# Kinematic Fitting at Future e⁺e⁻ Higgs Factories.

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## Kinematically Constrained Fitting.

**Lot of knowledge in e+e- events beyond the raw measurements:**

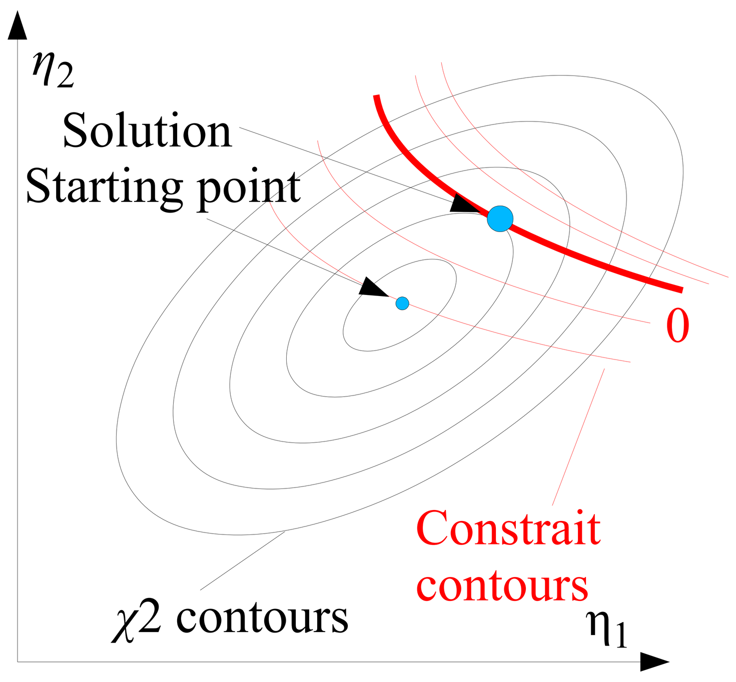
- known four-momentum of the initial state, e.g. Σpy = 0 → hard constraint
- masses of intermediate particles, e.g. M(jj) = $M_H$ or $M_Z$ → hard or soft constraint
- know which quantities are very well measured and which less so → error parametrisation

=> formulate hypothesis under which to interpret the event
=> test hypothesis by minimizing χ2 underconstraints by adjusting particle momenta

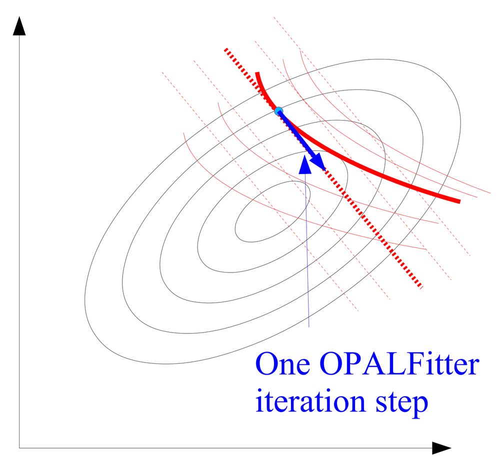
### Method of Lagrange Multipliers

$$\chi^2_T(\vec\eta,\vec\xi,\vec\lambda) = (\vec y - \vec\eta)^T \cdot V^{-1} \cdot (\vec y - \vec\eta) + 2\vec\lambda^T \cdot \vec f(\vec\eta,\vec\xi)$$

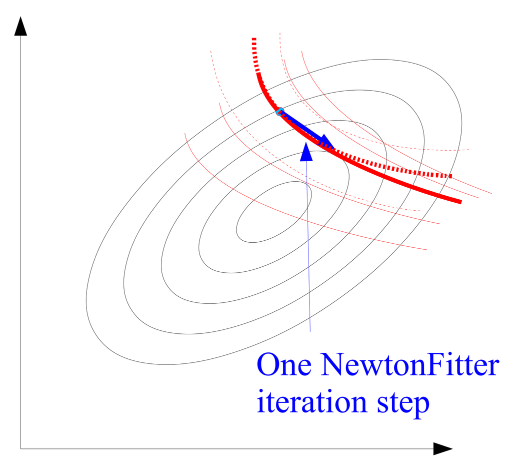
$$\begin{aligned}\nabla_\eta \chi^2_T &= -2V^{-1}\cdot(\vec y - \vec\eta) + 2\vec F_\eta^T \cdot \vec\lambda = \vec 0,\\ \nabla_\xi \chi^2_T &= \vec F_\xi^T \cdot \vec\lambda = \vec 0,\\ \nabla_\lambda \chi^2_T &= 2\vec f(\vec\eta,\vec\xi) = \vec 0,\end{aligned}$$




OPALFitter: Approximates constraint by tangential plane




NewtonFitter: Approximates constraint by tangential paraboloid




**Exploit this to**

- improve precision on observables, e.g. invariant masses
- determine unmeasured quantities (e.g. neutrino momentum)
- find best jet pairing
- select / reject events which match / don't match hypothesis

## Including ISR & Co.

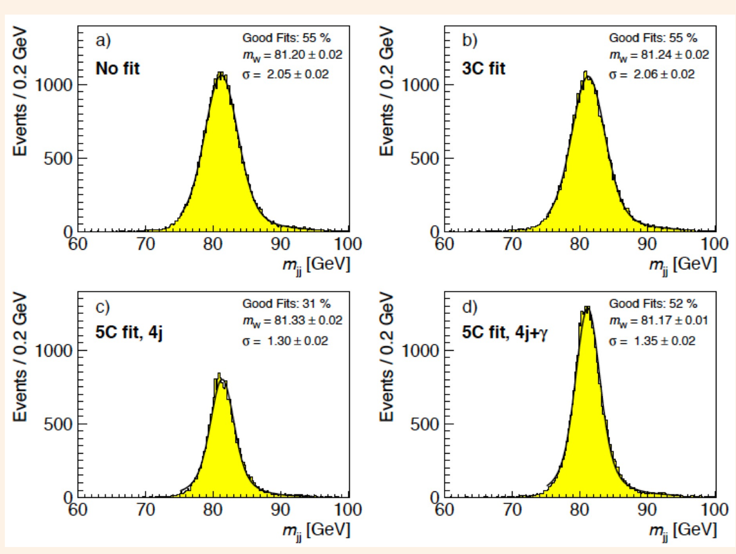
Additional FitObject with $p_z$ as pseudo-measured parameter:

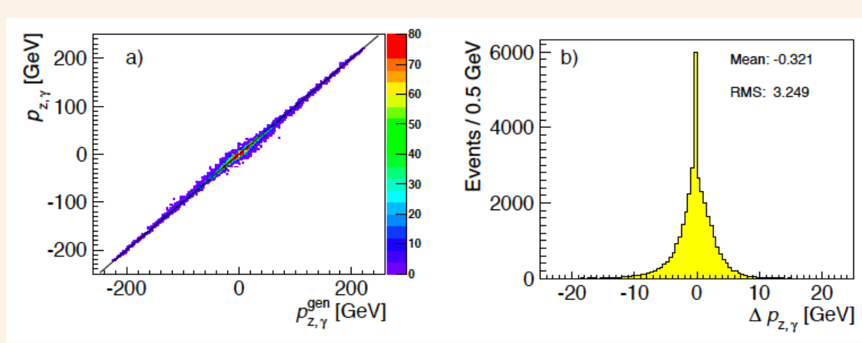
- "Measured" value = $p_z$ balance
- "Error": $\sigma$ of ISR spectrum transformed into a Gaussian

$$\mathcal{P}(p_{z,\gamma}) = \frac{\beta}{2E_{\max}} \cdot \left|\frac{p_{z,\gamma}}{E_{\max}}\right|^{\beta-1}$$

$$z = \mathrm{sign}(p_{z,\gamma}) \left(\frac{|p_{z,\gamma}|}{E_{\max}}\right)^\beta$$

$$\eta = \sqrt{2}\cdot \mathrm{erf}^{-1}(z)$$

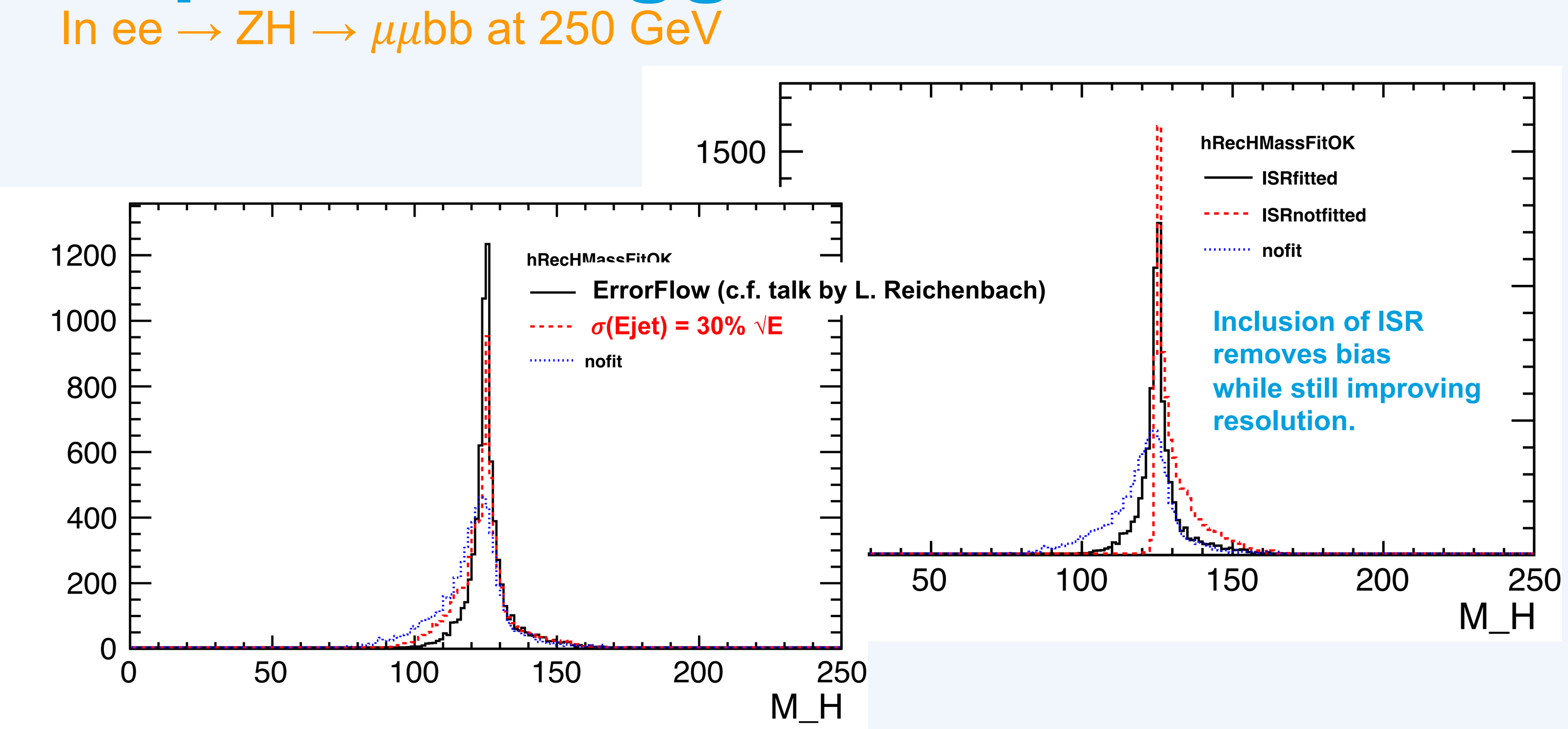



Quality of fitted photon $p_z$ in WW->4j @ 500 GeV

## Impact on Higgs reconstruction.

In ee → ZH → $\mu\mu$bb at 250 GeV

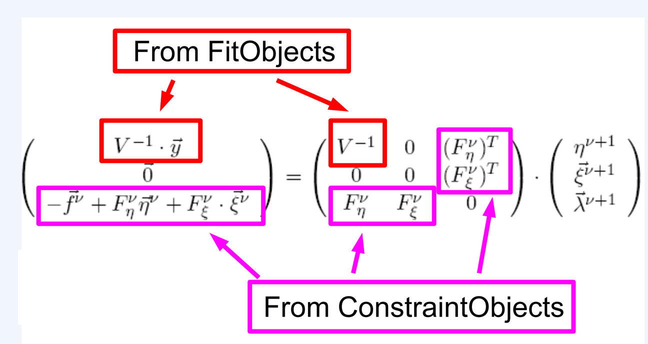



## Software Implementation.

**FitObject.** Encapsulates all details of the parametrization, calculates its own contributions to global $\chi$2 and its derivatives, calculates derivatives of 4-vector components wrt parameters.

**Constraint.** Calculates its value from 4-vectors of FitObjects and its derivatives wrt the 4-vector components of the FitObjects.

**Fitter.** Sets up and solves the system of equations, administers list of FitObjects and Constraints.




Create FitObjects (2 jets)
Create Constraints: $\Sigma p_x = 0$, $\Sigma p_y = 0$, Invariant mass = 90GeV
Tell constraints over which FitObjects they should sum
Create the Fitter Engine
Tell the Fitter which Objects are to be fitted, and which Constraints are to be observed
Perform the Fit

```
//                    E   theta phi    dE dtheta dphi mass
JetFitObject jet1 (44., 1.2, 0.087, 5.0, 0.2,  0.1, 0.);
JetFitObject jet2 (46., 1.8, 3.120, 5.0, 0.2,  0.1, 0.);

// Constraint 0*sum(E) + 1*sum(px) + 0*sum(py) + 0*sum(pz) = 0
MomentumConstraint pxconstraint (0, 1, 0, 0, 0);
pxconstraint.addToFOList (jet1);
pxconstraint.addToFOList (jet2);

// Constraint 0*sum(E) + 0*sum(px) + 1*sum(py) + 0*sum(pz) = 0
MomentumConstraint pyconstraint (0, 0, 1, 0, 0);
pyconstraint.addToFOList (jet1);
pyconstraint.addToFOList (jet2);

// Constraint total mass = 90
MassConstraint mconstraint (90);
mconstraint.addToFOList (jet1);
mconstraint.addToFOList (jet2);

OPALFitter fitter;

fitter.addFitObject (jet1);
fitter.addFitObject (jet2);

fitter.addConstraint (pxconstraint);
fitter.addConstraint (pyconstraint);
fitter.addConstraint (mconstraint);

fitter.initialize();
double prob = fitter.fit();
```

**MarlinKinfit.** https://github.com/iLCSoft/MarlinKinfit
**Example processors.** https://github.com/iLCSoft/MarlinKinfitProcessors
**Tutorial.** https://github.com/ILDAnaSoft/MarlinKinfitTutorial

## Next Steps.

- Transmit full ErrorFlow covariance matrix to FitObjects
- Implement correlations between FitObjects, e.g. to model jet clustering errors
- Optimisation of step length choice in NewtonFitter
- Fundamentally new minimizer, e.g. ML-based?
- Application to multi-jet analyses, e.g. ee → ZH, WW, tt, ZHH, …

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Learn more:

- M. Beckmann, B. List, J. List, Nucl.Instrum.Meth.A 624 (2010) 184-191, https://doi.org/10.1016/j.nima.2010.08.107
- B. List, J. List, LC-TOOL-2009-001, https://bib-pubdb1.desy.de/record/88030
- B. List, Constrained Fits, in Data Analysis in High Energy Physics: A Practical Guide to Statistical Methods, Wiley-VCH, ISBN 978-3527410583

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