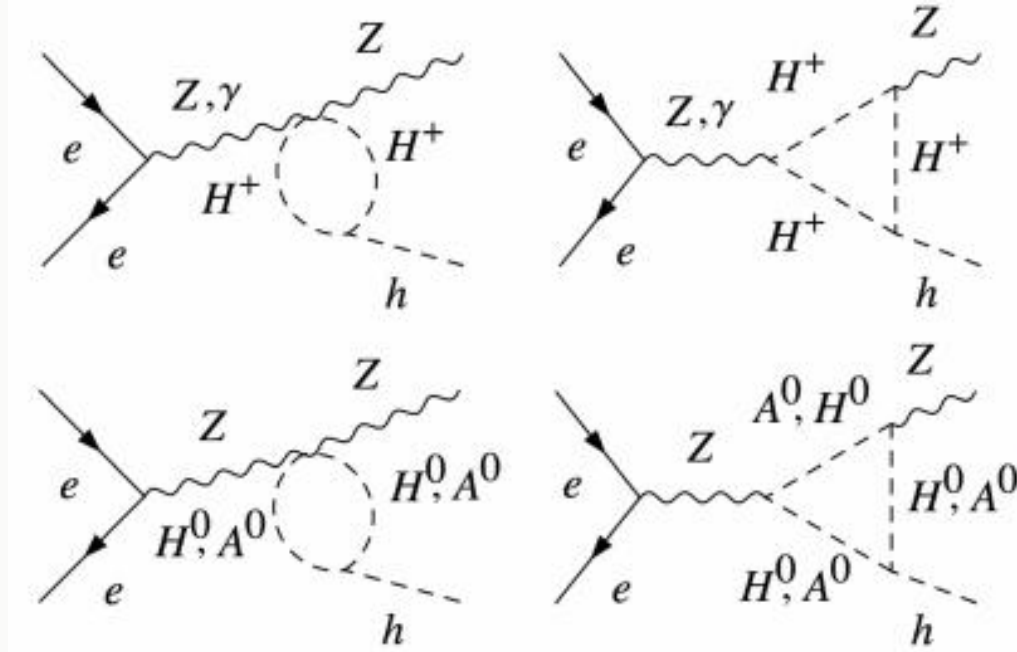
# Inert Doublet Model



$$\delta F_{L,R}^{S, \text{IDM}} = \frac{\alpha F_{L,R}^{\text{LO}}}{8\pi c_W^2 s_W^2 (S - m_Z^2)} \times \left\{ (c_W^2 - s_W^2)^2 \left[ 1 - \frac{2c_W s_W (1 - m_Z^2/S)}{g_{L,R}^{Ze} (c_W^2 - s_W^2)} \right] \times \left( 2B_{00}[S, m_{H^\pm}, m_{H^\pm}] - A_0[m_{H^\pm}] \right) + \left( 2B_{00}[S, m_{A^0}, m_{H^0}] - \frac{A_0[m_{H^0}] + A_0[m_{A^0}]}{2} \right) \right\},$$

$$(\delta F_{L,R}^{S, \text{IDM}})^{\text{ct}} = F_{L,R}^{\text{LO}} \left[ \frac{\delta m_Z^2}{S - m_Z^2} - \delta_Z - \frac{\delta_{AZ}}{2g_{L,R}^{Ze}} + \frac{\delta_{ZA}}{2g_{L,R}^{Ze}} \left( 1 - \frac{m_Z^2}{S} \right) \right]$$

# Inert Doublet Model



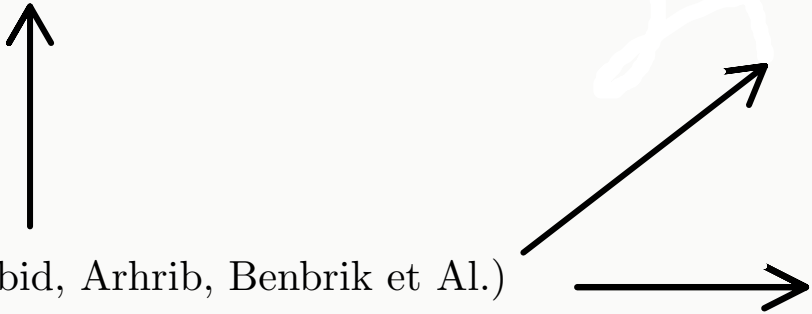
$$\delta F_{L,R}^{V, \text{IDM}} = \frac{F_{L,R}^{\text{LO}}}{16\pi^2} \left\{ (c_W^2 - s_W^2)^2 \left[ 1 - \frac{2c_W s_W (1 - m_Z^2/S)}{g_{L,R}^Z (c_W^2 - s_W^2)} \right] \right. \\
\times \lambda_3 \left( -B_0[m_h^2, m_{H^\pm}^2, m_{H^\pm}^2] \right. \\
\left. \left. + 4C_{00}[m_Z^2, m_h^2, S, m_{H^\pm}^2, m_{H^\pm}^2, m_{H^\pm}^2] \right) \right. \\
\left. + \frac{\lambda_3 + \lambda_4 + \lambda_5}{2} \left( -B_0[m_h^2, m_{H^0}^2, m_{H^0}^2] \right. \right. \\
\left. \left. + 4C_{00}[m_Z^2, m_h^2, S, m_{A^0}^2, m_{H^0}^2, m_{H^0}^2] \right) \right. \\
\left. + \frac{\lambda_3 + \lambda_4 - \lambda_5}{2} \left( -B_0[m_h^2, m_{A^0}^2, m_{A^0}^2] \right. \right. \\
\left. \left. + 4C_{00}[m_Z^2, m_h^2, S, m_{H^0}^2, m_{A^0}^2, m_{A^0}^2] \right) \right\},$$

$$(\delta F_{L,R}^{V, \text{IDM}})^{\text{ct}} = F_{L,R}^{\text{LO}} \left[ \frac{\delta h}{2} + \delta_Z + \delta_e + \frac{\delta m_Z^2}{2m_Z^2} \right. \\
\left. + \frac{c_W^2 - s_W^2}{2s_W^2} \left( \frac{\delta m_Z^2}{m_Z^2} - \frac{\delta m_W^2}{m_W^2} \right) \right]$$

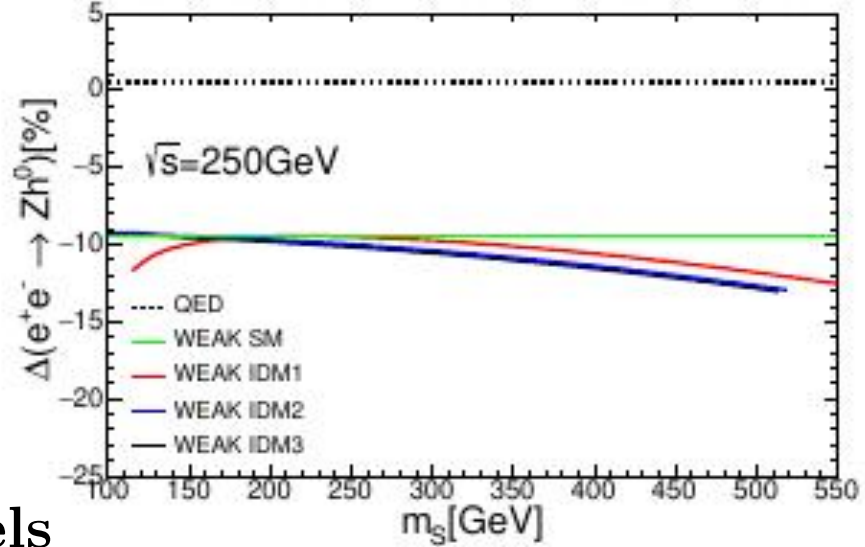
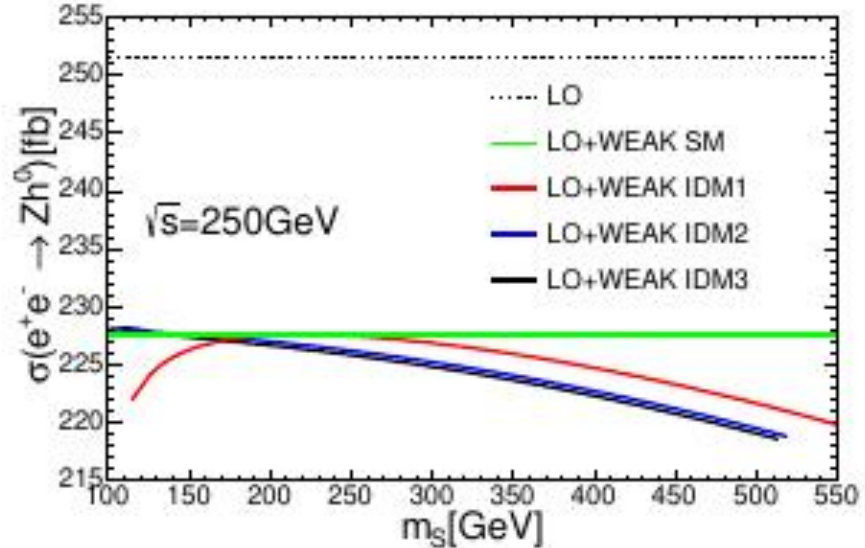
# Inert Doublet Model (past results)

”... it is observed that weak corrections in the IDM are typically negative and can reach 9%- 14% at  $\sqrt{S} = 250$  GeV...”

(from 2009.03250, Abouabid, Arhrib, Benbrik et Al.)



The IDM, a common IR-tail, would provide a large background for many NP models



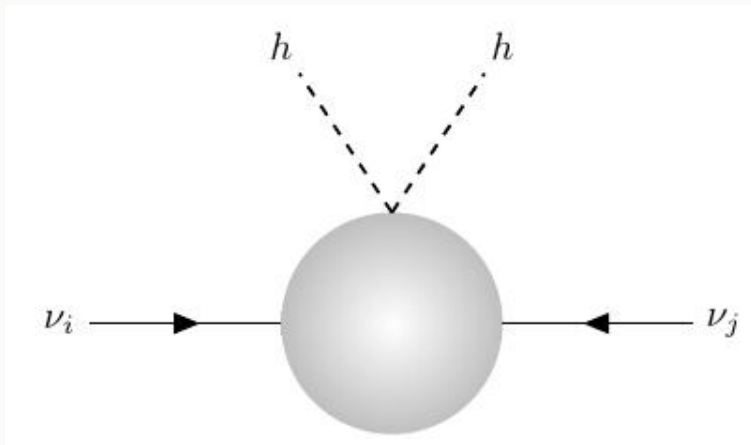
# Inert Doublet Model + sterile fermions $N_k$

A gift from the inert-doublet *portal*. By adding extra fermions:

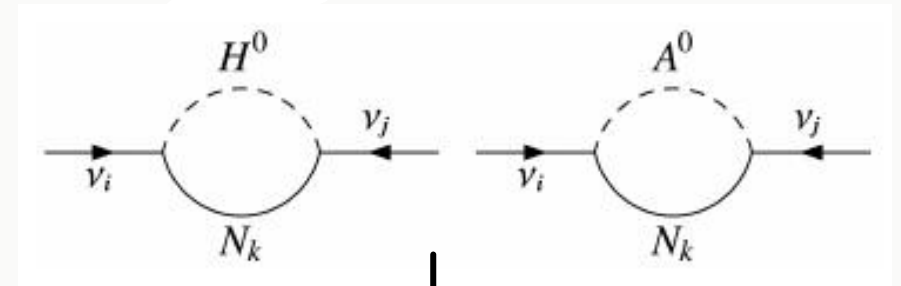
$$(\mathcal{L}^{\text{Scoto}} = \mathcal{L}^{\text{IDM}} + \mathcal{L}^{\text{RHN}})$$

$$\mathcal{L}^{\text{RHN}} = \bar{N}_k \gamma^\mu \partial_\mu N_k - y_{ik}^N \bar{L}_i \tilde{H}_2 N_k - \frac{m_{N_k}}{2} \bar{N}_k^c N_k + \text{h.c.}$$

Radiative realization of dim-5  $\bar{L}_i \cdot \tilde{H}_1 H_1^T \cdot L_j^c$



SSB  $\nearrow$



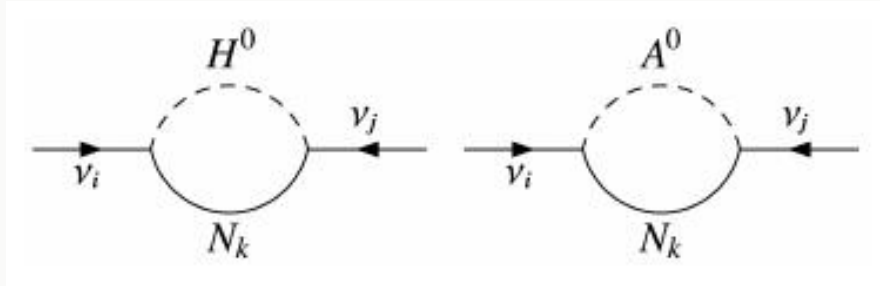
$\downarrow$   
massive neutrino

Scotogenic (dark) model - (Ernest Ma, 2006)



# Inert Doublet Model + sterile fermions $N_k$

Radiative realization of dim-5  $\bar{L}_i \cdot \tilde{H}_1 H_1^T \cdot L_j^c$



$$(m_\nu)_{ij} = (Y_N \Lambda Y_N^T)_{ij} = \sum_{k=1}^3 y_{ik}^N y_{jk}^N \Lambda_k$$

$$\Lambda_k = \frac{m_{N_k}}{16\pi^2} \left( B_0[0, m_{H^0}^2, m_{N_k}^2] - B_0[0, m_{A^0}^2, m_{N_k}^2] \right)$$

$H^0$  and  $A^0$  give equals but opposite in sign contributions:  
 neutrino masses controlled by mass splitting  $m_{H^0}^2 - m_{A^0}^2 = \lambda_5 v_h^2$

---

limit:  $m_0 \simeq m_{N_k}$   
 $\lambda_5 \rightarrow 0$

$$(m_\nu)_{ij} \simeq \frac{\lambda_5 v_h^2}{32\pi^2} \sum_{k=1}^3 \frac{y_{ik}^N y_{jk}^N}{m_{N_k}}$$

size of the see-saw scale reduced by a factor of

$$\lambda_5 v_h^2 / 32\pi^2$$

Scotogenic (dark) model - (Ernest Ma, 2006)

# Radiative neutrino model meets FCC-ee

$$\mathcal{L}^{\text{RHN}} = \bar{N}_k \gamma^\mu \partial_\mu N_k - y_{ik}^N \bar{L}_i \tilde{H}_2 N_k - \frac{m_{N_k}}{2} \bar{N}_k^c N_k + \text{h.c.}$$

..but also..

# Portal Doublet + secluded $N_k$ meet FCC-ee

$$\mathcal{L}^{\text{RHN}} = \bar{N}_k \gamma^\mu \partial_\mu N_k - y_{ik}^N \bar{L}_i \tilde{H}_2 N_k - \frac{m_{N_k}}{2} \bar{N}_k^c N_k + \text{h.c.}$$

and no neutrino masses..

# Radiative neutrino model meets FCC-ee

$$\mathcal{L}^{\text{RHN}} = \bar{N}_k \gamma^\mu \partial_\mu N_k - y_{ik}^N \bar{L}_i \tilde{H}_2 N_k - \frac{m_{N_k}}{2} \bar{N}_k^c N_k + \text{h.c.}$$

$N_{k=1,2,3}$

pick a generation number

Different Neutrino Pheno  
Different Lepton-Flavour signals

- 1) Neutrino Pheno
- 2) Survive LFV, address future LFV
- 3) Collider constraints (beyond oblique)
- 4) FCC-ee signal shaped by  $N_k$

*(Generalizing the Scotogenic model - Escribano, Reig, Vicente)*

*(LFV in the Scotogenic model - Toma, Vicente)*

# Radiative neutrino model meets FCC-ee

Neutrino observable	NH	IH
$\Delta m_{21}^2 [10^{-5} \text{eV}^2]$	$7.42^{+0.21}_{-0.20}$	$7.42^{+0.21}_{-0.20}$
$\Delta m_{3l}^2 [10^{-3} \text{eV}^2]$	$+2.515^{+0.028}_{-0.028}$	$-2.498^{+0.028}_{-0.029}$
$\sin^2 \theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.304^{+0.012}_{-0.012}$
$\sin^2 \theta_{13}$	$0.02220^{+0.00068}_{-0.00062}$	$0.02238^{+0.00064}_{-0.00062}$
$\sin^2 \theta_{23}$	$0.573^{+0.018}_{-0.023}$	$0.578^{+0.017}_{-0.021}$
$\delta_{\text{CP}} [^\circ]$	$194^{+52}_{-25}$	$287^{+27}_{-32}$

(Esteban, Gonzalez-Garcia, Maltoni, Schwetz, Zhou, 2007.14792)

- 1) **Neutrino Pheno**
- 2) Survive LFV, address future LFV
- 3) Collider constraints (beyond oblique)
- 4) FCC-ee signal shaped by  $N_k$

$$(m_\nu)_{ij} \simeq \frac{\lambda_5 v_h^2}{32\pi^2} \sum_{k=1}^3 \frac{y_{ik}^N y_{jk}^N}{m_{N_k}} \quad \text{must comply with data (PMNS): Casas-Ibarra reverse problem}$$

$$Y_N = U^{\nu*} \sqrt{\widehat{m}_\nu} R \sqrt{\Lambda_k}^{-1}$$

We have some freedom:  $R \in \mathbb{C}$  such  $R^T R = \mathbb{I}$

$$y_{ik}^N = \frac{\sqrt{m_1} U_{i1}^{\nu*} R_{1k} + \sqrt{m_2} U_{i2}^{\nu*} R_{2k} + \sqrt{m_3} U_{i3}^{\nu*} R_{3k}}{\sqrt{\Lambda_k}}$$

↓ escaping LFV!

# Radiative neutrino model meets FCC-ee

LFV Process	Current Limit	Future Limit
$\text{BR}(\mu \rightarrow e\gamma)$	$4.2 \times 10^{-13}$ (MEG at PSI)	$6 \times 10^{-14}$ (MEG II)
$\text{BR}(\tau \rightarrow e\gamma)$	$3.3 \times 10^{-8}$ (BaBar)	$5 \times 10^{-9}$ (Belle II)
$\text{BR}(\tau \rightarrow \mu\gamma)$	$4.4 \times 10^{-8}$ (BaBar)	$10^{-9}$ (Belle II)
$\text{BR}(\mu \rightarrow 3e)$	$1.0 \times 10^{-12}$ (SINDRUM)	$10^{-16}$ (Mu3e)
$\text{BR}(\tau \rightarrow 3e)$	$2.7 \times 10^{-8}$ (Belle)	$5 \times 10^{-10}$ (Belle II)
$\text{BR}(\tau \rightarrow 3\mu)$	$2.1 \times 10^{-8}$ (Belle)	$5 \times 10^{-10}$ (Belle II)
$\text{BR}(Z \rightarrow \mu e)$	$7.5 \times 10^{-7}$ (LHC ATLAS)	$10^{-10} - 10^{-8}$ (FCC-ee)
$\text{BR}(Z \rightarrow \tau e)$	$9.8 \times 10^{-6}$ (LEP OPAL)	$10^{-9}$ (FCC-ee)
$\text{BR}(Z \rightarrow \tau\mu)$	$1.2 \times 10^{-5}$ (LHC DELPHI)	$10^{-9}$ (FCC-ee)
$\text{BR}(h \rightarrow \mu e)$	$6.1 \times 10^{-5}$ (LHC CMS)	—
$\text{BR}(h \rightarrow \tau e)$	$4.7 \times 10^{-3}$ (LHC CMS)	—
$\text{BR}(h \rightarrow \tau\mu)$	$2.5 \times 10^{-3}$ (LHC CMS)	—

- 1) **Neutrino Pheno**
- 2) **Survive LFV, address future LFV**
- 3) **Collider constraints (beyond oblique)**
- 4) **FCC-ee signal shaped by  $N_k$**

Need to compute:

$$\text{BR}(l_i \rightarrow l_j \gamma), \text{BR}(l_i \rightarrow 3l_j) \text{ and } \text{BR}(Z(h) \rightarrow l_j l_j)$$

(LFV in the Scotogenic model - Toma, Vicente)

Most severe constraint from  $\mu \rightarrow e\gamma$  MEG experiment.

Use  $R$  to elude  $\mu \rightarrow e\gamma$ .

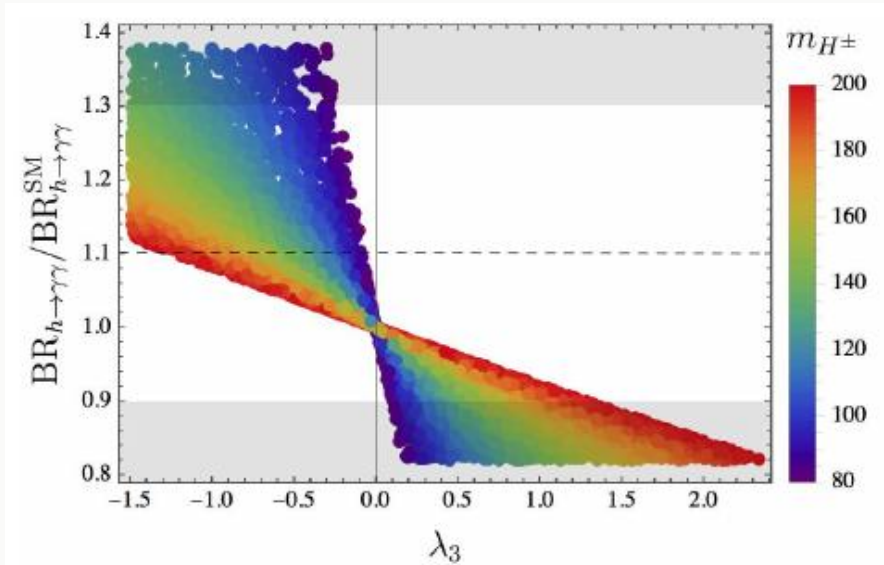
Fix its value within MEG II experiment.

# Radiative neutrino model meets FCC-ee

Sc.M. inherits IDM sector constraints

*one more time:*

Important bound on  $\lambda_3$  from  $H^\pm$  in  $h \rightarrow \gamma\gamma$ .



( $H^\pm$  must be included to appreciate  $N_k$ )

- 1) **Neutrino Pheno**
- 2) **Survive LFV, address future LFV**
- 3) **Collider constraints (beyond oblique)**
- 4) **FCC-ee signal shaped by  $N_k$**

Unsuppressed (but perturbative)  
NP couplings to light fermions +  
NP scale comparable to  $M_Z$  and  $M_W$  :

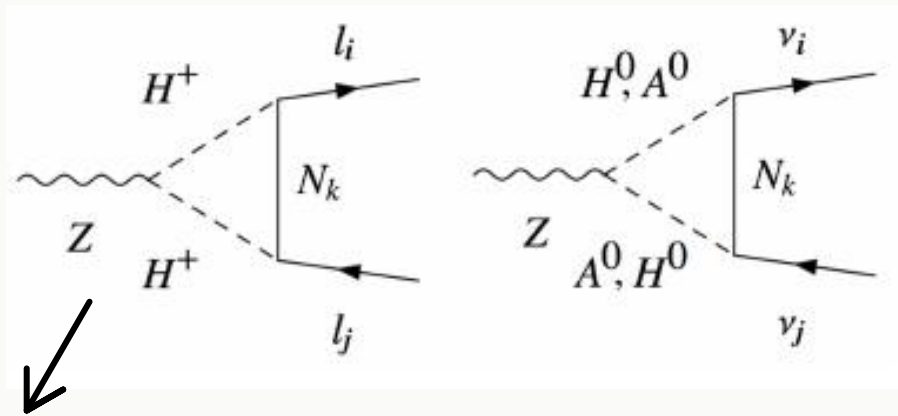


Explicit computation lepton-diagonal decays  
 $Z \rightarrow l^+l^-$ ,  $Z \rightarrow \nu_l\nu_l$  and  $W^\pm \rightarrow l^\pm\nu_l$

# Radiative neutrino model meets FCC-ee

Explicit computation lepton-diagonal decays

$Z \rightarrow l^+l^-$ ,  $Z \rightarrow \nu_l\nu_l$  and  $W^\pm \rightarrow l^\pm\nu_l$



$$\begin{aligned} \delta F_L^{Z \rightarrow l_i l_i} &= \frac{eg_L^{Ze}}{8\pi^2} \sum_k |y_{ik}^N|^2 \\ &\times C_{00} [m_Z^2, 0, 0, m_{H^\pm}^2, m_{H^\pm}^2, m_{N_k}^2], \\ (\delta F_L^{Z \rightarrow l_i l_i})^{ct} &= eg_L^{Ze} \left[ \frac{\delta Z}{2} + \delta_{l_{Li}} + \delta e + \frac{\delta_{AZ}}{2g_L^{Ze}} \right. \\ &\left. + \frac{1}{4s_W^3 c_W g_L^{Ze}} \left( \frac{\delta m_W^2}{m_W^2} - \frac{\delta m_Z^2}{m_Z^2} \right) \right] \end{aligned}$$

- 1) **Neutrino Pheno**
- 2) **Survive LFV, address future LFV**
- 3) **Collider constraints (beyond oblique)**
- 4) **FCC-ee signal shaped by  $N_k$**

$$\begin{aligned} \delta F_L^{Z \rightarrow \nu_i \nu_i} &= \frac{eg_L^{Z\nu}}{8\pi^2} \sum_k |y_{ik}^N|^2 \\ &\times C_{00} [m_Z^2, 0, 0, m_{A^0}^2, m_{H^0}^2, m_{N_k}^2], \\ (\delta F_L^{Z \rightarrow \nu_i \nu_i})^{ct} &= eg_L^{Z\nu} \left[ \frac{\delta Z}{2} + \delta_{\nu_{Li}} + \delta e \right. \\ &\left. + \frac{c_W^2 - s_W^2}{2s_W^2} \left( \frac{\delta m_W^2}{m_W^2} - \frac{\delta m_Z^2}{m_Z^2} \right) \right] \end{aligned}$$

Full Sco.M. reno required! (Same for  $W$  decays)

# Radiative neutrino model meets FCC-ee

Proper Sco.M. corrections: need for  $H^\pm$

$$\delta F_L^{V,\text{RHN}} = -\frac{m_Z(c_W^2 - s_W^2)}{16\pi^2} \sum_{k=1}^3 |y_{1k}^N|^2$$

$$\times \left( \frac{4\pi\alpha}{c_W^2 s_W^2 (S - m_Z^2)} C_{00} [S, 0, 0, m_{H^\pm}^2, m_{H^\pm}^2, m_{N_k}^2] \right.$$

$$- \lambda_3 C_1 [m_h^2, T, 0, m_{H^\pm}^2, m_{H^\pm}^2, m_{N_k}^2]$$

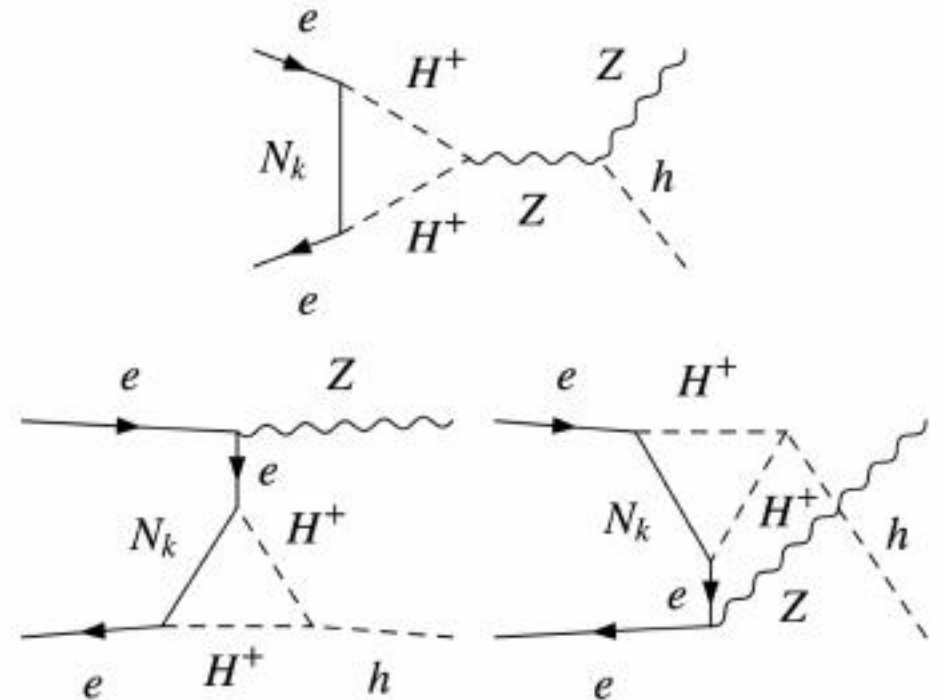
$$\left. + \lambda_3 C_1 [m_h^2, U, 0, m_{H^\pm}^2, m_{H^\pm}^2, m_{N_k}^2] \right)$$

and properly OS-renormalized

$$(\delta F_L^{V,\text{RHN}})^{ct} = F_L^{\text{LO}} \left[ \frac{\delta Z}{2} + \delta_{eL} + \delta_e + \frac{\delta_{AZ}}{2g_L^{Ze}} \right.$$

$$\left. + \frac{1}{2s_W^2} \left( \frac{\delta m_W^2}{m_W^2} - \frac{\delta m_Z^2}{m_Z^2} \right) \right]$$

- 1) **Neutrino Pheno**
- 2) **Survive LFV, address future LFV**
- 3) **Collider constraints (beyond oblique)**
- 4) **FCC-ee signal shaped by  $N_k$**





# Radiative neutrino model meets FCC-ee

Proper Sco.M. corrections: need for  $H^\pm$

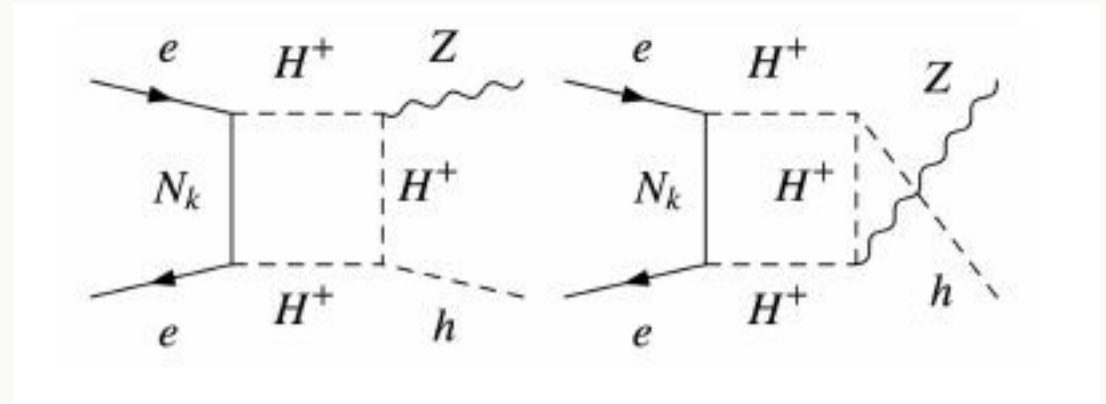
- 1) **Neutrino Pheno**
- 2) **Survive LFV, address future LFV**
- 3) **Collider constraints (beyond oblique)**
- 4) **FCC-ee signal shaped by  $N_k$**

And the finite box contributions:

$$\delta F_L^{B,\text{RHN}} = -\lambda_3 \frac{m_Z(c_W^2 - s_W^2)}{8\pi^2} \sum_{k=1}^3 |y_{1k}^N|^2$$

$$\times \left( D_{00} [m_h^2, m_Z^2, 0, 0, S, U, m_{H^\pm}^2, m_{H^\pm}^2, m_{H^\pm}^2, m_{N_k}^2] \right.$$

$$\left. + D_{00} [m_Z^2, m_h^2, 0, 0, S, T, m_{H^\pm}^2, m_{H^\pm}^2, m_{H^\pm}^2, m_{N_k}^2] \right)$$



(approximation! Full formulas and missing amplitudes in the paper)

# Radiative neutrino model meets FCC-ee

- 1) **Neutrino Pheno**
- 2) **Survive LFV, address future LFV**
- 3) **Collider constraints (beyond oblique)**
- 4) **FCC-ee signal shaped by  $N_k$**

**caveat N1**

$\lambda_5 = 0$ , YES, but forget massive  $\nu_l$

$\lambda_5 \neq 0$ , challenging!

(theory and pheno) constraints +  $\sigma_{Zh}^{scoto}$ :  
can we "see" the RHN?

big  $\lambda_5$ , too big scale of RHN

target: enhancing

$$\delta\sigma_{Zh}^{\text{RHN}} = \frac{\sigma_{Zh}^{\text{Scoto}} - \sigma_{Zh}^{\text{IDM}}}{\sigma_{Zh}^{\text{SM}}}$$

need tiny  $\lambda_5$ ,  $\leq 10^{-7}$ , protected by  $U(1)_{PQ}$

# Radiative neutrino model meets FCC-ee

- 1) **Neutrino Pheno**
- 2) **Survive LFV, address future LFV**
- 3) **Collider constraints (beyond oblique)**
- 4) **FCC-ee signal shaped by  $N_k$**

**caveat N2**

Cannot send to zero the IDM SE  
in a full degenerate scenario ( $\lambda_3 = 0$ )

we tried.

Big IDM background  
prerequisite for sizable  $\delta\sigma_{Zh}^{RHN}$

We need large  $\lambda_3$  and light  $H^\pm$

(theory and pheno) constraints +  $\sigma_{Zh}^{scoto}$ :  
can we "see" the RHN?

target: enhancing

$$\delta\sigma_{Zh}^{RHN} = \frac{\sigma_{Zh}^{Scoto} - \sigma_{Zh}^{IDM}}{\sigma_{Zh}^{SM}}$$

# Radiative neutrino model meets FCC-ee

Big IDM background  
prerequisite for sizable  $\delta\sigma_{Zh}^{RHN}$

The search for favourable  
points is split in two stages:

## Stage 1

Scan within IDM parameter space.  
We find IDM outposts with sizable  $\sigma_{Zh}$   
surviving theo and pheno constraints.

$m_{H^\pm, H^0} = \{80, 200\} \text{ GeV}$	$\lambda_2 = \{0, 4\pi/3\}$
$m_{H_0} - m_{A_0} = \{10^{-9}, 10^{-7}\} \text{ GeV}$	$\lambda_3 = \{-1.49, 1.4\}$

## Stage 2

Over surviving benchmarks, full exploration of Yukawa and RH masses

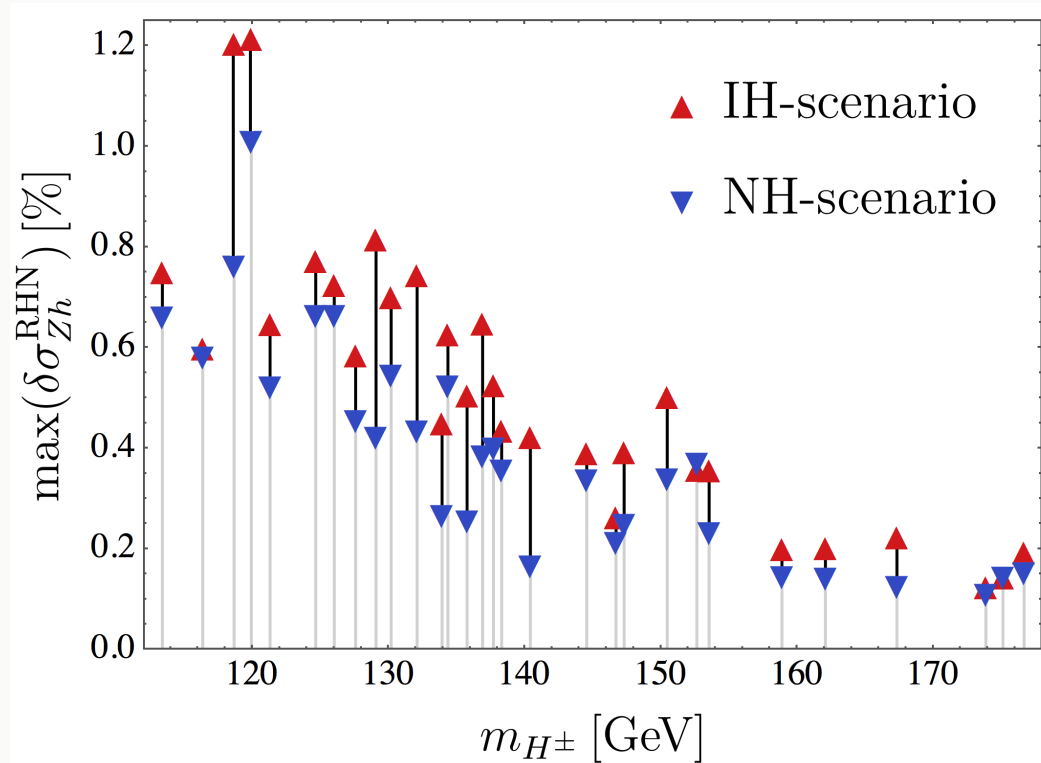
$$m_{N_i} \in \{0.01, 2\} m_{H^\pm}$$

$$m_{\nu_{\text{inf}}} \in \{10^{-7}, 10^{-1}\} \text{ eV}$$

Generating  $Y_N$  via Casas-Ibarra for NH and IH .  
BR( $\mu \rightarrow e\gamma$ ) tuned to future probing!

# Results: can we see the RHN?

The Sc.M. reduces the negative (compared to SM) IDM contribution.



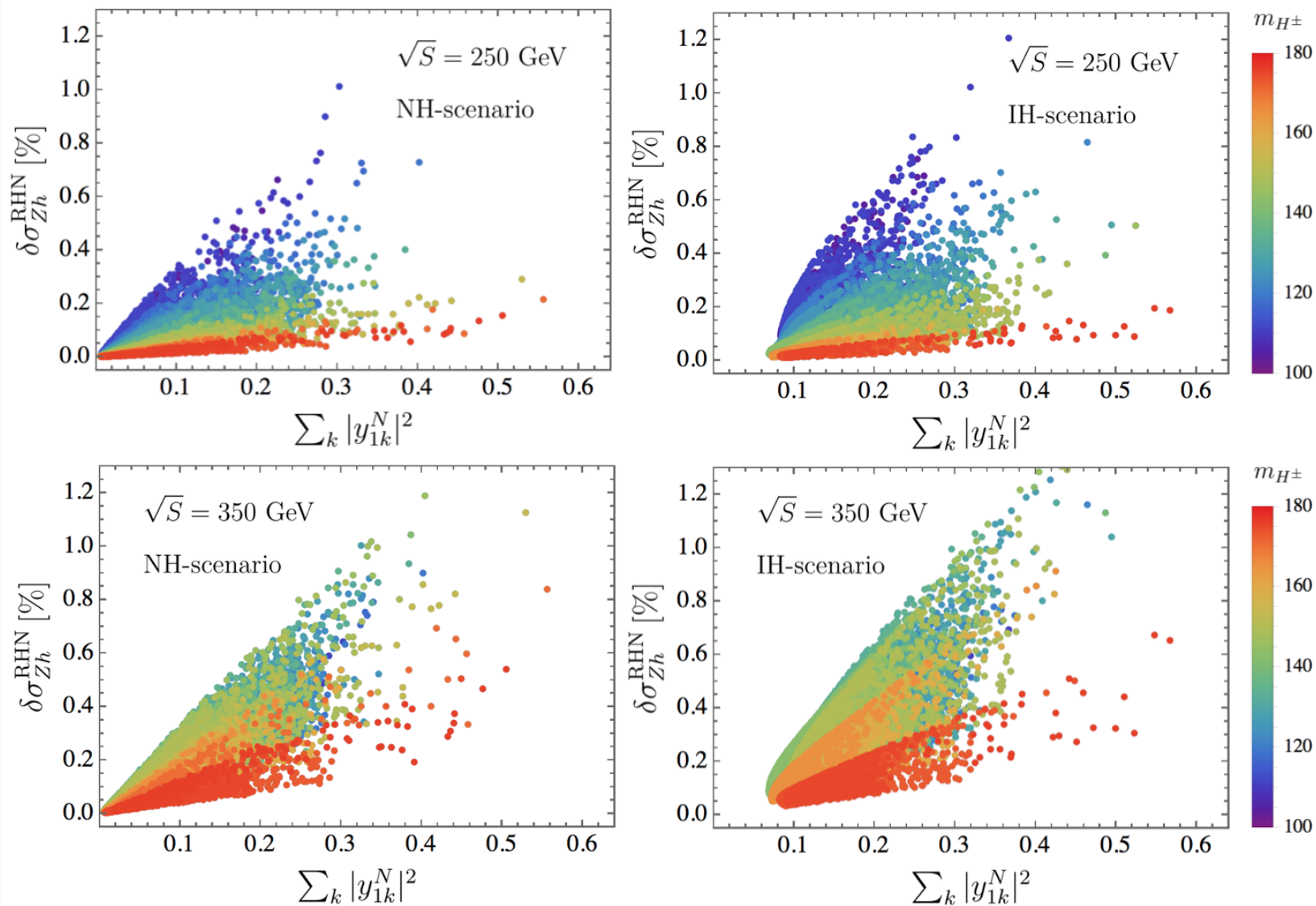
$$\sqrt{S} = 250 \text{ GeV}$$

prerequisite:  
 $\delta\sigma_{Zh}^{IDM} > 2\%$

the maximal values reached at  $m_{H^\pm} \simeq 120 \text{ GeV}$ :  
 NH-scenario it amounts to  $\delta\sigma_{Zh}^{\text{RHN}} \simeq 1.01\%$   
 while in the IH scenario, we reach up to  $\delta\sigma_{Zh}^{\text{RHN}} \simeq 1.22\%$ .

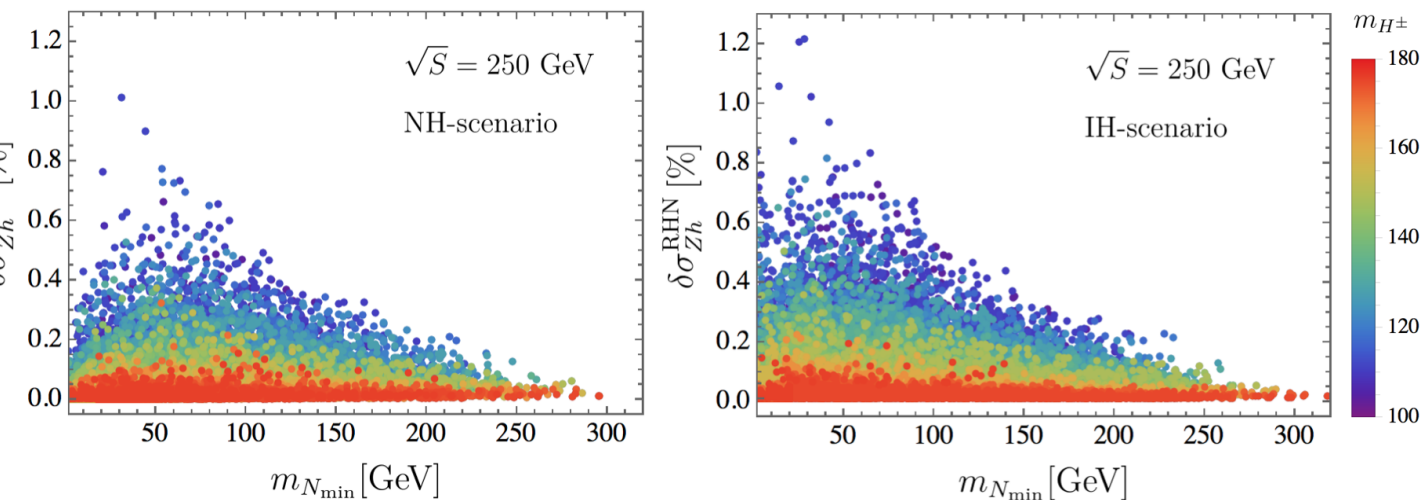
Point with the largest  $\delta\sigma_{Zh}^{\text{RHN}}$  for NH-scenario (black points) and IH-scenario (red points). The blue and red points along a vertical line share the same values of  $\lambda_2, \lambda_3, m_{H^0}, m_{A^0}$  and  $m_{H^\pm}$ .

# Results: can we see the RHN?



$N_{i=1,2,3}$  : challenging  
but possible!

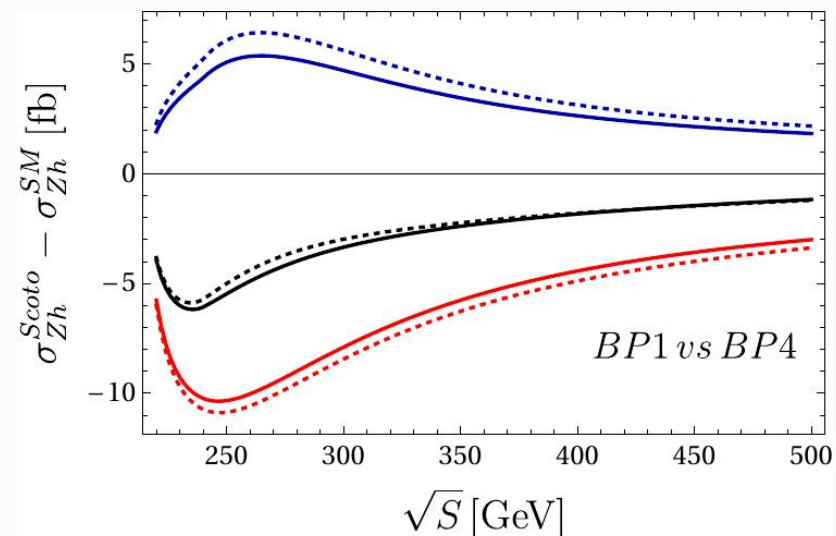
# Results: can we see the RHN?



$N_{i=1,2,3}$  : challenging  
but possible!

Mapping  $e^+e^- \rightarrow ZH$  to future LFV: selecting Sc.M. benchmarks

Cross-section	BP1	BP2	BP3	BP4	BP5	BP6
	NH	NH	IH	IH	IH	IH
$\sigma_{Zh}^{\text{Scoto}}(250 \text{ GeV})$ [fb]	223.261	220.897	222.085	223.729	221.060	222.157
$\sigma_{Zh}^{\text{IDM}}(250 \text{ GeV})$ [fb]	220.946	219.693	221.593	220.946	219.693	221.593
$\delta\sigma_{Zh}^{\text{RHN}}(250 \text{ GeV})$	1.0122%	0.5262%	0.2150%	1.2168%	0.5973%	0.2464%
$\sigma_{Zh}^{\text{Scoto}}(350 \text{ GeV})$ [fb]	120.986	120.702	120.973	121.138	120.868	121.121
$\sigma_{Zh}^{\text{IDM}}(350 \text{ GeV})$ [fb]	120.191	119.464	119.983	120.191	119.464	119.983
$\delta\sigma_{Zh}^{\text{RHN}}(350 \text{ GeV})$	0.6442%	1.0016%	0.8024%	0.7678%	1.1377%	0.9222%
$\delta\Gamma(Z \rightarrow ee)^{\text{RHN}}$ [MeV]	0.04388	0.08528	0.09608	0.04026	0.08453	0.09552
$\delta\Gamma(Z \rightarrow \mu\mu)^{\text{RHN}}$ [MeV]	0.04502	0.08861	0.09738	0.04390	0.08892	0.09693
$\delta\Gamma(Z \rightarrow \tau\tau)^{\text{RHN}}$ [MeV]	0.04459	0.08518	0.09687	0.04803	0.3916	0.08615
$\delta\Gamma(Z \rightarrow \text{inv.})^{\text{RHN}}$ [MeV]	0.1678	0.3724	0.4419	0.1734	0.3754	0.4318



# Mapping $e^+e^- \rightarrow ZH$ to future LFV: selecting Sc.M. benchmarks

LFV process	BP1	BP2	BP3	BP4	BP5	BP6
	NH	NH	IH	IH	IH	IH
BR( $\mu \rightarrow e\gamma$ )	$3.24 \times 10^{-13}$	$1.02 \times 10^{-13}$	$2.28 \times 10^{-13}$	$2.43 \times 10^{-13}$	$6.22 \times 10^{-14}$	$1.05 \times 10^{-13}$
BR( $\tau \rightarrow e\gamma$ )	$1.38 \times 10^{-8}$	$1.09 \times 10^{-8}$	$3.06 \times 10^{-8}$	$1.16 \times 10^{-8}$	$3.10 \times 10^{-8}$	$8.39 \times 10^{-9}$
BR( $\tau \rightarrow \mu\gamma$ )	$3.60 \times 10^{-9}$	$2.09 \times 10^{-8}$	$1.02 \times 10^{-9}$	$9.90 \times 10^{-9}$	$3.78 \times 10^{-8}$	$3.89 \times 10^{-9}$
BR( $\mu \rightarrow 3e$ )	$3.52 \times 10^{-13}$	$1.63 \times 10^{-13}$	$5.71 \times 10^{-14}$	$5.37 \times 10^{-13}$	$4.13 \times 10^{-13}$	$1.00 \times 10^{-13}$
BR( $\tau \rightarrow 3e$ )	$3.32 \times 10^{-10}$	$2.73 \times 10^{-10}$	$4.96 \times 10^{-10}$	$3.11 \times 10^{-10}$	$8.80 \times 10^{-10}$	$1.44 \times 10^{-10}$
BR( $\tau \rightarrow 3\mu$ )	$5.29 \times 10^{-11}$	$5.43 \times 10^{-11}$	$1.02 \times 10^{-11}$	$1.55 \times 10^{-10}$	$1.54 \times 10^{-10}$	$3.33 \times 10^{-11}$
BR( $Z \rightarrow \mu e$ )	$1.80 \times 10^{-15}$	$3.71 \times 10^{-15}$	$1.07 \times 10^{-16}$	$1.01 \times 10^{-15}$	$4.21 \times 10^{-15}$	$4.89 \times 10^{-16}$
BR( $Z \rightarrow \tau e$ )	$1.04 \times 10^{-12}$	$8.92 \times 10^{-13}$	$2.16 \times 10^{-12}$	$9.47 \times 10^{-13}$	$2.59 \times 10^{-12}$	$6.26 \times 10^{-13}$
BR( $Z \rightarrow \tau\mu$ )	$1.39 \times 10^{-13}$	$2.15 \times 10^{-12}$	$3.51 \times 10^{-14}$	$4.98 \times 10^{-13}$	$2.00 \times 10^{-12}$	$1.61 \times 10^{-13}$
BR( $h \rightarrow \mu e$ )	$7.75 \times 10^{-15}$	$2.18 \times 10^{-14}$	$4.91 \times 10^{-16}$	$5.05 \times 10^{-15}$	$2.47 \times 10^{-14}$	$2.53 \times 10^{-15}$
BR( $h \rightarrow \tau e$ )	$4.33 \times 10^{-9}$	$4.11 \times 10^{-9}$	$8.80 \times 10^{-9}$	$3.94 \times 10^{-9}$	$1.16 \times 10^{-8}$	$2.50 \times 10^{-9}$
BR( $h \rightarrow \tau\mu$ )	$8.12 \times 10^{-10}$	$9.27 \times 10^{-9}$	$1.99 \times 10^{-10}$	$2.55 \times 10^{-9}$	$1.12 \times 10^{-8}$	$8.35 \times 10^{-10}$

Barely out of LFV limits . Future LFV ready.

Combined signals for revealing Sc. M. .

(in particular  $l_i \rightarrow l_j\gamma$  and  $\mu \rightarrow 3e$ )

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# *Conclusions:*

*Extra inert scalars are a common IR tail for many BSM proposals.*

*They provide a possible background ( $\sim 10\%$  signal) from which different models need to be distinguished.*

*Theoretical NLO control over the precise Higgs-strahlung determination is a great opportunity, greatly enhanced by parallel signatures in different observables.*

*Studying interplay with LFV, the radiative-neutrino-mass paradigm can be tackled.*

# *Future:*

*Inclusion of complex parameters (CP), polarization effects ...*