

# *Future e+e- collider measurements of the W mass & width* experimental aspects

Expert Team:

Paolo Azzurri, Josh Bendavid, Martin Beneke, Jorge de Blas, Stefan Dittmaier, Ayres Freitas, Adrian Irlles, Andreas Meyer, Simon Plätzer, Matthias Schott, Raimund Ströhmer, Graham Wilson

Second ECFA Workshop on e+e- Higgs/EW/Top Factories,  
Paestum October 11, 2023

# future e+e- mW digest

1. from WW **threshold** cross sections at  $E_{\text{CM}} \simeq 157.5\text{-}162.5$  GeV  
 $\rightarrow \Delta m_W = 0.3$  MeV [10/ab]

Syst : Theory calculations /  $E_{\text{CM}}$  / acceptance / background

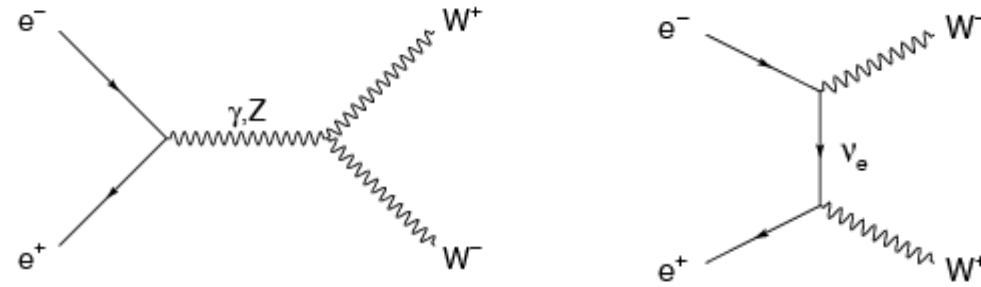
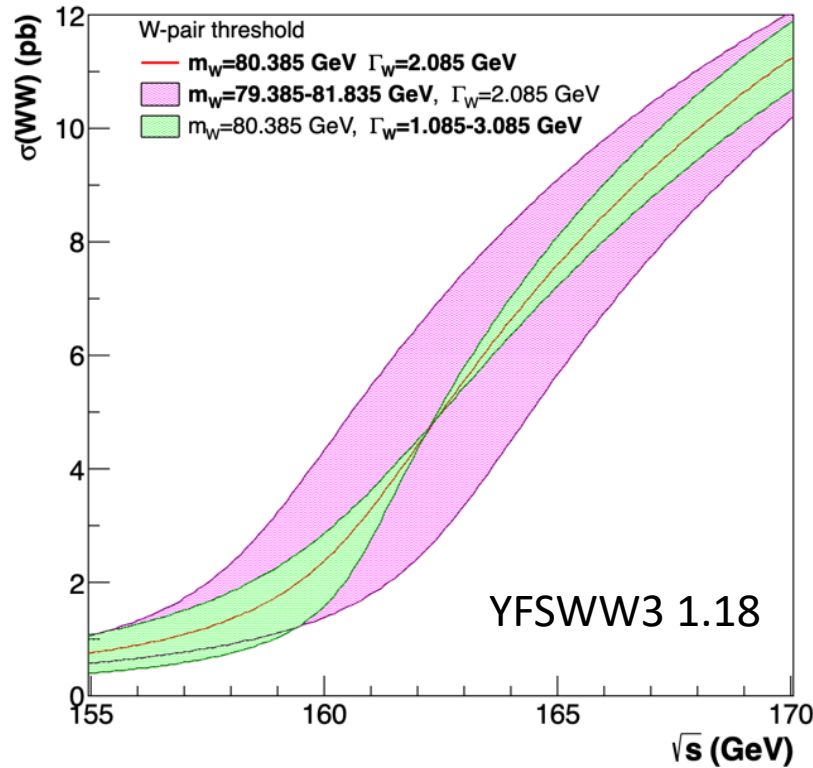
2. from decay **kinematics** mostly at  $E_{\text{CM}} \simeq 240$  GeV **and**  $E_{\text{beam}}$  (LEP2)  
 $\rightarrow \Delta m_W = 1\text{-}0.5$  MeV (stat) [2-5/ab] : **2-5 MeV (syst) ?**

Syst : Theory modeling (NP QCD) /  $E_{\text{CM}}$  / det calibration /

3. from **lepton decay kinematics** and hadronic decays **without**  $E_{\text{beam}}$   
 $\rightarrow \Delta m_W = 2$  MeV (stat) : **2-5 MeV (tot) ?**

Syst : det calibration / Theory modeling (NP QCD)

# The WW threshold lineshape and the W mass



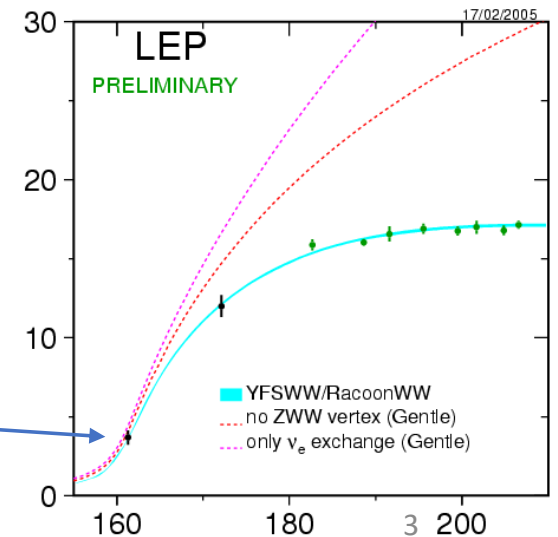
WW cross section rise  $\beta = \sqrt{1 - 4m_W^2/s}$  driven by t-channel production

Extract the W mass inverting the  $m_W$  dependence

$$\sigma(m_W, E)$$

$$m_W = \sigma^{-1}(E)$$

$$\Delta m_W = \left( \frac{d\sigma}{dm_W} \right)^{-1} \Delta \sigma$$



[ALEPH Phys.Lett.B 401 \(1997\) 347](#) with 10/pb  $m_W = 80.14 \pm 0.34$  GeV  
 stat extrapolation to 10/ab  $\Rightarrow \Delta m_W = 0.34$  MeV

# WW threshold : W mass precision requirements

Conditions to achieve  $\Delta m_W(\text{syst}) < \Delta m_W(\text{stat}) = \mathbf{0.3 \text{ MeV}}$   
with a single point WW threshold measurement

current theory precision  
 $\Rightarrow \Delta m_W = 3 \text{ MeV}$

$$\Delta m_W(B) = \left( \frac{d\sigma}{dm_W} \right)^{-1} \left( \frac{\Delta\sigma_B}{\varepsilon} \oplus \Delta\sigma_{TH} \right)$$

Background and Theory

$$\Delta\sigma_{TH} < \mathbf{1fb} \quad (\Delta\sigma_{TH}/\sigma_{TH} < 2 \cdot 10^{-4})$$

$$\Delta\sigma_B/\varepsilon < \mathbf{1fb} \quad (\Delta\sigma_B/\sigma_B < 4 \cdot 10^{-3})$$

$$\Delta m_W(\varepsilon) = \sigma \left( \frac{d\sigma}{dm_W} \right)^{-1} \left( \frac{\Delta\varepsilon}{\varepsilon} + \frac{\Delta L}{L} \right)$$

Acceptance and Luminosity

$$\left( \frac{\Delta\varepsilon}{\varepsilon} \oplus \frac{\Delta L}{L} \right) < \mathbf{2 \cdot 10^{-4}}$$

$$\Delta m_W(E) = \left( \frac{d\sigma}{dm_W} \right)^{-1} \left( \frac{d\sigma}{dE} \right) \Delta E \leq \frac{1}{2} \Delta E$$

Collision energy

$$\Delta E_b < \mathbf{0.3 \text{ MeV}} \quad (\Delta E_b/E_b < 4 \cdot 10^{-6})$$

# The WW threshold : background syst

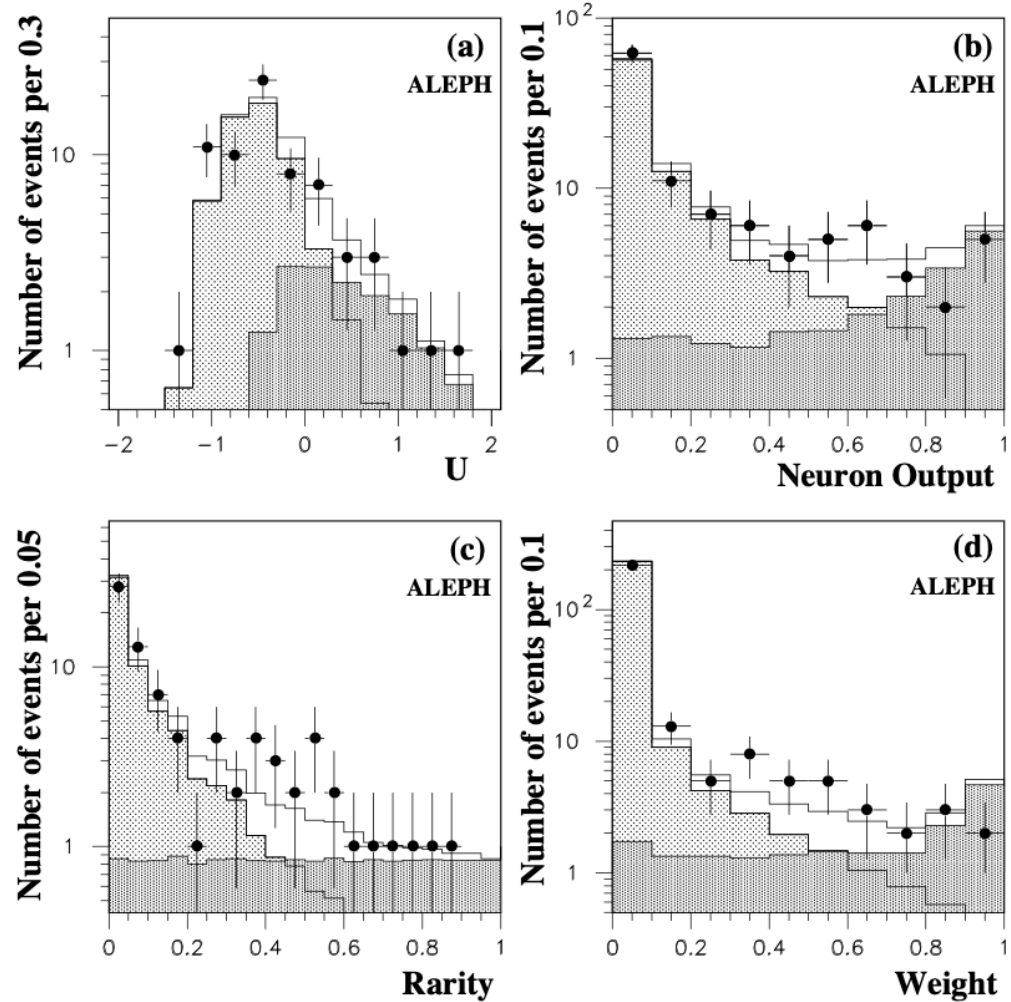
almost all bkg in the 4q channel

Selection	Expected signal	Expected background	Observed
$W^+W^- \rightarrow q\bar{q}q\bar{q}$	$9.6 \pm 1.0$	$3.44 \pm 0.39$	14
$W^+W^- \rightarrow q\bar{q}e\bar{\nu}_e$	$3.89 \pm 0.44$	$0.18 \pm 0.27$	3
$W^+W^- \rightarrow q\bar{q}\mu\bar{\nu}_\mu$	$4.19 \pm 0.46$	$0.27 \pm 0.15$	2
$W^+W^- \rightarrow q\bar{q}\tau\bar{\nu}_\tau$	$2.32 \pm 0.28$	$0.96 \pm 0.34$	7
$W^+W^- \rightarrow \ell^+\nu_\ell\ell'^-\bar{\nu}_{\ell'}$	$2.58 \pm 0.28$	$0.19^{+0.12}_{-0.04}$	2
Combined	$22.6 \pm 2.4$	$5.0 \pm 0.6$	28

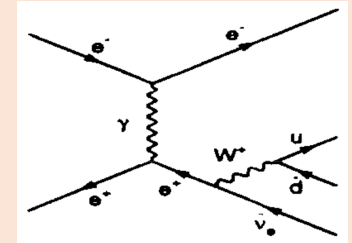
OPAL Phys. Lett. B 389 (1996) 416.

[Phys.Lett.B 401 \(1997\) 347](#)

purity ~95% achieved in the last bins



4-fermion-CC03  
interference effects



positive & negative  
effects (10-50 fb)  
reported in the various  
channels, within the LEP  
analyses acceptance

# WW threshold : acceptance syst

Syst unc at higher  $E_{CM}$  (207 GeV) on  $\sigma_{WW}$  ( $\sim 16\text{pb}$ )

Source	uncertainty (fb)			
	$l\nu l\nu$	$l\nu q\bar{q}$	$q\bar{q}q\bar{q}$	total
Tracking	4	19	31	50
Simulation of calorimeters	-	9	26	31
Hadronization models	-	27	8	35
Z peak $q\bar{q}$ fragmentation	-	-	20	20
inter-W final state interaction	-	-	28	28
Background contamination	9	5	31	35
Lepton identification	1	2	-	3
Beam-related background	10	17	37	22
$\mathcal{O}(\alpha)$ corrections DPA	2	9	12	6
Luminosity	8	35	44	87
Simulation statistics	6	20	14	25
Total	17	57	87	126

*ALEPH* [Eur.Phys.J.C 38 \(2004\) 147](#)

Source	$\sigma_{WW}^{q\bar{q}q\bar{q}}$ (pb)	$\sigma_{WW}^{q\bar{q}l\nu}$ (pb)	$\sigma_{WW}^{l\nu l\nu}$ (pb)
Four-jet modelling	$\pm 0.051$	$\pm 0.014$	-
Background cross-sections	$+0.009$	$+0.016$	$\pm 0.006$
Fragmentation	$\pm 0.045$	$\pm 0.038$	-
Final state interactions	$\pm 0.025$	-	-
Radiative corrections	$\pm 0.008$	$\pm 0.008$	$\pm 0.002$
Luminosity (theor)	$\pm 0.011$	$\pm 0.010$	$\pm 0.002$
Luminosity (exp)	$\pm 0.045$	$\pm 0.043$	$\pm 0.011$
Detector effects	$\pm 0.045$	$\pm 0.053$	$\pm 0.033$
Monte Carlo statistics	$\pm 0.005$	$\pm 0.014$	$\pm 0.033$

*DELPHI* [Eur.Phys.J.C 34 \(2004\) 127](#)

can roughly scale/4 for equivalent  $\epsilon$  effects at threshold  $\sigma_{WW}$  ( $\sim 4\text{pb}$ )

target : bring table items below  $4\text{fb}/4=1\text{fb}$

20-30fb on tables  $\Rightarrow \Delta m_W = 1.5\text{-}2\text{ MeV}$

**NP QCD effects** have important impacts on both  $q\bar{q}q\bar{q}$  and  $q\bar{q}l\nu$

need improvements in fragmentation and hadronization modeling plus constraints from control data ( $Z \rightarrow q\bar{q}$ )

less worrisome than using jet properties for kin reco

# WW threshold @ ILC

[arXiv:1603.06016](https://arxiv.org/abs/1603.06016) & [arXiv:1908.11299](https://arxiv.org/abs/1908.11299)

**ILC polarised collisions** : enhance (x4) t-channel  
WW production or suppress it to control background

Channel	Efficiency (%)	$\sigma_{\text{bkgd}}^U$ (fb)	$A_{\text{LR}}^B$	Eff. syst. (%)	Bkgd syst.	$A_{\text{LR}}^B$ syst.
lvlv	87.5	10	0.15	0.1	free	0.025
qqlv	87.5	40	0.30	0.1	free	0.012
qqqq	83.5	200	0.48	0.1	free	0.005

Table 3: Experimental assumptions for the WW event selection near threshold using a polarized scan

with 100 fb<sup>-1</sup>

Fit type	Uncertainty source	$\Delta M_W$ [MeV]	$\Delta M_W$ (syst.) [MeV]
fixbkg	Background	3.20	2.30
fixpol	Polarization	3.73	1.27
fixeff	Efficiency	3.86	1.18
fixlum	Luminosity	3.76	0.78
fixALRB	$A_{\text{LR}}^B$	3.86	0.80
fixall	Statistical	2.43	3.10
	Systematic		
standard	Total Error	3.94	

$$\Delta m_W (\text{MeV}) = 2.4 (\text{stat}) \oplus 3.1 (\text{syst}) \oplus 0.8 (\sqrt{s}) \oplus \text{theory}$$

fitted  $\Delta\varepsilon \sim 10^{-3}$  and  $\Delta\sigma_B \sim 6$  fb  
additional impact of pol uncertainty

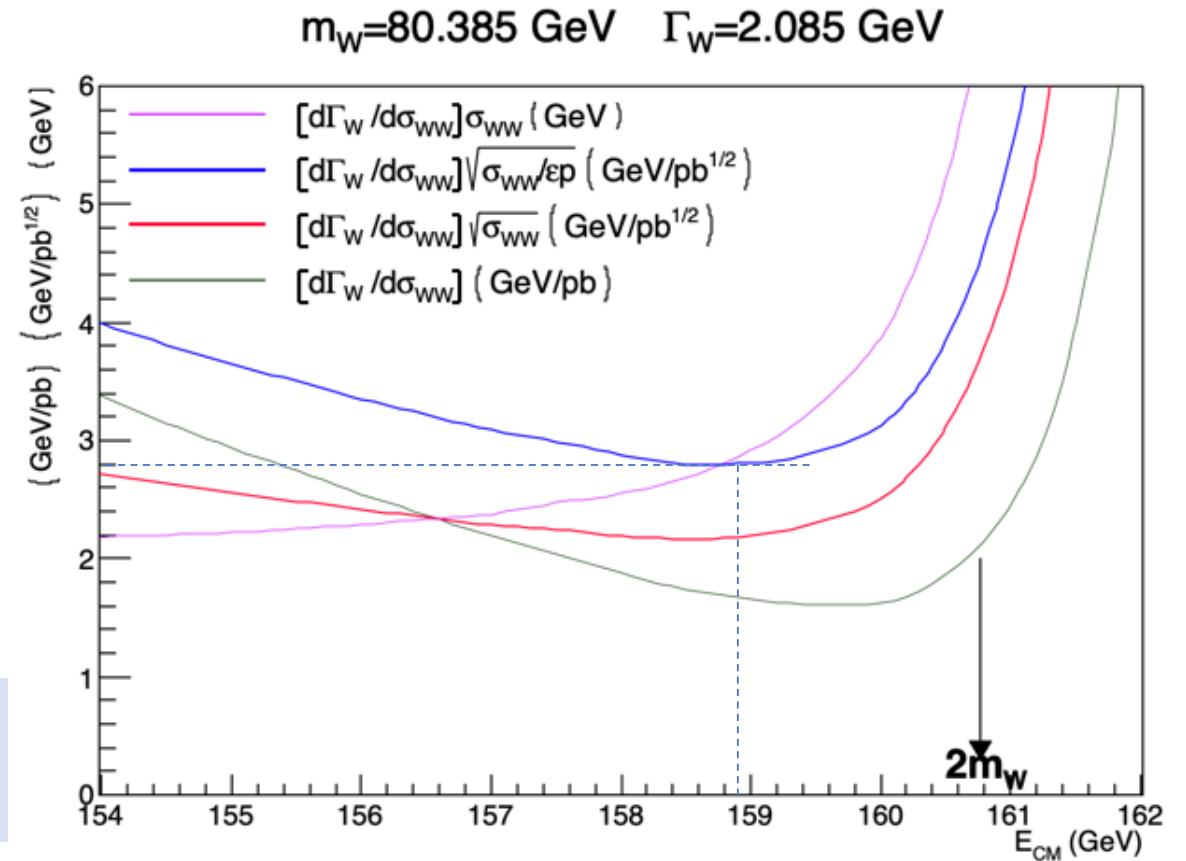
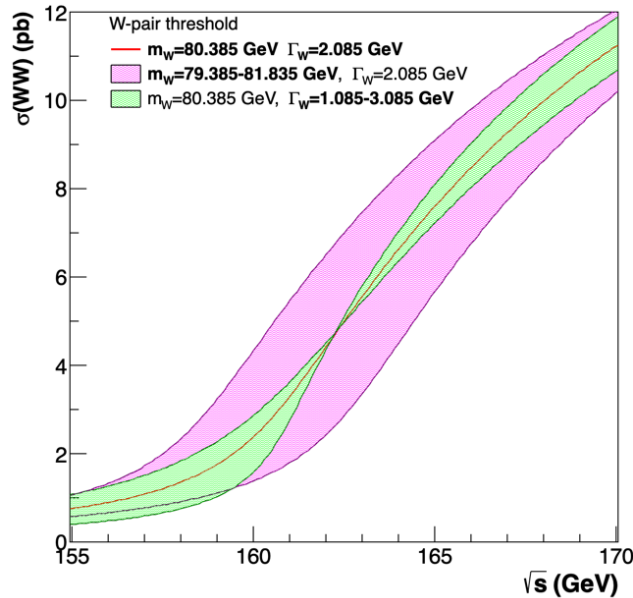
$\sqrt{s}$ (GeV)	L (fb <sup>-1</sup> )	$f$	$\lambda_e - \lambda_{e^+}$	$N_{ll}$	$N_{lh}$	$N_{hh}$	$N_{RR}$
160.6	4.348	0.7789	--+	2752	11279	12321	926968
		0.1704	+--	20	67	158	139932
		0.0254	+++	2	19	27	6661
		0.0254	---	21	100	102	8455
161.2	21.739	0.7789	--+	16096	67610	73538	4635245
		0.1704	+--	98	354	820	697141
		0.0254	+++	37	134	130	33202
		0.0254	---	145	574	622	42832
161.4	21.739	0.7789	--+	17334	72012	77991	4639495
		0.1704	+--	100	376	770	697459
		0.0254	+++	28	104	133	33556
		0.0254	---	135	553	661	42979
161.6	21.739	0.7789	--+	18364	76393	82169	4636591
		0.1704	+--	81	369	803	697851
		0.0254	+++	43	135	174	33271
		0.0254	---	146	618	681	42689
162.2	4.348	0.7789	--+	4159	17814	19145	927793
		0.1704	+--	16	62	173	138837
		0.0254	+++	10	28	43	6633
		0.0254	---	46	135	141	8463
170.0	26.087	0.7789	--+	63621	264869	270577	5560286
		0.1704	+--	244	957	1447	838233
		0.0254	+++	106	451	466	40196
		0.0254	---	508	2215	2282	50979

Table 1: Illustrative example of the numbers of events in each channel for the standard 100 fb<sup>-1</sup> 6-point ILC scan with 4 helicity configurations. Columns give the center-of-mass energy,  $\sqrt{s}$ , the apportioned integrated luminosity, the fraction for each helicity configuration,  $\lambda_e - \lambda_{e^+}$ , and the numbers of events observed in each channel.

# WW threshold : W mass and width

[arXiv:1703.01626](https://arxiv.org/abs/1703.01626)

[arXiv:2107.04444](https://arxiv.org/abs/2107.04444)

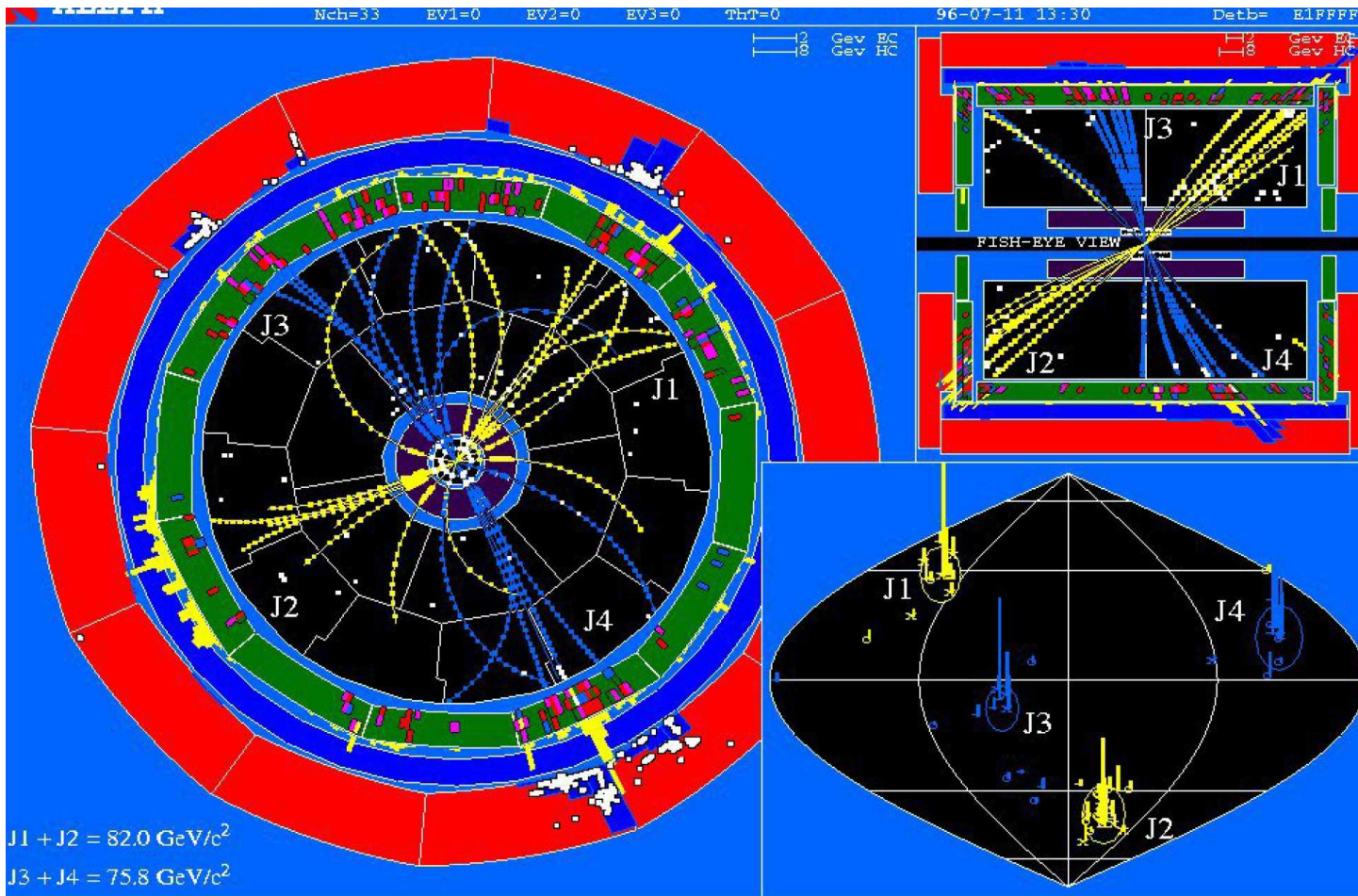


Max stat sensitivity at  $E_{CM} \sim 2m_W - \Gamma_W$

$$\left[ \left( \frac{d\sigma}{d\Gamma_W} \right)^{-1} \frac{\sqrt{\sigma}}{\sqrt{\epsilon p}} \right]_{min} \cong 2.8 \frac{GeV}{pb^{1/2}} = 2.8 \frac{MeV}{ab^{1/2}}$$



# W mass from decay kinematics



Threshold four jet event

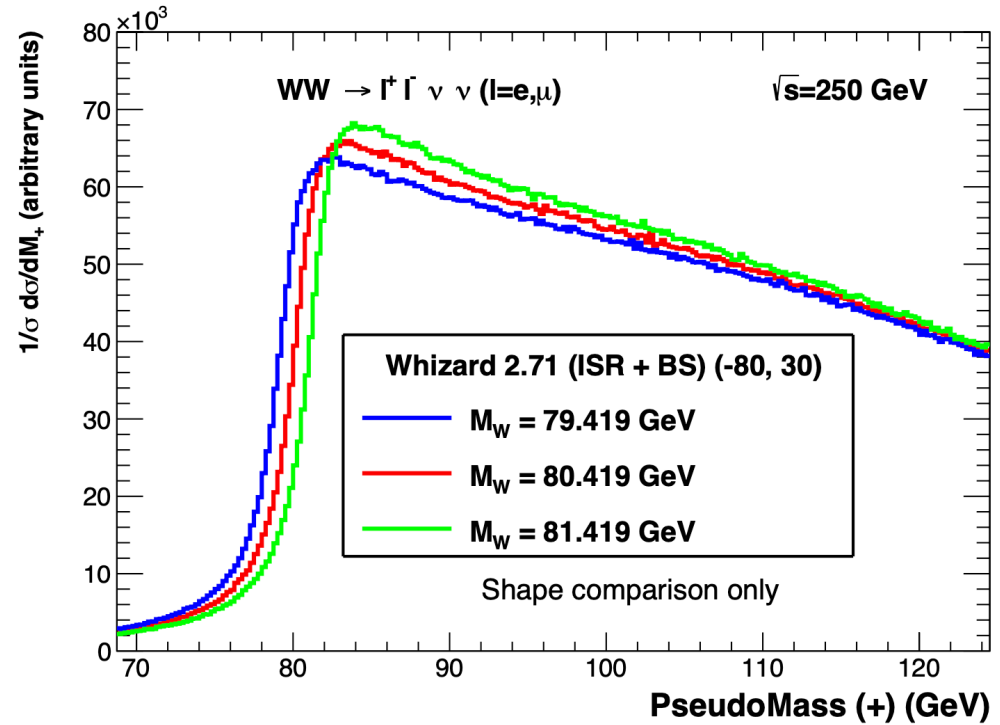
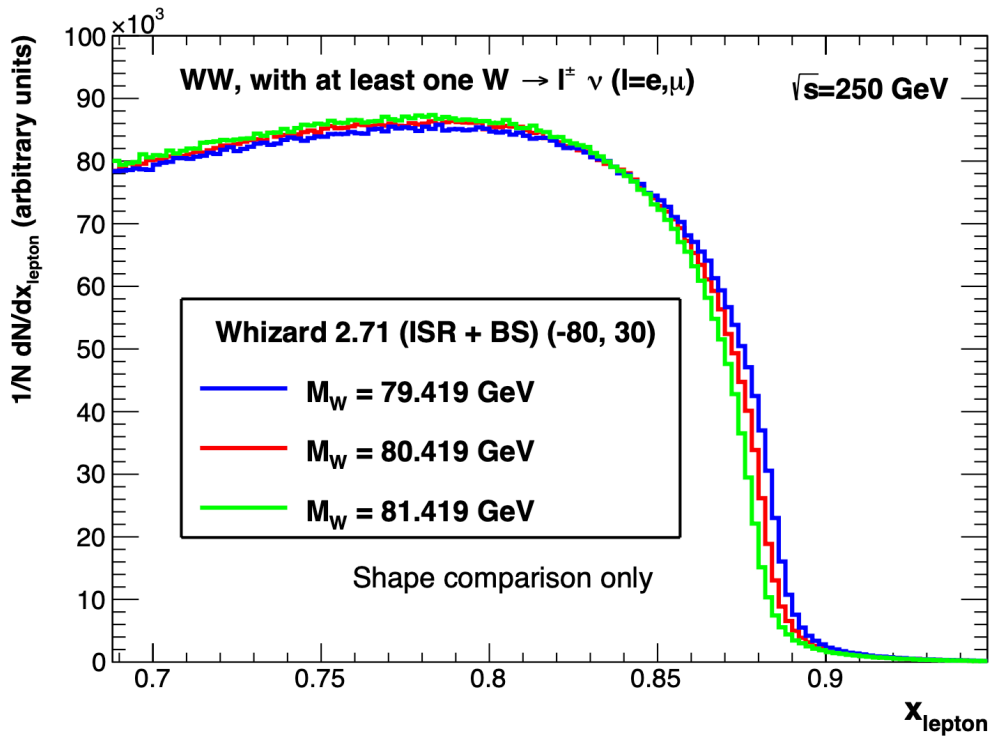
# LEP combined results

Direct Reconstruction			
Experiment	$W^+W^- \rightarrow q\bar{q}l\nu_\ell$ $m_W[\text{GeV}]$	$W^+W^- \rightarrow q\bar{q}q\bar{q}$ $m_W[\text{GeV}]$	Combined $m_W[\text{GeV}]$
Published			
ALEPH	$80.429 \pm 0.060$	$80.475 \pm 0.080$	$80.444 \pm 0.051$
DELPHI	$80.339 \pm 0.075$	$80.311 \pm 0.137$	$80.336 \pm 0.067$
L3	$80.212 \pm 0.071$	$80.325 \pm 0.080$	$80.270 \pm 0.055$
OPAL	$80.449 \pm 0.063$	$80.353 \pm 0.083$	$80.416 \pm 0.053$
LEP combination			
ALEPH	$80.429 \pm 0.059$	$80.477 \pm 0.082$	$80.444 \pm 0.051$
DELPHI	$80.339 \pm 0.076$	$80.310 \pm 0.101$	$80.330 \pm 0.064$
L3	$80.217 \pm 0.071$	$80.324 \pm 0.090$	$80.254 \pm 0.058$
OPAL	$80.449 \pm 0.062$	$80.353 \pm 0.081$	$80.415 \pm 0.052$

Source	Systematic Uncertainty in MeV			
	on $m_W$			on $\Gamma_W$
	$q\bar{q}l\nu_\ell$	$q\bar{q}q\bar{q}$	Combined	
ISR/FSR	8	5	7	6
Hadronisation	13	19	14	40
Detector effects	10	8	9	23
LEP energy	9	9	9	5
Colour reconnection	—	35	8	27
Bose-Einstein Correlations	—	7	2	3
Other	3	10	3	12
Total systematic	21	44	22	55
Statistical	30	40	25	63
Statistical in absence of systematics	30	31	22	48
Total	36	59	34	83

# W mass from lepton Energy and Pseudomass

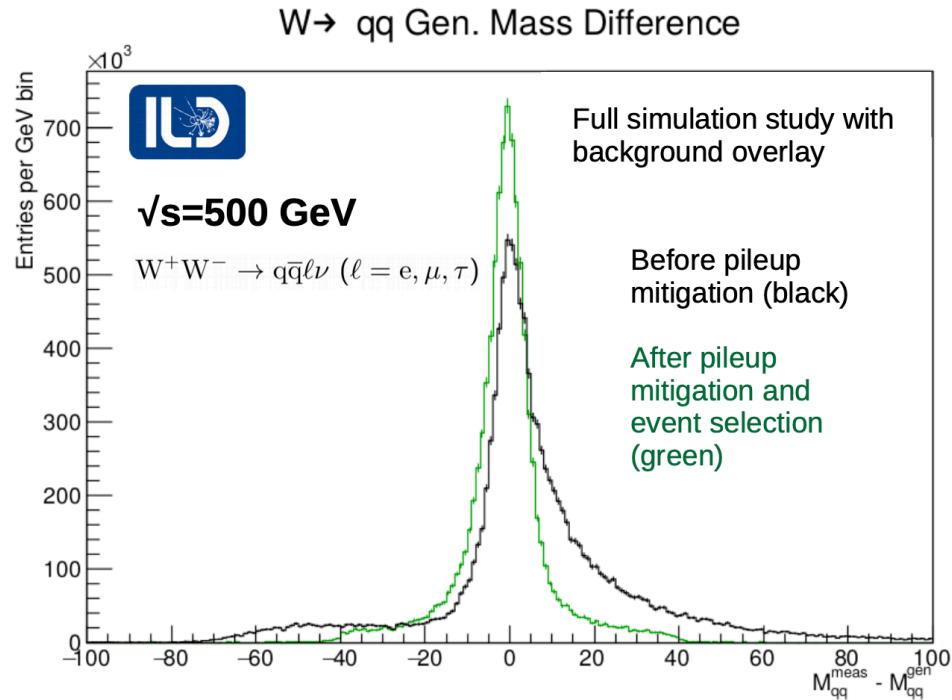
Endpoints in the lepton (or jet) energy a  
 $E_\ell = E_{CM}(1 \pm \beta)$  where  $\beta$  is the W velocity



expected statistical  $\Delta m_W = 4.4$  MeV with  $2/ab@250$  GeV  
 experimental syst from lepton energy calibration

# W mass from the hadronic mass

[arXiv:2011.12451](https://arxiv.org/abs/2011.12451)



$\Delta M_W$ [MeV]	ILC	ILC	ILC	ILC
$\sqrt{s}$ [GeV]	250	350	500	1000
$\mathcal{L}$ [ $\text{fb}^{-1}$ ]	500	350	1000	2000
$P(e^-)$ [%]	80	80	80	80
$P(e^+)$ [%]	30	30	30	30
jet energy scale	3.0	3.0	3.0	3.0
hadronization	1.5	1.5	1.5	1.5
pileup	0.5	0.7	1.0	2.0
total systematics	3.4	3.4	3.5	3.9
statistical	1.5	1.5	1.0	0.5
total	3.7	3.7	3.6	3.9

«.. dominated by the systematic uncertainties from the effective **jet energy scale** which is a challenging demand.. »

$$\Delta m_W = 0.3-0.4 \text{ MeV} \quad \Delta \Gamma_W = 1 \text{ MeV}$$

# ways ahead : WW threshold

- Evaluate theory requirements on total cross sections in the 157-162 GeV range
  - theory uncertainty evolution and correlation , 4f-interference effects
- Explore in more detail the **systematic uncertainties (cancellation) effects with multi-point ( $n \geq 3$ ) cross section measurements**. Evaluate benefits of additional model independence.
  - reduction / cancellation of **acceptance & luminosity systs** is of particular interest
- Design a realistic a modern analysis with event classifiers, evaluate performances and the corresponding **impact of systematic uncertainties**. Feedback to theory and detector design.
- ....

# ways ahead : W decay kinematics 1

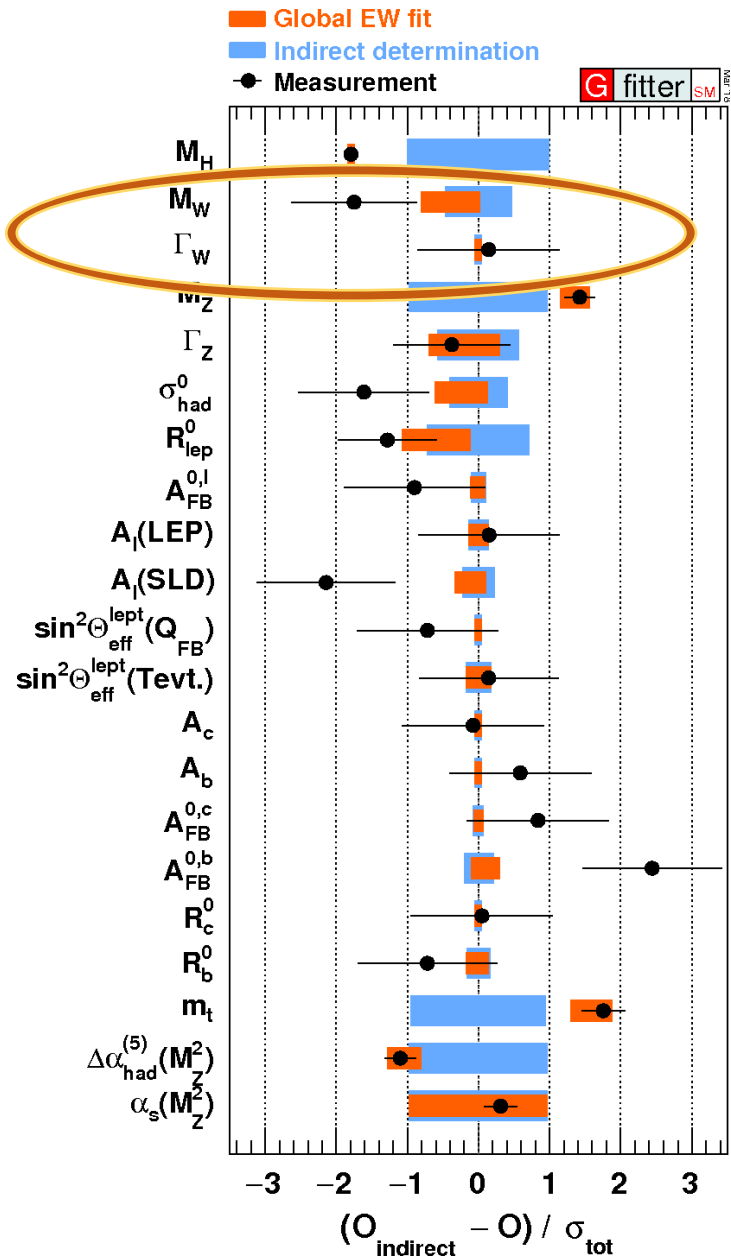
- Studies with a LEP-style  $m_W$  measurement : verify stat potential with different  $E_{CM}$  data and study the **impact of systematic uncertainties in detail** : feedback to theory and detector design
- Ultimate **simultaneous analysis and fit** of diboson events (WW, ZZ and  $Z\gamma$ ) to extract  $m_W/m_Z$  with potential cancellations of systematic uncertainties both theoretical and experimental
- ...

# ways ahead : W decay kinematics 2

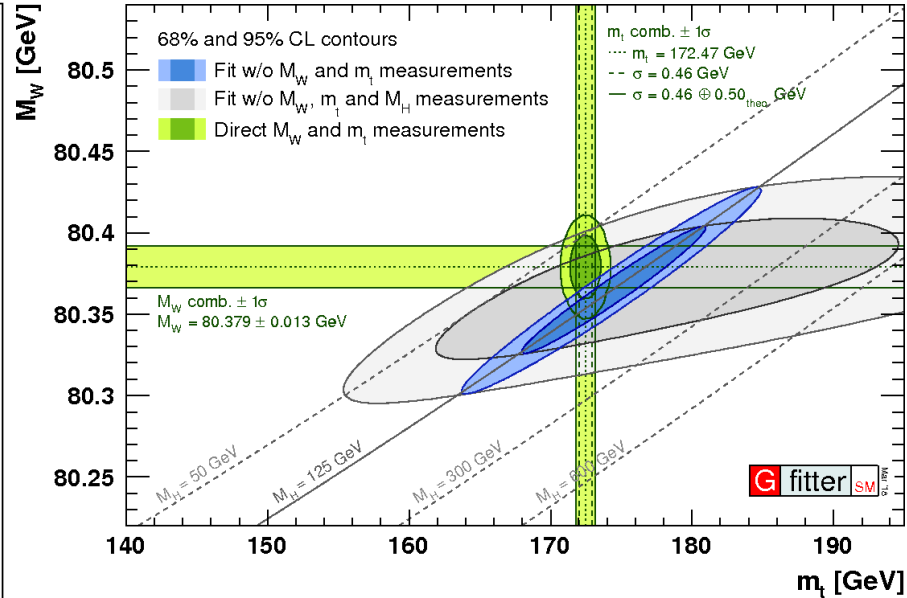
- kinematic reconstruction methods that do not make use of  $E_{\text{CM}}$ . Most demanding on experimental systs (energy & momentum calibration of jets and leptons) .  $\rightarrow$  Detector requirements
- ...
- dedicated discussion on 4-jet final state interconnection effects
  - different impact of effects with or without  $E_{\text{cm}}$  kin fits ?
  - what will be the impact of CR (BE) effects ? Can it be avoided/reduced with dedicated strategies (pcut, cone, ...)
  - How will CR (BE) be measured/constrained in situ ( inter-jet WW- $\rightarrow$ 4jets activity) and in other hadronic final states eg Z- $\rightarrow$  multijets . Viable models ?

# Additional Slides

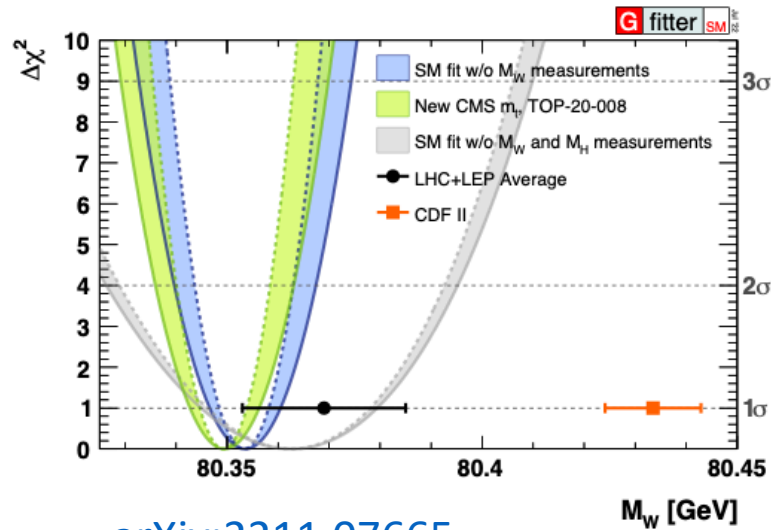
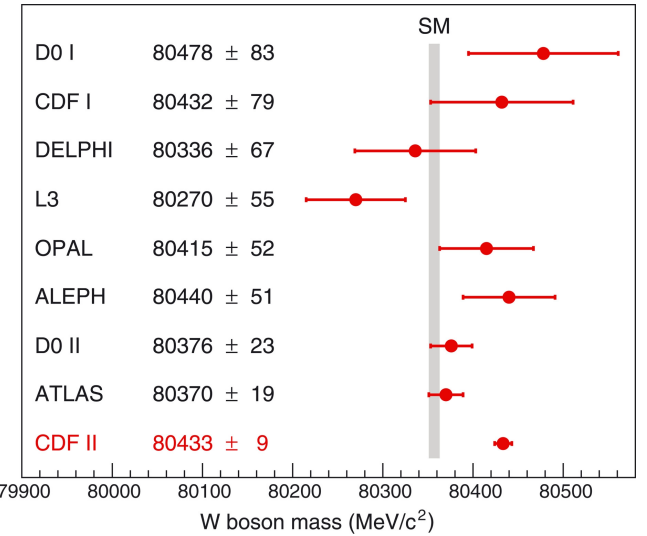




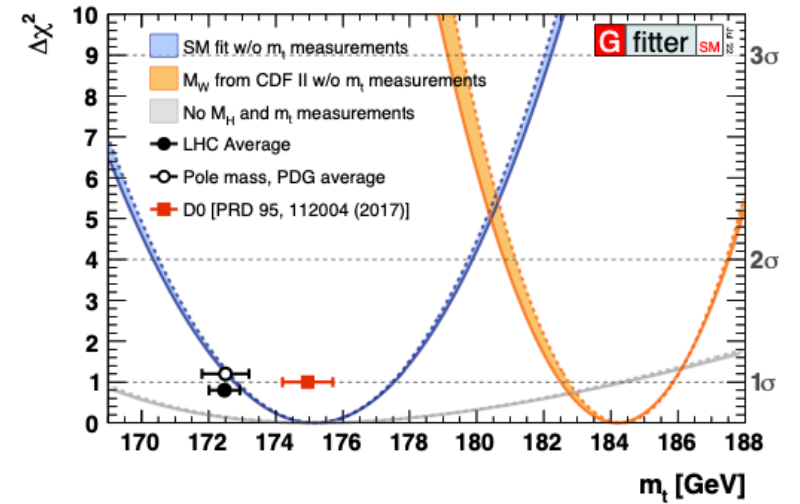
arXiv:1803.01853



Science 376 (2022) 170

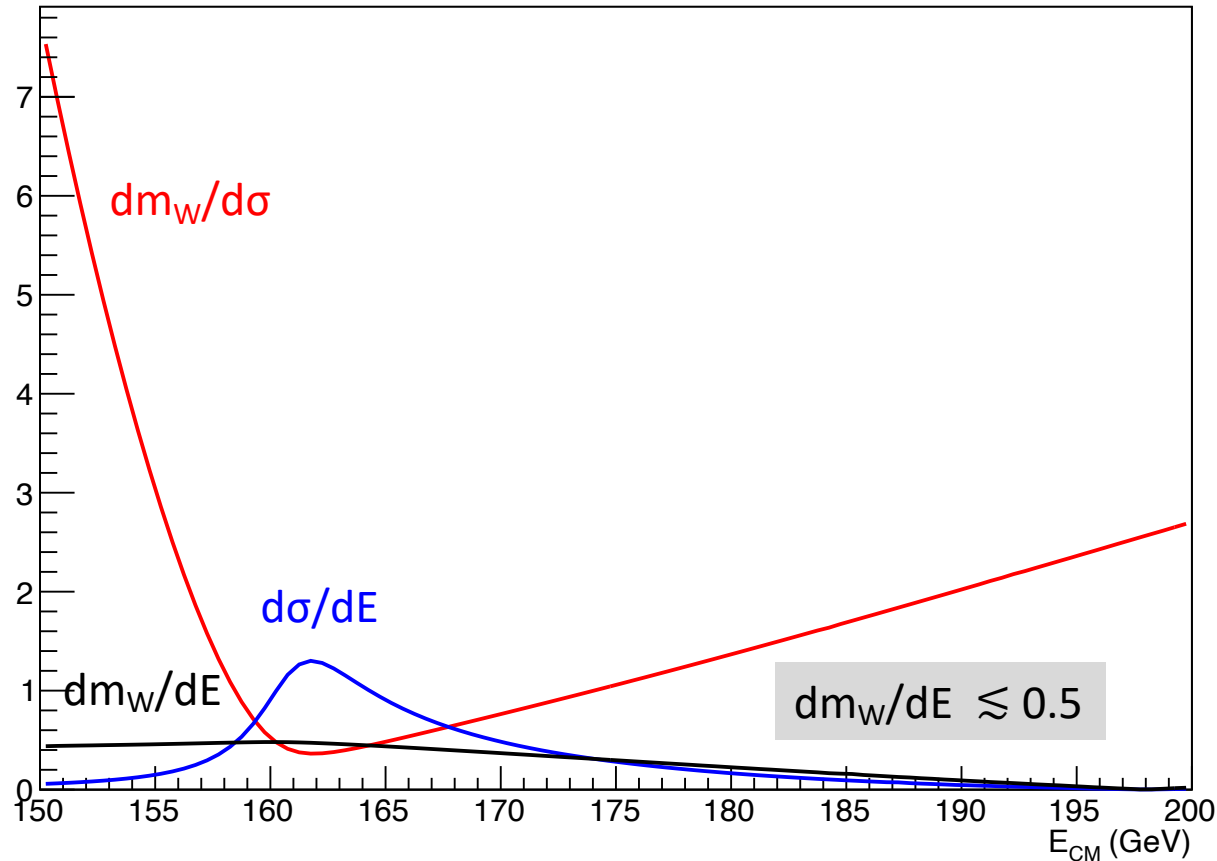


arXiv:2211.07665



EW fit  $p = 0.34 \rightarrow 10^{-7} (\gtrsim 5\sigma)$

# The WW threshold W mass : beam energy



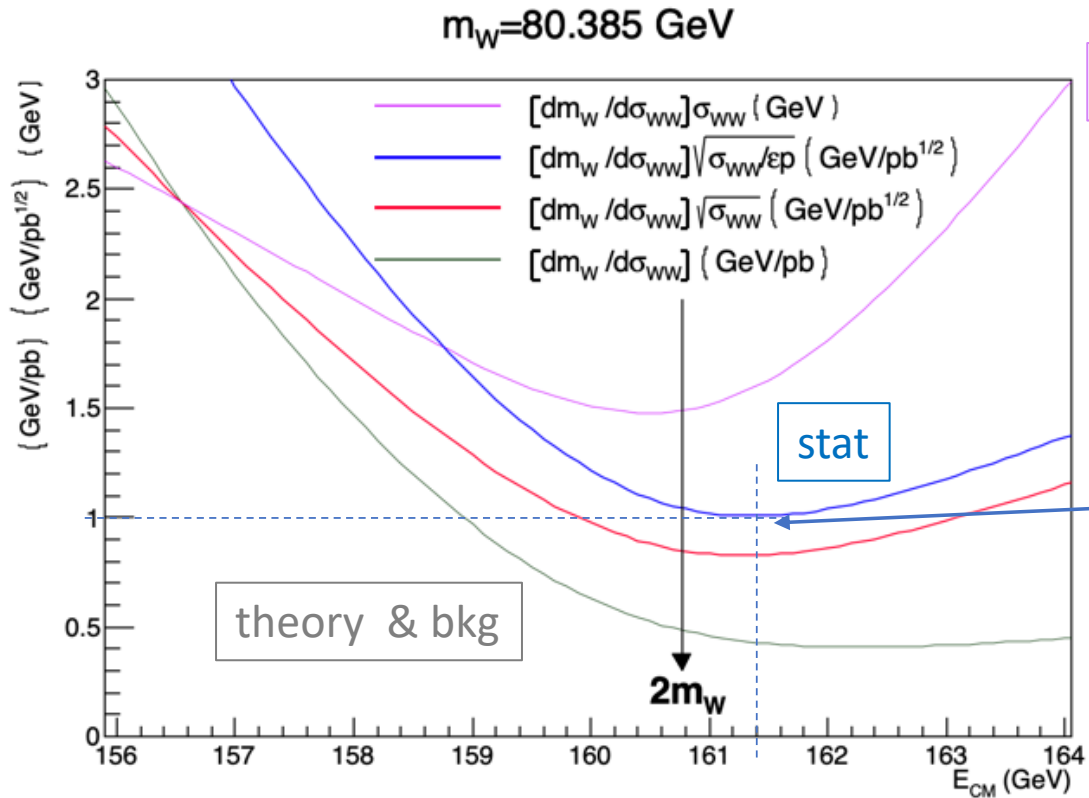
$$\Delta m_W(E) = \left( \frac{d\sigma}{dm_W} \right)^{-1} \left( \frac{d\sigma}{dE} \right) \Delta E \leq \frac{1}{2} \Delta E$$

Uncertainty on beam energy  $\Delta E_b = \frac{1}{2} \Delta E$   
 translates directly to  $m_W$

$$\Delta E_b \cong \Delta m_W$$

Very limited variations of the  $dm_W/dE$  coefficient with  $E_{CM}$  in the threshold region

# The WW threshold : W mass optimal $E_{CM}$



acceptance & lumi

optimal for stat is also close to optimal for syst contributions

stat uncertainty assuming event selection quality  
 $Q = \sqrt{\epsilon p}$  with fixed  $\epsilon = 0.75$  and  $\sigma_B = 0.3 \text{ pb}$

Max stat sensitivity at  $E_{CM} \sim 2m_W + 0.6 \text{ GeV}$

$$\left[ \left( \frac{d\sigma}{dm_W} \right)^{-1} \frac{\sqrt{\sigma}}{\sqrt{\epsilon p}} \right]_{min} \cong 1 \frac{\text{GeV}}{\text{pb}^{1/2}} = 1 \frac{\text{MeV}}{\text{ab}^{1/2}}$$

With  $L = 12/\text{ab} \Rightarrow \Delta m_W(\text{stat}) = 0.3 \text{ MeV}$

# The WW threshold : W mass uncertainties

$$\sigma = \left( \frac{N}{L} - \sigma_B \right) \frac{1}{\varepsilon}$$

$$\Delta m_W(\text{stat}) = \left( \frac{d\sigma}{dm_W} \right)^{-1} \frac{\sqrt{\sigma}}{\sqrt{L}} \frac{1}{\sqrt{\varepsilon p}}$$

Statistical

$$\Delta \sigma_{WW} = \frac{\Delta \sigma_B}{\varepsilon}$$

$$\Delta m_W(B) = \left( \frac{d\sigma}{dm_W} \right)^{-1} \left( \frac{\Delta \sigma_B}{\varepsilon} \oplus \Delta \sigma_{TH} \right)$$

Background and Theory

$$\Delta \sigma_{WW} = \sigma \left( \frac{\Delta \varepsilon}{\varepsilon} \oplus \frac{\Delta L}{L} \right)$$

$$\Delta m_W(\varepsilon) = \left( \frac{d\sigma}{dm_W} \right)^{-1} \left( \frac{\Delta \varepsilon}{\varepsilon} + \frac{\Delta L}{L} \right)$$

Acceptance and Luminosity

$$\Delta m_W(E) = \left( \frac{d\sigma}{dm_W} \right)^{-1} \left( \frac{d\sigma}{dE} \right) \Delta E \leq \frac{1}{2} \Delta E$$

Collision energy

# WW threshold : W mass and width

With cross section  $\sigma_1 \sigma_2$  measurements at two energies  $E_1 E_2$  : uncertainty propagation

$$\begin{cases} \sigma_1 = \sigma_{WW}(E_1, m_W, \Gamma_W) \\ \sigma_2 = \sigma_{WW}(E_2, m_W, \Gamma_W) \end{cases} \quad \begin{cases} \Delta\sigma_1 = a_1 \Delta m + b_1 \Delta\Gamma \\ \Delta\sigma_2 = a_2 \Delta m + b_2 \Delta\Gamma \end{cases} \quad \begin{matrix} a_1 = \frac{d\sigma_1}{dm} & b_1 = \frac{d\sigma_1}{d\Gamma} \\ a_2 = \frac{d\sigma_2}{dm} & b_2 = \frac{d\sigma_2}{d\Gamma} \end{matrix}$$

$$\Delta m = - \frac{b_2 \Delta\sigma_1 - b_1 \Delta\sigma_2}{a_2 b_1 - a_1 b_2}$$

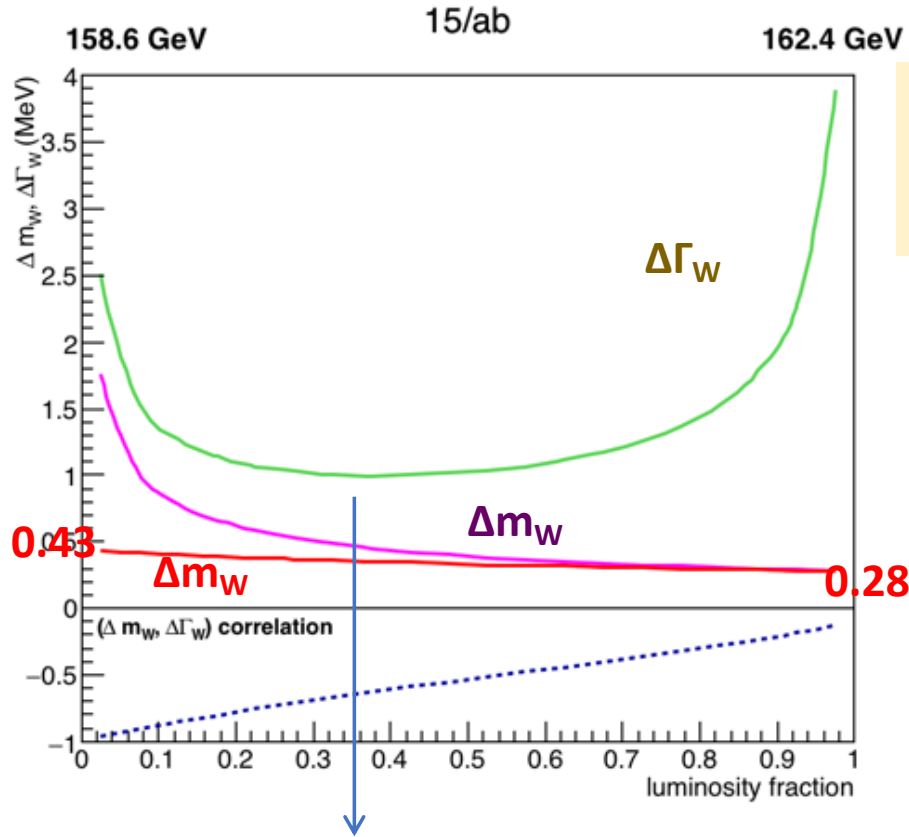
$$\Delta\Gamma = \frac{a_2 \Delta\sigma_1 - a_1 \Delta\sigma_2}{a_2 b_1 - a_1 b_2}$$

$\Delta m, \Delta\Gamma$  linear correlation with uncorrelated  $\Delta\sigma_1, \Delta\sigma_2$

$$r = - \frac{1}{\Delta m \Delta\Gamma} \frac{a_2 b_2 \Delta\sigma_1^2 + a_1 b_1 \Delta\sigma_2^2}{(a_2 b_1 - a_1 b_2)^2}$$

# WW threshold : W mass and width

Scans of possible  $E_1$   $E_2$  data taking energies and luminosity fractions  $f$  (at the  $E_2$  point)



$\Delta m_W = 0.45 \text{ MeV}$  ,  $\Delta \Gamma_W = 1 \text{ MeV}$  ( $r = -0.6$ )  
 $\Delta m_W = 0.35 \text{ MeV}$

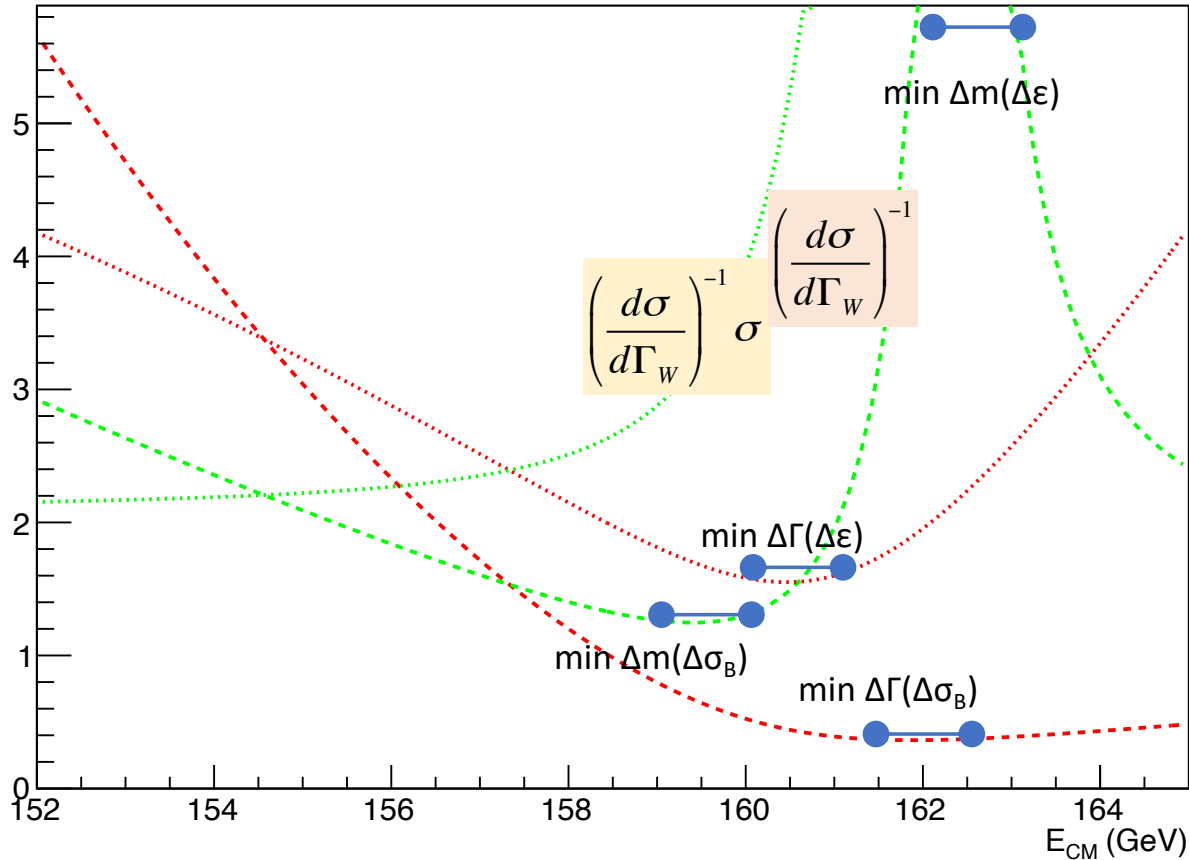
A - minimum of  $\Delta \Gamma_W = 0.91 \text{ MeV}$  with  $\Delta m_W = 0.55 \text{ MeV}$   
 taking data at  $E_1 = 156.6 \text{ GeV}$   $E_2 = 162.4 \text{ GeV}$   $f = 0.25$   
 yields  $\Delta m_W = 0.47 \text{ MeV}$  (as single par)

B- minimum of  $\Delta m_W = 0.28 \text{ MeV}$   $\Delta \Gamma_W = 3.3 \text{ MeV}$  with  
 $E_1 = 155.5 \text{ GeV}$   $E_2 = 162.4 \text{ GeV}$   $f = 0.95$   
 yields  $\Delta m_W = 0.28 \text{ MeV}$  (as single par)

C- minimum of  $\Delta \Gamma_W = 0.96 \text{ MeV} + \Delta m_W = 0.41 \text{ MeV}$  with  
 $E_1 = 157.5 \text{ GeV}$   $E_2 = 162.4 \text{ GeV}$   $f = 0.45$   
 yields and  $\Delta m_W = 0.37 \text{ MeV}$  (as single par)

$\Delta m_W$  ,  $\Delta \Gamma_W$ : error on W mass and width from fitting both  
 $\Delta m_W$ : error on W mass from fitting only  $m_W$

# WW threshold : W mass and width



Scans of  $(E_1, E_2, f)$  data taking **assuming limiting syst uncertainties**, either  $\Delta\varepsilon + \Delta L$  or  $\Delta\sigma_B + \Delta\sigma_{TH}$

More complex situation, depends very much on the correlation of uncertainties between the energy points (that can be quite large)

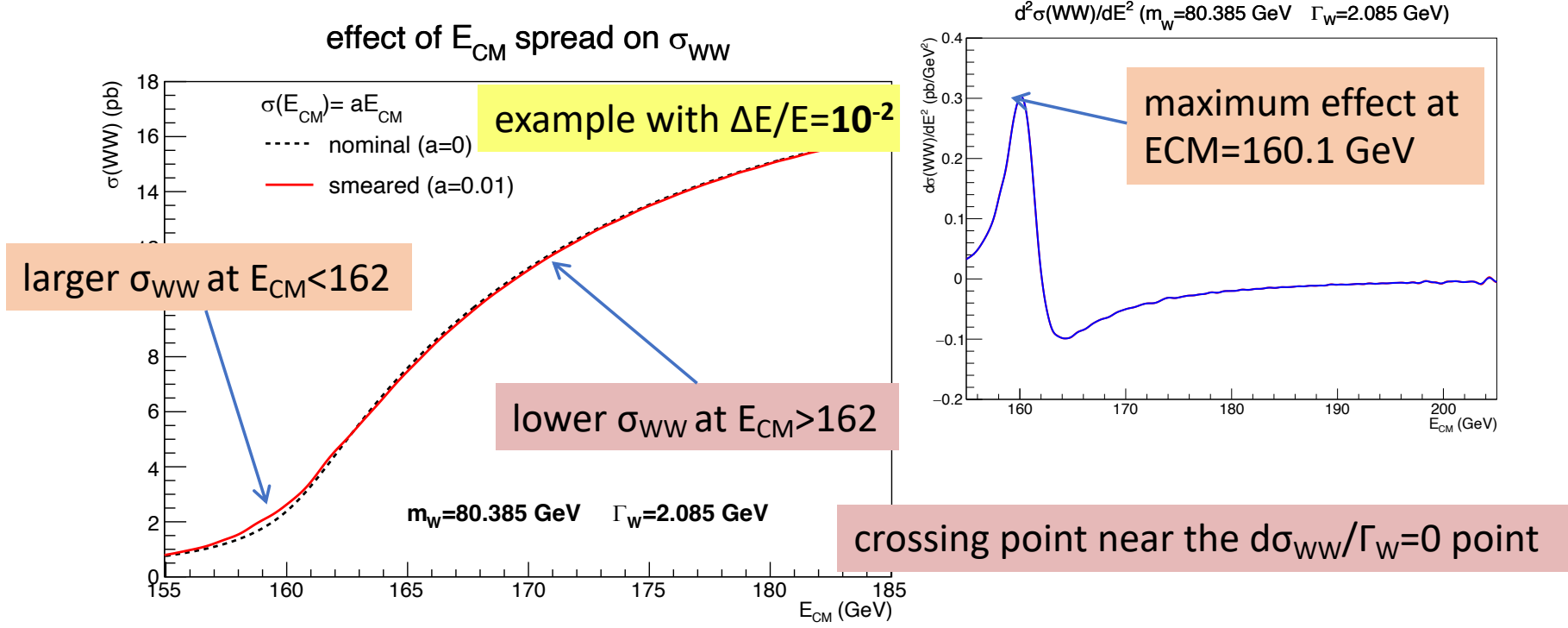
Correlated syst can cancel taking data at different  $E_{CM}$  points where the relevant differential factors are equal (around their minima)

>2 energy points will be beneficial to reduce the impact of (correlated) systematic uncertainties  
careful choice of additional points recommended

optimal E points with limiting correlated systs

partially explored in [Eur. Phys. J. C 80 no. 1, \(2020\) 66](https://doi.org/10.1007/s00526-019-1601-1)

# WW threshold : energy spread effects



$\sigma(E_{CM}) = (0.47-1.10) 10^{-3} E_{CM}$

Optimal  $m_W$  &  $\Gamma_W$  points @  $E_{CM}=157.3$  &  $162.6 \text{ GeV}$

- $\Delta\sigma_{WW} = +(0.24-1.3) \text{ fb}$  &  $-(0.18-1.0) \text{ fb}$
- $\Delta m_W = -(0.09-0.48) \text{ MeV}$
- $\Delta\Gamma_W = +(0.6-3.3) \text{ MeV}$

Maximum effects are at the level of  $\Delta m_W(\text{stat})$  and  $2x \Delta\Gamma_W(\text{stat})$  so that control on the beam energy RMS <50% is required to avoid additional syst contributions from this source



# W mass from kinematics with 4P fit (LEP2)

Formula for 2-jets final state from  $ee \rightarrow Z\gamma \rightarrow qq\gamma$

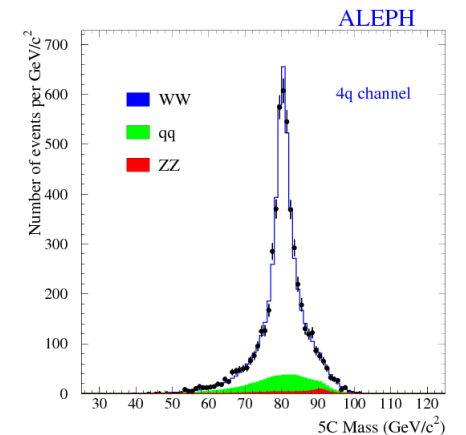
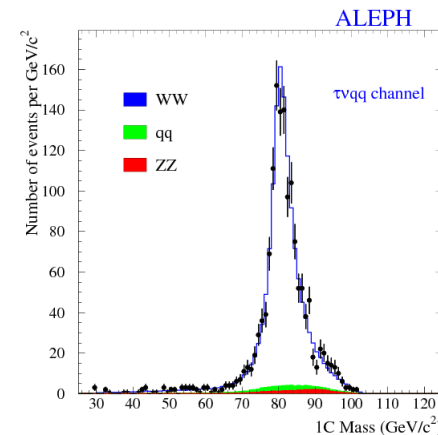
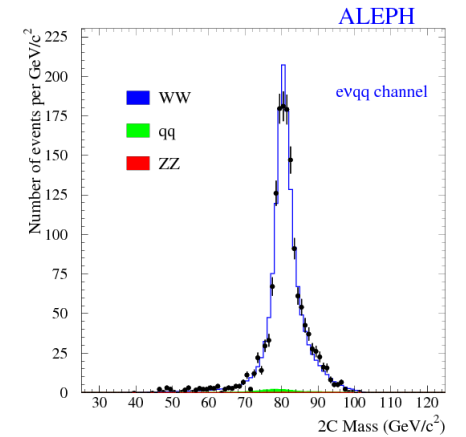
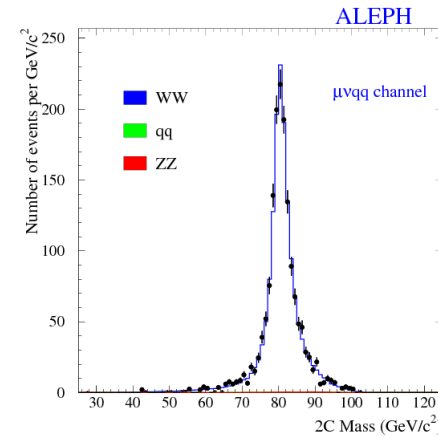
$$M_Z^2 = s \frac{\beta_1 \sin \theta_1 + \beta_2 \sin \theta_2 - \beta_1 \beta_2 |\sin(\theta_1 + \theta_2)|}{\beta_1 \sin \theta_1 + \beta_2 \sin \theta_2 + \beta_1 \beta_2 |\sin(\theta_1 + \theta_2)|}$$

$E_{\text{CM}}$  is again a main ingredient: sets jet energy scale  
 other main ingredients are the jets (and lepton) **angles**  
 secondary ingredients are the **jet velocities** ( $\beta = p/E$ )

statistical uncertainties ALEPH LEP2  $\rightarrow$  FCCee extrapolated

Stat uncertainty	$\Delta m_W$	$\Delta \Gamma_W$
evqq	87 MeV $\rightarrow$ 0.9 MeV	200 MeV $\rightarrow$ 2 MeV
$\mu\nu$ qq	82 MeV $\rightarrow$ 0.8 MeV	200 MeV $\rightarrow$ 2 MeV
$\tau\nu$ qq	121 MeV $\rightarrow$ 1.2 MeV	320 MeV $\rightarrow$ 3.2 MeV
qqqq	70 MeV $\rightarrow$ 0.7 MeV	120 MeV $\rightarrow$ 1.2 MeV
combined	43 MeV $\rightarrow$ 0.4 MeV	90 MeV $\rightarrow$ 0.9 MeV

LEP2 (ALEPH) from  $\sim 10\text{k}$  WW @  $E_{\text{CM}} = 183\text{-}209$  GeV



# W kinematic fit : systematics

EPOL  $\Delta E_{CM}=0.3$  MeV at  $E_{CM}=162.6$  GeV  
 [with  $\Delta m_W$  (stat)(162) $\sim 1$  MeV ]

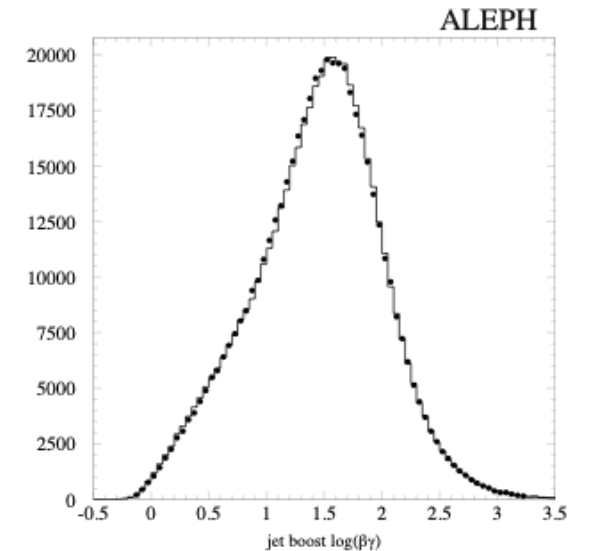
For larger  $E_{beam}$  at  $E_{CM}=240-365$  GeV can make use of  
 radiative Z-returns ( $Z\gamma$ ) and ZZ events  
 $\Delta E_{CM}(240\text{GeV})\sim 2$  MeV &  $\Delta E_{CM}(365\text{ GeV})\sim 10$  MeV

Table 9: Summary of the systematic errors on  $m_W$  and  $\Gamma_W$  in the standard analysis averaged over 183-209 GeV for all semileptonic channels. The column labelled  $\ell\nu q\bar{q}$  lists the uncertainties in  $m_W$  used in combining the semileptonic channels.

Source	$\Delta m_W$ (MeV/ $c^2$ )				$\Delta \Gamma_W$ (MeV)			
	$e\nu q\bar{q}$	$\mu\nu q\bar{q}$	$\tau\nu q\bar{q}$	$\ell\nu q\bar{q}$	$e\nu q\bar{q}$	$\mu\nu q\bar{q}$	$\tau\nu q\bar{q}$	$\ell\nu q\bar{q}$
e+ $\mu$ momentum	3	8	-	4	5	4	-	4
e+ $\mu$ momentum resoln	7	4	-	4	65	55	-	50
Jet energy scale/linearity	5	5	9	6	4	4	16	6
Jet energy resoln	4	2	8	4	20	18	36	22
Jet angle	5	5	4	5	2	2	3	2
Jet angle resoln	5	2	5	5	5	7	8	7
Jet boost	17	17	20	17	3	3	3	3
Fragmentation	10	10	15	11	22	23	37	25
Radiative corrections	5	2	5	5	5	2	2	2
LEP energy	9	9	10	9	7	7	10	8
Calibration ( $e\nu q\bar{q}$ only)	10	-	-	4	20	-	-	9
Ref MC Statistics	3	3	5	2	7	7	10	5
Bkgnd contamination	3	1	6	2	5	4	19	7

total 54(stat)+25(syst) = **60** MeV

lepton and jet uncertainties  
 from (Z) calibration data

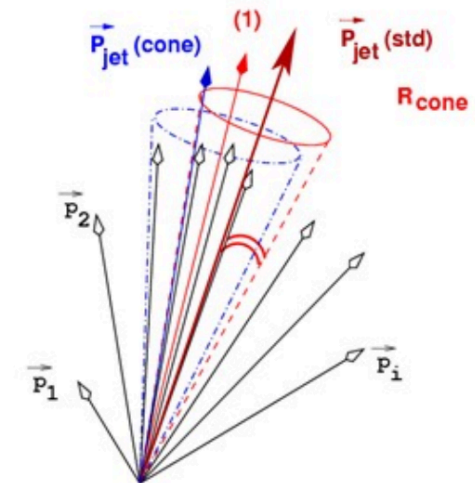
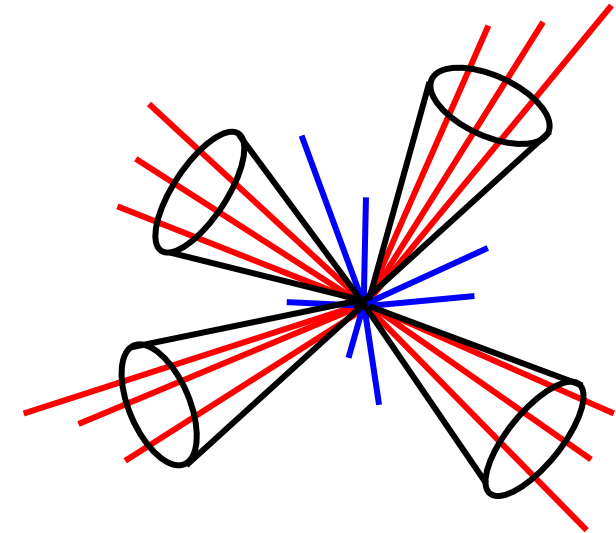


# W kinematic fit : systematics in 4q

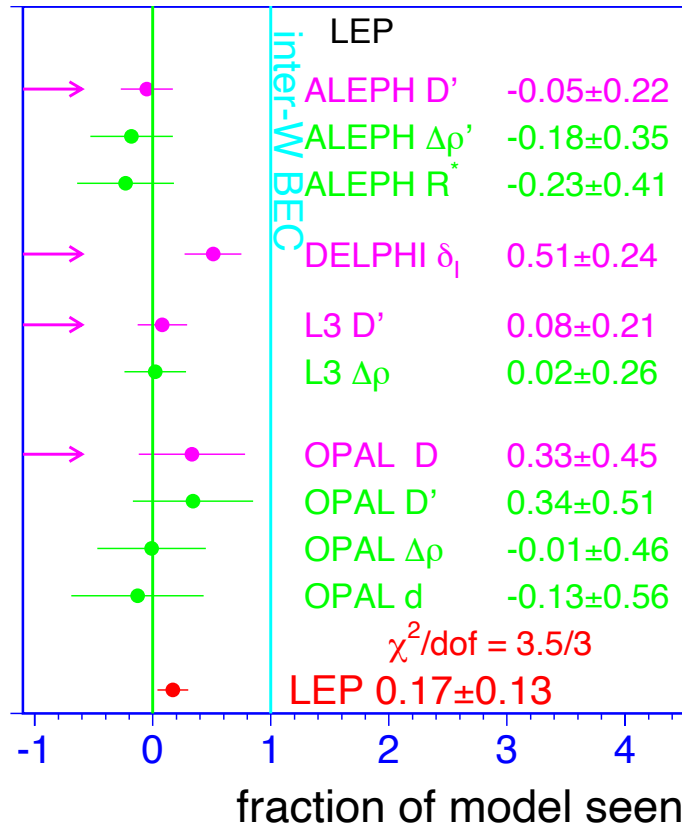
Table 8: Summary of the systematic errors on  $m_W$  and  $\Gamma_W$  averaged over 183-209 GeV in the  $q\bar{q}q\bar{q}$  channel for the standard, PCUT (= 3.0 GeV/c) and CONE (R=0.4) reconstructions.

Source	$\Delta m_W$ (MeV/c <sup>2</sup> )			$\Delta \Gamma_W$ (MeV)		
	standard	PCUT	CONE	standard	PCUT	CONE
Jet energy scale/linearity	2	2	3	2	12	4
Jet energy resolu	0	1	0	7	9	10
Jet angle	6	6	6	1	3	3
Jet angle resolu	1	3	2	15	18	9
Jet boost	14	15	11	5	5	4
Fragmentation	10	20	20	20	40	40
Radiative Corrections	2	2	2	5	7	7
LEP energy	9	10	10	7	7	7
Ref MC Statistics	2	3	3	5	7	7
Bkgnd contamination	8	5	5	20	31	32
Colour reconnection	79	28	36	104	24	45
Bose-Einstein effects	0	2	3	20	10	10

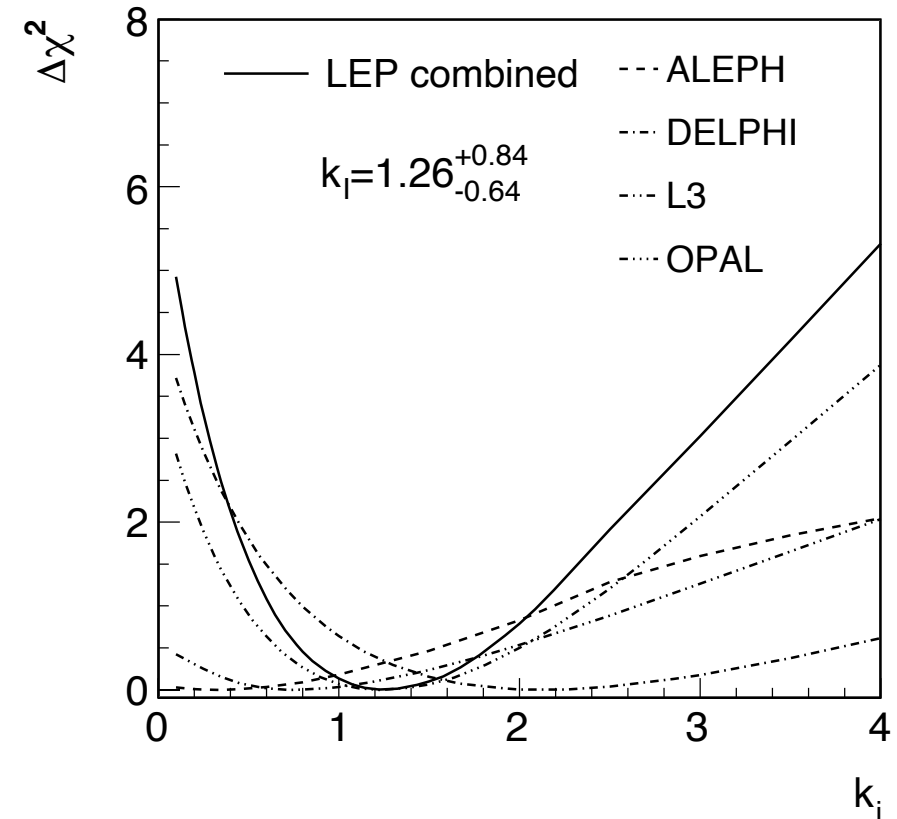
total 70(stat)+28(syst) +28(FSI) = **80 MeV**



# W kinematic fit : FSI systematics in 4q



Measured BEC expressed as the relative fraction of the model with inter-W correlations.



Individual and LEP combined  $\Delta\chi^2$  curves for the measurement of the CR parameter  $k_l$  in the SK1 model.